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NOTE ON THE K-MESON NUCLEON INTERACTION

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It is to be expected that the experimental study of K-meson production, scattering, and absorption processes will shed light on the fundamental interactions of the strange particles. The most promising method of analysis for determining the coupling constants and relative parities of the particles exploits the analyticity properties of various reaction amplitudes.[1, 2] The recently measured angular distribution of the K^+-proton scattering cross-section at various intermediate energies[3] and the K^--proton scattering parameters[4, 5] enable us to make a more accurate study of the K-nucleon-hyperon coupling terms than has previously been possible. Unfortunately, the accuracy is not sufficient to lead to a definite conclusion about the K-meson parity, but we do conclude that the K^--proton force is attractive. We hope that the present discussion also has value because it will make clear some of the difficulties that must be faced.

The numerical work is carried out most easily if one uses the observations to calculate the function $f(\omega)$ of K-meson laboratory energy ω, $(\hbar = c = m_K = 1)$

$$f(\omega) = \frac{1}{\pi}\int_1^4 \frac{A_+(\omega')}{\omega' - \omega}\, d\omega' + \frac{1}{\pi}\int_{\omega_{\Lambda\pi}}^4 \frac{A_-(\omega')}{\omega' + \omega}\, d\omega' , \qquad (1)$$

where $A_\pm(\omega)$ are the imaginary parts of the $K^\pm$-proton forward scattering amplitudes and $\omega_{\Lambda\pi}$ is the (non-physical) threshold for the reaction

$$K^- + p \rightarrow \Lambda + \pi \qquad (2)$$

Since the integrals in Eq. (1) have been cut off, it is necessary to use a dispersion relation that does not weight the high-energy region. We have used this expression for the real parts $D_{\pm}(\omega)$ of the forward scattering amplitudes:

$$\omega_o D_+ (\omega) - \frac{1}{2} (\omega_o + \omega) D_+(\omega_o) - \frac{1}{2} (\omega_o - \omega) D_-(\omega_o)$$

$$\cong \omega_o f(\omega) - \frac{1}{2} (\omega_o + \omega) f(\omega_o) - \frac{1}{2} (\omega_o - \omega) f(-\omega_o) \tag{3}$$

$$- \omega_o(\omega_o^2 - \omega^2) \left\{ \frac{p_\Lambda X_\Lambda}{(\omega_\Lambda^2 - \omega_o^2) (\omega_\Lambda + \omega)} + \frac{p_\Sigma X_\Sigma}{(\omega_\Sigma^2 - \omega_o^2) (\omega_\Sigma + \omega)} \right\} .$$

Here $p_\Lambda(p_\Sigma)$ is the parity of $K^-\Lambda(K^-\Sigma)$ relative to the nucleon and $X_\Lambda(X_\Sigma)$, proportional to the $K^-\Lambda N(K^-\Sigma N)$ coupling constant $g_\Lambda^2(g_\Sigma^2)$, is the magnitude of the residue at the pole $\omega = \omega_\Lambda(\omega_\Sigma)$; these quantities are given by the expressions ($\lambda = \Lambda, \Sigma$)

$$\omega_\lambda = \frac{m_\lambda^2 - m_p^2 \quad m_K^2}{2m_p}$$

$$X_\lambda = \frac{g_\lambda^2}{4\pi} \left| \frac{(m_\lambda + p_\lambda m_p)^2 - m_K^2}{4m_p m_\lambda} \right| , \tag{4}$$

in terms of the masses m_λ, m_p and m_K of the hyperous, protons, and K-mesons respectively.

The energy ω_o in Eq. (3) is one at which experimental information is available. By using a value substantially different from the meson rest energy, one can suppress the importance of the non-physical region and of the amplitude $D_-(\omega_o)$, which decreases with energy.

The optical theorem is used to obtain $A_\pm(\omega)$ from the total cross-section measurements in the physical region. The K^+-proton total cross-sections were taken from the data obtained by the use of nuclear emulsions[6] for energies below 200 Mev, the results of our experiment[3], and the data of Burrowes et al[7] for the higher energies. A curve was fitted to the data within an estimated statistical error of 10%. The low energy K^--proton total cross-sections have been measured in a hydrogen bubble chamber[8]. At higher energies the values are $\sigma_- = 52 \pm 9$ mb at $\omega = 2.08$ [9] and $\sigma = 60 \pm 20$ mb at $\omega = 2.52$. [10] We assumed the value $\sigma = 45$ at $\omega = 4$, and passed a smooth curve through all the points. The statistical error of the fit was estimated at 15%.

The values of $A_-(\omega)$ in the non-physical region were obtained by extrapolating with the parameters of Dalitz and Tuan[4, 5]. The integral is very sensitive to the sign of $D_-(1)$, the real part of the scattering amplitude at zero energy.[4] The combination of the integral and the scattering amplitude that occurs in Eq. (3), however, is not sensitive to this sign as long as the zero-range approximation gives the correct sign and a reasonably accurate value for $D_-(\omega_o)$. This is the case because the zero-range formula gives an analytic expression for the scattering amplitude which then essentially satisfies a dispersion relation by itself.

The results for the integration are given in Table I. They were obtained with a positive choice of $D_-(1)$. In the fourth column are the measured values[3] of $D_+(\omega)$; their magnitudes are simply related to the total cross-sections, because it was found that the scattering is isotropic at $\omega = 1.46$ (225 Mev) and it was assumed that it is isotropic at the lower energies. In the last column is the value of D_- obtained from the zero range parameters, which represent the data adequately.[4] An examination of the bubble chamber results[8] indicates that most of the elastic scattering is indeed a diffraction effect, (i.e., the real part of the scattering amplitude is small) and that there is accordingly some tendency toward forward scattering. Considering the experimental accuracy, one may estimate $| D_-(1.17) | < 2$ and $| D_-(1.285) | < 2$, consistent with Table I.

Since it is clearly not possible to solve for X_Λ and X_Σ separately, we have calculated the "average" coupling $\overline{pX}$ at an "average" pole $\bar{\omega} = \frac{1}{2}(\omega_\Lambda + \omega_\Sigma)$,

$$\overline{pX} = \frac{1}{2}(\bar{\omega}^2 - \omega_o^2)(\bar{\omega} + \omega)\left[\frac{p_\Lambda X_\Lambda}{(\omega_\Lambda^2 - \omega_o^2)(\omega_\Lambda + \omega)} + \frac{p_\Sigma X_\Sigma}{(\omega_\Sigma^2 - \omega_o^2)(\omega_\Sigma + \omega)}\right] \qquad (5)$$

for various choices of ω and ω_o in Eq. (3). The results are listed in Table II. We conclude that this determination of $\overline{pX}$ leads to the value

$$\overline{pX} = 0.0 \pm 0.5 , \qquad (6)$$

where we have included some provision for the uncertainity in $D_-(\omega_o)$.

Our conclusion, then, is that the K^+-proton interaction, at the present accuracy, is not sufficiently sensitive to the coupling parameters X_Λ and X_Σ to permit their evaluation from the present data. This is not too surprising, because Eq. (3) depends only on the difference of amplitudes at closely spaced energies that are quite distant from the poles ω_Λ and ω_Σ. We must also point out an important weakness of this type of analysis: only measurements on the $K^\pm$-neutron system can make possible a separation of the $K\Lambda P$ and $K\Sigma P$ coupling terms by their different isotopic properties. The determination of the K-meson parity from the angular dependence of associated $K\Lambda$ and $K\Sigma$ photoproduction[2] does not suffer from this defect and may, therefore, lead to a definitive result sooner.[11]

A few words are in order about the amplitude $D_-(\omega)$.[12] If it deviates substantially from its zero-range value at the energies of interest then Eq. (6) will be modified somewhat (the rhs contains $D_-(\omega_o)$ with a coefficient -0.2). The characteristic feature of the zero-range approximation, that the scattering is very strong in one of the two isotopic spin states[4] depends essentially only on the fact that the large K^--p elastic cross-section is accompanied by a substantial charge-exchange cross-section at low energy. Each satisfactory set of parameters includes one scattering length whose real part exceeds 1.5f in magnitude. We should like to add that only an extremely unlikely change in the experimental situation could reduce this value to 1 f. Since the elastic K^--proton interaction cannot have a range exceeding $\frac{\hbar}{2m_\pi c} \sim .7$ f, it follows that the interaction is attractive in the isotopic state with the large

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scattering length. If $D_-(1)$ is negative, then the attraction is so strong as to give rise to a "bound" state which will be unstable to decay into a pion-hyperon system. $D_-(\omega)$ can be expected to change sign near $\omega \sim 1.2$, a meson laboratory momentum of 300 Mev/c. If $D_-(1)$ is positive, then the attraction is not strong enough to give rise to a "bound" state. In this case $D_-(\omega)$ will decrease monotonically from its low energy value; near $\omega \sim 1.2$, it will already be quite small. Since the forces are attractive and strong in both of these cases, one cannot expect to distinguish between them by a study of the nuclear "optical model" potential for K^--mesons, which is then not simply related to the low-energy scattering amplitude. A measurement of the coulomb interference effects in the angular distribution of K^--proton scattering, will, of course, decide the matter.

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TABLE CAPTIONS

Table I. Integrals and scattering amplitudes used in the dispersion relation Eq. (3). The K-meson energy is ω measured in units of the rest energy. The unit for the other quantities is the K-meson compton wave-length.

Table II. "Average" couplings determined from Eq. (3) by the use of data at the energies ω and ω_o. The results are independent, to the accuracy of this work, of the choice of sign for $D_-(\omega)$.

Table I

ω	$f(\omega)$	$f(-\omega)$	$D_+(\omega)$	$D_-(\omega)$
1.00	3.0 ± 0.3	--	- 1.25 ± 0.14	--
1.17	3.1 ± 0.3	5.2 ± 1.0	- 1.24 ± 0.14	+ .56
1.285	3.1 ± 0.3	4.9 ± 0.8	- 1.23 ± 0.14	+ .40
1.46	3.0 ± 0.3	--	- 1.20 ± 0.08	--

Table II

ω	ω_o	$\overline{pX}$
1.46	1.285	- 0.2 ± 0.6
1.46	1.17	- 0.1 ± 0.4
1.00	1.285	+ 0.1 ± 0.4
1.00	1.17	+ 0.3 ± 0.6

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11. The determination made in reference 2 leads to $p_\Lambda X_\Lambda = 0.1 \pm 0.2$. This would suggest that both $p_\Lambda X_\Lambda$ and $p_\Sigma X_\Sigma$ are small, not only their "average".

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