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ABSTRACT

A Fuzzy logic controller (FLC) is designed and implemented in real time on a Toyota Celica test vehicle to achieve control of the lateral motion of the vehicle. The structure of the FLC is modularized as feedback, preview, and gain scheduling rule base. The parameters of the FLC are tuned manually using information from characteristics of human driving operation and an existing controller. Three feedback FLCs with different feedback variables are designed. A fuzzy preview rule base is developed to utilize preview information regarding the upcoming radii of curvature. Also, a gain scheduling rule base is designed to choose the appropriate controller based on the velocity of the vehicle. These fuzzy logic control strategies are implemented on the test vehicle, automatically following a multiple curved track using discrete magnetic markers as a lateral error reference system. The test results of the FLCs are shown with variations in driving conditions, and a comparison is made to similar tests conducted using the frequency shaped linear quadratic (FSLQ) controller with preview control.
1 INTRODUCTION

An important component of intelligent vehicle highway systems (IVHS) is lateral motion control of a vehicle in lane following maneuvers as well as lane change maneuvers. This investigation focuses on lane following, where the objective is to track the center of a lane with smooth ride quality under a range of operating conditions such as vehicle speed, wind gusts, and road conditions.

Under the direction of the PATH (Partners for Advanced Transit and Highways) program [12,13], a linear lateral guidance controller using the frequency shaped linear quadratic (FSLQ) optimal control and preview control theory has been developed and successfully implemented on a specially modified Toyota Celica, developed by IMRA America, Inc., [7,8,9,10]. The present effort is to explore other aspects in the design of the lateral controller. We propose using a fuzzy logic controller (FLC) based on fuzzy set theory [14,15] to control the lateral motion of the vehicle. There are many features of lateral guidance of a vehicle which motivate the use of a FLC. At the heart of the FLC is a rule base consisting of inference rules, which provides a natural environment for incorporating human knowledge and engineering judgment of vehicle steering to the controller. Also, the rule base can be designed to handle the substantial nonlinearities found in vehicle dynamics, especially in the tire characteristics.

We propose a control structure with feedback action and feedforward action. The feedback loop operates on the tracking error of the system, while the feedforward loop utilizes the preview information regarding upcoming road curvature to calculate a preview front wheel steering angle component. Motivation for the inclusion of feedforward action is provided by Roland and Sheridan, who suggest that preview information of an upcoming curve is vital for human drivers [11]. Also, experiments conducted by Donges [1] suggest that human drivers anticipate curvature changes and initiate front wheel steering before the curve begins.

A FLC for lane following vehicle lateral control was proposed in [4]. The rule base was developed based on engineering judgment. Controller parameters which define membership functions and the consequent expressions were tuned manually. The theme of this FLC was to formulate inference rules based on PID (proportional, integral, and derivative) control techniques. Therefore, the inputs to the FLC were chosen to be the lateral error between a sensor mounted on the vehicle and the center of the road, y, change
in the lateral error with respect to time, \( \dot{y} \), and the integral of the lateral error with respect to time, \( \int y \). The output of the FLC was the absolute command for the front wheel steering angle, \( \delta \). In addition to this feedback control loop, \([4]\) proposed a feedforward controller to utilize preview information regarding upcoming road curvature. This feedforward control term was based on a steady state estimate of front wheel steering angle as a function of the radius of curvature based on an expression obtained in \([7]\). This steering term was gradually implemented at curve transitions based on human observations obtained in \([2]\).

Another FLC for lane following vehicle lateral control was proposed in \([5]\). The rule base was also developed based on engineering judgment. Controller parameters which define membership functions and the consequent expressions were tuned manually. The inputs to this FLC were chosen to be the lateral error between a sensor mounted on the vehicle and the center of the road, \( y \), change in the lateral error with respect to time, \( \dot{y} \), and the yaw rate of the vehicle relative to the desired yaw rate, \( \dot{\epsilon} - \dot{\epsilon}_d \). The output of the FLC was the command for the rate of change of the front wheel steering angle, \( \dot{\delta} \). In addition, \([5]\) proposed a method to utilize preview information regarding upcoming road curvature estimating the lateral error at a specified look-ahead time.

Here, we extend the feedback FLC development by investigating variations of the choice of control inputs from the set, \( \{ y \ \dot{y} (\epsilon - \epsilon_d)(\dot{\epsilon} - \dot{\epsilon}_d) \} \), where \( \epsilon \) and \( \epsilon_d \) represent the actual and desired yaw angle, respectively. In addition, we consider using fuzzy logic to implement a preview-type controller to utilize the preview information for road curvature. The next section details the fuzzy algorithm used to implement the fuzzy controller. We then describe the design of the FLC, followed by a section showing the experimental results of the FLC implemented on the IMRA Test Vehicle.
2 FUZZY LOGIC CONTROL ALGORITHM

Fuzzy set theory has a variety of applications, among which is system control. Fuzzy sets can be thought of as a generalization of a crisp set. An element either belongs to or does not belong to a crisp set. For instance, if we define the crisp set \( S = \{ y: 0 \leq y \leq 0.3 \} \), the element \( y_1 = 0.1 \) belongs to \( S \), but \( y_2 = 0.5 \) does not belong to \( S \). In contrast, a fuzzy set considers elements which have a certain degree of membership to a particular set. For example, consider a fuzzy subset, \( A \), called “Positive Big Lateral Error”, and an element \( y = 0.2 \) meters, among the set of all possible lateral errors, \( Y \). A membership function \( \mu_A(y) \), is a mapping \( \mu: Y \rightarrow [0, 1] \), where 1 implies full membership to the fuzzy subset, \( A \), and 0 implies no membership. Therefore, \( \mu_A(y) \) is the degree to which \( y \) belongs to \( A \). For example, one may obtain \( \mu_A(0.2) = 0.5 \).

As with crisp subsets, set operations such as unions and intersections can be performed on fuzzy subsets. Since fuzzy subsets are characterized by membership functions, so are the fuzzy set operations. Although many consistent methods are available for these two operations, the min-max model, proposed by Zadeh [15], is the most common. For example, if \( A \) and \( B \) are two fuzzy subsets, the degree to which \( y \) belongs to \( A \) AND \( B \) is \( \min\{ \mu_A(y), \mu_B(y) \} \), while the degree to which \( x \) belongs to \( A \) OR \( B \) is \( \max\{ \mu_A(y), \mu_B(y) \} \). Refer to [15] for more details of these fuzzy set operations.

Fuzzy set operations are the mathematical tool required to evaluate inference rules in a fuzzy rule based controller. Several methods are available to develop such a rule base such as observations of human operators of the given system, fuzzy modeling, and our choice for this investigation, rule development based on engineering judgment and human driving characteristics.

The advantage of a FLC is its ability to develop and utilize rules which make intuitive sense and can be expressed in linguistic terms. Using vehicle guidance as a working example, consider two rules.

\[
\text{IF } LE \text{ is } PB \text{ and } CLE \text{ is } PS \quad \text{THEN } A \text{ is } NB
\]

\[
\text{IF } LE \text{ is } PB \text{ and } CLE \text{ is } NIL \quad \text{THEN } A \text{ is } Nit4
\]
LE is the linguistic variable for lateral error, y. CLE and A represent the change of lateral error with respect to time, \( \dot{y} \), and the front wheel steering angle command, \( \delta_e \), respectively. Also, \( P \Rightarrow \) positive, \( N \Rightarrow \) negative, \( B \Rightarrow \) big, \( A4 \Rightarrow \) medium, and \( NIL \Rightarrow \) near zero.

If one imagines the scenario of these two rules, they should make intuitive sense. The computational process takes the following form as seen in Fig. 1.

Various system variables are measured or estimated. For each of these variables, the relevant membership functions are evaluated. For instance, if the lateral error, y, is measured to be 0.25 meters, one relevant fuzzy subset is PB. The degree to which y belongs to PB is 0.6 (see Fig. 2).

---

**Figure 1: Flow of Rule Computation**

**Figure 2: Membership Function for Lateral Error**
A similar process determines the degree to which \( y \) belongs to \( PS, NIL, \) etc. It should be noted that membership functions can take any suitable form as long as the range interval is \([0, 1]\). In our development we use triangular membership functions which take the form:

\[
\mu(y) = \begin{cases} 
0 & ; |y - y_0| \geq \alpha \\
1 - \frac{|y - y_0|}{\alpha} & ; \text{otherwise}
\end{cases}
\] (3)

where \( y_0 \) and \( 1/\alpha \) are the center and slope of the triangular membership function.

A *rule* truth is the degree to which the antecedent is true. This computation for two rules (Eqn. 1 and 2) is carried out as follows:

\[
r_1 = \min(\mu_{LE}^{PB}(y), \mu_{CLE}^{PS}(\hat{y}))
\] (4)

\[
r_2 = \min(\mu_{LE}^{PB}(y), \mu_{CLE}^{NIL}(\hat{y}))
\] (5)

Once the rule truth has been calculated for each rule, the final step is *defuzzification*, the process of determining a crisp value for the front wheel steering angle command, \( \delta_c \), based on the evaluation of the fuzzy inference rules. A simple method of defuzzification is the weighted average method, using singletons (crisp values) for each consequent. For example the consequent of Eqn. 1 is “THEN A is NB”, which corresponds to the singleton equivalent of “THEN \( \delta^1_c = -4.0 \) degrees”, for rule 1. Thus, to calculate the defuzzified crisp control output, \( \delta_c \), if the number of rules is \( n_r \), we insert the rule truths \( (r_s \text{ for } s = 1, 2, \ldots n_r) \) along with the corresponding consequent values \( (\delta^s_c \text{ for } s = 1, 2, \ldots n_r) \) into Eqn. 6.

This defuzzification stage concludes the computation of the control law based on the fuzzy rule base control algorithm.

\[
\delta_c = \frac{\sum^n_{s=1} \delta^s_c r_s}{\sum^n_{s=1} r_s}
\] (6)
3 FUZZY LOGIC CONTROLLER DESIGN BY MANUAL TUNING

Figure 3 provides a block diagram of the vehicle and the fuzzy rule base controller. Note that the system assumes the existence of a *preview data base*, which provides the location, direction and radius of curvature for each curved section on the reference track. In an effort to make the development of the rule base and tuning easier, the controller is modularized into three separate rule bases, 1) feedback rules, 2) preview rules, and 3) gain scheduling rules. The parameters for the membership functions, the crisp singleton values, and the inference rules are manually tuned for each rule base. The following three subsections provide the details for the design of each rule base.

![Figure 3: Feedback Control Block Diagram](image)

### 2.1 Feedback Rule base

The inputs to the feedback rule base are some combination of the errors in the states of the system. That is, the control inputs are taken from the set \{y \dot{y}(\varepsilon - \varepsilon_d)(\dot{\varepsilon} - \dot{\varepsilon}_d)\}. In our investigation, four distinct feedback rule bases were designed by choosing the following three combinations of state errors:

\[
x_1 = [y \dot{y}(\varepsilon - \varepsilon_d)(\dot{\varepsilon} - \dot{\varepsilon}_d)]'
\]

\[
x_2 = [y \dot{y}(\dot{\varepsilon} - \dot{\varepsilon}_d)]^T
\]

\[
x_3 = [y \dot{y}(J_y)(\dot{\varepsilon} - \dot{\varepsilon}_d)]^T
\]
The input vector, $x_1$, was chosen to create rules using full state feedback from a simplified linear model of the vehicle [7]. The choice of input vector, $x_2$, eliminates relative yaw angle, $(\varepsilon - \varepsilon_d)$, from the rule base. This was done to investigate the necessity of using the state, $(\varepsilon - \varepsilon_d)$, as an input to the controller since $\varepsilon$ is difficult to measure without additional sensors. The input vector, $x_3$, was chosen to consider the effects of including an integral term for lateral error.

Here, we consider the simplest control input vector, $x_2$. Therefore, we apply the fuzzy algorithm detailed in the previous section to a feedback rule base consisting of inference rules with linguistic variables $LE$ (corresponding to $y$), $CLE$ (corresponding to $\dot{y}$), and $YWR$ (corresponding to $(\dot{\varepsilon} - \dot{\varepsilon}_d)$) in the antecedents and $A$ (corresponding to $\delta_e$) in the consequent terms. This feedback rule base is made up of rules taking the following form:

$$IF\, LE\, is\, A,\, AND\, CLE\, is\, A,\, AND\, YWR\, is\, A,\, THEN\, A\, is\, A, \quad (10)$$

where $A_i$ represents linguistic values of linguistic variable, $i$,

$$A_{LE} \in \{NB,NS,NIL,PS,PB\}$$
$$A_{CLE} \in \{NB,NS,NIL,PS,PB\}$$
$$A_{YWR} \in \{NB,NS,NIL,PS,PB\}$$
$$A_A \in \{NH,NB,NM,NS,NIL,PS,PM,PB,PH\}$$

(for example: $NB \Rightarrow$ Negative Big and $NH \Rightarrow$ Negative Huge)

The inference rule base was generated by considering human driving techniques. The initial consequent singletons were generated by observing the states of the system and the corresponding FSLQ control law. The parameters of the membership functions and the singletons were manually tuned to obtain the final feedback FLC. Note that the number of fuzzy subsets of $LE$ (i.e. the number of different linguistic values of $LE$) is 5. That of $CLE$ and $YWR$ are also 5 and 5, respectively. Therefore, there are $5 \times 5 \times 5 = 125$ rules in this feedback rule base. Appendix A provides the details of the membership function definitions.
2.2 Preview Rule base

The preview rule base is developed based on the structure used in preview control theory applied to vehicle guidance [4,9], which considers a preview time window, \( t_w \), that the vehicle can look ahead. A front wheel steering angle preview term, \( \delta_{pr} \), is generated by integrating a preview weight, \( p \), multiplied by the inverse of the future radius of curvature, \( \rho \), over the preview time window, \( t_w \), given by

\[
\delta_{pr}(t) = \int_0^{t_w} \frac{p(\tau)}{\rho(\tau+t)} d\tau
\]

For example, if a vehicle is traveling on a straight roadway which becomes curved at \( t = t_* \), \( \delta_{pr} \) may respond as in Fig. 4.

For simplification, we assume that the radius of curvature, \( \rho \), remains constant over every curved section. This implies that our preview time window, \( t_w \), contains at most two radii of curvature, \( \rho_c \) and \( \rho_n \), the current and next radii of curvature, respectively (see Fig. 5).

Thus, the preview term for the front wheel steering angle, \( \delta_{pr} \), takes the form

\[
\delta_{pr} = \frac{\rho_c}{\rho_c} + \frac{\rho_n}{\rho_n}
\]
where \( p_c \) and \( p_n \) are preview parameters for the current and next radii of curvature, respectively. The parameters \( p_c \) and \( p_n \) depend on \( \min(t_2, t_w) \), where \( t_2 \) is the estimated time for the vehicle to reach the next curve transition and \( t_w = 1 \) second as suggested in [9]. A preview data base provides the location and radius of curvature, \( \rho \), for each curved section on the track. Thus, given the velocity of the vehicle, \( v \), we can determine the time for the vehicle to get to the next curve, \( t_2 \).

The preview rule base has \( t_2 \) as its only input variable, and \( p_c \) and \( p_n \) as the two output variables. Since the preview parameters are rather complex functions of \( t_2 \), we choose 26 fuzzy subsets to describe \( t_2 \), as seen in Appendix A. The present scheme to realize preview action is based on preview control theory by Peng [8] and is different from the method of utilizing preview information discussed in [5].

### 2.3 Gain Scheduling Rule base

Both the feedback rule base and the preview rule base are sensitive to system parameters such as cornering stiffness, which quantifies the traction between the road and tires, and the velocity of the vehicle, \( v \). For example, at low \( v \) or on a dry roadway, we can infer more aggressive steering commands, while at high \( v \) or on a slippery roadway, we should infer more gentle steering commands.

Here, we focus on \( v \). To keep the rule base structure modular, a gain scheduling rule base was designed to infer the final front wheel steering angle command, \( \delta_c \), from \( v \). Both the feedback and preview rule bases were designed at three velocities, \( v \), of 5.0, 12.5, and 20.0 m/s. Thus, the control block diagram takes the form of Fig. 6.
For example, a gain scheduling rule is

\[
\text{IF Vis LOW THEN } \delta_c = \delta_{fb} + \delta_{pr}
\]  

(13)
4 EXPERIMENTAL RESULTS

3.1 Experimental System

The reference/sensing system consists of discrete magnetic markers embedded in the roadway, forming a predetermined path, and magnetometers mounted on the front of the vehicle [3,16]. The vertical and horizontal components of the magnetic field of each magnetic marker are measured by the magnetometers and are subsequently translated to lateral displacement, \( y \), by a computer algorithm.

A nonlinear mathematical model of the test vehicle [7] was used to simulate the fuzzy logic control scheme, which was then implemented in real time. Figure 7 is the diagram of the real time system. A DSP control board obtains yaw rate sensor data and the magnetic field from the magnetometer. This magnetic field measurement is processed by software to calculate the lateral distance between the front-center of the vehicle and the reference magnets in the roadway. The DSP sends the fuzzy rule base control output, \( \delta_e \), to the internal hydraulic control system. A complete description of the real time system is given in [9].

![Figure 7: Real time System](image)

The state variables \( \dot{y} \) and \( (e-e_d) \) were estimated using a Luenberger observer discussed in [9]. The longitudinal velocity, \( v \), of the vehicle was estimated by counting the timer interrupts on the control computer between the magnetic markers, which were spaced one meter apart in the nominal case. The command signal to the front wheel steering actuator
was updated every 21 milliseconds, which is the shortest time allowable by the hydraulic actuator system. Table 1 provides a listing of key system parameters.

Table 1: Key System Parameters

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$ (kg)</td>
<td>vehicle mass</td>
<td>1573</td>
</tr>
<tr>
<td>$I_z$ (kg-m$^2$)</td>
<td>moment of inertia (Z-axis)</td>
<td>2783</td>
</tr>
<tr>
<td>$v$ (km/hr)</td>
<td>vehicle velocity</td>
<td>30-60</td>
</tr>
<tr>
<td>$l_1, l_2$ (meter)</td>
<td>distances from c. g. to front and rear axles</td>
<td>1.034, 1.491</td>
</tr>
<tr>
<td>$d_c$ (meter)</td>
<td>distance from c. g. to magnetometers</td>
<td>1.4</td>
</tr>
<tr>
<td>$T_s$ (second)</td>
<td>control update time</td>
<td>0.021</td>
</tr>
<tr>
<td>$s_m$ (meter)</td>
<td>marker spacing</td>
<td>1.0</td>
</tr>
<tr>
<td>$t_m$ (second)</td>
<td>look-ahead time window</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The vehicle was driven on a test track shown in Fig. 8.

3.2 Closed Loop Experimental Results and Discussion

The test results for the three cases of FLCs (i.e. $x_1$, $x_2$, and $x_3$, see Eqns. 7-9) are presented in this section. The test plan called for experiments at vehicle speeds of 30, 40, 50, and 60 km/hr, but since the vehicle speed was controlled by a human driver, the target test velocities were only approximate. The robustness of the closed loop control systems were investigated by testing the vehicle under non-ideal conditions: increased load, reduced tire pressure, misaligned markers, and two meter marker spacing, $s_m$. 
3.2.1 Nominal Case

Experimental results were obtained under nominal conditions over the range of velocities for the three controllers distinguished by their input vectors, $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$ (see Eqns. 7-9). Figures 9-12 show the results for $\mathbf{x}_3$ over the velocity range of 30 m/s to 60 m/s. Since the FLCs using $\mathbf{x}_1$ and $\mathbf{x}_2$ show similar trends as $\mathbf{x}_3$ for different velocities of the vehicle, the three FLCs are compared only at 40 m/s. Figures 13 and 14 show the results for $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively, at 40 m/s. Table 2 lists the peak tracking errors for each test run. The peak tracking errors listed exclude large errors due to initial conditions and errors at the end of the track caused by saturation in the hydraulic steering actuator. The peak lateral accelerations did not vary among the three different cases ($\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$). The lateral acceleration was typically 0.5 m/s$^2$ at a vehicle speed of 30 km/hr and 1.5 m/s$^2$ at a vehicle speed of 60 km/hr. The magnitudes of the lateral error and lateral acceleration were comparable to the results of the FSLQ lateral controller [9]. However, the FSLQ control law appeared to provide a more smooth steering command which resulted in a more smooth lateral acceleration as compared to the FLC controllers that we tested.

<table>
<thead>
<tr>
<th>velocity (km/hr)</th>
<th>$\mathbf{x}_1$</th>
<th>$\mathbf{x}_2$</th>
<th>$\mathbf{x}_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>6</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>40</td>
<td>7</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>50</td>
<td>10</td>
<td>13</td>
<td>11</td>
</tr>
<tr>
<td>60</td>
<td>16</td>
<td>16</td>
<td>15</td>
</tr>
</tbody>
</table>

3.2.2 No Preview Action

The test results in Figures 15-18 show the case of omitting preview action in the controller at the four test velocities for $\mathbf{x}_3$. Table 3 lists the peak tracking errors for each test run. At a vehicle speed of 60 km/hr (Fig. 18), the controller was unable to stay within the range of the magnetometer sensors, which is ±40 centimeters. As can be seen from Table 3, there is a strong need for preview action in this FLC controller. In addition, the lateral acceleration was near 0.5 m/s$^2$ at vehicle speeds of 30 km/hr (Fig. 15), but approached 2.0 m/s$^2$ at vehicle speeds of 60 km/hr (Fig. 18), before the vehicle lost lateral control.
### Table 3: Peak Tracking Error in centimeters with no Preview Control Action

<table>
<thead>
<tr>
<th>Velocity (km/hr)</th>
<th>With Preview ( x_i )</th>
<th>No Preview ( x_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>8</td>
<td>20</td>
</tr>
<tr>
<td>40</td>
<td>8</td>
<td>22</td>
</tr>
<tr>
<td>50</td>
<td>11</td>
<td>38</td>
</tr>
<tr>
<td>60</td>
<td>15</td>
<td>+40</td>
</tr>
</tbody>
</table>

3.2.3 Increased Load

The test results in Figures 19-21 show the effect of increasing the load of the vehicle to a higher weight than the controllers were designed for in each of the three cases: \( x_1, x_2, \) and \( x_3 \). All tests were performed at vehicle speeds of 40 km/hr. The weight of the vehicle was increased from 1573 kg to 1733 kg for this test. Table 4 lists the peak tracking errors for each test run. There were only slight increases in the lateral error due to the increased load. The lateral acceleration was near 0.8 m/s\(^2\), which is comparable to the lateral acceleration of the nominal cases at vehicle test velocities of 40 km/hr.

### Table 4: Peak Tracking Error in centimeters with Variations in Vehicle Load with \( v = 40 \) km/hr

<table>
<thead>
<tr>
<th>Vehicle load (kg)</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1573</td>
<td>7</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>1733</td>
<td>8</td>
<td>12</td>
<td>10</td>
</tr>
</tbody>
</table>

3.2.4 Reduced Tire Pressure

The test results in Figures 22-24 show the effect of reducing the pressure in each of the four tires of the vehicle to a lower pressure than the controllers were designed for in each of the three cases: \( x_1, x_2, \) and \( x_3 \). All tests were performed at vehicle speeds of 40 km/hr. The air pressure in each of the four tires was decreased from 32 psi to 22 psi for this test. Table 5 lists the peak tracking errors for each test run. The lower tire pressure resulted in larger tracking errors due to the reduced cornering stiffness of the tires. This larger lateral error subsequently resulted in larger steering command signals. The lateral acceleration was near 0.8 m/s\(^2\), which is comparable to the lateral acceleration of the nominal case.
Table 5: Peak Tracking Error in centimeters with Variations in Tire Pressure with \( v = 40 \text{ km/hr} \)

<table>
<thead>
<tr>
<th>Tire Pressure (psi)</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>7</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>22</td>
<td>10</td>
<td>14</td>
<td>15</td>
</tr>
</tbody>
</table>

3.2.5 Misaligned Markers

The test results in Figures 25-27 show the effect of measurement noise in the magnetic marker reference/sensing scheme for each of the three cases: \( x_1, x_2, \) and \( x_3 \). All tests were performed at vehicle speeds of 40 km/hr. Measurement noise was injected by software into the lateral displacement measurement as a zero mean Gaussian noise with a variance of 1 cm². The steering command became more oscillatory as compared to the nominal case due to the more oscillatory lateral error term with the injected measurement noise. Table 6 lists the peak tracking errors for each test run. The lateral acceleration was near 0.8 m/s², which is comparable to the lateral acceleration of the nominal case.

Table 6: Peak Tracking Error in cm with Injected Lateral Error Measurement Noise with \( v = 40 \text{ km/hr} \)

<table>
<thead>
<tr>
<th>Injected Gaussian Noise Variance (cm²)</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>14</td>
<td>15</td>
</tr>
</tbody>
</table>

3.2.6 Two Meter Marker Spacing

The test results in Figures 28-30 show the effect of increasing the magnetic marker spacing, \( s \), from 1.0 meter to 2.0 meters in each of the three cases: \( x_1, x_2, \) and \( x_3 \). All tests were performed at vehicle speeds of 40 km/hr. The spacing between magnetic markers was increased from 1 meter to 2 meters. The result was a slightly larger oscillation in the steering command. Table 7 lists the peak tracking errors for each test run. For the case of using \( x_1 \) as the feedback vector, there was very little effect on the lateral error as compared to \( x_2 \) and \( x_3 \), which, unlike \( x_1 \) do not have access to \( (\varepsilon - \varepsilon_d) \). The lateral acceleration was near 0.8 m/s², which is comparable to the lateral acceleration of the nominal case.
Table 7: Peak Tracking Error in cm with 2.0 meter marker Spacing with $v = 40$ km/hr

<table>
<thead>
<tr>
<th>Marker Spacing (meters)</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>11</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>13</td>
<td>13</td>
</tr>
</tbody>
</table>
5 CONCLUSION

To properly compare the fuzzy logic controllers (FLCs) and the frequency shaped linear quadratic (FSLQ) controller, one must consider the design stages as well as the real time response. In the real time response each of the three FLCs (i.e. $x_1, x_2, x_3$) performed as well as the FSLQ in terms of tracking the reference track. However, the steering angle commands of the FSLQ controller were smoother than the FLCs and resulted in a smoother ride in terms of lateral acceleration. The parameters of the FLCs tested in this investigation were tuned manually. Although the tuning process was time consuming, the rule base could be adjusted and tuned by the designer based on engineering judgment. Thus, knowledge about the vehicle, itself, was not used explicitly in the rule development for the FLCs. However, the tuning of the parameters of the FLCs did make use of a computer simulation of the vehicle in order to develop an FLC with a satisfactory response. In contrast, the FSLQ uses a linear model of the vehicle explicitly in its design. In addition, although the structure of the FLC is designed to mimic human reasoning through inference rules, it must be noted that a human driver sees and feels more information that can be input to a FLC.

One advantage of a FLC over a conventional linear controller or a nonlinear controller, such as a full-state linearized feedback controller, is its ability to make use of a variety of inputs. The gain scheduling technique based on longitudinal velocity of the vehicle can be extended to other quantities such as cornering stiffness and tire pressure, for example.

6 ACKNOWLEDGMENT

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NOMENCLATURE

\( y \): lateral error from magnetometer to road center
\( \dot{y} \): lateral velocity with respect to the center of the road
\( \int y \): the integral approximation of lateral error, \( y \)
\( \delta_c \): front wheel steering angle command
\( \varepsilon \): yaw angle of the vehicle
\( \varepsilon_d \): desired yaw angle
\( \varepsilon \): yaw rate of vehicle
\( \dot{\varepsilon} \): desired yaw rate
\( \rho \): radius of curvature of the roadway
\( p \): preview parameter
\( n \): subscript denoting the next curve
\( c \): subscript denoting the current curve
\( v \): longitudinal velocity
\( t_s \): the time for the vehicle to reach the next marker
\( t_w \): the time window that the vehicle can look ahead
\( LE \): linguistic variable for lateral error, \( y \)
\( CLE \): linguistic variable for the relative yaw angle, \((\varepsilon - \varepsilon_d)\)
\( YAW \): linguistic variable for change in lateral error, \( \dot{y} \)
\( YWR \): linguistic variable for the relative yaw rate, \((\dot{\varepsilon} - \dot{\varepsilon}_d)\)
\( A \): linguistic variable for \( \delta_c \)
\( n \): number of rules
\( m \): vehicle mass
\( I_z \): moment of inertia about vertical axis at the center of gravity \((c.g.)\) of the vehicle
\( I_1 \): \( I_2 \): distances from c. g. to front and rear wheel axis, respectively
\( d \): distance from c. g. to magnetometers
\( T \): control update time
\( s_m \): spacing of the magnetic markers
REFERENCES

Appendix A: Membership Definitions

Figure A.1: Membership Functions, $\mu$, for $x_1$

Figure A.2: Membership Functions, $\mu$, for $x_2$
Figure A.3: Membership Functions, $\mu$, for $x_3$

Figure A.4: Membership Functions, $\mu$, for $t_2$

Figure A.5: Membership Functions, $\mu$, for $v$
FIGURES OF EXPERIMENTAL RESULTS

**Figure 9: Nominal Case, 30 km/hr, x_3 Test**
Figure 10: Nominal Case, 40 km/hr, x₃ Test
Figure 11: Nominal Case, 50 km/hr, x3 Test
Figure 12: Nominal Case, 60 km/hr, $x_3$ Test
Figure 13: Nominal Case, 40 km/hr, x, Test
Figure 14: Nominal Case, 40 km/hr, $x_2$ Test
Figure 15: Nominal Case, Without Preview, 30 km/hr, $x_3$ Test
Figure 16: Nominal Case, Without Preview. 40 km/hr, x3 Test
Figure 17: Nominal Case, Without Preview, 50 km/hr, x_j Test
Figure 18: Nominal Case, Without Preview, 60 km/hr, x3 Test
Figure 19: Increased Load Case, 1573 to 1733 kg, 40 km/hr, x₁ Test
Figure 20: Increased Load Case, 1573 to 1733 kg, 40 km/hr, $x_2$ Test
Figure 21: Increased Load Case, 1573 to 1733 kg, 40 km/hr, x3 Test
Figure 22: Reduced Tire Pressure Case, 32 to 22 psi, 40 km/hr, x₁ Test
Figure 23: Reduced Tire Pressure Case, 32 to 22 psi, 40 km/hr, $x_2$ Test
Figure 24: Reduced Tire Pressure Case, 32 to 22 psi, 40 km/hr, x₃ Test
Figure 25: Perturbed Marker Case, 40 km/hr, $x_1$ Test
Figure 26: Perturbed Marker Case, 40 km/hr, $x_2$ Test
Figure 27: Perturbed Marker Case, 40 km/hr, x₃ Test
Figure 28: Increased Marker Spacing Case, 1 to 2 meters, 40 km/hr, x₁ Test
Figure 29: Increased Marker Spacing Case, 1 to 2 meters, 40 km/hr, \( x_2 \) Test
Figure 30: Increased Marker Spacing Case, 1 to 2 meters, 40 km/hr, \( x \), Test