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USING KAON-NUCLEON DISPERSION RELATIONS

Terence W. Rogers

September 23, 1968

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CALCULATION OF THE COUPLING CONSTANTS $g_{\Lambda KN}^2$ and $g_{\Sigma KN}^2$
USING KAON-NUCLEON DISPERSION RELATIONS*

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September 23, 1968

ABSTRACT

We apply forward kaon-nucleon dispersion relations to determine whether the known energy dependence of the low-energy (Kp) elastic scattering amplitude uniquely and consistently prescribes the coupling constants, $g_{\Lambda K^-p}^2$ and $g_{\Sigma^0 K^-p}^2$.

In one application we check the consistency of the multichannel effective-range continuation below the physical threshold by calculating the coupling constants over a range of energies.

In the second application we use a theorem which places bounds on the energy dependence of certain types of unknown dispersion integrals, to find bounds on the coupling constants.

[†] Harkness Fellow of the Commonwealth Fund.

I. INTRODUCTION

In 1957, P. T. Mathews and A. Salam¹ wrote down the following dispersion relations for the forward elastic scattering amplitudes of kaons on protons:

$$\begin{aligned} \text{Re}[M^\pm(\omega)] &= \sum_{Y=\Lambda, \Sigma} B_Y^\pm(\omega) + \frac{1}{\pi} \int_{-\omega}^K \frac{\text{Im}[M_{\text{abs}}^-(\omega')]}{\omega' \pm \omega} d\omega' \\ &+ \frac{1}{\pi} \int_K^\infty \left[\frac{\text{Im}[M^+(\omega')]}{\omega' + \omega} + \frac{\text{Im}[M^-(\omega')]}{\omega' \pm \omega} \right] d\omega' + C, \quad (1.1) \end{aligned}$$

where the (\pm) signs refer to $K^{\pm}p$ scattering, ω is the lab. energy of the kaon, C is an undetermined constant, and B_Y^\pm represents the hyperon pole terms with the structure

$$B_Y^\pm = \frac{g_{YKN}^2}{2N} \frac{X_Y}{\omega_Y \pm \omega}; \quad (1.2a)$$

$$X_Y = \frac{(Y - N)^2 - K^2}{2N}; \quad (1.2b)$$

$$\omega_Y = \frac{Y^2 - N^2 - K^2}{2N}. \quad (1.2c)$$

We use capital letters to stand for the mass of the corresponding particle. An exactly analogous representation exists for the forward elastic kaon-neutron scattering amplitude.

Although these relations are divergent as they stand, all other useful KN dispersion relations may be derived from them by taking suitable combinations.

Every attempt to use these equations is plagued by the existence of the integral over the unphysical region, which extends from the beginning of the $\pi\Lambda$ cut, at $\omega = -\bar{\omega}$, to the physical threshold at $\omega = -K$. We know that there are resonances which should contribute to $\text{Im } M_{\text{abs}}^-$ in this region, and the crudest models suggest that this part of the integral will give a very important contribution. This is especially true for the problem which occupies the remainder of this paper, namely the calculation of the ΛKN and ΣKN coupling constants. In this problem the closeness of the poles to the unphysical region forces one to take into account the unphysical integral.

There are two ways in which one might do this:

(i) Write down explicit forms for $\text{Im } M_{\text{abs}}^-$ as derived from some model.

(ii) Utilize only very general properties of $\text{Im } M_{\text{abs}}^-$.

The latter may not seem very useful, since we know that the calculated value of the coupling constants, using a dispersion relation at one energy, depends very sensitively on the exact form we use in the unphysical region. However, it is also possible to correlate via the dispersion relation the energy dependence of the pole terms with that of the unphysical integral, and this latter dependence may be fairly insensitive to the exact form of $\text{Im } M_{\text{abs}}^-$.

A technique for doing this is given in Section III, and there we do impose useful limits on the coupling constants.

The model calculations of $\text{Im } M_{\text{abs}}^-$ have been gradually refined until at present there are two which seriously compete for attention. They both involve parametrizations of the low energy K^-p data in a form suitable for continuation below the physical threshold, and they may be referred to as

- (i) the constant scattering-length approximation (CSL),²
- (ii) the multichannel effective-range approximation (MER).³

These have been extensively described elsewhere and we shall not reproduce the details here. For our purposes it is sufficient to note that a "world average" of the results of these two models might be given as follows. Let

$$g_{\Lambda}^2 = g_{AK^-p}^2 ; \quad g_{\Sigma}^2 = g_{\Sigma^0 K^-p}^2 = \frac{1}{2} g_{\Sigma^- K^-n}^2 . \quad (1.3)$$

Then

$$\text{MER} : g_{\Lambda}^2 \approx 13.5 \pm 2.5 , \quad g_{\Sigma}^2 \leq 1 ;$$

$$\text{CSL} : g_{\Lambda}^2 \approx 6.5 \pm 2.5 , \quad g_{\Sigma}^2 \lesssim 3 .$$

The MER values are consistent with an $SU(3)$ mixing parameter α [$\alpha = F/(F + D)$] of approximately 0.35, whereas there is no value of α consistent with the pair of values given by CSL.

However, the quoted errors are derived purely from experimental errors, and give no indication of the uncertainty due to the approximations inherent in the models themselves. These could be extremely large, because a small variation in the shape of $\text{Im } M_{\text{abs}}^-$ changes the

couplings considerably.³ Therefore, before accepting either model as giving a useful result, we should at least check to see whether it is self-consistent.

Since both the CSL and the MER models give expressions for the real and the imaginary parts for $\bar{\omega} < \omega < 0.7$ GeV, one may check that the values of g_{Λ}^2 and g_{Σ}^2 calculated from the dispersion relation are independent of the energy at which the equation is evaluated.

Tests of this type have been hindered by the lack of reasonably precise experimental values for the real parts. Typical errors are at least 25%. However, a recent phase shift analysis⁴ of K^+p scattering for the K^+ lab. momentum less than 1500 MeV/c has supplied us with some real parts which we might expect to be reasonably accurate.

Together with these real parts we may use those derived from low energy K^-p data, ($\omega < 0.7$ GeV), and expressible in terms of the parameters of the MER model. Finally we may compute the MER extrapolated real parts and use these also. In this way we may make many different estimates of the coupling constants and check their consistency. This is done in Section II.

Before doing this, we should like to note that a more restricted consistency test has already been carried out in Ref. 5. Briefly the argument is as follows.

If $M(\omega)$ satisfies (1.1), then the function $M(\omega)/\alpha(\omega)$ where

$$\alpha(\omega) = (\omega + \bar{\omega})^{\beta} (\omega + K)^{1-\beta}, \quad (1.4)$$

and

$$0 < \beta < 1 ,$$

satisfies the new equation,

$$\begin{aligned} \operatorname{Re} \left\{ \frac{M^+(\omega)}{\alpha(\omega)} \right\} = & \sum_{Y=\Lambda, \Sigma} \frac{X_Y}{\alpha(-\omega_Y)} \cdot \frac{g_Y^2}{2N} \cdot \frac{1}{(\omega + \omega_Y)} + \frac{1}{\pi} \int_K^\infty \frac{\operatorname{Im} [M^+(\omega')]}{\alpha(\omega')(\omega' - \omega)} \\ & + \frac{\operatorname{Im} [M^-(\omega')]}{\alpha(-\omega')(\omega' + \omega)} d\omega' + \frac{1}{\pi} \int_\omega^K \frac{\operatorname{Re} [M^-(\omega')] \sin(\pi\beta) + \operatorname{Im} [M^-(\omega')] \cos(\pi\beta)}{(\omega' - \omega)^\beta (K - \omega')^{1-\beta} (\omega' + \omega)} d\omega' . \end{aligned} \quad (1.5)$$

This is valid for all ω , and β in the range $0 < \beta < 1$. (There is a concealed subtraction in this equation, due to a divergence in the unphysical integral.)

In Ref. 5, (1.5) was tested for one value of ω and β in the range $(0,1)$, and it was found that the MER model gave values of the coupling constants which were relatively stable against β , while those of the CSL model were not. We shall therefore restrict ourselves to checking further the self-consistency of the MER model.

II. THE CALCULATION OF $\frac{g^2}{g_\Lambda}$

In this section, we carry out the detailed calculation of the ΛKN coupling constant, using the MER parametrization of the low energy K^-p data and the unphysical region. The basic equation is derived from (1.1) by performing a subtraction at $\omega_S = -y$, with $y > 0$, and setting $\omega = x$ ($x > 0$):

$$\begin{aligned} \text{Re } M^+(x) - \text{Re } M^-(y) &= - (x + y) \sum_{Y=\Lambda, \Sigma} \frac{a_Y}{(\omega_Y + x)(\omega_Y - y)} \\ &+ \frac{(x + y)}{\pi} \int_{\omega}^{\infty} \frac{\text{Im } M^+(\omega')}{(\omega' - x)(\omega' + y)} - \frac{\text{Im } M^-(\omega')}{(\omega' + x)(\omega' - y)} d\omega' , \end{aligned} \quad (2.1)$$

where

$$a_Y = \frac{X_Y}{2N} g_Y^2 . \quad (2.2)$$

In the physical region $\text{Im } M^\pm$ will be calculated from the total cross sections for $K^\pm p$ scattering, according to the formulae

$$\text{Im } M^\pm(\omega) = \frac{k}{4\pi} \sigma^\pm(\omega) , \quad (2.3)$$

where k is the K^\pm lab momentum.

To isolate the Λ pole, we should need to consider the isospin zero combination of the Kp and Kn amplitudes, but we do not do this for the following reasons.

Using the MER model one finds that g_{Σ}^2 is less than unity; also the ratio of the coefficients of g_{Λ}^2 and g_{Σ}^2 is constant to within a few percent, for all the energies which we use. Since we are primarily interested in the constancy of the couplings, we may consider a composite coupling \bar{g}_{Λ}^2 , given by

$$\bar{g}_{\Lambda}^2 = g_{\Lambda}^2 + \frac{X_{\Sigma}(\omega_{\Lambda} + x)(\omega_{\Sigma} - y)}{X_{\Lambda}(\omega_{\Sigma} + x)(\omega_{\Lambda} - y)} g_{\Sigma}^2. \quad (2.4)$$

If g_{Λ}^2 and g_{Σ}^2 are independent of energy, then \bar{g}_{Λ}^2 will be also, to within about 1%.

This might be unnecessary if the Kn data were not far less accurately known than the Kp data, but this loss of accuracy compensates any gain in knowledge of g_{Λ}^2 which one might achieve by eliminating g_{Σ}^2 .

To check the consistency of the MER parametrization, we set $x = 0.494$ GeV, and calculate \bar{g}_{Λ}^2 for various values of y in the range 0.264 GeV $\leq y \leq 0.714$ GeV. This includes both the unphysical and the low energy K^-p regions, since the physical threshold occurs at 0.494 GeV.

Secondly we check that the MER model is consistent with the latest phase shift analysis of the low energy K^+p data.⁴ Thus we set $y = 0.464$ GeV and calculate \bar{g}_{Λ}^2 for 0.494 GeV $\leq x \leq 1.64$ GeV.

We call these two calculations solutions I and II respectively, and we plot the results in Figs. 1 and 2.

Before any meaningful analysis of the results may be made, we must look carefully at the errors, which may be classified into three types.

(a) Errors in σ^\pm .

For energies below 20 GeV, σ^+ is known to within 3% accuracy.⁶ Although some experiments claim much better accuracy, e.g. Ref. 7, we feel that the apparent discontinuities where one experimental set adjoins another obliges us to be more cautious.

The total cross section, σ^- , is less accurately known in the low energy regions. In the range $0.5 \text{ GeV} \leq \omega \leq 0.85 \text{ GeV}$, there are no precise measurements and we feel obliged to attribute to σ^- the large error of 15%. From 0.85 GeV to 1.0 GeV we give an error of 5%, and above 1.0 GeV, 3%.

The extrapolation of σ^\pm beyond 20 GeV was carried out by use of the Regge poles and parameters given in Ref. 8. The error here is fairly arbitrary and we chose 3%. Due to the cancellation between the terms involving σ^+ and σ^- , the net contribution is small but not insignificant.

(b) Errors in the real parts.

For the real parts calculated from the phase shift analysis an estimate of the error was obtained as follows. There are three sets of phase shifts which account for the data, and at any energy these yield three different estimates of the real part. The maximum discrepancy was taken as the error, and the mean value used as the real part. The

error is typically of the order of 5%, and its contribution to the final error in \bar{g}_Λ^2 is usually small.

In calculating the K^-p real parts from the MER model, we have made use of the parameter uncertainties quoted in Kim's analysis,³ to obtain an estimated error. As one would expect, this gives a small error near the physical threshold, but allows considerable uncertainty at the $\pi\Lambda$ threshold.

(c) Error in the integral over the unphysical region.

We have no way of calculating this, but it is useful to see how a given error reflects in the error on \bar{g}_Λ^2 . We have calculated the uncertainty in \bar{g}_Λ^2 due to an uncertainty of 20% in the integral.

In Figs. 1 and 2 we show a smooth curve representing \bar{g}_Λ^2 as a function of the energies x or y . Also shown are some typical errors, derived only from sources (a) and (b) above, and labelled in the figures accordingly.

From Fig. 1 it would appear that \bar{g}_Λ^2 is not constant, even within the errors allowed to us by the model. The parameters of the model therefore require some modification, and this could be achieved in one of two ways.

(i) It is conceivable that the energy dependence of the amplitude can be altered in such a way that the value near the physical threshold is approximately unchanged, while the value near the $(\pi\Lambda)$ threshold changes considerably. Thus the predictions of the mass and width of the $Y_0^*(1405)$ would remain unchanged, and, since this resonance dominates the unphysical integral, the values of

\bar{g}_Λ^2 near the physical threshold would not change either. In this way it might be possible to have \bar{g}_Λ^2 constant at the value of approximately 14.

(ii) It may be that the change in the unphysical absorptive part is sufficient to produce a significant change in the unphysical integral, and it is therefore interesting to have some idea as to how important this might be.

With this in mind we have computed the change in \bar{g}_Λ^2 (Δg^2) for a 20% change in the unphysical integral. The results are shown in Table I. They suggest that the value of \bar{g}_Λ^2 is very sensitive to changes in the unphysical integral, especially when evaluated near the physical threshold.

Thus it is very much a complicated computational problem as to whether one may remove the apparent energy dependence of \bar{g}_Λ^2 without significantly altering its value at the physical threshold.

The results of Fig. 2 are more reassuring. They indicate that the predicted values of the unphysical absorptive part are consistent with the K^+p real parts. One might worry that there is a certain degree of circularity, which arises because of the developing habit of using dispersion relations to eliminate the well-known Yang ambiguity inherent in phase shift determinations.

However, if some of the terms in the dispersion relation have very large errors, the constraint will be very weak, and will only decide between two types of phase shift, and not strongly limit their energy dependence. This was true of the analysis whose results we use.

The assumed coupling constants, unphysical absorptive part, and s-wave K^+p scattering length were all given a large error, and the real part calculated from the dispersion integral entered the analysis with a very low statistical weight. The phase shifts are more or less completely determined by the other experimental information, and it is by no means certain that they will be compatible with the MER absorptive part and an energy-independent coupling constant.

To conclude this section, we may say that there is still some doubt about the reliability of the MER model for the purpose of calculating \bar{g}_Λ^2 . Within present errors it is consistent with the known experimental real parts, but is not completely self-consistent, to the extent that the extrapolated real parts appear to diverge from those calculated via the dispersion relation.

III. THE DETERMINATION OF UPPER AND LOWER BOUNDS ON \bar{g}_Λ^2

In this section we attempt to put bounds on \bar{g}_Λ^2 which are independent of any particular model for the unphysical region. The only information used is the positivity of the unphysical absorptive part of the $K\bar{p}$ elastic amplitude, and the experimentally determined real and imaginary parts.

The positivity condition is by no means guaranteed to be satisfied, and in Ref. 1 it was shown that an approximate calculation suggests that $\text{Im } M_{\text{abs}}$ changes sign in the unphysical region if the relative KA parity is even, and does not do so if that parity is odd. With the accepted odd KA parity, no model or physical condition requires that $\text{Im } M_{\text{abs}}$ should change sign, and we feel that our assumption is very probably justified.

Since the method requires fairly accurate real parts at several different energies, we have had to use the results of the phase shift analysis already quoted. However, the results of this section strongly reinforce our remarks in Section II, namely that the resulting phase shifts limit the possible values of the coupling constants to a much smaller range than was assumed at the start of the phase shift analysis.

The technique which we use is based upon some relations given in a paper by G. Tiktopoulos and S. Treiman.⁹ We extract from that paper the following results.

Consider the integral

$$\psi(\lambda) = \int_{-1}^1 \frac{\sigma(\alpha)}{1 - \alpha\lambda} d\alpha, \quad (3.1)$$

where $\sigma(\alpha) \geq 0$ for $-1 \leq \alpha \leq 1$, and we restrict λ to the range $-1 < \lambda < 1$. Suppose that we evaluate the integral at a set of M points, so that we have

$$\psi_i = \psi(\lambda_i) \text{ with } i = 1, 2, \dots, M. \quad (3.2)$$

We may then show that the ψ_i must satisfy certain relations among themselves, which are summarized as follows. If out of the set $\{\psi_i\}$ we choose a subset containing m members, ($m \leq M$), then the elements of that subset satisfy either condition (a) or (b) below. We order the parameters λ_i so that $-1 < \lambda_1 < \lambda_2 < \dots < \lambda_m < 1$.

(a) $m = 2N$, N integral.

Define

$$D_{2N}[\psi(\lambda)] \equiv (-1)^N \begin{vmatrix} \psi_1, \lambda_1 \psi_1, \dots, \lambda_1^{N-1} \psi_1, 1, \lambda_1, \dots, \lambda_1^{N-1} \\ \psi_2, \lambda_2 \psi_2, \dots, \dots, 1, \lambda_2, \dots, \lambda_2^{N-1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \psi_{2N}, \end{vmatrix} \quad (3.3)$$

then the conditions are given by

$$D_{2N}[(1 - \lambda) \psi(\lambda)] \leq 0 \leq D_{2N}[(1 + \lambda) \psi(\lambda)]. \quad (3.4)$$

(b) $m = 2N + 1$, N integral.

Define

$$D_{2N+1}[\psi(\lambda)] \equiv (-1)^N \begin{vmatrix} \psi_1, \lambda_1 \psi_1, \dots, \lambda_1^N \psi_1, 1, \lambda_1, \dots, \lambda_1^{N-1} \\ \psi_2, \lambda_2 \psi_2, \dots \\ \vdots \\ \psi_{2N+1} \end{vmatrix} \quad (3.5)$$

The required conditions are given by

$$\begin{aligned} D_{2N+1}[(1 + \lambda) \psi(\lambda)] &\geq 0 && ; \\ D_{2N+1}\{[(1 - \lambda) \psi(\lambda)]^{-1}\} &\geq 0 && . \end{aligned} \quad (3.6)$$

Furthermore, setting $m = 1, 2, \dots, M$ and choosing all possible subsets for a given m , we deduce a hierarchy of relations, all of which must be satisfied. As an illustration, and for future reference, we work out the results for $m = 1, 2, 3$.

One-Point Conditions

Every element of the set $\{\psi_i\}$ satisfies

$$(1 + \lambda_i) \psi_i \geq 0$$

and

$$\frac{1}{(1 - \lambda_i) \psi_i} \geq 0 \quad (3.7)$$

Two-Point Conditions

Every pair of elements ψ_j, ψ_k from $\{\psi_i\}$, such that $-1 < \lambda_j < \lambda_k < 1$, satisfies

$$\begin{aligned} (1 + \lambda_j) \psi_j &\leq (1 + \lambda_k) \psi_k & ; \\ (1 - \lambda_j) \psi_j &\geq (1 - \lambda_k) \psi_k & . \end{aligned} \quad (3.8)$$

Three-Point Conditions

Every possible triplet $\psi_j, \psi_k, \psi_\ell$ from $\{\psi_i\}$, such that $-1 < \lambda_j < \lambda_k < \lambda_\ell < 1$, satisfies

$$\frac{\lambda_\ell - \lambda_k}{(1 + \lambda_j) \psi_j} + \frac{\lambda_k - \lambda_j}{(1 + \lambda_\ell) \psi_\ell} + \frac{\lambda_j - \lambda_\ell}{(1 + \lambda_k) \psi_k} \leq 0$$

and

$$\begin{aligned} (\lambda_\ell - \lambda_k)(1 - \lambda_j) \psi_j + (\lambda_k - \lambda_j)(1 - \lambda_\ell) \psi_\ell \\ + (\lambda_j - \lambda_\ell)(1 - \lambda_k) \psi_k \leq 0 . \end{aligned} \quad (3.9)$$

The one-point conditions are trivially satisfied, and in the rest of this paper we shall make use of only the two-point conditions. When used in conjunction with experimental quantities which have appreciable errors, the higher inequalities give meaningless results.

To use the two-point conditions we proceed as follows.

Subtracting (1.1) at $\omega = \omega_s < 0$, we obtain

$$\frac{\operatorname{Re} M^+(\omega) - \operatorname{Re} M^-(-\omega_S)}{\omega - \omega_S} = - \sum_{Y=\Sigma, \Lambda} \frac{X_Y g_Y^2}{2N} \frac{1}{(\omega_Y + \omega)(\omega_Y + \omega_S)} + \frac{1}{\pi} \int_{\bar{\omega}}^{\infty} \frac{\operatorname{Im} M^+(\omega')}{(\omega' - \omega)(\omega' - \omega_S)} - \frac{\operatorname{Im} M^-(-\omega')}{(\omega' + \omega)(\omega' + \omega_S)} d\omega' \quad (3.10)$$

From the right-hand side of (3.10) we extract the unphysical integral and rewrite it as

$$- \int_{\bar{\omega}}^K \frac{\operatorname{Im} M_{\text{abs}}^-(\omega')}{(\omega' + \omega)(\omega' + \omega_S)} d\omega' = \int_{\bar{\omega}}^K \frac{\chi(\omega')}{\omega' + \omega} d\omega' \equiv I(\omega) \quad (3.11)$$

If $\operatorname{Im} M_{\text{abs}}^- \geq 0$ and $\omega_S < -K$, then

$$\chi(\omega) = \frac{\operatorname{Im} M_{\text{abs}}^-(\omega)}{-(\omega + \omega_S)} \geq 0$$

We reduce $I(\omega)$ to the form required, by the transformations

$$\alpha = \frac{2}{(K - \bar{\omega})} \left\{ \omega' - \frac{K + \bar{\omega}}{2} \right\} ; \quad \frac{1}{\lambda} = \frac{2}{(K - \bar{\omega})} \left\{ -\omega - \frac{K + \bar{\omega}}{2} \right\}, \quad (3.12)$$

so that

$$I(\omega) = \lambda \int_{-1}^1 \frac{\chi(\alpha)}{\lambda\alpha - 1} d\alpha \quad (3.13)$$

Applying the two-point conditions to a pair of values of $I(\omega)$, we obtain

$$\begin{aligned} (\omega_1 + \bar{\omega}) I_1 &\leq (\omega_2 + \bar{\omega}) I_2 \quad ; \\ (\omega_1 + K) I_1 &\geq (\omega_2 + K) I_2 \quad , \end{aligned} \quad (3.14)$$

for $\omega_2 > \omega_1 > K$ and $\omega_S < -K$.

If (3.10) is written in the form

$$C(\omega) \bar{g}_\Lambda^2 = G(\omega) + I(\omega)$$

we have

$$\bar{g}_\Lambda^2 \{C(\omega_1)(\omega_1 + \bar{\omega}) - C(\omega_2)(\omega_2 + \bar{\omega})\} \leq (\omega_1 + \bar{\omega}) G(\omega_1) - (\omega_2 + \bar{\omega}) G(\omega_2) \quad (3.15)$$

and

$$\bar{g}_\Lambda^2 \{C(\omega_1)(\omega_1 + K) - C(\omega_2)(\omega_2 + K)\} \geq (\omega_1 + K) G(\omega_1) - (\omega_2 + K) G(\omega_2) \quad (3.16)$$

It is easy to show that the coefficient of \bar{g}_Λ^2 in these inequalities is positive, so that (3.15) sets an upper bound on \bar{g}_Λ^2 and (3.16) sets a lower bound. These bounds are expressible in terms of experimental real parts and integrals over experimentally determined quantities.

For these relations to be useful, it is imperative that the pole terms give sizeable contributions to the dispersion relation, and

that $G(\omega)$ can be measured to significantly better accuracy than its change over some energy range.

Our bounds are derived by setting $\omega_S = -0.614$ BeV, $\omega_2 = 1.62$ GeV, and varying ω_1 in the range $0.55 \text{ GeV} \leq \omega_1 \leq 1.5 \text{ GeV}$. The results for the lower bound (g_{\min}^2) are shown in Fig. 3, where we have interpolated the calculated points with a smooth curve. Representative errors are shown at three energies, and we have distinguished the error arising from the real parts from that due to the integrals. The results for the upper limit (g_{\max}^2) follow a similar curve, but vary from a value of 35 to 150. These latter numbers are not useful and we do not comment on them further.

The conclusions concerning g_{\min}^2 are, on the contrary, very interesting. The most reliable estimate we can form of g_{\min}^2 is 14 ± 4 , this being the lower limit of $g_{\Lambda}^2 + 1.05 g_{\Sigma}^2$. The errors derive almost equally from those on the total cross sections and on the real parts. We have tried to be cautious and to allow larger errors than might seem necessary, and the major source of uncertainty lies in the validity of the particular phase shift parameters we use. Perhaps further effort in phase shift analyses will result in more reliable real part estimates.

The important point, however, is that the problem is an experimental one, and can one hopes be tackled in the near future.

In principle the method is equally applicable to the amplitudes of definite isospin, and limits could be found on g_{Λ}^2 and g_{Σ}^2

separately. However, this will require knowing K_p and K_n real parts over a common range of energies to a high degree of accuracy. We suspect that this will not be achieved for some time.

IV. CONCLUSIONS

We have tried to decide whether the multichannel effective range parametrization of the low energy (K^-p) scattering amplitude gives a completely self-consistent calculation of the Λ and Σ coupling constants, $g_{\Lambda K^-p}^2$ and $g_{\Sigma^0 K^-p}^2$. In Section II we show that as yet it does not, but that it may well be possible to make minor modifications to achieve this, without significantly altering the presently calculated values of those constants.

Secondly we have shown that it is possible to put bounds on the coupling constants, and that these may be calculated from a knowledge of experimental quantities only. At the present stage of experimental knowledge, this gives a useful result only for the lower bound of a combination of the Λ and Σ coupling constants. The result is consistent with the MER model, and indicates that in the future it should be possible to use the method to distinguish the $SU(3)$ -satisfying values and the $SU(3)$ -violating values.

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FOOTNOTES AND REFERENCES

- * This work was supported in part by the U.S. Atomic Energy Commission.
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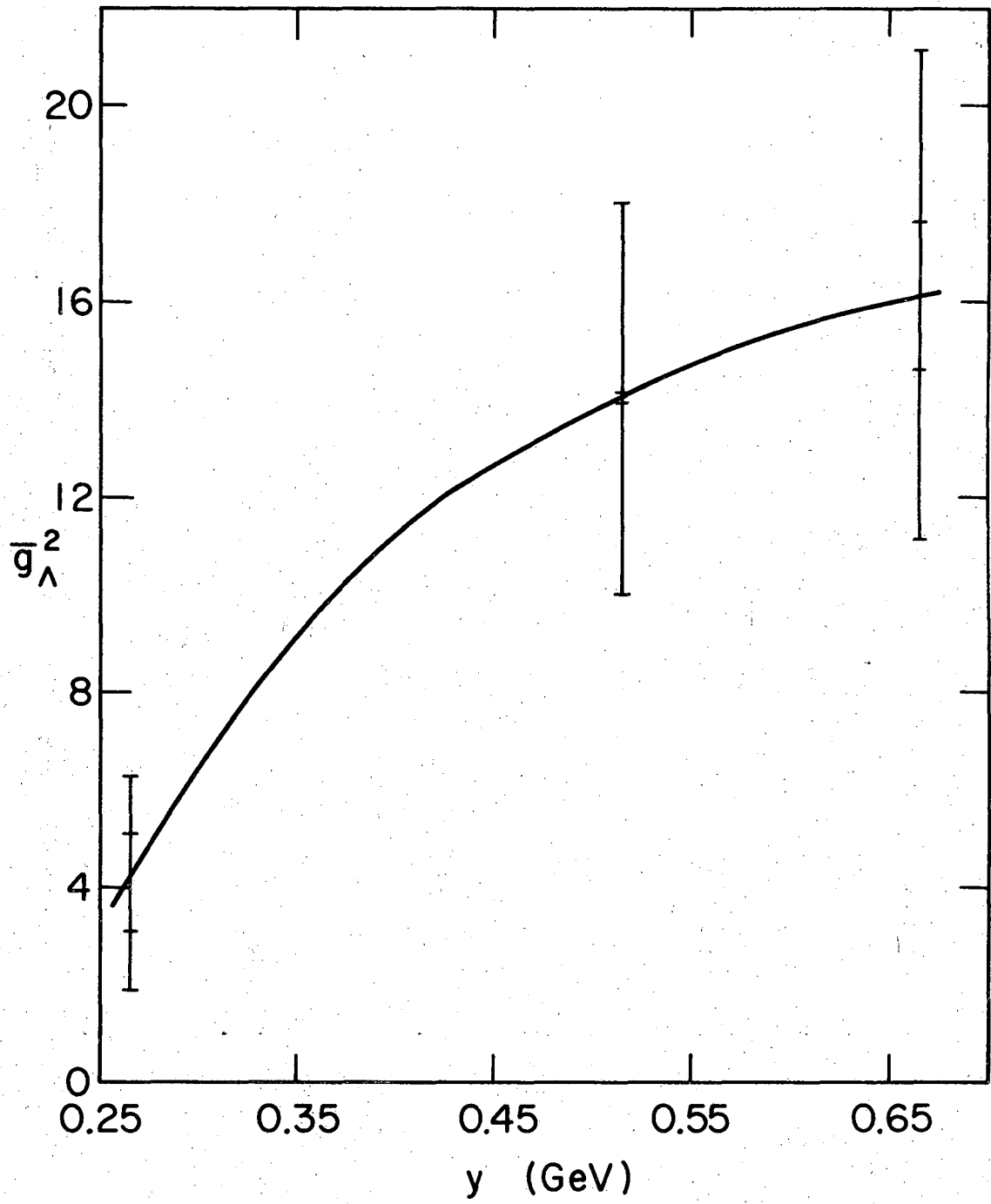
TABLE I.

The change (Δg^2) in $\frac{g^2}{\Lambda^2}$ caused by changing the value of the unphysical region contribution by 20% .

y BeV	Δg^2
0.264	2
0.514	8
0.714	3.5

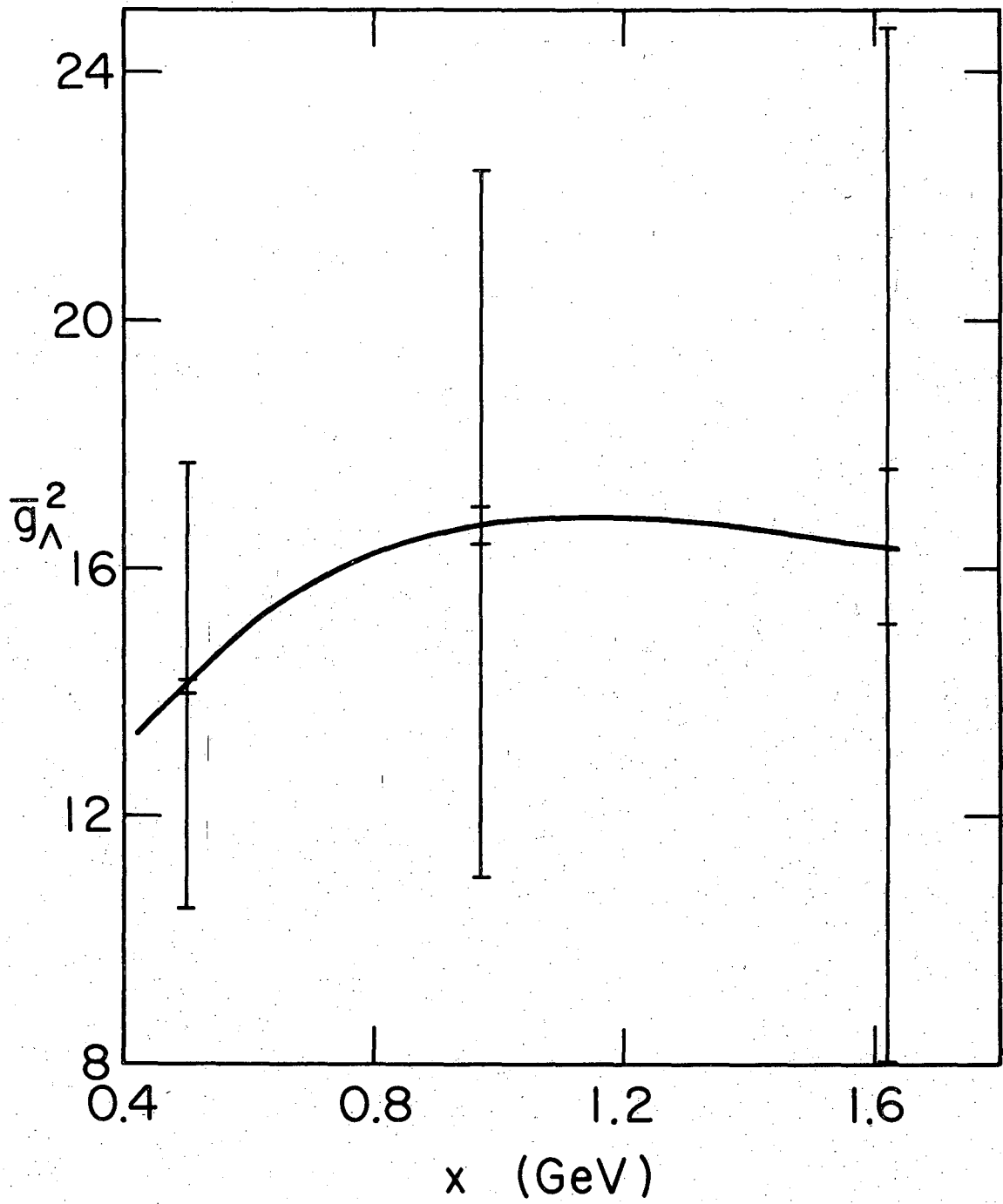
FIGURE CAPTIONS

- Fig. 1. A smooth curve through the values of \bar{g}_Λ^2 , which are calculated from the dispersion relation (2.1) at different values of the energy y . The double error bars distinguish the source of the errors as discussed under (a) and (b) of Section II. The larger error bar shows the combined error from (a) and (b); the smaller error bar that from (b) alone.
- Fig. 2. A smooth curve through the values of \bar{g}_Λ^2 , which are calculated from the dispersion relation (2.1) at different values of the energy x . The double error bars distinguish the source of the errors as discussed under (a) and (b) of Section II. The larger error bar shows the combined error from (a) and (b); the smaller error bar that from (b) alone.
- Fig. 3. A smooth curve through the values of the lower limit of \bar{g}_Λ^2 , calculated from (3.16) for various values of ω_1 . The double error bars distinguish the source of the errors as discussed under (a) and (b) of Section II. The larger error bar shows the combined error from (a) and (b); the smaller error bar that from (b) alone.



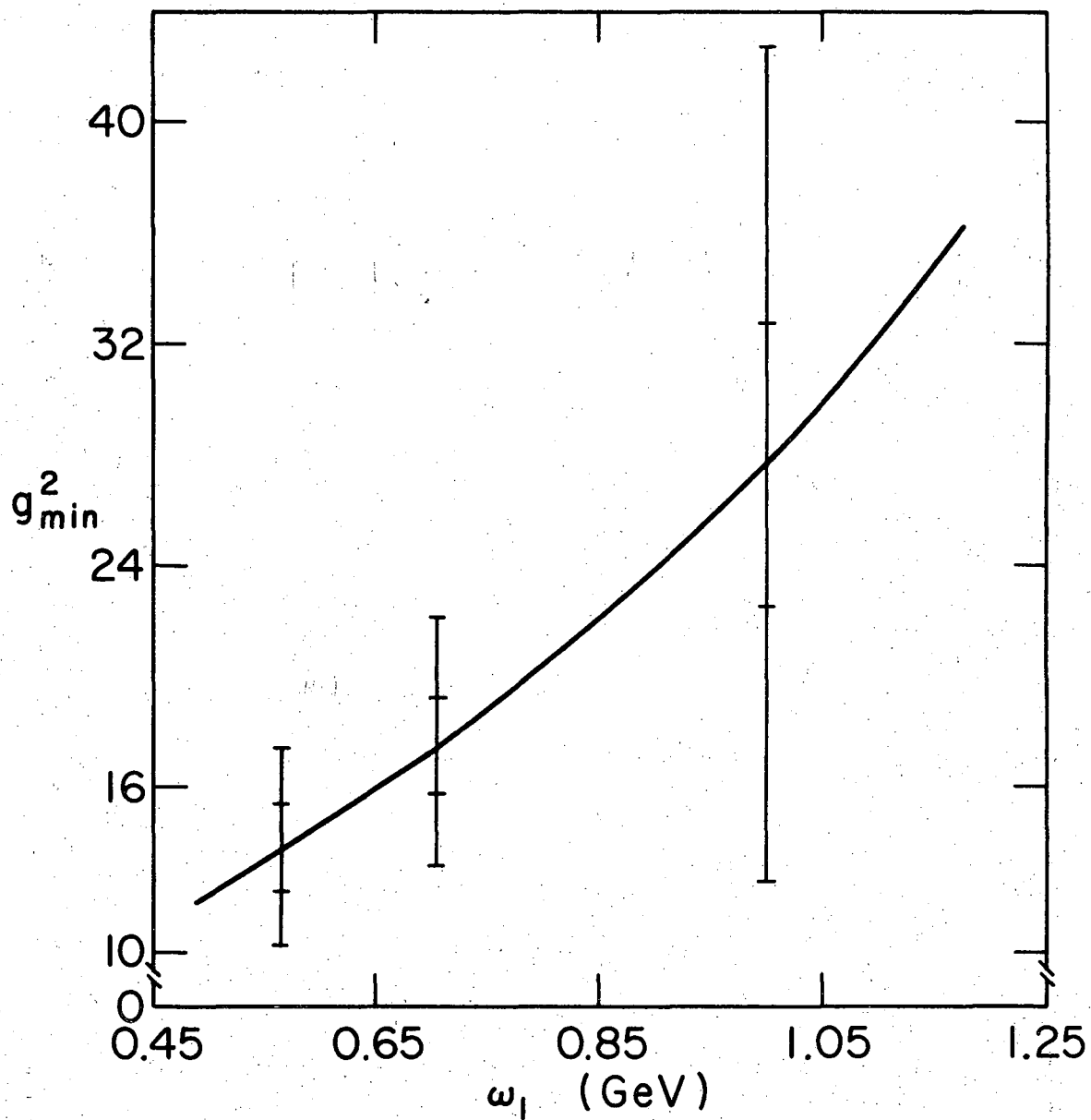
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Fig. 1



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Fig. 2



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Fig. 3

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