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FRACTAL CHARACTERISTICS OF FRACTURE ROUGHNESS AND APERTURE DATA

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ABSTRACT

In this study mathematical expressions are developed for the characteristics of apertures between rough surfaces. It shown that the correlation between the opposite surfaces influences the aperture properties and different models are presented for these different surface correlations. Fracture and apertures profiles measured from intact fractures are evaluated and it is found that they qualitatively follow the mathematically predicted trends.

INTRODUCTION

Flow of fluids through fractures in rocks is a significant transport mechanism in geological systems. The rate of groundwater and contaminant migration and the extent of spreading is largely governed by fracture networks and corresponding fracture apertures. The variation of these apertures controls the transport properties of the fracture such as its permeability and capillarity. The characteristics of this aperture variation are determined by the characteristics of the opposite surfaces of natural fractures and the correlation between them.

In this study apertures between fractal surfaces are analyzed. Mathematical expressions are developed that relate the aperture characteristics to those of the fracture faces. Surfaces of fractures exhibit two distinct properties that make them amenable for analysis by the application of the theory of fractal geometry.[1] The first is that roughness profiles of the surfaces seem to be nowhere differentiable though they are continuous. The other is that the profile is either self-similar or self-affine at different scales over a large range of length scales. The characteristics of the aperture distribution depend on the fractal surfaces and their correlation.[2,3] If the two surfaces are uncorrelated then the aperture distribution will have a fractal character which may or may not be different than that of the opposing faces. Conversely, if the the opposing faces are either displaced mirror images or are correlated at larger wavelengths then the aperture distribution may not be fractal or may be fractal only over certain length scales.

Fracture tracings of intact fracture from cores from the Nevada Test Site are also examined. It is found that the roughness profiles exhibit fractal characteristics over certain frequencies, but the characteristics of the aperture cannot be clearly associated with a unique fractal dimension over all scales. This is indicative of fractures between surfaces that are correlated at higher wavelengths. Opposite surfaces of natural fractures in geologic media are expected to be correlated at higher wavelengths leading to a mean value of aperture on which lower wavelength perturbations are superimposed.

SURFACE FRACTAL ANALYSIS

Self-affine or self-similar fractal distribution of heights of rough surfaces are characterized by correlation over many scales and by continuous, but not differentiable, profiles. Such profiles have a power spectrum of the following form[4,5]

$$S(\omega) \propto \frac{1}{\omega^{5-2D}} \; , \quad 1 < D < 2 \; , \qquad (1)$$

where D is the fractal dimension of the profile. As the fractal dimension increases, the heights of nearby points become more independent of each other and the profile becomes increasingly jagged. Due to the singular nature of the power spectrum $S(\omega)$ as $\omega \to 0$ it is frequently required to introduce a lower cut-off frequency γ which corresponds to the largest distance L measurable. Similarly, higher frequency cut-off corresponding to lowest measurable distance may also defined. Equation (1) is a sufficient condition for fractality, *i.e.*, if the power spectrum exhibits a linear variation in log-log representation and the slope is between -3 and -1, then the profile is fractal. Fractal analysis of surfaces will not be discussed further since most details can be found elsewhere.[1-5]

APERTURE ANALYSIS

The characteristics of the aperture are strongly a function of the characteristics of the opposing rough surfaces that form them. The fractal nature of the surfaces as well as the correlation between them influence the properties of the aperture and determine whether or not the aperture distribution is fractal. Thus three different cases are discussed below, each of which yield different fractal characteristics. A schematic depiction of the surfaces and apertures is shown in figure 1.

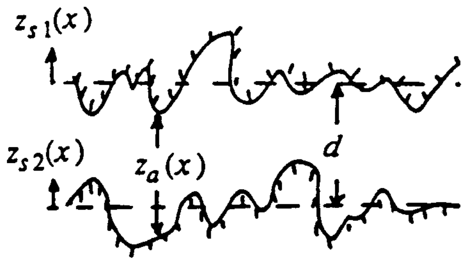

Figure 1. Schematic depiction of apertures.

A. Uncorrelated Surfaces

If the opposing surfaces of the fracture are completely uncorrelated over all scales the resultant aperture will be the difference of these two completely random uncorrelated height distributions. This implies that

$$\sigma_a^2 = \sigma_{s1}^2 + \sigma_{s2}^2 \ , \quad \sigma_a^2 = 2\sigma_s^2 \ \ (\text{if } \sigma_{s1} = \sigma_{s2} = \sigma_s) \ , \quad (2)$$

where σ is the standard deviation of aperture and surface profile heights and σ^2 is the variance. Here subscripts a, $s1$, $s2$ refer to aperture, surface 1, and surface 2, respectively. The variances σ_a^2 and σ_s^2 are proportional to the area under the power spectra $S_a(\omega)$ and $S_s(\omega)$ respectively, where ω is the angular frequency.

If the two uncorrelated surfaces $s1$ and $s2$ are fractal then the distribution of the apertures between the surfaces is also fractal. By fractal theory the power spectrums of the fractal aperture and surface profiles are given by equation (1) where $D = D_a$ or D_s, $S = S_a$ or S_s, and D_a and D_s are the fractal dimensions of the aperture and surface profiles, respectively. Assuming the constants of proportionality in equation (1) to be C_a and C_s respectively for the aperture and surfaces, and obtaining the variances by computing the areas under the power spectrum curve as

$$\sigma^2 = \int_{\gamma}^{\infty} S(\omega) d\omega \ , \quad \gamma = \frac{2\pi}{L} \ , \quad (3)$$

where L is the largest dimension under consideration, yields $D_a = D_s$ and $C_a = 2C_s$. Here γ is the lowest (cut-off) frequency of the physical system. By applying the fact that the relation between fracture surfaces and apertures should be independent of the largest length scale L under consideration, the result that the surface and aperture fractal dimensions are similar is obtained. Thus for apertures between completely uncorrelated similar surfaces of similar fractal dimension D_s, the fractal dimension D_a of the aperture is identical to that of the surfaces D_s, *i.e.*,

$$S_a(\omega) = 2S_s(\omega) \ , \quad (4)$$

where both S_a and S_s are fractal and obey equation (1).

If the two uncorrelated fractal surfaces have different characteristics then the fractal characteristics of the aperture are not as easy to obtain. For the case where $D_{s1} = D_{s2} = D_s$ and $C_{s1} \neq C_{s2}$ the value of C_a is $C_{s1} + C_{s2}$ and $D_a = D_s$. However if the fractal dimensions of the two surfaces are not identical the characteristics of the aperture are complicated to extract. This is because the power spectrum, and hence the amplitudes, of the surface with lower fractal dimension fall faster with increasing frequency than those on the one with higher fractal dimension, and this leads to a non-unique determination of the aperture dimension.

B. Partially Correlated Surfaces

In a situation where a strong correlation exists between the opposite faces at small frequencies (large wavelengths), the fractal relation developed in the previous section will hold in the uncorrelated region. The aperture distribution will thus be fractal only in the range where no correlation exists and the corresponding fractal dimensions of the surfaces and aperture will be the same as that obtained in the previous section. At the lower frequencies (larger wavelengths), where correlation between the opposing surfaces is strong, the surfaces will still be fractal but not so the aperture distribution. Due to correlation at the smaller frequencies (larger wavelengths) the power spectrum $S_a(\omega)$ of the aperture distribution over the correlated range will be smaller in magnitude than if it were fractal.

For opposing similar surfaces ($D_{s1} = D_{s2} = D_s$, $C_{s1} = C_{s2} = C_s$) that are correlated at smaller frequencies (larger wavelengths) the following relation is proposed:

$$S_a(\omega) = 2(1 - \langle \cos\phi(\omega) \rangle) S_s(\omega) \ , \quad (5a)$$

where $\phi(\omega)$ is the phase difference between the opposing faces at the frequency ω. The value of $<\cos\phi>$ is obtained as

$$<\cos\phi(\omega)> = \int_{-\pi}^{\pi} \cos(\phi) P_\omega(\phi) d\phi \quad . \tag{5b}$$

Here $P_\omega(\phi)$ is the probability of the phase difference of the Fourier components of the opposite faces to be ϕ at frequency ω. For completely uncorrelated frequencies, $P_\omega = 1/2\pi$, *i.e.*, any angle between $-\pi$ and π can be found with equal probability. In such a case equation (5) equals equation (4). If the phase difference is zero, implying that the two surfaces are identical at the frequency ω and that the amplitudes of the two surfaces will cancel, then $P_\omega(\phi) = 2\pi\delta(\phi)$, where δ is the Dirac delta function. This leads to $<\cos\phi> = 1$, and thus $S_a(\omega) = 0$ for this frequency ω.

C. Surfaces with Displacement

Yet another case is possible, that when the two surface profiles are mirror images of each other but are displaced by a small displacement distance x_c. This distribution can be shown to have the following power spectrum[3]

$$S_a(\omega) = 2(1 - \cos\omega x_c) S_s(\omega) \quad , \tag{6a}$$

$$= \omega^2 x_c^2 S_s(\omega) \quad , \quad \omega x_c \rightarrow 0 \quad . \tag{6b}$$

Clearly the aperture distribution is not fractal since its power spectrum $S_a(\omega)$ does not follow the functional form required by equation (1). From equation (6b) and using the functional form of the power spectrum $S_s(\omega)$ of the fractal surface from equation (1) yields

$$S_a(\omega) \rightarrow C_s \frac{x_c^2}{\omega^{(3-2D)}} \quad , \quad \omega \rightarrow 0 \quad , \quad D = D_s \quad . \tag{6c}$$

This implies that if the fractal dimension D_s is less than 1.5 the power spectrum of the aperture at small ω increases with decreasing ω and if D_s is greater than 1.5 the power spectrum decreases with decreasing ω. For fractal dimension D_s equal to 1.5 the power spectrum of the aperture asymptotes to a constant value as ω decreases. This power spectrum (for D_s = 1.5) is schematically depicted in figure 2.

INTACT FRACTURE DATA

The rock samples were obtained from the G-tunnel of the Nevada Test Site. The imprints of the intact fracture traces were obtained by using pencil lead on paper pressed against the core surface. This method was employed since measurements of *in-situ* apertures of intact fractures were desired and thus the fractures could not be opened to expose their surfaces. Only the visually exposed traces of the fracture on the core surfaces were available for measurement. Other sophisticated measurement techniques such as using profilometers and castings can only be used if the surface of the fracture is available, and such was not possible for this analysis. The tracings obtained were digitized using a thousand line per inch digitizing board and a contouring program. More than fifty such tracings were obtained from five different cores. However, most were hairline fractures and were not considered for this study since they

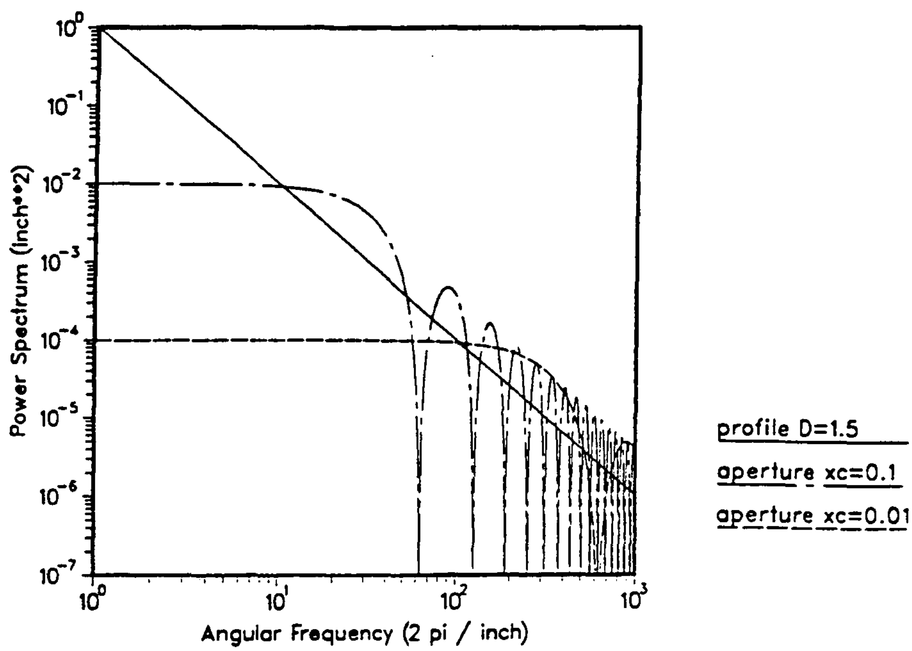

Figure 2. Power spectrum of apertures created by displacing mirror image surfaces that match perfectly by a displacement distance x_c.

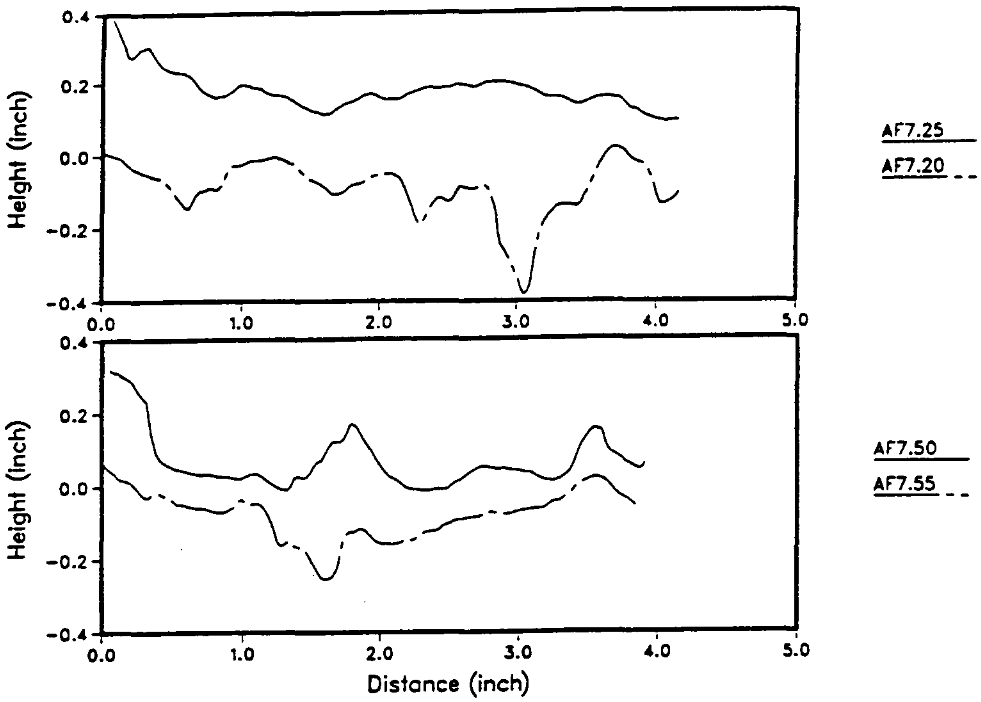

Figure 3. Two aperture tracings from one single fracture in core U12G-AF-7.

did not yield any measurable aperture. Some other large aperture fractures were damaged by the coring process. Of the remaining, four representative aperture data sets corresponding to fractures with measurable apertures and no discernable damage are presented here. They are from two different fractures and each set was measured from diametrically opposite parallel locations on two cylindrical core surfaces. The circumferance of the first core (U12G-AF-7) was approximately 31.8". The corresponding digitized tracings are presented in figure 3. The second core (U12G-AF-8) had a circumferance of approximately 31.6" and the tracings are presented in figure 4.

RESULTS

The power spectra of the two apertures from the first core U12G-AF-7 (figure 3) are presented in figure 5. A visual inspection of the aperture tracings indicates that the opposite faces of the fracture do not seem highly correlated at any wavelength or frequency. This is verified by the power spectrum of the apertures which indicates fractal behavior. However, the fractal dimension of the aperture is not same as that of the profiles. The power spectra of the profiles (not shown here) exhibit a power spectrum of the mathematical form indicated by equation (1), where the mean value of dimension D_s, found by fitting a straight line through the spectral data of the various profiles, is approximately 1.35. The mean dimension D_a of the two apertures from core AF-7 is approximately 1.2. This aperture exhibits characteristics of the type discussed under uncorrelated surfaces.

The apertures from the second core U12G-AF-8 (figure 4) appear visually to be correlated at large wavelengths. This is reflected in the power spectrum of the aperture distributions where a fall-off is obtained at smaller frequencies (larger wavelengths), see figure 6. The fractal dimension of the aperture at larger frequencies is approximately 1.2 whereas that of the surface profiles is 1.35. The characteristics of the two apertures from core AF-8 follow the results predicted by mathematical model developed for partially correlated surfaces.

CONCLUSIONS

In summary, the roughness data from intact fractures indicates that the fracture roughness profiles exhibit fractal power spectrum behavior. The fractal study of the aperture data indicates that the aperture distribution also follows a fractal behavior at higher frequencies (small wavelengths) and, for some apertures, displays a power fall-off at larger wavelengths indicating that the larger wavelength components of the opposing faces are correlated while the smaller ones are not. However, the mean fractal dimension of the aperture distribution corresponding to the smaller wavelengths is not similar to that of the fracture walls. Four reasons exist that can explain why these fractal dimensions are non-similar. The first is that because of the method of data collection a precise imprint of the profiles could not be obtained, and this introduced errors. The fracture aperture, which is obtained from the difference of the measurements of the two surfaces, has thus a higher error. Secondly, the range of frequencies spanned by the data is less

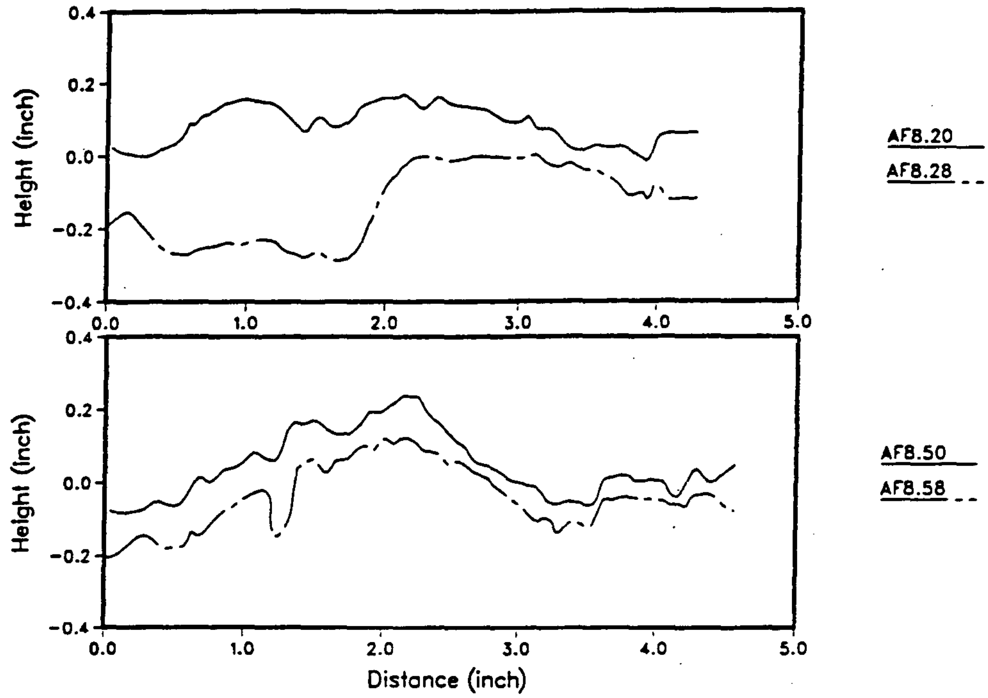

Figure 4. Two aperture tracings from one single fracture in core U12G-AF-8.

than two orders of magnitude and this hampers the accurate assessment of the surface and aperture characteristics. The third is that just two apertures per core sample are not enough to obtain the general characteristics of the surface roughness and apertures of the entire fracture. And lastly, even though the profiles and apertures considered are along one line, the effects related to the two-dimensionality of the surface may also introduce errors. The two surfaces may be highly correlated in a direction that is not parallel to the direction of measurement. In such a case the results along the direction of measurement will exhibit different characteristics than those along the correlation direction for which the mathematical expressions have been developed here.

The mathematical expressions developed for apertures between uncorrelated surfaces, partially correlated surfaces, and similar displaced surfaces are for ideally fractal surfaces. Deviations in geologic systems are expected due to erosion and physical constraints on the geology. However, the expressions developed provide insight to the dependence of aperture characteristics on the surface correlation and characteristics, and can be used in the modeling of fracture transport properties.

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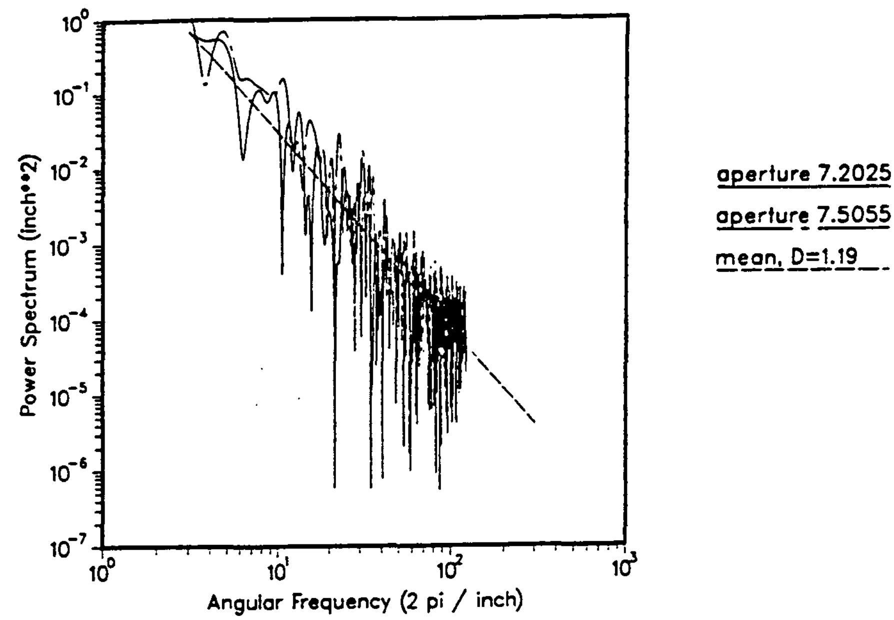

Figure 5. Power spectrum of the two apertures from U12G-AF-7.

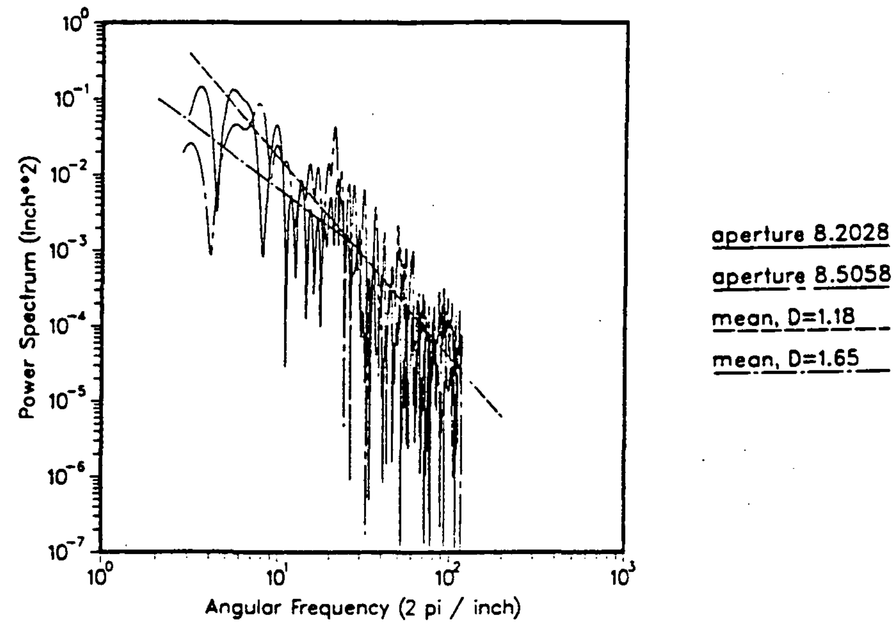

Figure 6. Power spectrum of the two apertures from U12G-AF-8.

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