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Dynamics of a Wavy Axisymmetric Swimmer

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Abstract

A novel underwater locomotion method is proposed. In this method, the propulsion system is designed as an empty tube with two open ends. The side wall of this tube is deformable so that the diameter of the tube varies both spatially and temporally following a wavy pattern that travels from head to tail. By using a fluid dynamics model based on the axisymmetric Navier-Stokes equations, we numerically demonstrate that this design is able to generate thrust for self-propelled swimming. The sources of the thrust have been identified. Specifically, the physical mechanisms that assist with the swimming performance are: 1) the momentum flux through the front and back openings, and 2) the normal stress at the front and back openings, most notably the suction force at the front opening due to negative pressure there. Moreover, a parametric study has been conducted to investigate the effect of geometric and kinematic parameters upon the performance of the system.

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Nomenclature

L	=	body length
D	=	mean body diameter
A_f, A_b	=	front opening or back opening
m_b	=	body mass
F_b	=	thrust on body
x_b	=	displacement of the center of mass in the swimming direction
u_b	=	$\dot{x}_b$ swimming speed
u_0	=	speed of an incoming flow introduced to keep the swimmer within the domain
P	=	power expenditure
CoT	=	cost of transport
L_w	=	wavelength on body
a	=	amplitude of body wave
T	=	period of body wave
c	=	L_w/T phase speed of body wave
μ	=	dynamic viscosity of fluid
ρ	=	density of fluid
p	=	pressure
p_∞	=	background pressure
σ	=	normal stress
σ_ν	=	viscous normal stress
R_e	=	Reynolds number based on ρ , μ , L , and c
$\mathbf{u}$	=	(u_x, u_r) flow velocity
$\hat{u}_x$	=	spatially averaged value of u_x at an opening
$\mathbf{f}$	=	(f_x, f_r) force exerted on the fluid by the immersed structure
$\mathbf{F}$	=	(F_x, F_r) force exerted on the structure by the fluid
F_{out}	=	hydrodynamic force on the outer side of the shell in the swimming direction
F_{in}	=	hydrodynamic force on the inner side of the shell in the swimming direction
F_{jf}, F_{jb}	=	thrust due to momentum flux through the front opening or back opening
$F_{\sigma f}, F_{\sigma b}$	=	thrust due to normal stress at the front opening or back opening
F_m	=	thrust due to momentum change in the chamber
Ω	=	fluid domain
Ω_I	=	fluid domain inside the chamber
Γ	=	immersed structure
s	=	Lagrangian coordinate along Γ
$\mathbf{V}, \mathbf{U}$	=	structural velocity and fluid velocity at the fluid-structure interface

1 Introduction

Soft-bodied robots are gaining popularity due to their adaptability in complicated conditions and mitigated impact upon the environment. In the aquatic world, the deformable bodies of these robots have two opposite effects on their swimming capability. On one hand, these robots are able to perform multiple-degree-of-freedom deformations that are far beyond the capacity of rigid-body robots, allowing more freedom in the optimization of locomotion. On the other hand, the structural flexibility severely limits the amount of hydrodynamic load they can sustain and subsequently their ability for high-speed swimming is compromised.

Marine invertebrates, which lack supporting structures, provide inspirations for the locomotion method of soft-bodied robots. Among these creatures, cephalopods such as squid are known for ultra-fast burst swimming powered by jet propulsion [1,2]. This locomotion mode is achieved via repeated refilling and discharging of the mantle cavity inside the body - as the body expands the volume of the cavity increases so that water is sucked in through the mantle aperture, whereas when the body shrinks water is expelled through the funnel tube to form a jet [1].

A handful of prototypes have been developed imitating the locomotion mechanism of squid [3]. With few exceptions [4], these designs feature a single opening on the body which works as both the inlet and the outlet. This leads to significant energy waste since the kinetic energy of the refilling flow cannot be utilized. The issue can be partially solved with the following measures:

1. By slowing down the refilling procedure, the speed and subsequently the kinetic energy of the refilling flow are reduced so that the energy waste is also mitigated

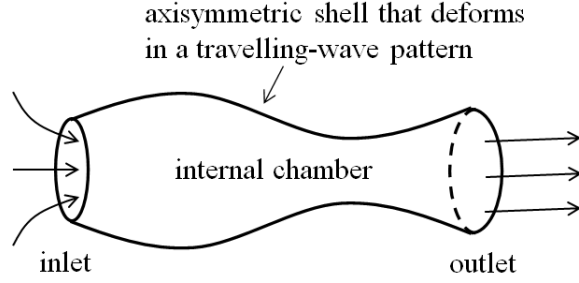


Figure 1: Three-dimension depiction of the axisymmetric swimmer.

- [5]. This method, however, may reduce the swimming speed.
2. Using an adjustable nozzle which expands during refilling and shrinks during jetting, the kinetic energy of the refilling flow is reduced for less energy waste [6].
 3. Using a valve-controlled auxiliary inlet that opens during refilling and closes during jetting, the overall area of the inlet is increased so that the kinetic energy of the ingoing flow is reduced.
 4. By putting the inlet and the outlet at different ends of the body, part of the refilling flow can be used to strengthen the jet [7, 8].

There are still issues in these designs. For instance, most of them involve adjustable valves or nozzles to control the sizes of the inlet/outlet or the directions of the refilling/discharging flows. This significantly increases the mechanical complexity and the potential of malfunction. Moreover, the refilling-discharging cycles in this method lead to variations in the magnitude, and in many cases the direction, of the flow in and out of the internal chamber. A direct consequence is that there are usually considerable oscillations in the forward speed of such a swimmer [5, 6, 8].

To overcome these issues, we envisage the development of a bio-inspired propulsion

method that uses the same physical principle of jet propulsion yet the design does not come directly from any existing creatures (Fig. 1). To avoid the energy waste and unsteadiness in thrust associated with the reversal of internal flow direction, the flow inlet (or the front opening) is positioned at the front end and the flow outlet (or the back opening) is at the back end, resembling the of a salp (a marine invertebrate) [9]. Therefore the basic design features an empty tube with two open ends and a thin shell. Unlike the salp, the refilling and discharging actions are not driven by inflation and deflation of the body with valves for flow control. Instead, in our proposed system a special body deformation is employed, in which the diameter of the cylindrical shell along the longitudinal axis varies in both space and time following a pattern of a travelling wave that propagates from the head towards the tail. This type of body deformation induces axial flow in the internal chamber and create thrust. Additional thrust can be generated at the outer surface of the shell by the travelling wave that propagates downstream, similar to the thrust generated by a wavy wall [10].

To certain extent, the system we propose also resembles undulating swimmers, especially anguilliform swimmers, which propel themselves through travelling waves along their bodies [11–13]. One notable difference is that an anguilliform swimmer has finite size in the dorsal-ventral direction so that there are three-dimensional effects that might affect swimming performance, e.g., induced drag due to tip vortices. The axisymmetric system, on the other hand, does not have these 'tips'. Moreover, the axisymmetric configuration guarantees no force generation in the lateral directions. The lateral forces produced by fish-like swimmers not only lead to energy waste but also compromise controllability.

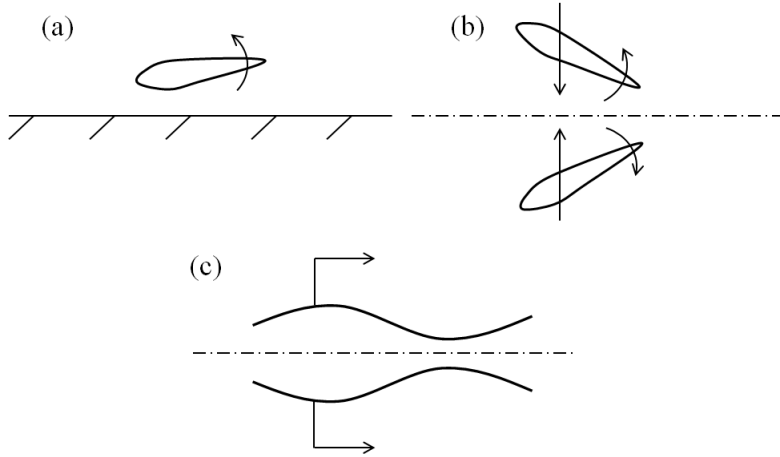


Figure 2: Scenarios of thrust generation with ground effect: (a) a flapping foil near ground [14], (b) a pair of flapping foils with opposite phases in a side-by-side formation [15], and (c) an axisymmetric body whose shell deforms in a propagating-wave pattern viewed in a cross section that goes through the axis of geometry (the dash-dotted line).

Another potential advantage of this design is that it might be able to use ground effect to enhance performance. As shown in Fig. 2a, when a foil undergoes periodic pitch motion near a wall, the thrust it generates increases monotonically as the distance between the wall and the foil decreases [14]. A similar effect was observed in the problem with two flapping foils conducting opposite-phase oscillations in a side-by-side formation [15] (see Fig. 2b). There is similarity between the cross-sectional view of our system (Fig. 2c) to these scenarios. It is thus reasonable to expect that the thrust generation capacity of our system may benefit from the hydrodynamic interaction among different parts of the shell. The effect will be more pronounced when the diameter of the body is small.

In the current study we use a numerical model based on the immersed-boundary framework to simulate the dynamics of the system described in Fig. 1. The purpose is to prove its feasibility and illustrate underlying physics. In this model the body

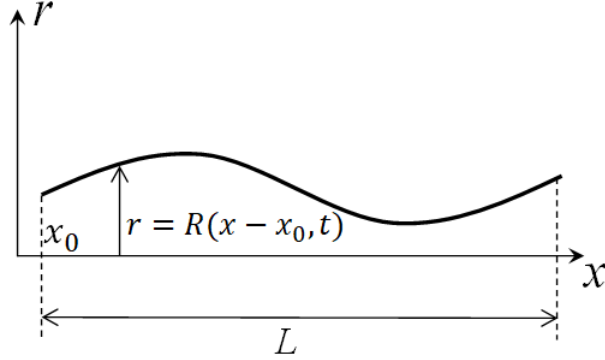


Figure 3: Cross-sectional view of the problem to be solved.

deformation, i.e., the propagating wave along the shell, is prescribed. The hydrodynamic force on the body is obtained by solving the Navier-Stokes equations. The forward motion is then calculated by invoking Newton's equation so that the self-propelled swimming of the system is simulated. We concentrate on the swimming speed that can be achieved this way, the energy efficiency, and the steadiness of the forward speed.

The rest of the paper is organized in the following manner. We start by depicting the geometry and (prescribed) deformation of the swimmer. This is followed by a brief description of the mathematical formulations and the numerical approach. In the results section we present the dynamics of the system, the near-body flow field, and the underlying physics. The last section is the conclusion.

2 Problem description

Under the axisymmetric assumption, the physical problem we aim at solving is displayed in Fig. 3. In this figure a cylindrical coordinate system $\mathbf{x} \equiv (x, r)$ is defined,

where x is the longitudinal axis and r the radial axis. The curve $r = R(x - x_0, t)$ represents the location of the zero-thickness shell (x_0 is the x -location of the front opening). The body deformation follows a sinusoidal wave propagating in the x direction as

$$r = R(x - x_0, t) = D/2 + a \sin [2\pi (x - x_0) / L_w - 2\pi t / T], \quad (1)$$

where D is the mean diameter of the tube, L_w is the wave length, and T is the period. The phase speed of the propagating wave is $c = L_w / T$. The Reynolds number based on the body length L and the phase speed c is defined as $Re = \rho L c / \mu$, where ρ and μ are the fluid density and dynamic viscosity, respectively.

In this design there is no need of mobile valves for flow control [7, 8]. However, it requires accurate control of the body deformation so that advanced actuation technology is necessary [16–18].

3 Methodology

3.1 Governing equations

By assuming that the flow is incompressible and axisymmetric, we invoke the axisymmetric Navier-Stokes equations and the continuity equation as the governing equations so that

$$\begin{aligned}
\rho \left(\frac{\partial u_x}{\partial t} + \mathbf{u} \cdot \nabla u_x \right) + \frac{\partial p}{\partial x} &= \mu \left[\frac{\partial^2 u_x}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_x}{\partial r} \right) \right] - f_x = 0, \\
\rho \left(\frac{\partial u_r}{\partial t} + \mathbf{u} \cdot \nabla u_r \right) + \frac{\partial p}{\partial r} &= \mu \left[\frac{\partial^2 u_r}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) - \frac{u_r}{r^2} \right] - f_r = 0, \\
\frac{\partial u_x}{\partial x} + \frac{1}{r} \frac{\partial(r u_r)}{\partial r} &= 0.
\end{aligned} \tag{2}$$

In these equations u_x and u_r are the x - and r -components of the velocity vector $\mathbf{u}$. p is the pressure. $\mathbf{f} \equiv (f_x, f_r)$ is the force exerted on the fluid by the immersed structure. When the structure exerts force $\mathbf{f}$ on the surrounding fluid, there is also a fluidic force $\mathbf{F} \equiv (F_x, F_r)$ on the structure itself. By using the immersed-boundary approach, these two forces are related through a Dirac Delta function δ . We have

$$\mathbf{f}(\mathbf{x}, t) = \int_{\Gamma} \mathbf{F}(s, t) \delta(\mathbf{X}(s, t) - \mathbf{x}) ds, \tag{3}$$

where Γ is the solid structure, i.e., the shell of the swimming body. s is a Lagrangian coordinate on Γ .

To obtain $\mathbf{F}$, we use a penalty method based on two numerical parameters, α and β . We have

$$\mathbf{F}(s, t) = \alpha \int_0^t [\mathbf{U}(s, \tau) - \mathbf{V}(s, \tau)] d\tau + \beta [\mathbf{U}(s, t) - \mathbf{V}(s, t)], \tag{4}$$

where $\mathbf{V}$ is the structural velocity, and $\mathbf{U}$ is the fluid velocity at the fluid-structure interface obtained as

$$\mathbf{U}(s, t) = \int_{\Omega} \mathbf{u}(\mathbf{x}, t) \delta(\mathbf{x} - \mathbf{X}(s, t)) d\mathbf{x}, \tag{5}$$

where Ω is the fluid domain.

Essentially, this method keeps neighboring fluidic and structural particles together to satisfy the no-slip and the no-flux boundary conditions at the fluid-structure interface. In the model these particles are virtually connected through a spring whose stiffness and damping property are represented by α and β . If the values of these parameters are properly chosen (in practice they are determined through numerical tests), they do not affect the results.

Once $\mathbf{F}$ is determined, its longitudinal component F_x is integrated over the body to get the thrust F_b . We have

$$F_b = - \int_{\Gamma} 2\pi r(s) F_x(s, t) ds. \quad (6)$$

The positive direction of F_b is the $-x$ direction, which is the swimming direction. The forward motion of the body is then obtained using Newton's equation as

$$m_b \frac{d^2 x_b}{dt^2} = F_b, \quad (7)$$

where m_b is the mass of the swimming body. x_b is the displacement of the center of mass in the $-x$ direction. For convenience we assume that the location of the center of mass is not affected by the body deformation. In real systems the effect of body deformation on the location of the center of mass depends on the detailed mechanical design. For example, if the shell consists of annular elements distributed in the longitudinal direction and these elements expand or shrink in the radial direction to create the wavy motion, the center of mass will not be affected. Other factors, e.g., the design of the actuation system, may also cause variations in the location of the

center of mass.

The performance of the system will be characterized by the following parameters: 1) the time-averaged forward speed $\bar{u}_b$; 2) the time-averaged power expenditure $\bar{P}$, where $P = \int_{\Gamma} 2\pi r \mathbf{F} \cdot \mathbf{U} ds$; 3) the cost of transport CoT, defined as the energy expenditure divided by the swimming distance; 4) the fluctuation in the forward speed δu_b , defined as $\delta u_b = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} (u_b - \bar{u}_b)^2 dt}$. All of these values are calculated within a cycle (t_0 to $t_0 + T$) after the steady state is established - in our simulations $t_0 = 5T$. For simplicity in all the plots of the time histories and snapshots of the flow field we shift the starting time from $t_0 = 5T$ to $t_0 = 0$.

3.2 Numerical approach

We use the finite-difference method to solve the set of equations in §3.1. It combines a second-order method for spatial discretization and a Crank-Nicholson method which is also second order in accuracy for time integration. The details of this method can be found in the literature [19]. The validity and accuracy of this method have been tested and corroborated in our previous studies [20, 21].

4 Physical insight through a control volume analysis

For insight into the underlying physical mechanisms of thrust generation, we conduct a control volume analysis to identify various force components and their contributions.

Firstly, the hydrodynamic force on the body can be separated into two parts, the force on the outer surface of the shell F_{out} and the one on the inner surface of the

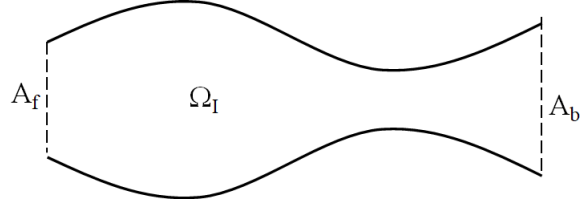


Figure 4: Control volume analysis of the internal flow field.

shell F_{in} . The outer flow problem is similar to the scenario of a wavy plate in a semi-infinite space, in which the dynamics depends on the parameter c/U (c is the phase speed of the wave and U is the incoming flow speed or the swimming speed as in our case) [10]. When the travelling wave propagates in the same direction of the incoming flow and $c/U > 1$, flow separation on the plate surface is suppressed. As c/U increases, the drag force decreases until it eventually switches to thrust. We expect a similar behavior in our case, although in the current immersed-boundary method F_{out} cannot be directly obtained.

The behavior of F_{in} , on the other hand, is expected to be more complicated, mostly due to the effect of the confined space as illustrated in Fig. 2. We invoke a control volume method to analyze the various mechanisms that contribute to this force. Towards this end, we define a control volume Ω_I enclosed within the shell, the front opening A_f , and the back opening A_b (see Fig. 4).

Following the same steps in [22], a control volume analysis of volume Ω_I with

moving boundaries leads to the following equation for F_{in}

$$\begin{aligned}
F_{in} &= \rho \frac{\partial}{\partial t} \iiint_{\Omega_I} u_x dV - \rho \iint_{A_f} u_x (u_x + u_b) dS \\
&\quad + \rho \iint_{A_b} u_x (u_x + u_b) dS - \iint_{A_f} (-\sigma) dS + \iint_{A_b} (-\sigma) dS \\
&= F_m + F_{jf} + F_{jb} + F_{\sigma f} + F_{\sigma b},
\end{aligned} \tag{8}$$

where $u_b = dx_b/dt$ is the forward speed of the body. $F_m = \rho \frac{\partial}{\partial t} \iiint_{\Omega_I} u_x dV$ refers to the force on the wall due to the momentum change of the fluid inside the chamber. $F_{jf} = -\rho \iint_{A_f} u_x (u_x + u_b) dS$ is thrust attributed to the momentum flux through the front opening A_f , and $F_{jb} = \rho \iint_{A_b} u_x (u_x + u_b) dS$ is the one due to the momentum flux through the back opening A_b . $F_{\sigma f} = -\iint_{A_f} (-\sigma) dS$ is the contribution from the normal stress σ at A_f ($\sigma = -p + p_\infty + \sigma_\nu$, where p is the pressure, p_∞ is the background pressure, and σ_ν the viscous normal stress.). $F_{\sigma b} = \iint_{A_b} (-\sigma) dS$ is the contribution from the normal stress σ at A_b .

With periodic body deformations, the time-averaged contribution of F_m is found to be negligibly small in comparison with the other components of F_{in} [5].

5 Results

Following sensitivity tests conducted in previous studies [20, 22], we choose the size of the computational domain to be $5L \times L$ (see Fig. 5). In practice, to prevent the swimmer from moving out of the computational domain a uniform incoming flow u_0 is introduced, whose speed is determined through numerical tests. The effect of this incoming flow is removed in post processing so that all the results presented are

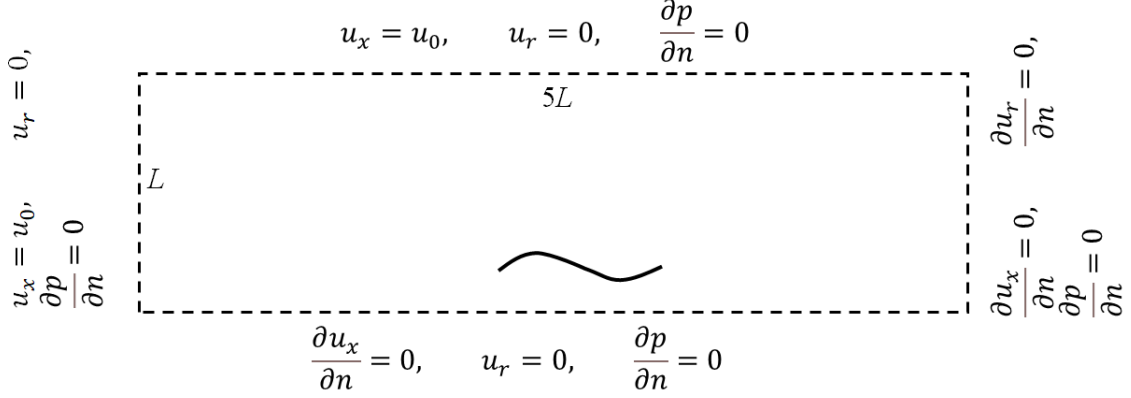


Figure 5: Computational domain and boundary conditions (n is the out-pointing normal direction at the boundary).

measured in a space-fixed coordinate system without any background flow.

The problem will all be presented in normalized forms. We choose the fluid density ρ , the wave period T , and the body length L as repeating variables. Thus, in the following results the lengths are normalized by L , the speeds by LT^{-1} , the time by T , the masses by ρL^3 , the forces by $\rho L^4 T^{-2}$, the power by $\rho L^5 T^{-3}$, and the vorticity by T^{-1} . For convenience, we do not change the symbols of the variables defined earlier, although hereafter they are all dimensionless.

For the simulations, the body mass m_b is chosen to be 0.05 and the fluid viscosity μ is 0.005. Unless otherwise specified, the wave amplitude a is also fixed as $a = 0.05$.

To further prove the validity and accuracy of our computational approach, numerical tests have been conducted to demonstrate the sensitivity of the results to numerical parameters. Towards this end we study three different meshes, a coarse mesh with $\Delta x = \Delta r = 0.006$ and $N_s = 225$, a medium one with $\Delta x = \Delta r = 0.0045$ and $N_s = 300$, and a fine one with $\Delta x = \Delta r = 0.003$ and $N_s = 450$. Here Δx and Δr are the sizes of the fluidic grids in the x and r directions, respectively. N_s is

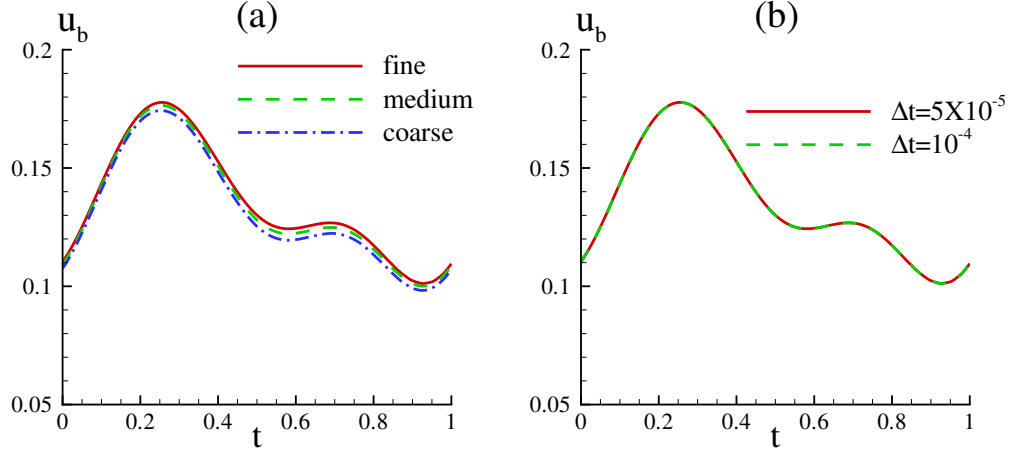


Figure 6: (a) Time histories of u_b predicted with different meshes and $\Delta t = 10^{-4}$. (b) Time histories of u_b predicted with the fine mesh and different values of Δt . $L_w = 1$, $D = 0.2$.

the number of grids on the structure. Two values of time step, $\Delta t = 5 \times 10^{-5}$ and $\Delta t = 10^{-4}$, are also tested. The sensitivity tests are conducted with a case in which $L_w = 1$ and $D = 0.4$. As shown in Fig. 6a, the time histories of u_b obtained with the three different meshes are close to each other. The mean forward speeds $\bar{u}_b$ predicted using these three meshes are 0.132 (coarse mesh), 0.134 (medium mesh), and 0.135 (fine mesh). Figure 6b suggests that the result is not sensitive to the choice of Δt as well. Indeed, the curves obtained with the two different values of Δt are graphically indistinguishable, and in both cases $\bar{u}_b = 0.135$. Moreover, to test the effect of the size of the computational domain, we have used two additional domains (10×2 and 15×3), the influence on the results is negligibly small. Based on these tests, hereafter we choose the fine mesh and $\Delta t = 10^{-4}$ with a computation domain shown in Fig. 5.

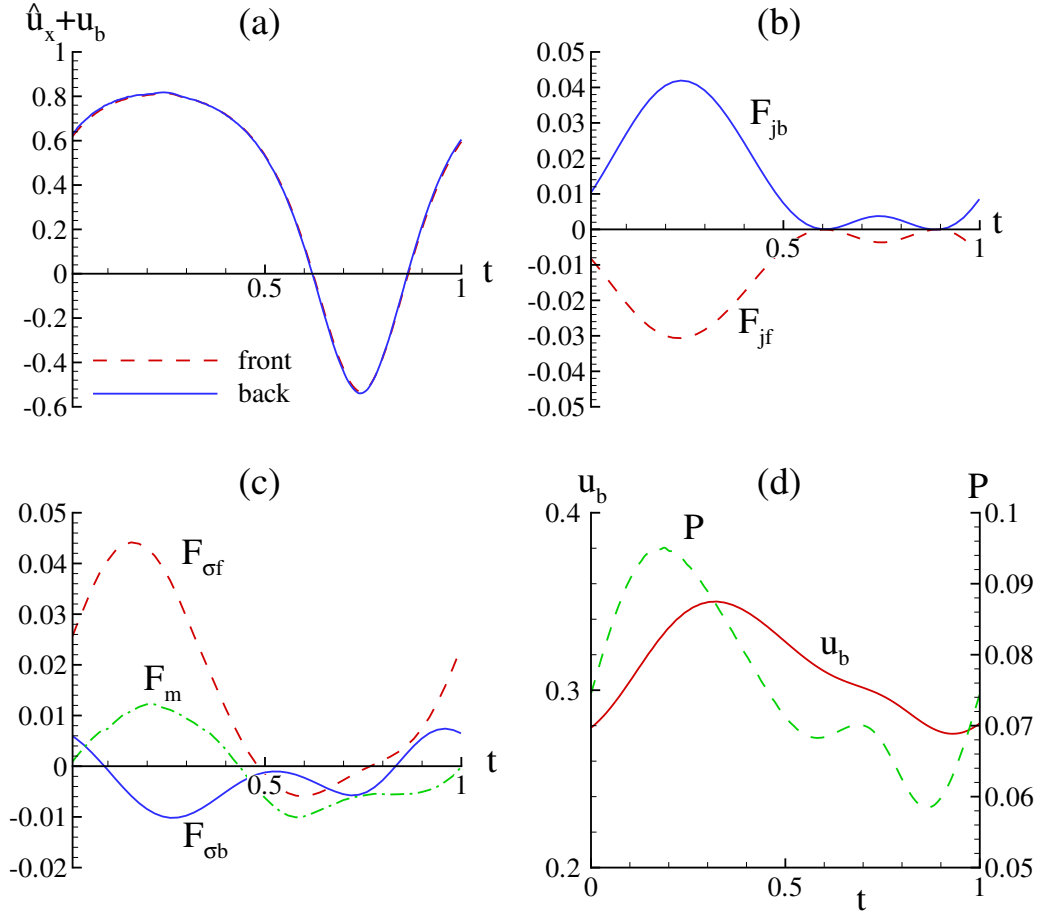


Figure 7: Time histories of (a) the relative speed between the spatially averaged flow speed $\hat{u}_x$ and the body at the front and the back openings, (b) thrust components due to momentum flux at the front and the back openings, (c) thrust components due to normal stress at the front and the back openings and the one due to acceleration of fluid inside the chamber, and (d) swimming speed u_b and power expenditure P . $L_w = 1$, $D = 0.2$.

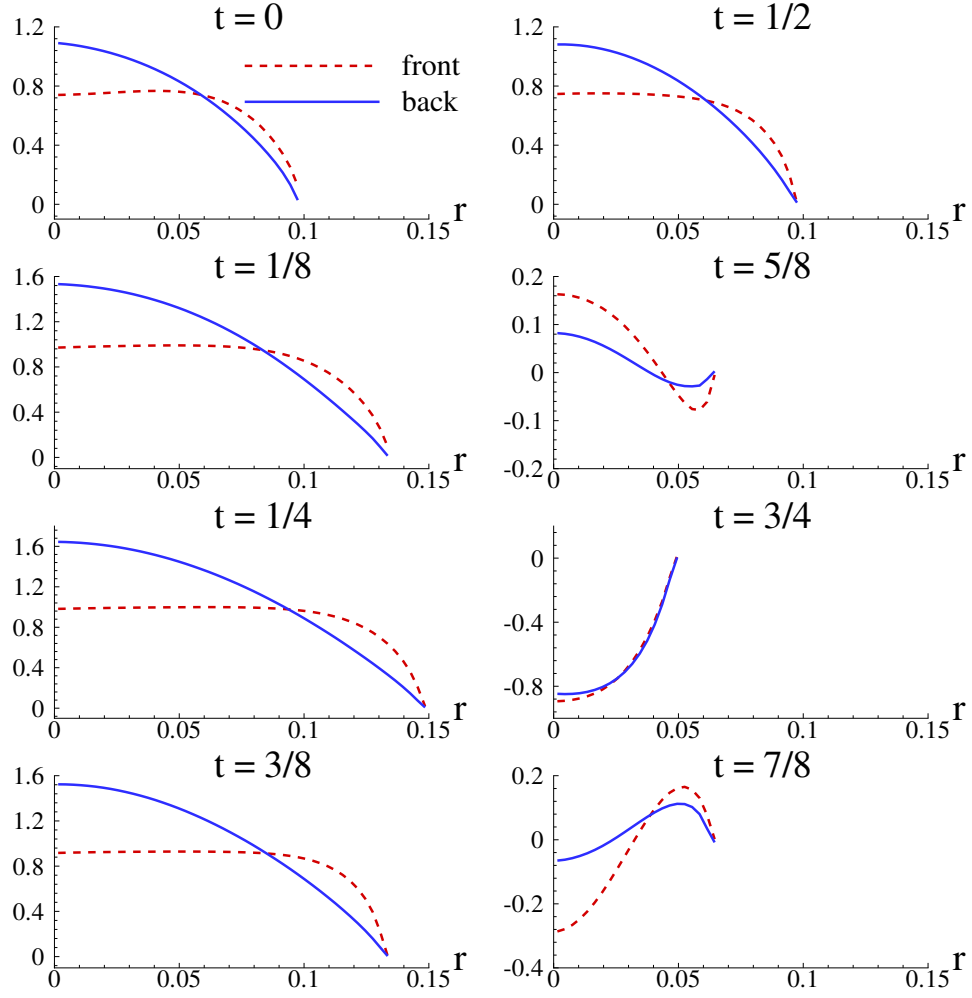


Figure 8: Snapshots of the distribution of the relative flow speed $u_x + u_b$ at the front and back openings. $L_w = 1$, $D = 0.2$.

5.1 Characteristics of the system dynamics

To study the basic characteristics of the system, we consider two typical cases, one with $L_w = 1$ and the other with $L_w = 2$. These two cases represent two scenarios: in the first scenario $L/L_w = n$ (n is a positive integer) so that there are n wave lengths along the shell, the volume of the internal chamber remains constant, and the sizes of the front and back openings are the same. In the second scenario the volume of the internal chamber varies with time and the sizes of the front and back openings can be different. These two cases have the same mean body diameter ($D = 0.2$). Based on the phase speed of the wave and the body length, the Reynolds numbers of these two cases are 200 (for $L_w = 1$) and 400 (for $L_w = 2$).

In Fig. 7a we plot the time histories of the relative flow speed $\hat{u}_x + u_b$ at the openings of the internal chamber in the case with $L_w = 1$. Here $\hat{u}_x$ is the spatially averaged flow speed at the front opening or the back one. It is seen that in this case the flow at the two openings is mostly uni-directional, i.e., entering the chamber from the front opening and leaving through the back opening. The reversal of flow direction occurs when t is between 0.62 and 0.86, roughly 24% of a cycle.

The detailed distributions of axial flow speed at the front and back openings in a cycle are shown in Fig. 8. It is clear that at the front end the speed is more evenly distributed in the r direction until it is very close to the shell. The flows are mostly in the $+x$ direction except for the instants when $t = 3/4$ and $7/8$. When $t = 5/8$, the flow speed close to the shell is also in the negative x direction. This is consistent with the time histories of $\hat{u}_x + u_b$ shown in Fig. 7a.

Figure 9 displays snapshots of the flow field visualized through the vorticity contour in the current case. There is almost no vorticity above the outer surface of the

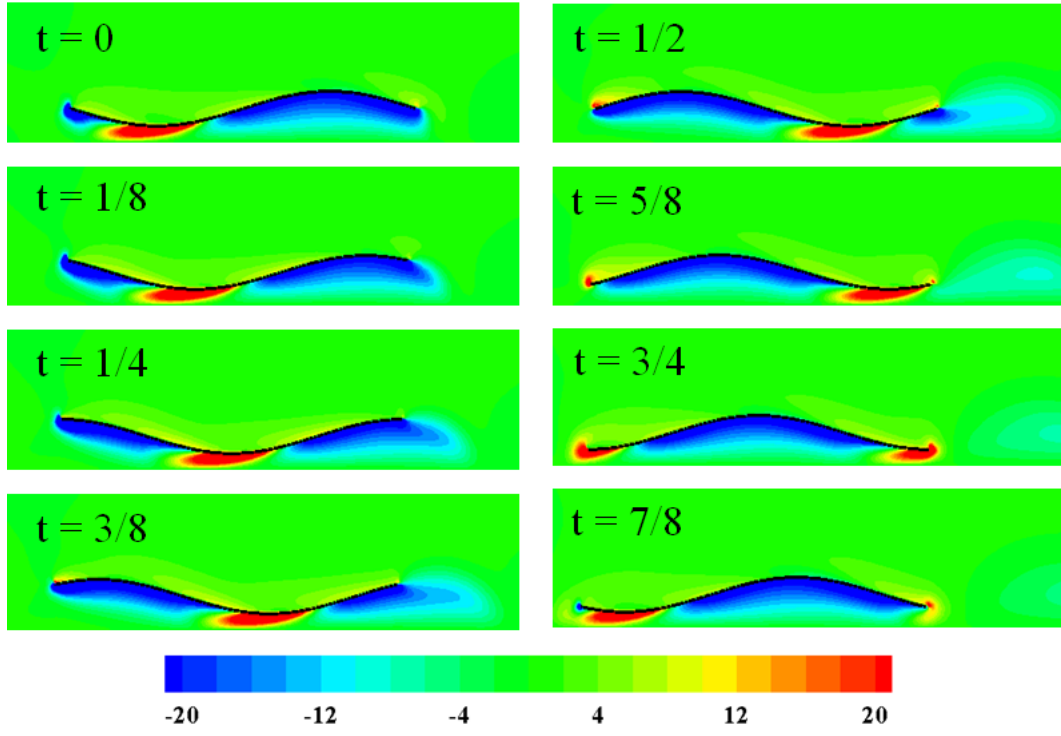


Figure 9: Snapshots of the vorticity (defined as $\partial u_x / \partial r - \partial u_r / \partial x$) contour in a swimming cycle. $L_w = 1$, $D = 0.2$.

shell as the separation there is suppressed by the wavy motion (the ratio between the phase speed of the travelling wave and the mean forward of the body in this case is above 2.76). However, inside the chamber significant vorticity behavior is observed. Concentrations of negative vorticity, which usually correspond to counter-clockwise vortices, are present in the vicinity of the peaks of the traveling wave. Near the troughs of the wave, there are concentrations of positive vorticity, usually corresponding to clockwise vortices. These regions of concentrated vorticity travel from the head to the tail following the propagating body wave, and eventually shed into the wake at the back opening. When negative vorticity occupies the openings, the flow velocity there relative to the body is predominantly in the $+x$ direction. On the other hand, when the vorticity at these locations is positive, the relative flow velocity is in the $-x$ direction.

A more illustrative way to show the flow field is by using the streamlines, as plotted in Fig. 10. This figure demonstrates that among the areas with concentrated vorticity shown in Fig. 9, only the ones that occur near the troughs, i.e., the concentration of positive vorticity, correspond to fluid in rotational motion. When these areas pass through the front or back openings, e.g., at $t = 3/4$ and $7/8$, the flow direction there is reversed. On the other hand, areas of negative vorticity does not affect the direction of the flow. This is consistent with the characteristics shown in Fig. 8.

As mentioned earlier, in this particular case the volume of the internal chamber remains unchanged. Moreover, the size of the front opening equals that of the backing opening. Due to conservation of mass the spatially averaged flow speed $\hat{u}_x$ at the front opening should always be the same as that at the back opening, which is confirmed in Fig. 7a. However, there is still significant difference between the distributions of

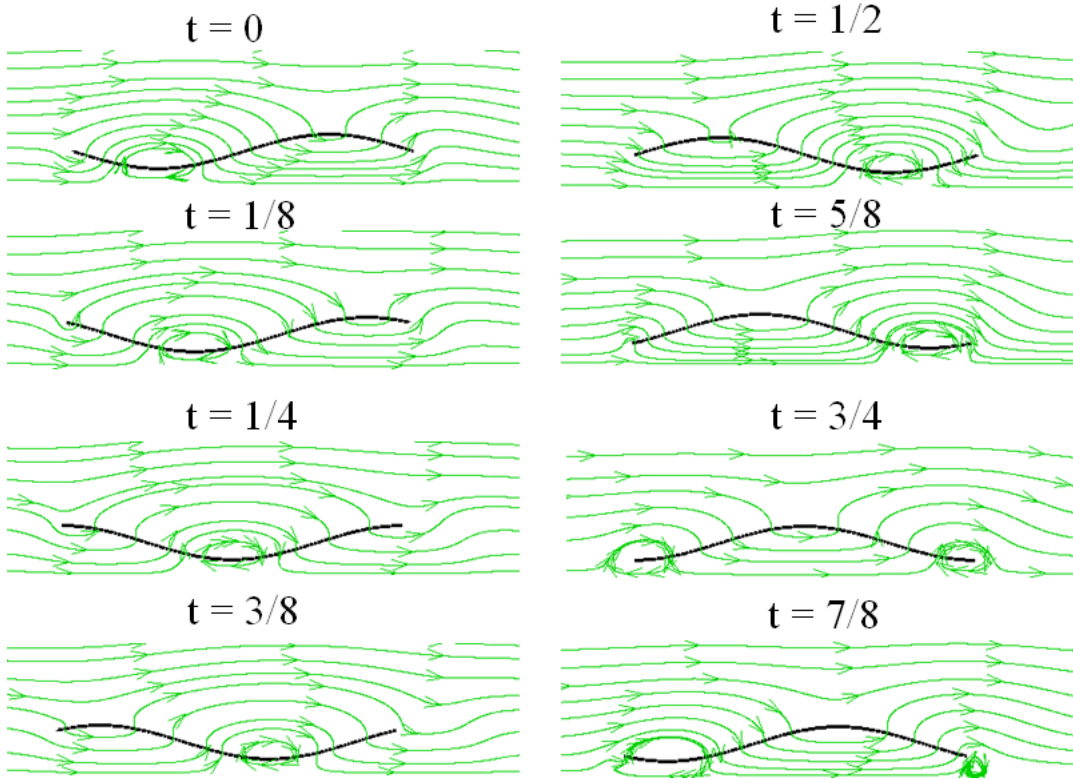


Figure 10: Snapshots of the streamlines in a reference system that travels leftwards with $\bar{u}_b$ during a swimming cycle. $L_w = 1$, $D = 0.2$.

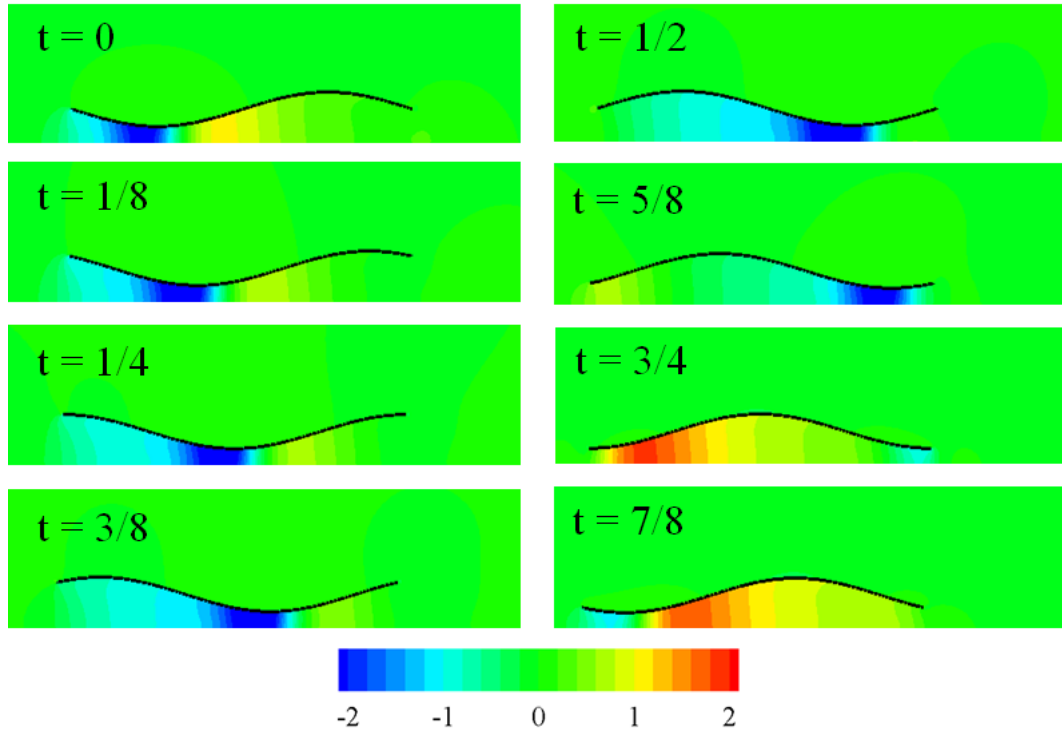


Figure 11: Snapshots of the relative pressure distribution $(p - p_\infty)$ in a swimming cycle. $L_w = 1$, $D = 0.2$.

u_x at the front and back openings (see Fig. 8). This difference leads to different (absolute) values of the thrust components F_{jf} and F_{jb} , as shown in Fig. 7b.

According to Fig. 7b, the value of F_{jb} remains positive even though the flow at the back opening is not uni-directional. This is explained by the fact that F_{jb} represents the momentum change in the internal chamber due to momentum flux in or out of the back opening. When there is flow out of the opening, it takes away negative momentum (with respect to the swimming direction). When there is flow in, it brings in positive momentum. Therefore, in terms of the momentum change inside the chamber these two processes have similar effect. For the same reason, F_{jf} remains negative.

Since the absolute value of F_{jb} is usually larger than that of F_{jf} , the overall contribution of momentum flux to thrust is positive. The time-averaged value of $F_{jf} + F_{jb}$ is 3.79×10^{-3} .

Another important source of thrust, the one that is related to the normal stress at the front and back openings, is shown in Fig. 7c. The most pronounced effect is the positive contribution from $F_{\sigma f}$ during the first half of the cycle. As the dominant part in normal stress is the relative pressure $p - p_\infty$, in Fig. 11 we plot the distribution of this quantity at several instants within a cycle. It is seen that during the first half of the cycle there is indeed negative pressure near the front opening. This negative pressure, together with the fact that the front opening reaches its maximum size during the first half of the cycle, explains the large suction force that is a major contributor to F_{in} . The time-averaged value of $F_{\sigma f} + F_{\sigma b}$ is 1.23×10^{-2} , which is much higher than the contribution from the momentum flux in this case. In Fig. 7c we also plot the time history of F_m . Its instantaneous value is not that small

in comparison with the other components. However, the time-averaged value of F_m over a complete period is only -8.9×10^{-5} .

Figure 7d shows the time histories of the forward speed u_b and the power expenditure P . In this case the mean forward speed $\bar{u}_b$ is 0.312. Its fluctuation δu_b normalized by $\bar{u}_b$ is 0.079. The cost of transport CoT is 0.244.

Now we consider the case with a longer wave length of $L_w = 2$. Similar to what has been done in the previous case, we plot the time histories of $\hat{u}_x + u_b$ at the openings of the internal chamber in Fig. 12a. Since in this case the volume of the chamber is not constant and the diameters of the front and back openings can be different, these two speeds are different from each other. Another difference between Fig. 12a and Fig. 7a is that at both the front and the back openings the speed $\hat{u}_x + u_b$ can be negative (i.e., in the $-x$ direction) within a more significant portion of the cycle (35% of the cycle at the front opening and 38% at the back opening). This could lead to higher unsteadiness in thrust generation and thus more significant oscillation in the swimming speed u_b .

Figure 13 includes snapshots of the flow field visualized through vorticity contour. Similar to the previous case, we see concentrations of negative vorticity near the peak of the travelling wave and positive vorticity near the trough. When there is a concentration of negative vorticity at an opening, the relative flow motion there is in the $+x$ direction. A concentration of positive vorticity corresponds to relative flow motion in the $-x$ direction. This is consistent with the finding in the previous case when $L_w = 1$.

Similar to the previous case, it is discovered that the streamlines provide better illustration of the actual flow field, especially areas of rotating flow (Fig. 14). Specif-

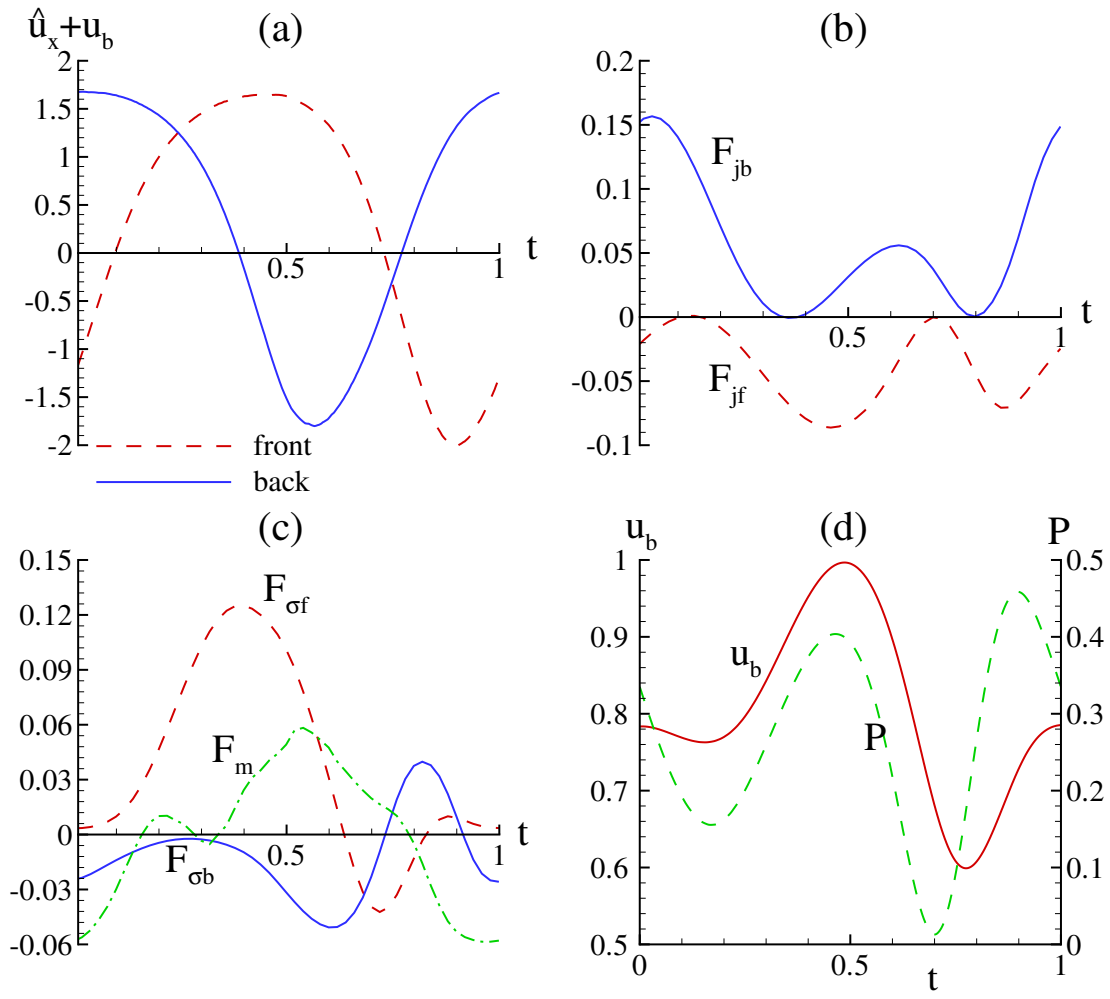


Figure 12: Same as Fig. 7 except that $L_w = 2$.

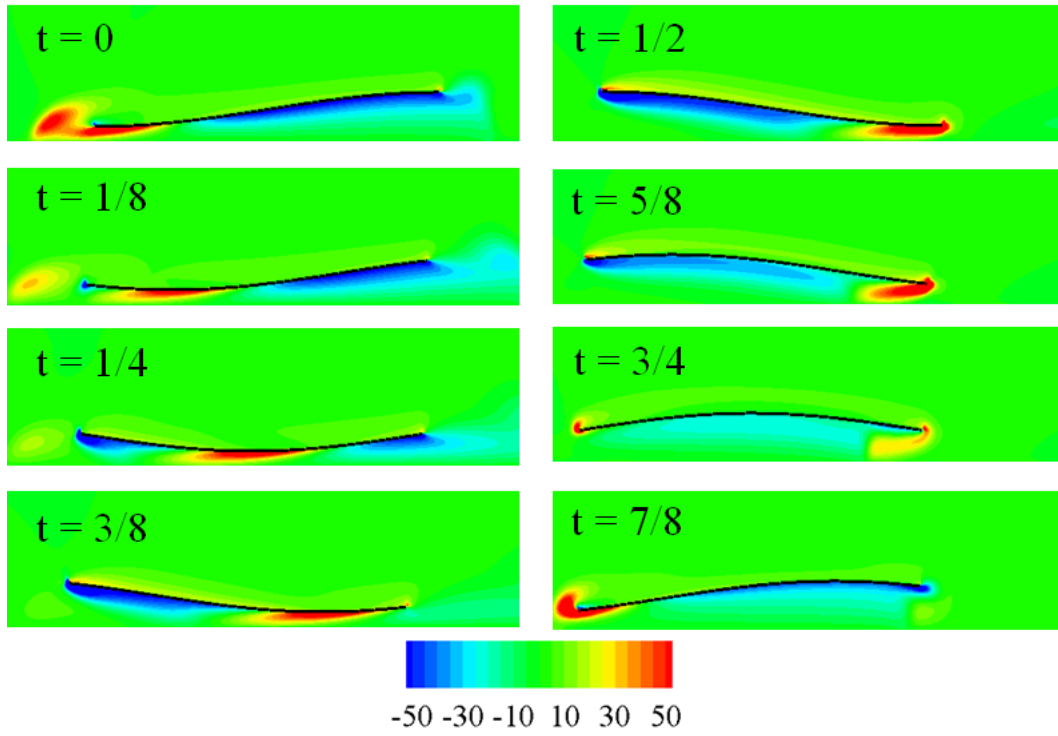


Figure 13: Snapshots of the vorticity $(\partial u_x / \partial r - \partial u_r / \partial x)$ contour in a swimming cycle. $L_w = 2$, $D = 0.2$.

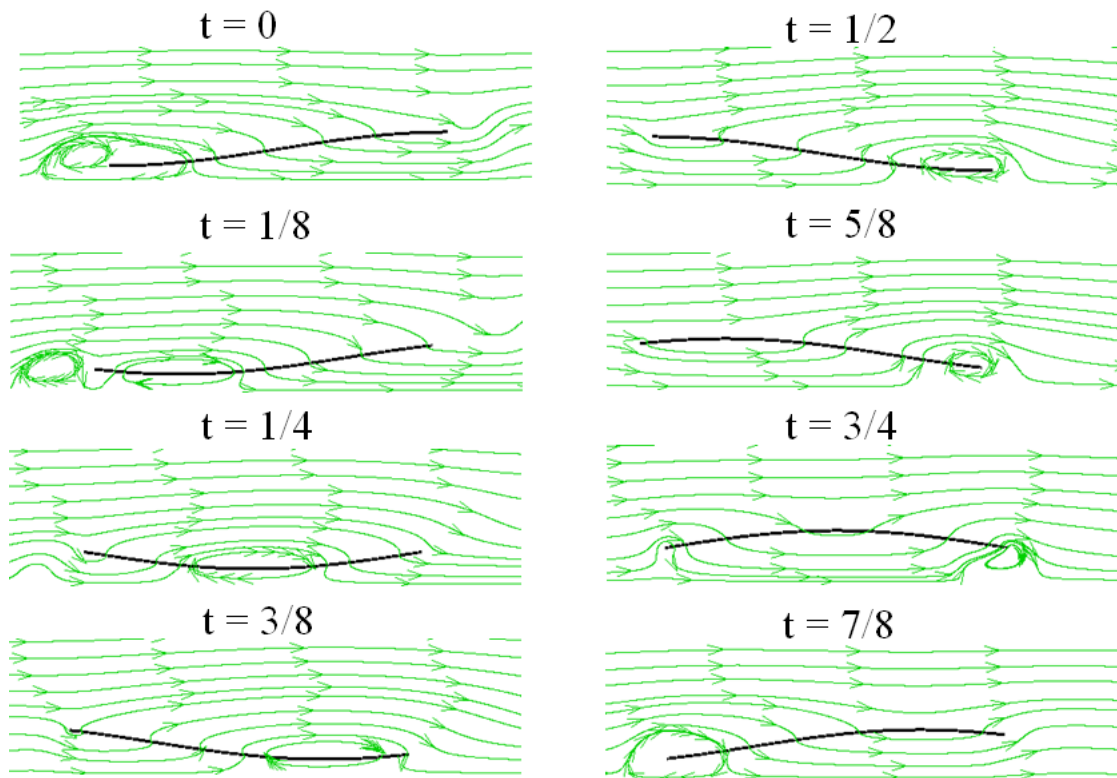


Figure 14: Snapshots of the streamlines in a reference system that travels leftwards with $\bar{u}_b$ during a swimming cycle. $L_w = 2$, $D = 0.2$.

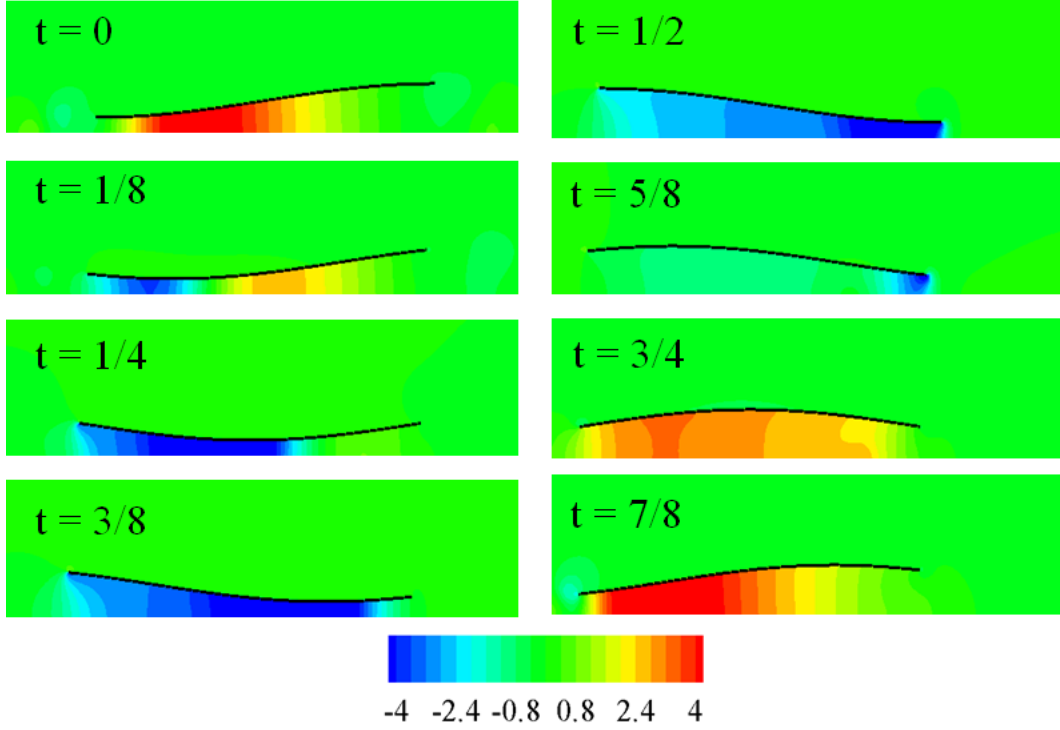


Figure 15: Snapshots of the relative pressure distribution ($p - p_\infty$) in a swimming cycle. $L_w = 2$, $D = 0.2$.

ically, only concentrations of positive vorticity can cause rotational flow and thus reversal of flow direction when they pass through the front or back openings.

The time histories of F_{jf} and F_{jb} are demonstrated in Fig. 12b. Within the cycle F_{jf} remains negative, while F_{jb} is positive. This is again consistent with the results in the previous case. As to the contributions from the normal stress at the front and back openings (Fig. 12c), the front edge suction force $F_{\sigma f}$ still plays a pivotal role in thrust generation. This can be visually illustrated in Fig. 15, where the distribution of $p - p_\infty$ (the dominant component in the normal stress σ) is displayed. Negative pressure area occurs near the front opening when t is between $1/8$ and $1/2$. This coincides with the rise of the diameter of the front opening. It explains the large

positive force $F_{\sigma f}$ in the same time interval.

Overall, in this case when $L_w = 2$ the time-averaged value of $F_{jf} + F_{jb}$ is 1.66×10^{-2} , and the one for $F_{\sigma f} + F_{\sigma b}$ is 2.61×10^{-2} . The time-averaged value of F_m is only -4.2×10^{-4} , even though it still contributes to the instantaneous thrust generation (Fig. 12c). Therefore, the momentum flux plays a more important role in thrust generation in comparison with the previous case when $L_w = 1$.

Cases with other values of L_w have also been examined. The characteristics are summarized below:

1. Inside the chamber, there are areas of negative vorticity near peaks and positive vorticity near troughs.
2. Concentrations of positive vorticity corresponds to counter-clockwise vortices. When these vortices pass through an opening, they could induce reversal of flow direction with the spatially averaged flow speed relative to the body in the $-x$ direction.
3. $F_{jf} \leq 0$, while $F_{jb} \geq 0$. Since the absolute value of F_{jb} is usually larger than that of F_{jf} , the overall contribution from momentum flux to the thrust is positive.
4. Inside the chamber, there is negative pressure in regions where $\partial R / \partial x < 0$ and positive pressure in regions where $\partial R / \partial x > 0$.
5. An important contributor to the thrust is the suction force due to negative pressure near the front opening.

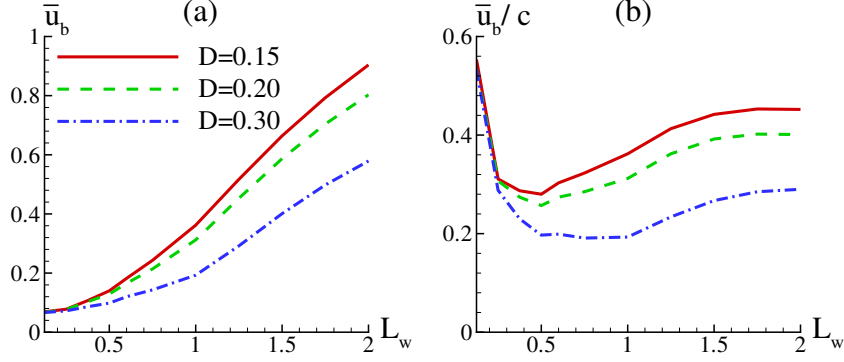


Figure 16: Dependencies of (a) the mean swimming speed $\bar{u}_b$ and the ratio between the mean swimming speed and the wave speed $\bar{u}_b/c$ upon L_w and D .

5.2 Parametric effects

In this part we consider the effect of parameters, hereby L_w and D , upon the dynamics of the system. The range of L_w , the wave length on the body, is between 0.125 and 2.0. Correspondingly, the Reynolds number Re in the cases we study is between 25 and 400. Three values of D , 0.15, 0.2, and 0.3, are considered.

Figure 16 shows the mean swimming speed $\bar{u}_b$ and the ratio between it and the phase speed of the travelling wave c as functions of L_w at these three different values of D . It is seen that $\bar{u}_b$ increases monotonically with rising L_w . However, since c is proportional to L_w , the ratio $\bar{u}_b/c$ reaches minima when L_w is between 0.5 and 1.

In cases with small L_w (e.g., $L_w = 0.125$), the diameter D has very little effect on $\bar{u}_b$. As L_w increases, the effect of D becomes more pronounced. The trend is that $\bar{u}_b$ increases in cases with smaller D . We note that as D decreases, the internal chamber shrinks and the boundary effect becomes more significant. Therefore, our finding is reminiscent of the reported increase in thrust generation of a flapping foil when the wall effect is increased [14, 15].

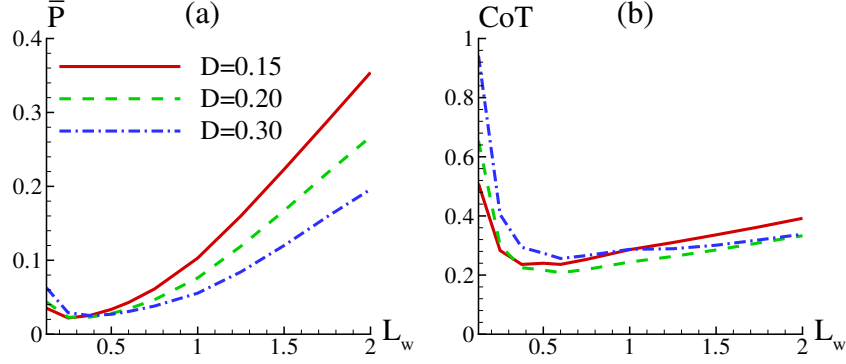


Figure 17: Dependencies of (a) the mean power expenditure $\bar{P}$ and (b) the cost of transport CoT upon L_w and D .

Following the increase of L_w , the mean power expenditure $\bar{P}$ first decreases. After $L_w > \sim 0.02$, however, it rises monotonically with L_w . The dependance of $\bar{P}$ upon D is quite complicated. When $L_w < 0.02$, larger D leads to larger $\bar{P}$. On the other hand, when L_w is above that range, the tendency is reversed, i.e., larger D leads to smaller $\bar{P}$.

The cost of transport, which with our normalization is the ratio between $\bar{P}$ and $\bar{u}_b$, is shown in Fig. 17b. In terms of this parameter, the optimal performance occurs when $D = 0.2$ and $L_w \sim 0.6$. The corresponding value of CoT is 0.21.

In Fig. 18 we plot the fluctuation in the forward speed δu_b normalized by the mean forward speed $\bar{u}_b$ at different conditions. Despite some oscillations, the general tendency is that this parameter increases with L_w and D . Within the whole range of parameters, it remains relatively small (below 0.18) so that the unsteadiness of the forward speed is relatively minor in comparison with other pulsed jetting methods (e.g., the salp-inspired systems [8]), especially if L_w is sufficiently small.

Finally, we examine the effect of the wave amplitude a on the locomotion perfor-

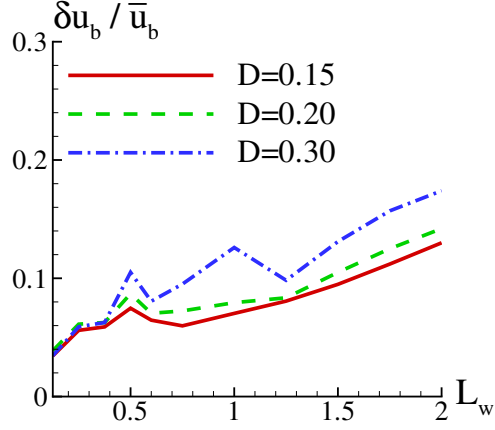


Figure 18: Dependence of the fluctuation in forward speed δu_b upon L_w and D .

mance. For this purpose we focus on a case in which $L_w = 0.5$ and $D = 0.2$, where relatively small CoT has been found. In Fig. ??? we plot $\bar{u}_b$ and CoT as functions of a . It is seen that the speed $\bar{u}_b$ increases (almost) linearly with a , although in terms of CoT the best performance is achieved when a is around 0.03. For optimization of the design, more comprehensive studies are necessary to document the performance of the system within the multi-dimensional parametric space.

6 Conclusion

By using a numerical model based on the immersed-boundary method, we have examined the feasibility and performance of a novel jet-propulsion system including an empty and open tube whose shell undergoes deformations that follow a travelling-wave pattern. In comparison with other undulating or jet-propulsion mechanisms, the advantages of the proposed design are: 1) no lateral force generation, 2) reduced oscillations in the swimming speed, and 3) mechanical simplicity since no valves are

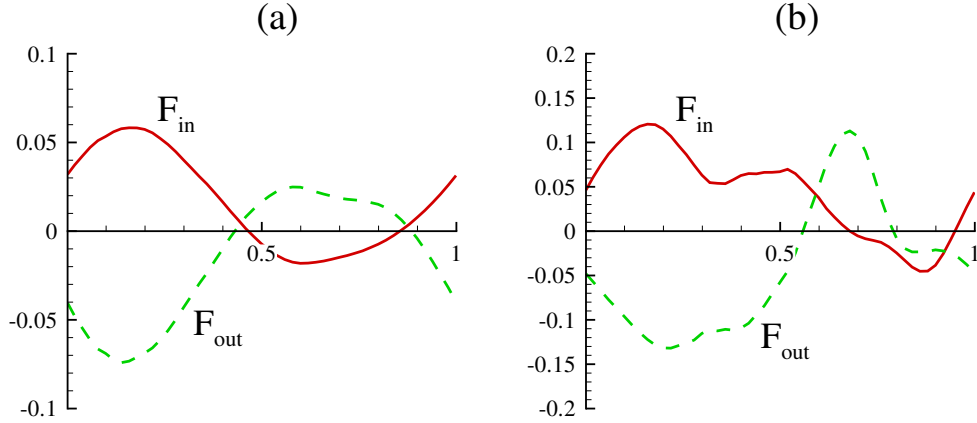


Figure 19: Time histories of F_{in} and F_{out} in steady swimming when (a) $L_w = 1$ and (b) $L_w = 2$. $D = 0.2$.

needed.

In our model the body deformation is prescribed. The forward motion, on the other hand, is obtained by solving the dynamic equations including the axisymmetric Navier-Stokes equations and Newton's equation. The numerical results confirm that this method is capable of thrust generation for self-propelled swimming. A detailed analysis illustrates the following mechanisms that helps its functioning:

1. The momentum fluxes through the front and back openings create a positive thrust.
2. The normal stress at the front and back openings also contributes to thrust generation. Most notably, the negative pressure near the front opening creates a suction force, which is a major contributor to the thrust.

In addition, the traveling wave suppresses separation so that it is expected to reduce drag on the outer surface of the shell. Although the method we use does not explicitly predict the force F_{out} on the outer side of the shell, its value can be

indirectly calculated as $F_{out} = F_b - F_{in}$, where F_{in} is the thrust force on the internal surface of the shell whose value is $F_{jf} + F_{jb} + F_{\sigma f} + F_{\sigma b} + F_m$ (by definition, F_b , F_{in} and F_{out} are positive if they point at the $-x$ direction). In Fig. 19 we plot time histories of F_{in} and F_{out} over a cycle in the two representative cases in §5.1. It is seen that F_{in} is mostly positive while F_{out} is mostly negative so that $\overline{F}_b = 0$. Indeed, in all the cases we study, the time-averaged value $\overline{F}_{in}$ remains positive based on the control volume analysis. Thus the conclusion is that the inner side of the shell generates mostly thrust yet the outside generates mostly drag. This is evidence that the confined-flow effect in the chamber plays a pivotal role in the swimming performance of the system.

With the period of the traveling wave, the body length, and the physical properties of the fluid (density and viscosity) fixed, the performance of the system depends on the diameter of the tube and the wave length of the traveling wave. The general trend is that smaller body diameter and longer wave lead to faster swimming. In terms of the energy efficiency, hereby represented by the cost of transport, the best performance is achieved when the traveling wave length is around $0.5L$ in the cases considered hereby. However, other factors, e.g., the detailed pattern of the wave (e.g., non-sinusoidal waves), may further enhance the performance. Comprehensive parametric studies are needed for optimization.

In this study the maximum Reynolds number, which is defined by using the phase speed of the body wave as the characteristic speed, is 400. On the other hand, using the mean forward speed of the body $\bar{u}_b$ as the characteristic speed, the Reynolds numbers in all the cases we study are below 180 so that the flow field remains axisymmetric. However, at higher Reynolds numbers symmetry-breaking instability

is expected to occur in the wake, which may cause non-zero lateral force on the body. To understand this effect, three-dimensional (rather than axisymmetric) simulations are necessary.

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