Title
COURSE IN THE THEORY AND DESIGN OF PARTICLE ACCELERATORS -LECTURE VI

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Course in the Theory and
Design of Particle Accelerators

LECTURE VI
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Table of Definitions and Units--See Last Page.

CYCLOTRON LAWS

The curvature of a particle in a magnetic field, discussed in Lecture V, is one of the fundamental cyclotron laws:

$$B\ e\ R = E\beta = E\frac{v}{c} \tag{1}$$

or rearranged

$$R = \frac{Ev}{Bec} \tag{2}$$

The cyclotron fundamental law, or Larmour Frequency, the expression of angular velocity, ω, can be derived as follows:

$$\omega = \frac{v}{R} \text{ by definition} \tag{3}$$

$$\frac{v}{R} = \frac{Bec}{E} = \frac{Be}{mc} \quad \text{rearrangement of (1) and subs } E = mc^2 \tag{4}$$

$$\omega = \frac{v}{R} = \frac{Bec}{E} = \frac{Be}{mc} \quad \text{substitute (4) in (3)} \tag{5}$$

The Rotation Frequency of ions circulating in the cyclotron is:

$$f = \frac{\omega}{2\pi} = \frac{Bec}{2\pi E} = \frac{Bec}{2\pi}\left[E_o + E_k\right] \tag{6}$$

From equation (2) it is evident that as the particle increases its velocity the ion path radius increases and spirals outward in ever-widening circles.

Equation (5) shows that the angular velocity is independent of the ion orbit radius; i.e., that an ion requires the same time to complete one revolution whether moving near the ion source or close to the deflector. This means that the R.F. accelerating voltage on the dee can have a constant frequency as expressed in equation (6).

BETATRON LAW

The Betatron Principle is described in Lecture III, page 6. The Basic Law may be stated as follows:

The ratio of rate of change in field at the orbit, dB/dt, to the rate of change of flux through the core, dØ/dt, must be constant to produce acceleration and maintain the ion path at a constant radius.

The Law may be derived as follows:

First: From the curvature of a particle in a magnetic field: (Referring to the "Energy Triangle"—Fig. 1)

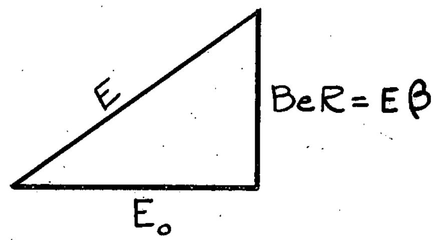


Fig 1

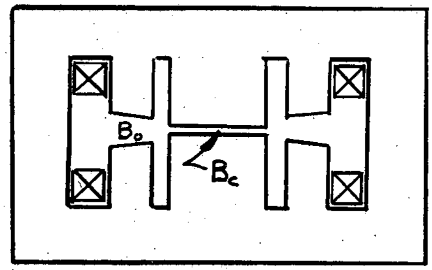


Fig 2

$$E^2 - E_o^2 = B^2e^2R^2 \tag{1}$$

Differentiating:

$$2EdE = 2B_oe^2R^2dB \tag{2}$$

$$\frac{dE}{dB_o} = \frac{B_oe^2R^2}{E} \tag{3}$$

$$f = \frac{B_oec}{2\pi E} \qquad \text{From Page 1 - (6)} \tag{4}$$

$$f = \frac{dn}{dt} \tag{5}$$

$$\frac{dt}{dn} = \frac{2\pi E}{B_oec} \tag{6}$$

$$\frac{dE}{dB_o} \cdot \frac{dt}{dn} = \frac{B_oe^2R^2}{E} \cdot \frac{2\pi E}{Bec} = \frac{2\pi eR^2}{c} \qquad \text{Combine (3)-(6)} \tag{7}$$

$$\frac{dE}{dn} = \frac{2\pi eR^2}{c} \cdot \frac{dB_o}{dt} \tag{8}$$

Equation (8) expresses the change in energy per turn, dE/dn, required to keep the particle at a constant radius when the magnetic "turning" field is changing at the rate, dB/dt.

Next, it is required to find an expression for the change in energy per turn, dE/dn, developed by the magnetic induction accelerating force, F, which may be expressed in terms of the rate of change of flux, dØ/dt, threading the ion orbit.

$$\frac{dE}{dn} = 2\pi RF = 2\pi R\,\frac{Bev'}{c} - (\text{Radiation Loss}) \tag{9}$$

Where:

v' is the radial velocity with which the flux crosses the orbit to provide the change in flux inside of the orbit, see Fig. 3.

Neglecting radiation loss:

$$d\phi = 2\pi R B v' dt \quad (10)$$

$d\phi$ may be shown analogous to a volume as shown in Fig. 4.

$$\frac{d\phi}{dt} = 2\pi R B v' \quad (11)$$

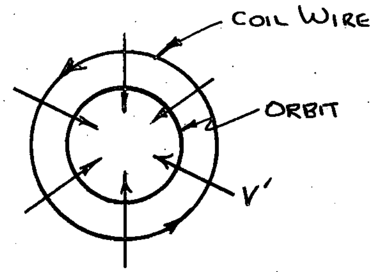


FIG 3

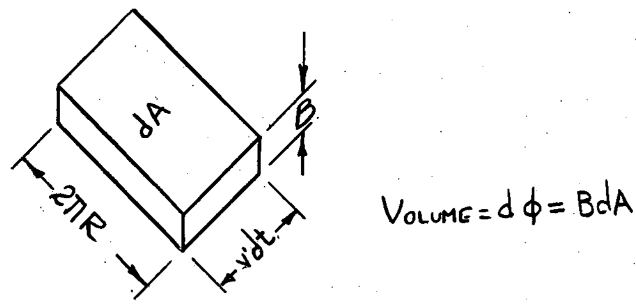


FIG 4

$$\frac{dE}{dn} = \frac{e}{c} \cdot \frac{d\phi}{dt} \quad (12)$$

Combining equation (8) and (12)

$$\frac{dE}{dn} = \frac{2\pi e R^2}{c} \cdot \frac{dB_o}{dt} = \frac{e}{c} \cdot \frac{d\phi}{dt} \quad (13)$$

$$d\phi = 2\pi R^2 \, dB_o$$

Total flux in central field, $\phi = \pi R^2 B_c$

Then:

$$d\phi = \pi R^2 dB_c = 2\pi R^2 dB_o$$

and:

$$\Delta B_c = 2 \, \Delta B_o \qquad \underline{\text{AND}} \qquad R = \frac{1}{2\pi} \sqrt{\frac{d\phi}{dB_o}}$$

A typical curve for the change of magnetic field strength with time is shown in Figure 5.

When the magnet is adjusted such that $\Delta B_c = 2 \Delta B_o$, then the radius that the particle takes is constant. If, for example, the magnet is adjusted such that the constant is greater than 2, this would mean that the central field, B_c, which is the accelerating field was changing too rapidly for the bending field, or orbit field, B_o, to maintain a constant radius and the ions would be spiral outward on an ever increasing radius until they would strike some mechanical interference such as the vacuum tube wall. Likewise, if the constant were less than 2, the ions would be bent too rapidly and they would spiral inward.

The fact that the Betatron requires a central field with a flux density twice that of the orbit field fixes the design to a magnet of the same general shape as shown in Figure 2 which is an iron central core and allows high flux densities. This shape is costly in iron as compared for example, with the magnet shape of the Synchrotron

The central flux density, B_c, is limited by the saturation of the iron which in turn limits the orbit flux density to approximately 10,000 gauss.

The $d\phi/dt$ of the Betatron magnet says that it is "AC" and must be laminated to reduce eddy current losses in the iron. This increases the cost further.

The Betatron Magnet utilization can be improved by direct current biasing of the central core. This is done by starting the accelerating cycle with the core magnetized in the opposite direction so that the core swings from Maximum negative magnetization through zero to a maximum positive magnetization as shown on Figure 5 and 6. This means that for the same magnet core, the change in orbit flux density, B_o, may be doubled by biasing; and hence, the Kinetic Energy given to the particle by the machine is doubled, as seen in the expression, $E_k = B_o eR$

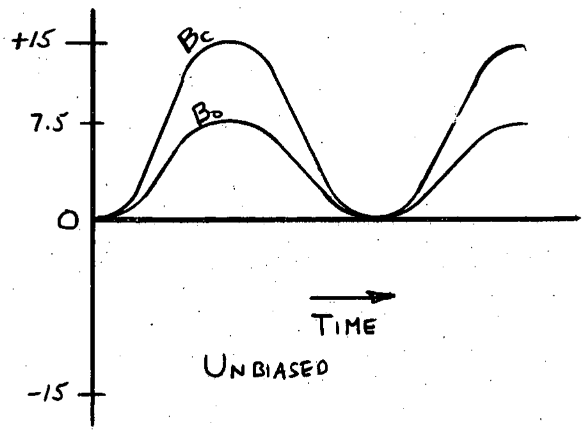


Fig 5

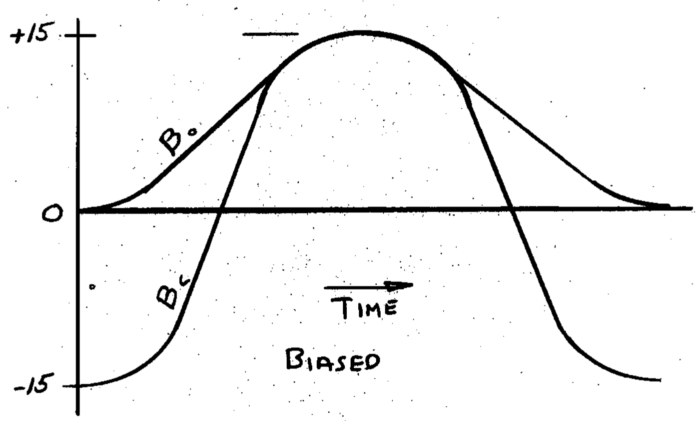


Fig 6

The ratio of central flux to orbit field is usually controlled by adjusting the reluctance of the two flux paths in parallel. However, the flux ratio can also be controlled by separate windings on the two parts of the core. In this case, the ratio can be adjusted at high energies to provide additional accelerating force to correct for radiation loss.

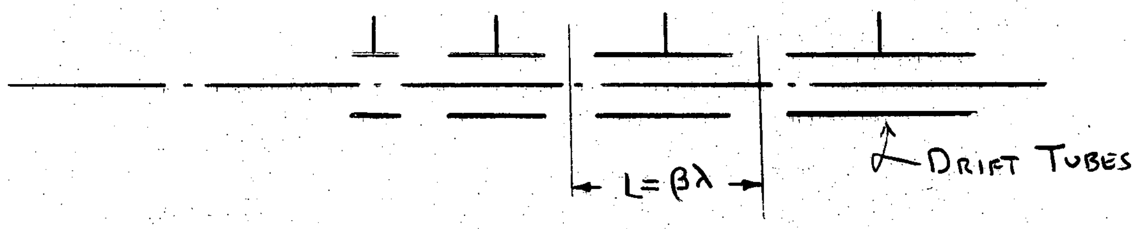
The term "radiation loss" which was neglected in the deviation is the energy radiated by particles moving in a closed orbit. This energy varies as $(E/E_o)^4(1/R)$ and becomes appreciable over a few hundred million volts. The "two to one" flux condition must then be modified to provide a more rapid rate of change of central flux at high energies. The rapid increase of the radiation loss with energy probably sets a practical energy limit for Betatrons in the region of 500 MEV.

LINEAR ACCELERATOR

DRIFT TUBE NUMBER

The Drift Tube Number is the number of cycles or Drift Tubes required to move a particle from rest to a given energy.

The particle starts in the injector and so the first Drift Tube of a machine may be, say, the theoretical number 10 Drift Tube or even an irrational number as 10.42. In this case, the second tube would be number 11.42 and so on.



$$F = d\,\frac{(mv)}{dt} \text{ or } Fdt = d(mv) \tag{1}$$

In the case in which the force F is constant along the machine:

$$Ft = mv \;\ldots\; t \text{ being the time from rest} \tag{2}$$

also

$$t = \frac{N}{f} \tag{3}$$

then

$$F\,\frac{N}{f} = mv \qquad \text{substitute 3 in 2} \tag{4}$$

$$m = m_o\,\frac{1}{\sqrt{1-\beta^2}} \tag{5}$$

$$\frac{FN}{f} = \frac{m_o v}{\sqrt{1-\beta^2}} \qquad \text{substitute 5 in 4} \tag{6}$$

$$\beta = \frac{v}{c} \quad \text{or} \quad V = \beta c \tag{7}$$

$$\frac{FN}{f} = \frac{m_o c\beta}{\sqrt{1-\beta^2}} = \frac{m_o c^2\beta}{c\sqrt{1-\beta^2}} = \frac{E_o\beta}{c\sqrt{1-\beta^2}} \tag{8}$$

$$N = \frac{E_o\beta f}{Fc\sqrt{1-\beta^2}} \tag{9}$$

$$dE = Fdl \text{ or } F = \frac{dE}{dl}, \text{ the Energy Gradient} \tag{10}$$

$$f = \frac{c}{\lambda} \tag{11}$$

$$N = \frac{E_o\beta}{\frac{dE}{dl}\,\lambda\sqrt{1-\beta^2}}$$

At low Energies

$B \ll 1$

Then

$$N = \frac{E_o\beta}{\frac{dE}{dl}\,\lambda}$$

The energy gradient on the UCRL Linear Accelerator is:

$$\frac{dE}{dL} = \frac{32-4}{40} = .7 \text{ MEV/ft. Length}$$

Where:

32 MEV = Final Energy
4 MEV = Injector Energy

and the wave length of the exciting frequency is:

$$\lambda = \frac{c}{f} = \frac{3 \times 10^{10}}{202 \times 10^{6}} = 150 \text{ cm.}$$

This analysis is strictly correct only if the energy gradient is constant along the length of the machine. This is the condition for the machines at Berkeley and Minnesota, however, it is not necessarily true for any machine. If a machine is designed with a varying energy gradient along its length F must be expressed mathematically or graphically as a function of length and $\int F dt$ properly evaluated.

TABLE OF DEFINITIONS AND UNITS

	DEFINITION	UNITS
F	Force	Electron ESU Volts/cm
B	Flux Density	Gauss
B_o	Orbit Flux Density	Gauss
B_c	Betatron Central Flux Density	Gauss
E	Total Energy	Electron ESU Volts
E_o	Rest Energy	Electron ESU Volts
E_k	Kinetic Energy	Electron ESU Volts
R	Orbit Radius	Cm
N	Drift Tube Number	
n	Turns	
m_o	Rest Mass	$\frac{\text{Electron ESU Volt sec}^2}{\text{cm}^2}$
m	Total Mass	$\frac{\text{Electron ESU Volt sec}^2}{\text{cm}^2}$
a	Acceleration	cm/sec^2
v	velocity	cm/sec
t	time	seconds
f	frequency	cycles/second
l	length	cm
β	$\frac{v}{c}$	dimensionless
ω	$\frac{v}{R}$	Radians/sec
λ	Wave Length	cm
ϕ	Total Flux	Maxwells = gauss cm^2 or lines

Rest Energy of Electrons - 0.5108 MEV Million Electron Practical Volts.
Rest Energy of Proton (Hydrogen Nucleus) 938 MEV
Rest Energy of Deuteron (Deuterium Nucleus) 1874 MEV
Rest Energy of Alpha Particle (Helium Nucleus) 3729 MEV

300 Practical Volts = 1 ESU Volts