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Continuous quantum measurement with observer: pure wavefunction evolution instead of decoherence

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We consider a continuous measurement of a two-level system (double-dot) by weakly coupled detector (tunnel point contact nearby). While usual treatment leads to the gradual system decoherence due to the measurement, we show that the knowledge of the measurement result can restore the pure wavefunction at any time (this can be experimentally verified). The formalism allows to write a simple Langevin equation for the random evolution of the system density matrix which is reflected and caused by the stochastic detector output. Gradual wavefunction “collapse” and quantum Zeno effect are naturally described by the equation.

The problem of quantum measurements has a long history, however, it still attracts considerable attention and causes discussions and even some controversy, mainly about the wavefunction “collapse” postulate [1–4]. One of the leading modern ideas is to replace this postulate by the gradual decoherence of the density matrix due to the interaction with the detector, so that this decoherence can be described by the Schrödinger equation and, hence, no additional postulate is necessary (see, e.g. Ref. [5]). Let us also mention the “hidden variables” idea [6] and the approach of a stochastic evolution of the wavefunction [7–15]. The renewed interest to the measurement problem is justified by the development of the experimental technique, which allows more and more experimental studies of the quantum measurement in optics and mesoscopic structures [16–20]. The problem has also close connection to the rapidly growing fields of quantum cryptography [21] and quantum computing [22].

In the recent experiment [19] with “which-path” interferometer the suppression of Aharonov-Bohm interference due to the detection of which path an electron chooses, was observed. The weakly coupled quantum point contact was used as a detector. The interference suppression in this experiment can be quantitatively described by the decoherence (dephasing [23]) due to the measurement process [24–27] (see also Refs. [28,29]).

In the present paper we consider somewhat different setup which is simpler and more basic in the context of quantum measurements: two quantum dots occupied by one electron and a weakly coupled detector measuring the position of the electron. As a detector we assume small tunnel contact (“point contact”) close to the

double-dot so that the detector barrier height depends on the electron position. This setup was analyzed in Ref. [24] (see also Ref. [27]) in which the equations for the double-dot density matrix evolution affected by the decoherence due to the measurement process, were derived. However, the decoherence approach cannot describe the detector output that is a separate interesting problem [27,30] analyzed in the present paper. We answer two related questions: how the detector current looks like (as a function of time) and what is the proper double-dot density matrix for particular detector output. (Notice that our result for the first question contradicts the point of view presented in Ref. [27].)

We show that the decoherence rate derived in Refs. [24–27] coincides with the lower bound determined by the knowledge about the system gradually acquired during the continuous measurement (thus proving that the considered model of point contact corresponds to an “ideal” detector). This lower bound is derived assuming that the system can be still described by the pure wavefunction after each particular realization of the random detector output. Hence, the fact that the lower bound is really achieved for the point contact as a detector, leads us to the conclusion that the decoherence in this case is just a consequence of ignoring the measurement result, i.e. averaging over all possible realizations. The observer who follows the detector output is able to obtain the complete knowledge about the system: he knows the wavefunction at each moment of time.

The measurement process modifies the wavefunction, for example, leading to gradual localization. From the observer’s point of view the evolution of the wavefunction can be described as a stochastic process related to the detector output. We develop a simple formalism of this evolution and briefly discuss the philosophical aspect of the presented result. The formalism can be applied to more general case of a two-level system measured by weakly coupled detector; however, for the definiteness we speak about the double-dot and point contact.

Similar to Ref. [24] let us describe the double-dot system and the measuring point contact by the Hamiltonian

$$\mathcal{H} = \mathcal{H}_{DD} + \mathcal{H}_{PC} + \mathcal{H}_{int}, \quad (1)$$

where

$$\mathcal{H}_{DD} = \frac{\epsilon}{2}(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1) \quad (2)$$

is the standard Hamiltonian of the double-dot system [31] with tunneling coupling H between dots (H is assumed to be real),

$$\mathcal{H}_{PC} = \sum_l E_l a_l^\dagger a_l + \sum_r E_r a_r^\dagger a_r + \sum_{l,r} T(a_r^\dagger a_l + a_l^\dagger a_r) \quad (3)$$

describes the tunneling through the point contact (for simplicity T is real and does not depend on energies), and the coupling between the double-dot and the detector is assumed to be

$$\mathcal{H}_{int} = \sum_{l,r} \Delta T c_2^\dagger c_2 (a_r^\dagger a_l + a_l^\dagger a_r), \quad (4)$$

i.e. the tunneling matrix element for the point contact is T when the first dot is occupied while it is $T + \Delta T$ when the electron is in the second dot. The voltage V across the point contact is sufficiently large, $eV \gg T^2 \rho$ (ρ is the density of states), so that the simple description of the point contact is possible (see Ref. [24]). Basically we can say that the average current $I_1 = 2\pi T^2 \rho_l \rho_r e^2 V / \hbar$ flows through the detector when the electron is in the first dot, and the current is $I_2 = I_1 + \Delta I = 2\pi(T + \Delta T)^2 \rho_l \rho_r e^2 V / \hbar$ when the second dot is occupied.

We make an important assumption of weak coupling between the double-dot and the detector (actually, it would be better to call it "weakly responding" detector),

$$|\Delta I| \ll I_0 = (I_1 + I_2)/2, \quad (5)$$

so that many electrons ($N \gtrsim (I_0/\Delta I)^2$) should pass through the point contact before the observer is able to distinguish which dot is occupied (i.e. when the uncertainty due to the detector shot noise becomes less than ΔI). This assumption is necessary to allow the classical description of the detector, namely to neglect the coherence between the quantum states with different number of electrons passed through the detector (we implicitly assume that the corresponding "collapse" happens on the time scale $t \ll (e/I_0)(I_0/\Delta I)^2$, much faster than typical evolution of the double-dot density matrix).

One of the main results of Ref. [24] is the equation for the decoherence rate Γ_d of the nondiagonal element $\sigma_{12}(t)$ of the double-dot density matrix due to the measurement by the point contact: $\Gamma_d = (\sqrt{I_1/e} - \sqrt{I_2/e})^2/2$. In the weakly-coupled limit (5) it can be replaced by

$$\Gamma_d = \frac{1}{8} \frac{(\Delta I)^2}{eI_0}. \quad (6)$$

The decoherence has an obvious relation to the low frequency shot noise in the detector (the origin can be traced to Eq. (4)), so let us write Eq. (6) in the form

$$\Gamma_d = \frac{1}{4} \frac{(\Delta I)^2}{S_I}, \quad (7)$$

where $S_I = 2eI_0$ is the usual Schottky formula for the shot noise spectral density S_I . Equation (7) has been also obtained in Refs. [25,26] for the quantum point contact as a detector, the difference in that case is $S_I = 2eI_0(1 - \mathcal{T})$ where \mathcal{T} is the transparency of the channel [32] (while in the case considered above we implicitly assumed $\mathcal{T} \ll 1$ [33]). As shown in Ref. [27], Eq. (7) should be modified (decoherence rate increases) if the phase of transmitted and reflected electrons in the detector is sensitive to the double-dot state; we assume that there is no such a dependence in our case.

Concluding the introductory part of the paper let us write the full equation for the double-dot density matrix in the decoherence approach:

$$\dot{\sigma}_{11} = -\dot{\sigma}_{22} = \frac{iH}{\hbar}(\sigma_{12} - \sigma_{21}), \quad (8)$$

$$\dot{\sigma}_{12} = \frac{i\epsilon}{\hbar}\sigma_{12} + \frac{iH}{\hbar}(\sigma_{11} - \sigma_{22}) - \frac{1}{4} \frac{(\Delta I)^2}{S_I} \sigma_{12}. \quad (9)$$

Notice that the decoherence rate (7) was derived in Refs. [24–27] without any account of the information provided by the detector, implicitly assuming that the measurement result is just ignored. Now let us study how this additional information affects the double-dot density matrix.

We start with the completely classical case when there is no tunneling between dots ($H = 0$) and the initial density matrix of the system does not have nondiagonal elements, $\sigma_{12}(0) = 0$ (then obviously $\sigma_{12}(t) = 0$ for any $t > 0$). We can assume that the electron is actually located in one of the dots, but we just do not know exactly in which one, and that is why we use probabilities $\sigma_{11}(0)$ and $\sigma_{22}(0) = 1 - \sigma_{11}(0)$. The detector output is the fluctuating current $I(t)$. The fluctuations grow when we examine $I(t)$ at smaller time scales, so we need some averaging in time ("low-pass filtering"), at least in order to neglect the problem of individual electrons passing through the point contact. Let us always work at sufficiently low frequencies, $f \ll S_I/e^2$, for which the low frequency limit S_I for the spectral density is well achieved.

Provided that i th dot is occupied, the probability to have a particular value for the current averaged over time τ , $\langle I \rangle = \int_0^\tau I(t) dt$, is given by the Gaussian distribution

$$P_i(\langle I \rangle, \tau) = (2\pi D)^{-1/2} \exp(-(\langle I \rangle - I_i)^2/2D), \quad (10)$$

$$D = S_I/2\tau.$$

Notice that this equation obviously does not change if we divide the time interval τ into pieces and integrate over all possible average currents for each piece (to consider only positive currents the typical timescale τ should be sufficiently long, $S_I/\tau \ll I_i^2$, that is always satisfied within the assumed low frequency range). After the measurement during time τ the observer acquires additional

knowledge about the system and should change the probabilities σ_{ii} according to the standard Bayes formula. (It says that a posteriori probability $p'(A)$ of an event A after the knowledge that the event F has happened, is equal to $p'(A) = p(A)p(F|A)/\sum_B[p(B)p(F|B)]$ where $p(A)$ is a priori probability and $p(F|A)$ is the conditional probability of event F given event A .) Hence,

$$\begin{aligned}\sigma_{11}(\tau) &= \sigma_{11}(0) \exp[-(\langle I \rangle - I_1)^2/2D] \\ &\quad \times \left\{ \sigma_{11}(0) \exp[-(\langle I \rangle - I_1)^2/2D] \right. \\ &\quad \left. + \sigma_{22}(0) \exp[-(\langle I \rangle - I_2)^2/2D] \right\}^{-1}, \\ \sigma_{22}(\tau) &= 1 - \sigma_{11}(\tau).\end{aligned}\quad (11)$$

Notice that we do not use any ‘‘collapse’’ postulate here because we speak so far about the classical measurement.

Now let us assume that the initial state was fully coherent, $\sigma_{12}(0) = \sqrt{\sigma_{11}(0)\sigma_{22}(0)}$ (while still $H = \epsilon = 0$). Since the detector is sensitive only to the position of electron, the detector current will behave exactly the same way [34] and the probability of a particular value $\langle I \rangle$ is still given by

$$P(\langle I \rangle, \tau) = \sigma_{11}(0)P_1(\langle I \rangle, \tau) + \sigma_{22}(0)P_2(\langle I \rangle, \tau). \quad (12)$$

After the measurement during time τ we should obviously assign the same values for $\sigma_{11}(\tau)$ and $\sigma_{22}(\tau)$ as in Eq. (11), but the question is not so trivial for the nondiagonal element $\sigma_{12}(\tau)$. Nevertheless, we can easily write the upper bound:

$$\text{Re } \sigma_{12}(\tau) \leq |\sigma_{12}(\tau)| \leq \sqrt{\sigma_{11}(\tau)\sigma_{22}(\tau)}. \quad (13)$$

Let us imagine the observer who does not want to know the result of the measurement (which actually exists!). Then using the probability distribution of different outcomes given by Eq. (12) and the upper bound (13) for each realization, he can calculate the upper bound for σ_{12} (disregarding the actual result):

$$\begin{aligned}\langle \text{Re } \sigma_{12}(\tau) \rangle &\leq \int \sqrt{\sigma_{11}(\tau)\sigma_{22}(\tau)} P(\langle I \rangle, \tau) d\langle I \rangle \\ &= \sqrt{\sigma_{11}(0)\sigma_{22}(0)} \exp\left(-\frac{(\Delta I)^2\tau}{4S_I}\right).\end{aligned}\quad (14)$$

This upper bound exactly coincides with the actual result given by decoherence approach (9). This fact forces us to accept somewhat surprising statement that Eq. (13) gives not only the upper bound, but the true value of the nondiagonal matrix element, i.e. the pure state remains pure after the measurement (no decoherence occurs) if we know the measurement result [35].

Simultaneously, we prove that the point contact detector considered in Refs. [24–26] causes the slowest possible decoherence of the measured system (disregarding the measurement result), and hence represents an ideal detector in this sense. In contrast, the result of Ref. [29]

shows that a single-electron transistor with large tunnel resistances and biased by relatively large voltage, is not an ideal detector (for the same amount of the back-influence on the system it provides an observer with less information than an ideal detector). Similarly, the generalization of the quantum point contact considered in Ref. [27] describes a non-ideal detector.

If the initial state of the double-dot is not purely coherent, $|\sigma_{12}(0)| < \sqrt{\sigma_{11}(0)\sigma_{22}(0)}$, we can treat it as the statistical combination of purely coherent and purely incoherent states with the same $\sigma_{11}(0)$ and $\sigma_{22}(0)$. Then instead of Eq. (13) we have

$$\sigma_{12}(\tau) = \sigma_{12}(0) \frac{[\sigma_{11}(\tau)\sigma_{22}(\tau)]^{1/2}}{[\sigma_{11}(0)\sigma_{22}(0)]^{1/2}}. \quad (15)$$

Eq. (15) together with Eq. (11) is the central result of the present paper; these equations give the density matrix of the measured system with account of the measurement result [36].

The measurement should lead to the localization of the wavefunction in one of the dots. This is a random process, and the observer who continuously follows the detector output can describe it as the random evolution of the wavefunction (provided the pure initial state), or more generally the random evolution of the density matrix. Eqs. (10)–(12) and (15) allow to simulate this evolution. For example, we can use Monte-Carlo method and do the following. First we choose the timestep τ satisfying inequalities $e^2/S_I \ll \tau \ll S_I/(\Delta I)^2$ and draw a random number for $\langle I \rangle$ according to the distribution (12). Then we update $\sigma_{11}(t)$ and $\sigma_{22}(t)$ using this value of $\langle I \rangle$ and repeat the procedure many times (the distribution for the current averaged over the interval $\Delta t = \tau$ is new every timestep because of changing $\sigma_{ii}(t)$ which are used in Eq. (12)). The nondiagonal matrix element can be calculated at any time using Eq. (15).

This Monte-Carlo procedure can be equivalently described by the nonlinear Langevin-type equation for the density matrix evolution (equation for σ_{11} is sufficient):

$$\dot{\sigma}_{11} = \mathcal{R} = -\sigma_{11}\sigma_{22} \frac{2\Delta I}{S_I} \left[\frac{\sigma_{22} - \sigma_{11}}{2} \Delta I + \xi(t) \right], \quad (16)$$

where the random process $\xi(t)$ has zero average and the spectral density $S_\xi = S_I$ (only low-frequency limit matters). The term in square brackets is equal to $I(t) - I_0$, so it is directly related to the detector output. One can easily check that calculation of actual $\sigma_{11}(t)$ evolution for known detector output $I(t)$ using Eq. (16) coincides with the direct result given by Eq. (11).

Equation (16) is closely connected with the Quantum State Diffusion approach of Refs. [8–11] (for review, see Ref. [15]). Actually, it is possible to obtain mathematically such a stochastic differential equation for any equation for the density operator [8–10]. In our treatment,

however, we derived Eq. (16) using only basic physical reasoning.

Figure 1 shows a particular result of the Monte-Carlo simulation for the symmetric initial state, $\sigma_{11}(0) = \sigma_{22}(0) = 1/2$ (notice that $\sigma_{12}(0)$ does not affect the evolution if $H = 0$). Thick line shows the random evolution of $\sigma_{11}(t)$. Equation (16) describes the gradual localization in one of the dots (first dot in case of Fig. 1). Let us define the typical localization time τ_{loc} as $\tau_{loc} = 2S_I/(\Delta I)^2$ (we choose the exponential factor at $\sigma_{11} = \sigma_{22} = 1/2$). Then it is exactly equal to the time $\tau_{dis} = 2S_I/(\Delta I)^2$ necessary for the observer to distinguish between two states (defined as the relative shift of two Gaussians by two standard deviations), and $\tau_{loc} = \tau_d/2$ where $\tau_d = \Gamma_d^{-1}$. The probability of final localization in the first dot is equal to $\sigma_{11}(0)$ (as it should be) that can be easily proven because the procedure described above conserves $\sigma_{11}(\tau) - \sigma_{22}(\tau)$ averaged over realizations. The detector current basically follows the evolution of $\sigma_{11}(t)$ but the additional noise is large and depends on the bandwidth. The dashed line in Fig. 1 shows the detector current averaged over the “running window” with the duration $\Delta t = S_I/(\Delta I)^2$ while the thin solid line is current $\langle I \rangle$ averaged starting from $t = 0$.

Our result for the detector current contradicts the statement made in Ref. [27] that the detector output in each particular realization should correspond to the average double-dot population, $\langle I \rangle \simeq \sum_i I_i \sigma_{ii}(0)$, which we believe is incorrect as well as the statement that σ_{ii} can be measured in a single experiment “without a collapse of wavefunction”.

Now let us consider the general case of the double-dot system with non-zero tunneling H between dots. If the frequency Ω of “internal” oscillations in the double-dot is sufficiently low so that the low-frequency limit for the detector shot noise is well achieved,

$$\Omega = (4H^2 + \epsilon^2)^{1/2}/\hbar \ll S_I/e^2, \quad (17)$$

then we can use the same formalism just adding the slow evolution due to finite H (the product $\Omega\tau_{loc}$ can be both larger or smaller than unity, so in this sense the coupling between double-dot and the detector can be arbitrary large). The particular realization can be either simulated by Monte-Carlo procedure similar to that outlined above [now update of $\sigma_{12}(t)$ using Eq. (15) should be necessarily done at each timestep] or equivalently described by the corresponding coupled Langevin equations which are the counterpart of Eqs. (8)–(9):

$$\dot{\sigma}_{11} = -\dot{\sigma}_{22} = \frac{-2H}{\hbar} \text{Im}(\sigma_{12}) + \mathcal{R}, \quad (18)$$

$$\begin{aligned} \dot{\sigma}_{12} = & \frac{i\epsilon}{\hbar}\sigma_{12} + \frac{iH}{\hbar}(\sigma_{11} - \sigma_{22}) + \frac{\sigma_{22} - \sigma_{11}}{2\sigma_{11}\sigma_{22}} \mathcal{R}\sigma_{12} \\ & - \gamma_d\sigma_{12}, \end{aligned} \quad (19)$$

where \mathcal{R} is given by Eq. (16) and the last term in Eq. (19) will be discussed later ($\gamma_d = 0$ for an ideal detector).

Figure 2 shows particular results of the Monte-Carlo simulations for the double-dot with $\epsilon = H$ and different strength of the interaction with an ideal detector. The electron is initially located in the first dot, $\sigma_{11}(0) = 1$. The dashed line shows the evolution of σ_{11} with no detector. Notice that because of the energy asymmetry, the initial asymmetry of the electron location remains in this case for infinite time. When the interaction with detector, $\mathcal{C} = \hbar(\Delta I)^2/S_I H$, is relatively small (top solid line), the evolution of σ_{11} is close to that without the detector. However, the electron gradually “forgets” the initial asymmetry and the evolution can be described as the slow variation of the phase and amplitude of oscillations (recall that the wavefunction remains pure). In the decoherence approach (averaging over realizations) this corresponds to $\sigma_{11} \rightarrow 1/2$ at $t \rightarrow \infty$ [24].

When the coupling with the detector increases, the evolution significantly changes (middle and bottom curves in Fig. 2). First, the transition between dots slows down (Quantum Zeno effect [37]; see also Refs. [8,11,14,16,24,29,30]). Second, while the frequency of transitions decreases with increasing interaction with detector (at sufficiently strong coupling), the time of a transition (sort of “traversal” time) decreases, so eventually we can say about uncorrelated “quantum jumps” between states. The case $\mathcal{C} \gg 1$ is completely analogous to the standard description of the quantum Zeno effect with frequent wavefunction reductions.

In a regime of small coupling with detector, $\mathcal{C} \ll 1$, the detector output is too noisy to follow the evolution of σ_{ii} . It does not give an accurate information about the electron position and, correspondingly, only slightly affects the oscillations. On contrary, when $\mathcal{C} \gg 1$ the detector accurately informs about the position of electron and the jumps between states, and simultaneously destroys the internal oscillating dynamics of the system.

Equations (18)–(19) can be generalized for a nonideal detector, $\Gamma_d > (\Delta I)^2/4S_I$ (as in Ref. [29]), which gives an observer less information than possible in principle. Let us model a nonideal detector as two ideal detectors “in parallel”, so that observer can read the output of the first of them while the output of the second detector is disregarded. Then the information loss can be represented by the extra decoherence term $-\gamma_d\sigma_{12}$ in Eq. (19) where $\gamma_d = \Gamma_d - (\Delta I)^2/4S_I$. The limiting case of a nonideal detector is the detector with no output (just an environment) or with disregarded output. Then the evolution equations reduce to the standard decoherence case described by Eqs. (8)–(9).

For nonideal detector it is meaningful to keep our old definition of the localization time, $\tau_{loc} = \tau_{dis} = 2S_I/(\Delta I)^2$ while decoherence (in decoherence approach) occurs faster, $\tau_d < 2\tau_{loc}$. Actually, this means that if another observer is able to get more information (to read

the output of the second detector in a model above), then for him the localization time will be shorter. In other words, we define localization time not as a real physical quantity (that is meaningless because observer cannot check it) but as a quantity related to observer's information. Similarly, we can define the effective decoherence time as $\tau_d' = \gamma_d^{-1}$.

The main point of the present paper is that Eqs. (18)–(19) can be used not only to simulate the measurement process, but also to obtain the actual evolution of the density matrix in an experiment provided the known detector output $I(t)$ (high-frequency component of the output can be suppressed) and initial condition $\sigma_{ij}(0)$. For this purpose the term \mathcal{R} given by Eq. (16) should be replaced by

$$\mathcal{R} = -\sigma_{11}\sigma_{22} \frac{2\Delta I}{S_I} [I(t) - I_0]. \quad (20)$$

Notice that even if the initial state is completely random, $\sigma_{11} = \sigma_{22} = 1/2$, $\sigma_{12} = 0$, the nondiagonal matrix element appears during the measurement because of acquired information, so that sufficiently long observation with an ideal detector leads to almost pure wavefunction (of course, this wavefunction does not have direct relation to the initial state but emerges during the measurement).

Let us briefly discuss the philosophical aspect of the developed formalism. The statement that the pure wavefunction remains pure during the continuous measurement by an ideal detector (with known result) may seem surprising at first, however, we easily recognize that this is a direct analogy to the “orthodox” situation of a “sharp” measurement (the wavefunction is pure after the “collapse”). Another important point is that the density matrix is in some sense observer-dependent. If an observer disregards the detector output, he can either average over all possible detector outcomes or assume the decoherence; both ways give the same result. Now if two observers have different level of access to the detector information (as, for example, in the model of nonideal detector considered above), then the density matrix for them will be different. Nevertheless, the observer with less information can safely use his density matrix for all purposes; the only difference – he will be able to make less accurate predictions than the observer with complete knowledge of the detector output. There is no sense to speak about “actual” density matrix, it is meaningful to speak only about “accessible” density matrix. This statement obviously contradicts the point of view that the density matrix represents the objective reality. Simultaneously, this statement is completely consistent with the “orthodox” (Copenhagen) point of view that in quantum mechanics the reality is closely related to our knowledge about it, so the density matrix represents the indivisible mixture of the reality and our information about it.

If the knowledge of the detector output is not used in the experiment, then the post-measurement density matrices should be averaged, leading (equivalently) to decoherence. On contrary, one can devise an experiment in which the subsequent system evolution depends on the preceding measurement result; then the only proper description is the pure wavefunction (for simplicity we assume ideal detector).

For example, let us consider the double-dot with $H = 0$ and fully coherent symmetric initial state. According to our formalism, after the measurement during some time τ (most interesting case is $\tau \lesssim \tau_{loc}$) the wavefunction remains pure but becomes asymmetric (Eqs. (11) and (15)). This means that if an experimentalist can switch off the detector at $t = \tau$, reduce the barrier between the dots (create finite H) and change the relative energies of the dots in a proper way, then after some definite time period the electron can be moved to the first dot with the probability equal to unity (the corresponding parameters can be easily calculated using $\sigma_{ij}(\tau)$ [38]) that can be checked by the detector switched on again. Alternatively, using the knowledge of $\sigma_{ij}(\tau)$ an experimentalist can exactly prepare the ground state of the coupled double-dot system and check it, for example, by the photon absorption. Another experimental idea is to start with completely random state of the double-dot with finite H and then gradually (most interesting case is $\Omega\tau_{loc} \lesssim 1$) obtain almost pure wavefunction using the detector output $I(t)$ and Eqs. (18)–(19). The final test of the wavefunction is similar to that considered above.

An experiment of this kind would be able to verify the formalism developed in the present paper. While such an experiment is still a challenge for the present-day technology, we can hope that it will become realizable in the nearest future.

In conclusion, we developed a simple formalism for the evolution of double-dot density matrix with account of the result of the continuous measurement by weakly coupled (weakly responding) point contact. The formalism is suitable for any two-level system measured by weakly coupled detector.

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 [34] Actually, this statement is not so trivial but follows from the general quantum mechanical ideas.
 [35] If we formally apply a similar approach to the case $\Delta I \sim I_0$, we obtain $\langle \text{Re}\sigma_{12}(\tau) \rangle / (\sigma_{11}(0)\sigma_{22}(0))^{1/2} \leq [2\sqrt{S_{I1}S_{I2}} / (S_{I1} + S_{I2})]^{1/2} \exp[-\tau(\Delta I)^2 / 2(S_{I1} + S_{I2})]$.

FIG. 1. Thick line: particular Monte-Carlo realization of σ_{11} evolution in time during the measurement of uncoupled dots, $H = 0$. Initial state is symmetric, $\sigma_{11}(0) = \sigma_{22}(0) = 1/2$, while the measurement leads to gradual localization. Initially pure wavefunction remains pure at any time t . Thin line shows the corresponding detector current $\langle I \rangle$ averaged over the whole time interval starting from $t = 0$ while the dashed line is the current averaged over the running window with duration $S_I/(\Delta I)^2$.

FIG. 2. Random evolution of σ_{11} (particular Monte-Carlo realizations) for asymmetric double-dot, $\epsilon = H$, with the electron initially in the first dot, $\sigma_{11}(0) = 1$, for different strength of coupling with detector: $\mathcal{C} = \hbar(\Delta I)^2/S_I H = 0.3, 3, \text{ and } 30$ from top to bottom. Dashed line represents $\mathcal{C} = 0$ (unmeasured double-dot). Increasing coupling with detector destroys the quantum oscillations (while wavefunction remains pure at any t), slows down the transitions between states (Quantum Zeno effect), and for $\mathcal{C} \gg 1$ leads to uncorrelated jumps between well localized states.

Then the result of Ref. [24] is smaller than our upper bound for large τ , while it is larger for $\tau \lesssim \Gamma_d^{-1} \sim e/I_0$. The latter unphysical situation is because the detector cannot be described classically in this case.

- [36] Notice that Eqs. (11) and (15) can be readily obtained from the standard “reduction” procedure, $\sigma(\tau) = \mathcal{A}/\text{Tr}\mathcal{A}$, $\mathcal{A} = \mathcal{P}(\langle I \rangle, \tau)\sigma(0)\mathcal{P}(\langle I \rangle, \tau)$, if the generalized “projection” operator $\mathcal{P}(\langle I \rangle, \tau)$ is defined as $\mathcal{P}_{ii} = [P_i(\langle I \rangle, \tau)]^{1/2}$, $\mathcal{P}_{12} = \mathcal{P}_{21} = 0$.
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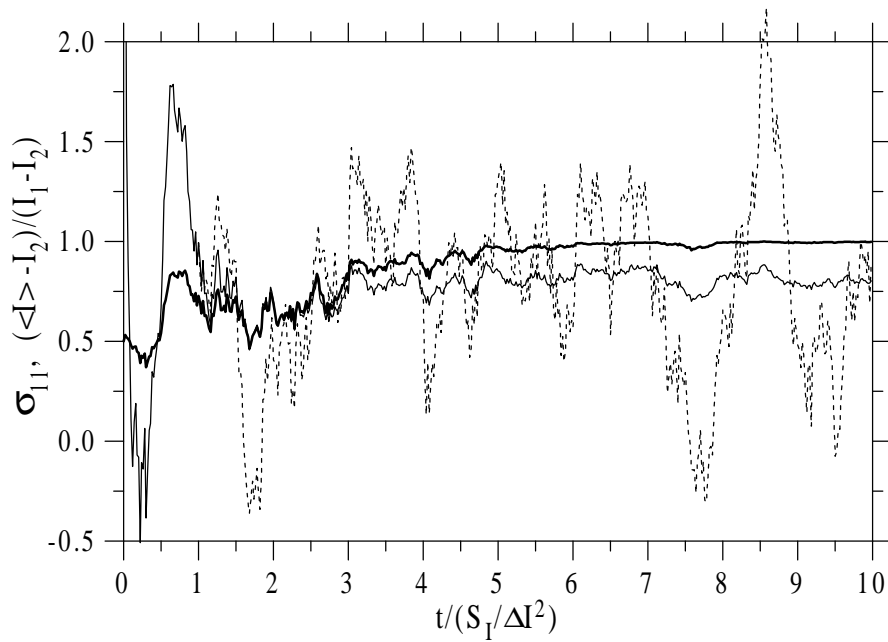


Fig. 1

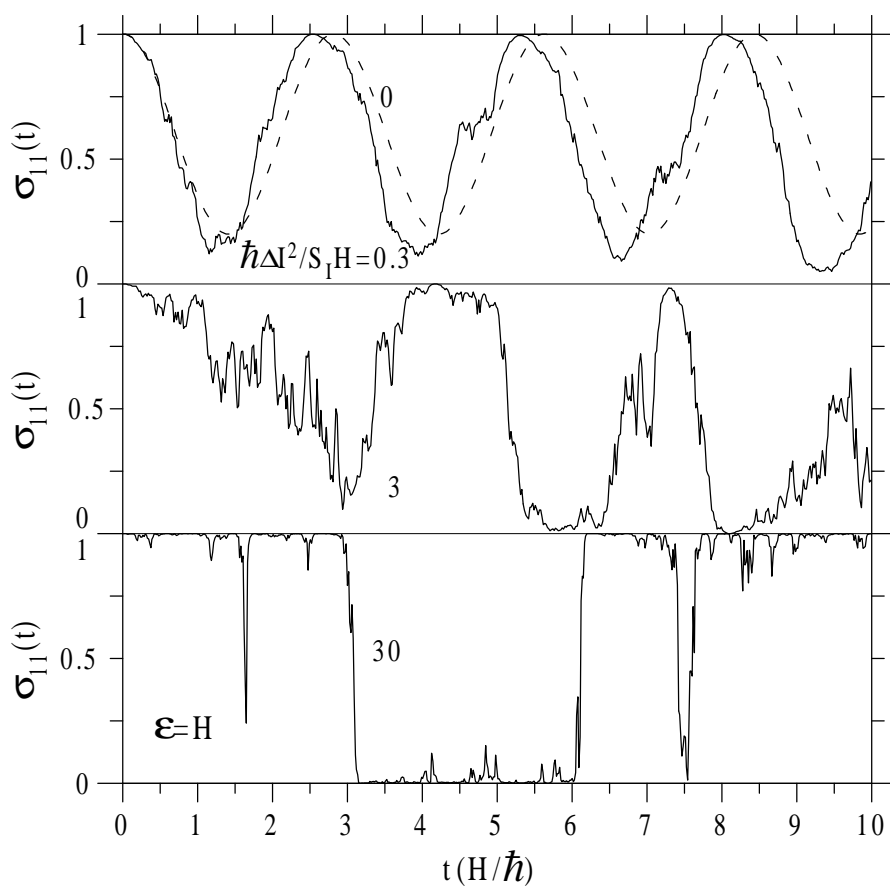


Fig. 2