A Temperature-Dependent Model for Small-Strain Shear Modulus of Unsaturated Soils

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ABSTRACT

Near-surface soils in geotechnical and geoenvironmental applications are often unsaturated, and natural or imposed changes in temperature may lead to a softening effect at constant suction that causes a change in stiffness. To capture thermal effects on the stiffness of unsaturated soils, this paper presents an effective stress-based, temperature-dependent model for the small-strain shear modulus of unsaturated soils, with an emphasis on silts. Temperature dependency of the model is accounted for by employing temperature-dependent functions for matric suction and effective saturation characterized using the soil-water retention curve. To validate the proposed model, laboratory tests using a modified triaxial apparatus with bender elements are carried out on Bonny silt to measure the small-strain shear modulus at 23 and 43°C for varying matric suctions of 0 to 110 kPa. The results from the proposed model are in a reasonable agreement with the experimentally measured values, and demonstrate the importance of considering temperature effects on the shear modulus of unsaturated soils. The accuracy of the model is further validated by comparing the predicted values with laboratory test results on silts reported by two independent studies reported in the literature.

KEYWORDS:

Unsaturated soils; Temperature; Shear modulus; Silt; Stiffness; Suction; Effective stress

INTRODUCTION AND BACKGROUND

Under working stress conditions, geotechnical structures like retaining walls, pavements, foundations experience shear strains ranging from 0.001% to 1%, with shear strains equal or smaller than 0.001% representing linear elastic conditions (e.g., Atkinson and Sallfors, 1991; Mair et al., 1993; Atkinson, 2000; Clayton, 2011; Likitlersuang et al., 2013; Ng et al., 2016). The shear modulus and Young's modulus defined at these small strain magnitudes (also referred to as elastic moduli) are important soil properties that establish the elastic stress-strain relationships used extensively in the analysis of geotechnical structures, including immediate settlement of footings and embankments, pavement subgrade deformation response, soil-structure interaction, and foundation vibration response (e.g., Viggiani and Atkinson, 1995; Kramer, 1996; Rampello et al., 1997; Likitlersuang et al., 2013; Yang and Gu, 2013). Several experimental studies have reported that the elastic moduli of soils greatly depend on particle size, void ratio, compaction energy, matric suction, effective saturation, stress history, and net normal stress (e.g., Hardin and Black, 1969; Cho and Santamarina, 2001; Mitchell and Soga, 2005; Oh et al., 2009; Sawangsuriya et al., 2009; Khosravi and McCartney, 2012; Oh and Vanapalli, 2014).

Various attempts have been made to experimentally investigate and develop analytical models for the elastic moduli of unsaturated soils that capture the effects of suction and effective saturation (e.g., Fredlund et al., 1975; Edil and Motan, 1979; Edil et al., 1981; Mancuso et al., 2002; Costa et al., 2003; Inci et al., 2003; Khoury and Zaman, 2004; Sawangsuriya et al., 2005; Khosravi and McCartney, 2012; Dong et al., 2016, 2018). Most of the models developed in these studies are extensions of models developed for dry or saturated soils by Hardin and his colleagues (e.g., Hardin and Black, 1969; Hardin, 1978) to unsaturated conditions. Previous studies have shown that elastic moduli increase with matric suction due to corresponding increases in the average skeleton stress and stabilization effects of suction on the soil skeleton (e.g., Edil and Motan, 1979; Mancuso et al., 2002; Costa et al., 2003; Khoury and Zaman, 2004; Sawangsuriya et al., 2005; Khosravi et al., 2016). An added complication with unsaturated soils is that hydraulic hysteresis will lead to changes in elastic modulus because of suction-induced hardening (Khosravi and McCartney, 2012). Another issue is that the shear modulus of soils, in general, will decrease with the applied shear strain magnitude, and several empirical and semi-empirical models have been proposed in the literature to establish matric suction-dependent relationships for shear and Young's moduli of unsaturated soils at larger shear strain magnitudes (e.g., Vanapalli et al., 2008; Sawangsuriya et al., 2009; Oh et al., 2009; Lu and Kaya, 2014; Dong et al., 2018).

In many of the geotechnical and geoenvironmental applications mentioned above, changes in temperature may occur, which have an additional effect on the elastic moduli of unsaturated soils. Further, other geotechnical applications involving elevated temperatures include earthen structure-atmospheric interaction under a changing climate, storage of nuclear waste, energy piles, soil-borehole thermal energy storage systems, buried high voltage cables, and thermally active earthen structures (e.g., Gens and Olivella, 2001; Laloui and Di Donna, 2013; Robinson and Vahedifard, 2016; Vahedifard et al., 2015, 2016; 2017; 2018a; McCartney et al., 2016; Başer et al., 2018; Thota et al., 2019; Shahrokhabadi et al., 2020). Several experimental studies have illustrated the effects of temperature on the shear strength, volume change and stiffness of saturated and unsaturated soils (e.g., Cekerevac and Laloui, 2004; Uchaipichat and Khalili 2009; Coccia et al., 2013; Alsherif and McCartney, 2015; Zhou and Ng, 2016; Ng et al., 2017). Their studies have provided useful insights through the study of unsaturated constitutive relationships under at temperatures that rely on different stress state variables (e.g., Bishop's mean effective stress, matric suction, and deviator stress) and state variables (e.g., specific volume and effective saturation) for defining temperature-dependent elastic moduli (e.g., Cekerevac and Laloui, 2004; Ng et al., 2016; Zhou and Ng, 2017). However, more work is needed to enhance our understanding of the combined effects of temperature, effective stress state, anisotropic stress conditions, void ratio, and stress history on the elastic moduli (e.g., McCartney et al., 2019). Specifically, temperature may affect the soil-water retention curve (SWRC), which is a key component in the prediction of the effective stress of unsaturated soils (e.g., Lu et al., 2010). Temperature-induced changes in the SWRC and effective stress can alter the stiffness of unsaturated soils.

Advances in equipment and methodologies for testing unsaturated soils under temperature-controlled and suction-controlled conditions at various scales have been employed to gain an improved understanding regarding the effect of temperature on soil stiffness and underlying mechanisms. However, the impact of temperature on the elastic moduli of unsaturated soils still poses a complex problem, leading to dissimilar trends reported by various investigators. For example, a group of studies (e.g., Dumont, 2010; Zhou et al., 2015) reported that elastic moduli decrease with increasing temperature due to reduction of the air-water surface tension. On the other hand, several studies (e.g., Tanaka et al., 1996; Cekerevac and Laloui, 2004; Laloui and Cekerevac, 2008) reported that elastic moduli increase with increasing temperature due to thermal hardening and more interaction between particles. The difference in the reported trends can possibly be attributed to differences in drainage conditions, mean effective stress and soil mineralogy (e.g., Uchaipichat, 2005; Uchaipichat and Khalili, 2009; François and Ettahiri, 2012; Alsherif and McCartney, 2015). For instance, during undrained heating, there may be an increase in pore-water pressure that leads to a decrease in effective stress and softening, which could result in decreases in shear modulus. For drained conditions, heating may cause a drying effect leading to suction hardening, which could result in increase in shear modulus (e.g., McCartney et al. 2019). The majority of existing thermo-mechanical or thermo-hydro-mechanical constitutive models for unsaturated soils assume the elastic moduli (including the shear modulus) to be independent of temperature to simplify formulations (e.g., Thomas and He, 1997; Loret and Khalili, 2002; Laloui et al., 2003; Bolzon and Schrefler, 2005; Nuth and Laloui, 2008; Zhou and Ng, 2016). Instead, many thermo-mechanical models assume that the temperature only affects the mean effective preconsolidation stress (Laloui and Cekerevac 2008). However, results of several experimental studies (e.g., Cekerevac and Laloui, 2004; Alsherif and McCartney, 2015; Zhou and Ng, 2016; Ng et al., 2017) suggest that considering temperature-dependent elastic moduli can lead to more accurate simulations of the mechanical response of unsaturated soils under elevated temperature.

Gaps and unanswered questions remain in the literature regarding the development of unified models for elastic moduli of unsaturated soils, particularly under elevated temperatures. Ideally, such models should properly account for all, or the majority of, underlying mechanisms of through which the temperature affects the elastic response of unsaturated soil under elevated temperatures. To address the aforementioned gaps, this study presents a closed-form relationship to determine the temperature-dependent small-strain (i.e., 0.001% or lower strain) shear modulus of unsaturated soils, with an emphasis on silts. For this purpose, a general functional form is proposed based upon a suction stress-based representation of effective stress incorporating three primary variables of net normal stress, matric suction, and effective saturation. Temperature dependency of the model is accounted for by employing temperature-dependent functions for matric suction and effective saturation characterized using the SWRC. A set of laboratory tests using a modified triaxial test setup are performed to measure the small-strain shear modulus of Bonny silt at two different temperatures for varying matric suctions. The proposed model is validated against the measured data obtained in the current study as well as those inferred from two other independent experimental studies performed on silts reported in the literature.

THEORY AND FORMULATIONS

General Functional Form

Hardin and Richart (1963) performed a set of micromechanical analyses and showed that the small-strain shear modulus of soils can be reasonably fitted with an effective stress-dependent power functional form as follows (Hardin and Richart, 1963):

$$G = A_1\, f(e)\left[p'\right]^{n_1} \tag{1}$$

where A_1 and n_1 are fitting parameters, p' is the mean effective stress, and $f(e)$ is the void ratio function, which can be expressed for sands and clays as (Hardin, 1978):

$$f(e) = \frac{1}{0.3 - 0.7e^2} \tag{2}$$

The Hardin and Richart (1963) equation (Eq. 1) is applicable to saturated soils since it is a function of Terzaghi's effective stress. For unsaturated soils, several studies have built upon

Hardin's model and proposed new models for small-strain shear modulus primarily as a function of net normal stress and matric suction (e.g., Sawangsuriya et al., 2009; Khosravi and McCartney, 2009; Khosravi and McCartney, 2012; Ghayoomi et al., 2013; Oh and Vanapalli, 2014; Dong et al., 2016). The majority of the previous models (e.g., Mancuso et al., 2002; Mendoza et al., 2005; Sawangsuriya et al., 2009; Khosravi and McCartney, 2012; Oh and Vanapalli, 2014) are developed using a form of Bishop's effective stress (Bishop, 1959), which is primarily dominated by matric suction and effective saturation (Lu et al., 2010). Based on these observations, we propose the following general functional form for small-strain shear modulus of unsaturated soils:

$$G = A\, f(e) P_a \left[\frac{p_n + S_e^{\kappa_{ref}} \psi}{P_a} \right]^n \quad (3)$$

where P_a is the atmospheric pressure used as a normalizing parameter, and *A* and n are fitting parameters, p_n is the mean net normal stress (equal to the difference between the total mean stress, p, and the pore-air pressure, u_a), S_e is the effective saturation, and ψ is the matric suction, which is equal to the difference between the pore-air pressure and the pore-water pressure (u_w), κ_{ref} is a fitting parameter that controls the impact of variation of water content. The effective saturation to the κ_{ref} power is used to represent Bishop's effective stress parameter, χ (Bishop 1959) as suggested by Vanapalli and Fredlund (2000). It should be noted that a similar functional form is used by several studies and extensively validated against experimental tests performed on various soils at ambient temperature (e.g., Sawangsuriya et al., 2009; Oh and Vanapalli, 2014; Dong et al., 2016). The variables used in the proposed functional form represent external confining level (by mean net normal stress), soil hardening or softening (by effective saturation) and interparticle contact forces (by effective stress) (Dong et al., 2016). The proposed function allows to distinctly account for the effect of effective saturation and matric suction, which, while interrelated, are shown to possibly have independent effects on soil hardening or softening and effective stress (Khalili et al., 2004; Dong et al., 2016).

In this study, temperature dependency of the small-strain shear modulus is considered by incorporating temperature-dependent functions for matric suction and effective saturation, which is characterized using the SWRC, into the proposed functional form (Eq. 3). Temperature-dependency of matric suction is accounted for by quantifying the role of temperature on the surface tension, soil-water contact angle, and adsorption by the enthalpy of immersion. Similar formulations are employed by Vahedifard et al. (2018b) and Vahedifard et al. (2019) to consider the effects of temperature on the SWRC and effective stress, respectively.

Temperature-Dependent Matric Suction

The temperature dependency of matric suction, commonly used to represent capillary pressure in unsaturated soils, is well established in the literature (e.g., Young, 1805; Grant and Salehzadeh, 1996; Lu and Likos, 2004), and arises from changes in the air-water surface tension and the water-solid contact angle with temperature. For example, the temperature-dependent matric suction can be defined as (Grant and Salehzadeh, 1996):

$$\psi = \psi_{T_r}\left(\frac{\beta + T}{\beta_{T_r} + T_r}\right) \tag{4}$$

where ψ_{T_r} is the matric suction at the reference temperature, T and T_r are arbitrary and reference temperatures, respectively, and β_{T_r} is a regression parameter defined at the reference temperature T_r. The parameter β can be estimated as follows (Grant and Salehzadeh, 1996):

$$\beta = \frac{-\Delta h T_r}{-\Delta h + a'\left(\cos\alpha\right)_{T_r} + b\left(\cos\alpha\right)_{T_r} T_r} \tag{5}$$

where a' and *b* are fitting parameters that can be estimated from the work of Dorsey (1940) and Haar et al. (1984) to be $a' = 0.11766\ \mathrm{Nm}^{-1}$ and $b = -0.0001535\ \mathrm{Nm}^{-1}K^{-1}$, α is the soil-water contact angle, and Δh is the enthalpy of immersion per unit area, which can be determined by experimental measurements or by using the differential enthalpy of adsorption of the vapor.

Grant and Salehzadeh (1996), which is used as the basis of the formulations for temperature-dependent matric suction in this study, did not consider the effect of temperature on the enthalpy of immersion. However, previous studies like Watson (1943) demonstrated that temperature can affect the enthalpy of immersion as well. In this study, we used the following equation developed by Watson (1943) to account for the reduction of enthalpy with increasing temperature:

$$\Delta h = \Delta h_{T_r} \left(\frac{1 - T_r}{1 - T} \right)^{0.38} \tag{6}$$

where Δh_{T_r} is the enthalpy of immersion per unit area at the reference temperature. Further discussion about the enthalpy of immersion is presented in the Appendix.

The temperature-dependent soil-water contact angle is given as follows (Grant and Salehzadeh, 1996):

$$\cos\alpha = \frac{-\Delta h + TC_1}{a' + bT} \tag{7}$$

where C_1 is a constant that can be determined as follows (Grant and Salehzadeh, 1996):

$$C_1 = \frac{\Delta h_{T_r} + a'(\cos\alpha)_{T_r} + b(\cos\alpha)_{T_r} T_r}{T_r} \tag{8}$$

Considering the Young-Laplace equation, the matric suction (or capillary pressure) is a function of surface tension and contact angle at a given pore size. These parameters, which control the matric suction, are sensitive to temperature. Fig. 1 depicts temperature effects on surface tension, enthalpy of immersion, contact angle, and matric suction at various pore sizes. As shown in Fig. 1a, the surface tension decreases with an increase in temperature. This could be due to a reduction in attractive forces because of an increase in molecular thermal sensitivity (e.g., Gardner, 1955; Grant and Bachmann, 2002). Fig 1b depicts the variation of enthalpy of immersion with temperature for various values of enthalpy of immersion at reference temperature (see typical enthalpy values for different minerals in Table A1 of Appendix). The temperature-dependent contact angles are calculated for different enthalpy values and are shown in Fig. 1c. Based on the proposed model, the enthalpy of immersion (Fig. 1b) and the contact angle (Fig. 1c) increase with temperature. Soils with higher contact angle and enthalpy of immersion are more sensitive to temperature. The predicted trends of contact angle and enthalpy of immersion shown in Fig. 1 are consistent with the existing experimental data in the literature (e.g., Watson, 1943; Bachmann et al., 2002; Grant and Bachmann, 2002). Fig. 1d illustrates the variation in matric suction with temperature for different pore sizes. Typically, a pore size of 0.10 mm represents fine sand, 0.02 mm represents silt, and 0.0015 mm represents clay (Nimmo, 2004). For example, at r = 0.02 mm, the matric suction decreases approximately by 18%, 36%, 54%, and 72% when the soil temperature increases incrementally from 20°C to 40°C, 60°C, 80°C, and 100°C, respectively. The results show the importance of considering temperature effects on the surface tension, contact angle, and enthalpy of immersion. The predicted thermal effects are consistent with the trends reported from laboratory tests (Watson, 1943; She and Sleep, 1998; Bachmann et al., 2002), but are commonly ignored in the majority of existing temperature-dependent analytical and numerical simulations.

Temperature-Dependent Effective Saturation

In this study, we employ the SWRC to characterize the effective saturation. Following the work by Grant and Salehzadeh (1996), all temperature-dependent SWRC formulations are developed as a function of matric suction at the reference temperature. Accordingly, Eq. 5 is rearranged to obtain matric suction at the reference temperature and is incorporated into the SWRC model proposed by van Genuchten (1980). The temperature-dependent effective saturation can be obtained using the temperature-dependent extension of the van Genuchten SWRC model as (Vahedifard et al., 2018b):

$$S_e = \left(1 + \left(\alpha_{VG}\psi\left(\frac{\beta_{T_r} + T_r}{\beta + T}\right)\right)^{n_{VG}}\right)^{-m_{VG}} \tag{9}$$

where α_{VG} is a fitting parameter inversely related to the air-entry suction (1/kPa), n_{VG} is the pore-size distribution fitting parameter, and m_{VG} is a fitting parameter representing the overall geometry of the SWRC assumed to be equal to 1-1/n_{VG}. A key feature of Eq. 10 is that the formulation only requires the SWRC fitting parameters (α_{VG} and n_{VG}) to be defined at the reference (ambient) temperature with Δh_{T_r} being the only additional parameter needed to account for the effect of temperature.

Vahedifard et al. (2018b) employed similar formulations for matric suction and effective saturation to develop temperature-dependent SWRC models. They validated the proposed formulations versus three laboratory tests on sand, silt, and clay at different temperatures. To avoid redundancy and keep the focus of this study on shear modulus, we do not repeat the entire validation results and related discussion for matric suction and SWRC in this paper. For completeness and using the data presented in Table 1, Fig. 2 shows the predicted effective saturation from the van Genuchten SWRC model versus measured data for Bonny silt reported by Alsherif and McCartney (2014, 2015) at temperatures 23°C and 64°C. Results from the proposed formulation, in general, show good agreement with the measured effective saturation at different temperatures. Interested readers are referred to Vahedifard et al. (2018b) for further details regarding validation of the matric suction and effective saturation formulations.

To demonstrate the effect of temperature, the extended van Genuchten SWRC model is used to study the temperature dependency of effective saturation for three silts: Bonny silt (Alsherif and McCartney, 2015), Bourke silt (Uchaipichat and Khalili, 2009) and a completely decomposed tuff classified as silt (Zhou et al., 2015). Table 1 shows the SWRC parameters used for determination of temperature-dependent effective saturation for these silts. The SWRC fitting parameters at ambient temperature are obtained using the measured SWRC data reported by Alsherif and McCartney (2015), Uchaipichat and Khalili (2009), and Zhou et al. (2015). The parameter Δh_{T_r} is assumed to be the same for all silts and was assumed to be the same as a silty soil tested by Grant and Salehzadeh (1996).

Fig. 3 depicts the changes in the effective saturation at various temperatures ranging from 20°C to 100°C for the three silts. For comparison purposes, the temperature-induced changes in the effective saturation at matric suction of 150 kPa are examined. For Bonny silt, the effective saturation decreases by approximately 16%, 28%, 38%, and 55% when increasing the temperature from 20°C to 40°C, 60°C, 80°C, and 100°C, respectively. For the same temperatures, the decreases in effective saturation are approximately 13%, 23%, 32% and 49% for Bourke Silt and 11%, 20%, 28% and 43% for completely decomposed tuff. These decreases in effective saturation can be due to thermal effects on surface tension, contact angle, and enthalpy (Vahedifard et al., 2018b). Further, the results for all silts suggest that increasing temperature leads to a smaller air-entry suction. This finding can contribute to more representative simulations of unsaturated soils under elevated temperatures. As mentioned before, the proposed formulations only need the SWRC parameter representing the air entry suction at the reference temperature. Employing temperature-dependent formulations for the contact angle and enthalpy of immersion captures the impact of elevated temperature on reducing the air entry suction (Vahedifard et al., 2019).

Closed-Form Equation for Temperature-Dependent Shear Modulus

Using S_e obtained from the extended van Genuchten SWRC model and by substituting Eqs. (4), (5), and (10) into Eq. (3), one can obtain the following closed-form model for the temperature-dependent shear modulus of unsaturated soils:

$$G = A\,f(e)P_a \left\{ \frac{p_n + \left(\left(1 + \left(\alpha_{VG}\psi \left(\frac{\beta_{T_r} + T_r}{\beta + T} \right) \right)^{n_{VG}} \right)^{-(1-1/n_{VG})} \right)^{\kappa_T} \psi \left(\frac{\beta_{T_r} + T_r}{\beta + T} \right)}{P_a} \right\}^{n} \tag{10}$$

where κ_T is a parameter which controls the impact of effective saturation on the effective stress and depends on temperature as follows:

$$\kappa_T = \kappa_{ref} + \frac{T - T_r}{T_r} \mathrm{e}^{(\kappa_{ref} m)} \tag{11}$$

where m is a fitting parameter equivalent to m_{VG}. For ambient temperature conditions, κ_T degenerates to κ_{ref} but the value of κ_T increase as temperature elevates. Physically, Eq. 11 captures the changes in effective saturation caused by variation of temperature. Heat-induced reductions in water content can cause changes in confinement and, therefore, stiffness of the soil mass. At a given matric suction, increases in confinement cause sharper reductions in water content with temperature. Similar to the proposed temperature-dependent SWRC, all the fitting parameters ($\alpha_{VG}, n_{VG}, n, A, \kappa_{ref}$) used in Eq. (11) are those determined at the reference (ambient) temperature and Δh_{T_r} is the only additional parameter needed to account for the effect of temperature. This feature can facilitate the use of the proposed model as it does not require many additional parameters.

The proposed formulations for temperature-dependent suction and effective saturation can be used to extend other existing models for small-strain shear modulus (e.g., Sawangsuriya et al., 2009; Dong et al., 2016) to temperature-dependent conditions. The model presented in this study does not consider possible effects of temperature on net normal stress, which may occur due to the impact of temperature on pore air pressure. Further, hydraulic hysteresis is not modeled but can be considered by following the approach of Khosravi and McCartney (2012). They incorporated the ratio of the mean apparent yield stress to the current mean effective stress (equal to the overconsolidation ratio, OCR for saturated or dry soils) into the model for the small-strain shear modulus to consider suction hardening during hydraulic hysteresis.

In general, capillarity and adsorption are two main soil-water retention mechanisms (Lu, 2016), which also control the soil stiffness (Lu, 2018). The model proposed in this study is developed based upon capillarity being the dominant soil-water retention mechanism. This assumption is legitimate for most soil types including silts, which are the main focus of this study. For clays, it is prudent to consider both capillarity and adsorption mechanisms in the development of a shear modulus model. Following this rationale, Lu (2018) proposed a generalized model for Young's modulus of unsaturated soils at ambient temperature explicitly considering capillarity and adsorption mechanisms. Temperature can differently affect adsorption and capillarity, an aspect that needs to be taken into consideration when developing a temperature-dependent model of small-strain shear modulus including both mechanisms. This can be done by following the approach outlined by Vahedifard et al. (2018b, 2019) for the development of temperature-dependent SWRC and effective stress models, respectively.

Temperature may affect the small-strain shear modulus through inducing changes in parameters other than matric suction and effective saturation as well. However, capturing all relevant temperature-induced mechanisms is certainly not feasible using a closed-form model (as intended in this study) and warrants employing more complex numerical models. Even with such numerical models and despite major advances in constitutive modeling of coupled processes in unsaturated soils, it is still hard to argue that there is a single constitutive model in the literature than can capture all of the relevant temperature effects. Nevertheless, the proposed model provides a simple yet reliable tool to account for the temperature effect on the small-strain shear modulus of unsaturated soils. To the best of the authors' knowledge, this work is the first study presenting such a closed-form model. Although major elements used in the development of the proposed model (i.e., temperature-dependent matric suction, SWRC) are already part of the literature, there has been no such an attempt in the literature to make use of all these elements to develop an analytical model to capture the temperature effect on the shear modulus of unsaturated soils in the form and details presented in this study.

VALIDATION AGAINST EXPERIMENTALLY MEASURED DATA

The accuracy of the proposed model is validated by comparing the predicted values with experimentally measured results attained from: (a) laboratory tests performed in this study on Bonny silt using a modified triaxial apparatus with bender elements, and (b) laboratory tests on silts reported by two independent studies reported in the literature. For each set of data, the validation process involves two steps: 1) Calibrating the model at ambient temperature to determine the fitting parameters (n, A, and κ_{ref}) leading to minimum error using the least square optimization, and 2) Using the calibrated model to predict the shear modulus at elevated temperatures and comparing against results from the laboratory tests.

Comparison with Laboratory Measurements using Bender Elements

A set of laboratory tests is performed to measure the small strain shear modulus of Bonny silt at different suctions and temperatures. The tests are carried out using a modified Bishop-Wesley triaxial apparatus with bender elements. The apparatus is set up to measure shear wave velocities at different matric suctions for a specific temperature and net normal stress. Fig. 4 shows the schematic diagram of the complete test setup. Three individual systems are included in the test setup to measure temperature, matric suction, and shear wave velocities. First, a pressure panel is used to apply confining, air and water pressures to the specimen. Second, the temperature controller and circulating pump are used to control and mix the water in the cell to achieve a desired specimen temperature. In addition, a thermocouple sensor is installed to measure the temperature in the cell. Third, bender elements are embedded to the top and bottom caps to send and receive wave signals and therefore measure shear wave velocities.

Table 2 displays the index properties of Bonny silt. The specimens used in the tests are prepared with a thickness of 25 mm and a diameter of 76 mm. The specimen is compacted under a water content of 10.5% (dry side of optimum) with a void ratio of 0.68 (Alsherif and McCartney, 2015). The compacted specimen is placed in the cell and saturation is achieved by reaching a minimum B-value of 0.95 at regular intervals of confining and pore water pressures. The axis-translation technique is used to apply matric suction to the specimen. The air pressure on the top of the specimen is maintained constant and the water pressure at the bottom is reduced to apply different matric suction in the specimen. The first matric suction is applied after making sure there is no change in water levels in the pressure panel for at least 12 hours. The matric suction is applied in intervals from zero to 110 kPa at two constant temperatures 23°C and 43°C. The next step of matric suction is applied after the specimen reaches a steady or equilibrium state. For both tests, the specimens were confined at a constant net normal stress of ~ 50 kPa. To assess the variability of results, multiple wave velocity measurements are made at a given suction after reaching suction equilibrium. The measurements are found to be identical implying zero variability. The suction is increased to the next level only after the wave velocities remain constant for at least 12 hours.

The measured shear wave velocities at different matric suctions and temperatures obtained from the experimental tests are used to determine the small-strain shear modulus as:

$$G = \rho V^2 \tag{12}$$

where ρ is the total density of the soil and V is the shear wave velocity of the soil.

Fig. 5 depicts the measured and predicted small-strain shear modulus versus matric suction at T = 23°C and T = 43°C. The experimentally measured data demonstrate that the shear wave velocities, and therefore the small-strain shear modulus, are affected by matric suction and temperature. At a given temperature, the shear modulus increases with an increase in matric suction. At a given matric suction, the shear modulus decreases with an increase in temperature.

The effect of temperature on shear modulus is more pronounced in higher matric suctions. For example, at matric suction of 40 kPa, the reduction of shear modulus is approximately 16% by increasing temperature from 23 to 43°C. At higher matric suction of 100 kPa, the reduction of shear modulus is approximately 39% by increasing temperature from 23 to 43°C. This could be due to variation in effective stress and in turn the stiffness at a higher temperature depending on the range of matric suction. At low range of matric suction, the trend of effective stress with temperature is similar to the one shown in Vahedifard et al. (2019). For the proposed model, the input parameters A = 1000, n = 2.1, α_{VG} = 0.05 kPa^{-1}, n_{VG} = 2.2, κ_{ref} = 0.35, Δh_{T_r} = -0.516 Jm^{-2} are used to calibrate and predict the shear modulus at ambient and elevated temperatures. The root mean square error (RMSE) values of the model with respect to the measured data are 22 MPa and 22 MPa at 23°C and 43°C, respectively. As seen, the model shows a reasonable match with the measured values for the elevated temperature case (T = 43°C). The only exception where the model shows a relatively high over estimation is at matric suction of 75 kPa and T = 23°C. The large difference can be possibly due to the measured shear modulus being somehow lower than expected at this point, which can be due to testing issues and limitations.

Comparison with Experimental Data Reported in the Literature

There is no experimental data in the literature directly reporting the small-strain shear modulus of unsaturated soils under elevated temperatures. Nevertheless, data from tests at higher shear strain amplitudes are used to extrapolate trends in the small-strain shear modulus available data in the literature for further validation of the proposed model. For this purpose, we use results from suction-controlled temperature-controlled triaxial tests on Bourke silt (reported by Uchaipichat and Khalili, 2009) and completely decomposed tuff (reported by Zhou et al., 2015). Following the procedure explained below, we infer the shear modulus of the tested soils at a shear strain of 0.001% and use for validation against predictions of the proposed model. Table 3 presents a summary of the experimental testing matrix used for calibration and validation purposes in this section.

For the results presented by Uchaipichat and Khalili (2009), the finite-strain Young's modulus at an axial strain of 1% is obtained from the reported deviatoric stress-axial strain curve for each tested combination of net normal stress, temperature, and suction. The corresponding finite-strain shear modulus is calculated using a Poisson's ratio of 0.25 (e.g., Alsherif and McCartney, 2015), and is then scaled to 0.001% strain using the scaling equation proposed by Dong et al. (2018) as follows:

$$\frac{G^*}{G} = \frac{1}{1+\left[\dfrac{\gamma}{\alpha_{VG}(p_n + S_e\psi)\gamma_{ref}}\right]} \tag{13}$$

where G^* is the finite-strain modulus, γ_{ref} is the reference shear strain. The reference shear strain can be defined as (Dong et al., 2018):

$$\gamma_{ref} = \eta\theta^{\xi} \tag{14}$$

where η is a multiplier parameter, ξ is the power factor for water content, θ is the volumetric water content, which is related to S_e as follows:

$$S_e = \frac{\theta - \theta_r}{\theta_s - \theta_r} \tag{15}$$

where θ_s and θ_r are the saturated and residual volumetric water contents, respectively. For the data reported by Uchaipichat and Khalili (2009), the following parameters are used to scale the measured finite-strain shear moduli to small-strain conditions: θ_r = 0.1258, θ_s = 0.55, γ = 1%, η = 0.0027, and ξ = 1.857.

Zhou et al. (2015) reported the measured secant shear modulus at several shear strains ranging from 0.003% to 1%. Using regression analysis to find the best nonlinear fit passing through the data, we employ the regression equation to infer the shear modulus at 0.001% strain,

$G_{0.001}$. Fig. 6 shows the measured shear moduli versus shear strain for the tested silt reported by Zhou et al. (2015) and best-fit curves through the measured data.

For calibration, the SWRC parameters given in Table 1 are used in the model. Table 4 summarizes the calibrated fitting parameters of the proposed shear modulus model at ambient temperature for the two silts. Fig. 7 provides a comparison between the inferred shear moduli at 0.001% strain ($G_{0.001}$) against calibrated results at ambient temperature and predicted results at elevated temperatures. For both Bourke silt and the decomposed tuff, the results of the proposed model are in good agreement with the experimental data. It is noted that more tests at higher matric suctions are needed to better understand and model the shear stiffness of unsaturated soils at high matric suctions under elevated temperature.

CONCLUSIONS

Capturing the temperature dependency of small-strain shear modulus can be an important aspect of modeling the behavior of unsaturated soils subjected to varying temperatures. This study presented a closed-form model to determine the temperature-dependent small-strain shear modulus of unsaturated soils, with an emphasis on silts. An effective stress-based general functional form was proposed, and temperature dependency of the model was considered by incorporating temperature-dependent functions for matric suction and effective saturation. The effective saturation was presented by analytical expressions in which the effects of temperature were considered on the surface tension, soil-water contact angle, and adsorption by the enthalpy of immersion per unit area. The proposed formulations were used to extend the SWRC model originally developed by van Genuchten (1980), which was then used to develop the equations for temperature-dependent shear modulus of unsaturated soils at small strains. Further, a series of experimental tests were conducted to measure the small-strain shear modulus of unsaturated Bonny silts at elevated temperatures. The proposed formulation was compared and validated against the experimental data from the current study and two other independent studies reported the literature. The results of the proposed model showed good match against the measured data.

The model presented in this study can contribute toward an improved understanding of the temperature effect on the mechanical response of unsaturated soils. Experimental measurements of elastic moduli of unsaturated soils under different temperatures require time-consuming tests and certain expertise. Hence, empirical or semi-empirical models such as that developed in this study can facilitate the implementation of temperature-dependent analyses in the geotechnical engineering practice by providing a reasonable estimation of the small-strain shear modulus of unsaturated soils at elevated temperatures. The proposed formulation offers a generalized model and involves constitutive relationships that are needed in a coupled heat transfer and water flow model. As the degree of saturation, suction, and temperature changes during a transient flow process, the model should still provide accurate predictions. Thus, the model can be incorporated as a constitutive relationship into both steady-state and transient flow and heat analyses. This study is the first attempt in the literature to experimentally measure and predict the small-strain shear modulus of unsaturated soils at elevated temperatures. For future studies, more experimental tests are recommended to examine and further validate the proposed model for different soil types and wider ranges of suction and temperature.

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APPENDIX: ENTHALPY OF IMMERSION AT REFERENCE TEMPERATURE

The enthalpy of immersion at the reference temperature (Δh_{T_r}) is a key input parameter in the proposed formulations and is defined by the International Union of Pure and Applied Chemistry

(IUPAC) as the difference between the enthalpy of a solid completely immersed in a wetting fluid and that of the solid and the liquid taken separately (Grant and Salehzadeh, 1996). The value of Δh_{T_r} must be specified whether the solid in the initial state is in contact with vacuum or with the vapor of the liquid at a given partial pressure. According to Everett (1972), the measurements of the enthalpy of wetting of a solid equilibrated with varying relative pressures of the vapor of a pure wetting liquid may be used to derive the differential enthalpy of adsorption. Jaroniec and Madey (1988) showed that the enthalpy of immersion is proportional to the average adsorption potential and can be calculated using the parameters characterizing the energetic heterogeneity of microporous solids. Table A1 presents Δh_{T_r} of different materials reported in the literature.

The enthalpy of immersion at the reference temperature can be determined based on experimentally measured variables. For example, as per Harkins and Jura (1944), Δh_{T_r} can be calculated as:

$$\Delta h_{T_r} = -\left[\sigma\left(\cos\alpha\right)\right]_{T_r} \tag{16}$$

where σ is the air-water surface tension at T_r. There are several studies in the literature that experimentally measure σ and $\cos\alpha$ (e.g., She and Sleep, 1998; Bachmann et al., 2002). Further, previous studies have proposed several empirical models for Δh_{T_r} (e.g., Stoeckli and Kraehenbuehl, 1981; Watson, 1943; Kahr et al., 1990). Kahr et al. (1990) proposed the following expression for the enthalpy of immersion of sodium and calcium bentonites as a function of initial total water content (θ) at the reference temperature of 293 K:

$$\Delta h_{T_r} = A\exp(-B\theta - C\theta^2) \tag{17}$$

where A, B, and C are fitting parameters.

DATA AVAILABILITY STATEMENT

All data, models, and code generated or used during the study appear in the submitted article.

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Table 1. SWRC parameters used for calculating temperature-dependent effective saturation

Soil Type	Δh_{T_r} (Jm^{-2})	T_r (K)	α_{VG} (kPa^{-1})	n_{VG} (-)
Bonny silt			0.330	1.61
Bourke silt	-0.516	298.15	0.021	1.54
Completely decomposed tuff			0.023	1.46

Table 2. Index and compaction properties of Bonny silt (after Alsherif and McCartney, 2015)

Property (unit)	Magnitude
Liquid limit (%)	25
Plastic limit (%)	21
Specific gravity	2.65
Maximum dry unit weight (kN/m^3)	16.3
Initial void ratio	0.68
Optimum moisture content (%)	13.6

Table 3. Experimental tests from the literature used for calibration and validation

Soil Type	Reference	Net Normal Stress (kPa)	Temperature (°C)	Suction (kPa)
Bourke silt	Uchaipichat and Khalili (2009)	150	25	100
				300
			40	100
				300
			60	100
				300
Completely decomposed tuff	Zhou et al. (2015)	200	20	1
				150
			60	1
				150

Table 4. Calibrated fitting parameters for the proposed shear modulus model at ambient temperature.

Soil Type	*n*	*A*	κ_{ref}
Bourke silt	0.35	40	1.3
Completely decomposed tuff	0.95	220	0.5

Table A1. Enthalpies of immersion per unit area of different materials reported in the literature

Material	*T* of observation (°C)	Δh (mJ/ m^2)	Reference
Silica	35	-195	Khalil (1978)
	35	-202	Khalil (1978)
	35	-278	Khalil (1978)
	35	-309	Khalil (1978)
Quartz	31	-505	Partyka et al. (1979)
	31	-510	Partyka et al. (1979)
	25	-120	Whalen (1961)
	25	-120	Whalen (1961)
Anatase (untreated)	25	-510	Harkins and Jura (1944)
Anatase (coated with Al_2O_3)	25	-630	Harkins and Jura (1944)
Na-bentonite	20	-400	Kahr et al. (1990)
Ca-bentonite	20	-750	Kahr et al. (1990)
Plano silt loam	25	-516	Grant and Salehzadeh (1996)
Elkmound sandy loam	25	-285	Grant and Salehzadeh (1996)
Kaolinite	25	-358	Brooks (1960)
Bentonite	25	-575	Zettlemoyer et al. (1955)

LIST OF FIGURES:

Fig. 1. Temperature effects on: (a) surface tension, (b) enthalpy of immersion with different values at the reference temperature, (c) contact angle at different values of enthalpy of immersion at the reference temperature, and (d) matric suction for various pore sizes.

Fig. 2. Comparison of predicted versus measured effective saturation for Bonny silt at T = 23°C and 64°C (measured data from Alsherif and McCartney, 2015).

Fig. 3. Effective saturation versus matric suction at different soil temperatures using the extended van Genuchten SWRC model: (a) Bonny silt, (b) Bourke silt, and (c) Completely decomposed tuff.

Fig. 4. Schematic diagram showing the experimental setup.

Fig. 5. Measured and proposed variation in small strain shear modulus with matric suction at T = 23 °C (with calibrated model values) and 43 °C (with predicted model values).

Fig. 6. Measured shear moduli versus shear strain for completely decomposed tuff reported by Zhou et al. (2015) and best fit curves through the measured data.

Fig. 7. Comparison of inferred shear modulus at 0.001% strain ($G_{0.001}$) with calibrated shear modulus at ambient temperature and predicted shear modulus at elevated temperatures.

Figure 1

Click here to access/download;Figure;Fig.1.PDF

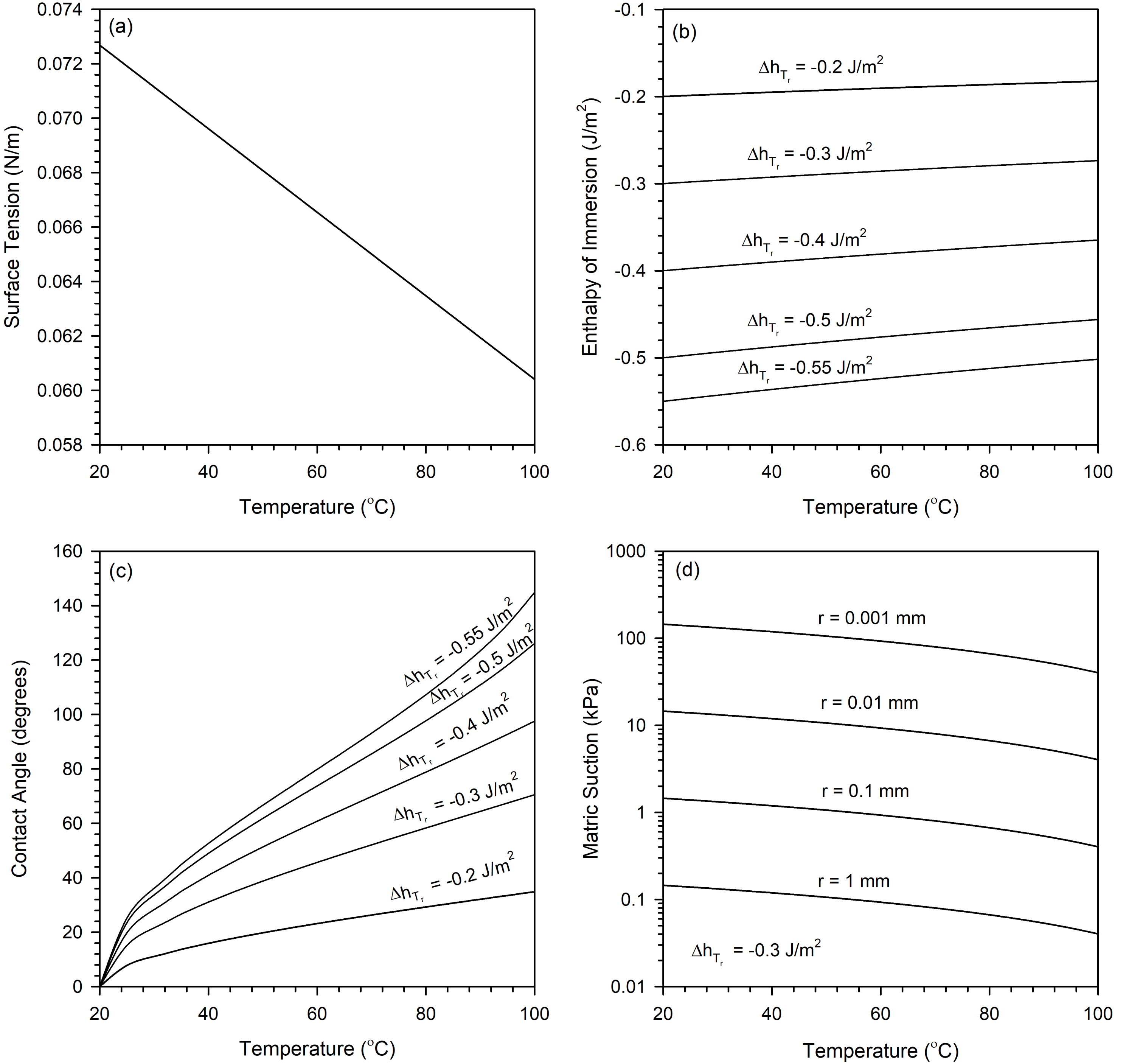

Figure 2

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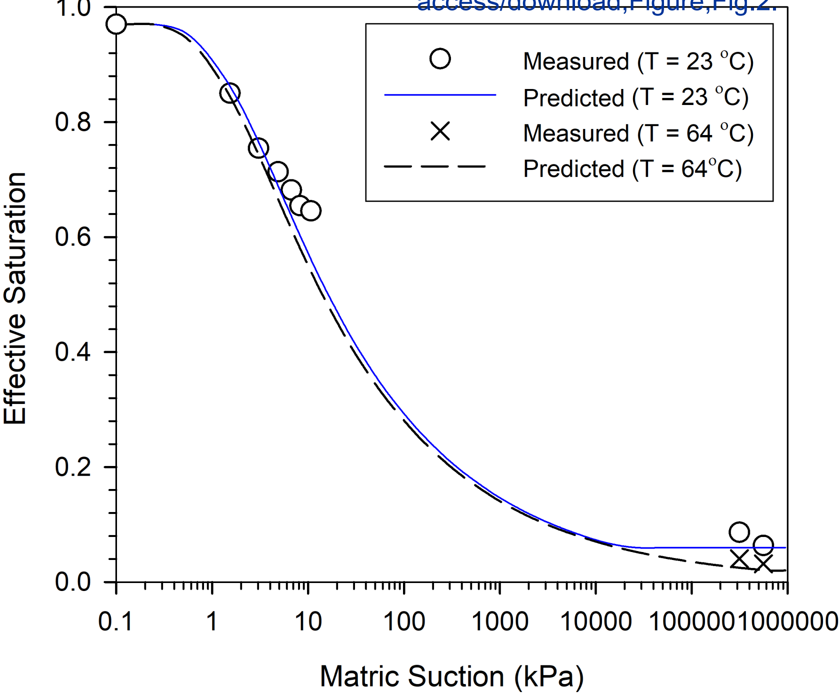

Figure 3

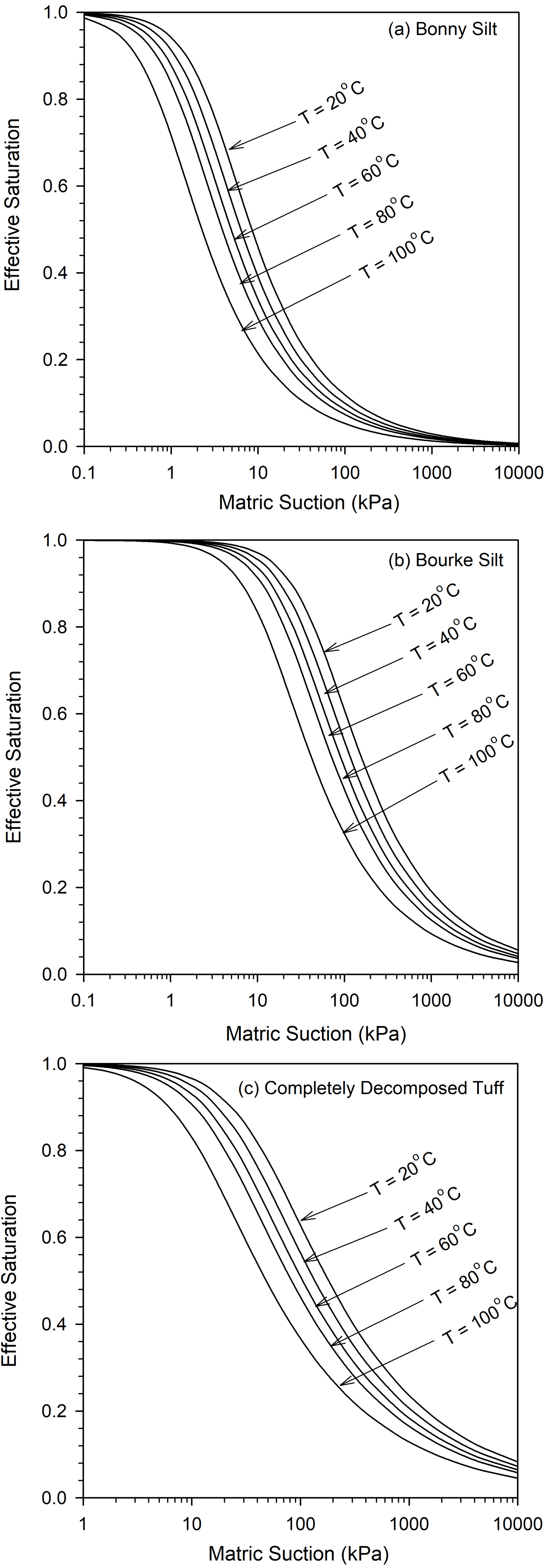
(a) Bonny Silt
T = 20°C
T = 40°C
T = 60°C
T = 80°C
T = 100°C
Effective Saturation
1.0
0.8
0.6
0.4
0.2
0.0
0.1
1
10
100
1000
10000
Matric Suction (kPa)
(b) Bourke Silt
T = 20°C
T = 40°C
T = 60°C
T = 80°C
T = 100°C
Effective Saturation
Matric Suction (kPa)
(c) Completely Decomposed Tuff
T = 20°C
T = 40°C
T = 60°C
T = 80°C
T = 100°C
Effective Saturation
Matric Suction (kPa)

Figure 4

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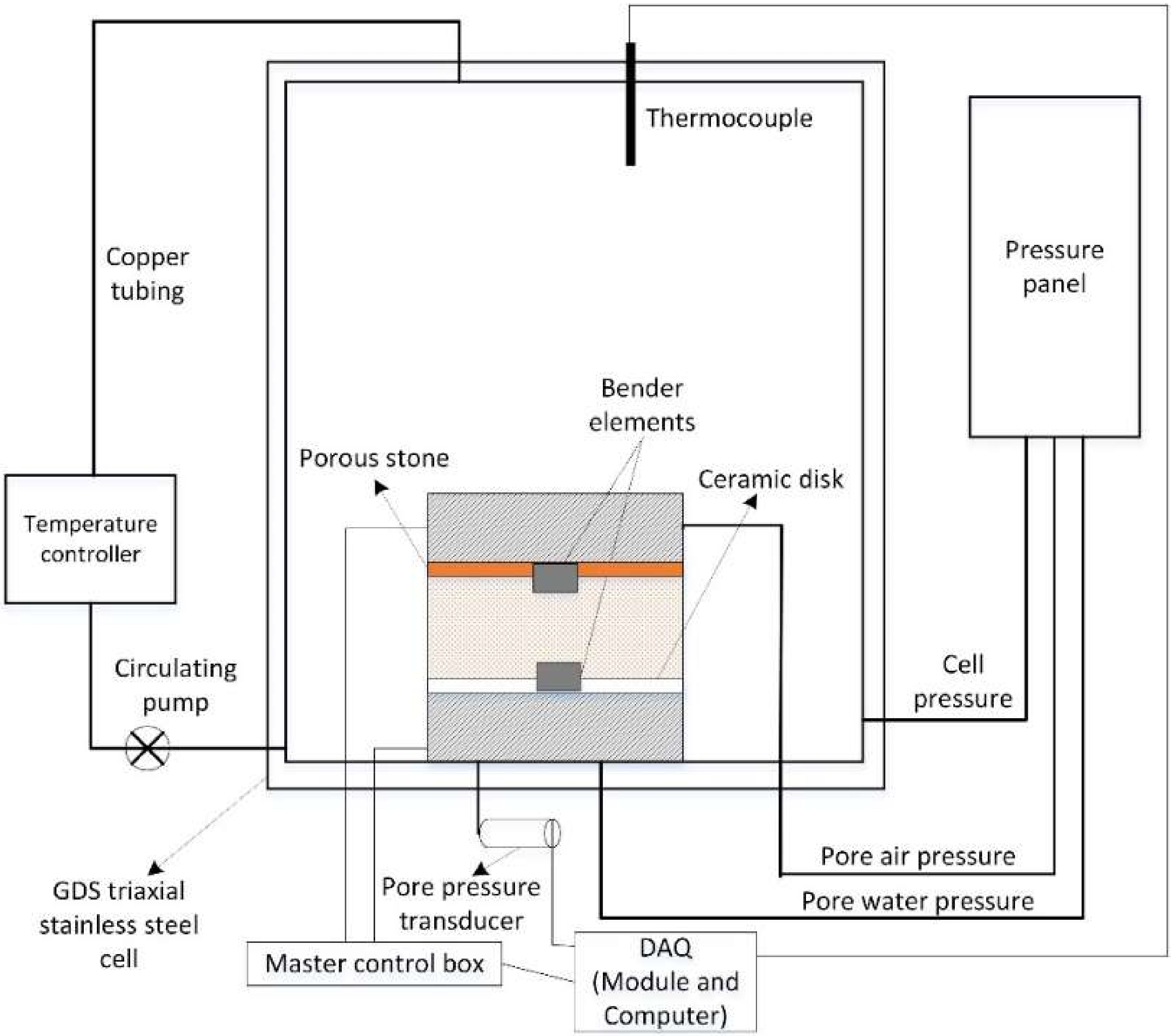
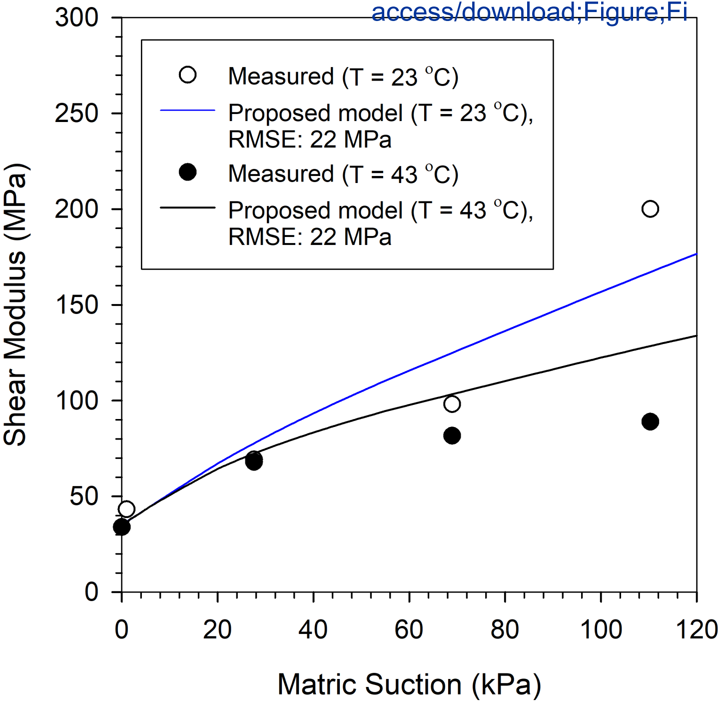


Figure 5



300
250
200
150
100
50
0
Shear Modulus (MPa)
0
20
40
60
80
100
120
Matric Suction (kPa)
Measured (T = 23 °C)
Proposed model (T = 23 °C), RMSE: 22 MPa
Measured (T = 43 °C)
Proposed model (T = 43 °C), RMSE: 22 MPa

Figure 6

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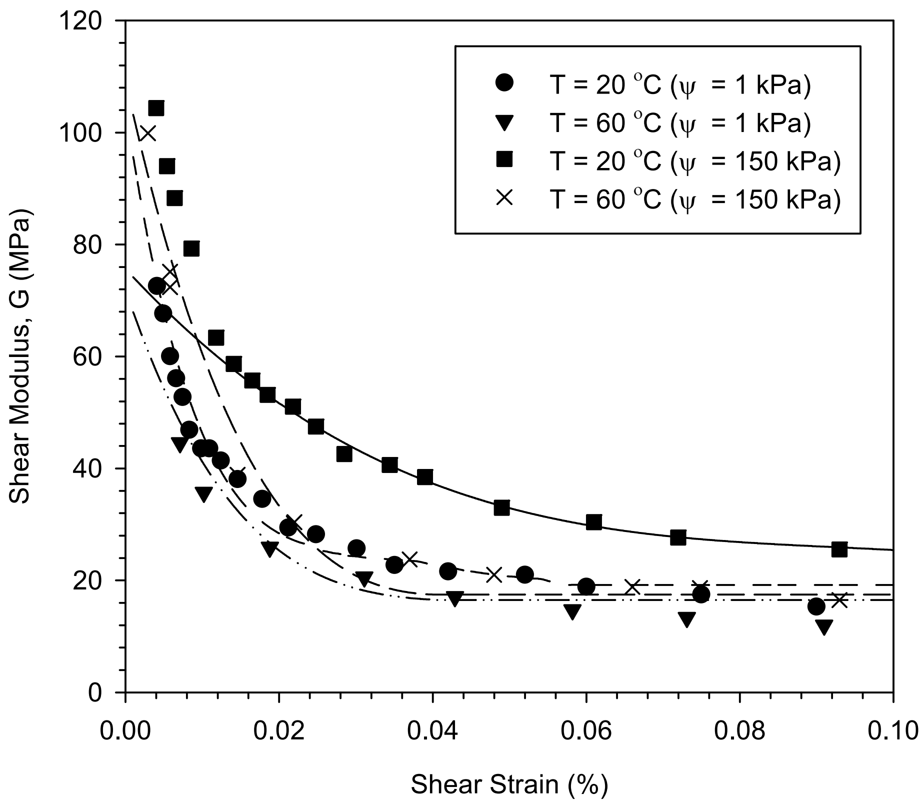

Figure 7

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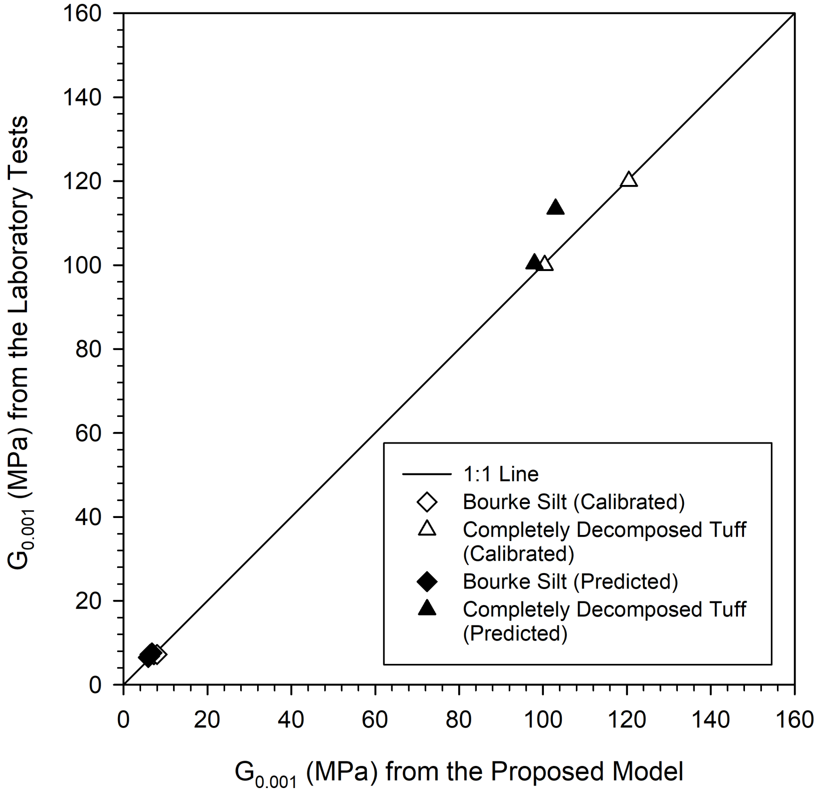

Response to Review Comments by Reviewer #2 (Manuscript MS GTENG-8804)

This manuscript is a major revision of a previous submission to JGGE. The major comments from this reviewer have been addressed.

Response/Change: We thank the reviewer for taking the time to review our new submission. We are glad to hear the reviewer found our changes and responses to the comments satisfactory. Please see below for our itemized response, and associated changes made in the revised manuscript.

The only minor issue is that the descriptions about Fig. 6 are confusing. Please clarify: are the lines in this figure produced by the proposed closed-form model using one set of parameters? or each curve uses a different set of parameters? If the former, please present the parameters. If it is the latter, there is not much point to show Fig.6 because one may just fit it with a Hardin's equation as well.

Response/Change: We didn't use our proposed model to generate curves shown on Fig. 6. Our proposed equation is for small strains (<0.001%), where the shear modulus is strain independent. Fig. 6 shows the measured values of shear modulus at different strain levels from Zhou et al. (2015). They reported shear modulus values for strain levels of 0.003% to 1%. However, to compare their results against our model at small strains (<0.001%), we fitted each of their measured data set with a polynomial function and then extrapolated to infer the "measured" values at small strains (i.e., <0.001%) using their data. The measured small strain shear modulus values obtained after extrapolation are compared against the "predicted" values from our proposed equation (see Fig. 7). We clarified this aspect in the paper.