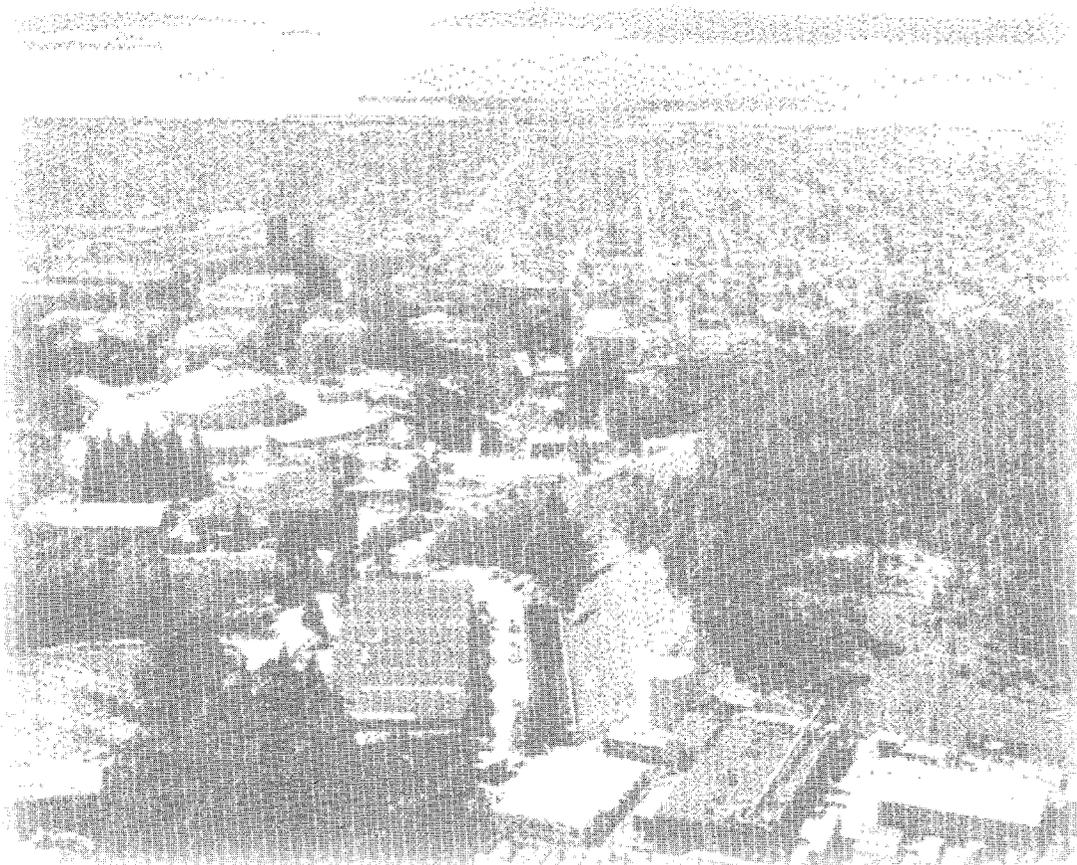# ERNEST ORLANDO LAWRENCE
BERKELEY NATIONAL LABORATORY

Fields in Multilayer Beam Tubes

Glen R. Lambertson

**Accelerator and Fusion
Research Division**

August 1999

DISCLAIMER

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Fields in Multilayer Beam Tubes*

Glen R. Lambertson

Accelerator and Fusion Research Division
Lawrence Berkeley National Laboratory
University of California
Berkeley, CA 94720

August 1999

Fields in Multilayer Beam Tubes*

Glen R. Lambertson

Lawrence Berkeley National Laboratory
University of California
Berkeley, California 94720
(26 February, 1999)

Abstract. Equations are presented for calculating the fields from a bunched beam that penetrate into the layers of a beam tube of circular cross section. Starting from the radial wave impedance of an outer surface, the wave functions in inner layers are calculated numerically to obtain field strengths or the longitudinal beam impedance. Examples of a vertex-detector region and of an injection kicker are given.

Introduction

The vacuum tubes that enclose particle beams, while usually of thick metal, have at places regions with thin metal or ceramic surrounded by metallic or magnetic structures. Fields from the beam current penetrate these walls, particularly at low frequencies. Calculation of the fields is needed to know the strengths of the fields outside and to determine the beam impedance presented by the multilayer structure. Penetration of the fields is affected by dissipative media and by the necessary matching of boundary conditions at the interfaces between layers. Because the beam is moving often at relativistic speed, it is important to use the field equations for waves propagating axially at the beam velocity along the tube. Reflections at the boundaries between materials are strongly dependent upon the relativistic factor $\beta\gamma$ and upon the ratio of radius of the layer to the wavelength.

The first section below gives equations for calculations using Bessel functions for the circularly cylindrical geometry. These may be used directly for numerical computation. A second section examines the wave functions in various media and gives approximations applicable for the usual parameter ranges. Following are example calculations of some actual cases.

1

Field equations and method

Assuming cylindrical symmetry about the beam axis, the fields within each of the layers may be obtained from axial TM Hertz vectors (Ref. 1,2,3). These vectors are sums

of the modified Bessel functions

$$\psi_+ = K_0(hr)e^{j(\omega t - k_0 z/\beta)} \tag{1a}$$

$$\psi_- = I_0(hr)e^{j(\omega t - k_0 z/\beta)} \tag{1b}$$

where

$$h^2 = \frac{\omega^2}{v^2} - \omega^2 \mu\varepsilon + j\omega\mu\sigma$$

$$h^2 = k_0^2\left(\frac{1}{\beta^2} - \mu_r\varepsilon_r\right) + j\frac{2}{\delta^2}. \tag{2}$$

If the material is a metal, the term containing the skin depth δ is usually the greater. I have subscripted the ψ^s with + or − to indicate that K_0, infinite at r = 0, is similar to an outgoing wave and I_0 an incoming field. While the axial phase velocity is always βc, wave fronts in media with $\mu_r\varepsilon_r \neq \beta^{-2}$ proceed with some radially outward or inward component.

In a structure without axial variation, only the three TM components of the fields arise from the potentials:

$$E_z = -h^2\psi \qquad E_r = -j\frac{k_0}{\beta}\frac{\partial\psi}{\partial r} \qquad H_\phi = -\left(j\frac{k_0}{Z_0}\varepsilon_r + \sigma\right)\frac{\partial\psi}{\partial r} \tag{3}$$

Z_0 is $\mu_0 c = 120\pi$ *ohm*. The boundary conditions between layers are simply that E_z and H_ϕ be continuous, and we shall not need the radial E-component. It will also be convenient to omit the exponential z- and t- dependence when writing the fields.

In the interior of the beam tube with beam current $I_e^{j(\omega t - k_0 z/\beta)}$, we have for radii greater than the beam radius

$$E_z = j\frac{IZ_0}{2\pi}\frac{k_0}{(\beta\gamma)^2}K_0\left(\frac{k_0 r}{\beta\gamma}\right) + B_0 I_0\left(\frac{k_0 r}{\beta\gamma}\right)$$

$$\hspace{10cm}(4)$$

$$H_\phi = \frac{I}{2\pi}\frac{k_0}{\beta\gamma}K_1\left(\frac{k_0 r}{\beta\gamma}\right) + jB_0\frac{\beta\gamma}{Z_0}I_1\left(\frac{k_0 r}{\beta\gamma}\right).$$

The constant B_0 is to be determined. These expressions appear more familiar if we make an approximation for the usual case of $k_0 r/\beta\gamma \ll 1$:

$$E_z \cong -j\frac{IZ_0}{2\pi r}\frac{k_0 r}{\beta\gamma}\ell n\frac{k_0 r}{\beta\gamma} + B_0 \hspace{4cm}(4a)$$

$$H_\phi \cong \frac{I}{2\pi r}\left[1 + \frac{1}{2}\left(\frac{k_0 r}{\beta\gamma}\right)^2\left(\ell n\frac{k_0 r}{\beta\gamma} + 0.616\right)\right] + jB_0\frac{k_0 r}{2Z_0}$$

In the medium of layer n we shall let $\psi = A_n K_0(hr) + B_n I_0(hr)$ and represent the fields as a vector F that is the product of matrix M_n and amplitudes A_n and B_n:

$$F \equiv \begin{pmatrix} E_+ + E_- \\ H_+ + H_- \end{pmatrix} = M_n\begin{pmatrix} A_n \\ B_n \end{pmatrix} \hspace{3cm}(5a)$$

with

$$M_n = \begin{pmatrix} -h^2 K_0 & -h^2 I_0 \\ \left(j\dfrac{k_0}{Z_0}\varepsilon_r + \sigma\right)hK_1 & -\left(j\dfrac{k_0}{Z_0}\varepsilon_r + \sigma\right)hI_1 \end{pmatrix} \hspace{2cm}(5b)$$

Within a layer the fields at the inner radius, e.g. r = a, are obtained from the fields at their outer radius, e.g. r = b, using matrix M and its inverse:

3

$$F(a) = M(a)[M(b)]^{-1} F(b) \tag{6}$$

At the inner surface of the outermost medium to be considered, we must specify the ratio of fields E_z and H_ϕ. For a compound structure such as a magnet, this may not be simple and may introduce azimuthal variations not strictly provided for in this analysis. Perhaps a suitable approximation can be found. In what follows, I shall choose the simpler case of a uniform exterior medium of infinite radial extent, such as vacuum or a magnetic or conducting material. In this case, only outgoing fields will exist in the material and a sufficient potential is;

$$\psi_e = A_e K_0(h_e r) \tag{7}$$

The ratio Z = E/H is then given by

$$Z_e = \frac{-h k_0(hr)}{\left(j \dfrac{k_0}{Z_0} \varepsilon_r + \sigma \right) K_1(hr)} \tag{8}$$

and the field vector F_e at this outer boundary may be written

$$F_e = \begin{pmatrix} E_e \\ H_e \end{pmatrix} = \begin{pmatrix} Z_e \\ 1 \end{pmatrix} A_e \tag{9}$$

where A_e may be determined from the analysis if knowledge of this outermost field is desired.

By successively applying the matrices M and M^{-1} for the layers as in Eq. (6) a matrix-product may be constructed that transforms fields F_e to the fields at the inner surface of the beam tube, at e.g. radius a.

$$\begin{pmatrix} E_z(a) \\ H_\phi(a) \end{pmatrix} = \left(M\text{-}product \right) \begin{pmatrix} Z_e \\ 1 \end{pmatrix} A_e \equiv \begin{pmatrix} FE \\ FH \end{pmatrix} A_e \tag{10}$$

The amplitude A_e is still unknown, but only the values of FE and FH (or their ratio) are needed to solve for the constant B_0 in Eq. (4), which is given by

$$B_0 = \frac{I}{2\pi} \frac{FEhK_1(ha) - jFH\dfrac{Z_0 h^2}{k_0} K_0(ha)}{FHI_0(ha) - jFE\dfrac{k_0}{Z_0 h} I_1(ha)} \tag{11}$$

where $h = k_o/\beta\gamma$. The longitudinal beam impedance per unit length, Z'_B, that adds to any space-charge term is

$$Z'_B = -\frac{B_0}{I}. \tag{12}$$

The magnetic field at $r = a$ is given by

$$H(a) = \frac{I}{2\pi a} \frac{FH}{FHI_0(ha) - jFE\dfrac{k_0}{Z_0 h} I_1(ha)} \tag{13}$$

At the outer surface, the field F_e of Eq. (9) is found using amplitude

$$A_e = \frac{H(a)}{FH} \tag{14}$$

Media matrices

In vacuum or in any medium where the value of hr is less than 0.1 and real, an approximation of Eq. (5) is

$$M \approx \begin{pmatrix} h^2(\ell n\, hr - 0.116) & -h^2 \\ j\dfrac{k_0}{Z_0 r}\left[1 + \dfrac{1}{2}(hr)^2(\ell n\, hr - 0.616)\right] & -j\dfrac{k_0}{2Z_0}h^2 r \end{pmatrix}. \tag{15}$$

Of use in calculating a symbolic inverse is the value of the determinant (valid for any value of hr):

$$|M| = j\frac{k_0 h^2}{Z_0 r} \tag{16}$$

If we discard terms of order $(hr)^2$, we may find a simple product matrix to transform through one layer of vacuum from radius b to smaller radius a:

$$M(a)[M(b)]^{-1} \approx \begin{pmatrix} 1 & 0 \\ -j\frac{k_0}{Z_0}\frac{b^2-a^2}{2a} & \frac{b}{a} \end{pmatrix} \tag{17}$$

In a layer of dielectric or magnetic material, the value of h^2 (c.f. Eq. (2)) may be either positive or negative. For the negative case, it may be convenient to use Bessel functions and Hankel functions $H_n^{(2)} = J_n - jY_n$; for that we let $h = jg$, i.e.

$$g = k_0\left(\mu_r\varepsilon_r - \frac{1}{\beta^2}\right)^{1/2} \tag{18}$$

Then ψ becomes

$$\psi = A\left(-j\frac{\pi}{2}H_0^{(2)}(gr)\right) + BJ_0(gr) \tag{19}$$

and

$$E_z = g^2\psi \qquad H_\phi = -j\frac{k_0}{Z_0}\varepsilon_r\frac{\partial\psi}{\partial r} \tag{20}$$

The matrix M is

$$M = \begin{pmatrix} -j\frac{\pi}{2}g^2 H_0^2(gr) & g^2 J_0(gr) \\ \frac{\pi k_0}{2Z_0}\varepsilon_r g H_I^{(2)}(gr) & j\frac{k_0}{z_0}\varepsilon_r g J_1(gr) \end{pmatrix} \tag{21}$$

which becomes for small gr

6

$$M \approx \begin{pmatrix} -g^2 \left[\ell n(gr) - 0.116 + j\frac{\pi}{2} \right] & g^2 \\[2mm] j\frac{k_0 \varepsilon_r}{Z_0 r} \left[1 - \frac{(gr)^2}{2} \left(\ell n(gr) - 0.616 + j\frac{\pi}{2} \right) \right] & j\frac{k_0 \varepsilon_r}{2Z_0 r}(gr)^2 \end{pmatrix} \qquad (22)$$

The determinant of these is the same as Eq. (16) with $h^2 = -g^2$.

The functions change considerably if the material is a conducting metal with small skin depth δ. The quantity h becomes essentially

$$h \approx \frac{1+j}{\delta} \qquad (23)$$

and hr is usually very large. Using the asymptotic values of Bessel functions of $x\sqrt{j}$ the field matrix, omitting multiplicative constants, is

$$M = \sqrt{\frac{\delta}{r}} \begin{pmatrix} e^{-hr} & e^{hr} \\[2mm] -\frac{\sigma}{h}e^{-hr} & \frac{\sigma}{h}e^{hr} \end{pmatrix} \qquad (24)$$

The full transform inward across a layer of thickness $t = b - a$ between radii b and a becomes

$$M(a)[M(b)]^{-1} = \sqrt{\frac{b}{a}} \begin{pmatrix} \cosh(ht) & -\frac{h}{j}\sinh(ht) \\[2mm] -\frac{\sigma}{h}\sinh(ht) & \cosh(ht) \end{pmatrix} \qquad (25)$$

A double metal layer

The beam tube at the interaction point for beams in the PEP II collider has two thin layers of beryllium. These are 0.8mm and 0.4mm thick separated by 1.6mm. Outside these is a metal shield. It is desired to know the e.m. field from the beam that reaches silicon detectors that are in the gap between beryllium and the outer shield. Field penetration should be greatest at the low frequency that is present when the beam current is modulated at the orbital frequency $f_0 = 136kHz$. We shall calculate the field per ampere of current at frequency f_0 ignoring effects of the detectors and any axial reflections along the Be tube. Parameters for the calculations are given in Table I.

Beryllium fills the radial spaces from a to b and from c to d. The shield, assumed to be thick and of copper is at radius e.

<div align="center">Table I</div>

Radii in mm

a = 25.0	Z_0	120π ohm
b = 25.8	Orbital frequency f_0	136 kHz
c = 27.4	$\beta\gamma$	6.07×10^3
d = 27.8	Conductivity of Be	3×10^7 mho/m
e = 29.8	Conductivity of Cu	5.8×10^7 mho/m

From the values in Table I we calculate

h in vacuum $\quad = \dfrac{k_0}{\beta\gamma} = 4.692 \times 10^{-5} \, m^{-1}$

δ in $Be \qquad = 2.492 \times 10^{-4} \, m$

$\sigma\delta$ in $Cu \qquad = 1.039 \times 10^4 \, mho$

At $r = e$, the copper shield, the ratio of E_z to H_ϕ is

$$Z_e = -\frac{1+j}{\sigma\delta} = -\frac{1+j}{1.039 \times 10^4} \; ohm \qquad (26)$$

In the Be layers, use Eq. (25), designating in Eq. (27), $M1$ for the first layer a to b and $M3$ for the second layer c to d. In vacuum, use Eq. (15), designated $M_v(r)$. We can then calculate the value at $r = a$ of

$$\begin{pmatrix} FE \\ FH \end{pmatrix} = M1 \cdot M_v(b) \cdot [M_v(c)]^{-1} \cdot M3 \cdot M_v(d) \cdot [M_v(e)]^{-1} \begin{pmatrix} Z_e \\ 1 \end{pmatrix} \qquad (27)$$

and further find the value of H_ϕ using Eq. (13) for $I = 1 \; ampere$:

$$HO = 6.366 \; A/m \qquad (28)$$

This result is numerically essentially the same as $I/2\pi a$ indicating that the radially incoming field adds very little to the H-field. This is usually the case.

From Eq. (10) we find

$$A_e = \frac{HO}{FH} = \frac{6.366}{11.898 - j117.45} \tag{29}$$

The fields at the shield from $I = 1$ ampere are then, using Eqs. (3) and (9)

$$H_\phi(e) = A_e = (5.51 + j54)10^{-3} \, A/m$$

$$E_z(e) = Z_e A_e = (4.71 - j5.77)10^{-6} \, V/m$$

$$E_r(e) = \frac{Z_0}{\beta} H_\phi = 2.08 + j20.5 \, V/m$$

We see in Eq. (29) that the quantity FH is the attenuation factor for the H-field. This factor of only 116 is approximately the same as the simple exponential attenuation through the 1.2mm of beryllium, indicating that reflections and impedance mismatches at the surfaces have contributed little. This can be the case here because the impedances E/H of the outgoing waves in Cu, Be, and vacuum are in this example coincidentally alike within a factor of 1.4.

It is also of interest that if one were to assign infinite conductivity to the shield rather than that of copper, the fields H_ϕ and E_r calculate to be 1.7 times stronger. This is a caution about using this common characterization of a conducting surface.

A kicker magnet

This kicker assembly has a pulsed ferrite H-magnet that surrounds a ceramic beam tube. It is desired to know the longitudinal beam impedance as a function of frequency. On the inner surface of the ceramic at radius a = 31.7 mm is a thin coating of Kovar alloy. The coating has surface resistivity $1/\sigma t$ of *0.3 ohm*/square. The ceramic tube wall is 6.4 mm thick. The yoke, a square frame of ferrite, is 25 mm thick and there are copper windings between ferrite and ceramic. These windings are normally open circuit. While these features do not have cylindrical symmetry, I shall assume that the windings act as neutral spaces, do not carry any net current and do not modify the impedance that the ferrite frame presents to the beam fields inside. The ferrite, between radii c and d, is surrounded by a shield of, we shall assume, copper. The parameters used are given in Table II.

Table II

Radii in mm		
a = 31.7	Ceramic permittivity ε	9
b = 38.1	Coating surface resistivity	*0.3 ohm/sq*
c = 40.0	Coating conductivity	*3.5x10⁵ mho/m*
d = 65.0	Coating permeability	1000
	Ferrite permittivity	10
	Ferrite permeability	1300
	Copper conductivity	*5.8x10⁷ mho/m*
	Z_0	*120π ohm*
	$\beta\gamma$	*6.07x10³*

As in the previous example, the impedance Z_e of the thick outer shield of copper is $-(1+j)/\sigma\delta$; in this case expressed as a function of frequency. Matrices for the layers of ferrite, vacuum, ceramic, and Kovar follow Eq. (5), (21), and (25). Numerical calculation of the Z'_B then gives the result plotted in Fig. 1. This result is substantially the same as one would obtain from an equivalent-circuit diagram having the resistance of the coating in parallel with the inductance of the magnet yoke; the ceramic acts no differently from a radial spacer.
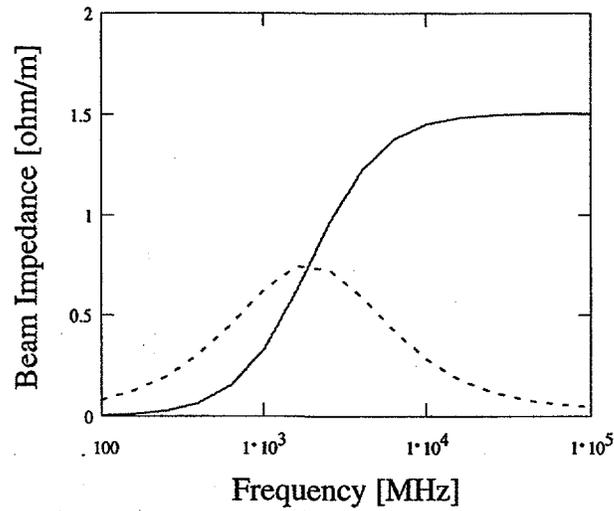
Figure 1: Beam impedance per unit length of the injection kicker. Solid and dashed curves are respectively the real and imaginary parts.

REFERENCES

1. Stratton, J.A., Electromagnetic Theory, McGraw Hill Co., New York, 1941, p. 360.
2. Keil, E., and Zotter, B., "Impedance of Layered Vacuum Chambers for Large Colliders," CERN SL/98-019(AP), June 1998.
3. Piwinski, A., "Penetration of the Field of a Bunched Beam through a Ceramic Vacuum Chamber with Metallic Coating," IEEE Trans. on Nuclear Science, Vol. NS-24, No.3, June 1977, p. 1364.