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Measurements of Secondary Cosmic Microwave Background Anisotropies with the
South Pole Telescope

by

Martin Lueker

A dissertation submitted in partial satisfaction of the
requirements for the degree of

Doctor of Philosophy

in

Physics

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

Professor William Holzapfel, Chair

Professor John Clarke,

Professor Geoffrey Bower

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Abstract

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University of California, Berkeley

Professor William Holzappel, Chair

The South Pole Telescope is a 10m millimeter-wavelength telescope for finding galaxy clusters via the thermal Sunyaev-Zel'dovich (tSZ) effect. This thesis is divided into two parts. The first part describes the development of the kilopixel SPT-SZ receiver and the frequency-domain multiplexor (fMUX). The second part describes the first SPT power spectrum measurement and the first detection of the tSZ power spectrum.

The SPT-SZ focal plane consists of 960 spiderweb coupled transition-edge sensors. Due to strong electro-thermal feedback, these devices have good sensitivity and linearity, though risk spontaneous oscillations. Adding heat capacity to these devices can ensure stability, so long as the loopgain, $\mathcal{L}$, is less than G_{int}/G_0 , the ratio between the thermal conductances linking the TES to the heat capacity and linking the heat capacity to the bath. I describe an experimental technique for measuring the internal thermal structure of these devices, allowing for rapid sensor evaluation.

The fMUX readout system reduces wiring complexity in this receiver by AC-biasing each sensor at a unique frequency and sending signals from multiple bolometers along one pair of wires. The Series SQUID Arrays (SSAs) used to read changes in bolometer current are notably non-linear and extremely sensitive to ambient magnetic fields. The SSAs are housed in compact magnetic shielding modules which reduces their effective area to $80 \text{ m}\Phi_0/\text{gauss}$. The SSA are feedback with a flux-locked loop to improve their linearity and dynamic range, and decrease their input reactance. The FLL is bandwidth of 1 MHz with a measured loopgain of 10. In the current implementation, this bandwidth is limited between the SQUID input coil and other reactances, which I study in Chapter 4.

In the second part of the thesis I present power spectrum measurements for the first 100 deg^2 field observed by the SPT. On angular scales where the primary CMB anisotropy is dominant, $\ell \lesssim 3000$, the SPT power spectrum is consistent with the standard ΛCDM cosmology. On smaller scales, we see strong evidence for a point source contribution, consistent with a population of dusty, star-forming galaxies. I combine the 150 and 220 GHz data to remove the majority of the point source power, and

use the point source subtracted spectrum to detect Sunyaev-Zel'dovich (SZ) power at 2.6σ . At $\ell = 3000$, the SZ power in the subtracted bandpowers is $4.2 \pm 1.5 \mu\text{K}^2$, which is significantly lower than the power predicted by a fiducial model using WMAP5 cosmological parameters.

To my dear—and extremely patient—wife, Sirena.
Your support and encouragement made all of this possible.

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Chapter 1

Cosmological Background

In this chapter I provide an overview of the observational basis for the standard “ Λ CDM” cosmological model. All of the ideas here have been covered in a wide collection of textbooks, though given the tremendous amount of progress that has been made in the last 15 years, I would recommend starting with a recent textbook. I particularly enjoy the treatment by Dodelson (2003), and will use it as the basis for much of the foundation work here.

1.1 The Smooth Expanding Universe

The Λ CDM model is part of a class of cosmologies which are philosophically grounded in the Cosmological Principle: that there is no special vantage point in the universe, and on the largest scales, the properties of the universe appear the same for all observers. Based on this idea of a homogeneous isotropic universe, one can derive the Friedmann equations, which describe the dynamics of a universe filled with a homogeneous, fluid mixture. The inhomogeneities that give rise to the structure we see today can then be treated as variations around this bulk flow.

In the framework of General Relativity such an isotropic universe is described by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

$$ds^2 = -dt^2 + a^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) \quad (1.1)$$

Here and throughout this chapter we use natural units, setting $c = 1$. Changes in the scale parameter, $a(t)$, lead to an observed change in the separation between two objects. By convention, the scale parameter is defined to be unity at the current time: $a(t_0) = 1$. The time-derivative of the scale parameter represents the rate of cosmological expansion observed at any particular time, and is thus related to the Hubble parameter $H = \dot{a}/a$. We use the notation H_0 to represent the Hubble constant at the current time, t_0 .

The symbol k in Equation 1.1 represents the global curvature of space-time. The curvature depends on the energy density of the universe, and is predictive of the fate of the universe. A closed universe, one with $k > 0$, is gravitationally bound and will eventually lose momentum and collapse back in on itself. Meanwhile an open universe is one with $k < 0$ will expand forever. The value of k is related to the combined energy density of the universe. $k = \frac{H_0^2}{c^2}(1 - \sum_i \rho_i/\rho_{cr})$, where ρ_i is the energy density of each individual component (dark matter, baryons, radiation, etc.) and $\rho_{cr} = 3H_0/8\pi G$ is the critical density. As a shorthand, one often expresses component densities in units of the critical density:

$$\Omega_i \equiv \rho_i(t_0)/\rho_{cr}. \quad (1.2)$$

Thus Ω_m , Ω_r , and Ω_Λ represent the energy densities associated with matter, radiation and dark energy, all normalized to the critical density at the current time.

Given the relative simplicity of this metric the ten Einstein equations are reduced to two:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3} \quad (1.3)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G(\rho + 3p) + \frac{\Lambda}{3} \quad (1.4)$$

Here the dot over a indicates the a time-derivative, and G is the gravitational constant. The terms ρ and p are the instantaneous mass-energy and pressure associated with the contents of the universe. Non-relativistic matter, ρ_m , carries little pressure, $p_m \approx 0$, whereas the pressure of radiation or some highly relativistic gas is $p_r = \rho_r/3$.

The term Λ is referred to as the cosmological constant, without which the Friedmann equations predict a universe which is never stationary. Einstein was certain that such a term was needed to match his view of a static, unchanging universe and so he added the term Λ to allow for static solutions. Once Hubble observed that the Universe was indeed expanding, Einstein felt no need to keep this term. However, as we shall see in the next section, present experimental data not only supports a non-zero cosmological constant, this term, or some similar “Dark Energy” dominates the dynamics of the universe at the current time.

For a given fluid component be it baryonic gas or radiation, it is convenient to express the pressure-density relationship by its equation of state $w = p/\rho$. For instance, radiation has $w = 1/3$, while matter has $w = 0$. The cosmological constant is often represented as a fluid component with density, $\rho_\Lambda = \Lambda/8\pi G$, and equation of state $w = -1$. It can be shown by combining Equations 1.3 and 1.4 that each individual fluid component evolves as:

$$\rho_i(a) = \Omega_i \rho_{cr} a^{-3(1+w_i)}. \quad (1.5)$$

So the Friedmann equations can then be written in a more compact form:

$$\frac{H^2}{H_0^2} = \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_\Lambda + \Omega_k a^{-2}, \quad (1.6)$$

where the curvature term, $\Omega_k \equiv 1 - (\Omega_m + \Omega_\Lambda + \Omega_r)$, depends on the cumulative density of the other three components discussed here: radiation, matter and the cosmological constant.

1.2 Experimental Evidence for Dark Energy

In this section I describe the current experimental evidence for the existence of a non-zero cosmological constant, Λ , and the abundance of cold dark matter (CDM). Understanding the nature of these exotic components in this Λ CDM model is one of the major problems in physics today.

1.2.1 Type Ia Supernovae

The first observational evidence for dark energy came in 1999 from two independent different groups (Perlmutter et al., 1999; Riess et al., 1998), both working to refine measurements of the expansion history of the universe, though observations of the redshift-magnitude relation in high-redshift Type Ia supernovae. Type Ia supernovae erupt when a white dwarf has accreted enough mass to exceed the Chandrasekhar mass limit, beyond which a white dwarf can no longer be supported by electron degeneracy pressure. Given that these objects always erupt at the same mass, their luminosity is nearly constant, making them a “standard candle” for measuring cosmological distances based on their apparent magnitude.

The results were inconsistent with a flat, matter-dominated universe. Both groups both measured the deceleration parameter:

$$q_0 \equiv -\ddot{a}(t_0)/H_0^2 \approx \frac{\Omega_m}{2} - \Omega_\Lambda. \quad (1.7)$$

to be much less than the $\Omega_m = 1$ prediction of $q_0 = 0.5$. Moreover as shown in Figure 1.1, q_0 was found to be most likely negative: $q_0 = -1 \pm 0.4$ (Riess et al., 1998). Thus rather than being decelerated by gravitational self-attraction, the universe was observed to be actually *accelerating*.

Though profound, this measurement still left some degeneracy between Ω_m and Ω_Λ . For instance, if one ignored Dark Energy, fixing $\Lambda = 0$, the data were consistent with an open universe: $\Omega_m = 0.2 \pm 0.4$, whereas in a flat universe the data would be consistent with $\Omega_\Lambda = 0$. Fortunately, there are other observables that we can study to further constrain these parameters, such as the Cosmic Microwave Background (CMB).

1.2.2 The Cosmic Microwave Background

The CMB is noted for its extreme homogeneity. After the CMB was first detected by Penzias & Wilson (1965), it took over 20 years before the first anisotropies were

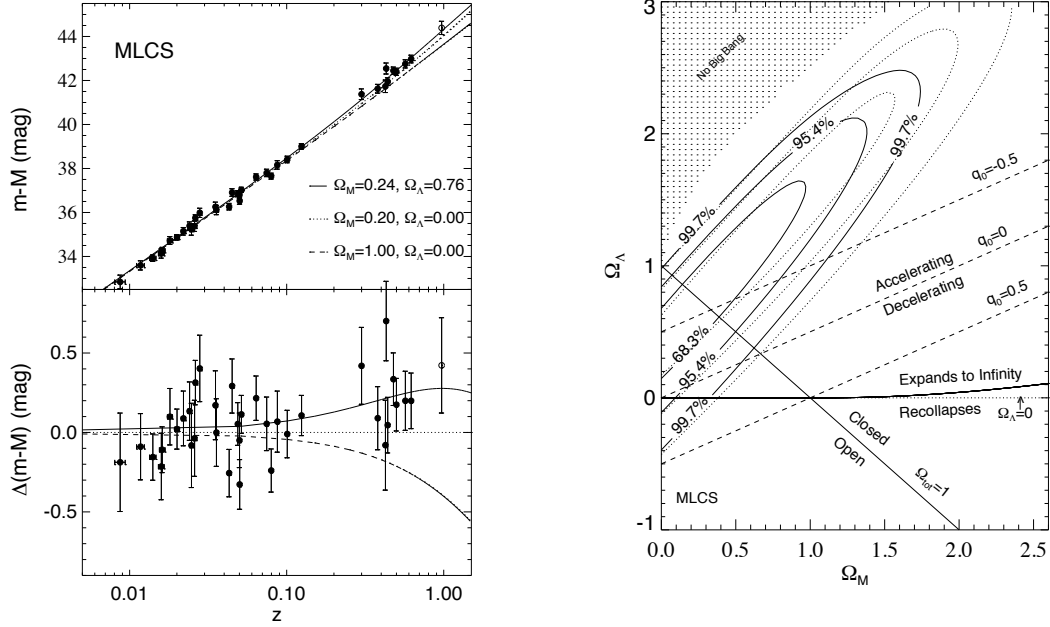


Figure 1.1: *Left Panel:* Magnitude-Redshift diagrams for distance Type Ia supernovae. Given the fixed luminosity of these objects the magnitude is used as proxy for the luminosity distance. The data strongly exclude a flat $\Omega_m = 1$, $\Omega_\Lambda = 0$ universe. *Right Panel:* Constraints on Ω_m and Ω_Λ . The data prefer a *negative* deceleration parameter, q_0 .

detected (Smoot et al., 1990). Though faint, these anisotropies are a direct record of density fluctuations at the surface of last scattering. Before the decoupling of matter and radiation, the pressure from radiation led to the production of acoustic waves in the primordial plasma, and it is a snapshot of these waves which are imprinted in the CMB. In particular, some waves would have had just enough time to reach their maximum amplitude at the epoch of decoupling, while others are at a null in their amplitude fluctuations. These acoustic waves lead to a sequence of peaks in the CMB power spectrum as shown in Figure 1.2. Since the universe was so nearly homogeneous at this time, the physical size corresponding to these peaks can be computed analytically, making them a *standard ruler* for cosmology.

The apparent angular scale of these peaks is an excellent measure of the curvature parameter, Ω_k (For an excellent review of how the spectrum of the CMB depends on Ω_k and other parameters see Hu & White, 1996). Such measurements have now been done with extreme precision (Hinshaw et al., 2009; Komatsu et al., 2009), and the

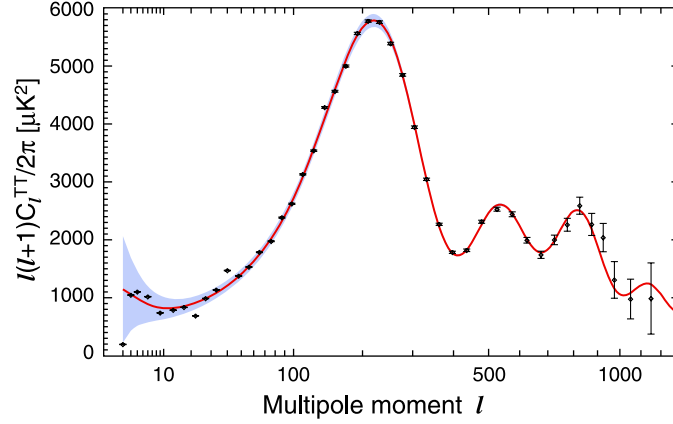


Figure 1.2: Large- to intermediate-scale CMB anisotropies. The best fit model is shown by the **red trace** whereas the **black data points** are from the WMAP experiment (Komatsu et al., 2009). The **grey shaded areas** indicate cosmic variance, the intrinsic sampling uncertainty associated with each monopole due to the limited number of observable modes.

constraints on Ω_m and Ω_Λ are shown in Figure 1.3. These measurements on their own exhibit some degeneracy with the H_0 , though adding in constraints on H_0 from other sources, such as the aforementioned supernovae measurements, confirms that the universe is flat with $\Omega_m + \Omega_\Lambda = 1$.

1.2.3 Large-Scale Structure and Baryon Acoustic Oscillations

Together, measurements of curvature and cosmic acceleration can be combined to produce a very peculiar story: the geometry of the universe is nearly flat—its total energy density is inferred to be nearly equal to the critical density. However 73% of this energy is associated with the cosmological constant or some other form of “dark energy”. This result is at once amazing and confounding, and may prompt us to question our understanding of the physics behind one or both of these results. The large-scale structure (LSS) of galaxy distributions is yet a third observable to either independently confirm or further constrain this model.

For instance, we can look at large-scale structure (LSS) in the distribution of galaxies and clusters of galaxies. The first way we constrain cosmology with LSS is to look at the underlying matter power-spectrum which underlies the galaxy distributions. This power spectrum depends strongly on the matter density, Ω_m , since it is the balance between matter and radiation which determines the extent to which structure is damped on certain scales. (Dodelson, 2003, chapter 7)

We can also use correlations in the galaxy distribution as a standard ruler. The acoustic oscillations which gave us our standard ruler in the CMB should also be

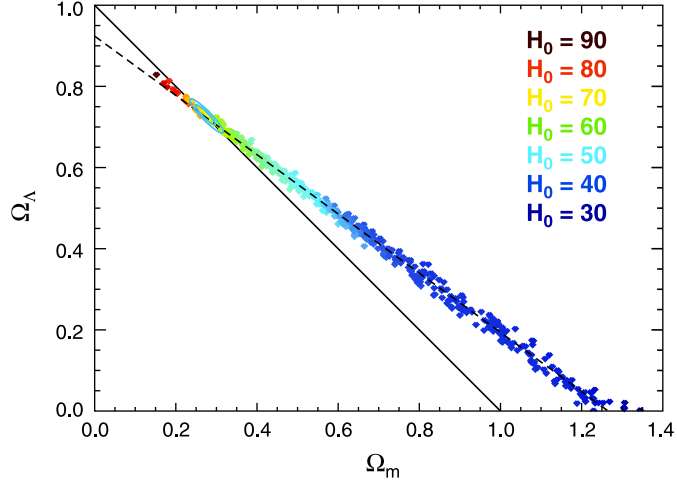


Figure 1.3: Constraints on Ω_m and Ω_Λ from Komatsu et al. (2009). The angular scale of the acoustic peaks mostly determines the curvature $\Omega_k = 1 - \Omega_m - \Omega_\Lambda$. However there is a slight additional dependence on Ω_Λ since the recent accelerated expansion also effects the apparent angular scale of the acoustic peaks, as for instance shown by Hu & White (1996)

apparent in the spatial distribution of galaxies. Such baryon-acoustic oscillations (BAO) in the galaxy-galaxy correlation function were first observed by the Sloan Digital Sky Survey (SDSS). We can study the evolution of the angular diameter distance as a function of redshift. Yielding constraints on Ω_m and Ω_Λ which are complementary to both Type Ia supernovae or the CMB.

The combined cosmological constraints from these three observations are shown in Figure 1.4. In this comparison all three results seem to agree on the same picture: that most of the energy in the universe is of a form that we don't understand, though it is consistent with Einstein's cosmological constant, Λ .

1.2.4 Beyond the Cosmological Constant: The Dark Energy Equation of State

Data from a wide variety of source confirm that the universe is flat and accelerating, but explaining the origin of all of the Dark Energy driving this acceleration is a vexing theoretical problem (For a summary of the different branches in the theoretical approach see e.g. Bean et al., 2005). For instance many field theory models have be devised to explain the amplitude of Ω_Λ , however most of them are can only be distinguished from a generic cosmological constant if it can be determined that w is time dependent, or at the very least $w \equiv p_\Lambda/\rho_\Lambda \neq -1$. From the three observables described in this section, the current best constraint is $w = -0.980 \pm 0.53$, consistent

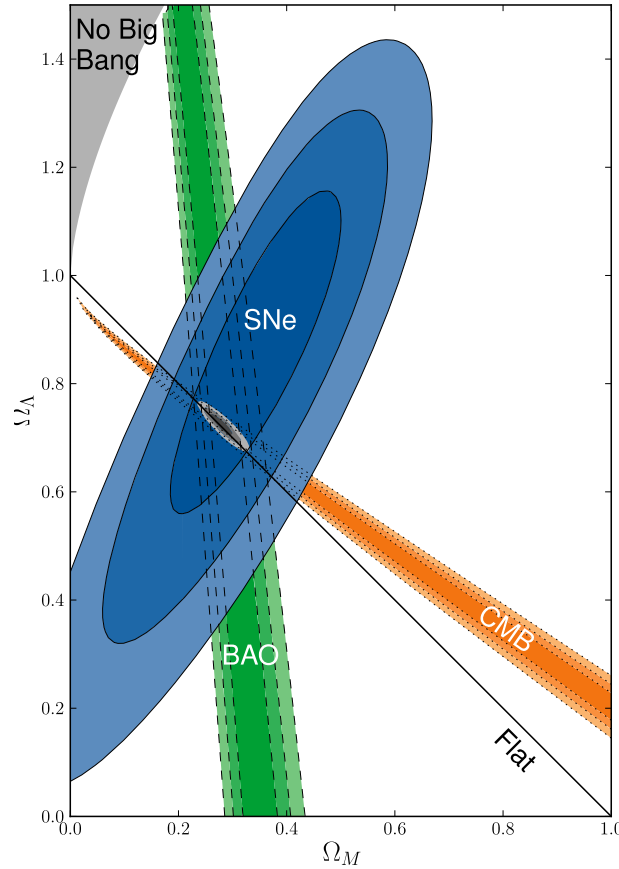


Figure 1.4: Constraints on Ω_m and Ω_Λ combining CMB, Type I SNe and BAO, from Amanullah et al. (2010). Note how all three measurements intersect at the same point, reinforcing the standard model.

with a cosmological constant.

1.3 The Non-linear Growth of Structure

It is plain to see that the universe is no longer homogeneous, except on very large scales. Planets, stars, galaxies and clusters of galaxies are all examples of objects which have collapsed under the force of gravity. However these objects all once started as small density fluctuations such as those seen in the CMB. Though based on the amazing homogeneity of the CMB we know that these fluctuations were once small compared to the mean density, $\bar{\rho}$, or in other words the density contrast, $\delta(\mathbf{x}) = (\rho(\mathbf{x}) - \bar{\rho})/\bar{\rho}$, was much smaller than unity during the epoch of recombination. At such early times when $\delta \ll 1$, the growth of structure and the evolution of δ is

described by linear equations and so the density contrast of any given region depends only on time. Thus the ratio of densities at two different times can be described by the growth function: $D(t; t_*) = \delta(\mathbf{x}, t)/\delta(\mathbf{x}, t_*)$. Typically one defines the growth function with respect to the current time: $D(t) = D(t, t_0)$. Thus when theoretically reporting density contrasts $\delta(\mathbf{x})$, one is reporting the density contrast that *would be* observed if one were to assume linear structure formation.

Obviously the density contrast is much larger than unity today, as one can see simply by comparing the density of the Earth or the interstellar medium to the critical density. All of the structures we observe today have arisen from *nonlinear* gravitational collapse. The number of collapsed structures of a given mass can be a powerful probe of cosmology. This is particularly true where at the largest mass scales where the astrophysics is dominated by the interplay between the gravitational forces and opposing pressure of Dark Energy. One can therefore study dark energy by measuring the mass function, $dn(M, t)/dM$, the number density of objects of mass M , which have collapsed by the time t .

The mass function was first computed for a flat matter-dominated universe by Press & Schechter (1974), and is nicely summarized by Lacey & Cole (1994). Within this cosmological model, we consider a spherical region within which the density is constant but higher than the critical density $\rho(\mathbf{x}) > \rho_{cr}$. This matter distribution is referred to as a “spherical tophat” distribution. Such a region is bound to eventually collapse. The time required to do so, t_c , will depend on the amplitude of the density contrast of that region and can be computed from the Friedmann equations. Another way of looking at this is to define a critical density, $\delta_{cr}(t)$, which expresses the density contrast of structures which are just collapsing at time t . For instance, if $\Omega_m = 1$, a spherical region with density contrast δ_* at some early time t_* , would collapse to a point at some later time (Lacey & Cole, 1994): $t \propto t_* \delta_*^{-3/2}$. Thus by this cosmological model, the critical density scales as:

$$\delta_c(t) = \delta_{cr}(t_0)(t/t_0)^{-2/3}. \quad (1.8)$$

As one would expect, more dense systems will collapse earlier.

Press & Schechter (1974) studied the number density of objects of a particular mass, $n(M)$ under the assumption that these objects arose from some gaussian-distributed density distribution, $\delta(\mathbf{x})$, and spherically collapsed by some time t . The number of collapsed objects is predicted by counting the density of regions enclosing a mass M , where the mean overdensity exceeds the $\delta_{cr}(t)$. The variances of such regions can be computed by smoothing the initial density distribution over a radius $R = \left(\frac{3M}{4\pi\bar{\rho}}\right)^{1/3}$:

$$\sigma(R) \equiv \sqrt{\left\langle \left[\int d^3\mathbf{x}' W_R(\mathbf{x} - \mathbf{x}') \delta(\mathbf{x}') \right]^2 \right\rangle}. \quad (1.9)$$

Here the filter kernel can be nearly any function with characteristic radius R , such as a simple Top Hat:

$$W_R(\mathbf{x}) = \begin{cases} 3/4\pi R^3 & |\mathbf{x}| < R \\ 0 & |\mathbf{x}| > R \end{cases} \quad (1.10)$$

At a time t , the number density of objects with mass M is then given by a gaussian distribution (Lacey & Cole, 1994):

$$\frac{dn}{dM}(t) \propto \frac{\bar{\rho}}{M^2} \frac{\delta_c(t)}{\sigma(M)} \left| \frac{d \log \sigma}{d \log M} \right| e^{-\delta_c^2(t)/2\sigma^2(M)} \quad (1.11)$$

The variance on large scales is smaller than that on small scales as a result of this smoothing in Equation 1.9, and thus on average these objects on larger scales take longer to collapse.

In Equations 1.8 and 1.11, one would expect any region with $\delta > 0$ to collapse eventually, and for large objects to continue to form up to the modern epoch. However one should remember that these equations as presented here are for a flat matter-dominated cosmology. Dark Energy tends to inhibit structure formation. In the standard Λ CDM model, any structures which have not formed by the time Λ -dominated era will be smoothed away by the cosmic acceleration. For more complicated cosmologies, numerical simulations are used to calibrate the expected scaling factor for Equation 1.11 and to compute variations in these scalings based on changes in cosmological parameters (For instance, see Tinker et al., 2008).

1.4 Constraining Dark Energy with the Galaxy Clusters and the Sunyaev-Zel'dovich effect

With masses as high as $10^{15}M_\odot$ or more, galaxy clusters are the most massive collapsed structures in the universe. They are also the latest to form, making them more sensitive to the latest phase of dark-energy-dominated expansion and the properties of dark energy. Also being so large, their overall composition is consistent with that of the universe as a whole. The gas component is extremely hot ($10^8 K$ or more) and ionized. This hot electron gas can then either be detected by its thermal bremsstrahlung emission or by its interaction with the microwave background.

As the CMB photons pass through a cluster, a small fraction of them Inverse Compton scatter off the hot intra-cluster gas, gaining energy in the process. An observer viewing the CMB through a cluster will see an excess of high-frequency photons which have been scattered into the line of sight, and a decrement of lower-frequency photons, where the cool background photons have been scattered away, an effect first described by Sunyaev & Zel'dovich (1972). This thermal Sunyaev-Zel'dovich (tSZ) effect is often calibrated as an equivalent temperature fluctuation in

the CMB:

$$\frac{\Delta T(\theta)}{T_{\text{CMB}}} = y(\theta) \left(x \frac{e^x + 1}{e^x - 1} - 4 \right), \quad (1.12)$$

where $x \equiv \frac{h\nu}{k_b T_{\text{CMB}}}$ is a dimensionless frequency parameter. Meanwhile the Compton y-parameter,

$$y(\theta) = \int n_e \sigma_T \left(\frac{kT_e}{m_e c^2} \right), \quad (1.13)$$

represents the total integrated electron-gas pressure along the line of sight. Since this effect manifests itself as a frequency distortion of the CMB, the intensity of the effect depends only on the properties of the clusters, and is independent of their redshift. Thus the tSZ effect is a nearly redshift-independent means of building a cluster catalog of all clusters above a certain mass threshold, M_{th} , to the redshift of their redshift of formation.

In order to constrain cosmological parameters from such a catalog, the observed clusters are sorted into redshift bins, and their density is reported as a surface density, or the number per unit solid angle per unit redshift: $\frac{dN}{dz d\Omega}$. This quantity depends not only on the co-moving abundance of clusters at redshift z , but also the scaling of the comoving volume element (Haiman et al., 2001), $\frac{dV}{dz d\Omega}$:

$$\frac{dN}{dz d\Omega} = \frac{dV}{dz d\Omega} \int_{M_{th}(z)}^{\infty} \frac{dn(z, M)}{dM}. \quad (1.14)$$

As mentioned before the SZ effect is only weakly dependent on redshift. So the detection threshold, $M_{th}(z)$, depends less on redshift and more on the sensitivity of the instrument and the length of observation.

1.5 The Power Spectrum of the tSZ Effect

Thus far I have focused on how galaxy cluster surveys can constrain the dark energy equation of state. This approach counts clusters with mass above some threshold of order $10^{14} M_{\odot}$. However there is a wealth of structure in the SZ effect due to objects below the mass threshold. The SZ flux from these objects may not be resolved individually, though they are detectable from a measurement of the SZ power spectrum. The shape of this power spectrum has been calculated analytically by Komatsu & Seljak (2002), and is shown in Figure 1.6. The power spectrum peaks at $\ell = 3000$, where the amplitude of the primary CMB spectrum is rapidly falling. The σ_8 -dependence of the SZ power-spectrum is expected to be extremely steep: $\mathcal{D}_{\text{SZ}} \propto \sigma_8^{\alpha}$, where Komatsu & Seljak (2002) estimate an exponent of $\alpha \sim 7$, though for lower values of σ_8 , α can be as steep as 9. Thus this spectrum can be used to set tight constraints on σ_8 which in turn refines the normalization of the matter power spectrum, lending more power to cosmological constraints from the CMB or cluster survey counts.

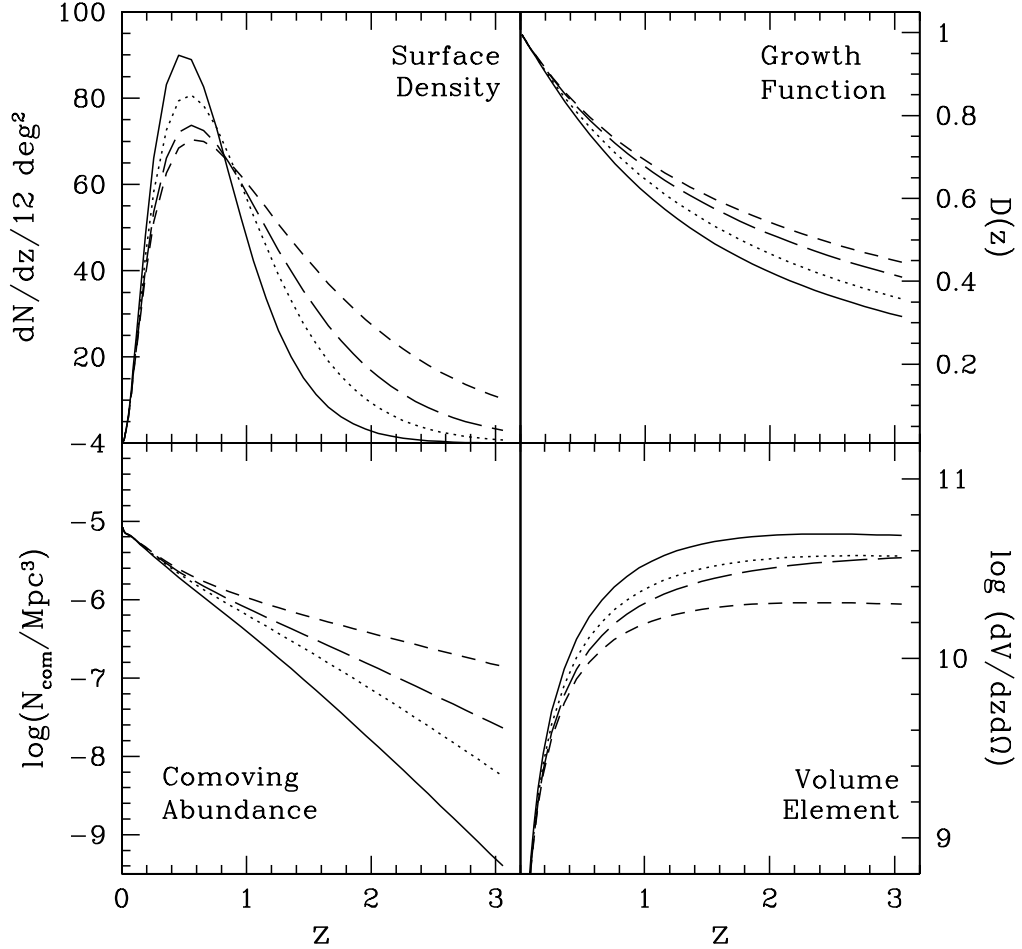


Figure 1.5: The abundance of galaxy clusters above a given mass threshold, M_{th} , as a function of w , as predicted by Haiman et al. (2001). In the top left panel, the **solid line** illustrates the cluster abundance predicted by the authors' fiducial cosmological model: $(\Omega_m, \Omega_\Lambda, h, \sigma_8, n, w) = (0.7, 0.3, 0.65, 0.9, 1, -1)$. Most recent observations have shifted these parameters leading to dramatically lower expectations for cluster abundance, though the paper still serves as good description of the physical motivation for performing galaxy cluster surveys. The **dotted curve** shows the same model with $w = -0.6$, the **short-dashed curve** shows $w = -0.2$, and the **long-dashed curve** shows an open cosmology model with $\Omega_m = 0.3$, and $\Omega_\Lambda = 0$. Also shown are the growth function, $D(z)$, the comoving abundance, $n(M_{th})$, and the volume element $\frac{dV}{dz d\Omega}$. All results are normalized to the locally observed cluster density at $z = 0$.

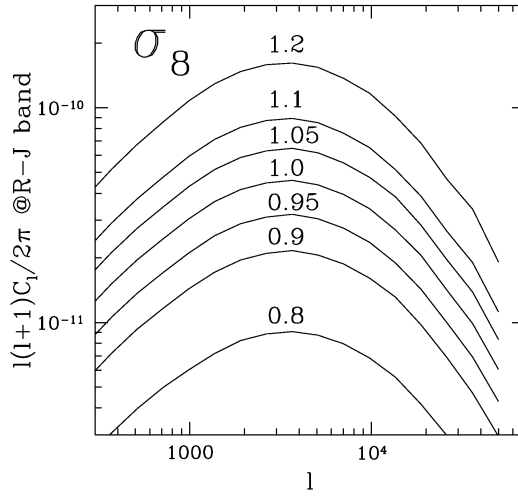


Figure 1.6: An analytical prediction of the tSZ power spectrum from Komatsu & Seljak (2002).

Perhaps what is most interesting about this spectrum is that it probes a unique set of galaxy clusters. Near $\ell = 3000$, where the tSZ power spectrum is the most pronounced, half of the power comes from clusters with mass $M < 2 \times 10^{14} M_{\odot}$, as shown in Figure 1.7. Meanwhile as a function of redshift, approximately half of the tSZ power spectrum comes from objects at high redshifts, $z > 1$. Thus a measurement of the tSZ power spectrum probes a sample of clusters which is unique from SZ cluster surveys, which probe high-mass systems, and X-ray cluster surveys, which are more sensitive at low redshifts.

There are some systematic uncertainties in interpreting measurements of the tSZ spectrum. The SZ-flux of these low-mass and/or high- z systems is harder to model due to astrophysical effects beyond just gravitational collapse. For instance, the electron pressure in lower mass clusters is more sensitive to non-gravitational heating effects such as star-formation or emission from active galactic nuclei (AGN). Meanwhile we have limited observational data on high-redshift clusters. That said, there is still some uncertainty as to the expected shape and amplitude of the tSZ power spectrum due to the current lack of knowledge of the properties of intracluster gas in low-mass and high-redshift galaxy clusters.

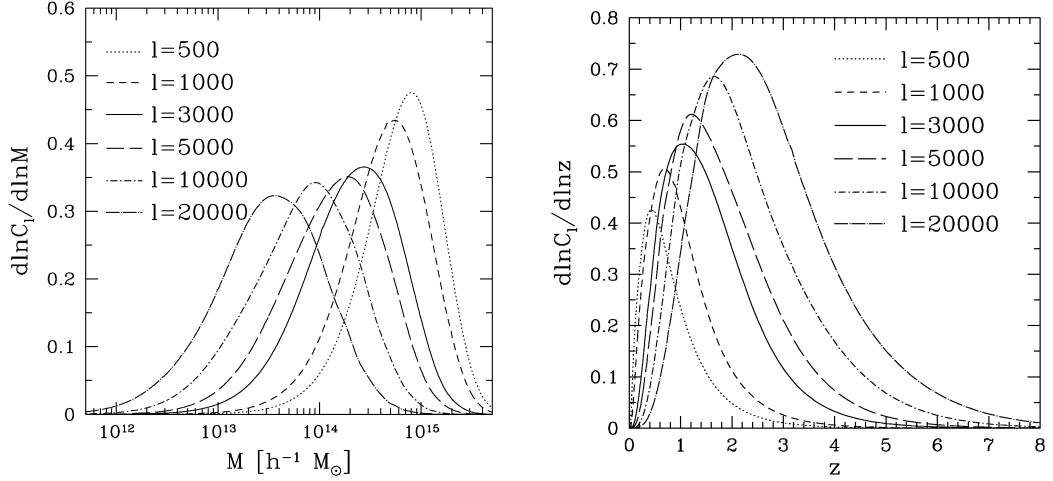


Figure 1.7: Another pair of figures from Komatsu & Seljak (2002) showing the populations probed by the tSZ spectrum, as a function of multipole. We are most interested in the spectrum near $\ell \sim 3000$, shown by the **solid traces**. This is where the tSZ spectrum is strongest relative to other sources such as the CMB primary anisotropies or emission from dusty galaxies. The **left panel** shows the dependence of the tSZ spectrum on mass, showing that in our range of interest much of this spectrum comes from $M < 2 \times 10^{14} M_{\odot}$ sources. The **right panel** shows the redshift dependence, illustrating our strong sensitivity to sources at $z > 1$.

1.6 Other Contributions to the Microwave Power Spectrum

We have already discussed in Section 1.2.2 the primary anisotropies in the CMB and their constraints to the Λ CDM cosmological model. Interactions between the CMB and intervening matter, such as the tSZ effect, are referred to as secondary anisotropies. On angular scales $\lesssim 10$ arcminutes, the primary CMB anisotropy is exponentially damped due to photon diffusion in the primordial plasma (Silk, 1968); the resulting decline in power with increasing multipole is known as the “damping tail”. The anisotropy on very small scales, which is only beginning to be explored experimentally, is instead dominated by foreground emission and secondary distortions. However on larger angular scales, multipoles below $\ell \sim 2000$, the primary anisotropies are expected to dominate and introduce uncertainty in the tSZ power spectrum at lower ℓ .

The scattered CMB photons also obtain a net Doppler shift when ionized matter is moving with respect to the rest frame of the CMB. This effect depends solely on the motion and density of free electrons. When the ionized gas is bound to a cluster

this effect is referred to as the kinetic Sunyaev-Zel'dovich (kSZ) effect (Sunyaev & Zeldovich, 1980). However this effect can also be generated on larger scales, by bulk matter flows after the epoch of reionization, in what is called the Ostriker-Vishniac (OV) effect (Ostriker & Vishniac, 1986). These two effects differ in origin and physical scale, though they are otherwise difficult to distinguish. For simplicity in this thesis, we refer to all anisotropies from ionized gas flows as the kSZ effect.

In contrast to the tSZ effect, the kSZ effect has contributions from electrons with temperatures as low as 10^4 K. Therefore higher-redshift epochs, before massive objects finish collapsing, are expected to have relatively larger contributions to the kSZ power. Recent simulations and analytic models also predict a sizable signal from the epoch of the first radiative sources which form ionized regions several tens of Mpc across, within a largely neutral Universe. Low-redshift galaxy clusters dominate the power on small angular scales, while high-redshift reionizing regions have their largest relative contribution on angular scales around $\ell = 2000$. At 150 GHz, the kinetic effect is expected to amount to tens of percent of the total SZ power.

In addition to the tSZ and kSZ effects, foreground emission is important on these small angular scales. After bright radio sources are removed, the most significant foreground at 150 and 220 GHz is expected to be a population of unresolved, faint, dusty, star-forming galaxies (DSFGs) with a rest frame emission spectrum that peaks in the far infrared. These sources have been studied at higher frequencies close to the peak of their emission spectrum¹ (e.g. Holland et al. (1999); Kreysa et al. (1998); Glenn et al. (1998); Viero et al. (2009)), however extrapolating their fluxes to 150 GHz remains uncertain. Adding to the challenge is the expected significant clustering of these sources (Haiman & Knox, 2000; Knox et al., 2001; Righi et al., 2008; Sehgal et al., 2010). IR emission from clustered DSFGs was first observed with the Spitzer telescope at $160\ \mu\text{m}$ (Lagache et al., 2007) and more recently this clustering has also been observed at sub-mm wavelengths by the BLAST experiment (Viero et al., 2009). The clustering of these DSFGs is expected to produce anisotropic power at 150 GHz with an angular power spectrum that is similar to that of the SZ effect. However, emission from DSFGs is spectrally separable from the SZ effect and the SZ power spectrum can be recovered by combining information from overlapping maps at 150 and 220 GHz.

1.7 The State of tSZ Power Spectrum Measurements Before the SPT

Anisotropy in the cosmic microwave background (CMB) has been well characterized on angular scales larger than a few arcminutes (Jones et al., 2006; Reichardt

¹Since these sources are typically brightest at sub-millimeter wavelengths they are also referred to in the literature as sub-millimeter galaxies (SMGs).

et al., 2009a; Nolta et al., 2009; Brown et al., 2009), but only a handful of experiments have had sufficient sensitivity and angular resolution to probe the damping tail of the CMB anisotropy. Early measurements at 30 GHz by CBI (Mason et al., 2003; Bond et al., 2005) reported a $> 3\sigma$ excess above the expected CMB power at multipoles of $\ell > 2000$. Observations with the BIMA array at 30 GHz (Dawson et al., 2006) also reported a nearly 2σ detection of excess power at $\ell = 5237$. However, more recently, the SZA experiment (also observing at 30 GHz) has published an upper limit of $149 \mu\text{K}^2$ at 95% confidence on excess power at these multipoles (Sharp et al., 2010) in apparent conflict with the previous CBI and BIMA results. For the relatively small patch (0.1 deg^2) observed by BIMA, the non-Gaussian nature of the SZ sky means that there is no significant tension between the BIMA and SZA results. The latest CBI measurements (Sievers et al., 2009) include more data, improved radio source removal, and a proper treatment of non-Gaussianity of the SZ sky. These measurements continue to suggest excess power but with a significance of only 1.6σ .

At 150 GHz, the ACBAR (Reichardt et al., 2009a) and QUaD (Friedman et al., 2009) experiments have both measured the damping tail of the primary CMB anisotropy at $\ell < 3000$ with high signal to noise. Either with or without the addition of the expected foreground and tSZ contributions, the power measured at the highest multipoles by both experiments is consistent with primary CMB anisotropy. In the last year, the results of 150 GHz observations out to $\ell = 10000$ made with the Bolocam (Sayers et al., 2009) and APEX-SZ (Reichardt et al., 2009b) experiments have been released. These experiments have been used to place upper limits on power above the primary CMB of $1080 \mu\text{K}^2$ and $105 \mu\text{K}^2$ respectively at 95% confidence. The constraints on σ_8 from these upper limits remain weak, in no small part due to the large, highly non-Gaussian sample variance of the tSZ effect on the small $\sim 1 \text{ deg}^2$ patches of sky observed by Bolocam and APEX-SZ. The cosmic variance of the tSZ effect will be significantly reduced in the on-going $\gtrsim 100 \text{ deg}^2$ surveys being conducted by next-generation experiments such as ACT (Fowler et al., 2007) and SPT.

In this work, I present measurements by the South Pole Telescope (SPT) which comprise the first significant detections of anisotropy power for $\ell > 3000$ at 150 and 220 GHz. The SPT has sufficient angular resolution, sensitivity and sky coverage to produce high-precision measurements of anisotropy over a range of multipoles from $\sim 100 < \ell < 9500$. However, for the immediate goal of measuring secondary CMB anisotropies, we start with the first bandpower at $\ell = 2000$ where primary CMB still dominates the power spectrum. We combine bandpowers from two frequencies to minimize the DSFG contribution and produce the first significant detection of the SZ contribution to the CMB power spectrum.

In Part I of this thesis, I describe many of the necessary ingredients toward building a successful instrument for mapping the SZ effect, covering the basics of superconducting transition-edge sensor (TES) bolometers, advances in the frequency-domain multiplexor readout needed to build a large TES array, and the design of the SPT itself. In Part II, I turn to measurements made by the SPT in the austral winter of

2008, the analysis of this data to generate the first SPT power spectrum results. I also cover the cosmological interpretation of this power spectrum, using multifrequency analysis to extract the tSZ signature from the DSFG emission.

Part I

Building a TES Bolometer Array for the South Pole Telescope

Chapter 2

Transition Edge Sensor Bolometers

At infrared or radio wavelengths there are two broad classes of radiation detectors. For coherent detectors, the sensor response is proportional to amplitude of the electromagnetic wave, whereas for direct detectors, the response is proportional to power. Coherent detectors are quite useful for interferometric observations or for other applications where high frequency-resolution is required. For wide bandwidth measurements, ($\Delta\nu/\nu_0 \approx 0.1$), direct detectors are more sensitive at wavelengths shorter than a few millimeters (Richards, 1994).

A bolometer is a type direct thermal detector which the radiation is coupled to a thermal isolated absorber, with heat capacity $C \equiv \frac{dQ}{dT}$, that is coupled to a thermal bath via a thermal conductance, $G \equiv \frac{dP}{dT}$. In the absence of feedback, a change in power, δP , leads to an instantaneous change in the bolometer temperature, $\frac{dT}{dt}(t=0) = \delta P/C$. With time, the temperature asymptotically approaches a new steady-state temperature, $\delta T = \delta P/G$. This shift in temperature with varying input power can then be measured, for instance by measuring the temperature-dependent resistance of a film mounted at the absorber as in Figure 2.1.

In order to achieve the maximal sensitivity, one chooses films with a very steep temperature-to-resistance dependence. Superconducting Transition-Edge Sensors (TES) make use of the fact that the resistance abruptly approaches zero at the transition temperature between the normal and superconducting states, as shown in Figure 2.2. The SPT devices consist of an AlTi bilayer with a transition temperature of $T_c \approx 550$.

As I will discuss in Section 2.2, the sensitivity of an individual bolometer has reached its fundamental limits, thus instrument sensitivity can only be increased by building larger bolometer arrays. Fabrication technologies for TES devices are now mature, and large arrays of these devices can be readily fabricated by standard microlithography techniques. The SPT contains a 960-element bolometer array, making it one of the most sensitive instruments for mapping the microwave background.

In rest of this Chapter I will discuss the principles of TES operation, such as the details of electrothermal feedback, and the sensitivity of such devices. This discussion

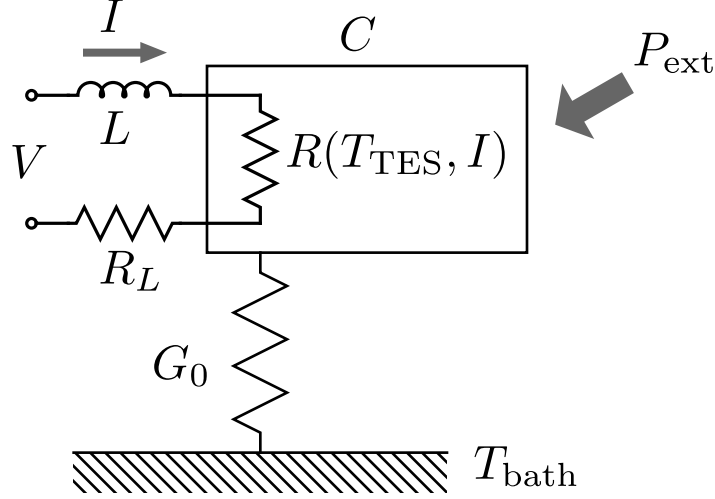


Figure 2.1: A Schematic Voltage-Biased Bolometer. A voltage bias is applied across the steeply temperature-dependent resistor $R(T_{\text{TES}}, I)$. The power dissipated by this voltage, P_{elec} , combined with any absorbed power from radiation, P_{ext} , must flow through a weak thermal link with conductance, G , leading to an elevated temperature, $T_{\text{TES}} > T_{\text{bath}}$. The response time of the bolometer is limited by the absorber heat capacity, C . In the case of the transition-edge sensors described in this chapter the dynamics also depend on the inductance L and the series load resistance, R_L .

will lay the foundation for future chapters where I discuss some to the details of the bolometer and readout design.

2.1 Electrothermal Feedback

Given that the the superconducting transition is so narrow, early experiments into the use of superconducting bolometers needed to use warm feedback electronics to keep the sensor within the active temperature range (Clarke et al., 1977). A fixed bias current would be pumped into the bolometer to heat it above the bath temperature, and into the superconducting transition. However it was also noted that the power dissipated in the sensor was also dependent upon resistance, and in the case of current bias, could lead to positive feedback and thermal runaway.

By contrast, it was later discovered that a bolometer placed under a fixed voltage bias, V , would experience *negative* electro-thermal feedback (Irwin, 1995). Decreases in optical power would lead to decreases in temperature and resistance, and thus an *increase* in the electrical bias power, $P_{\text{elec}} = \frac{V^2}{R}$. In the strong feedback limit, changes

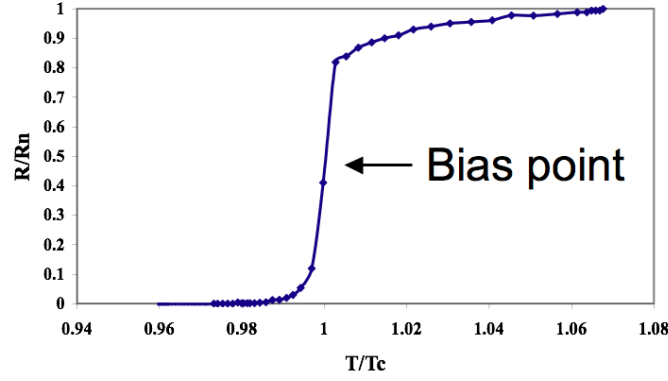


Figure 2.2: The resistance vs. temperature dependence of an Al-Ti bilayer near the superconducting transition temperature. When voltage-biased, the device is self-heating and for low-enough bias voltages the temperature is held near the critical temperature, T_c , by electro-thermal feedback. For the SPT devices, T_c is roughly 550mK.

in ambient power, δP_{ext} , are nearly matched by changes in electrical power: $\delta P_{\text{elec}} \approx -\delta P_{\text{ext}}$. The two power contributions cancel meaning that device holds itself in the superconducting transition, obviating the need for warm TES feedback electronics. In the strong-feedback limit the power-to-current responsivity, s_I , is simply $s_I = \frac{dI}{dP} \approx -\frac{dI}{dP_{\text{elec}}} = -1/V$. As one would expect for a strong negative feedback system, this simple form of the responsivity is independent of the bolometer properties, making these devices notably linear. The treatment of electrothermal feedback used in this chapter is based on Irwin & Hilton (2005), and will serve as a starting point for the discussion of TES operation in the SPT detectors.

The loopgain of this electro-thermal feedback network depends on steepness of the transition, as quantified by the logarithmic derivative of the resistance with respect to temperature: $\alpha \equiv \frac{\partial \log R}{\partial \log T} > 0$. Given that such a film is so near its transition temperature, it is also likely to have a very low critical current density. The resistance may therefore also be noticeably dependent on current, as quantified by the derivative $\beta \equiv \frac{\partial \log R}{\partial \log I} \geq 0$. These derivatives are usually calculated at the quiescent operating temperature, T_0 , and current I_0 . Small perturbations in the film temperature, δT or current, δI , lead to a change in resistance around the equilibrium value, R_0 :

$$\frac{\delta R}{R_0} = \alpha \frac{\delta T}{T_0} + \beta \frac{\delta I}{I_0} \quad (2.1)$$

So the change in electrical bias power, $\delta P_{\text{elec}} = \delta(I^2 R)$, due to such a perturbation is:

$$\delta P_{\text{elec}} = 2I_0 R_0 \delta + I_0^2 \delta R \quad (2.2)$$

$$= (2 + \beta) I_0 R_0 \delta I + \mathcal{L} G \delta T, \quad (2.3)$$

where in the second line I have introduced the ETF loopgain of the bolometer as $\mathcal{L} \equiv \frac{\alpha P_{\text{elec}}}{G T_0}$.

The equations of motion for δT and δI come from Ohm's Law:

$$L \frac{d}{dt} I = V - IR - IR_L, \quad (2.4)$$

and the conservation of energy equation:

$$C \frac{dT}{dt} = G(T - T_{\text{bath}}) + P_{\text{ext}} + P_{\text{elec}}. \quad (2.5)$$

It should be noted that the thermal transfer term, $P(T, T_{\text{bath}}) = G(T - T_{\text{bath}})$, is based on our approximation that the TES is near thermal equilibrium, $T \approx T_{\text{bath}}$. More generally, the power-temperature relationship is typically nonlinear in T and T_{bath} , and is represented as a polynomial:

$$P(T, T_{\text{bath}}) = K(T^{n+1} - T_{\text{bath}}^{n+1}) \quad (2.6)$$

Here the exponent n depends on the mechanism of thermal transport, and should be roughly 1 for electron transport, and 3 for phonon transport. The lead a temperature dependence in G , and leads to certain asymmetries in the analysis of bolometers with more complex thermal structure, as discussed in Appendix A.

The two equations of motion can also be linearly expanded in terms of δI and δT to obtain a pair of linear differential equations:

$$I_0 L \frac{d}{dt} \delta I = -G \mathcal{L} \delta T - (1 + \xi + \beta) I_0 R_0 \delta I + I_0 \delta V \quad (2.7)$$

$$C \frac{d}{dt} \delta T = (\mathcal{L} G - G) \delta T + (2 + \beta) I_0 R_0 \delta I + \delta P_{\text{ext}}. \quad (2.8)$$

Here we have replaced the stray-to-bolometer resistance ratio $\frac{R_L}{R_0} = \xi$ by a constant. These two equations now relate the current or temperature response to external perturbations in power, δP_{ext} or bias voltage, δV . In order to understand the solutions to these equations it is convenient to write them in matrix form:

$$\frac{d}{dt} \mathbf{v} = \mathbf{A} \mathbf{v} + \mathbf{p}, \quad (2.9)$$

where

$$\mathbf{v} \equiv \begin{pmatrix} L I_0 \delta I \\ C \delta T \end{pmatrix}, \quad \mathbf{p} \equiv \begin{pmatrix} I_0 \delta V \\ \delta P_{\text{ext}} \end{pmatrix} \quad (2.10)$$

and

$$\mathbf{A} \equiv \begin{pmatrix} -\tau_e^{-1} & -\mathcal{L}\tau^{-1} \\ \frac{2+\beta}{1+\beta+\xi}\tau_e^{-1} & (\mathcal{L}-1)\tau^{-1} \end{pmatrix} \quad (2.11)$$

Here, $\tau \equiv \frac{C}{G}$ and $\tau_e \equiv \frac{L}{(1+\beta+\xi)R_0}$ are thermal and electrical time constants. When $\mathcal{L} = 0$, these equations become decoupled. Then the current response to voltage perturbations and the temperature response to power are both well described by a single exponential decay. In this limit, τ describes the thermal decay time. That is, for a brief impulse in power, $\delta P(t) = \Delta P \Delta t \delta(t)$, the temperature response is $\delta T(t) = \frac{\Delta P \Delta t}{C} e^{t/\tau}$. Likewise, τ_e is the decay time associated with a voltage impulse ($\delta V(t) = \Delta V \Delta t \delta(t)$), for which the current response is $\delta I(t) = \frac{\Delta V \Delta t}{L} e^{t/\tau_e}$.

2.1.1 Frequency Response of a TES Bolometer

As non-zero loopgains, the generalized responsivity matrix, $\mathbf{A}$, is essential for understanding the frequency response, $I(\omega)$ or $T(\omega)$, of a detector to sinusoidal perturbations in voltage, ($\delta V = \Delta V e^{i\omega t}$), or power ($\delta P_{\text{ext}} = \Delta P_{\text{ext}} e^{i\omega t}$). By Equation 2.9 the frequency response will be:

$$\begin{pmatrix} I_0 R_0 I(\omega) \\ CT(\omega) \end{pmatrix} = (i\omega - \mathbf{A})^{-1} \begin{pmatrix} I_0 \Delta V \\ \Delta P_{\text{ext}} \end{pmatrix} \quad (2.12)$$

The power-to-current responsivity, $s_I = \frac{I(\omega)}{\Delta P_{\text{ext}}} (\Delta V = 0)$, is perhaps the most important quantity for converting the bolometer data to astronomical signals. At moderate loopgains, $\mathcal{L} \ll \frac{\tau}{\tau_e}$, s_I takes the simple form:

$$s_I(\omega) = -\frac{1}{I_0 R_0} \left(\frac{\mathcal{L}}{\mathcal{L}(1-\xi) + 1 + \beta} \right) \left(\frac{1}{1 + i\omega\tau_{\text{eff}}} \right) \left(\frac{1}{1 + i\omega\tau_e} \right) \quad (2.13)$$

Here τ_{eff} is the time-constant of the bolometer as sped up by feedback:

$$\tau_{\text{eff}} = \frac{\tau}{1 + \mathcal{L} \frac{1-\xi}{1+\xi+\beta}}. \quad (2.14)$$

At higher loopgain, the response still takes the form of a two-pole system, though feedback will lead to a more complicated interaction between the two time-constants (See Irwin & Hilton (2005) for details).

For detector diagnostic purposes, one commonly measures other such interesting quantities as the power-to-temperature responsivity, $S_T(\omega) \equiv \frac{dT}{dP_{\text{ext}}} = \frac{T(\omega)}{\Delta P}$, or the complex impedance of the sensor, $Z(\omega) = \frac{\Delta V}{I(\omega)} - i\omega L - \xi R_0$ (Irwin & Hilton, 2005). As we discuss in Chapter 5 such diagnostics can be very useful for understanding the detailed thermal structure of the bolometer.

2.1.2 Electrothermal Feedback Stability

The stability of a feedback system is classified in terms of its response to an impulse. A linear system is considered unstable if a small impulse drives it into exponentially growing oscillations. An underdamped system may oscillate in response to an impulse, but these oscillations will decay in amplitude with a time constant, τ_d . A system is called stable, or overdamped, if the perturbed system simply decays back to equilibrium.

For a system with two degrees of freedom, (such as our TES), the equation of motion takes the form of an exponential:

$$\mathbf{v}(t) = A\mathbf{v}_+e^{\lambda_+t} + B\mathbf{v}_-e^{\lambda_-t}, \quad (2.15)$$

where $\lambda_{\pm}$ are eigenvalues of $\mathbf{A}$.

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v}. \quad (2.16)$$

These eigenvalues can be written as:

$$\lambda_{\pm} = \frac{\text{Tr}(\mathbf{A})}{2} \pm \frac{1}{2}\sqrt{\text{Tr}(\mathbf{A})^2 - 4\text{Det}(\mathbf{A})}. \quad (2.17)$$

If the real portion of either eigenvalue is positive, then the detector will be unstable. So for stability $\text{Tr}(\mathbf{A}) < 0$, or $\mathcal{L} < \frac{\tau}{\tau_e} + 1$. The second requirement, $\text{Det}(\mathbf{A}) > 0$, leads to a constraint on the loopgain due to the series resistance, $R_L = \xi R_0$: $\mathcal{L} < \frac{1+\beta+\xi}{\xi-1}$. If the effective series resistance is low compared to the TES resistance, $|\xi| \ll 1$ then this requirement is satisfied.

If either eigenvalue has an imaginary component then the TES will become underdamped. Thus for stable, overdamped operation we require: $\text{Det}(\mathbf{A}) < \frac{\text{Tr}(\mathbf{A})^2 f}{4}$. In the high loopgain limit, $\mathcal{L} \gg 1$, and neglecting β or ξ , this requirement reduces to:

$$\mathcal{L} < \left(3 - 2\sqrt{2}\right) \frac{\tau}{\tau_e} \approx \frac{1}{5.8} \frac{\tau}{\tau_e} \quad (2.18)$$

This stability requirement is commonly cited and was published in Irwin et al. (1998). Violation of this requirement will often lead to resonances in the TES responsivity, and decreased sensitivity near said resonances.

For bolometers with complex thermal structure, this same approach can be applied, provided one knows how to calculate the generalize responsivity matrix, as we well see in Chapter 5.

2.2 TES Noise

Optimizing the signal-to-noise ratio is the primary goal when developing an instrument for the observing the CMB. In this section we give an overview of the sources

of noise. For a basic discussion of noise in bolometers in general see Richards (1994), Mather (1982). For a detailed discussion of TES noise in the strong ETF limit see Irwin & Hilton (2005).

When comparing different noise sources, it is important to include their relative amplitude to the input signal. For radiometric instruments, a common metric of signal-to-noise is the Noise Equivalent Power, which is defined as the level of absorbed power which would be observed with unity signal-to-noise over a 1 Hz bandwidth range. For sources of noise which originate as thermodynamic fluctuation in power on the sensor the NEP is simply equal to the square root of the power spectral density (PSD), S_P . For sources of noise which are more commonly expressed as fluctuation in current, e.g. Johnson noise, the NEP is calculated by dividing the square root of the current PSD, $S_I(\omega)$, by the power-to-current responsivity, $s_I(\omega)$:

$$NEP_I = \sqrt{S_I}/|s_I|. \quad (2.19)$$

For simple estimates of NEP, one can assume high loopgain: $|s_I| \leq -1/V$, in which case the NEP for current sources is roughly:

$$NEP_I^2 \geq V^2 S_I \quad (\text{High-loopgain approximation}) \quad (2.20)$$

The NEP is a very useful quantity for characterizing individual devices, though this metric does not take into account the efficiency of the optical system feeding the detectors. In order to compare different CMB experiments, one typically uses Noise Equivalent Temperature (NET) as the figure of merit. The NET is the change in celestial brightness temperature that can be measured with unity signal-to-noise in one second of integration. For a single-moded antenna system, and assuming a source with a Raleigh-Jeans (RJ) spectrum, the NET is directly related to the NEP by the optical efficiency, η and the effective bandwidth, $\Delta\nu$ of the optical system. In this case a change in temperature, ΔT_{RJ} is accompanied by a change in power $\Delta P = 2k\Delta T_{RJ}\eta\Delta\nu$, and so:

$$NET_{RJ} = \frac{NEP}{2\sqrt{2}k\eta\Delta\nu} \quad (2.21)$$

The additional factor of $\sqrt{2}$ arises from the fact that NEP is expressed as the square root of a single-sided PSD (with units of $W/\sqrt{\text{Hz}}$), while NET is expressed in terms of integration time (with units $K\sqrt{s}$).

In the more general case one requires more information about the frequency profile of the emission source, $\frac{dP}{dT}(\nu)$, as well as on the shape of $\tau(\nu)$, the optical transfer function, and the atmospheric opacity $\epsilon(\nu)$. For CMB applications, one is usually more interested in the sensitivity to fluctuations in the temperature of the CMB, δT_{CMB} . We assume a Planck spectral brightness:

$$B(\nu, T) = \frac{h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1} \quad (2.22)$$

In the case where the optical system is diffraction-limited ($A\Omega = \lambda^2$), the NET in CMB units is related to the NEP by the following expression:

$$NET_{\text{CMB}} = \frac{NEP}{\sqrt{2} \int d\nu \frac{dB(\nu, T_{\text{CMB}})}{dT_{\text{CMB}}} \frac{c^2}{\nu^2} \tau(\nu) (1 - \epsilon(\nu))} \quad (2.23)$$

In either limit, the somewhat obvious trend is that more optical efficiency across a wider bandwidth leads to more signal and thus a higher signal-to-noise ratio.

2.2.1 Photon Noise Terms

The fundamental limit to TES noise comes from photon-counting statistics. Individual photons absorbed at the detector lead to shot noise in the observed power. For instance, if the photons arriving at the detector are Poisson-distributed¹, with an average number of photons per second, n . The variance in the number of photons observed in one second will be $(\Delta n)^2 = n$. Thus for an input signal spectrum, P_ν , the noise power in 1 Hertz of signal bandwidth will be (Richards, 1994):

$$\text{NEP}_{\text{phot}}^2 = 2 \int d\nu P_\nu h\nu \quad (2.24)$$

Note that here P_ν , includes not just signal but also background power, from sources such as atmospheric emission, or internal loading within the cryostat. With background temperatures of $T_{\text{RJ}} \approx 250$ K, the atmospheric emission, which for the SPT is the dominant sources of loading, is strongly in the the Raleigh-Jeans limit at millimeter wavelengths. The NEP is thus:

$$\text{NEP}_{\text{phot}}^2 \approx 4hkT_{\text{atm}} \int d\nu \nu \tau(\nu) \epsilon(\nu), \quad (2.25)$$

where as before, $\tau(\nu)$, is the transfer function of the optical system (not including atmospheric absorption), and $\epsilon(\nu)$ is the atmospheric opacity. It is this background loading which sets the fundamental sensitivity limit for a single bolometer, also known as the background limited instrument performance (BLIP) limit.

With bolometer technology currently achieving the background limit, the only way to increase instrument sensitivity is to push for large arrays of TES sensors. The problem of optimizing a TES array instrument has been studied by Griffin et al. (2002), and requires careful consideration of tradeoffs such as the detector density vs. beam efficiency, as well as trade-offs between background loading vs. observing bandwidth.

¹Some corrections to the Poisson-distribution are anticipated due to Bose-Einstein statistics causing correlations between photon events (See Richards, 1994, for a review.), though these correlations are neglected in this discussion.

2.2.2 Thermal Fluctuation Noise

Designing an array of BLIP detectors is no trivial task and requires careful design of the bolometers themselves. After the background loading noise, the next source of noise to be considered is the thermal fluctuation noise (TFN). It is a fundamental result in statistical mechanics that a heat-capacity in contact with a thermal bath will undergo energy fluctuations: $(\Delta E)^2 = kT^2 C$ (See for example Kittel & Kroemer, 1980, chap. 3). Since this heat capacity also sets the natural bandwidth, τ , of a simple bolometer, the effective power fluctuations per unit bandwidth are:

$$\text{NEP}_{\text{TFN}}^2 = 4kT^2 G \times F(T, T_{\text{bath}}) \quad (2.26)$$

The function $F(T, T_{\text{bath}})$ is a function which describes the non-equilibrium nature of the link between the TES and the bath, for the TES is typically heated to above the bath temperature. This function ranges from 0.5 to 1, and depends on the nature of the thermal link between the sensor and the bath (For details see Mather, 1982).

2.2.3 Johnson Noise

The next most important noise term for TES sensors is Johnson noise. For any resistor, R , this noise source can be modeled as an equivalent series voltage fluctuation with power-spectral density (PSD), $S_V = 4kT_R R$, over a bandwidth, $\delta\nu = 1/2\pi\tau_e$. In the absence of electrothermal-feedback, these voltage fluctuations correspond to a current fluctuations. For the system shown in Figure 2.1 the frequency dependent PSD is

$$S_{I,\text{total}}(\omega) = \frac{4k(T_{R_L} R_L + T_{\text{TES}} R_{\text{TES}})}{|R_L + R_{\text{TES}} + i\omega L|^2}. \quad (2.27)$$

In an active TES, the power dissipation associated with these fluctuations will be reduced by electrothermal-feedback. The magnitude of the Johnson noise current then actually depends on whether the noise source is on the thermal island $S_{I,\text{int}}$, or external to the bolometer $S_{I,\text{ext}}$. For external fluctuations from the bias resistor, R_L , the magnitude of the current can just be calculated based on the complex impedance of the TES:

$$S_{I,\text{ext}}(\omega) = \frac{4kT_{R_L} R_L}{|R_L + Z_{\text{TES}}(\omega) + i\omega L|^2} \quad (2.28)$$

For Johnson noise from the TES itself, work done by the TES must be taken into account leading to the result from Irwin & Hilton (2005):

$$S_{I,\text{TES}} = 4kT_0 P_0 (1 + \omega^2 \tau^2) |s_I(\omega)|^2 / \mathcal{L}^2 \quad (2.29)$$

$$\approx 4kT_0 R_0 \frac{1}{\mathcal{L}(1 + \xi) + 1 + \beta} \left| \frac{1 + i\omega\tau}{(1 + i\omega\tau_{\text{eff}})(1 + i\omega\tau_e)} \right|^2 \quad (2.30)$$

In the second-line, I have used the moderate-loopgain approximation for s_I , Equation 2.13. Thus by Equation 2.19, the NEP for TES Johnson noise is

$$\text{NEP}_{\text{TES}} = \frac{4kT_0P_0}{\mathcal{L}^2}(1 + \omega^2\tau^2) \quad (2.31)$$

This Johnson Noise supression effect have been well-studied in DC-biased systems (Irwin & Hilton, 2005). In Appendix B, I demonstrate that this Johnson noise supression effect applies to AC-biased systems as well.

2.2.4 Readout Noise

The last noise contribution comes from the read-out (RO) electronics. These noise terms arise from a wide variety of sources such the SQUID current transducers, noise in the following amplifiers, and Johnson noise in the warm electronics. The sum of each of these noise terms can be expressed as an equivalent current noise at the SQUID input, $S_{I,\text{RO}}$. In the high-loopgain approximation, Equation 2.20, the NEP associated with these sources is:

$$\text{NEP}_{\text{RO}}^2 \geq V^2 S_{I,\text{RO}} = P_{\text{elec}} R_0 S_{I,\text{RO}} \quad (2.32)$$

The obvious method of limiting readout noise is to limit the amount of current noise from the readout system. Alternatively one can operate the detectors at low voltage bias by limiting either P_{elec} or R_0 , though P_{elec} is specified by the dynamic range requirements of the experiment. For this reason many TES bolometer systems choose low operating resistances, on the order of a few m Ω s. However, the bandwidth requirements for the multiplexor LRC filters (Chapter 3 set our target operating resistances in the range of $\sim 1\Omega$).

2.3 Summary

In this Chapter, I have covered the basic theory of TES bolometer performance, which will serve as the background for rest of the chapters in the first part of this thesis.

By virtue of electrothermal feedback, these detectors are notably linear over a wide range of input power. When incorporated into a well designed system are in the end limited only by the intensity of background loading, and the efficiency of the optical system. Due to this fundamental sensitivity limit, the only way to greatly increase the sensitivity of a bolometer system is to increase the number of detectors. In the next chapter we will discuss a multiplexing system for reading out many detectors with a single transducer to reduce the wiring complexity of a large bolometer array.

One note of caution, however is to be wary of instabilities in the electrothermal feedback network. In Chatper 5 I will discuss our efforts to improve stability of these devices when operated under electrothermal feedback.

Chapter 3

Frequency Domain Multiplexed SQUID Readout

3.1 System Overview

As seen in the last chapter, photon shot noise places a fundamental limit on signal-to-noise performance of single-moded detectors, and the only way to improve the sensitivity of a background-limited bolometer instrument is to increase the number of pixels. For this reason, most recent CMB bolometer experiments have all been designed for large focal planes with hundreds or even thousands of background limited detectors.

Such large arrays of detectors present a significant cryogenic challenge. The thermal load incurred by thousands of wires must be limited to avoid overloading the sub-Kelvin refrigeration systems, which provide limited cooling power. TES multiplexing systems reduce the number of wires required to read-out multiple TES's. Such multiplexing systems are typically divided into two broad classes: time-domain (Chervenak et al., 1998, 1999; Irwin, 2002) or frequency-domain (Lanting et al., 2003; Lanting et al., 2004; Lanting et al., 2005, 2006; Lanting, 2006).

The fMUX frequency-domain multiplexing system, developed at Berkeley, was first utilized by the APEX-SZ experiment, a precursor to the SPT. For a complete overview of the basic system the reader is referred to Lanting (2006). Though the fMUX system was successfully demonstrated before the development of APEX-SZ and SPT, the implementation in a full scale receiver highlighted some new challenges. In this chapter I elaborate on some of the details of the SQUID multiplexor system, particularly those details encountered when scaling up from an eight-bolometer test bed to a full scale kilopixel array. I will discuss details of the warm electronics. I will then focus on the operation of the Series SQUID Arrays (SSA's) themselves, the stability of the flux-locked loop, and the design of the SQUID housing.

The fMUX system is shown schematically in Figure 3.1. Variations in the optical

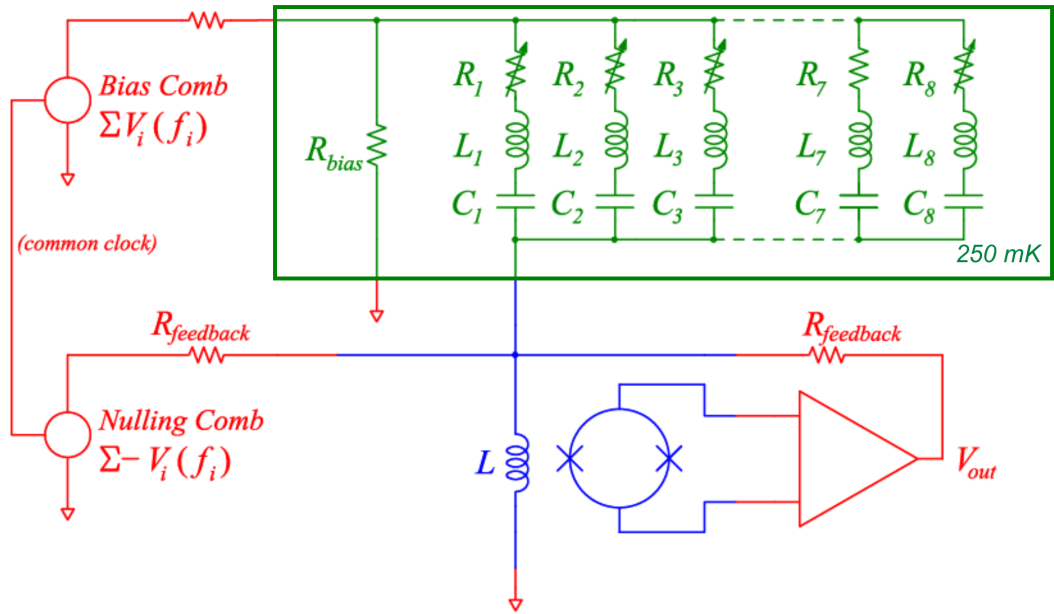


Figure 3.1: A schematic overview of the frequency-domain multiplexor. The **red** components indicate warm electronic elements. The Series SQUID Array (SSAs shown in **blue**) transducers are held near 4K. Lastly the LC-coupled bolometers are cooled to approximately 250 mK and are shown here in **green**.

power absorbed by the detector change the detector resistance thereby modulating the current produced by the high frequency bias voltage. This translates the low-frequency CMB signal to sidebands centered on the bias frequency. Each bolometer in a MUX group is biased at a different frequency, so the individual sensor signals are uniquely positioned in frequency space, which allows them to be combined in one wire. Each detector is connected in series with a resonant LC filter, which limits the current output of each bolometer to the appropriate bias source. A multiplexed group of detectors consists of N bolometer LCR legs connected in parallel. This group of detectors is then biased by a set of N sine-wave voltage generators (i.e. Direct Digital Synthesizers). Each oscillator is tuned to the unique resonant frequency of a particular LC-coupled bolometer. The unique resonant frequency of each bolometer in the group allows later the separation of each of the N signals absorbed by the bolometers. In the frequency-domain, the collective electrical response each of these tuned bolometer circuits appears as several periodically spaced peaks, and thus we often refer to each group of bolometers as a “comb”. The range of bias frequencies required to operate a single comb, the “bias-band”, typically ranges from 300kHz to 1 MHz.

All bolometers in the same group are read out by a single Series SQUID Array (Welty & Martinis, 1991, SSA). We feed the sum of the currents from all N bolometers into a SQUID Flux-locked Loop (FLL), which is designed to operate with good linearity over the full frequency range spanned by the biases. Finally the output of this FLL is processed and digitized by a bank of N demodulators one at each bias frequency.

The FLL has a fixed dynamic range, as we shall see in the next Section. Each bolometer channel sends a several μA bias current to the SSA, though we are only interested in changes in this current. Therefore, we eliminate most of the current at the SQUID input by injecting a second sinusoidal “nulling” signal in the SQUID input coil. This nulling signal is arranged to be 180° out of phase with the bias current so that the two currents largely cancel.

This conceptually simple multiplexing scheme has already been thoroughly analyzed and demonstrated (Lanting et al., 2004; Lanting et al., 2005, 2006; Lanting, 2006). However there are some details that must be carefully considered when implementing this technology in a full system. The fMUX system contains not just one, but two feedback loops (i.e. the TES ETF, and the SQUID flux-locked loop), each of which must be carefully controlled to prevent self-oscillation. There are many differences between a simple test cryostat and an actual kilopixel array. Not only are there more components in a kilo-pixel system, there are also differences in physical scale: in a real receiver the SQUIDs and the TES bolometers tend to be spaced farther apart, requiring longer interconnects and introducing more opportunities for stray reactances in the system. It is also essential for such a large system to be somewhat automated. Changes in ambient temperature or background loading may shift the optimum tuning parameters for the SQUIDs or the bolometers. Both the detectors and the SQUID readout must be sufficiently tolerant of variations in ambient condi-

tions that they can be tuned and diagnosed by efficient software algorithms. Finally, for the SPT cryostat, much attention has been paid to modularity in the design: the receiver should be not only efficient to operate, but also efficient to disassemble, upgrade and reassemble.

Much of my own work in the last decade has been devoted to making the improvements necessary to operate the multiplexor SQUID subsystem in a large observation cryostat. These improvements include the development of the SQUID Flux-locked loop (FLL), the 4 K packaging for the SQUIDs and much of the warm electronics testing and design.

3.2 Shunt-Feedback SQUID Controllers

3.2.1 SQUIDs as Current Transducers

Due to their low input impedance and tremendous current sensitivity, Superconducting QUantum Interference Devices (SQUIDs) have become the most commonly used transducer for measuring the TES response currents. Clarke (1996) provides a detailed review of SQUID devices, including the physics of Josephson junctions, SQUID noise performance, and fabrication details. In this section we give a very brief phenomenological overview SQUID transducers.

A SQUID consists of a superconducting loop which has been broken by two narrow insulating barriers, or Josephson junctions (Josephson, 1962; Stewart, 1968). The critical current, I_c , of a Josephson junction determine how much superconducting current the junction can sustain with zero voltage drop. Meanwhile the current around the loop is determined by the magnetic flux, Φ , impinging upon that loop. Deviations from an integer number of flux quanta, Φ_0 , induce a circular current in the loop: $I = (\Phi - n\Phi_0)/L$, where L is the inductance of the loop. These circulating currents reduce the critical current of one of the junctions on either side, and thus reduce the critical current of the SQUID as a whole. Thus when the across through the SQUID exceeds this reduced critical current ($I \gtrsim 2I_c - \Phi_0/L$), the voltage across the SQUID is non-zero and is periodic in the applied flux, Φ .

Current can then be coupled into the SQUID, via coils placed immediately above the SQUID loop. For a small change in the input current, δI_{coil} , the flux through the SQUID washer, $\delta\Phi$, is determined by the mutual inductance $M = \delta\Phi/\delta I_{\text{coil}}$. The voltage response of the bolometer then depends on the slope of the $V - \Phi$ curve: $V_\Phi \equiv \frac{\partial V}{\partial \Phi}$. The overall response of the SQUID is then given by the transimpedance, Z_{tr} :

$$Z_{tr} \equiv \frac{dV}{dI_{\text{coil}}} = MV_\Phi \quad (3.1)$$

Capacitance in the Josephson junctions can lead to hysteric behavior unless the junctions are damped by resistive shunt resistors, and these shunts limit the out-

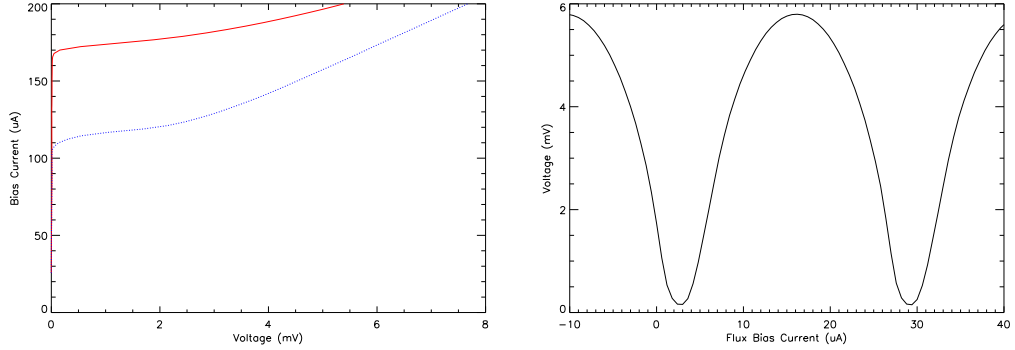


Figure 3.2: I – V and V – Φ curves for a NIST 8-turn series SQUID array (SSA) as measured with the fMUX SQUID tuning routines. In the *left panel*, the I – V curve curves are measured at integral (**red trace**) and half-integral (**blue trace**) flux quanta. In the *right panel*, the SSA is biased at a fixed current just above $I_c = 140 \mu\text{A}$, as current is swept through the input coil, modulating the flux and demonstrating the nearly sinusoidal response.

put impedance of the SQUID.

Typical SQUIDS have output impedances of about 1Ω , making them poorly matched to the transmission lines connecting them to the warm electronics. As we shall see in Chapter 4, this matching is very important for our application. One solution to this problem is to wire 100 SQUIDS in series, creating a series SQUID array (SSA) with output impedances closer to 100Ω (Welty & Martinis, 1991). For our application we use the NIST 8-turn SSAs on account of their larger mutual inductance. These devices have a total input coil inductance of approximately 160 nH , and peak trans-impedances of 500Ω or higher. Each SQUID in the array is coupled to the input coil with a mutual inductance of $M = 80 \text{ pH}$, corresponding to a modulation curve period of $\Phi_0/M = 26 \mu\text{A}$ at the input coil. Typical I – V and V – Φ curves for one of these devices are shown in Figure 3.2. In many regards, these devices behave much like single SQUIDS, though with larger impedances, and so we often use the term “SQUID” when referring to SSAs.

3.2.2 Properties of Shunt-Feedback SQUIDS

The SQUID arrays discussed in the last Chapter are indeed low-impedance low-noise current transducers. However they are very non-linear and have a very limited dynamic range since one can only input roughly $\pm\Phi_0/(4M) = 6.5 \mu\text{A}$ of current before the roughly sinusoidal SQUID V – Φ response turns over and the response becomes non-monotonic. SQUIDS are typically operated under feedback for these reasons. For AC-biased bolometers, the input inductance of the SQUID also presents a large

reactive load to the TES as well. The shunt-feedback circuit topology, shown in Figure 3.4, has the advantage that it inputs the feedback signal directly at the input coil, which also reduces the effective input inductance of the SQUID as well. This chapter starts by reviewing the properties of the SQUID Controller many of which have been detailed by Lanting (2006) and Spieler (2002). We will then go on to discuss the SQUID Controller implementation used in the field, and later in the chapter we will discuss threats to SQUID stability.

Unlike the TES electrothermal feedback, the SQUID flux feedback system has only one parameter, SQUID Flux, Φ . The strength of the feedback expressed in terms of SQUID loopgain $\mathcal{L}_{\text{SQ}}$, which is related to the slope of the V - Φ curve, Z_{tr} , the amplifier gain, $A(\omega)$ and the feedback resistance, R_{fb} .

$$\mathcal{L}_{\text{SQ}} \equiv \left(\frac{dI_{\text{fb}}}{dI_{\text{coil}}} \right)^{-1} \quad (3.2)$$

$$= \frac{Z_{\text{tr}} A(\omega)}{R_{\text{fb}}} \quad (3.3)$$

Unlike the ETF loopgain, $\mathcal{L}$ in Chapter 2, $\mathcal{L}_{\text{SQ}}(\omega)$, is a complex valued, frequency-dependent quantity which can be used on its own to predict the stability of the circuit. $\mathcal{L}$ would be analogous to $|\mathcal{L}_{\text{SQ}}(0)|$. The bias point of the SQUID is chosen such that Z_{tr} is negative and the SQUID is connected to the non-inverting input of the amplifier, so that $A(0)$ is real and positive. The overall loopgain at DC, $\mathcal{L}_{\text{SQ}}(0)$, is then real and negative which indicates negative feedback.

As an aside, it should be noted that the equality in 3.3 assumes that there are no other circuit elements in parallel with the SQUID input coil. Circuit elements in parallel with the SQUID input coil can draw current away from the SQUID input coil, modifying the loopgain. For instance, in Figure 3.1, from the point of view of the feedback network, the LRC filtered bolometers are in parallel with the SQUID input coil, as are all of the transmission lines between the SQUID input coil and 250 mK or 300 K. These shunting effects are particularly important when considering the stability of the flux-locked loop as shall be discussed in Section 4.4.

At higher frequencies, reactances in the system lead to phase shifts and variations in the overall amplitude of the loopgain. For instance, one of the most obvious sources of loopgain variability is the gain of the amplifier itself, which in even the most simple amplifier models has a single pole roll-off with a time constant τ_a :

$$A(\omega) = A(0)/(1 + i\omega\tau_a). \quad (3.4)$$

Other sources of phase shifts include delays in the cables the 4K SQUIDS to the warm electronics, and other filters placed in the flux-locked loop.

As shown in Figure 3.4, much of the input signal current is drawn away from the squid input into the feedback loop. The input current $I_{\text{in}} = I_{\text{sq}} + I_{\text{fb}}$ is then divided

between the SQUID I_{sq} and the feedback network I_{fb} . For a small SQUID current, the input voltage is:

$$V_{in} = i\omega L I_{sq}, \quad (3.5)$$

while the voltage is:

$$V_{out} = V_{in} A(\omega) \frac{Z_{tr}}{i\omega L} = A(\omega) Z_{tr} I_{sq}. \quad (3.6)$$

Since $I_{fb} = (V_{in} - V_{out})/R$ the SQUID current is related to the input current by the formula,

$$I_{sq} = \frac{I_{in}}{1 - \mathcal{L}_{SQ} + i\omega L/R_{fb}} \quad (3.7)$$

$$\approx \frac{I_{in}}{1 - \mathcal{L}_{SQ}} \quad (3.8)$$

For many of the equations below we neglect terms of order $i\omega L/R_{fb}$ since R_{fb} is always greater than $1k\Omega$, and so even at $10MHz$, this term is only about a 1% correction. In the presence of strong negative feedback ($|\mathcal{L}_{SQ}| \gg 1$) the current through the SQUID is greatly reduced, which means that the dynamic range of the SQUID is extended. The maximum allowable input current, I_{max} is then calculated by integrating I_{sq} from 0 to $\Phi_0/(4M)$, assuming a perfectly sinusoidal $V-\Phi$ response curve (Lanting, 2006):

$$I_{max} = \frac{\Phi_0}{M} \left(\frac{1}{4} - \frac{\mathcal{L}_{SQ}}{2\pi} \right) \quad (3.9)$$

The forward gain of the circuit, $Z_{forward} \equiv V_{out}/I_{in}$ can be found by combining Equations 3.6 and 3.8 we find:

$$Z_{forward} \approx -R_{fb} \frac{\mathcal{L}_{SQ}(\omega)}{1 - \mathcal{L}_{SQ}(\omega)} \quad (3.10)$$

Since most of the current travels through the feedback resistor it is not surprising that $Z_{forward} \approx R_{fb}$ under strong feedback.

One last quantity of interest is the input impedance $Z_{in} \equiv V_{in}/I_{in}$ of the feedback SQUID Controller, which by combining Equations 3.5 and 3.8, we predict to be:

$$Z_{in}(\omega) \approx \frac{i\omega L}{1 - \mathcal{L}_{SQ}(\omega)} \quad (3.11)$$

It is especially interesting to calculate the effect that poles in the feedback network has on the input impedance. We have already pointed out the pole in the amplifier response, τ_a , with an effective time constant may add with other transmission line delays, δt_i , to retard the feedback network. At low frequencies we can model the loopgain as a single pole function and neglect higher order terms in frequency:

$$\mathcal{L}_{SQ}(\omega) \approx \frac{\mathcal{L}_{SQ}(0)}{1 + i\omega\tau_{eff} + O((\omega\tau_{eff})^2)}, \quad (3.12)$$

where $\tau_{\text{eff}} \equiv \sum \tau_i + \sum \delta t_j$. To lowest order in $\omega\tau_{\text{eff}}/|\mathcal{L}_{\text{SQ}}(0)|$, the real and imaginary parts of the input impedance are:

$$\text{Re}(Z_{\text{in}}(\omega)) = -\omega^2 L \tau_{\text{eff}} \frac{|\mathcal{L}_{\text{SQ}}(0)|}{(1 - \mathcal{L}_{\text{SQ}}(0))^2} \quad (3.13)$$

$$\text{Im}(Z_{\text{in}}(\omega)) = \frac{\omega L}{1 - \mathcal{L}_{\text{SQ}}(0)} \quad (3.14)$$

Thus we expect a small negative resistive component to the SQUID input impedance. One can indirectly measure the input impedance of the SQUID Controller when LC-filtered bolometers are attached to the input. By injecting current into the SQUID input and measuring changes in the amplitude and phase of the SQUID Controller both on and off the LC resonant frequency. Such indirect measurements of this negative resistance indicate that the SQUID Controllers in the SPT are operating with input resistances of approximately -100Ω at 900 kHz, and reactances of $100i\Omega$. Both of which are rough agreement with SQUID loopgains of about $|\mathcal{L}_{\text{SQ}}| \approx 10$, $\tau_{\text{eff}} \approx 300\text{ns}$ and SQUID inductances of 160 nH.

3.2.3 Implementation: SQUID Controller

As with electro-thermal feedback described in the last chapter, too much loop gain or too much bandwidth in the feedback network can lead to instability (See chapter 4). These instabilities are strongly influenced by stray reactances in the feedback network, and so the SQUID Controller implementation requires careful layout and choice of components. In this section we describe the implementation of the SQUID Controller board, shown schematically in Figure 3.3. Each SQUID Controller board manages 8 SQUID channels.

The SQUID bias current and flux-operating point are tunable by software control. This is essential since with 120 series SQUID arrays in the entire experiment, manual set up of each SQUID is labor intensive. At the SPT, SQUID diagnostics and biasing are done during each cycle of the cryogenic refrigerators (roughly once every 36 hours, see Chapter 6 for details).

For each SQUID channel there are four software-programmable adjustments that can be made: the bias current, I_b ; the flux-bias current, I_{fb} ; the amplifier offset, V_{off} ; and a heater voltage, V_{heat} . Each adjustments are controlled by a digital-to-analog converter. With 8 channels, there are a total of 32 DACs on each SQUID Controller. The SQUID current bias, I_b , is chosen near I_c to maximize the peak-to-peak response of the SQUID V - Φ curve. When open loop, the flux-bias I_{fb} modulates the flux across the SQUID, and may be used to locate the flux-offset required to achieve the minimum amplitude of V_Φ (recall that we operate on the inverting edge where V_Φ is negative.) For a sinusoidal $V - \Phi$ response, the optimum value of V_Φ would be at a $I_{fb} = 3\Phi_0/4M$ midway between the peak of the $V - \Phi$ curve and the trough, on the

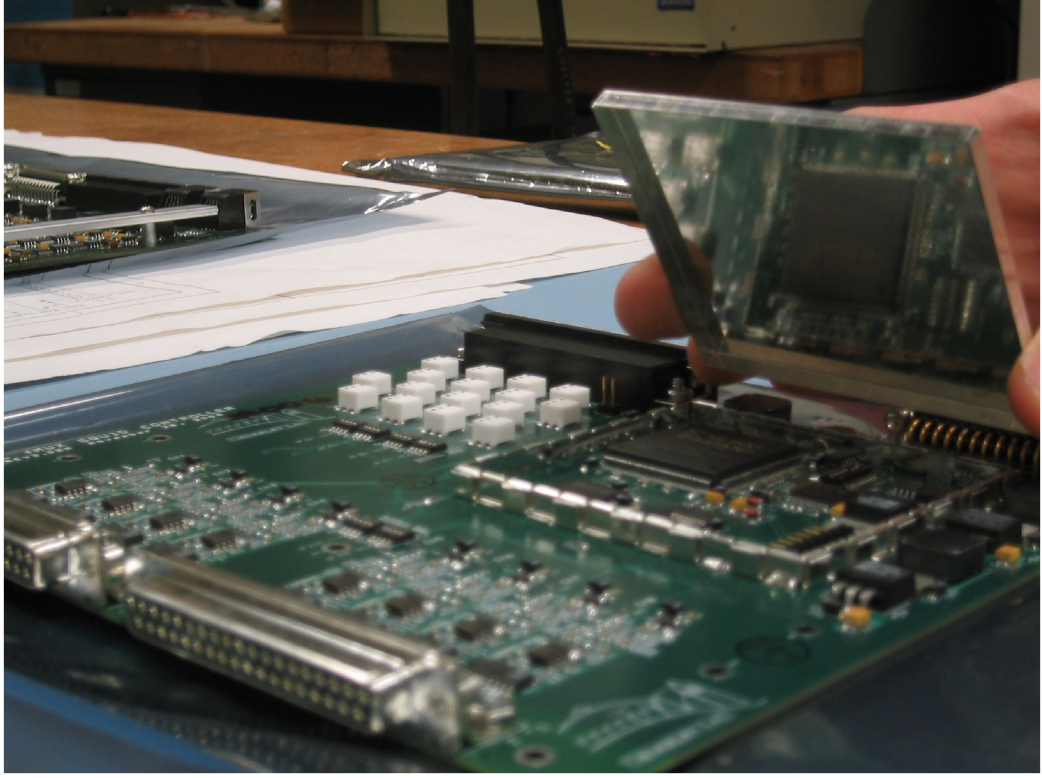


Figure 3.3: A photograph of the 8-channel SQUID Controller. The warm amplifiers and feedback circuits for the 8 flux-locked loops are shown in the foreground. In the background, the RF shield dividing the digital and analog portions of the board has been lifted to show the communications FPGA and the D/A converter which biases all the SQUIDs.

inverting edge. The flux bias however does not strictly set the operating point under feedback. The feed back amplifier will inject a flux, $\Delta\Phi$, into the SQUID input coil in order to keep its inputs the at the same voltage. Thus the operating flux through the SQUID under feedback, $\Phi_{\text{op}} = I_{\text{fb}}/M + \Delta\Phi$ is that flux which keeps the SQUID voltage equal to the offset voltage, V_{off} , applied to the inverting input:

$$V(\Phi_{\text{op}} + n\Phi_0) = V_{\text{off}} \quad (3.15)$$

In order to obtain the most dynamic range, V_{off} and I_{fb} are adjusted so that they both correspond to the same operating point, and thus $\Delta\Phi$ is kept near zero. Each SQUID is also placed near a 100Ω resistor which can provide heat should some flux quanta become trapped in the SQUID or the Nb shielding. This power for this is provided by a heater is provided by a voltage source, V_{heat} on each channel of the SQUID Controller.

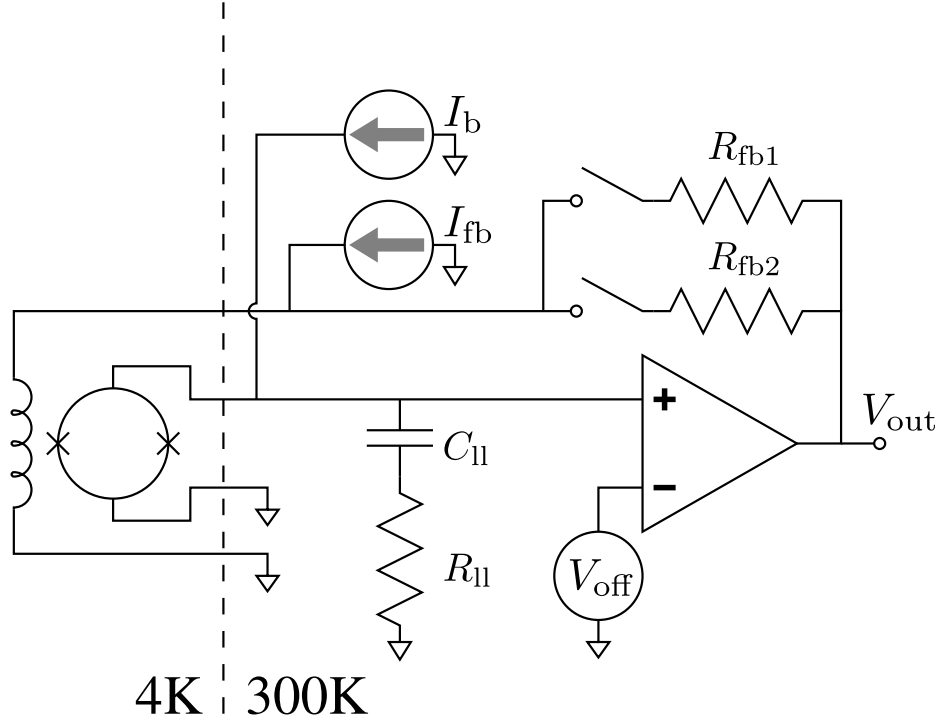


Figure 3.4: Simplified schematic of a single SQUID Controller channel (with SQUID Array) showing feedback resistors, $R_{fb,1}$ and $R_{fb,2}$, feedback switches, voltage offset DAC V_{off} , bias current DAC I_b , flux bias DAC, I_{fb} , and the lead-lag filter R_{ll} , C_{ll} . Not shown are the digital control electronics which communicate with control software to command the the DAC and switch settings. Also not shown is the heater DAC (one per channel) which pushes a voltage (as much as 8V) across a 100Ω resistor adjacent to each SQUID Array.

Since the SQUID modulation curves are periodic in flux this procedure the real operating flux may be shifted by a flux quantum. Sometimes transients or noise excursions can exceed the dynamic range of the flux-locked loop, forcing the loop to settle one Φ_0 over. When this happens the FLL output exhibits a flux-jump of $\Phi_0 R_{fb}/M$ and the amplifier pushes a full Φ_0 of additional current into the loop to maintain it at this new operating point. Flux-jumps severely limit the dynamic range of this system. Fortunately that can be undone by briefly injecting a large opposing flux into the loop, forcing the loop to flux-jump in the opposite direction.

The amplifier is a model OPA687¹ operational amplifier. This model was chosen for its large gain bandwidth product (3800MHz), and low input noise. We desire

¹formerly Burr-Brown, now supplied by Texas Instruments, Dallas, Texas 75265

high-loopgain over the entire 1MHz bias-bandwidth, so a local feedback network sets the gain of the amplifier this amplifier stage to 3500, setting the bandwidth to 1MHz. Such large gain is unwieldy however and can cause oscillations. Thus this amplifier is followed by an attenuator to reduce the effective gain to ~ 500 , though the option remains to rework this attenuator if more (or less) loop gain is desired.

The feedback resistance can be adjusted by means of programmable switches: Available options are 10k Ω , 5k Ω , or both in parallel, for a combined resistance of 3.3k Ω . With a SQUID transimpedances of 300 Ω , this allows for low-frequency loop-gains ($\mathcal{L}_{SQ}(0)$) ranging from 15 to 45 at 10k Ω or 3.3k Ω feedback respectively.

Commands come in from the oscillator-demodulator boards (Section 3.3) over four Low-Voltage Differential Signalling (LVDS) logic pairs. One pair is for the input data commands, while another is an output for data response. There are also two strobe pairs, one for input, one for output. The oscillator demodulator-board asserts 44 bits of data in sequence, strobing the input line after each bit. After each bit is received the SQUID Controller responds with one bit of reply, followed by a pulse on the output strobe. The FPGA has no internal clock to drive its state machine, and these strobe pulses are the only timing signals that the SQUID Controller FPGA receives. Thus when no communications are being received, this FPGA is completely passive and no digital transients occur on the board.

Being very sensitive, and having such a wide bandwidth the SQUID feedback loops respond dramatically to switching transients caused by digital activity on the SQUID Controller board. In addition to eliminating clock pulses when the digital electronics are passive, these circuits are also electrically shielded from analog circuits. The SQUID Controller has split analog and digital ground planes as well as isolated power regulators for each domain. The DAC outputs and the digital control lines are all heavily filtered as they pass from the digital to the analog circuit regions. Finally in order to minimize RF interference from the digital chips, the entire digital region is encapsulated within an own RF enclosure, as seen in Figure 3.3.

3.3 Oscillator Demodulator Boards

A second set of warm readout boards is responsible for generating the TES bias and nulling combs, and for demodulating and digitizing the output signals. Each board has 16 channels each with one oscillator and one demodulator. In the analog implementation used on the SPT, the demodulation and digitization scheme recovers only one component of the detector output, the component in phase with the bias oscillator. If a phase delay, δ , exists in the signal chain, then the signal at the output of the demodulator is attenuated by a factor of $\cos \delta$. Since many sources of noise exist in both the I-phase and the Q-phase of the carrier, this represents a penalty in the signal-to-noise ratio. For nulling operations and for diagnostics, it is also important to have access to the orthogonal phase component. Thus one additional oscillator-

demodulator chain per comb is allocated as an orthogonal-phase helper channel when necessary. This means that each board can accommodate two (2) combs with seven (7) channels each. One of the improvements of the later digital oscillator-demodulator system Dobbs et al. (2008), was the introduction of a phase-adjustable demodulator.

The fMUX readout electronics are designed to be used in both non-multiplexed and multiplexed systems, so the oscillator-demodulator boards can be reconfigured to operate 16 non-multiplexed bolometers attached to 16 SQUID arrays. In this mode, which was used for the APEX-SZ engineering run, each oscillator-demodulator board has two daughter SQUID Controllers. Multiplexed operation requires a factor of eight fewer SQUIDS, so for the SPT, 12 boards have SQUID Controllers that power eight arrays and another six have SQUID Controllers that power four. In non-multiplexed mode, each of the 16 chains has two outputs: a post-demodulator (AC) output and a pre-demodulator (DC) output. The predemodulator output is low-pass filtered at frequencies much less than the carriers, and is used to monitor the SQUID DC levels for diagnostic purposes and to watch for flux jumps. Multiplexed operation reduces the number of outputs to 16 AC channels (14 bolometers and two helpers) and two DC channels (one per SQUID array). As with the SQUID Controller, a FPGA is used to command the configurable components on the oscillator-demodulator boards. These components include the DDSes, potentiometers that attenuate the bias and nulling combs, the signal digitization chain, switchable bias and nulling gain resistors, another switchable gain for the demodulator output, and others; the FPGA also mediates the communication with the SQUID Controllers. The boards communicate with the receiver control computer via a bidirectional RS485 serial interface using the Modbus² protocol, and communicate with each other, the SQUID controllers, and the data acquisition systems using low-voltage differential signaling (LVDS).

The output of each demodulator and the pre-demodulator outputs are sampled and digitized by on-board analog-to-digital converters (ADC), before being digitally transferred to the data acquisition computer. The data transfer mechanism uses a “data-push” protocol, meaning that the digitized data is transferred on a strobed 8-bit bus, without any handshaking between the sender and receiver. This has the advantage that it reduces the need for buffering or handshaking logic on the board. However, conventional PC hardware has long latencies on the PCI I/O bus, meaning that fast data transfers can only be achieved by transferring the data to the CPU in large blocks. The data acquisition board (model PCI-6534³) comes equipped with 64MB of RAM, in which the data is buffered before being sent to the CPU in several kB blocks. The data format contains many delimiters and checksums to verify the integrity and contiguity of the data stream. Also, since there may be a delay between the transmission of the data and the reception at the CPU, the FPGA on the “master” oscillator demodulator board, marks each sample with a GPS time-stamp. This time-

²<http://www.modbus.com>

³National Instruments Corporation, Austin, Texas 78759

stamp is obtained via IRIG-B⁴, which guarantees timing accuracy at the $\sim 10\mu\text{S}$ level. The time-stamped bolometer data can then be accurately interleaved with the telescope attitude data.

3.4 Cold SQUID Housing

Strategies for magnetic shielding fall into 2 broad categories. The area to be shielded can be surrounded by a material of high magnetic permeability, μ . Depending on the geometry of the enclosure this material will draw magnetic flux into the permeable magnetic walls away from the enclosed area. The Meissner effect in Type I superconductors means that a superconducting enclosure will have no field in the enclosure walls. A completely sealed enclosure will have no flux in the interior. Type II superconductors are often preferred for their higher critical temperatures. Type II shields do not reject flux, rather they pin it, thus type II shields reject temporal *changes*, in the magnetic field.

The design used by the APEX-SZ, SPT, and Polarbear instruments contains both superconducting and permeable materials is shown in Figure 3.5. The SQUIDs are mounted on a printed circuit board, which mates to the cryostat wall, and connects the SQUIDs to the warm electronics. The SQUID traces travel to this edge connector via the shortest possible path in order to minimize the propagation delays, for the sake of maintaining stability. The resulting module is easy to insert or remove when servicing the cryostat. This board is shrouded by a ferromagnetic sheath, which both protects the SQUIDs and attenuates the ambient magnetic field. The outer sheath is fabricated from 0.058" thick Cryoperm10⁵ a ferromagnetic alloy optimized for use at 4K, with a relative permeability of 65000. Space constraints require that multiple modules be packed side-by-side in the experiment, and so the actual shape of the SQUID cavity is rectangular rather than cylindrical. As discussed in the next Chapter the signal traces need to be kept short in order to maintain stability of the feedback loop. So there is slot along one edge of the SQUID cavity where the PCB traces carrying the SQUID output and feedback signals exit the shielded area to mate directly to connectors on the 4K wall of the cryostat. Each board accommodates eight series SQUID array chips, each placed directly above a 9mm-square Nb foil⁶. As a type-II superconductor, this foil is intended to pin the residual field, and minimize interference due to temporal variations in either the ambient magnetic field or the SQUID orientation.

⁴irigb.com/IRIGB.standard.html

⁵Vacuumschmelze GmbH & Co. KG, D-63450 Hanau, Germany

⁶Goodfellow Corporation, Oakdale, PA 15071,

Part # NB000315, Niobium Foil 99.9%, annealed. 9 mm x 9 mm x 0.05mm

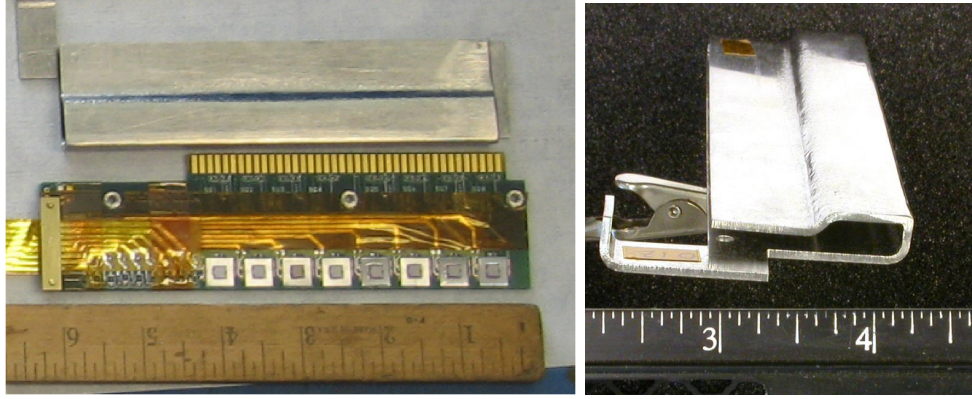


Figure 3.5: A magnetically shielded SQUID module. This module houses 8 SSAs mounted on a single PCB, shown on the lower half of the *left panel*. Each SSA is mounted above a 9mm niobium foil to pin the ambient magnetic flux. The PCB then slides into a sheath made of high-permeability sheath made of Cryoperm10 (also shown in the *left panel*). The *right panel* shows a profile view of the sheath, including the wide SQUID cavity on the right side and the narrow slot on the left side, where the PCB traces exit the sheath.

3.4.1 Expected Shielding Performance

The magnetic field inside a shield cavity is generally calculated by solving Laplace's equation. In the absence of explicit sources the magnetic field can be treated as the gradient of a scalar “magnetic potential”:

$$B(\mathbf{x}) = \nabla\phi, \quad (3.16)$$

where ϕ satisfies Laplace's equation: $\nabla^2\phi = 0$. The boundary conditions at infinity are chosen such that $\phi(\mathbf{x}) \approx \mathbf{B}_{\text{ext}} \cdot \mathbf{x}$. Near the interface between vacuum and a material with relative permeability, μ , the magnetic field must satisfy the boundary conditions:

$$B_{\perp,\text{vac}} = B_{\perp,\text{shield}} \quad (3.17)$$

for the component normal to the boundary, and

$$B_{\parallel,\text{vac}} = B_{\parallel,\text{shield}}/\mu_{\text{shield}} \quad (3.18)$$

for the parallel components. By contrast, for a type I superconductor the Meissner effect enforces the condition $B_{\perp} = 0$ near the surface. If the flux in a type II superconductor is well pinned this condition changes to $B_{\perp} = \text{constant}$. Using these equations one can solve for the shielding factor, $S \equiv B_{\text{ext}}/B_{\text{SQUID}}$. Generally the efficiency of a particular shield depends on the orientation of the magnetic field. Since we only

care about the magnetic flux through the SQUID, we calculate the shielding factor for magnetic fields oriented normal to the SQUID.

The geometry illustrated in Figure 3.5, is very complicated and hard to solve analytically, though we here we make some back-of-the-envelope estimates. We start by approximating the shield as a circular cylinder, for which by the transverse shielding efficiency is (Mager, 1970):

$$S = \frac{\mu}{4} (1 - D_i^2/D_o^2). \quad (3.19)$$

As before, μ is the relative permeability of the shield material, while D_i and D_o are the inner and outer diameter of the shield respectively. We take the inner “diameter” of the shield to be 0.540”, the largest dimension of the cavity. The outer diameter is larger by 0.058”, the thickness of the cryoperm. By this crude estimate, the shielding efficiency of the cryoperm should be about 3000.

The cryoperm sheath does not fully enclose the SQUIDS, and so it is important to consider the effect of fringing fields leaking to the SQUIDS from the open ends. For an long cylindrical opening the field amplitude falls off exponentially with the distance, x , from the opening (Mager, 1970): $B(x) \propto e^{-kx}$. Here, k is a geometrical factor which depends on the geometry and dimensions of the opening. For a circularly cylindrical opening with radius R , $k = 3.8/R$. For this reason the SQUIDS are placed at least 0.6” into the cylindrical cavity to ensure that the fringing fields entering from the end of the shield are no larger than $B_{\text{ext}}/3000$. We can also estimate the field fringing from the long slot where the PCB traces exit the shield. For a long narrow slot of width w , one can show by separation of variables that $k \approx \pi/w$. To match the expected shielding ratio of 3000, the slot through which the PCB traces exit must have an aspect ratio $\gtrsim 2.5$.

The field stability provided by the Niobium foil is calculated by approximating the foil as a 9mm diameter disk. Laplace’s equation can be solved in ellipsoidal coordinates around such a disk as done by Lamb (1895). The field at a point along the axis of the disk and separated from the disk by a distance z is a function of the ratio $\zeta = z/R$, where R is the radius of the disk.

$$B(\zeta) = B_0 \left[1 + \frac{2}{\pi} \left(\frac{\zeta}{\zeta^2 + 1} - \cot^{-1} \zeta \right) \right]$$

For small values of ζ this approximates to $B(\zeta)/B_0 \approx 4\zeta/\pi$. So for squids fabricated on a 0.5mm wafer above a 4.5mm-radius superconducting disk the shielding efficiency is expected to be roughly 7.

Though normal, non-ferromagnetic metals do little to attenuate static magnetic fields, time-varying electromagnetic fields are attenuated by electrical conductors (See e.g. Jackson, 1998). In the simplest geometry, the shielded area would be separated from the external field source by a flat sheet of material with resistivity, ρ , permeability, μ , and thickness, t . The frequency-dependent

attenuation is then:

$$\mathbf{B}_{\text{int}}(\omega)/\mathbf{B}_{\text{ext}}(\omega) = e^{-t/\delta(\omega;\rho,\mu)}, \quad (3.20)$$

where for good conductors, i.e. $\sigma \gg \omega\epsilon$, $\delta(\omega;\rho,\mu) = \sqrt{2\rho/\mu\omega}$. As an example, the skin depth for aluminum is of order 1" at 60 Hz and room temperature.

For a conductive circular cylinder of thickness t and diameter $D \gg t$ in the presence of a transverse magnetic field, Mager (1970) cites an approximate shielding factor in terms of the parameters, $\alpha \equiv D/\delta(\omega;\rho,\mu)$, and $\beta \equiv 2t/D$:

$$S = \sqrt{\frac{\alpha^4 + 8\alpha^2 + 4}{16\alpha^2} \cosh \beta - \frac{\alpha^4 - 8\alpha^2 + 4}{16\alpha^2} \cos \beta + \frac{2\alpha^2 + 1}{2\alpha} \sinh \beta - \frac{2\alpha^2 - 1}{2\alpha} \sin \beta}. \quad (3.21)$$

This approximation will be useful when examining the frequency dependence of our magnetic shielding in the next section.

3.4.2 Measured Performance

The shielding factor of this shield has been measured by applying a magnetic field to the shielded SQUID array using a circular copper-wire coil. The magnetic field generated at the center of this coil is measured to be 4 gauss per amp of input current. This coils is placed immediately outside the test cryostat, and normal to the SQUID array module. By the Biot-Savart law, the field at a point along the coil axis, but displaced from the center by a distance d , scales as $\mathbf{B}(d)/\mathbf{B}(0) = (1 + (d/R)^2)^{-3/2}$, where R is the radius of the coil. Given a coil radius of 7" and an approximate displacement of 9", the field generated at the SQUID module location is estimated to be approximately 0.9 gauss/amp.

We vary the current through the external test coil to generate an external field B_{ext} , and for each step in the magnetic field, we sweep the current applied to the SQUID-array input-coil, I_{coil} to measure the V - Φ_{coil} characteristic of the SQUID array. For an ideal SQUID array with an input-coil mutual inductance, M , effective area, A_{eff} and magnetic shielding S , the voltage observed at the SQUID array output should be

$$V_{\text{SQ}} \propto \cos \frac{2\pi}{\Phi_0} (M_{\text{coil}} I_{\text{coil}} + \Phi_{\text{ext}}), \quad (3.22)$$

where $\Phi_{\text{ext}} = \frac{B_{\text{ext}} A_{\text{eff}}}{S} + C$, and the constant C accounts for offsets in I_{coil} or terrestrial magnetic fields. We measure M and Φ_{ext} from the period and phase shift of the modulation curves. We then perform a linear fit to Φ_{ext} as a function of B_{ext} to constrain the ratio $X \equiv A_{\text{eff}}/S$, as shown in Figure 3.6.

Since the SQUID array modulation curves deviate slightly from a perfect sinewave, we can improve upon the fitting function in Equation 3.22 by adding a small non-linearity term. Into the quadratic function:

$$f_{\text{NL}}(x) \equiv A_{\text{pp}}(x - x^2/V_{\text{NL}}), \quad (3.23)$$

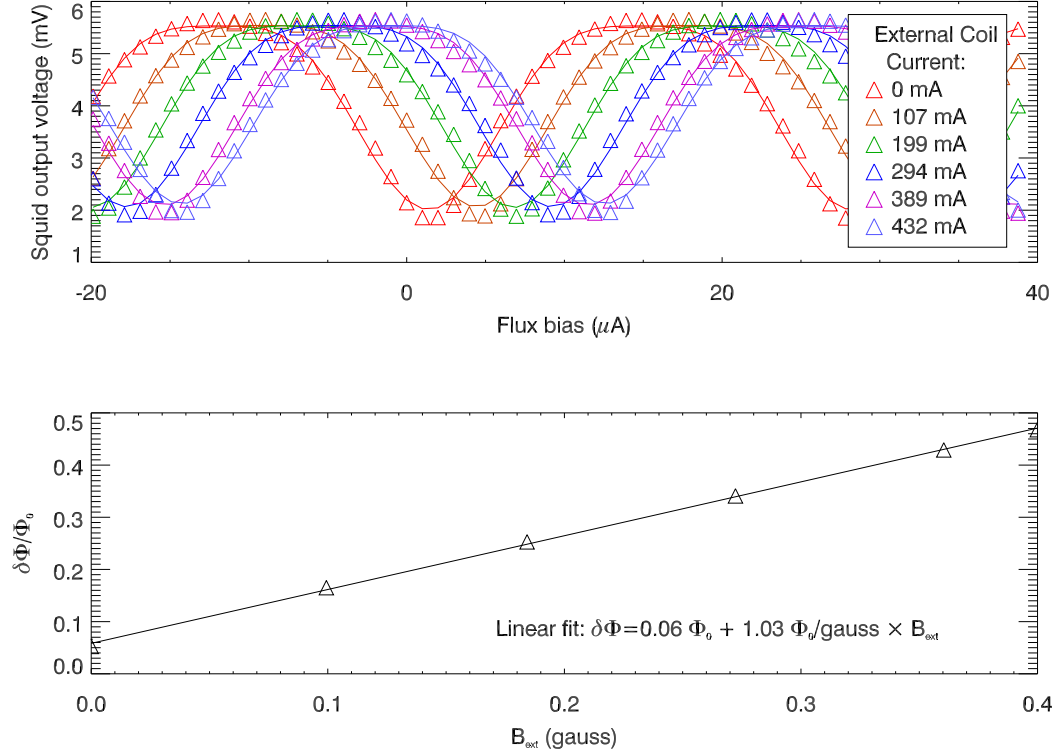


Figure 3.6: A measurement of the magnetic shielding efficiency of the fMUX SQUID module. By measuring the phase shift of the SQUID V - Φ curve, we can measure the magnetic flux observed by the SQUID. The slope of the observed flux-field relationship measures the ratio A_{eff}/S . This degeneracy between A_{eff} and S can be broken by measuring the field in a variety of different shielding configurations, i.e. shielded vs. unshielded.

we make the substitution:

$$V_{\text{SQ}} = f_{\text{NL}} \left(\frac{\cos \left(2\pi \frac{I_{\text{coil}}}{I_0} + 2\pi \frac{\Phi_{\text{ext}}}{\Phi_0} \right) + 1}{2} \right) + V_0. \quad (3.24)$$

We then perform a 5-parameter fit to determine A_{pp} , I_0 , $\Phi_{\text{ext}}(B_{\text{ext}})$, V_0 , and V_{NL} . An example of fit a measurement is shown in Figure 3.6. This procedure dramatically improves the quality of fit.

For a SQUID enclosed in the magnetic shield the overall sensitivity to magnetic fields is $79 \text{ m}\Phi_0/\text{gauss}$. This shielding is adequate in view of the fact that our detectors are AC-biased. With the linearity provided by the flux-feedback, low-frequency

magnetic variations do not directly appear in our data. Though low-frequency fluctuations may cause variations in the SQUID gain while open loop, these variations should be reduced by orders of magnitude when operating closed loop. Meanwhile, bias-band electro-magnetic fluctuations should be effectively blocked by the cryostat walls, based on the small skin depths at frequencies in the 100kHz range.

Our measurement of the slope of the $\Phi_{\text{ext}}-B_{\text{ext}}$ relation still leaves a degeneracy between the shielding efficiency S and the effective area A_{eff} . It also does not tell us the relative efficiency of the individual shielding components. To measure the effective area and the effectiveness of the individual shielding components, we perform the measurement in four magnetic shield configurations: without shielding, with only the Niobium foil, with only the Cryoperm and with both the Niobium and Cryoperm. We extract the A_{eff} of the SQUID from the unshielded case, and then use this area to determine the shielding efficiency of the other three cases. Based on the unshielded data the effective area of the Series SQUID arrays is 10^{-9} m^2 . The shielding efficiencies for the other combinations are plotted in Table 3.1.

Table 3.1: SQUID response and shielding efficiencies for a variety of magnetic shielding configurations

Shield Configuration	SQUID Array Flux ($\text{m}\Phi_0/\text{gauss}$)	Shielding Efficiency
Cryoperm + Nb foil	79	650
Cryoperm only	1000	52
Nb foil only	6.2×10^3	8.3
none	52×10^3	1

The observed shielding factor for the Niobium foil alone, $S = 8.3$, shows very good agreement with the expected value of 7. It is perhaps not surprising that the square foils perform slightly better than a circular foil due to the extra niobium around the square corners. The Cryoperm on the other hand provides a shielding factor of $S = 51$, which much less than the factor of 3000 assumed by our coarse cylindrical geometry approximation. There is also some suggestion from the frequency dependence of the Cryoperm shielding that the permeability at 4K is lower than the nominal value, though this difference can at best account for a factor of 3 reduction in shielding, and we are looking for a factor of 60.

It is interesting to note that the overall shielding provided by the combination of the niobium and the Cryoperm is 50% higher than the product of the shielding factors of either alone. Such an increase may be expected, given that in the presence of the niobium the field lines are deformed to be more parallel to the face of the Cryoperm which could lead to more of the field being drawn away from the SQUIDs. A detailed understanding of this mechanism would require a more complete numerical solution.

In order to measure the frequency dependence of the magnetic shielding the flux across the SQUID array is measured in the closed-loop configuration. The output voltage is $V_{\text{out}} = Z_{\text{forward}}/M \Phi_{\text{ext}}$, where by Equation 3.10 $Z_{\text{forward}} \approx R_{\text{fb}}$. The mutual inductance, $M \approx \Phi_0/26\mu\text{A} = 80\text{ pH}$, has been measured by the periodicity of the $V-\Phi$ curve. For a sinusoidally-varying magnetic field $B(\nu)$, the SQUID response is then expected to be $V_{\text{out}}(\nu) = A_{\text{eff}}B_{\text{ext}}(\nu)S(\nu)R_{\text{fb}}/M$. This measurement is repeated at several frequencies of the magnetic field. The SQUID response (which is proportional to the shielding factor) is shown in Figure 3.7 as a function of frequency. In order to more easily compare the frequency dependence of different shielding configurations, all traces are normalized to unity at low frequencies.

We can compare the frequency response of various configurations to determine the contribution of each component to the frequency-dependence. For instance, by comparing SQUIDS with and without a niobium foil (but both in a cryoperm sheath), we can confirm that—as expected—the niobium shielding has little frequency dependence in the range of frequencies tested. The completely unshielded configurations (no niobium or Cryoperm) did not behave well under this test, likely because the excitation signal exceeded the dynamic range of the SQUIDS. However, we can compare the shielding efficiencies measured with the Cryoperm sheath, S_{sheathed} , to the shielding observed without it, $S_{\text{unsheathed}}$. In both cases the measurement is shone with the niobium foils, since no significant frequency difference is observed when comparing SQUIDS with and without a foil. The frequency dependence of $S_{\text{unsheathed}}(\nu)$ is expected to depend only on the electromagnetic properties of the test cryostat, most notably the electrical conductivity and geometry of the dewar shell. On the other hand, the ratio $S_{\text{sheathed}}/S_{\text{unsheathed}}$ indicates the frequency response of the cryoperm sheath itself. The sheathed and unsheathed shielding factors, as well as their ratio, can both be found in Figure 3.7. Also shown in Figure 3.7 are some analytical estimates for the expected frequency dependence based on Equation 3.21.

Interestingly enough, the cryostat walls, as indicated by $S_{\text{unsheathed}}(\nu)$, accounts for almost all of the frequency dependence observed in these measurements. There is a good agreement also with the theoretical expectation, if we apply the geometry of the 0.5"-thick, 12"-diameter shell and assume a resistivity of $\rho = 40\text{ n}\Omega\text{-m}$ which is expected for Al-6061 at room temperature.⁷ Based on the electromagnetic properties of Cryoperm 10⁸ the sheath should have a similar transfer function. We assume a relative permeability of $\mu_r = 65000$ and a resistivity of $\rho = 350\text{ n}\Omega\text{-m}$. However from the ratio $S_{\text{sheathed}}/S_{\text{unsheathed}}$, we see that the transfer function of the Cryoperm is much weaker than expected at moderately high frequencies. This Cryoperm transfer function is more consistent with permeability of a $\mu_r = 20000$, or some combination of a lower permeability and a higher resistivity.

⁷<http://asm.matweb.com/search/SpecificMaterial.asp?bassnum=MA6061t6>

⁸<http://www.vacuumschmelze.de/index.php?id=136&L=2>

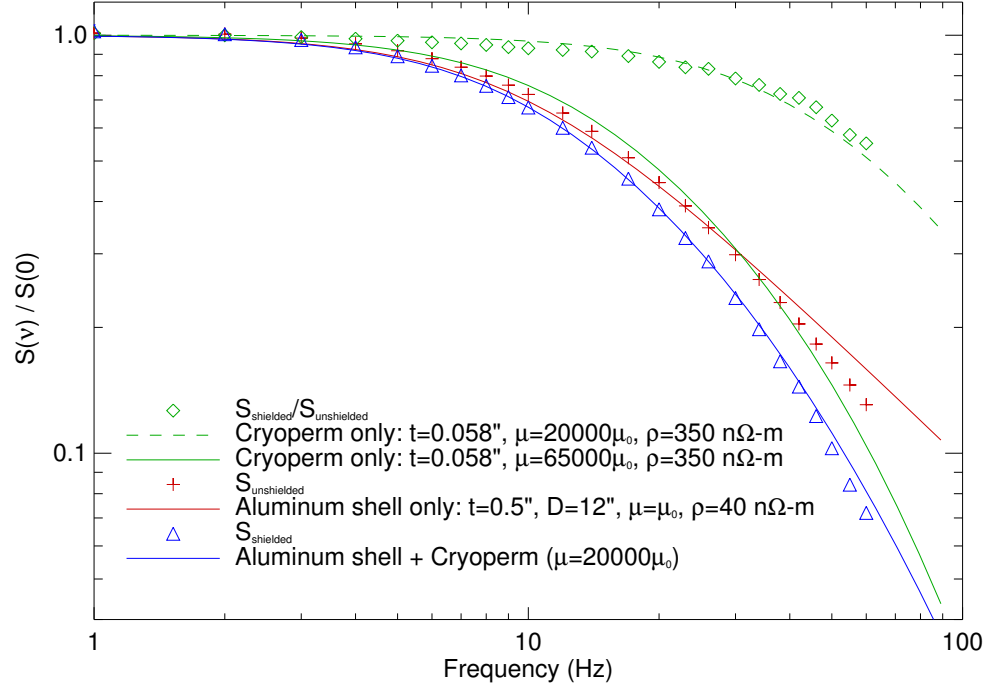


Figure 3.7: Frequency dependence of the Cryoperm shielding efficiency. The **blue triangles** show $S_{\text{sheathed}}(\nu)$, the shielding efficiency of the cryoperm sheath and niobium foil, as excited through the walls of the aluminum cryostat shell. The **red crosses** show data for $S_{\text{unshielded}}$ the attenuation of the dewar wall and the Niobium foil. Based on comparisons between SQUID's with and without the Niobium foil, the frequency variation of the Niobium shielding factor is known to be less than 10%, and so $S_{\text{unshielded}}$ is taken to be completely due to the cryostat walls. The **green diamonds** are the sheathed-to-unshielded shielding ratio, which indicates the frequency-dependence of the cryoperm sheath alone. The **red line** shows good agreement between the theoretical prediction of Equation 3.21 and the aluminum shell properties. However, the theoretical predictions for the Cryoperm **solid green line** indicate that the frequency dependence of the sheath is much weaker than expected. This may be consistent with a permeability that is low by a factor of 3 (**dashed green line**), or a resistivity that is high by a similar factor. The **blue line** shows the theoretical shielding transfer function for the combined system under the assumption that the permeability of the cryoperm is low.

Chapter 4

Flux-locked Loop Stability

We require a SQUID FLL with high loopgain over a wide bias band so that we can accommodate many bolometers per SQUID array with good linearity and dynamic range. In high-bandwidth feedback circuits containing reactive components some care is required to ensure stability, and the shunt feedback SQUID Controller described in the last chapter is no exception. The SSA has an input inductance of about 180 nH and strong negative feedback even at frequencies in the 10–100 MHz range. Over such a wide bandwidth, this input coil can resonate with reactances or stray poles within the feedback network and these strays can drive the SQUID into oscillations. Much of the early work in the SQUID Controller was focused on the problem of building a feedback circuit where the transducer and the feedback amplifier were separated by a length of transmission line. Resonances in these transmission lines were studied by Spieler (2003) and Lanting (2006) in the first demonstrations of the fMUX system, and a general program of keeping the 300K-4K lines short was adopted to minimize phase delays.

In the year before the deployment of APEX-SZ and SPT, the phase delays between the SQUID arrays and the *250 mK bolometers* were observed to be larger than anticipated when designing the demodulator boards. This led to a change in the sub-Kelvin bolometer wiring design, but this change led to new resonances in the system. So though the sub-Kelvin wiring is not an explicit part of the loop, these wires play a role in the stability of the system. Fortunately these new resonances were mitigated in time for the SPT deployment, and as we shall see, the solutions deployed on SPT were more than adequate for achieving our science goals. However, more improvements will be needed if we wish to extend the FLL bandwidth, or to push for more dynamic range or linearity.

In this chapter I review the different elements in the SQUID feedback network and how they effect stability. I will complex present measurements of the SQUID loopgain from SPT-like systems, and provide guidelines for future improvements.

4.1 Stability

The stability criterion for the flux-locked loop (FLL) is much simpler than the TES stability criteria discussed in Chapters 2 and 5. Since the state of the FLL is described by only one parameter, namely the flux in the SQUID, the stability of the system can be described entirely in terms of the loopgain. One can predict whether such a system is stable by means of the Nyquist stability criterion. This criterion is easiest to describe graphically. We consider a Nyquist plot which shows the contour traced out when $\mathcal{L}_{\text{SQ}}$ is plotted on the complex plane (see for example Figure 4.1) If this contour circumscribes the point $1 + 0i$, then the closed loop system is unstable. Otherwise it the closed loop system is stable, provided that the system is stable in the open loop configuration.

The Nyquist criterion is a firm predictor of stability (i.e. whether or not the system is prone to spontaneous oscillations), but it does not describe how robust the system is to variation in component values or how susceptible the system is to underdamped oscillations. One commonly cited estimate of the robustness of the stability is the concept of phase margin¹, or the phase of $\mathcal{L}_{\text{SQ}}$ when $|\mathcal{L}_{\text{SQ}}| = 1$. For negative feedback, the loopgain phase at zero frequency is $\arg(\mathcal{L}_{\text{SQ}}(0)) = 180^\circ$. Poles in the warm amplifier response or elsewhere in the system will lead to attenuation and phase delays, diminishing this phase. The Nyquist criterion usually requires that the phase margin be positive. However, in order to avoid ringing in the FLL a phase margin of 45° is preferred.

4.1.1 Poles, Delays, Resonances and Zeroes

In this chapter I will discuss stability in terms of linear phase delays, poles, zeroes and resonances. Later in the chapter we will examine SPICE simulations and measurements of the complex loopgain. However before we start, a brief analytical discussion may help understand the interaction between all of these features.

Linear phase delays arise when there is a temporal delay, δt , in the feedback network, for instance due to propagation delays in transmission lines. Introducing a delay into a system modifies the loopgain by adding a phase delay which is linear in frequency:

$$\mathcal{L}_{\text{SQ}}(\omega) \rightarrow \mathcal{L}_{\text{SQ},0}(\omega)e^{-i\omega \delta t}, \quad (4.1)$$

though the amplitude of the loopgain is unchanged.

Poles in the feedback loop, are expressed in terms of their time constant τ :

$$\mathcal{L}_{\text{SQ}}(\omega) \rightarrow \mathcal{L}_{\text{SQ},0}(\omega) \frac{1}{1 + i\omega\tau} \quad (4.2)$$

¹It should be noted that this definition of phase margin, $\arg(\mathcal{L}_{\text{SQ}})|_{|\mathcal{L}_{\text{SQ}}|=1}$, is subtly different than the definition cited by Lanting (2006): $\min(\arg(\mathcal{L}_{\text{SQ}}))|_{|\mathcal{L}_{\text{SQ}}|>1}$. The definition here is more in line with the strict definition of the Nyquist criterion, namely that the Nyquist contour must enclose the pole $\mathcal{L}_{\text{SQ}} = 1 + 0i$

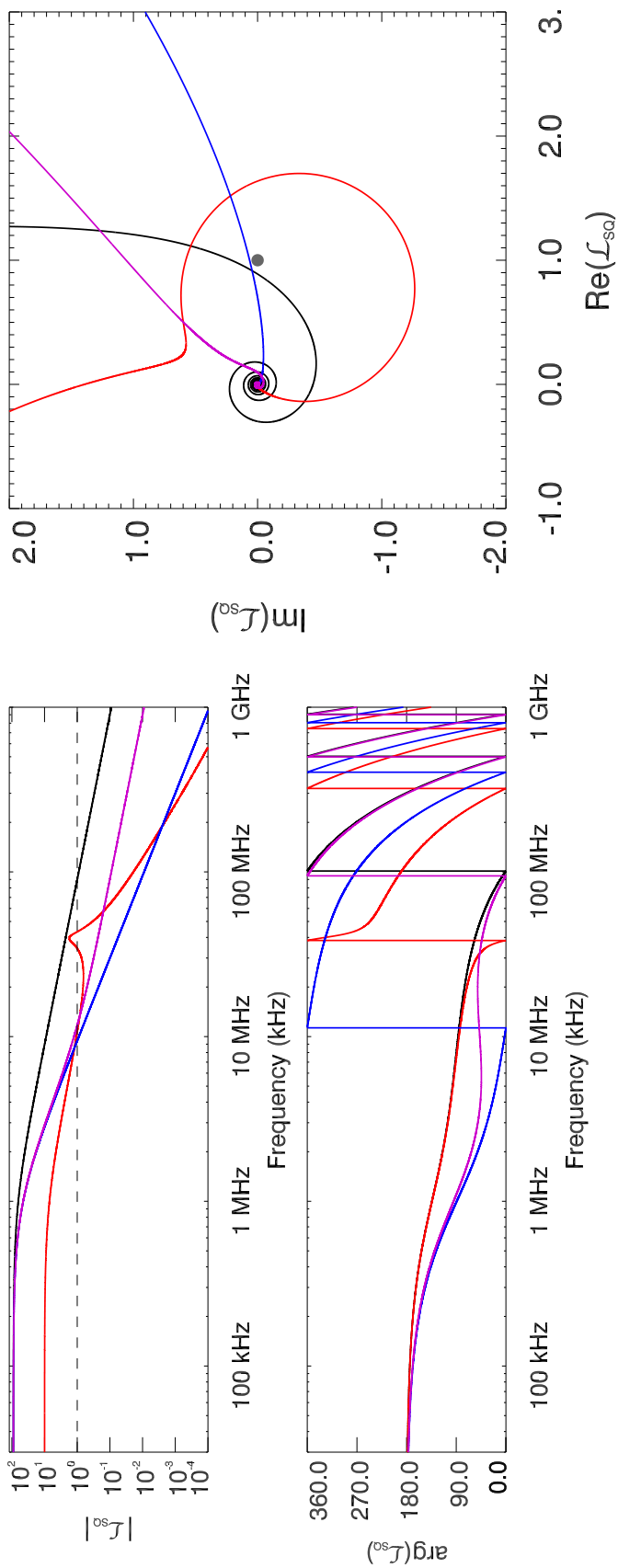


Figure 4.1: Bode and Nyquist plots showing the interplay of cable delays, poles, resonances and zeroes in determining the stability of the SQUID feedback loop. In the *left panel*, the Bode plot shows the amplitude and phase of the loopgain for four different circuit models. The *right panel* shows the Nyquist plot for the same four models. Models in which the loopgain contour encircles the point $1 + 0i$, shown as a **gray dot**, are unstable. All four models shown here have a total cable delay of 2.5 ns, which by Equation 4.9 limits the the loopgain-bandwidth product to 100 MHz. The **black trace** represents a system with a single pole at 1 MHz, and a loopgain of 90 in the passband. This trace does not enclose the gray dot meaning that it represents a stable system. The **blue trace** shows a system with a second pole, also at 1MHz. By merit of the low frequency of both poles, this system is also nominally stable, though it has very little phase margin. The **violet trace** shows a system also with a double pole at 1MHz and a zero at the $\omega_{1,2\text{pole}}/(2\pi) \approx 10$ MHz. The phase advance of the zero counteracts the delays from the poles and lends $\sim 45^\circ$ of phase margin to this curve as seen in the Nyquist plot. Finally, the **red trace** shows a system with loopgain-bandwidth product of only 10 MHz, but a $Q=7$ resonance at 40 MHz. This system would have a whopping 85° of phase margin if this resonance were not present. Although the resonance is well beyond the unity loopgain frequency, $\omega_1/(2\pi) = 10$ MHz, this resonance is strong enough to make this system oscillate, as indicated by the red loop surrounding the $(1 + 0i)$ point in the Nyquist plot. The failure of this otherwise very stable system demonstrates the importance of eliminating resonances in the SQUID feedback loop.

Though the attenuation introduced by each pole is small at frequencies much below the cutoff, $\omega \ll \omega_c \equiv 1/\tau$, their phase delays can be important. For a single pole these phase delays grows to -45° at $\omega = \omega_c$. For $\omega \gg \omega_c$ the loopgain amplitude falls as $|\mathcal{L}_{\text{SQ}}(\omega)| \propto \mathcal{L}_{\text{SQ}}(0) \times \omega_c/\omega$ while the phase delay asymptotically approaches 90° . Thus in a simple network where there is only a single pole the frequency, ω_1 , where the loopgain drops to unity amplitude, ($|\mathcal{L}_{\text{SQ}}(\omega_1)| = 1$) is roughly $\omega_1 \approx \omega_c \mathcal{L}_{\text{SQ}}(0)$. Since a single pole can consume up to 90° of phase margin, this leaves 90° ($\pi/2$ radians) for any propagation delays in the system. Thus in even the simplest system the maximum allowed propagation delay is constrained by the relation $\omega_1 \delta t \leq \pi/2$, or in other words:

$$\delta t \leq \frac{\pi}{2} \frac{\tau}{\mathcal{L}_{\text{SQ}}(0)} \quad (4.3)$$

In Figure 4.1, I illustrate a Nyquist plot showing the interaction between a single pole and a nearly critical propagation delay.

Second poles can be a problem. With two poles the combined phase delay above the second cutoff frequency asymptotes to 180° , leaving nearly no margin for propagation delays. If the second cutoff frequency occurs below ω_1 , the additional phase shift combined with cable delays can devour the phase margin, bringing instability. However, at frequencies well above ω_1 second poles do not pose an issue, since even though they add phase delays, the loopgain amplitude remains below unity and so these delays do not induce instability.

There is an exception to this if the network is designed so that both poles are at low frequencies. In that case the rapid attenuation from the second pole brings the loopgain to below unity before propagation delays come into effect leaving a system that is still marginally stable, as shown in Figure 4.1. In the special case where the two poles in the system appear with the same cutoff frequency, ω_c , then the loopgain drops to unity amplitude at a lower frequency: $\omega_{1,2\text{pole}} = \sqrt{\mathcal{L}_{\text{SQ}}\omega_c}$. Once the loopgain has crossed this threshold, additional poles at higher frequencies should present no problem.

Unlike additional poles, resonances can lead to increases in $|\mathcal{L}_{\text{SQ}}(\omega)|$ at the resonant frequency, thus even a resonance that occurs above ω_1 can lead to instability, (see Figure 4.1). A resonance at frequency ω_r with quality factor Q modifies the loopgain as:

$$\mathcal{L}_{\text{SQ}}(\omega) \rightarrow \mathcal{L}_{\text{SQ}}(\omega) \frac{1}{1 - \left(\frac{\omega}{\omega_r}\right)^2 + \frac{i\omega}{Q\omega_r}} \quad (4.4)$$

What makes a resonance even worse for stability is that they lead to an abrupt phase inversion at the resonant frequency, meaning that if they can push the $|\mathcal{L}_{\text{SQ}}|$ above unity they are extremely likely to cause oscillations. Thus resonances in the loop should be avoided whenever possible. Much of this chapter will be devoted to the topic of loopgain resonances and how to avoid them.

4.1.2 Zeroes and the Lead-Lag Filter

Zeroes in the loopgain network can cause phase advances and actually increase the phase margin. Analytically they take the form:

$$\mathcal{L}_{\text{SQ}}(\omega) \rightarrow \mathcal{L}_{\text{SQ}}(\omega)(1 + i\omega\tau) \quad (4.5)$$

In the fMUX shunt-feedback network, there is a lead-lag filter, shown schematically in Figure 4.2. The lead lag introduces both a pole and a zero to the filter response:

$$A_{ll}(\omega) = \frac{1 + i\omega\tau_{ll}/\chi}{1 + i\omega\tau_{ll}}, \quad (4.6)$$

where $\tau_{ll} \equiv (R_{\text{dyn}} + R_{ll})C_{ll}$ and $\chi \equiv \frac{R_{ll} + R_{\text{dyn}}}{R_{ll}}$. The properties of the lead-lag have been discussed by Lanting (2006). Due to the interaction of the pole and the zero, the phase delay of the lead-lag filter peaks at some maximum value:

$$\max(-\arg(A_{ll}(\omega))) = -\arg(A_{ll}(\omega_m)) = \tan^{-1} \left(\frac{1}{2} (\sqrt{\chi} - 1) \right), \quad (4.7)$$

For low values of χ , the peak phase delay is much less than the 90° phase delay introduced by a simple single pole.

In order to provide guidelines for choosing the lead-lag parameter it is easier to think about the effects of the lead-lag pole and zero separately. In Figure 4.1 it was shown that a second pole—with its full 90° phase delay—can be marginally stable even without an additional zero in the loop. Since the a zero introduces a 45° phase advance at the zero frequency, $\omega_z = \chi/\tau$, tuning the lead-lag such that this zero falls right at $\omega_{1,2\text{pole}}$ creates a system where the loopgain very quickly falls to unity amplitude, but then slows down just enough to leave 45° of phase margin. Thus for a system with DC loopgain $\mathcal{L}_{\text{SQ}}(0)$ and bandwidth $\Delta\nu$, setting $\tau_{ll} = 1/(2\pi\Delta\nu)$ and $\chi = \sqrt{\mathcal{L}_{\text{SQ}}(0)}$ will make for a very stable system as shown in Figure 4.1.

4.2 Simulating and Measuring $\mathcal{L}_{\text{SQ}}$

The complex loopgain of a system can be predicted easily in numerical simulations and if one is careful about preserving phase shifts, one can also directly measure the loopgain in the laboratory. In either case, a break is made in the the feedback circuit, be it real or simulated, for example as shown by the dotted line in the schematic shown in Figure 4.3. A sinusoidal voltage source is applied to V_{test} . The amplifier output voltage is then: $V_{\text{out}}(\omega) = \mathcal{L}_{\text{SQ}}(\omega)V_{\text{test}}(\omega)$.

For simulation of the loopgain, I have used NGSPICE AC analyses². Though the work by Spieler (2002) was done in PSPICE, the results should be very similar between

²<http://newton.ex.ac.uk/teaching/CDHW/Electronics2/userguide/sec1.html#1.1.2>

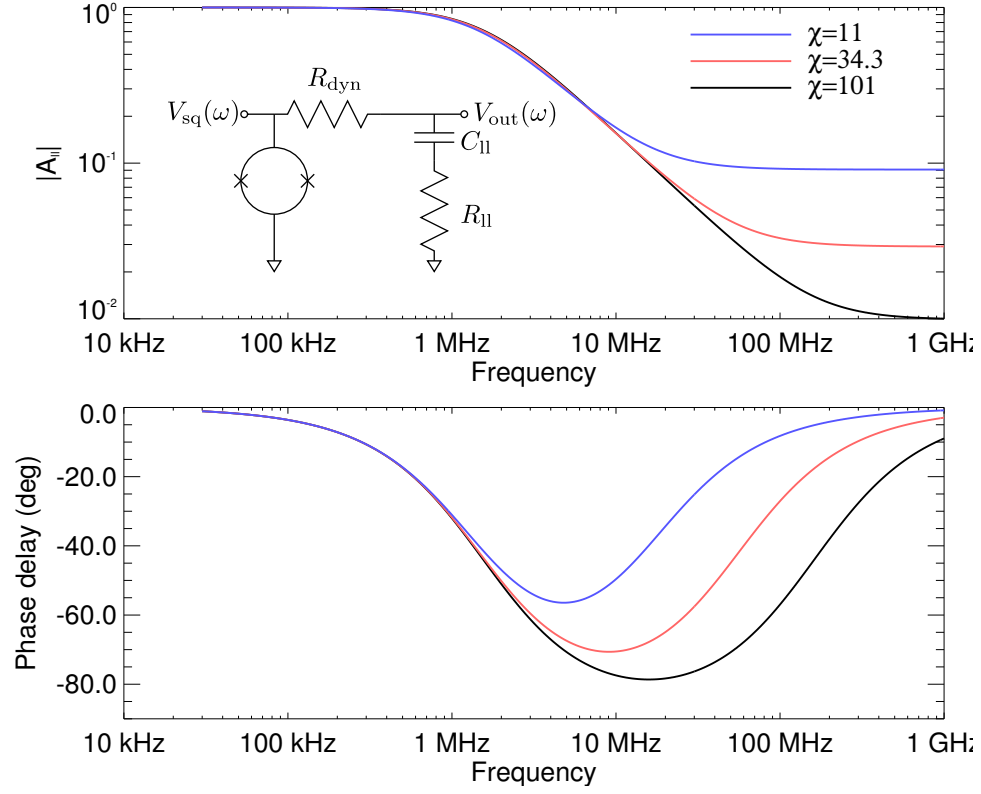


Figure 4.2: The topology and transfer function, $A_{II}(\omega) = V_{out}(\omega)/V_{sq}(\omega)$, of the lead lag filter. For historical reasons, the filter is currently implemented by loading the SQUID output with an RC network. The transfer function thus depends on the magnitude of the dynamic impedance, R_{dyn} .

the two packages. The voltage applied to V_{test} is a unity amplitude sinusoidal source with zero phase. The output voltage is then equal to the loopgain: $V_{out} = \mathcal{L}_{SQ}(\omega)$. For an AC sweep analysis, the frequency of the source is swept over the desired range, and the loopgain is recorded for each frequency.

In order to directly measure the loopgain in a real SQUID Controller, the feedback loop is broken by unsoldering a 0Ω surface-mount resistor which normally connects the amplifier output to the feedback switch. The solder mount pads on either side of this resistor are connected to BNC jacks in the SQUID Controller RF shield via a few inches of 50Ω mini-BNC cable. The shield of the BNC cable is connected to the SQUID Controller ground plane in order to prevent capacitive coupling between the test leads and the rest of the circuit. All of this is necessary in order to get a complete measurement of the loopgain, including any parasitics in the feedback switch. I also avoids confusing the loopgain with any gain from the amplifier stages which connect the output of the flux locked loop to the output of the SQUID Controller. By including

the feedback switches (Figure 3.4) in the loop one can adjust the amplitude of the DC loopgain $\mathcal{L}_{SQ}(0)$, or run the system “open-loop” (both switches open) to determine contributions in the measurement which may arise from stray coupling in the test set-up.

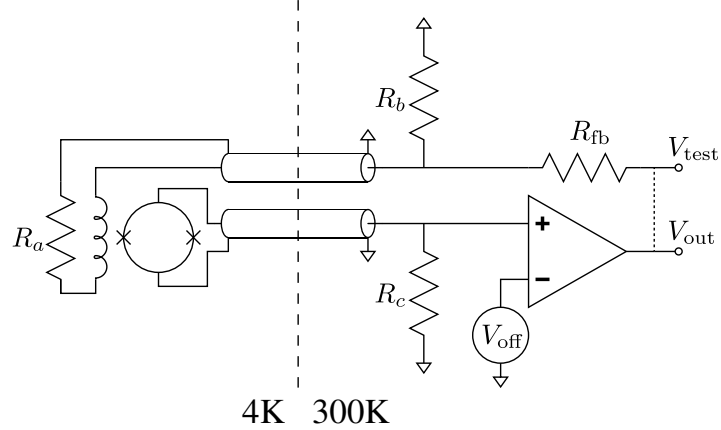


Figure 4.3: Schematic illustrating how to simulate or measure the loopgain $\mathcal{L}_{SQ}$, and various schemes for terminating the SQUID wiring between 4K and 300K. In either simulation or a real SQUID Controller measurement the loopgain can be inferred via the quantity V_{out}/V_{test} . This schematic in particular highlights the various sites for potential termination resistors on either end of the 4K–300K wiring. The cables between 4K and 300K are 5 inches long with an inductance of approximately $L_i = 25\text{nH/inch}$ and $Z_0 = 100\Omega$ for a capacitance of $C_i = 2.5\text{pF/inch}$. The wires with an approximate series resistance of $2\Omega/\text{inch}$. Though for the most part room temperature resistances add significantly to the noise of the system, some termination is required especially on the line going to the SQUID coil. The SQUID-to-amplifier cable is more robust to poor termination since the SQUID Arrays have a dynamic output impedance of order $R_{dyn} \approx 100\Omega$ making them well matched to the cables.

We use an HP 4195a network analyzer to sweep the frequency of the excitation source (emitted from the 4195a S-port), and record the transfer function of the feedback network. The output of the feedback amplifier, V_{out} is connected to the 4195a T-port to record the transmitted signal. In order to eliminate phase shifts due to cabling between the 4195a and the SQUID Controller under test, the 4195a reference (R-port) is also connected to the SQUID Controller PCB at the same point where the excitation source is connected, V_{test} . The cabling between V_{test} and the R-port is matched in length and impedance to the cabling between the V_{out} and the T-port. In this way the complex transfer function measured by the network analyser, $V_T/V_R = V_{out}/V_{test} = \mathcal{L}_{SQ}$, is the loopgain and is compensated to remove any phase

shifts due to cable delays between the SQUID Controller and the network analyzer.

An example of such a measurement of the loopgain is shown in Figure 4.4. The data shown in this Figure is for an SPT like system. It has the same cryogenic wiring, though no lead-lag filter. The open loop measurement shows parasitic coupling either in the PCB layout or cabling at frequencies above 40 MHz, setting a limit to the frequencies at which this particular measurement can be trusted. Based on the network analysis, the ripples below 2 MHz are associated with LRC coupled bolometers and appear in the Nyquist diagram as a series of tight spirals. Ideally this data would show a single-pole rolloff above 1MHz. Two soft resonances are visible at 6 and 10 MHz. In the Nyquist diagram these two resonances appear as wide arcs, at $\text{Im}(\mathcal{L}_{\text{SQ}}) = 2$ and 0 respectively. These resonances effectively limit the loopgain achievable with this system. However the addition of a lead-lag filter and a 50ω damping resistor shunting across the 250 mK transmission lines also buys more stability, as also shown numerically in Figure 4.4. With a lead-lag filter to suppress these resonances, this system was stable enough for observations with the SPT. However future receivers are demanding more bandwidth, or more loopgain. In the interest of developing future receivers it is important to understand the origin of these resonances.

4.3 Transmission lines: 4K to 300K

Most commercially available semiconductor amplifiers do not operate at cryogenic temperatures. So the feedback amplifier illustrated in Figure 3.4 is operated at 300K separated from the cryogenically cooled SQUIDS by some length of transmission line, as shown in Figure 4.3.

A short segment of transmission line of length, δl , is typically parametrized by its series inductance $L_i \delta l$ and its shunt capacitance $C_i \delta l$. Here L_i and C_i are the inductance and capacitance per unit length. For lossy transmission lines one should also consider the resistance per unit length, R_i . Lossless transmission lines are much easier to study analytically.

Viewed from one end, the impedance observed at the input of a transmission line of length l is parametrized in terms of the propagation delay:

$$\tau \equiv l \sqrt{L_i C_i},$$

and the characteristic line impedance:

$$Z_0 \equiv \sqrt{L_i / C_i}.$$

The observed impedance then depends on the termination load Z_L at the other end:

$$Z_{\text{line}}(Z_L, Z_0, \omega\tau) = Z_0 \frac{Z_L + iZ_0 \tan(\omega\tau)}{Z_0 + iZ_L \tan(\omega\tau)} \quad (4.8)$$

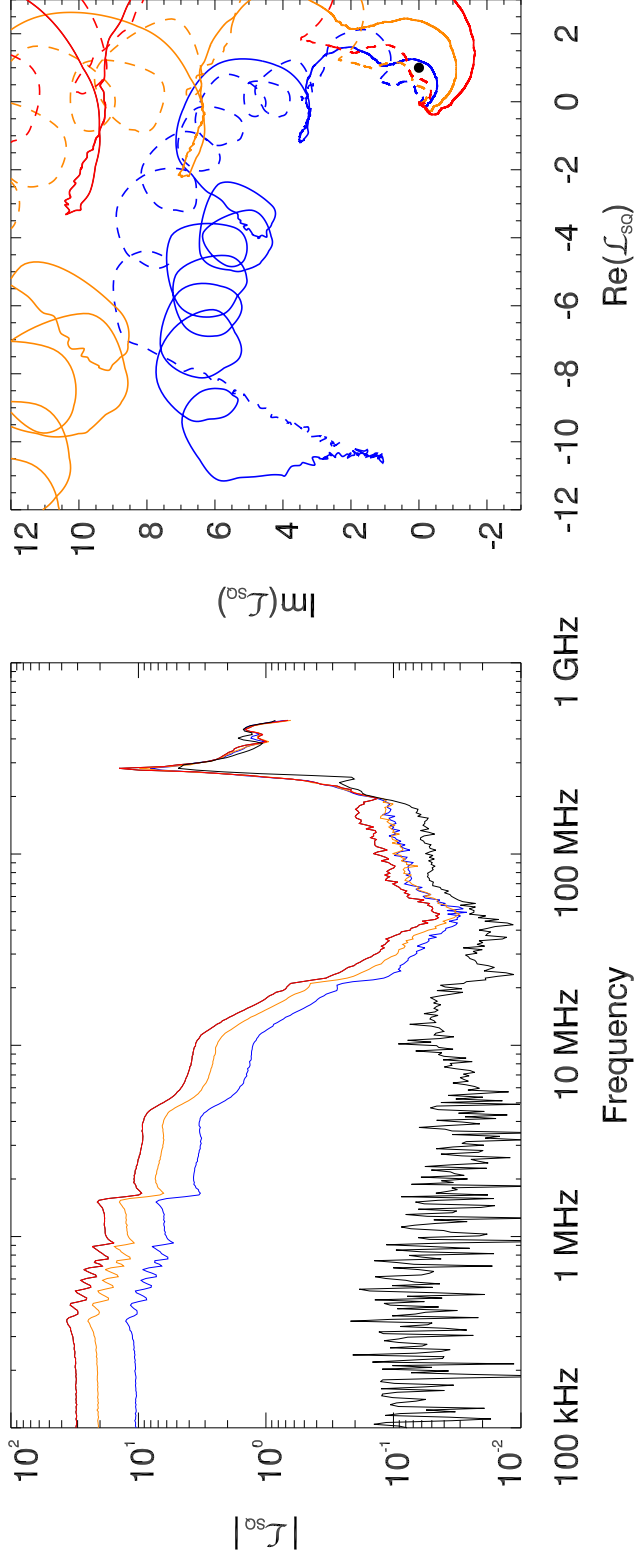


Figure 4.4: An example of a loopgain measurement in an SPT like system. The *left panel* shows the loopgain amplitude as a function of frequency. The open loop measurement (**black trace**) illustrates that the measurement is free of parasitic coupling up to 40 MHz. The **blue**, **orange** and **red** traces correspond to feedback resistances of 10kΩ, 5kΩ, 3.3kΩ respectively. In order to more clearly resolve the features in the loop the lead-lag filter has been disconnected in these measurements. The resonances below 2 MHz are associated with LRC-coupled bolometers which have been heated out of their transition. The resonances near 6 and 10 MHz are dangerous for stability as shown in the Nyquist plot in the *right panel*. Without a lead-lag, even the 10kΩ feedback setting is unstable. The **dashed curves** have been multiplied by a SPT-like lead-lag transfer function (Equation 4.6) $\tau_{ll} = 2.4$ ns and $\chi = 6$. This leads to good stability at 10kΩ feedback, and marginal stability at the 5kΩ and 3.3kΩ feedback settings.

A well-terminated transmission line, with $R_L = Z_0$, appears purely resistive at the input. Meanwhile a poorly terminated transmission line has a complex impedance, which depending on the frequency, has a reactance which may be either positive or negative. For instance, at low frequencies, a segment of transmission line appears as an inductor, lL_i , whereas a transmission line that is open at the far end looks like a capacitor, lC_i . As we will see through out this chapter, these transmission line reactances can then resonate with other reactances in the system. If these resonances cause the loopgain to spike above unity then the feedback network will likely be unstable, as illustrated in Section 4.1.1

However even well-terminated, these 4K-300K transmission lines introduce time-delays, δt_i into the feedback network. As a corollary to Equation 4.3, the loopgain bandwidth product, LGBWP, is limited by the cumulative phase shift in the system:

$$\text{LGBWP} \leq \frac{1}{4 \sum \delta t_i} \quad (4.9)$$

For this reason, these transmission lines are designed to be as short as possible. Both the cryostats for the APEX-SZ and SPT use the same wire harnesses from Tekdata³. The lines are made from twisted pairs of Manganin wire, with an approximate length of 5" (127mm). The impedance of these lines is roughly 100Ω . Based on indirect measurements, the equivalent inductance of these lines is estimated to be 25 nH/in, which implies a delay of 2.5 ns/in, or 12.5 ns for each line. Manganin is chosen for its low thermal conductivity, though it is somewhat electrically resistive and so these lines have resistances of nearly 10Ω . Based on this design and Equation 4.9, the theoretical peak-LGBWP is 100 MHz, though resonances or poor terminations lead to dramatically reduced performance. SPT and APEX-SZ started with a more modest goal of achieving peak loopgains of 15–40 over a bandwidth of 1 MHz. SPT currently operates at the lower end of this range, though with some further design and study it should be possible to get closer to the theoretical limit.

Properly terminating the transmission lines in Figure 4.3, is slightly complicated by noise and loading considerations. Termination resistors at 300K will inject excess noise into the system. Fortunately a transmission line that is properly terminated at one end appears purely resistive at the other, and so termination at one end is sufficient to eliminate transmission line resonances. In Figure 4.3 we illustrate three possible locations for termination resistors, R_a , R_b , and R_c .

In table 4.1 we list 5 different termination schemes for terminating these lines, and using SPICE simulations, we can evaluate the stability of each of these configurations. In the simulations, the SQUID is modelled as a linear device with a 180 nH input coil and a transimpedance of -500Ω . The dynamic output impedance of the SQUID, R_{dyn} , in these simulations is allowed to vary from 100Ω to 300Ω , depending on the model and the types of resonances being considered. The transmission lines are

³Cryoconnect, Div. of Tekdata, Stoke on Trent, Staffs. ST1 5SQ, United Kingdom

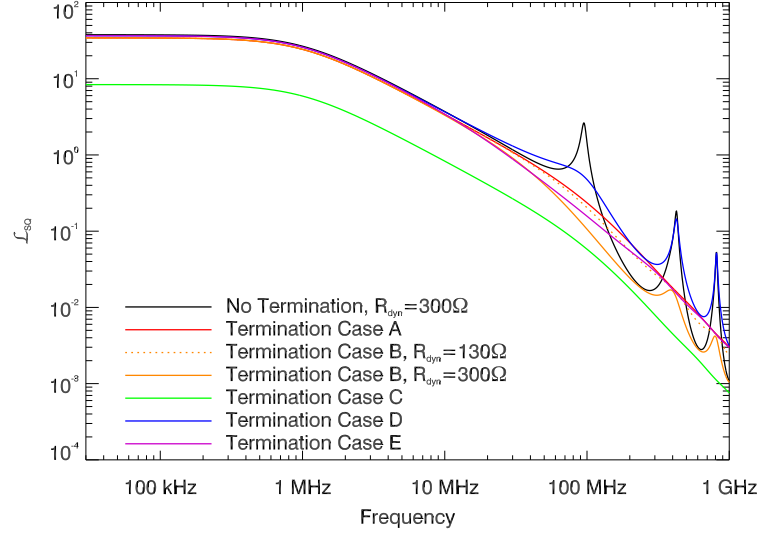


Figure 4.5: Transmission line resonances in the 4K wiring and termination schemes. See text for simulation details. The unterminated case shows a strong resonance near 100 MHz. Case A removes all resonances at the expense of adding additional current noise. Case B shows the effects of leaving the SQUID line unterminated. Case C shows the attenuation penalty of trying to terminate the SQUID at the 300K end. Case D is an attempt to terminate at the feedback line on the 4K end, though owing to the high impedance of the coil at relevant frequencies the resonance is underdamped. In Case E the feedback line is terminated, but the termination resistor is capacitively coupled to only allow noise to flow at frequencies above 40 MHz.

modelled using the SPICE3 LTRA lossy transmission model⁴, with $L = 25\text{nH/in}$, $C = 2.5\text{pF/in}$, $R = 2\Omega/\text{in}$, and a total length of 5in. The amplifier has a gain of 250 and a 1MHz bandwidth, while the feedback resistance is $R_{fb} = 3.3\text{k}\Omega$. The loopgain is measured by injecting a sinusoidal voltage at $V_{\text{test}}(\omega)$ (see Figure 4.3). The frequency is swept across the range of interest and at each frequency the relative phase and amplitude of the output voltage is recorded to calculate the loopgain: $\mathcal{L}_{\text{SQ}}(\omega) = V_{\text{out}}(\omega)/V_{\text{test}}(\omega)$. Thus we calculate the complex loop gain as a function of frequency, and generate Nyquist plots to estimate the stability for each termination scheme as shown Figures 4.5 and 4.6.

In the unterminated case, the transmission line resonances are clearly unacceptable. Figure 4.5 shows strong resonances at frequencies of 100 MHz and above. These strong resonances clearly push the loopgain amplitude to above unity amplitudes, and as shown in 4.6. Note that in the most pessimistic “unterminated” case we also assume that the SQUID array has a very high dynamic impedance of $R_{\text{dyn}} = 300\Omega$.

⁴<http://bwrc.eecs.berkeley.edu/classes/icbook/spice/UserGuide/elements.html#550>

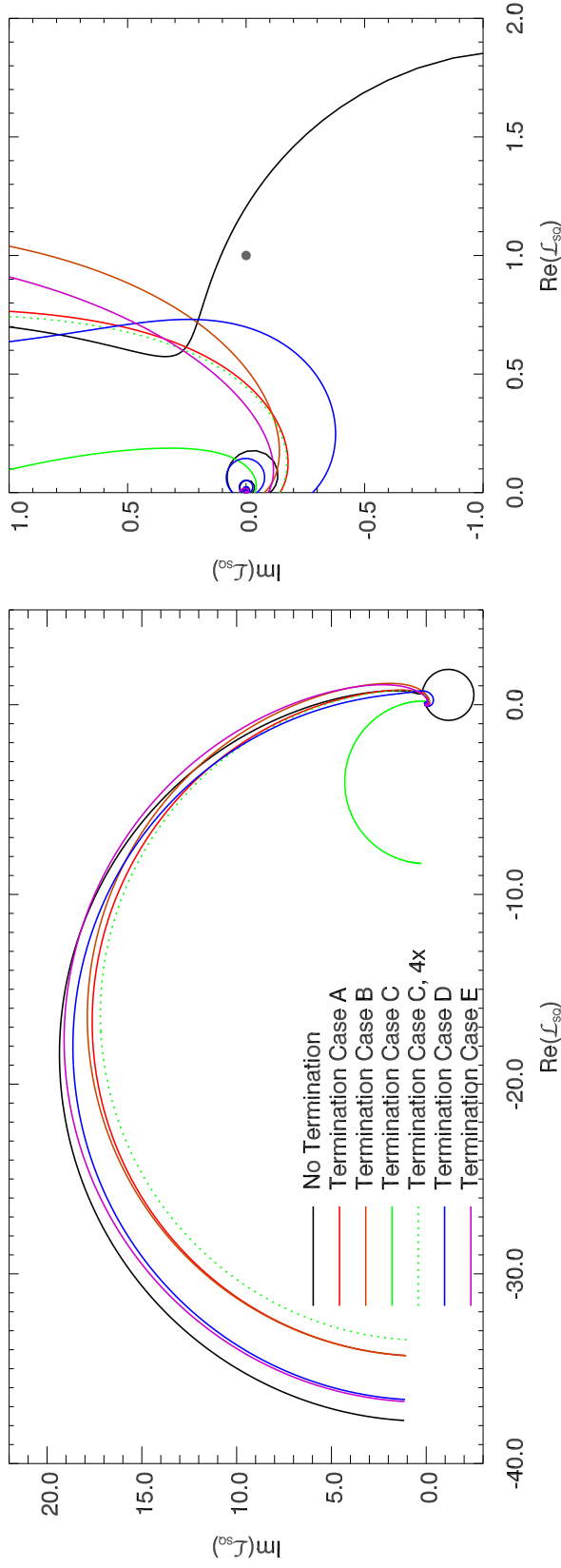


Figure 4.6: Nyquist diagrams corresponding to the loopgain amplitude diagrams in Figure 4.5. All of the attempts at termination schemes in Table 4.1 allow more stability than the unterminated case shown in **black**. Case A (**red**) is very effective though it is still noisy. Case B (**orange**) is stable, though it has slightly less phase margin. Thus even when R_{dyn} is much greater than 100Ω , the need for stability may not justify the extra noise incurred by terminating the SQUID line on the 300K side. Case C (**green**) shows how terminating the SQUID line on the 300K side also attenuates the loopgain, which means that more amplifier gain may be needed to meet the same loopgain (**dotted green trace**). Terminating the feedback line is much more important. Case D (**blue**) is an attempt at terminating this line at 4K. It allows stable operation though it is not very effective at high frequencies. The scheme in Case E (**violet**) is a very effective 300K termination which achieves low noise in the bias-band by capacitively coupling at low frequencies.

Table 4.1: Termination schemes explored in Figures 4.5 and 4.6. Z_a , Z_b , and Z_c refer to termination sites illustrated in Figure 4.3.

Case	Assumed R_{dyn}	Z_a	Z_b	Z_c	Notes
Baseline	100Ω	short	open	open	No explicit termination
A	100Ω	short	100Ω	open	Very effective feedback termination, but noisy
B	300Ω	short	100Ω	open	Illustrates effects of poor termination on SQUID line
C	300Ω	short	100Ω	100Ω	Terminates SQUID line, but suffers from loopgain attenuation and noise
D	100Ω	100Ω	open	open	Low noise, but feedback line is only terminated when $\omega \ll L_{\text{coil}}/(100\Omega)$
E	100Ω	short	$100\Omega + 1/(i\omega \times 40\text{pF})$	open	Effective feedback termination, low noise penalty for $\nu < 10\text{MHz}$

Termination schemes A, B, and C explore the different efficacies of different SQUID line termination schemes. In all three schemes we terminate the feedback line with $R_b = 100\Omega$ to study reflections in the feedback line alone. In case A we assume that we have the flexibility to operate the series SQUID array at a dynamic impedance of $R_{\text{dyn}} = 100\Omega$. This termination scheme is quite effective as it removes all resonances, rendering the circuit stable. However, when the SQUID is biased for optimal gain, the dynamic impedance may be slightly higher. For this reason we also simulate in case B, a pessimistic case in which the R_{dyn} is allowed to rise to 300Ω . This case shows broad resonances at frequencies of 250 MHz and above, though the stability is only somewhat diminished and the effect is even less discernable when the SQUID is operated at a less pessimistic value of $R_{\text{dyn}} = 130\Omega$. Termination at R_c removes these resonances, but it also loads the SQUID output, reducing the total gain of the system. Option C also adds an addition $1.3\text{ nV}/\sqrt{\text{Hz}}$ of voltage noise to the amplifier input, which is more than the noise of the amplifier itself. From these simulations it seems that additional SQUID termination at R_c is necessary and desirable only if the if the SQUID arrays are operating very far from their nominal $R_{\text{dyn}} = 100\Omega$.

Resonances in the feedback lines are a much bigger threat to stability. Termination schemes A, D and E test various schemes for terminating the feedback lines. In each of these case the R_{dyn} is assumed to be 100Ω , effectively terminating the SQUID transmission line so that we can study reflections in the feedback line alone. One of

the most straightforward approaches to termination would be to terminate the feedback line on the 300K side with $R_b = 100\Omega$, as in case A. This approach is very effective in eliminating the resonances and it does not effect the magnitude of the loopgain. However this warm termination resistor introduces an unacceptably large $13\text{pA}/\sqrt{\text{Hz}}$ noise current into the SQUID input. Placing a cold 100Ω at R_a , as in case D, is only somewhat effective since this location is in series with the 180 nH SQUID input, and does not provide good termination at frequencies near or above $\nu = R/2\pi L_{\text{coil}} \approx 90\text{ MHz}$. Case E is an attempt at a compromise. A 40 pF capacitor is placed in series with a 100Ω resistor at R_b . This capacitor blocks the flow of current at frequencies below 40 MHz , with a slightly more tolerable noise penalty of $3\text{pA}/\sqrt{\text{Hz}}$ at 10 MHz . Compromises such as Case E may be important for extending this system to larger loopgains or bandwidths.

4.4 Role of the SQUID input coil

In the above discussions of SQUID stability, the threats to stability have all been posed in terms of phase delays in the feedback loop. However, circuit elements in parallel with the SQUID input coil (as in Figure 4.7) can redirect or even invert the feedback current bringing the SQUID into oscillations. If, as shown in 4.7, one breaks the feedback loop and injects a voltage, V_{test} the loopgain can be related to the resulting output voltage:

$$\mathcal{L}_{SQ}(\omega) = \frac{V_{\text{out}}}{V_{\text{test}}} = \frac{V_{\text{coil}}}{V_{\text{test}}} \frac{Z_{tr} A_{\text{amp}}(\omega)}{i\omega L_{\text{coil}}} \quad (4.10)$$

$$\approx \frac{Z_{\text{par}}}{Z_{\text{par}} + i\omega L_{\text{coil}}} \widehat{\mathcal{L}}_{SQ}(\omega), \quad (4.11)$$

where $\widehat{\mathcal{L}}_{SQ}(\omega)$ is the “natural loopgain”, the loopgain that would have been observed in the same system if Z_{par} were open, (Equation 3.3). In the second line I have used the fact that $R_{fb} \gg |\omega L|$. For the upcoming discussion I will refer to this modification of the loopgain as the “loopgain boosting factor”:

$$\Xi(\omega) = \frac{\mathcal{L}_{SQ}(\omega)}{\widehat{\mathcal{L}}_{SQ}(\omega)} = \frac{Z_{\text{par}}(\omega)}{Z_{\text{par}}(\omega) + i\omega L_{\text{coil}}} \quad (4.12)$$

This loopgain boosting factor changes both the phase and magnitude of the loopgain.

Equation 4.12 can also be used to understand the resonances caused by the 300K-4K transmission lines. The feedback transmission line, and its associated terminations also appear as elements also in parallel with the SQUID coil. If we assume for noise optimization purposes that $R_{fb} \gg Z_0$, where Z_0 is the impedance of the line, then by Equation 4.8 we have that the effective impedance from the SQUID-coil end of the feedback-line is well-approximated as a capacitor, C_{line} , in parallel with the SQUID

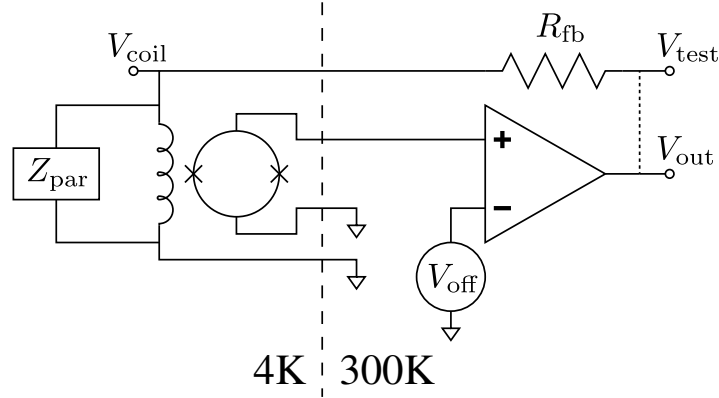


Figure 4.7: Schematic diagram illustrating how circuit elements in parallel with the SQUID coil, Z_{par} , can form parallel resonances and spikes in the effective loopgain.

input coil (See Figure 4.8). This resonance can be averted by properly terminating the transmission line, whereby the resonance at $\omega \approx 1/\sqrt{L_{\text{coil}}C_{\text{line}}}$ becomes at (much safer) pole at $\omega = Z_0/L_{\text{coil}}$. This figure clearly demonstrates the importance of well-terminating transmission lines at the ends *opposite* the SQUID coil, at both 300K and 250 mK.

4.5 Bolometers Gone Superconducting

If a TES goes unstable or if its voltage bias is completely removed it will become fully superconducting, and will no longer dissipate electrical power. The TES is thus latched in the superconducting state until it is heated above the transition temperature by some other source. Since the bolometer is coupled directly to the SQUID input coil (which for this system is also the feedback coil), this can effect the loopgain of the SQUID, potentially creating instabilities.

As one can see in Figure 3.1, current from the feedback resistor, I_{fb} is split between the SQUID input coil and the LC-coupled bolometers. If I label the total impedance of the LC-coupled bolometers as Z_{MUX} then the effective parallel impedance is $Z_{\text{par}} = Z_{\text{MUX}} + R_{\text{bias}}$. Due to the resonances in Z_{MUX} , it is reasonable to expect that these bolometers would impart resonant features to $\Xi(\omega)$ and $\mathcal{L}_{\text{SQ}}$.

Such resonant ripples in the loopgain are demonstrated by the simulation from Figure 4.9. This simulation depicts a SQUID attached to three LC-filtered bolometers. The 4K-300K transmission lines are terminated as in Case E, and the loopgain from that simulation is shown in this figure for reference. If the bolometers all have relatively large resistances, then the ripples in the loopgain are relatively small and the bolometers pose no risk to FLL stability. However as shown it the Nyquist plot

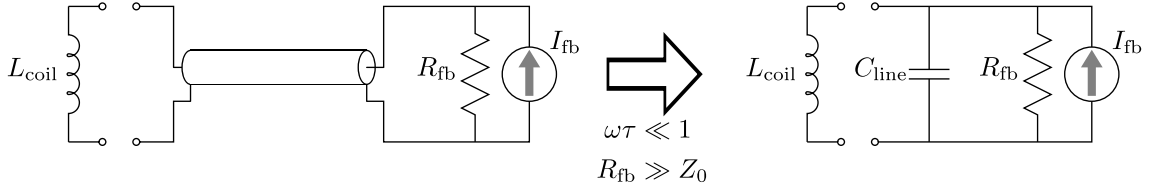


Figure 4.8: Equivalent circuit to the 300K-4K feedback line, showing the origin of the resonances in this line. The feedback amplifier and resistor are represented on the left hand side by the Norton equivalent circuit with $I_{fb} \equiv V_{out}/R_{fb}$, and output impedance of R_{fb} . Looking down the transmission line from the 4K end, it can be nearly approximated by the transmission line capacitance, C_{line} , since R_{fb} is much greater than Z_0 . This equivalent clearly looks like a resonant tank circuit when placed across the SQUID input coil. At high frequencies, $\omega \gtrsim \tau$, this approximation breaks down somewhat. However, so long as $\omega\tau < \tan^{-1}(R_{fb}/Z_0) < \pi/2$ the load will still look capacitive in nature, but the magnitude of the effective capacitance will be off.

in 4.9, if the resistance of a bolometer drops too low, these resonances can cause FLL instabilities.

TES instabilities like this are a common pitfall when operating with the fMUX system. Under normal operation, care should be taken to avoid electrothermal feedback instabilities. If a bolometer goes into electrothermal-feedback oscillations and latches superconducting this may in turn render the attached FLL unstable, meaning that the rest of the bolometers on the comb are unusable. This problem can be easily corrected by heating the bolometers, though this process can be time consuming and is an inefficient use of telescope time. This is one reason why we try to design our bolometers with some extra stability margin.

A natural question to ask is: What is the minimum tolerable bolometer resistance? Under what conditions is a shorted bolometer tolerable? An analytic discussion of this topic could proceed by calculating the loop gain and the cumulative phase shifts from the input inductance, the feedback amplifier and other phase delays. However in Equations 3.13 and 3.14 we have already seen how these delays conspire to determine the effective input impedance, and so we can use the input impedance to understand these instabilities. In Figure 4.10, we show a simple resonator in series with the SQUID impedance. This circuit will be unstable if, on resonance, the total real portion of the impedance is less than zero. Thus from Equation 3.13 we have the SQUID stability criterion:

$$R_{bolo} \geq -R_{bias} - \text{Re}(Z_{in}) \approx \omega_0^2 L_{coil} \left(\sum \tau_i \right) \frac{|\mathcal{L}_{SQ}(0)|}{(1 - \mathcal{L}_{SQ}(0))^2} - R_{bias} \quad (4.13)$$

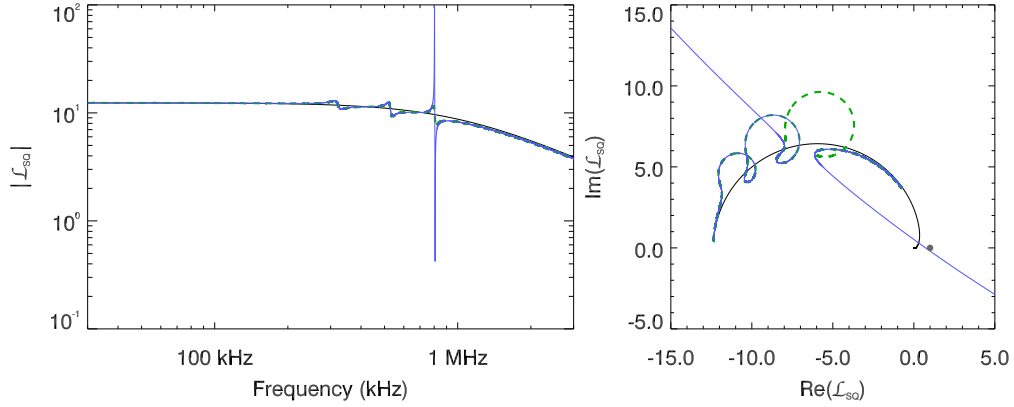


Figure 4.9: Nyquist diagrams showing how the loopgain changes in presence of the LC-coupled bolometers. The **black** trace shows the natural loopgain, $\widehat{\mathcal{L}}_{\text{SQ}}(\omega)$. The **dotted green** trace represents a system with three bolometers attached, all with 2Ω resistance and resonances at 320 kHz, 530 kHz and 606 kHz. In the **blue** trace, the 600 kHz bolometer has gone superconducting, so the Q of that particular resonance is only damped by the $R_{\text{bias}} = 30 \text{ m}\Omega$ bias resistor. By Equation 4.14 the critical frequency for this system 570 kHz, meaning that this bolometer short is sufficient to drive the FLL unstable, as indicated by the fact that the blue contour in circumscribes the point $1 + 0i$ (**grey dot**).

So bolometers that have latched superconducting will render the SQUID unstable if they are tuned to frequencies greater than the critical frequency:

$$\omega_{\text{crit}} = |1 - \mathcal{L}_{\text{SQ}}(0)| \sqrt{\frac{R_{\text{bias}}}{|\mathcal{L}_{\text{SQ}}(0)| L_{\text{coil}} \sum \tau_i}}. \quad (4.14)$$

For instance, in the simulation shown in Figure 4.9, $R_{\text{bias}} = 30 \text{ m}\Omega$, $\mathcal{L}_{\text{SQ}}(0) = -12.3$, $L_{\text{coil}} = 180 \text{ nH}$, and $\sum \tau_i = 185 \text{ ns}$. The critical frequency for this system is then about 570 kHz, and so the simulated short circuit near this frequency just barely encloses the critical pole and is unstable. This critical frequency can be raised by increasing the loopgain, increasing the the bias resistance, adding other resistance in series with the TES or, as always, limiting unnecessary phase delays, τ_i , in the system.

4.6 Other Sub-Kelvin Strays

On the sub-Kelvin side of the SQUID coil, the LC-coupled bolometers are not the only contributions to Z_{par} . There are also stray reactances, which may occur as lumped elements or may arise due to poor terminations in the 4K-250mK wiring.

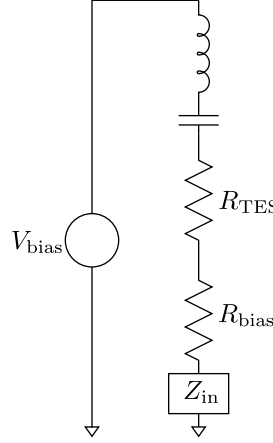


Figure 4.10: Equivalent circuit highlighting the mechanism by which a superconducting bolometer can lead to instability in the fMUX system. As shown by Equation 3.13, the real portion of the SQUID input impedance is made negative by the interaction of the SQUID input coil with the poles and delays in the system. If $\text{Re}(Z_{\text{in}}) + R_{\text{bias}} + R_{\text{TES}} < 0$ then the resonator will be unstable.

Like resonances in the 4K wiring, these reactances are a pernicious threat to stability. In this section, we explore the limits of stray capacitance or loading which can be tolerated by the FLL. I start by considering a simple single-pole system, before moving on to study the improvements offered by a lead-lag filter.

Resistive loads can lead to instability, or at the very least severely eat into the phase margin. By a resistive load, I do not mean the bolometers, since the LC-coupling makes them look inductive at high frequency, rather I am talking about resistors directly shunting the SQUID input coil. An example of such a shunt may be a line termination resistor at either 300K or 250mK. Consider a resistor directly attached to the input of the SQUID coil: $Z_{\text{par}} = R_{\text{load}}$. This crates an additional pole at the angular frequency, $\omega_p = R_{\text{load}}/L_{\text{coil}}$. As discussed in Section 4.1.1, this second pole will consume most if not all of the available phase margin unless $\omega_p \gg \omega_1 = 2\pi\text{LGBWP} = 2\pi \Delta\nu \mathcal{L}_{\text{SQ}}(0)$. This then sets a limit on the amount of loading that can be tolerated across the SQUID coil:

$$R_{\text{load}} \gg R_{\text{min}} \equiv 2\pi L_{\text{coil}} \Delta\nu \mathcal{L}_{\text{SQ}}(0) \quad (4.15)$$

In regard to reactive loads, inductive loads across the SQUID input coil are harmless. They may draw some current away from the coil, and reduce the overall loopgain, but if neccesary this can be easily corrected for by adding more gain in the warm amplifier. The one exception to this is if the inductive element in question also has a stray parallel capacitance as discussed by Lanting (2006). In this case the inductors

actually look like capacitors at some high frequency.

Capacitances are worse than resistors because they form sub-Kelvin resonators which are the bane of stability. Consider replacing the multiplexor circuitry with a single lumped capacitor: $Z_{\text{par}} = -i/(\omega C_{\text{stray}})$. This capacitance leads to a spike in $\Xi(\omega) = (1 - \omega^2 L_{\text{coil}} C_{\text{stray}})^{-1}$, at the resonant frequency $\omega_0 = (L_{\text{coil}} C_{\text{stray}})^{-\frac{1}{2}}$. Such a spike in the modified loopgain can lead to instabilities even at frequencies above the unity-loopgain frequency, ω_1 . Even worse, the sign of the loopgain is immediately inverted at this resonance. Thus without sufficient damping, these resonances lead to immediate instability, regardless of the resonant frequency.

One can damp such resonances by adding a resistor, R_{damp} , in parallel to with the stray capacitance. The quality factor of this damped resistance, $Q = R_{\text{damp}} \sqrt{\frac{C_{\text{stray}}}{L_{\text{coil}}}}$, should be set to be much less than unity. By the constraint that $R_{\text{damp}} > R_{\text{min}}$ this places a limit on the stray capacitance allowed in the system:

$$C_{\text{stray}} \ll C_{\text{max}} \equiv \frac{\tau_{\text{amp}}^2}{L_{\text{coil}} \mathcal{L}_{\text{SQ}}(0)^2} = (2\pi \Delta\nu \mathcal{L}_{\text{SQ}}(0))^{-2} L_{\text{coil}}^{-1} \quad (4.16)$$

This constraint is equivalent to demanding that the resonant frequency remain well above the unity-loopgain frequency. Figure 4.11 illustrates how a capacitance across the SQUID input can drive the system unstable even when trying to damp it.

As with the 300K-4K wiring in section 4.3, unterminated transmission lines in the sub-Kelvin wiring introduce stray reactance which, like the lumped capacitor in the last section, resonate with the input coils. Though the multiplexor will short the transmission line near the bolometer resonances, the transmission line is left largely open at frequencies in the 10MHz range. So at wavelengths longer than the length of the line, a transmission line which is left unterminated on the sub-Kelvin end will look to the SQUID coil just like a capacitor of value $C_{\text{line}} = \tau_{\text{line}}/Z_0$. As just discussed, this capacitance will cause instability in the FLL if left undamped. However, this transmission line is better than a lumped capacitor because this line can be terminated which leaves it completely resistive with $Z_{\text{line}} = Z_0$. Although as in Equation 4.15 it is important the combined parallel impedance of the transmission lines from the coil to the 300K, $Z_{0,300\text{K}}$ and to the sub-Kelvin stage, $Z_{0,\text{subk}}$, be greater than R_{min} :

$$(Z_{0,300\text{K}}^{-1} + Z_{0,\text{subk}}^{-1})^{-1} > R_{\text{min}}. \quad (4.17)$$

Otherwise, the resultant pole will lead to oscillations. It should be noted that when terminated properly, the stability of the flux-locked loop is independent of the length of the cabling, though very long cables will lead to high transmission-line inductances.

4.6.1 Enhancements From a Lead-Lag Filter

The lead-lag filter in Section 4.1.2 improves stability by more rapidly attenuating the SQUID output. If the loopgain is cut to below unity at low-enough frequency,

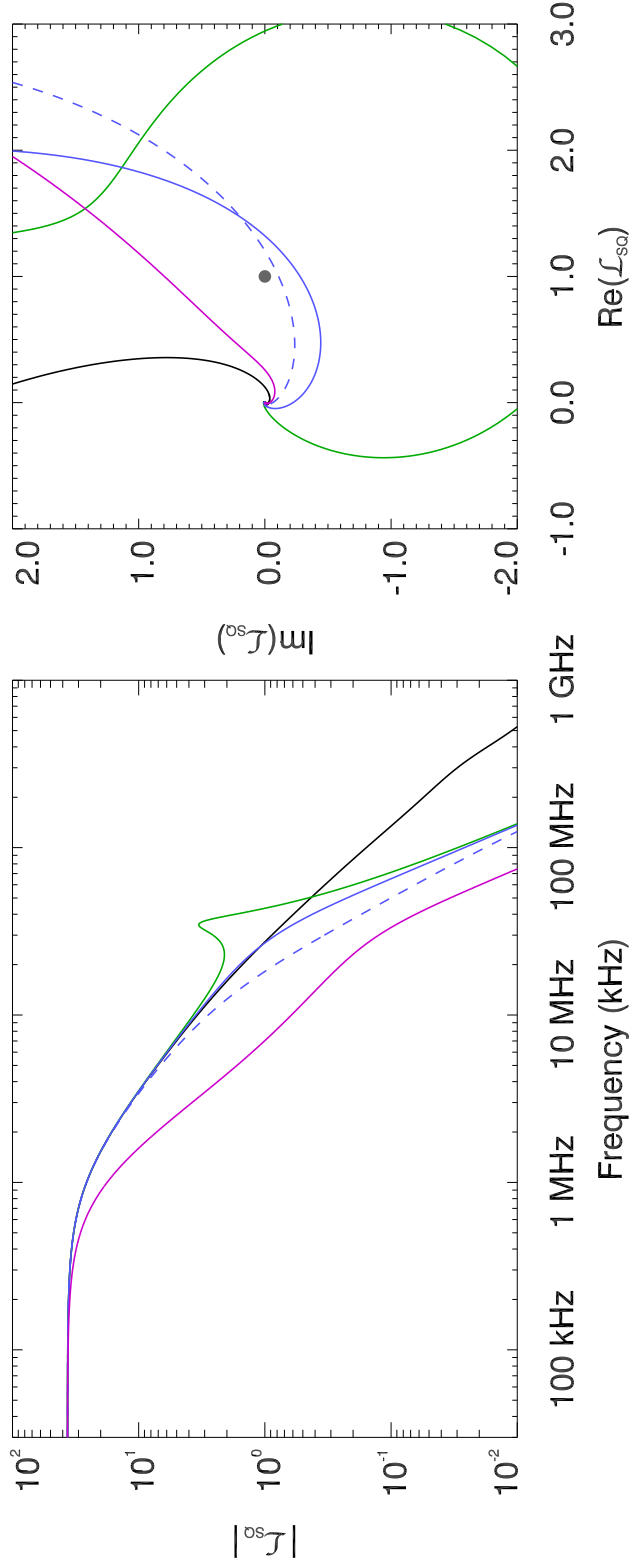


Figure 4.11: Loopgain amplitude (*left panel*) and Nyquist (*right panel*) Diagrams illustrating an instability caused by too much capacitance across the SQUID input coil. The **black trace** represents a simulated system with $\mathcal{L}_{\text{SQ}}(0) = 38$, a $\Delta\nu = 1$ MHz bandwidth, and using termination case E (Table 4.1) for the 300K wiring. By Equation 4.16 this system can stably support no more than 98 pF of stray capacitance across the input coil. The **green trace** shows the same system with a 100 pF capacitor directly across the coil, making the system unstable, as seen by the fact that this trace encircles the **grey dot** in the Nyquist plot. Attempts to damp this resonance by shunting the capacitor with a 42Ω (**solid blue trace**) or 15Ω (**dashed blue trace**) resistor do not help for this large capacitor. The **purple trace** shows the combined benefit of damping with 42Ω and using a lead-lag filter with $\tau_U = 1.59\text{ns} = 1/(2\pi \cdot 1\text{MHz})$ and $\chi = \sqrt{38}$. This choice of lead lag parameters dramatically increases the maximum allowed capacitance by a factor of 38. This improves stability of the system while leaving it with $\sim 45^\circ$ of phase margin.

instabilities due to resonances or additional poles can be effectively eliminated. Alternatively one can think of the lead-lag as enhancing the stability margin for stray loading or capacitances. The lead-lag will not diminish the bandwidth of the FLL, $\Delta\nu$, if one choose the pole to be $\tau_l = 1/(2\pi \Delta\nu)$. Meanwhile choosing $\chi = \sqrt{\mathcal{L}_{\text{SQ}}(0)}$, rapidly attenuates the loopgain to unity. In this case, the unity loopgain frequency is cut from $\omega_1 = \Delta\nu \mathcal{L}_{\text{SQ}}(0)$ to $\omega_{1,2\text{pole}} = \Delta\nu \sqrt{\mathcal{L}_{\text{SQ}}(0)}$. This means that the for stray poles or resonances can be tolerated at lower frequencies. The new minimum resistance threshold is reduced by a factor of $\sqrt{\mathcal{L}_{\text{SQ}}(0)}$.

$$R_{\text{load}} \gg R_{\text{min},l} \equiv 2\pi L_{\text{coil}} \Delta\nu \sqrt{\mathcal{L}_{\text{SQ}}(0)}. \quad (4.18)$$

Meanwhile the amount of shunt capacitance that can be tolerated is dramatically increased by a factor of $\mathcal{L}_{\text{SQ}}(0)$:

$$C_{\text{stray}} \ll C_{\text{max},l} \equiv \frac{\tau_{\text{amp}}^2}{L_{\text{coil}} \mathcal{L}_{\text{SQ}}(0)^2} = (2\pi \Delta\nu)^{-2} \mathcal{L}_{\text{SQ}}(0)^{-1} L_{\text{coil}}^{-1} \quad (4.19)$$

These enhancements, in addition to the enhancements in phase margin, make a properly tuned lead-lag an important tool for managing SQUID stability.

4.7 Summary

In this Chapter, I have attempted to shed some more light on the problem of FLL stability by providing quantitative guidelines for the allowable propagation delays, stray shunt impedances and lead-lag parameters. It should be emphasized that one of the best ways to ensure stability is to make sure that all resonances with the SQUID coil are damped and that all of the transmission lines connecting to the SQUID coil are terminated, preferably on the end opposite the coil. Though the SPT system is stable with its trial-and-error tunings, I hope that these guidelines will improve the performance of future receivers. Meeting the full potential of this system will require analysis of strays in the system combined with careful verification through measurements of the loopgain.

Chapter 5

Thermal Design of the SPT Pixels

In this Chapter we discuss the design of the SPT pixels. We start by discussing the absorber geometry, which determines the optical response time of the sensor. Meanwhile, the electrical response time is bounded by the multiplexor LC filters discussed in Chapter 3. Just as stray inductance can lead to electrothermal instabilities in DC biased sensors, so can these filters can cause instability unless the thermal time constant is larger than the electrical time constant. In this chapter we describe the thermal engineering done to these sensors to ensure electro-thermal stability. We also demonstrate a method for probing the power-to-current responsivity, s_I , which we used to directly probe of the thermal structure of our devices, giving us great insight into the origins of our thermal instability.

5.1 Spiderweb-coupled TES Bolometers

Our TES bolometers consist of an Al-Ti bilayer, with a critical temperature of 500 mK with a normal resistance in the range on of $1 - 1.5\Omega$, depending on the fabrication run. These films are suspended at the center of a spiderweb absorber as shown in 5.1. The spiderweb itself is a $1\mu\text{m}$ thick SiN mesh. The mesh is 3 mm in diameter and is suspended from the Si substrate by 0.5 mm-long legs. To couple this mesh to incoming radiation, gold is deposited to bring the web surface impedance to roughly $100\Omega/\square$. This geometry has the advantage that it is efficient at millimeter wavelengths yet has a low cross-section to cosmic rays (Bock et al., 1995). This absorber design has been used a large number of CMB experiments such as ACBAR (Runyan et al., 2003), MAXIMA (Rabii et al., 2006), BOOMERANG (Crill et al., 2003), BICEP (Yoon et al., 2006) and Planck (Lamarre et al., 2003). In our implementation these devices are completely fabricated by photolithography techniques (Gildemeister et al., 1999), which makes this design well suited for large-array fabrication. Erik Shirokoff designed and fabricated these arrays.

The response-time of spiderweb coupled TESs is limited by the thermal diffusion

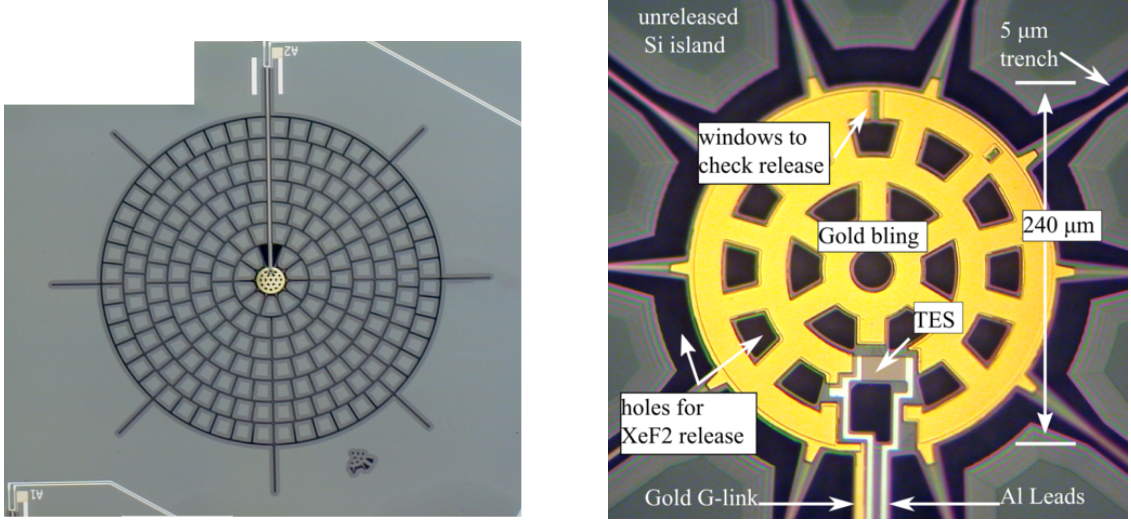


Figure 5.1: On the **left-hand side** an example of one the 4mm-diameter spiderweb absorber bolometers deployed on the SPT. The dark lines indicate regions where the underlying silicon has been etched away leaving the spiderweb membrane suspended, although the web itself is too fine to seen in this image. On the **right-hand side**, a yet closer view of the center of the spiderweb pixel, showing the AlTi TES film, and the Gold BLING which provides heat capacity to keep the detector stable.

time of the web itself, τ_{web} . Thus even though the effective time-constant, τ_{eff} , of the bolometer itself becomes shorter with increased loop gain, the optical response time is ultimately limited by τ_{web} (Gildemeister et al., 1999). For the devices shown in 5.1 the optical time-constant is of order 10ms.

5.2 ETF Stability

As discussed in Chapter 2, strong electrothermal feedback offers a wide number of advantages, such as improved linearity and more rapid response times. However the threat of instability places limits on the amount of loopgain that we can achieve. The first batch of detectors designed for the APEX-SZ and SPT experiments were not stable even at low loopgains. Typical thin film heat capacities together with the G 's required to meet our dynamic range requirements led to intrinsic time-constants of $\tau \leq 100\mu\text{s}$. With LRC filter time-constants of $\tau_e = 32\mu\text{s}$ these devices were underdamped even at unity loopgain (see Equation 2.18).

Given the 7–10 ms absorber response time, this speed was unnecessary. The decision was made to increase the heat capacity of these devices, slowing them down. We added large heat-capacity, $3\mu\text{m}$ -thick, gold features to the bolometers. These *Bandwidth Limiting Interfaces* (Normally made of Gold), also known as BLING,

thermally slow down our devices to natural time constants of roughly $\tau \approx 20$ ms. Though this intrinsic time-constant is slower than the web, it theoretically allows for overdamped operation at ETF loopgains of $\mathcal{L} = 100$ or higher, making the effective TES time-constant, $\tau_{\text{eff}} \approx \tau/(\mathcal{L} + 1)$, much faster than the absorber. The BLING geometry for the SPT detectors is shown in Figure 5.1

5.2.1 Bound Thermal Oscillations

By Equation 2.18, these bandwidth-limited devices should be stable at a loopgain of at least 100. However at loopgains of about 6–8, these new devices exhibit a more subtle class of spontaneous thermal oscillations, shown in Figure 5.2. These oscillations are characterized by rapid anharmonic fluctuations (e.g. periodic exponential spikes followed by a decay) in the current amplitude. The fluctuations typically occur with a period of $\sim 100 \mu\text{s}$, though the period and qualitative structure of the oscillations varied with operating point.

These oscillations have the unique feature that they are bound rather than exponentially increasing in amplitude. Rather than growing exponentially until the device latches, the resistance fluctuations reach a maximum amplitude. This resistance fluctuation amplitude is a large fraction of R_n . The anharmonic nature of these oscillations is a form of non-linear behavior, though given the relatively large amplitude of these fluctuations, this is perhaps not unexpected. However, the nonlinear nature of these oscillations also makes them hard to understand from a small-signal perspective. Thus we employ SPICE models to numerically simulate the behavior of the TES. Simulations have shown bound oscillations can arise from weak coupling between the TES and the BLING. Such a device is schematically illustrated in Figure 5.3, while the a simulation of the oscillations is shown in Figure 5.4

5.2.2 BLING Coupling Requirements for ETF Stability

One can analytically estimate the conditions for stability for a TES that is weakly coupled to the heat capacity, such as the model shown in Figure 5.3 Compared to the simple bolometer considered in Chapter 2, this new model has an extra degree of freedom, the temperature of the intermediate node. It also has two new parameters, the ratio of the BLING to TES heat capacity, $\eta \equiv C_0/C_{\text{TES}}$, and the strength of the TES-BLING coupling, relative to the coupling to the bath, $\gamma \equiv G'/G_0$. Under the assumption that the TES has very little heat capacity, we define the thermal time constant, τ , in terms of the BLING heat capacity: $\tau = \frac{C_0}{G_0}$. The effective thermal conductance of this bolometer is: $G_{\text{eff}} = G_0(1 + \gamma^{-1})^{-1} = \frac{\gamma}{\gamma+1}G_0$. Based on the formalism laid out in Appendix A, the equations of motion for this system are:

$$\frac{d}{dt}\mathbf{v} = \mathbf{A}\mathbf{v} + \mathbf{p} \quad (5.1)$$

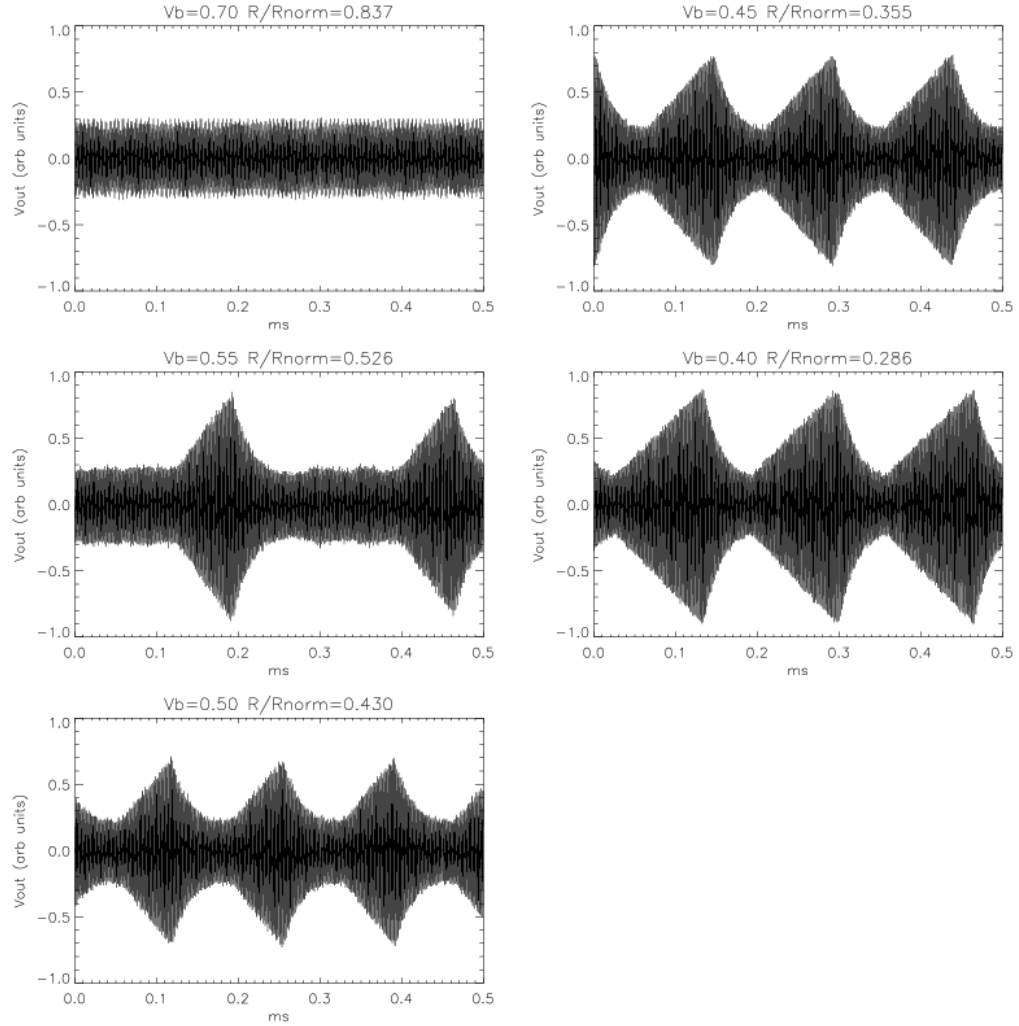


Figure 5.2: Bound electrothermal oscillations observed in detectors with additional heat capacity. Each panel is taken at successively lower voltage bias, V_b , indicated above each panel. The mean resistance, $V_{\text{rms}}/I_{\text{rms}}$, is also indicated, though with the instantaneous current amplitude fluctuating so dramatically these values should be treated with suspicion. These resistances are not indicative of the steady-state resistance that one would normally expect for a stable bolometer at the same bias voltage. Operated at a bath temperature of 0.25 K, the device shown here is stable down to an operating resistance of approximately $0.8R_{\text{normal}}$. Below this resistance, the bolometer exhibits periodic surges in the amplitude of the AC-bias current. At yet lower voltage biases, the bolometer enters into periodic non-sinusoidal resistance oscillations, with the general trend that the oscillation period gets longer at lower voltage biases.

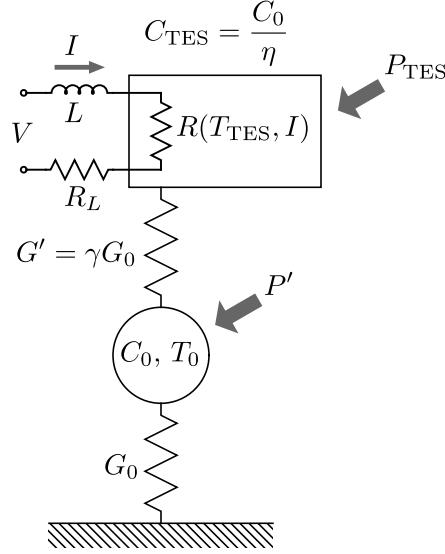


Figure 5.3: Two-Body bolometer model. The TES, the top node, is coupled to an intermediate node, which here represents the BLING, with heat capacity, C_0 . The BLING in turn is coupled to the bath, via a thermal conductance, G_0 . The coupling between the TES and intermediate node is stronger than the coupling to the bath by a factor γ . The heat capacity of the TES is assume to be smaller than C_0 by a factor η .

where:

$$\mathbf{v} \equiv \begin{pmatrix} LI\delta I \\ \frac{C_0}{\eta}\delta T_{\text{TES}} \\ C_0\delta T_0 \end{pmatrix}, \mathbf{p} \equiv \begin{pmatrix} I\delta V \\ \delta P_{\text{TES}} \\ \delta P_0 \end{pmatrix} \quad (5.2)$$

and

$$\mathbf{A} \equiv \begin{pmatrix} -\tau_e^{-1} & -\eta\frac{\gamma}{\gamma+1}\mathcal{L}\tau^{-1} & 0 \\ \frac{(2+\beta)}{1+\beta+\xi}\tau_e^{-1} & \eta(\frac{\gamma}{\gamma+1}\mathcal{L} - \gamma)\tau^{-1} & \gamma\tau^{-1} \\ 0 & \eta\gamma\tau^{-1} & -(\gamma+1)\tau^{-1} \end{pmatrix} \quad (5.3)$$

As in Chapter 2, one can determine stability of the TES by computing the eigenvalues of this matrix. The device will be stable, or at worst underdamped, if all of the eigenvalues of $\mathbf{A}$ have a negative real portion. One necessary condition then is that the sum of the eigenvalues should be less than zero: $\text{Tr}(\mathbf{A}) < 0$. In the limit that TES has low heat capacity and the electrical time constant is very small: $\tau_e \ll \tau/\eta \ll \tau$, we have the constraint: $\mathcal{L} < \gamma + 1 + \frac{C_{\text{TES}}}{G_{\text{eff}}\tau_e}$. The bolometer may be stable if the heat capacity of the TES is large enough to keep the bolometer stable on its own, i.e. if $\frac{C_{\text{TES}}}{G_{\text{eff}}} > \mathcal{L}\tau_e$. Otherwise the loopgain is limited to $\gamma+1$, the relative strength of the thermal coupling to the BLING as compared to the thermal coupling to the bath. Thus γ is an important quantity to measure if we wish to understand the these

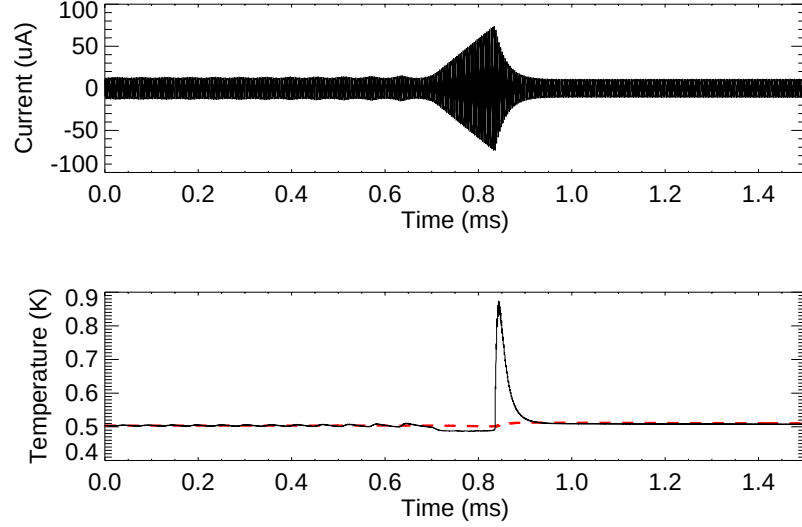


Figure 5.4: Current and temperature profiles from a *simulated* AC-biased TES. The thermal circuit is illustrated in Figure 5.3, with $C_{\text{TES}} = 0$, $T_0 = 500\text{mK}$, $G_0 = 370\text{pW/K}$ and $C_0 = 3.7\text{pJ/K}$ and $\gamma = 15$. The inductor in Figure 5.3 has been replaced with a 300kHz LC resonator. The **top panel** shows the TES current, while the **black trace** in the **bottom panel** shows the simulated TES temperature. The **dashed red** trace meanwhile shows the BLING temperature. Much like in the real detectors high in the transition, the oscillations start with ripples in the the current amplitude (compare the simulated current to the center left panel in Figure 5.2). These ripples grow until the detector settles below the BLING temperature, which is nearly constant. During this period the current surges, eventually blasting the detector out of the transition. There is then a period of calm as the detector cools back into the transition.

instabilities in our devices.

5.3 Measuring of the Internal Thermal Structure of the TES

In order to verify this model, and to obtain engineering feedback about our devices, it is important to be able to measure the internal thermal coupling G' of the SPT devices. One natural test would be to measure the generalized thermal conductance, $G(\omega)$, or the ratio of the sinusoidal external power fluctuation, $P_{\text{ext}}(\omega)$, to the temperature response $T_{\text{ext}}(\omega)$, or the inverse of the power-to-temperature responsivity

at zero-loopgain:

$$G(\omega) \equiv \frac{P(\omega)}{T(\omega)} = (s_T(\omega)|_{\mathcal{L}=0})^{-1} \quad (5.4)$$

This complex quantity represents not just the amplitude of the response but also any the phase delays due to internal heat capacities.

Given a model of the thermal structure it is very easy to calculate, $G(\omega)$. For instance, the generalized thermal conductance for a simple bolometer (Figure 2.1) with a single thermal time-constant can be expressed as:

$$G(\omega) = G_0 + i\omega C \quad (5.5)$$

For the more complicated bolometer model illustrated in Figure 5.3, $G(\omega)$ is:

$$G(\omega) = G_0 \frac{\gamma}{1 + \gamma} \left(\frac{1 + i\omega \frac{C}{G_0}}{1 + i\omega \frac{C}{(\gamma+1)G_0}} \right) \quad (5.6)$$

Even though $G(\omega)$ describes the power-temperature relation only at the TES, it contains enough information to determine the stability of ETF.

5.3.1 Measuring $s_I(\omega)$

One can extract $G(\omega)$ from a measurement of the power-to-current responsivity, $s_I(\omega)$, since for a fixed voltage bias, fluctuations in current are proportional to fluctuations in temperature.¹ In the appendix, we show that $G(\omega)$ can be related to $s_I(\omega)$ by the formula:

$$s_I(\omega) = \frac{\mathcal{L}}{I_0 R_0} \left(\left(\mathcal{L} - \frac{G(\omega)}{G_{\text{eff}}} \right) (1 + \xi + \beta)(1 + i\omega\tau_e) - \mathcal{L}(2 + \beta) \right)^{-1} \quad (5.7)$$

If we for a moment neglect nuisance terms such as β , or ξ , this simplifies to:

$$s_I(\omega) = -\frac{\mathcal{L}}{I_0 R_0} \left(\frac{G(\omega)}{G_{\text{eff}}} (1 + i\omega\tau_e) + \mathcal{L}(1 - i\omega\tau_e) \right)^{-1} \quad (5.8)$$

One nice feature of such a measurement would be that, when done at low loopgain, the response is nearly proportional to $G(\omega)^{-1}$, making for easy interpretation of $S_I(\omega)$ measurements, even if the exact value of the loopgain is not known. However even at higher loopgains, one can use 5.7 to extract a thermal model curve, $G(\omega)$ if the loopgain is well known.

In order to quickly evaluate our devices, we have developed a method to measure $s_I(\omega)$ directly using our AC-bias readout system. The measurement technique

¹Here we neglect noise terms and assume the detector is operating stably.

requires the use of an extra sine-wave oscillator as well as an additional demodulator. For us, this technique is particularly convenient, because this equipment is built into the fMUX readout hardware, and can be done on any fMUX system with very little adjustment of hardware.

Consider a TES biased at a frequency ω_0 , with a bias voltage of amplitude V_0 such the equilibrium resistance is held at R_0 . Take δP and $\delta\omega$ to be the amplitude and frequency of some perturbation in the external power (i.e. $P_{\text{ext}}(t) = \delta P \cos(i\delta\omega t)$). This perturbation in power will in turn cause a modulation in the current amplitude, $I(t) = (I_0 + 2|s_I(\omega)| \cos(i\delta\omega + \phi)\delta P) e^{i\omega t}$. The amplitude of the the current in either sideband is then:

$$I_{sb,\pm} = |s_I(\omega)|\delta P \quad (5.9)$$

So a measurement of the current in either sideband is a direct measurement of $|s_I(\omega)|$.

Unfortunately direct stimulation of the TES can be tricky. For instance with the SPT detectors one can only apply optical power through the spiderweb absorber, which deletes any high frequency information about the response of the detector. Building a heater into the bolometer may present similar problems, since the heater may not couple directly to the TES. Another approach is to sinusoidally perturb the bolometer bias voltage, V , making measurements of the complex impedance, $Z(\omega)$. (Lindeman et al., 2004). However at low-loopgains the bolometer impedance looks almost entirely like a resistor, with only a small thermal response superimposed on a larger electrical current. At high-loopgain the thermal response is much more strong, though in order to compare the complex impedance to a thermal model, one must always include a term for the additional electrical response.

Our AC-bias system gives us a convenient mechanism for applying a power perturbation. We sum the output of a second voltage oscillator of amplitude V' and frequency $\omega - \delta\omega$ to the original voltage carrier, at frequency ω_0 , and amplitude, V_0 . The power dissipated in the TES is then:

$$P(t) = \frac{V_0^2}{2R_0} + \frac{V'V \cos(\delta\omega t + \phi)}{R_0 \sqrt{1 + \omega^2 \tau_e^2}} + O(V'^2) \quad (5.10)$$

The factor of $1 + \omega^2 \tau_e^2$ in the denominator arises from the fact that the LC filter attenuates the voltage off-resonance. The TES is perturbed with a power $\delta P = V_0 V' / R_0 \sqrt{1 + \omega^2 \tau_e^2}$. Since V' is much smaller than V_0 , most of the feedback power comes from the main carrier, V_0 . As in (5.3.1), ETF modulations in the carrier create current at both sideband frequencies, which is given by Equation 5.9. Since there is no external voltage applied at frequency $\omega + \delta\omega$, we set our second demodulator channel to this frequency. The current $I_{sb,+}$ measured at this frequency contains only the response to our power perturbation, and no passive current from the bolometer. This current at this frequency is therefore a direct measurement of $|s_I(\omega)|$:

$$|s_I(\omega)| = \frac{R_0}{V_0 V'} |I_{sb,+}| |1 + i\omega \tau_e| \quad (5.11)$$

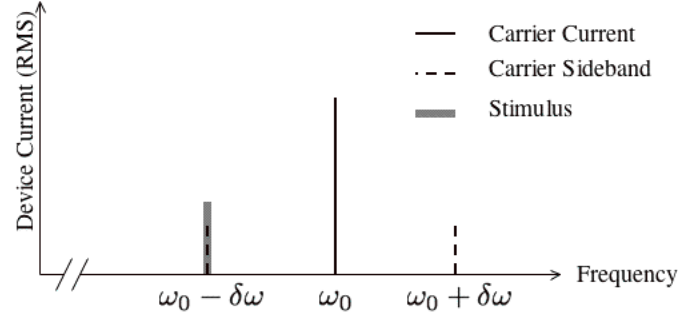


Figure 5.5: A technique for measuring $s_I(\omega)$. We stimulate the bolometer by applying a sinusoidal voltage perturbation, V' , at a frequency, $\omega - \delta\omega$. This voltage beats against the carrier voltage at frequency ω_0 , causing a modulation in the electrical power at a frequency, $\delta\omega$. The bolometer responds by amplitude modulating the bias current, and these amplitude modulations appear as current in both sidebands. A measurement of the current in the opposite sideband, at the frequency $\omega + \delta\omega$ yields a direct measurement of $s_I(\delta\omega)$.

In order to highlight the principle behind this technique, we have been somewhat lax about keeping track of phases. It is possible to use the *instantaneous phase differences* between the carrier and both sidebands to also extract the phase of $s_I(\omega)$. This would allow one to directly calculate $G(\omega)$ from $s_I(\omega)$, rather than performing a fit. Due to the 1kHz sampling rate in the analog oscillator-demodulator boards used by SPT, measuring the instantaneous phase shift between three signals is very difficult unless $\delta\omega$ is less than a few 100 Hz. This restriction should be much more relaxed in the subsequent generation of digital oscillator-demodulators boards, which are capable of much faster sampling rates. Using these new boards, a phase sensitive improvement will be pursued in a future work.

It is important to determine the loopgain accurately if we wish to extract $G(\omega)$ from a measurement of $s_I(\omega)$. At low loopgain, measurements of $s_I(\omega)$ and $G(\omega)$ have roughly the same temperature dependence. Though with low loopgain, these preliminary measurements may exhibit low signal-to-noise. The measurement of γ can be done most efficiently in two measurements. A first measurement is done very high in the transition to measure the intrinsic time constant of the device, $\tau_0 = C_0/G_0$. Follow-up measurements can be done to measure the speed up of the device and thus constrain the loopgain deeper in the transition, allowing us to compare $s_I(\omega)$ and fit to models for $G(\omega)$. These follow-up measurements will also have higher signal to noise, making it easier to measure the decoupling time constant, $\tau_{\text{int}} = C_0/G_0 = \tau_0/(\gamma + 1)$.

This procedure relies on the non-linear electrical nature of the TES, else it would not be able to mix current from one sideband into the other. In our linear approximations above, we have made the assumption that $V' \ll V_0$. However it is important to

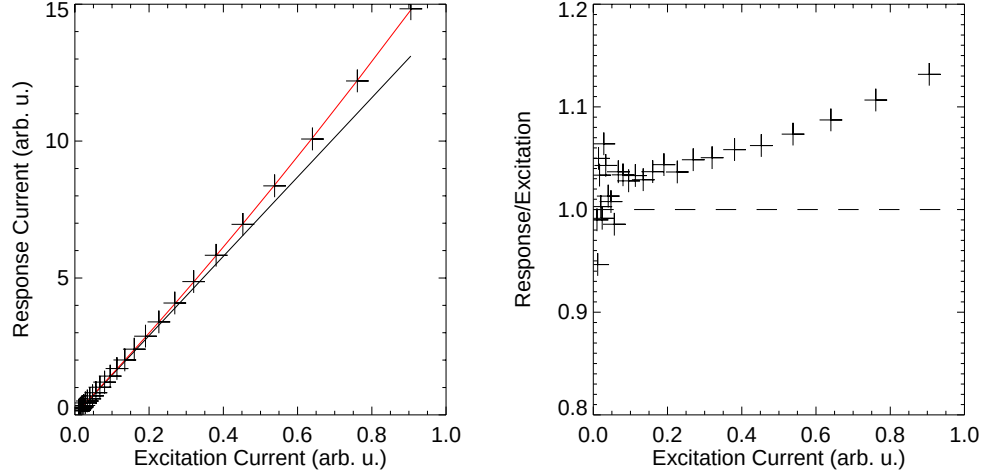


Figure 5.6: A test to measure the linearity of the $s_I(\omega)$ measurement. In the *left panel*, the detector modulation response (**black crosses**) is measured as a function of the excitation voltage. The fit to a quadratic is shown by the **red line**. The linear extrapolation of this model from zero amplitude is shown by the **black line**. The models diverge at high excitations indicating some non-linearity. The ratio of the data to the linear extrapolation is shown in the *right panel*, and the choice of excitation voltage is limited to excitations where the quadratic contribution is no more than 5% of the linear contribution.

measure the linearity between V' and $I_{sb,+}$, in order to ensure that $I_{sb,+}(\omega)/I_{sb,+}(0)$ is an accurate representation of $s_I(\omega)/s_I(0)$. Before each measurement we measure the amplitude of the response, $I_{sb,+}$ as function of the stimulus, V' , as shown in figure 5.6 In order to quantify the degree of nonlinearity, we fit the excitation-response to a quadratic,

$$I_{sb,+} = BV' + CV'^2.$$

We require that the amplitude response does not deviate from a linear portion $I_{sb,\text{linear}} = BV'$ by more than 5%. Though one would like to increase the amplitude of V' to increase the signal-to-noise of our measurement, this linearity requirement thus sets an upper limit on the size of our stimulus.

Using this technique, we measure $s_I(\omega)$ for some of the devices we have fabricated for SPT. One of our older generation devices, P12, exhibits oscillations at modest loop-gains. Meanwhile the more recent devices SA13 and Q11 were redesigned for more thermal stability, with the bling closer to the TES. Q11 in particular can be operated at loop-gains above 40. The measurements for these three different detectors are shown in Figure 5.7. For each device, the rolloff below 100Hz indicates the thermal time constant τ_0 , with the speedup from ETF (2.14). The inflection points near 300

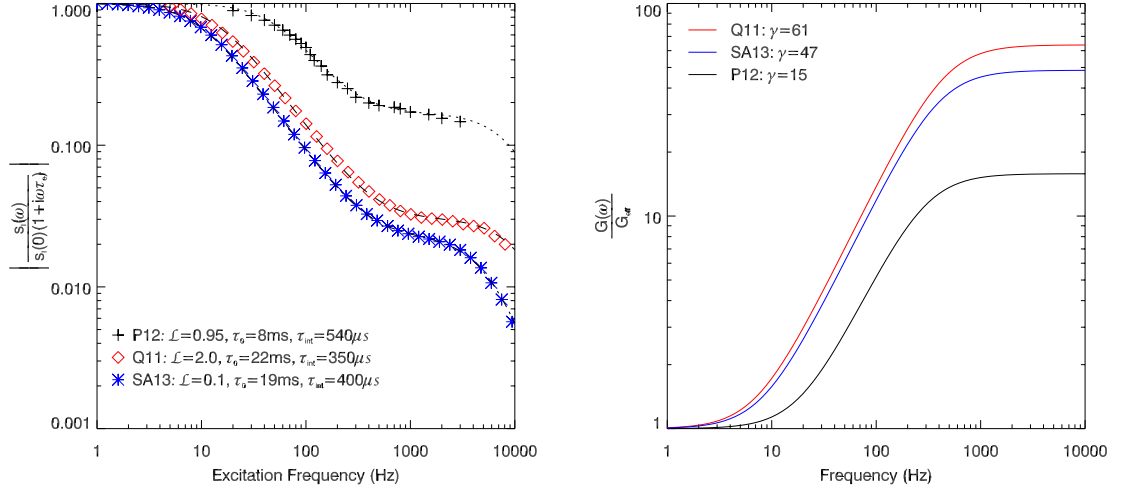


Figure 5.7: In the *left panel*, the thermal responsivity, $s_I(\omega)$ is measured for three different detector wedges. In each case, $s_I(\omega)$ is normalized to unity at zero frequency. For each detector a fit is performed to the two-body model illustrated in Figure 5.3. The best fit thermal model is shown by the **dotted line** after correcting for the loopgain, which is measured by the ETF speedup in τ_{eff} . In the *right panel*, the generalized thermal conductance corresponding to each detector is shown without the loopgain correction. The height of the high-frequency plateau is an indication of higher γ .

Hz indicate the internal BLING decoupling time, τ_{int} . The second rolloff near 5kHz is an artifact of the MUX LRC filters. The inferred magnitude of $G(\omega)$ is shown as a function of frequency. From Equation 5.6, the value of $G(\omega)/G_0$ should asymptote to $\gamma + 1$ at high frequencies, which as discussed in Section 5.2.2, is a figure of merit for good stability. The more stable detectors Q11 and SA13, have γ ratios of 61 and 48 respectively as compared to the less stable P12, with a coupling ratio of 15. This method proved invaluable in evaluating subsequent bling-TES coupling schemes for the SPT-SZ detectors.

5.4 Summary

The bandwidth-defining filters set requirements on the thermal response times of our bolometers. We meet these requirements by adding a high heat capacity node to our detectors, which we refer to as BLING. We then operate these devices at high loopgain for a rapid thermal response.

The interface between the BLING and TES must have a very high thermal con-

ductivity in order to maintain stability at high loop-gain. In order to study this interface, we designed a rapid technique for directly probing the internal thermal structure of our devices. This technique takes advantage of our AC-bias readout and makes it possible for us to quickly evaluate the internal thermal coupling of each new bolometer prototype. Consequently, the current generation of detectors benefits from stable operation at higher loop-gains

Not only has this technique been critically important the design of the SPT-SZ detectors, it is now being applied to detectors for a wide variety of AC-biased TES experiments such as SPTPol, EBEX, and Polarbear, each of which is dramatically different in terms of optical and thermal design. One interesting future avenue of research may be to do a more complete comparison between this technique and the complex impedance techniques used by the DC-biased TES groups.

Chapter 6

The South Pole Telescope

In this chapter we describe the South Pole Telescope (SPT), an off-axis Gregorian telescope with a 10-m diameter primary mirror located at the South Pole. The telescope is optimized to perform high resolution surveys of low surface brightness sources. In this chapter we describe the SPT-SZ receiver, which is designed to identify a mass-limited sample of galaxy clusters. The first clusters discovered by the were reported in Staniszewski et al. (2009, hereafter S09). This receiver is equipped with a 960-element array of superconducting TES bolometers, and makes use of all of the technologies reported in the previous two chapters. The focal plane is split between three frequency bands centered at 95 GHz, 150 GHz, and 220 GHz. This multifrequency coverage enables the separation of the tSZ effect from the primary CMB anisotropy and astronomical foregrounds.

6.1 Atmospheric Conditions at the South Pole

The South Pole is a high, dry site with exceptional atmospheric transparency and stability at millimeter and submillimeter wavelengths. The median atmospheric zenith opacity at 150GHz is $\epsilon \sim 0.03$.

Though other molecular species in the atmosphere are well-mixed, water vapor is problematic since it tends to clump leading to a spatially- and temporally- varying atmospheric signal as the telescope rasters across the sky. In winter, the median precipitable water vapor is ~ 0.25 mm Chamberlin (2001) (See figure 6.1 for a comparison between the South Pole and other terrestrial sites). This is at least an order of magnitude better than other established terrestrial sites (Peterson et al., 2003).

Lay & Halverson (2000) have modeled the water vapor emission as a being confined to a turbulently mixed atmospheric layer at height h_{av} , and thickness Δh . Based on scaling and geometric arguments the projected atmospheric water vapor emission

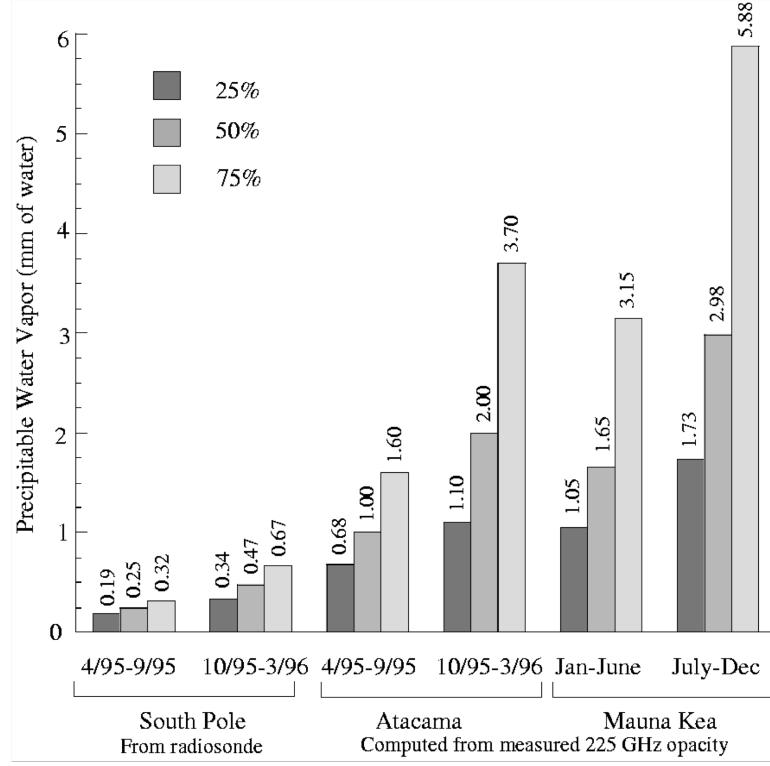


Figure 6.1: Comparison of precipitable water vapor (PWV) levels for three terrestrial observing sites, from Lane (1998). Mauna Kea and the Atacama desert are both well known for their strong atmospheric transmission at mm-wavelengths, however the South Pole has consistently lower moisture than these other sites, making the atmospheric emission more stable and uniform.

should have a PSD of the form:

$$\langle \tilde{T}^2(q_x, q_y) \rangle = \begin{cases} Ah_{av}^{\frac{5}{3}}(\alpha_x^2 + \alpha_y^2)^{-\frac{11}{6}} & (\alpha_x^2 + \alpha_y^2)^{-1} \ll 2\Delta h \quad (\text{“3D limit”}) \\ A'h_{av}^{\frac{2}{3}}(\alpha_x^2 + \alpha_y^2)^{-\frac{8}{6}} & (\alpha_x^2 + \alpha_y^2)^{-1} \gg 2\Delta h \quad (\text{“2D limit”}) \end{cases} \quad (6.1)$$

By this model, the atmospheric signal is more strongly observed on large angular scales. Bussmann et al. (2005), have shown that data from the ACBAR experiment is consistent with the 3D limit of Lay & Halverson (2000) ($\langle \tilde{T}^2(\alpha) \rangle \propto |\alpha|^{-11/3}$), and measured the median brightness fluctuation power at $\lambda = 2$ mm to be $Ah^{\frac{5}{3}} \sim 31 \text{ mK}^2 \text{ rad}^{-\frac{5}{3}}$ in CMB temperature units. Due to wind combined with the slewing of the telescope during observations, these fluctuations appear as temporal fluctuations in the time-ordered data. In section 6.5.3 we will discuss the observed noise in the SPT data and in Chapter 7 we will discuss analysis techniques to filter out these fluctuations.

6.2 Telescope and Optical Design

The SPT optical design is optimized for an SZ survey at 150 GHz (2mm wavelength). The design calls for: (a) 1' resolution at 150GHz in order to match the characteristic angular scale of SZ clusters (b) a wide, 1° , field of view, and (c) minimal signal contamination or loading due to stray radiation. The telescope is an off-axis Gregorian design, schematically shown in Figure 6.2, and is described in detail by Padin et al. (2008). Compared to other optical systems the SPT is a simple optical system with only two mirrors with a weak lens.

The large field of view requirement combined with the limited space available for a cryogenically cooled focal-plane means that the secondary mirror must have a short focal length. This however means that the image of the primary mirror is poor, since objects not located near one of the foci of the fast Gregorian secondary will suffer from severe aberrations. This feature leads to some unique features for this optical system.

There is no flat chopping mirror to steer the beams across the sky. Though chopping mirrors are typical for millimeter-wavelength telescopes, they must be placed at the image of the primary and are thus impractical for this fast optical system. The lack of a chopping tertiary is not a problem since TES bolometers are much less sensitive to microphonic pickup than their high-impedance bolometer counterparts. This means that we can steer the beams across the sky by slewing the entire telescope without signal interference.

Also without a clear image of the primary, there is no preferred location for the optical stop. Thus the secondary mirror is also chosen as the location the exit pupil. This stop is realized by surrounding the secondary with a baffle made of microwave absorber. In order to minimize the loading on the detectors this stop must be cryogenically cooled. Both the secondary and the cold stop are cooled to 10K. The secondary optics cryostat is described in §6.3.1.

The angular response of a radiometer is often calculated by considering the illumination pattern that would be generated if the sensors were operated as transmitters. For a well focused system, the angular beam pattern is approximated by the Fourier transform of the radiation illumination pattern on the primary. Thus it is this illumination pattern which sets the beamsize. For a diffraction-limited optical system, the mirror diameter must be at least 7.5 m to achieve the specified 1' beamsize at 150GHz. Since the stop is located at the secondary, rather than at an image of the primary, the illumination pattern is offset for off-axis pixels. Thus the primary mirror must also enclose an additional 0.5m perimeter around the central illumination pattern.

In order to ensure that the wavefront is planar at the final focus, there is a weak lens located just before the focal plane. In order to minimize reflections, azimuthal grooves are cut into the lens surface to create a gradual transition in refractive index (Plagge, 2009).

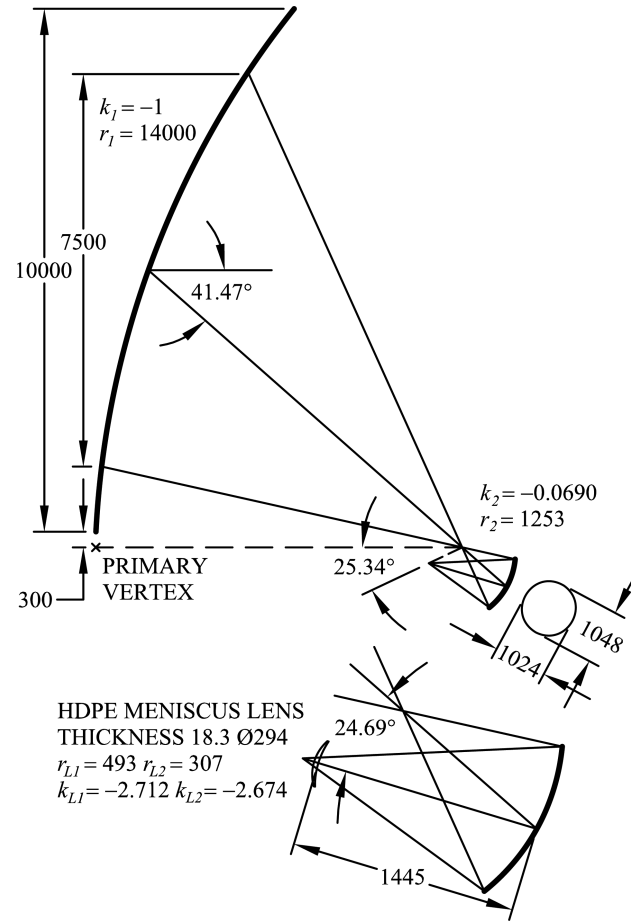


Figure 6.2: The optical design for the SPT, from Padin et al. (2008). The telescope is an off-axis Gregorian design. The primary mirror has a 7.5m illumination pattern, with a 1m guard ring to accommodate the edge pixels. The secondary mirror inset also serves as the stop in the system. Finally a weak HDPE lens establishes a telecentric wavefront allowing for a flat detector plane.

6.3 Cryogenics

The focal plane must be cooled to below the 500 mK TES transition temperature. Meanwhile the readout SQUID arrays must be cooled to 4K, and several optical elements (e.g. the secondary mirror and the associated baffles, the lens and several thermal blocking filters) must be cryogenically cooled to limit the optical load on the detectors. The cryogenic system for the SPT consists of two cryostats in a shared vacuum space. The optics cryostat is responsible for cooling the secondary mirror, and the cold stop. The receiver cryostat cools the focal plane, and the SQUID modules. Each cryostat is equipped with its own refrigeration system. This arrangement has the advantage that receiver cryostat can be removed and operated independently for testing purposes.

6.3.1 The Optics Cryostat

The secondary mirror is a lightweighted (20 kg), 1m-diameter mirror, made of aluminum 7075-T6. The original surface error of the secondary mirror was $11\text{ }\mu\text{m}$ rms at room temperature. However, the surface error after cooldown is $50\text{ }\mu\text{m}$.

In order to effectively terminate stray radiation, and to define the optical stop in the system, the optical path from the window to the secondary, and from the secondary to the receiver, is absorptive baffle. This baffle is constructed of a pair of aluminum coned which are then covered in flexible microwave absorber (HR-10¹).

In order to limit the loading on the detectors the secondary baffle is cooled to roughly 10K. The optics cryostat was designed and fabricated at Case Western Reserve University, and is described further in Padin et al. (2008). The optics cryostat is cooled by a PT410 pulse tube cooler from Cryomech². This two-stage refrigeration unit has a cooling capacity of 10 W at 10K and 80 W at 70K.

Radiation enters the secondary cryostat through a 100mm thick window made of an expanded polypropylene foam (Zotefoam PPA-30³). Infrared blocking metal mesh filters (Tucker & Ade, 2006) are placed just behind the window at the opening of the cold stop. The optics cryostat shares a common vacuum space with the receiver cryostat, and radiation exits the optics cryostat at the flange which defines the boundary between the two vessels.

6.3.2 Receiver Cryostat

The focal plane is mounted in the receiver cryostat, designed by Brad Benson and assembled at Berkeley. The cryostat consists of three roughly rectangular shells, each nested within the other. The outer shell serves as the vacuum jacket, while the

¹Emerson-Cumming, Billerica MA 01821

²Cryomech Inc., Syracuse NY 13211

³Zotefoams PLC, Croydon CR9 3AL UK

inner two shells provide radiative shielding for the focal plane and SQUID modules. These radiation shields are respectively cooled to 45K and $\lesssim 4$ K using a PT415 model pulse-tube, also from Cryomech. The PT415 has a cooling capacity of 1.5W at 4K and 40W at 45K.

The pulse tube also serves as the backing stage for the sorption refrigerator which is responsible for cooling the focal plane to its 280 mK operating temperature. We use a 3-stage $^4\text{He}^3\text{He}^3\text{He}$ sorption refrigerator (Bhatia et al., 2000) from Simon Chase.⁴ A sorption refrigerator operates by pumping on a volume of liquid (here either ^3He or ^4He), to reduce the vapor pressure, thereby reducing the boiling point of the liquid. The Chase fridge design is shown in figure 6.3.2. This sorption fridge has two cooling heads. The “inter-head” contains two reservoirs one for ^4He and the other for ^3He . The “ultra-head” contains one reservoir for ^3He . Each reservoir is separated from a charcoal sinter by a narrow stainless steel tube. When cold, this sinter collects the helium and provides the pumping action. There is also a copper heat-exchanger which provides a high thermal conductivity contact between the stainless steel necks on each reservoir.

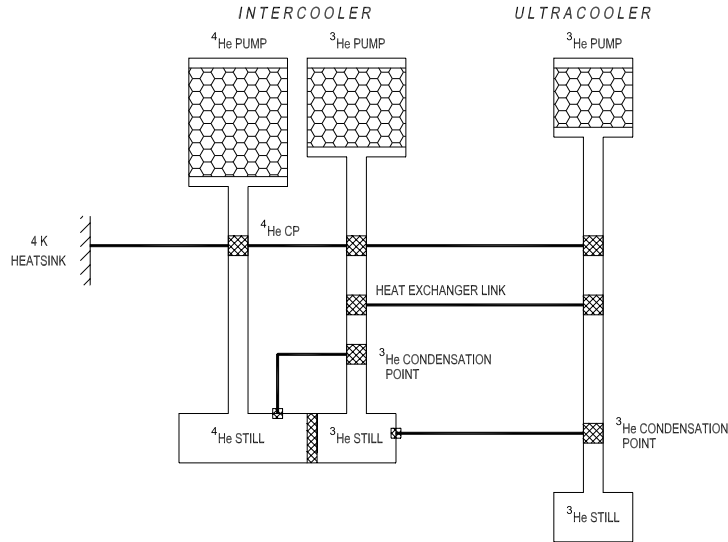


Figure 6.3: A schematic of the three-stage subkelvin sorption refrigerator, from Bhatia et al. (2000). Each stage consists of a reservoir which is separated by a stainless tube from a charcoal sinter. When cold the charcoal adsorbs helium from the reservoir, reducing the pressure and the boiling point of the helium below. The ^4He stage serves as backing stage for the next “inter-cooler” stage allowing ^3He to condense, the inter-cooler in turn serves as a backing stage for the final “ultra-cooler” stage.

⁴Chase Research Cryogenics Ltd., Sheffield S10 5DL UK

A typical cycle of the Chase refrigerator starts by heating the charcoal pumps, releasing helium gas into all three reservoirs. Once the ^4He has condensed, the ^4He pump is cooled to 4K via a gas gap heat switch, engaging the pump. This lowers the temperature of the inter-head to 900 mK, allowing ^3He to condense in the inter-head. This also cools the heat-exchanger to roughly 1.3 K, starting the condensation of ^3He in the ultra-head. Once the ^4He has fully evaporated both the ^3He pumps attached are engaged, cooling the the inter- and ultra-heads to their respective nominal base temperatures of 350 mK and 250 mK. This fridge can sustain a loading of $100\mu\text{W}$ at 380 mK and $3\mu\text{W}$ at 250 mK⁵. In the SPT receiver, the complete cycle executes in 3 hours and holds for approximately 36 hours.

The receiver cryostat also contains many of the SQUID array modules described in Section 3.4. In order to limit the propagation delays in the flux-locked loop, the SQUID controllers should be placed as close to the SQUID modules as possible (See Section 4.3). For this reason the SQUID controllers are mounted directly to outer wall of the cryostat, and the SQUID modules are located along one wall of the 4K shield, directly opposite the SQUID controllers. With a multiplexing factor of 8, each 160 bolometer wedge requires 20 SSA's or $2\frac{1}{2}$ 8-SQUID modules. Since splitting a single SQUID module between two wedges would create additional wiring complexity, each wedge is connected to three SQUID modules, one of which is only half populated. Thus 6 of the 18 SQUID modules contain only 4 SQUIDs.

6.4 Focal Plane Module Design

The general design of a single SPT bolometer pixel is described in Section 5.1. The SPT receiver is a 960 element TES bolometer array, though only 7 out of 8 bolometers are read-out for reasons described in Chapter 3.3. The array is laid out in a hexagonal geometry, and is divided into six triangular wedges, each with 160 bolometers per silicon wedge, as shown Figure 5.1. Each wedge is packaged as an independent module which contains its own set of waveguides and filters, which tune each wedge to a particular observing band (95, 150, or 220 GHz), as well as the MUX-LC filters.

These spider web bolometers are placed in an cavity behind a circular waveguide which opens up to a smooth-walled conical feed horn, illustrated in Figure 6.4. The feed horn determines the profile of the beam pattern. The waveguide diameter determines the cutoff frequency for the lowest-frequency mode, the TE_{11} mode, and hence determines the lower edge of the detector band. The upper edge of each detector frequency band is defined by a stack of metal-mesh filters (Ade et al., 2006) placed at the mouth of the feed horn. The final bands are shown in Figure 6.5. The bands are chosen to avoid atmospheric absorption lines. The upper limit of each frequency

⁵See Bhatia et al. (2000) for load curves

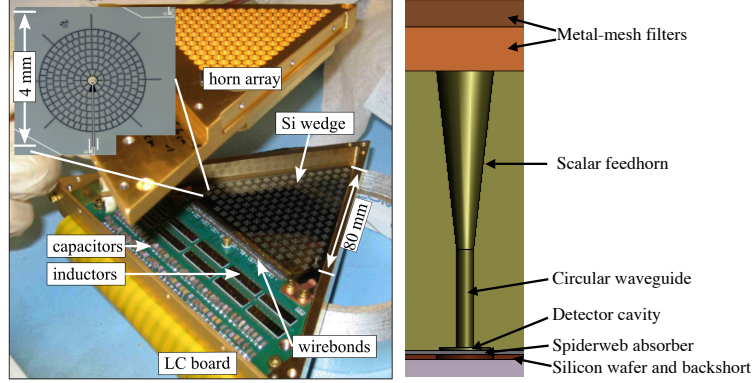


Figure 6.4: Inside a detector module. On the **left-hand side**, a 160-bolometer wedge, shown with its complement of LC filters, and the horn array which defines the beam profile. In the **left-hand side, inset**, a close up view of one of the pixels, showing the spiderweb absorber. On the **right-hand side**, a schematic view of a single scalar feedhorn. Incoming radiation is low-pass filtered by a stack of metal mesh filters at the entrance of the horn. (The second filter is to block transmission at harmonics of the first filters cut-off frequency). The lower edge of the band is set by the cutoff frequency of the waveguide behind the horn. The backside of the wafer is gold plated to provide a reflective backshort for the detector. The thickness of each wafer is tuned to be a quarter-wavelength at the band center frequency.

band is also chosen to be below the next higher-frequency waveguide mode, the TM_{01} mode, thereby making the beam-pattern single-moded.

The detector wafers are also tuned for absorption at either 95, 150 or 220 GHz. In order to maximize the absorption efficiency within the detector cavity, a reflective gold backshort layer is placed on the reverse side of the detector wafer. The wafer thickness,

$$t = \frac{c}{4n_{\text{Si}}\nu_c},$$

corresponds to a quarter wavelength in silicon. Here ν_c is the desired bandcenter, and n_{Si} is the refractive index of Silicon at ~ 250 mK. Thus, the incident and reflected wavefronts interfere constructively at the absorber. Based on this criterion, the wafer thicknesses are chosen to be $230 \pm 5 \mu\text{m}$, $150 \pm 5 \mu\text{m}$ and $105 \pm 5 \mu\text{m}$ at 95 GHz, 150 GHz, and 220 GHz, respectively. Such thin wafers are relatively fragile, and much harder to process than the $500 \mu\text{m}$ wafers typically utilized in the Berkeley Microlab. For this reason, these wafers are bonded to a silicon “backing wafer” after the Au backshort is deposited. Computational simulations have shown that the alternative solution, namely tripling the backshort distance to $\frac{3}{4}$ -wavelength, would lead to reduced optical efficiency and larger stray coupling between neighboring detectors (Mehl, 2009),

The modules are pre-assembled and then attached to the focal plane backing

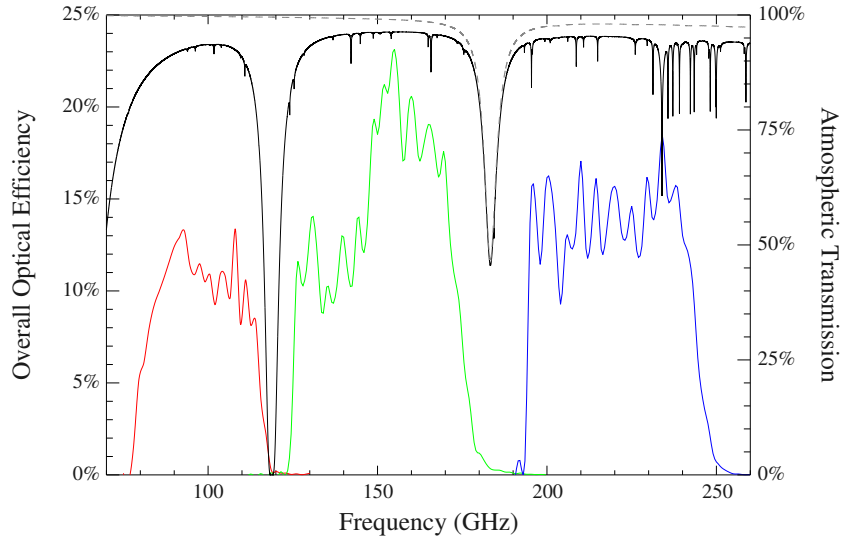


Figure 6.5: Measured bandpasses for the three SPT bands. The **green** and **blue** traces respectively illustrate the 150 GHz and 220 GHz bands, as measured for the 2008 detectors. The 90 GHz wedge was upgraded for the 2009 observing season, and the spectrum for this band is shown by the **red** trace. The bands are chosen to avoid atmospheric emission (**solid black** trace). For instance the 90 GHz band is separated from 150 GHz by an oxygen line. Since most species are well mixed in the atmosphere, atmospheric emission at 90 GHz is fairly constant. Meanwhile the water contribution to the spectrum, shown by the **black dashed trace**, is very poorly mixed and leads to time-varying signals as clouds blow across the telescope’s field of view.

structure. This simple modular design makes it easy to reconfigure the focal plane in the field. Modules are typically assembled and tested before they are shipped to the South Pole and installed in the instrument.

6.5 Instrument Performance

6.5.1 Bandpass Performance

Fourier-transform spectroscopy (FTS) measurements of the SPT receiver were performed in the austral summer of 2008-2009. The frequency response of the three SPT band passes are shown in Figure 6.5. The average 150 GHz and 220 GHz bands are shown for the 2008 detector array. The average bandwidth for the 150GHz

detectors is $\delta\nu = 35.6$ GHz, with a band center of $\nu_0 = 152$ GHz. The average bandwidth for the 220 GHz detectors is $\delta\nu = 42.6$ GHz with a bandcenter of $\nu_0 = 220$ GHz. In 2008, there was one 90GHz wedge. This wedge suffered from low detector yield, and was not useful for science data. Thus, in the analysis Chapter of this thesis we will only be discussing results for the 150GHz and 220GHz datasets. However, in the austral summer of 2008-2009, the 90GHz wedge was replaced, and the spectrum for this new 90 GHz wedge is also shown in Figure 6.5.

6.5.2 Calibration and Optical Efficiency

The calibration of the SPT data is tied to the superb WMAP5 absolute calibration through a direct comparison of 150 GHz SPT maps with WMAP5 V and W-band (61 and 94 GHz) maps (Hinshaw et al., 2009) of the same sky regions. The WMAP5 maps are resampled according to the SPT pointing information, and the resulting TOD are passed through the SPT analysis pipeline to capture the effects of TOD filtering. The ratio of the cross-spectra of the filtered WMAP and SPT maps after correcting for the instrumental beams,

$$c = \frac{\langle a_{\ell m, WMAP_i}^* a_{\ell m, WMAP_j} \rangle}{\left\langle a_{\ell m, SPT}^* a_{\ell m, WMAP_j} \left(\frac{B_{\ell}^{WMAP_i}}{B_{\ell}^{SPT}} \right) \right\rangle}, \quad (6.2)$$

is used to estimate the relative calibration factor between the two experiments. A similar procedure was used to calibrate the Boomerang, ACBAR, and QUaD experiments (Jones et al., 2006; Reichardt et al., 2009a; Brown et al., 2009). Dedicated SPT calibration scans of four large fields totaling 1250 deg^2 of sky were obtained during 2008. The results for these four fields are combined to achieve an absolute temperature calibration uncertainty of 3.6% at 150 GHz.

The 150 GHz calibration is transferred to 220 GHz through the overlapping coverage of SPT's high S/N maps. We calculate the relative calibration by examining the ratio of the cross-spectra between the 150 and 220 GHz maps to the auto-spectra of the 150 GHz map after correcting for the beam and filtering differences. We estimate the relative calibration uncertainty to be 6.2% and the final absolute calibration uncertainty of the 220 GHz temperature map to be 7.2%.

In addition to the CMB calibration scans, the SPT also performs scans of the galactic HII region, RCW38. This source is observed several times daily between science observations, and the observed flux is compared to 150 GHz and 220 GHz observations of RCW38 made by ACBAR (Runyan et al., 2003) and 90 GHz observations from BOOMERANG (Coble et al., 2003). These daily calibrations measure relative changes in detector gain as well as drifts in atmospheric opacity.

Based on our measurements of RCW38 we can then obtain an end-to-end measurement of the optical efficiency of each detector. This overall efficiency is also

illustrated for each band in Figure 6.5. This optical efficiency measurement includes not only the efficiency of the absorption cavity and the detectors themselves (see Section 6.4), but is also includes losses due to scattering at mirrors, absorption by the IR-blocking filters, loss due to beam spillover at the secondary mirror stop, and atmospheric extinction. Of these effects, the beam spillover at the stop is one of the largest sources of loss, with spillover efficiencies of 0.50, 0.80 and 0.93 at 90 GHz, 150 GHz and 220 GHz respectively. In band, the remaining IR-blocking filters and mirrors are all expected have transmissions of 95% or greater. However with multiple filter stages at 4K, 10K and 77K the cumulative effect of the other optical elements cannot be ignored.

6.5.3 Noise and Sensitivity

The observed PSD for the SPT timestreams is shown in Figure 6.6. This data had been studied by Brad Benson. The data is fit to a 4-component model:

$$NEP_{\text{obs}}(\nu)^2 = A_{\text{white}} + \frac{A_{\text{photon}}}{1 + 2\pi\nu^2\tau_{\text{opt}}^2} + A_{\text{red}}\nu^{-\alpha_{\text{red}}} + A_{\text{pink}}\nu^{-1} \quad (6.3)$$

The red component is attributed to atmospheric contributions, which based on discussion in Section 6.1 the exponent α_{red} should be expected to be close to 11/3 based on the analysis by Bussmann et al. (2005). The pink component is an attempt at quantifying the low-frequency noise in the readout electronics. The photon component is a measurement of the photon shot noise discussed in Section 2.2.1. The optical time-constant τ_{opt} is mostly due to the spiderweb absorbers and is measured for each individual detector using a chopped thermal source in the optics cryostat. The “white component” is a catchall for each of the other noise components, (Sections 2.2.2, 2.2.3, and 2.2.4). In this high-TES-loopgain limit, these components are for simplicity treated as mostly white.

Table 6.1: Photon shot noise and white noise levels after removal of atmospheric and 1/f electronics noise.

Wedge	Frequency (GHz)	A_{photon} (aW/ $\sqrt{\text{Hz}}$)	A_{white} (aW/ $\sqrt{\text{Hz}}$)
X12	150	46 ± 12	60 ± 9
X15	150	46 ± 16	69 ± 8
X17	150	42 ± 12	84 ± 12
X6	220	59 ± 16	59 ± 7
X9	220	40 ± 23	72 ± 17

Average fits to the “white” and “photon” components are shown in Table 6.1. The photon noise terms are consistent are slightly surpassed by the combined “white”

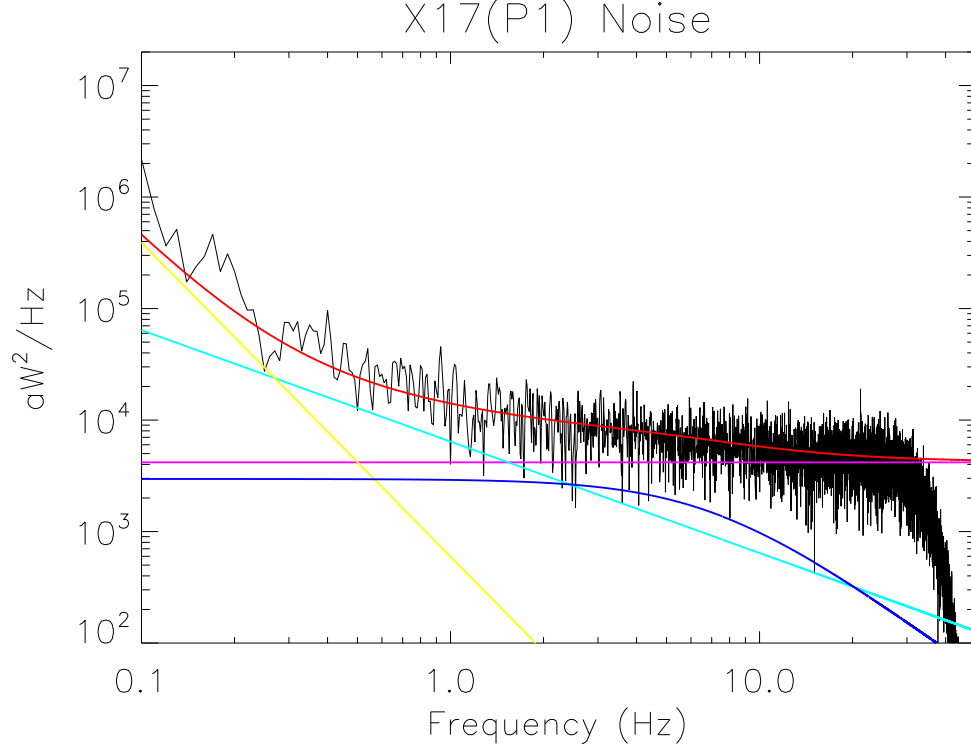


Figure 6.6: Noise PSD from one of the SPT detectors, referred to units of input power. Figure courtesy of Tom Plagge. The yellow line is the “red” atmospheric model. The light blue trace is the best fit to the “1/f” noise expected from the readout. The dark blue trace is the best fit to the photon noise contribution and the violet trace represents the best fit to the remaining noise terms which are expected to have a roughly white spectrum at these frequencies.

noise terms. The photon noise terms are all consistent with the $50 \text{ aW}/\sqrt{\text{Hz}}$ expected for the 15 pW of radiative power observed in these detectors. However the other “white” noise terms are higher than the expected $44 \text{ aW}/\sqrt{\text{Hz}}$, pushing them above the photon noise. The expected “white” noise should be dominated by $36 \text{ aW}/\sqrt{\text{Hz}}$ of TFN noise (Section 2.2.2), with the other terms (Johnson Noise, Readout Noise etc) expected to contribute another $24 \text{ aW}/\sqrt{\text{Hz}}$ in quadrature. This prediction includes ETF Johnson noise suppression as calculated in Appendix B, as well as a “Harmonic correction factor” of $\pi/2\sqrt{2} = 1.11$, which takes into account the fact the fMUX demodulator is sensitive to noise around the odd harmonics of the bias frequency, mixing more power into the demodulator output.

The red and 1/f noise components completely dominate the raw noise spectrum at frequencies below few Hz. The best fit to the red exponent is $\alpha_{\text{red}} = 3.0 \pm 0.6$ averaged

over the entire bolometer array. Though this is lower than the Lay & Halverson (2000) 3D model exponent of $11/3$, it is still within the limits posed by the Lay & Halverson (2000) 2D model. Fortunately, the atmospheric component is well correlated between detectors and can be removed by a simple spatial-mode subtraction (see Section 7.2.2). The pink electronic noise component likely arises from an instability in the output stage of the DDS oscillators. The magnitude of A_{pink} varies wildly across the array with an average amplitude of $80 \pm 40 \text{ aW}/\sqrt{\text{Hz}}$. Combined with the observed photon noise and “white” noise components this corresponds to a $1/f$ knee frequency of roughly 1 Hz.

As discussed in Section 2.2, the true measurement of instrument performance is the sensitivity to CMB temperature fluctuations, the Noise Equivalent Temperature (NET). By combining the photon and the “white” noise terms from Table 6.1 with the FTS spectra shown in Figure 6.5, I calculate the NET for each wedge in Table 6.2. The overall NEP is calculated from Table 6.1: $\text{NEP}^2 = A_{\text{photon}}^2 + A_{\text{white}}^2$. This is then converted to an NET using Equation 2.23.

Table 6.2: Noise equivalent temperatures. The value of NEP comes from the photon noise and white noise fits as tabulated in Table 6.1. The optical efficiency, $\eta(\nu)$, comes from Figure 6.5.

Wedge	Frequency (GHz)	Overall NEP ($\text{aW}/\sqrt{\text{Hz}}$)	NET ($\mu\text{K}_{\text{CMB}}\sqrt{\text{s}}$)
X12	150	75	470
X15	150	83	460
X17	150	93	590
X6	220	83	970
X9	220	82	830

6.5.4 Beam Measurements

The SPT beams are measured by combining maps of three sources: Jupiter, Venus, and the brightest point source in the 100 deg^2 field. Observations of Jupiter are used to measure a diffuse, low-level sidelobe in the range $15' < r < 40'$, where r is the radius to the beam center. Although this sidelobe has a low amplitude (-50 dB at $r=30'$), it contains approximately 15% of the total beam solid angle. A measurement of this sidelobe is necessary for the cross-calibration with WMAP described in §6.5.2. The observations of Jupiter show signs of potential non-linearity in the response of the detectors for $r < 10'$. For this reason we only use the observations of Jupiter to map the sidelobe at $r > 15'$. Observations of Venus are used to measure the beam in the region $4' < r < 15'$. The angular extent of Jupiter or Venus has a negligible effect on the measurement of the relatively smooth beam features present at these

large radii. The brightest point source in the map of the 100 deg² field is used to measure the beam within a radius of 4'. In this way, the random error in the pointing reconstruction (7'' RMS) and its impact on the effective beam are taken into account. The pointing error has a negligible effect on the relatively smooth outer ($r > 4'$) region of the beam.

A composite beam map, $B(\theta, \phi)$, is assembled by merging maps of Jupiter, Venus, and the bright point source. The TOD are filtered prior to making these maps, in order to remove large-scale atmospheric noise. Masks with radii of 40', 25', and 5' are applied around the locations of Jupiter, Venus, and the point source, respectively. These masks ensure that the beam measurements are not affected by the filtering.

Using the flat-sky approximation, we calculate the Fourier transform of the composite beam map, $B(\ell, \phi_\ell)$. From this, we compute the azimuthally-averaged beam function,

$$B_\ell = \text{Re}\left\{\frac{1}{2\pi} \int B(\ell, \phi_\ell) d\phi_\ell\right\}. \quad (6.4)$$

We note that averaging $|B(\ell, \phi_\ell)|^2$ instead of $B(\ell, \phi_\ell)$ would result in a percent-level noise bias in B_ℓ at very high multipoles due to the presence of noise. The results in this work assume an axially symmetric beam, which is only an approximation for SPT. We simulate the effects of ignoring the asymmetry on the bandpowers and find that the errors introduced by making this assumption are negligible.

Although the measured beam function B_ℓ is used for the bandpower estimation, an empirical fit is used to quantify the errors on B_ℓ . B_ℓ is fit to the empirical model

$$B_\ell = ae^{-\frac{1}{2}(\sigma_b \ell)^{1.5}} + (1 - a)e^{-\frac{1}{2}(0.00292*\ell)^{1.8}}. \quad (6.5)$$

There are two components: a main lobe (first term) and a diffuse shelf (second term). The form of the model and the numerical values of the slopes of the exponents were constructed to provide a good fit to the measured B_ℓ . We note that B_ℓ^{150} and B_ℓ^{220} are measured and fitted separately. The RMS difference between the model and measured B_ℓ s is approximately 1%. Two parameters remain free: σ_b , which describes the width of the main lobe, and a , which sets the relative normalization between the main lobe and the diffuse shelf. These parameters are left free to quantify the uncertainty in B_ℓ . The uncertainty in the values of these parameters directly translates to an uncertainty in B_ℓ .

There are a number of factors that limit the accuracy of the measurement of B_ℓ . These include residual map noise, errors associated with the map-merging process, and spectral differences between the CMB and the sources used to measure the beam. The final uncertainties in the beam model parameters σ_b and a are constructed as the quadrature sum of the estimated uncertainties due to each of these individual sources of error. To a good approximation, the uncertainties on σ_b and a can be taken to be Gaussian and uncorrelated.

In practice, a change in the value of a is equivalent to a change in the overall calibration for $\ell > 700$. After the beam uncertainties are estimated, the uncertainty

in a is folded into the estimated uncertainty on the absolute calibration and the parameter a is fixed to the best-fit value of 0.85. The quoted beam uncertainties in the CosmoMC⁶ (Lewis & Bridle, 2002) data files in Section 7.5 include only the uncertainty on σ_b . Figure 6.7 shows the measured beam functions for 150 and 220 GHz, along with the 1σ uncertainties in the main lobe beam width σ_b .

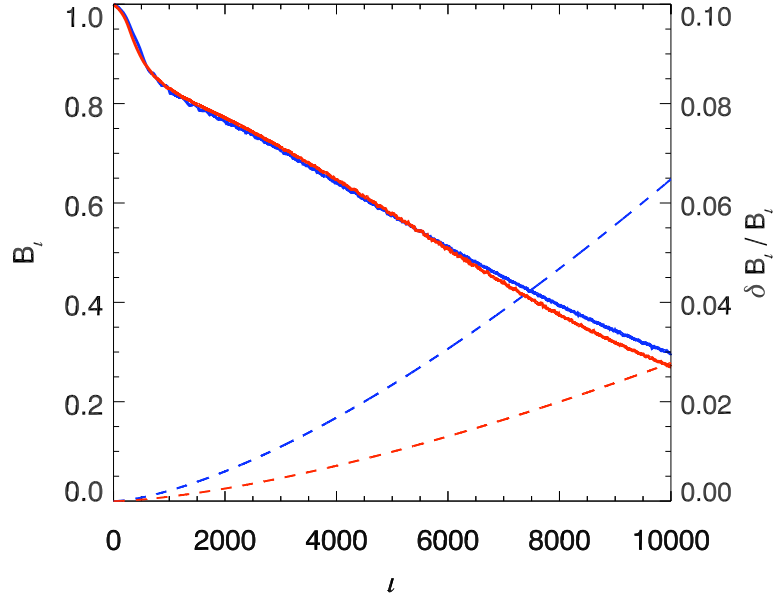


Figure 6.7: Average beam functions and uncertainties for SPT. *Left axis:* The SPT beam function for 150 GHz (**red**) and 220 GHz (**blue**). *Right axis:* The 1σ uncertainties on the beam function for each frequency. The beam uncertainties shown here include only uncertainties on the main lobe beam width, σ_b , since the uncertainty of the sidelobe amplitude has been subsumed into the calibration uncertainty.

6.6 Summary

The SPT is one of the most sensitive ground based instruments on Earth for studies of the fine-scale anisotropies in the CMB. It is a 3-band instrument with $1'$ angular resolution. Drawing on the technologies from the last few Chapters this instrument has a receiver with nearly 1000 pixels. The performance of this receiver is

⁶<http://cosmologist.info/cosmomc>

nearly background limited, though there may be some room for improvement in the other noise terms.

Part II

SZ Power Spectrum Constraints

Chapter 7

The High- ℓ SPT Power Spectrum

This Chapter describes the first power spectrum results obtained with the SPT. The discussion starts with a description of the first set of observations performed in 2008. Next we describe the analysis techniques used to generate maps from the time-ordered data (TOD), followed by the techniques for computing the power spectrum. After a brief discussion of the systematic checks performed on the data, I present the band-averaged power spectra (bandpowers) associated with the 150 and 220 GHz data. This work, started in 2009, has very recently been incorporated into a paper by Lueker et al. (2010).

7.1 2008 Observations

For this analysis, we use data at 150 and 220 GHz from one 100 deg² field centered at right ascension 5^h30^m, declination -55° (J2000) observed by SPT in the first half of the 2008 austral winter. The location of the field was chosen for overlap with the Blanco Cosmology Survey (BCS)¹ optical survey and low dust emission. We observed this field for a total of 779 hours, of which 575 hours is used in the analysis after passing data quality cuts. The final map noise is 18 μ K-arcmin² at 150 GHz and 40 μ K-arcmin at 220 GHz. This field accounts for half the sky area observed in 2008 and one eighth of the total area observed by SPT to date. The SPT maps of this field are shown in Figure 7.1.

The scanning strategy used for these observations involves constant-elevation scans across the 10° wide field. After each scan back and forth across the field, the telescope executes a 0.125° step in elevation. A complete set of scans covering the entire field takes approximately two hours, and we refer to each complete set as an observation.

¹<http://cosmology.illinois.edu/BCS>

²For the rest of this thesis, the unit K refers to equivalent fluctuations in the CMB temperature, i.e., the level of temperature fluctuation of a 2.73 K blackbody that would be required to produce the same power fluctuation. The conversion factor is given by the derivative of the blackbody spectrum, $\frac{dB}{dT}$, evaluated at 2.73 K.

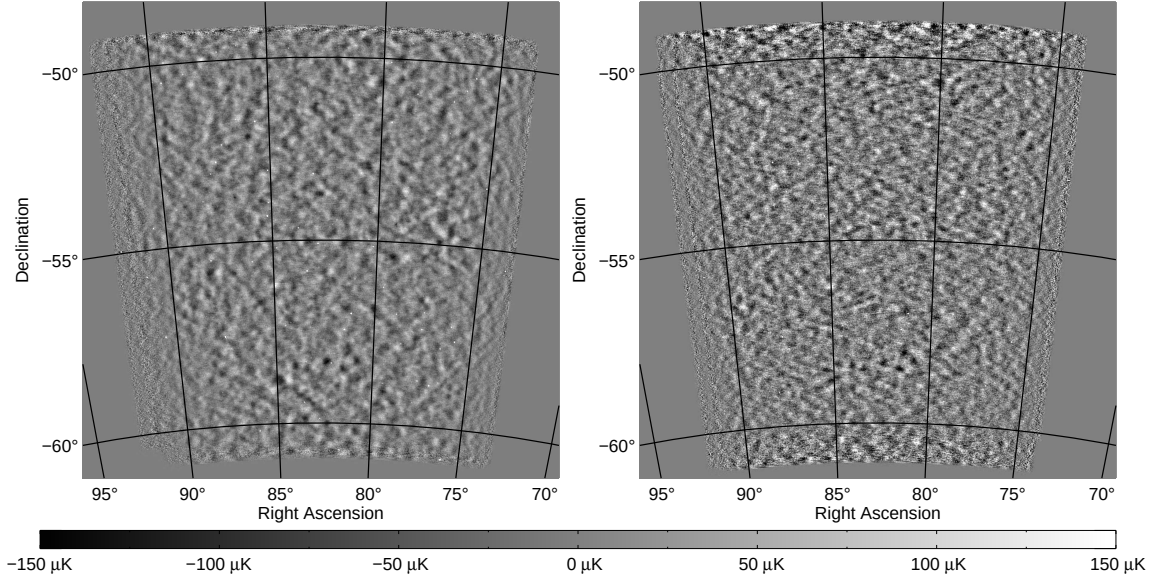


Figure 7.1: Maps of the field used in this analysis. The 150 GHz map is shown in the (**left panel**), the 220 GHz data is in the (**right panel**). See Section 7.2 for a description of the map-making pipeline. Near the center of each map the depth is fairly uniform, with a noise RMS of roughly $18 \mu\text{K-arcmin}$ at 150 GHz, and $40 \mu\text{K-arcmin}$ at 220 GHz. The apparent apodization on the left and right edges is due to the polynomial filter which has been applied to the time-ordered data. For presentation purposes, the maps have been low-pass filtered at $\ell = 11000$. Point sources have not been masked.

Successive observations use a series of different starting elevations to ensure even coverage of the field. Of the 300 observations used in this analysis, half were performed at an azimuth scanning speed of 0.44° per second and half at a speed of 0.48° per second.

Given the small sky area analyzed here, we use the flat-sky approximation. We generate maps in a flat-sky projection and analyze these maps using Fourier transforms. Thus, the discussion of filtering and data-processing techniques refer to particular modes by their corresponding angular wavenumber $\mathbf{k}$ in radians. Note that in these units $|\mathbf{k}| = \ell$.

7.2 Timestream processing and Map-making

We generate one map per frequency band for each two-hour observation of the field. The map-making pipeline used for this analysis is very similar to that used by S09, with some small modifications. An overview of the pipeline is provided below,

emphasizing the aspects of data processing and map-making that are most important for understanding the power spectrum analysis.

7.2.1 Data Selection

Variations in observing conditions and daily receiver setup can affect the performance of individual bolometer channels. The first step in the data processing is to select the set of bolometers with good performance for each individual observation. The criteria for selecting these detectors are primarily based on responsivity (determined through the series of calibrations performed prior to each field observation) and noise performance. Detectors are rejected if they have a low signal-to-noise response to a chopped thermal calibration source or atmospheric emission (modulated through a short elevation scan). They are also rejected if their noise power spectrum is heavily contaminated by readout-related line-like features. More detail on the bolometer cuts can be found in S09. In addition, bolometers that have a responsivity or inverse-noise-based weight more than a factor of three above or below the median for their observing band are rejected. On average, 286 bolometers at 150 GHz (out of 394 possible) passed all cuts for a given observation. At 220 GHz, an average of 161 (out of 254 possible bolometers) passed all cuts.

In a given observation, segments of the TOD corresponding to individual scans may be rejected due to readout or cosmic-ray induced features, anomalous noise, or problems with pointing reconstruction. The scan-by-scan data cuts are the same as those described in S09 and remove approximately 10% of the TOD.

The SPT detectors exhibit some sensitivity to the receiver’s pulse-tube cooler. Bolometer noise power spectra are occasionally contaminated by narrow spectral lines corresponding to the pulse-tube frequency and its harmonics. As in S09, we address this by applying a notch filter to the affected data. This filter removes less than 0.4% of the total signal bandwidth.

In addition to the cuts above, we remove a small number of observations that have either incomplete coverage of the field or high noise levels. Observations taken under poor atmospheric conditions are rejected in this way. The application of these cuts remove 36 out of the 336 total observations of the field.

7.2.2 Time-Ordered Data (TOD) Filtering

Let $d_{\alpha j}$ be a measurement of the CMB brightness temperature by detector j at time α . Contributions to $d_{\alpha j}$ are the celestial sky temperature, $s_{\alpha j}$, the atmospheric temperature, $a_{\alpha j}$, and instrumental noise, $n_{\alpha j}$. The instrumental noise is largely uncorrelated between detectors. However, the atmospheric signal is highly correlated across the focal plane due to the overlap of individual detector beams as they pass through turbulent layers in the atmosphere.

The signals in the data have been low-pass filtered by the bolometers' optical time constants. These single-pole bolometer transfer functions are measured by the method described in S09. We deconvolve the transfer functions for each bolometer on a by-scan basis, as the first step in TOD filtering. At the same time, we apply a low-pass filter with a cutoff at 12.5 Hz to remove noise above the Nyquist frequency associated with the final map pixelation. These operations can be represented as linear operation $\mathbf{\Pi}^{\text{fft}}$ on the TOD,

$$d'_{\alpha j} = \Pi_{\alpha\beta}^{\text{fft}} d_{\beta j}, \quad (7.1)$$

where a sum is implied over repeated indices.

To remove 1/f noise and atmospheric noise in the scan direction, we project out a 19th order Legendre polynomial from the TOD for each detector on a scan-by-scan basis. This operation $\mathbf{\Pi}^{\text{t}}$ can be described by

$$d''_{\alpha j} = \Pi_{\alpha\beta}^{\text{t}} d'_{\beta j}. \quad (7.2)$$

The effect on the signal is similar to having applied a one-dimensional high-pass filter in the scan direction at $\ell \gtrsim 300$.

Significant atmospheric signal remains in the data after this temporal polynomial removal. Because the atmospheric power is primarily common across the entire focal plane, we can exploit the spatial correlations to remove atmosphere without removing fine-scale astronomical signal. The method used in this analysis is to fit for and subtract a plane, $a_0 + a_x \mathbf{x} + a_y \mathbf{y}$, across all detectors in the detector array at each time sample, where $\mathbf{x}$ and $\mathbf{y}$ are set by the angular separation of each pixel from the boresight of the telescope. All detectors are equally weighted in this fit. This operation can be represented as a linear operation on all detectors at each time sample α to produce an atmosphere-cleaned dataset, $\mathbf{d}'''$,

$$d'''_{\alpha j} = \Pi_{jk}^{\text{s}} d''_{\alpha k}. \quad (7.3)$$

We mask the brightest point sources in the map before applying the polynomial subtraction and the spatial-mode filtering. The 92 sources that were detected above 5σ in a preliminary 150 GHz map have been masked. A complete discussion of the point sources detected in this field can be found in Vieira et al. (2010, hereafter V10).

7.2.3 Map-making

The data from each bolometer is inverse noise weighted according to the calibrated, pre-filtering detector PSD in the range 1-3 Hz, corresponding to $1400 < \ell < 4300$. This multipole range covers the angular scales where we expect the most significant detection of the SZ effect. We represent the mapping between time-ordered bolometer

samples and celestial positions with the pointing matrix $\mathbf{L}$, which we apply to the cleaned TOD to produce a map $\mathbf{m}$,

$$\mathbf{m} = \mathbf{L}\mathbf{\Lambda}_w\mathbf{\Pi}^s\mathbf{\Pi}^t\mathbf{\Pi}^{\text{fft}}\mathbf{d}. \quad (7.4)$$

$\mathbf{\Lambda}_w$ is a diagonal matrix encapsulating the detector weights. Information on the pointing reconstruction can be found in S09 and Carlstrom et al. (2009).

7.3 Maps to Bandpowers

We use a pseudo- C_ℓ method to estimate the bandpowers. In pseudo- C_ℓ methods, bandpowers are estimated directly from the Fourier transform of the map after correcting for effects such as TOD filtering, beams, and finite sky coverage. We process the data using a cross spectrum based analysis (Polenta et al., 2005; Tristram et al., 2005) in order to eliminate noise bias. Beam and filtering effects are corrected for according to the formalism in the MASTER algorithm (Hivon et al., 2002). We report the bandpowers in terms of $\mathcal{D}_\ell$, where

$$\mathcal{D}_\ell = \frac{\ell(\ell+1)}{2\pi}C_\ell. \quad (7.5)$$

As the first step in our analysis, we calculate the Fourier transform, $\tilde{m}_{\mathbf{k}}$, of the single-observation maps for each frequency. Each map is apodized using the same window $\mathbf{W}$, and thus $\tilde{m}_{\mathbf{k}}^{(\nu,A)} \equiv \text{FT}(\mathbf{W}\mathbf{m}^{(\nu,A)})$, where the first superscript, ν , indicates the observing frequency, the second superscript, A , indicates the observation number, and the subscript denotes the angular frequency. Cross-spectra are computed for every map pair, and averaged within the appropriate ℓ -bin b :

$$D_b^{\nu_i \times \nu_j, AB} \equiv \frac{1}{N_b} \sum_{\mathbf{k} \in b} \frac{k(k+1)}{2\pi} \tilde{m}_{\mathbf{k}}^{(\nu_i, A)} \tilde{m}_{\mathbf{k}}^{(\nu_j, B)*}. \quad (7.6)$$

We use the abbreviation, $D^{\nu_i} \equiv D^{\nu_i \times \nu_i}$, when referring to single-frequency auto-spectra. Recall that $|\mathbf{k}| = \ell$ in the flat-sky approximation.

The FFT of a map, $\tilde{m}_{\mathbf{k}}^{(\nu,A)}$, is linearly related to the FFT of the astronomical sky, $a_{\mathbf{k}}^\nu$. It also includes a noise contribution, $n_{\mathbf{k}}^{(\nu, Ai)}$, which is uncorrelated between the 300 observations:

$$\tilde{m}_{\mathbf{k}}^{(\nu,A)} = \widetilde{W}_{\mathbf{k}-\mathbf{k}'} G_{\mathbf{k}'} \left(B_{\mathbf{k}'} a_{\mathbf{k}'}^\nu + n_{\mathbf{k}'}^{(\nu,A)} \right). \quad (7.7)$$

Here $G_{\mathbf{k}}$ is the 2-dimensional amplitude transfer function, which accounts for the TOD filtering as well as the map-based filtering described in §7.3.2. $\widetilde{W}_{\mathbf{k}}$ is the Fourier transform of the apodization mask.

Following the treatment by Hivon et al. (2002), we take the raw spectrum, D_b^{AB} , to be linearly related to the true spectrum by a transformation, $K_{bb'}$. The K transformation combines the power spectrum transfer function, F_ℓ , which includes the

effects of TOD-based and map-based filtering (§7.3.3), the beams, B_ℓ (§6.5.4), and the mode-coupling matrix, $M_{kk'}[\mathbf{W}]$ (§7.3.1). The mode-coupling matrix accounts for the convolution of the spectrum due to the apodization window, $\mathbf{W}$.

$$K_{bb'}^{\nu_i \times \nu_j} = P_{bk} \left(M_{kk'}[\mathbf{W}] F_{k'}^{\nu_i \times \nu_j} B_{k'}^{\nu_i} B_{k'}^{\nu_j} \right) Q_{k'b'}. \quad (7.8)$$

Here, P_{bk} is the re-binning operator (Hivon et al., 2002):

$$P_{bk} = \begin{cases} \frac{k(k+1)}{2\pi\Delta k_{(b)}} & k_{(b-1)} < k < k_{(b)} \\ 0 & \text{otherwise} \end{cases} \quad (7.9)$$

while the inverse of the re-binning operator is Q_{kb} :

$$Q_{kb} = \begin{cases} \frac{2\pi}{k(k+1)} & k_{(b-1)} < k < k_{(b)} \\ 0 & \text{otherwise} \end{cases}. \quad (7.10)$$

In the rest of the paper, we will often refer to band averaged quantities, such as $C_b = P_{bk} C_k$. Cross-spectra between observations that have been corrected for apodization and processing are denoted as,

$$\hat{D}_b^{\nu_i \times \nu_j, AB} \equiv (K^{-1})_{bb'} D_{b'}^{\nu_i \times \nu_j, AB}. \quad (7.11)$$

The final bandpowers are then computed as the average of all cross-spectra,

$$q_b^{\nu_i \times \nu_j} = \frac{1}{n_{\text{obs}}(n_{\text{obs}} - 1)} \sum_A \sum_{B \neq A} \hat{D}_b^{\nu_i \times \nu_j, AB}. \quad (7.12)$$

We make the simplifying assumption that the noise properties of each observation are statistically equivalent, hence the uniform weighting chosen here. The data selection criteria in §7.2.1 ensure that these observations target the same region of the sky, have roughly the same number of active bolometers, and have similar noise properties. Therefore, this uniform weighting should be unbiased and only slightly sub-optimal.

The cross-spectrum bandpowers, $D_b^{\nu_i \times \nu_j, AB}$, generated from the 300 observations are used in conjunction with signal-only Monte Carlo bandpowers to generate empirical covariance matrices, as described in Appendix C. The variance of the Monte Carlo bandpowers is used to estimate the sample variance contribution to the covariance matrix. Meanwhile, we use the variance in the spectra of the real data to estimate the uncertainty due to noise in the maps.

7.3.1 Apodization Mask and Calculation of the Mode-mixing Kernel

Since we have only mapped a fraction of the full sky, the angular power spectra of the maps are convolutions of the true C_ℓ s with an ℓ -space, mode-mixing kernel that

depends on the map size, apodization, and point source masking. We calculate this mode-mixing kernel, $M_{kk'}[\mathbf{W}]$, following the derivation in Hivon et al. (2002) for the flat sky case:

$$M_{kk'}[\mathbf{W}] \equiv \frac{1}{(2\pi)^2} \int d\theta_k d\theta_{k'} \left| \widetilde{W}_{\mathbf{k}-\mathbf{k}'} \right|^2. \quad (7.13)$$

If the mask is smooth on fine angular scales, then the mode coupling kernel can be approximated by a delta function at high- k :

$$M_{kk'}[\mathbf{W}] \approx w_2 \delta_{kk'}. \quad (7.14)$$

Here we use the notation $w_n \equiv \langle \mathbf{W}^n \rangle$ to represent the n^{th} moment of the apodization window. In this limit, the coupling kernel serves the purpose of re-normalizing the power spectrum to account for modes lost due to apodization. As we discuss in Appendix C, the coupling kernel also plays an important role in determining the shape and normalization of the covariance matrix.

We avoid areas of the map with sparse or uneven coverage in any single observation. Thus, the apodization window is conservative in its avoidance of the map edges. We also mask 144 point sources detected in the 150 GHz data above a significance of 5σ , which corresponds to a flux of 6.4 mJy. This source list is from a more refined analysis than the preliminary list used in §7.2.2; the differences are in sources near the 5σ detection threshold. Each point source is masked by a 2'-radius disk, with a Gaussian taper outside this radius. Many different tapered mask shapes were tested for both efficacy in removing point source power and noise performance. Given the relatively small area masked by point sources, varying the shape of the point source mask has little effect on the final spectrum. The effective sky area of the mask is 78 deg^2 . Simulations have been performed to test whether the application of this mask will bias the inferred power and we see no bias with a 5σ cut. As could be expected, we do observe a mild noise bias when using a more aggressive 3σ point source mask. The same 5σ mask is used for all maps at both frequencies and for all observations.

7.3.2 Fourier Mode Weighting

The maps produced by the steps described in §7.2.2 have anisotropic signal and noise. In particular, the map noise PSD rises steeply at spatial frequencies corresponding to low spatial frequencies in the scan direction (low k_x). The covariance of the power estimated in a given ℓ -bin depends on the second power of the map noise PSD for all the Fourier modes in that ℓ -bin. In the presence of either non-uniform noise or signal, applying an optimal mode-weighting when calculating the mean bandpower may significantly reduce the final noise covariance matrix of the power spectrum bandpowers. In the case of SPT, we find that for the purposes of

measuring the $\ell \gtrsim 2000$ power spectrum, a simple, uniform selection of modes at $k_x > 1200$ is close to optimal, and we apply this mode weighting in calculating the SPT bandpowers.

7.3.3 Transfer Function Estimation

In order to empirically determine the effect of both the TOD-based filtering and the Fourier mode-weighting, we calculate a transfer function, F_k , as defined in Hivon et al. (2002). Note that this power spectrum transfer function is distinct from the amplitude transfer function G_k . Specifically, the transfer function accounts for all map- and TOD-processing not taken into account by the mode-coupling kernel or the beam functions.

We created 300 Monte Carlo sky realizations at 150 and 220 GHz in order to calculate the transfer function of the filtering. The simulations also serve as an input for the covariance matrix estimation. These simulations contain two components: a CMB component and a point source component. The CMB component is computed for the best-fit WMAP5 lensed Λ CDM model.

The point source component includes two different populations of dusty galaxies, a low- z population and a high- z population. For each population we generate sources from a Poisson distribution. Sources are generated in bins of flux, S , ranging from 0.01 to 6.75 mJy. This upper limit in flux is close to the 5σ detection threshold in the 150 GHz maps. In each flux bin, the 150 GHz source density, dN/dS , of each population is taken from the model of Negrello et al. (2007). We relate the flux of each source at 220 GHz to its flux at 150 GHz with a power law in intensity, $S_\nu \propto \nu^\alpha$. The power law spectral index, α , for each source is drawn from a normal distribution. We use spectral indices of $\alpha = 3 \pm 0.5$ for the high- z protospheroidal galaxies, and $\alpha = 2 \pm 0.3$ for the low- z IRAS-like galaxies. As with any Poisson distribution of point sources, the power spectrum of these maps is white ($C_\ell^{\text{ps}} = \text{constant}$) and related to the flux cutoff, S_0 , by:

$$C_\ell^{\text{ps}} = \left(\frac{dB_\nu}{dT} \right)_{\text{T}_{\text{CMB}}}^{-2} \int_0^{S_0} S^2 \frac{dN}{dS} dS. \quad (7.15)$$

The power spectra of these simulated point source maps are well represented by a constant $C_\ell^{\text{ps}} = 1.1 \times 10^{-5} \mu\text{K}^2$ at 150 GHz and $C_\ell^{\text{ps}} = 6.8 \times 10^{-5} \mu\text{K}^2$ at 220 GHz.

These simulated maps are smoothed by the appropriate beam. From each map realization, we construct simulated TOD using the pointing information. Each realization of the TOD is processed using the low-pass filter, polynomial removal, and the spatial-mode subtraction described in §7.2.2. No time-constant deconvolution is applied, since these realizations of the TOD have not been convolved by the bolometer time constants. The filtered, simulated TOD are then converted into maps according to equation 7.4.

We apply the same apodization mask and Fourier mode weighting to these map realizations as is used on the actual data. We then compute the Monte Carlo pseudo-power spectrum, $(D_k)_{\text{MC}}$, for each map. The simulated transfer function is calculated iteratively, by comparing the Monte Carlo average, $\langle D_k \rangle_{\text{MC}}$, to the input theory spectrum, C_k^{theory} .

For the single-frequency bandpowers, we start with an initial guess of the transfer function:

$$F_k^{\nu,(0)} = \frac{\langle D_k^\nu \rangle_{\text{MC}}}{w_2 B_k^{\nu 2} C_k^{\nu,\text{theory}}}. \quad (7.16)$$

The superscript, (0), indicates that this is the first iteration in the transfer function estimates. For this initial guess, we approximate the coupling kernel as largely diagonal as in equation 7.14. Thus, the factor w_2 is the normalization factor required by the apodization window. We then iterate on this estimate using the full mode-coupling kernel:

$$F_k^{\nu,(i+1)} = F_k^{\nu,(i)} + \frac{\langle D_k^\nu \rangle_{\text{MC}} - M_{kk'} F_{k'}^{\nu,(i)} B_{k'}^{\nu 2} C_{k'}^{\nu,\text{theory}}}{B_k^{\nu 2} C_k^{\nu,\text{theory}} w_2}. \quad (7.17)$$

We iterate on this estimate five times, although the transfer function has largely converged after the first iteration.

This method may misestimate the transfer function if the simulated spectrum is significantly different from the true power spectrum. The primary CMB anisotropy has been adequately constrained by previous experiments, however, the foreground power spectrum is less well known. We repeated the transfer function estimation using an input power spectrum with twice the nominal point source power. The resulting transfer function was unchanged at the 1% level, giving confidence that the transfer function estimate is robust.

The transfer function for a multifrequency cross-spectrum is taken to be the geometric mean of the two individual transfer functions:

$$F_k^{\nu_i \times \nu_j} = \sqrt{F_k^{\nu_i} F_k^{\nu_j}}. \quad (7.18)$$

This treatment is only strictly correct for isotropic filtering. As a cross check, we have also computed the cross-spectrum transfer function directly. For the angular multipoles reported here, the geometric-mean transfer function estimate is in excellent agreement with the estimate obtained using $\langle D_k^{\nu_1 \times \nu_2} \rangle_{\text{MC}}$ in equations 7.16 and 7.17.

7.3.4 Frequency-differenced Spectra

We are interested in the power spectra of linear combinations of the 150 and 220 GHz maps designed to remove astronomical foregrounds. One method for obtaining such power spectra would be to directly subtract the maps after correcting the maps for differences in beams or processing. In such a map subtraction, the scaling

of the 220 GHz map, x , can be adjusted to optimally remove foregrounds. The differenced maps can then be processed using our standard pipeline to obtain spectra with a reduced foreground contribution.

Equivalently, the differenced spectrum can be generated from the original bandpowers, q_b^i , using a linear spectrum transformation, ξ :

$$q_b^{150-x \times 220} = \sum_i \xi^i(x) q_b^i. \quad (7.19)$$

Here, the index, i , denotes the 150 GHz auto-spectrum, 220 GHz auto-spectrum, or 150×220 GHz cross spectrum. This transformation is computationally fast and takes advantage of the fact that the cross-frequency bandpowers include information on the relative phases of each Fourier component in the map. In this way, the difference spectrum can be represented in terms of the three measured spectra:

$$\begin{aligned} q_b^{\nu_i - x \times \nu_j} &= \frac{1}{(1-x)^2} \sum_{\mathbf{k} \in b} |a_{\mathbf{k}}^{\nu_i} - x a_{\mathbf{k}}^{\nu_j}|^2 \\ &= \frac{1}{(1-x)^2} \sum_{\mathbf{k} \in b} |a_{\mathbf{k}}^{\nu_i}|^2 - 2x \text{Re}(a_{\mathbf{k}}^{\nu_i} a_{\mathbf{k}}^{\nu_j*}) + x^2 |a_{\mathbf{k}}^{\nu_j}|^2 \\ &= \frac{1}{(1-x)^2} \left(q_b^{\nu_i} - 2x q_b^{\nu_i \times \nu_j} + x^2 q_b^{\nu_j} \right). \end{aligned} \quad (7.20)$$

The overall normalization is chosen such that the CMB power is unchanged in the subtracted spectrum. For clarity, we have momentarily avoided here the complications of beams and filtering and have expressed the bandpowers in terms of the Fourier transform of the celestial sky, $a_{\mathbf{k}}$. For a given proportionality constant, x , the values of $\xi(x)$ are:

$$\begin{aligned} \xi^{150}(x) &= \frac{1}{(1-x)^2}, \\ \xi^{150 \times 220}(x) &= \frac{-2x}{(1-x)^2}, \\ \xi^{220}(x) &= \frac{x^2}{(1-x)^2}. \end{aligned} \quad (7.21)$$

In order to compute covariances for the subtracted spectrum, one needs to know the correlations between bandpowers measured at different frequencies. Since both frequencies cover the exact same area of sky, the 150 GHz, 220 GHz and cross spectra are nearly completely correlated at low ℓ 's where the errors are dominated by sample variance. At higher ℓ -ranges the cross spectrum bandpowers are also partially correlated with both the 150 GHz and 220 GHz bandpowers, since the instrumental contribution to the cross-spectrum variance originates from the 150 GHz and 220 GHz

noise. We represent the correlation between ℓ -bins, b and b' , measured at frequencies, i and j as:

$$\mathbf{C}_{bb'}^{(i,j)} \equiv \langle q_b^i q_{b'}^j \rangle - \langle q_b^i \rangle \langle q_{b'}^j \rangle \quad (7.22)$$

As before, the superscripts i and j may stand for 150 GHz, 220 GHz, or the cross-spectrum 150 $\times$ 220 GHz. The method for computing the multi-frequency covariance matrix is discussed in the appendix. By combining equations (7.19) and (7.22), one can calculate the covariance matrix for the subtracted bandpowers from the multi-frequency covariance matrices:

$$\begin{aligned} \mathbf{C}_{bb'}^{150-x \times 220} &\equiv \langle q_b^{150-x \times 220} q_{b'}^{150-x \times 220} \rangle \\ &\quad - \langle q_b^{150-x \times 220} \rangle \langle q_{b'}^{150-x \times 220} \rangle \end{aligned} \quad (7.23)$$

$$= \sum_{i,j} \xi^i(x) \mathbf{C}_{bb'}^{(i,j)} \xi^j(x) \quad (7.24)$$

In this way, we account for correlations between bandpowers of different frequencies when computing the subtracted-spectrum covariance matrix. Ignoring these correlations can lead to a gross over-estimate of the uncertainty in the subtracted spectrum. For the SPT bandpowers presented in §7.5, neglecting these correlations leads to a 100% over-estimate of δC_ℓ at $\ell = 2000$ for $x = 0.325$.

7.4 Systematic checks

We apply a set of jack-knife tests to the SPT data to search for possible systematic errors. In a jack-knife test, the data set is divided into two halves, based on features of the data associated with potential sources of systematic error. The two halves are differenced to remove any astronomical signal, and the resulting power spectrum is compared to zero. Significant deviations from zero would indicate either a systematic problem or a noise misestimate. We implement the jack-knives in the cross-spectrum framework by differencing single pairs of observations and applying the cross-spectrum estimator outlined in Section 7.3 to the set of differenced pairs. In total, we perform 13 different jack-knife tests.

Six jack-knives are based on the observing parameters, such as time, scan direction and azimuthal range. The data can be split based on when it was taken to search for systematic changes in the calibration, beams, detector time constants, or any other time variable effect. The “first half - second half” jack-knife probes variations on month time scales, while an “even-odd” jack-knife differencing every other observation looks for variations on hourly time scales. Results for the “first half - second half” jack-knife are shown in the top panel of Figure 7.2. The data can also be split based on the direction of the scan in a “left-right” jack-knife (panel 2 of Figure 7.2). We would expect to see residual power here if the detector transfer function had been improperly de-convolved, if the telescope acceleration at turn-arounds induces a

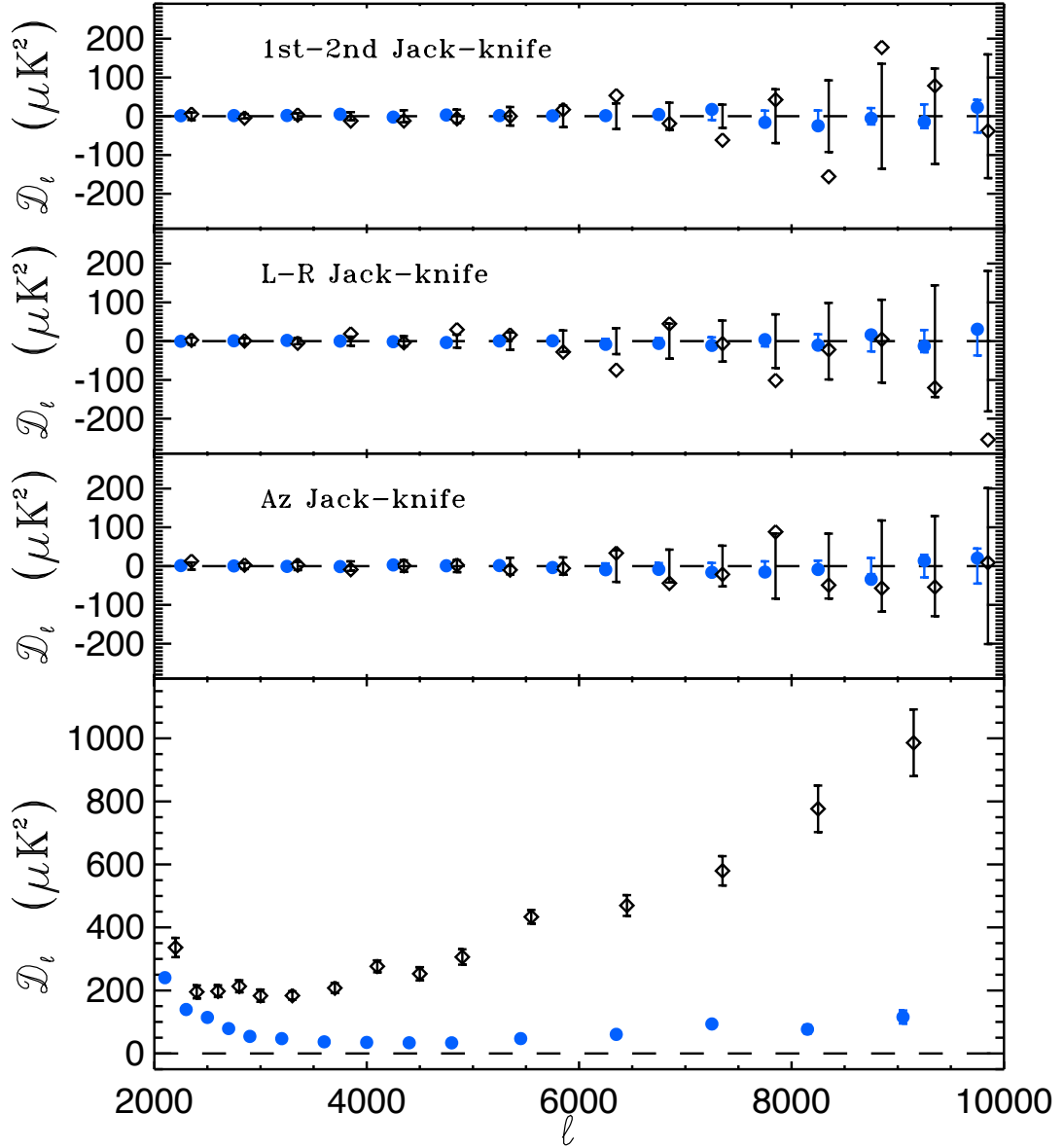


Figure 7.2: Jack-knives for the SPT data set at 150 GHz (**blue circles**) and 220 GHz (**black diamonds**). For clarity, the 220 GHz jack-knives have been shifted to the right by $\Delta\ell = 100$. *Top panel:* Bandpowers of the “first half - second half” jack-knife compared to the expected error bars about zero signal. Disagreement with zero would indicate either a noise misestimate or a time-dependent systematic signal. *Second panel:* Power spectrum of the left-going minus right-going difference map. This test yields strong constraints on the accuracy of the detector transfer function deconvolution and on possible directional systematics. *Third panel:* Bandpowers for the difference map when the data is split based on azimuth. Signals fixed in azimuth such as side-lobe pickup from the nearby support building would produce non-zero power. We see no evidence for ground-based pickup. The cumulative probability to exceed the χ^2 observed in these three tests is 76% at 150 GHz and 22% at 220 GHz. *Bottom panel:* The un-differenced SPT power spectra at each frequency for comparison.

signal through sky modulation or microphonics, or if the wind direction is important. We observed this field at two scan speeds different by 10%. We check for systematic differences related to the scan speed, such as a mirror wobble, in a “scan speed” jack-knife. Side-lobe pickup could potentially introduce spurious signals into the SPT maps from the moon or features on the ground. We test for moon pickup by splitting the data based on whether the moon is above or below the horizon. We test for ground pickup by splitting on the mean azimuth of the observation. To maximize the sensitivity to ground pickup, the azimuthal ranges are selected to be centered on and directly opposite the closest building to the telescope, which is the most likely source of ground signal. The azimuthal jack-knife is shown in the third panel of Figure 7.2.

We also perform jack-knives based on four noise and observation-quality statistics of the 150 GHz data. The first is based on the overall RMS in the maps, which is affected by atmospheric conditions and detector noise. The second is based on the average raw detector PSD in the range 9-11 Hz, which is a measure of the detector “white” noise level. The third is the RMS near $\ell = 3000$ where the S/N on the SZ power spectrum is highest. The fourth is based on the number of bolometers active in each observation.

There are also a number of line-like spectral features in the SPT TOD that could potentially affect the power spectrum bandpowers, and we perform three jackknife tests for sensitivity to these features. Some of these line features appear at harmonics of the receiver pulse-tube frequency, and are typically correlated across many bolometer channels. These lines are filtered from the data as described in §7.2.2. In addition, some channels exhibit occasional line-like features at other frequencies, which are not filtered in the data processing. We search for residual effects in a jack-knife based on the average number of line-like features for all 150 GHz bolometers, as well as a jack-knife based on the bandwidth affected by the lines. Finally, we do an additional split using the average number of lines in an observation that appear to be related to the pulse-tube cooler.

We calculate the χ^2 with respect to zero for each jack-knife over the range $\ell \in [2000, 10000]$ in bins with $\Delta\ell = 500$. Some of the tests are highly correlated. For example, we changed scan velocities approximately midway through the observations so splitting the data based on scan velocity is nearly identical to splitting the data between the first and second halves of the season. We calculate a correlation coefficient between the different tests by adding $1/N_{obs}$ for each common observation in a half, and subtracting $1/N_{obs}$ for each distinct observation in a half. This algorithm returns unity for two identical sets and zero for two random sets, as we expect 50% of the observations to be in common for two random selections. The correlation coefficients between the 13 jack-knives range from 0 to 0.83, with the maximum correlation being for the previously mentioned scan velocity and first half - second half jack-knives. We invert the jack-knife correlation matrix, $\mathcal{C}$, and calculate $\chi^2 = v_{i\alpha}(\mathcal{C}^{-1})_{ij}v_{j\alpha}$. Here $v_{i\alpha}$ is the ratio of the bandpower over the uncertainty for the i^{th} jack-knife and α^{th} ℓ -bin. The probability to exceed the measured χ^2 for the complete set of thirteen jack-knives

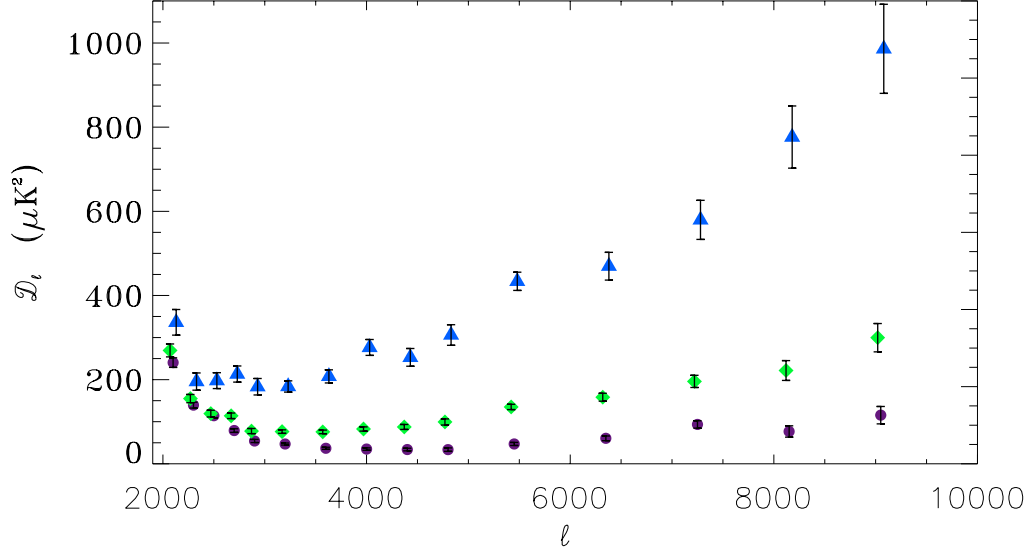


Figure 7.3: The SPT 150 GHz (**purple circles**), 150×220 GHz (**green diamonds**) and 220 GHz (**blue triangles**) bandpowers. The damping tail of the primary CMB anisotropy is apparent below $\ell = 3000$. Above $\ell = 3000$, there is a clear excess with an angular dependence consistent with point sources. These sources have low flux (as sources with > 6.4 mJy at 150 GHz have been masked) and a rising frequency spectrum, consistent with our expectations for Poisson distributed DSFGs. The point source population and resulting contributions to anisotropy power are discussed in more detail in H09.

is 77% for the 150 GHz data, 32% for the 220 GHz data and 57% for the combined set with both frequencies. We thus find no evidence for systematic contaminants in the SPT data set.

7.5 Power Spectrum

Figure 7.3 shows the bandpowers we compute by applying the analysis methods described in Section 7.3 to one 100 deg^2 field observed by SPT at 150 and 220 GHz. The bandpowers for the two frequencies and their cross-spectrum are tabulated in Table 7.1. The bandpower uncertainties are derived from the combination of simulations and the measured intrinsic variations within the SPT data as described in Section 7.3. The bandpowers can be compared to theory using the associated window functions (Knox, 1999). The bandpowers, uncertainties, and window functions may now be downloaded from the SPT website.³

³<http://pole.uchicago.edu/public/data/lueker09/>

Table 7.1: Single-frequency bandpowers

ℓ range	l_{eff}	150 GHz		150 $\times$ 220 GHz		220 GHz	
		q (μK^2)	σ (μK^2)	q (μK^2)	σ (μK^2)	q (μK^2)	σ (μK^2)
2001 - 2200	2058	240.5	11.4	269.5	15.3	336.2	30.4
2201 - 2400	2276	139.4	7.0	155.0	9.8	195.4	20.5
2401 - 2600	2474	114.3	5.2	119.2	8.1	197.6	18.8
2601 - 2800	2677	79.0	4.3	114.0	6.7	213.3	19.2
2801 - 3000	2893	54.1	3.7	77.7	6.1	183.1	19.5
3001 - 3400	3184	47.0	2.4	76.5	4.2	183.7	13.4
3401 - 3800	3581	36.9	2.4	75.8	4.3	207.7	15.6
3801 - 4200	3992	35.0	2.8	82.7	5.0	276.5	18.8
4201 - 4600	4401	33.9	3.2	87.5	5.9	252.8	21.0
4601 - 5000	4789	33.6	4.2	99.4	7.3	306.1	24.5
5001 - 5900	5448	47.1	3.5	135.1	6.3	433.6	21.7
5901 - 6800	6359	60.5	5.5	158.2	9.4	469.6	33.0
6801 - 7700	7255	93.5	8.8	195.8	14.5	579.7	46.4
7701 - 8600	8161	76.8	13.5	221.8	23.6	776.5	73.9
8601 - 9500	9059	115.6	20.7	299.7	33.7	986.1	105.9

Band multipole range and weighted value ℓ_{eff} , bandpower q_B , and uncertainty σ_B for the 150 GHz auto-spectrum, cross-spectrum, and 220 GHz auto-spectrum of the SPT field. The quoted uncertainties include instrumental noise and the Gaussian sample variance of the primary CMB and the point source foregrounds. The sample variance of the SZ effect, beam uncertainty, and calibration uncertainty is not included. Beam uncertainties are shown in Figure 6.7 and calibration uncertainties are quoted in §6.5.2. Point sources above 6.4 mJy at 150 GHz have been masked out in this analysis. This flux cut substantially reduces the contribution of radio sources to the bandpowers, although DSFGs below this threshold contribute significantly to the bandpowers.

These single frequency spectra have been studied by Hall et al. (2010), who have decomposed the bandpowers into primary CMB, flat $\mathcal{D}_\ell$ and Poisson terms, and have studied the implications for the properties of DSFGs. The SPT bandpowers are dominated by the damping tail of the primary CMB anisotropy on angular multipoles $2000 < \ell < 3000$. At these multipoles, the bandpowers are in excellent agreement with the predictions of a Λ CDM model determined from CMB observations on larger angular scales. On smaller scales, the SPT bandpowers provide new information on secondary CMB anisotropies and foregrounds which dominate the primary CMB anisotropy. The SPT data presented here represent the first highly significant detection of power at these frequencies and angular scales where the primary CMB anisotropy is sub-dominant. After masking bright point sources, the total signal-to-noise ratios on power in excess of the primary CMB are 55, 55, and 45 at 150, 150×220 , and 220 GHz respectively. The majority of the high- ℓ power can be attributed to a Poisson distribution of point sources (likely DSFGs) on the sky. The largest source of secondary CMB anisotropy at 150 GHz is expected to be the SZ effect, and we investigate SZ constraints in the following Chapter.

Chapter 8

Cosmological Interpretation of the SPT Power Spectrum

In this Chapter we interpret the power spectra computed in the last Chapter in order to determine the amplitude of the tSZ power spectrum and to use this amplitude to constrain the normalization of the matter power spectrum, σ_8 . This measurement requires separating the tSZ signal from the other astrophysical signals in our data which include primary CMB anisotropy (including lensing effects), DSFGs (both Poisson and clustered components), and anisotropy due to the kSZ effect. The primary CMB anisotropy and Poisson point source component can be separated from a tSZ-like component on account of the distinct angular power spectra of these three signals. However the tSZ, kSZ, and clustered DSFG components are all expected to be roughly flat in $\mathcal{D}_\ell$, and we depend on their distinct frequency dependences to separate them. We use a combination of the two frequency bands to remove the DSFG contribution to the power spectra. We address the remaining degeneracy between the tSZ and kSZ effects by repeating the analysis for a range of assumed kSZ models.

In Section 8.1, we discuss the expected contribution of DSFGs to SPT power spectrum. In Section 8.2 we combine the two-frequency bandpowers computed in the last Chapter to generate a set of DSFG-subtracted bandpowers. In Section 8.3 we describe the Monte Carlo Markov chain (MCMC) analysis used to estimate the tSZ power spectrum amplitude, parametrized as the normalization A_{SZ} of a model template, from the DSFG-subtracted bandpowers. In Section 8.4 we discuss the implications of the measured tSZ power spectrum amplitude for σ_8 and modeling of the tSZ effect.

8.1 Foregrounds

The main foregrounds at frequencies near 150 GHz are expected to be galactic dust emission, radio sources, and dusty star forming galaxies (DSFGs). Note, however,

that the SPT field is selected to target one of the cleanest regions on the sky for galactic dust emission, and in the Finkbeiner et al. (1999) model, dust emission is primarily on large angular scales. The contribution for the selected field on arcminute scales is insignificant. The primary foregrounds of consideration for this analysis are radio sources and DSFGs.

Tens of bright radio sources are detected in the SPT maps at $> 5\sigma$, and contribute substantial amounts of power at both 150 and 220 GHz. Information on the fluxes and spectral indices of these and other sources significantly detected in the SPT maps can be found in V10. Without masking, the measured point source power is $C_\ell^{\text{unmasked}} = 2.1 \times 10^{-4} \mu\text{K}^2$ at 150 GHz and $C_\ell^{\text{unmasked}} = 1.6 \times 10^{-4} \mu\text{K}^2$ at 220 GHz. These estimates are dominated by the brightest few sources and thus subject to very large sample variance. We mask all sources with 150 GHz fluxes above the 5σ detection threshold, $S = 6.4 \text{ mJy}$. By masking these sources we reduce the radio source contribution to the SPT bandpowers by several orders of magnitude. According to the de Zotti et al. (2005) model source counts, after masking bright sources, we expect a residual radio source contribution of $C_\ell^{\text{radio}} = 8.5 \times 10^{-7} \mu\text{K}^2$ at 150 GHz.

The point source masking will remove the SZ contribution from only a few galaxy clusters, leading to negligible reduction of the SZ power. This is because the large majority of radio sources and DSFGs reside outside of SZ clusters. Most of the masked sources are identified as radio sources in V10. Extrapolations from lower frequency observations imply that, at 150 GHz, less than 3% of clusters contain radio source flux exceeding 20% of the tSZ flux decrement (Lin et al., 2009; Sehgal et al., 2010). The masked sources are selected as increments at 150 GHz and therefore have fluxes much greater than 20% of the tSZ of any associated galaxy cluster. The number of clusters masked by the radio source masking should then be much less than 3% and negligible. We also compare the tSZ power spectrum in the Sehgal et al. (2010) simulated sky maps with and without masking $> 6.4 \text{ mJy}$ sources and find the difference to be $\ll 1\%$. A small number (six) of the masked sources are identified as DSFGs (V10). Galaxy clusters have a DSFG abundance only twenty times larger than the field (Bai et al., 2007), although they exceed the mass density of the field by a factor of 200 or more. Given the relative rarity of galaxy clusters, it follows that only a small fraction of DSFGs can live in galaxy clusters. Therefore, the number of clusters masked along with the DSFGs should be much smaller than six and negligible.

Both the DSFG and radio source arguments above depend implicitly on the impact of potentially masking $\lesssim 10$ clusters. As a worst-case study, Shaw et al. (2009) consider the impact of masking the most massive ten clusters in the field and show that even in this extreme case, the tSZ power spectrum at $\ell = 3000$ is reduced by only 11%. Of course, the point source masking will not select the most massive clusters and is highly unlikely to remove as many as ten clusters. Hence, the true impact will be significantly less.

After we mask sources above the 5σ threshold, DSFGs are the dominant point source population in the SPT maps. These sources have been extensively studied at

higher frequencies by SCUBA (Holland et al., 1999), MAMBO (Kreysa et al., 1998), Bolocam (Glenn et al., 1998), LABOCA (Siringo et al., 2009), AzTEC (Scott et al., 2008), SCUBA-2 (Holland et al., 2006), and BLAST (Pascale et al., 2008), and there have been some preliminary indications of their contribution in previous small-scale power spectra at 150 GHz (Reichardt et al., 2009a,b). The flux of these galaxies has been observed to scale to higher frequencies as $S_\nu \propto \nu^{2.4-3.0}$ (Knox et al., 2004; Greve et al., 2004; Reichardt et al., 2009b), with the exact frequency dependence a function of the dust emissivity, the dust temperature, and the redshift distribution of the galaxy population. This range of spectral indices corresponds to point source amplitude ratios, $\delta T_{220}^{\text{ps}}/\delta T_{150}^{\text{ps}}$, of 2.1-2.6 when expressed in units of CMB temperature. The measured spectral index of the DSFGs in the SPT maps is discussed extensively in Hall et al. (2010).

In order to obtain an unbiased estimate of the SZ power spectrum, it is essential that we take these sources into account in our fits and modeling. After masking the bright point sources, we significantly detect a Poisson distributed power at 150 GHz of $C_\ell^{\text{ps}} = 7.1 \pm 0.5 \times 10^{-6} \mu\text{K}^2$ (H09). This unclustered point source power climbs with increasing ℓ to be comparable to the SZ effect by $\ell = 2500 - 3000$, and is the dominant astronomical signal in the maps at arcminute scales.

The distribution of DSFGs on the sky is also expected to be clustered, resulting in a significant increase in power at $\ell \lesssim 3000$. BLAST recently detected this clustered term for DSFGs at 600 - 1200 GHz (500 - 250 μm) (Viero et al., 2009). Extrapolating the measured clustering to 150 GHz, we expect the clustered contribution to be comparable to the tSZ effect. Hall et al. (2010) have analyzed the SPT 150, 220, and 150×220 GHz bandpowers presented in §7.5, and have also found that the amplitude of the clustered component is indeed comparable to the tSZ amplitude. Discriminating between clustered DSFGs and the tSZ effect would be extremely difficult for a single-frequency instrument as the angular dependencies are very similar. However, the two frequencies used in this analysis allow the spectral separation of the these two astronomical signals.

8.2 DSFG-subtracted Bandpowers

Our immediate goal is to measure the amplitude of the tSZ power spectrum. However, several signals in these maps have similar angular power spectrum shapes, and the single-frequency maps only constrain the sum of the power from these sources. For instance, the 150 GHz data effectively constrain the sum of the tSZ, kSZ, and clustered DSFG power.

As discussed earlier, each of these components has a distinct frequency dependence so a linear combination of SPT's two frequency bands can be constructed (following Section 7.3.4) to minimize any one of them. Hall et al. (2010) find significant evidence for a clustered DSFG power contribution to the single-frequency bandpowers listed in Table 7.1 with an amplitude comparable to that of the tSZ effect. We expect the kSZ effect to be smaller than the tSZ effect at 150 GHz on theoretical grounds. Additionally, due to the frequency dependencies of the components, removing the kSZ effect would inflate the relative contribution of clustered DSFGs with respect to the tSZ effect. Therefore, we choose to remove DSFGs from the SPT bandpowers.

For a mean DSFG spectral index, α , the proper weighting ratio, x , to apply to the 220 GHz spectrum for DSFG removal would be:

$$x = \frac{S_{150}/(\frac{dB_\nu}{dT_{\text{CMB}}}|_{150})}{S_{220}/(\frac{dB_\nu}{dT_{\text{CMB}}}|_{220})} = (150/220)^\alpha \frac{\frac{dB_\nu}{dT_{\text{CMB}}}|_{220}}{\frac{dB_\nu}{dT_{\text{CMB}}}|_{150}}. \quad (8.1)$$

The spectrum in Table 8.1 and Figures 8.2 and 8.1 is produced with a weighting factor of $x = 0.325$ corresponding to a mean spectral index of $\alpha = 3.6$. The contribution from DSFGs can be completely removed only if every galaxy has the same spectral index. However, the comparative closeness of SPT's two frequency bands ensures that power leakage into the difference maps remains small despite some expected scatter in the spectral index of the dusty galaxies. In Section 8.2.1, we motivate this choice of x and discuss predictions for the residual point source power.

The power spectra for the two SPT bands were presented in Section 7.5. We combine these multi-frequency bandpowers as described in Section 7.3.4 to produce the 'DSFG-subtracted' bandpowers listed in Table 8.1. This power spectrum is compared to the results from WMAP5, ACBAR, and QUaD in Figure 8.2. The best-fit model to this combined data set including the primary CMB, kSZ, tSZ, and a Poisson point source contribution is shown for reference. The primary CMB anisotropy is estimated for a spatially-flat, Λ CDM model, which includes gravitational lensing.

We assume that there is no tSZ contribution to the 220 GHz data as the 220 GHz band is designed to be centered on the SZ null. Fourier transform spectroscopy measurements of the 220 GHz band pass confirm that the tSZ amplitude in the 220 GHz band will be $\lesssim 5\%$ of the 150 GHz amplitude. Any error incurred by subtracting roughly one third of the 220 GHz amplitude from the 150 GHz data would be less than 3%, far below the present $\sim 40\%$ statistical uncertainty on A_{SZ} (see Table 3).

Table 8.1: DSFG-subtracted Bandpowers

ℓ range	l_{eff}	q (μK^2)	σ (μK^2)
2001 - 2200	2058	221.3	16.9
2201 - 2400	2276	130.2	11.2
2401 - 2600	2474	126.5	10.3
2601 - 2800	2677	60.2	7.7
2801 - 3000	2893	50.4	8.0
3001 - 3400	3184	36.6	5.9
3401 - 3800	3581	21.0	6.5
3801 - 4200	3992	22.9	8.4
4201 - 4600	4401	8.1	9.5
4601 - 5000	4789	3.0	11.3
5001 - 5900	5448	11.2	10.5
5901 - 6800	6359	16.0	16.2
6801 - 7700	7255	60.3	27.9
7701 - 8600	8161	32.1	42.9
8601 - 9500	9059	54.7	63.8

Band multipole range and weighted value ℓ_{eff} , bandpower q_B , and uncertainty σ_B for the DSFG-subtracted maps of the SPT field. These bandpowers correspond to a linear combination (see Section 7.3.4) of the 150, 150×220 , and 220 GHz power spectra, optimized to remove emission from DSFGs below the point source detection threshold of SPT. Point sources above 6.4 mJy at 150 GHz have been masked out in this analysis. The quoted uncertainties include instrumental noise and Gaussian sample variance of the primary CMB and point source foregrounds. The sample variance of the SZ effect, beam uncertainty and calibration uncertainty is not included. Beam and calibration uncertainties are quoted in Section 6.5.4 and Section 6.5.2 and shown in Figure 8.1.

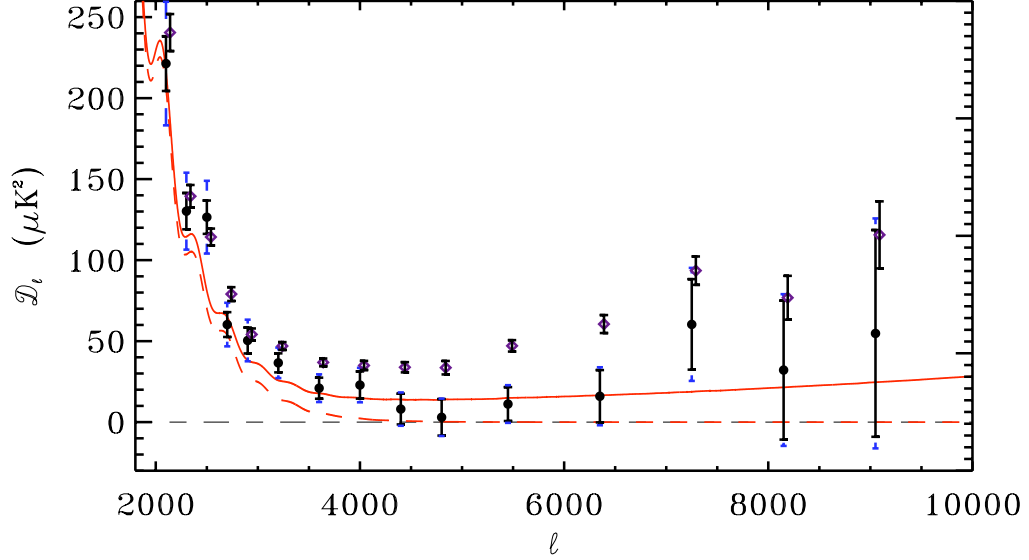


Figure 8.1: The SPT 150 GHz (**purple diamonds**) and DSFG-subtracted (**black circles**) bandpowers over-plotted on the best-fit models to the DSFG-subtracted bandpowers. The best-fit, lensed Λ CDM cosmological model for the primary CMB anisotropy is shown by the **dashed red line**, while the sum of the best-fit Λ CDM model, kSZ, tSZ and point source terms is shown by the **solid red line**. The primary CMB anisotropy alone is a poor fit to the SPT data. The uncertainties on the DSFG-subtracted bandpowers are larger for two reasons. First, the normalization convention inflates the uncertainties by a factor of $1/0.675^2$, and second, these bandpowers also include the more noisy 220 GHz data. Beam and calibration uncertainties are marked by a second blue error bar for the DSFG-subtracted bandpowers only. Note that the calibration and beam uncertainties are correlated between ℓ -bins. The 150 GHz data has been shifted to the right by $\Delta\ell = 40$ for clarity. Point sources above 6.4 mJy at 150 GHz have been masked in this analysis.

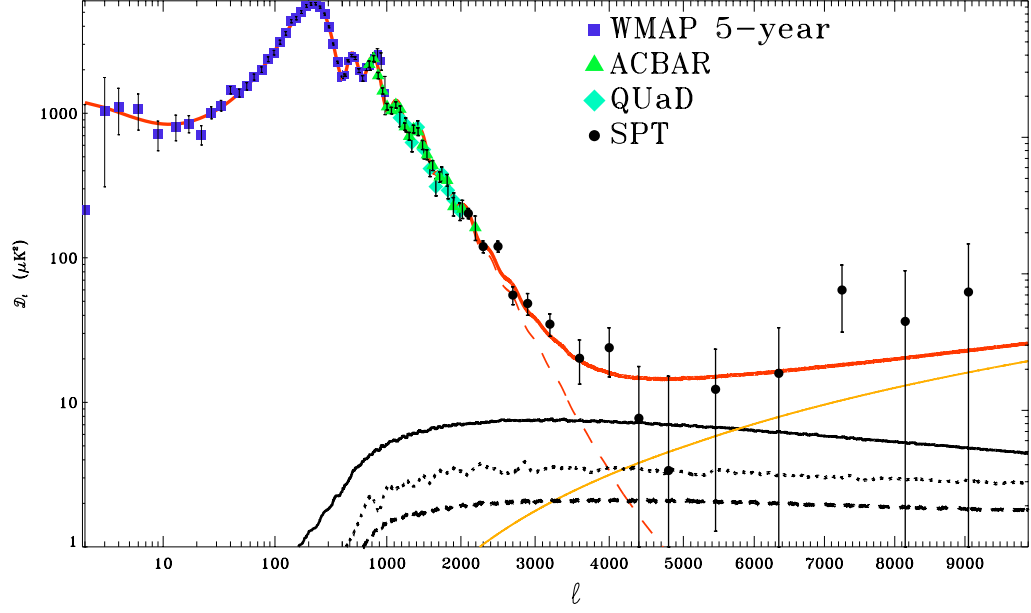


Figure 8.2: WMAP5 (**blue squares**), ACBAR (**green triangles**), QUaD (**turquoise diamonds**) and the SPT (**black circles**) DSFG-subtracted SPT bandpowers are plotted over the best-fit, lensed Λ CDM cosmological model (**dashed red line**), best-fit tSZ power spectrum (**solid black line**), homogeneous kSZ model (**dashed black line**), and residual Poisson-distributed point source contribution (**solid orange line**). The combined best-fit model is shown by the **solid red line**. The plotted SPT bandpowers have been multiplied by the best-fit calibration factor of 0.92. Point sources above 6.4 mJy at 150 GHz have been masked. The patchy kSZ template is also shown for reference (**dotted black line**). The DSFG-subtracted bandpowers are normalized to preserve the amplitude of the primary CMB anisotropies.

It is important to note that the apparent tSZ power in the DSFG-subtracted bandpowers will be a factor of $1/(1-x)^2 = 2.2$ higher than at 150 GHz as the differenced bandpowers have been normalized to preserve the amplitude of the primary CMB anisotropy. In this work, we report SZ amplitudes scaled to 153 GHz which is the effective band center of the 150 GHz band for a t SZ spectrum.

8.2.1 Residual Point Source Power

The DSFG-subtracted maps have substantially less power due to both unclustered and clustered point sources as seen in Figure 8.1. However, we include a Poisson point source amplitude in all fits since we expect a small fraction of the point source power to remain in the DSFG-subtracted maps. The best-fit amplitude is consistent with zero and unphysical negative values of the Poisson point source power are allowed due to noise. To prevent this, we place a prior on the residual Poisson point source power based on what we know about the observed DSFG population from Hall et al. (2010) and radio source population from V10 and de Zotti et al. (2005).

The residual power in the subtracted map due to Poisson-distributed DSFGs, C_ℓ^{DSFG} , depends on the scatter in spectral indices σ_α , the accuracy to which the mean spectral index $\bar{\alpha}$ is known, and an estimate of the Poisson DSFG power in the 150 GHz band, $C_\ell^{\text{ps},150}$. For a given combination of these parameters, this residual DSFG power will be:

$$C_\ell^{\text{DSFG}} = C_\ell^{\text{ps},150} \times \left(\sigma_\alpha^2 [\ln(\nu_{150}/\nu_{220})]^2 + \left(1 - \frac{x}{x_{\text{true}}} \right)^2 \right), \quad (8.2)$$

where ν_{150} and ν_{220} are the effective bandcenters of the 150 and 220 GHz bands, and x and x_{true} are the assumed and true values of map weighting ratio.

We examine the residual Poisson point source amplitudes for a broad range of weighting ratios to estimate the optimal x value and the error in that estimate. For each x , we estimate the probability that $C_\ell^{\text{ps}}(x)$ is less than $C_\ell^{\text{ps}}(x = 0.325)$ using the MCMC chains described in Section 8.3. The resulting probability distribution is taken to be the likelihood function for x_{true} . As shown in Figure 8.3, there is a broad maximum for $x = 0.25$ to 0.4 and we adopt the best fit, $x = 0.325$, for the following results. In the absence of a direct measurement, we place a conservative uniform prior on the scatter in DSFG spectral indices, $0.2 < \sigma_\alpha < 0.7$, as discussed in Hall et al. (2010).

Our expectation for the residual radio contribution to the DSFG-subtracted bandpowers, C_ℓ^{radio} , is based on the de Zotti et al. (2005) radio source count model. This model is in excellent agreement on the high flux end with the SPT source counts (V10). As discussed in Hall et al. (2010), the residual radio source power after masking is expected to be a small fraction of the DSFG power at 150 GHz. However,

this small radio contribution may be comparable to the residual DSFG power in the DSFG-subtracted spectrum. The radio source power can be calculated from the integral of $S^2 dN/dS$ for the de Zotti et al. (2005) counts model from zero to the flux masking threshold of 6.4 mJy. We compute the power these sources contribute to the optimal DSFG-subtracted spectrum to be $3.9 \times 10^{-7} \mu\text{K}^2$ by assuming an average spectral index of $\alpha = -0.5$ based on the detected sources in V10. This power level is nearly identical to that predicted by the Sehgal et al. (2010) simulations. To allow for a variation in spectral index as well as uncertainty in the model normalization when extended to lower flux sources, we assign a conservative uncertainty of 50% on the predicted residual radio source power in the prior.

We combine this information to create a prior on the residual point source power in the DSFG-subtracted maps $C_\ell^{\text{ps}} = C_\ell^{\text{DSFG}} + C_\ell^{\text{radio}}$. This prior spans the range $C_\ell^{\text{ps}} \in [3.5, 9.0] \times 10^{-7} \mu\text{K}^2$ at 68% confidence and $[1.2, 13.9] \times 10^{-7} \mu\text{K}^2$ at 95% confidence. The best-fit value of the residual Poisson component before applying the prior is $C_\ell^{\text{ps}}(\text{no prior}) = (6.2 \pm 6.4) \times 10^{-7} \mu\text{K}^2$ and lies at the middle of our assumed prior range. The upper end of the 95% range is approximately 20% of the best-fit value of the Poisson point source power in the undifferenced 150 GHz bandpowers. This suggests that we have subtracted over 80% of the point source power from the 150 GHz spectrum, with the residual point source power largely from radio sources. Without this prior on the Poisson point source amplitude, the uncertainty on the A_{SZ} detection presented in the next section would increase by $\sim 50\%$.

8.2.2 Residual Clustered Point Source Power

We assume that the contribution of clustered point sources is insignificant in the DSFG-subtracted bandpowers. Using a combination of the SPT bandpowers at 150, 220, and 150×220 GHz as well as the DSFG-subtracted spectrum, Hall et al. (2010) argue that the residual clustered DSFG component in the DSFG-subtracted bandpowers is less than $0.3 \mu\text{K}^2$ at 95% confidence. This is several percent of the SZ power spectrum but negligible at the current detection significance of $\lesssim 3\sigma$. We also argue in Section 8.1 that clustered radio sources are negligible. Therefore, residual power from clustered point sources will not bias SZ constraints from the DSFG-subtracted bandpowers.

The above argument holds if the point sources are uncorrelated with the SZ signal. However, if the clustered term was completely anti-correlated with the SZ signal, the measured SZ power in Table 8.2 could underestimate the true SZ power by 38%. This is unlikely for two reasons. First, the residual after DSFG subtraction should be uncorrelated as long as the spectral dependence of cluster member DSFGs is similar to the general DSFG population. Second, as argued in Section 8.1, most DSFGs are not galaxy cluster members. We also look at the correlations between the Sehgal et al. (2010) DSFG and tSZ simulated sky maps. The Sehgal et al. (2010) DSFG model scales the number density of DSFG cluster members linearly with cluster mass; this

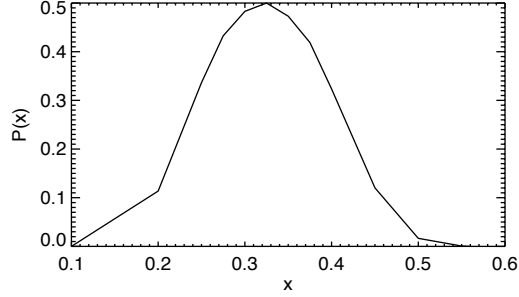


Figure 8.3: Probability that the residual point source power in the DSFG-subtracted map constructed by $\bar{m}_{150} - x\bar{m}_{220}$ is lower than the value at $x = 0.325$ as a function of x . We can interpret this as the probability that a given value of x is the true value. There is a broad maximum centered at 0.325, which corresponds to a spectral index of 3.6 between the 150 and 220 GHz bands. This is consistent with the ratio of point source power between single-frequency fits to the 150, 150 $\times$ 220, and 220 GHz bandpowers. We estimate the x which minimizes residual point source power to be 0.325 ± 0.08 .

is a substantially stronger scaling than observed (Bai et al., 2007). Therefore, estimating the cluster-DSFG correlation from the Sehgal et al. (2010) should be overly conservative. We calculate the anti-correlation coefficient between the Sehgal et al. (2010) simulated tSZ maps at 148 GHz and a linear combination of the 148 and 219 GHz simulated IR source maps with the same weighting as used for the SPT DSFG-subtracted bandpowers. We find the anti-correlation coefficient between the tSZ effect and total DSFG power to be 21% using the power measured at $\ell = 3000$. This highly conservative upper limit of 21% on the anti-correlation implies that the true SZ power is underestimated by less than 4%. Based on these arguments, we assume that correlations between SZ signals and emission from cluster member galaxies is negligible in this analysis.

Separately, one might worry about correlations between radio sources and SZ clusters. Radio sources could suppress the tSZ signal by “filling in” the tSZ decrements. However, the work by Lin et al. (2009) and Sehgal et al. (2010) shows the number of cluster-correlated radio sources is expected to be small. For instance, we can examine this correlation in the simulated tSZ and radio source maps produced by Sehgal et al. (2010). We look at the power spectrum at $\ell = 3000$ for the tSZ map, the tSZ+radio-source map, and the radio source only map after masking sources above 6.4 mJy. We find an anti-correlation coefficient of 2.3% for the two components. Given the expected radio source power level of $\sim 0.6 \mu\text{K}^2$ at $\ell = 3000$ (§8.2.1), radio source-cluster correlations should not affect the results in this work.

8.3 Markov Chain Analysis

The DSFG-subtracted bandpowers presented in Section 8.2 detect at high significance a combination of the primary CMB anisotropy, secondary SZ anisotropies, and residual point sources. In this section, we use an MCMC analysis to separate these three components and to produce an unbiased measurement of the tSZ power spectrum amplitude.

8.3.1 Elements of the MCMC Analysis

We fit the DSFG-subtracted bandpowers to a model including the lensed primary CMB anisotropy, secondary tSZ and kSZ anisotropies, and a residual Poisson point source term. We use the standard, six-parameter, spatially flat, lensed Λ CDM cosmological model to predict the primary CMB temperature anisotropy. The six parameters are the baryon density Ω_b , the density of cold dark matter Ω_c , the optical depth to recombination τ , the angular scale of the peaks Θ , the amplitude of the primordial density fluctuations $\ln[10^{10}A_s]$, and the scalar spectral index n_s . To fit the high- ℓ power, we extend the basic, six-parameter model with two additional parameters: the amplitude of a tSZ power spectrum template, A_{SZ} , and a constant, C_ℓ^{ps} , such as would be produced by a Poisson distribution of point sources on the sky. We also explore the potential impact of the kSZ effect on these parameters by using three different kSZ models.

Gravitational lensing of CMB anisotropy by large scale structure tends to increase the power at small angular scales, with the potential to influence a SZ power spectrum measurement. The calculation of lensed CMB spectra out to $\ell = 10000$ proved prohibitively expensive in computational time. We avoid this computational limitation by calculating the lensing contribution for the the best-fit cosmological model, and adding this estimated lensing contribution to the unlensed CMB power spectrum calculated at each step in the Markov chain. Given the small allowed range of Ω_m with current CMB data, we predict that using the fixed lensing contribution will misestimate the actual lensing by less than 30%. We have checked this assertion on a sampling of parameter sets drawn from the chains. The lensing contribution to the high- ℓ spectrum is $\sim 1.5 \mu\text{K}^2$ while the modeled tSZ spectrum for the same best-fit WMAP5 cosmology averages $\sim 8.6 \mu\text{K}^2$ near $\ell = 3000$ where SPT has the highest S/N on the SZ spectrum. In the differenced spectra used to derive the A_{SZ} constraints (see Section 8.2), the ratio of lensing to the tSZ effect is suppressed by a factor of $(1 - x)^2 = 0.46$. This reduction occurs because the subtracted spectrum is normalized such that the CMB power is unaffected, though the SZ power is enhanced. As a result, we expect possible lensing misestimates to introduce a negligible error of $\lesssim 3\%$ on the A_{SZ} constraints. Of course, this error will be larger for smaller values of A_{SZ} .

The tSZ template we use as a fiducial model is based on simulations by Sehgal

et al. (2010).¹ The simulations are for a WMAP5 cosmology with $\sigma_8 = 0.80$ and $\Omega_b h = 0.0312$ and an observing frequency of 148 GHz. We rescale the template to 153 GHz, the effective center frequency of the SPT 150 GHz band for signals with a tSZ spectrum.

For a limited number of cases, we also compare the results of the Sehgal numerical tSZ template to those of the Komatsu & Seljak analytic template with WMAP5 parameters² (Komatsu & Seljak, 2002) and the numerical template by Shaw et al. (2009). The primary difference between the Shaw and Sehgal simulations is the value of the energy feedback parameter in the Bode et al. (2007) intracluster gas model. The Shaw simulations have the feedback parameter reduced to 60% of the Sehgal value. Increasing the feedback parameter causes the gas distribution in clusters to ‘puff-out’ or inflate; in low mass clusters the gas may become unbound altogether. The overall effect is to reduce the predicted tSZ power spectrum, especially at small angular scales. We limit our analysis of specific tSZ models to these three models.

An independent measurement of the kSZ spectrum is outside the scope of this work, however, it is necessary to take the kSZ effect into account when inferring the tSZ amplitude from the data. We consider three kSZ cases based on two published kSZ models. The kSZ effect is assumed to be zero in the first case (“no kSZ”). As a second, intermediate case, we use the model by Sehgal et al. (2010) (referred to as “homogeneous kSZ”). This model includes kSZ contributions from a homogeneous reionization scenario, but does not include the additional kSZ power produced by patchy reionization scenarios. We take this case to be the fiducial kSZ model. We include an estimate of patchy reionization in the third kSZ case. For the patchy reionization phase, we use the “brief history” model B from Zahn et al. (2005), recalculated for WMAP-5 best fit cosmological parameters. The sum of the homogeneous kSZ model and the patchy contribution will be referred to as the “patchy kSZ” model. We do not scale these templates for different cosmological parameter sets as we expect the kSZ theoretical uncertainty to be at least as large as the cosmological dependence. Finally, we include a residual Poisson point source component in all chains. The construction of the point source prior is outlined in Section 8.2.1.

Previous CMB experiments have produced exceptional constraints on the primary CMB anisotropy, and we use the bandpowers from WMAP5 (Dunkley et al., 2009), ACBAR (Reichardt et al., 2009a), and QUaD (Brown et al., 2009) at $\ell < 2200$ in all parameter fitting. We refer to this collection along with the SPT DSFG-subtracted bandpowers as the ‘CMBall’ data set. It is important to note that the two-parameter extension to the Λ CDM model for point sources and the tSZ effect is only restricted to the SPT bandpowers. This restriction is imposed for two reasons. First, the point source contributions to each experiment may be different due to the different frequencies and flux cuts for masking sources. Arguably, the point source power is

¹http://lambda.gsfc.nasa.gov/toolbox/tb_cmbsim_ov.cfm

²http://lambda.gsfc.nasa.gov/product/map/dr3/pow_sz_spec_get.cfm

likely to be similar in the 150 GHz SPT, ACBAR, and QUaD results, but we would be unable to use frequency information to discriminate between the SZ effect and clustered DSFGs. Second, the primary CMB is dominant below $\ell \sim 3000$ and the other experiments lack sufficient statistical weight at high- ℓ to improve upon SPT's measurement of SZ effect and point source power.

Parameter estimation is performed by MCMC sampling of the full multi-dimensional parameter space using an extension of the CosmoMC package (Lewis & Bridle, 2002). We include the code extension produced by the QUaD collaboration (Brown et al., 2009) to handle uncertainties on a non-Gaussian beam in CosmoMC. CMB power spectra for a given parameter set are calculated with CAMB (Lewis et al., 2000). We use the WMAP5 likelihood code publicly available from <http://lambda.gsfc.nasa.gov>. After the burn-in period, each set of four chains is run until the largest eigenvalue of the Gelman-Rubin test is smaller than 0.0005. Wide uniform priors are used on all six parameters of the Λ CDM model. A weak prior on the age of the Universe ($t_0 \in [10, 20]$ Gyrs) and Hubble constant ($h \in [0.4, 1]$) is included in all chains, but should not affect the results. We use a uniform prior on A_{SZ} over a wide range from -1 to 10 times the value expected for $\sigma_8 = 0.80$.

8.3.2 Constraints on SZ amplitude

We fit for the normalization factor of a fixed tSZ template and, in Table 8.2, report both this template-specific normalization, A_{SZ} , and the total inferred SZ-power at $\ell = 3000$, near the multipole with maximum tSZ detection significance. This estimate of the SZ power includes both tSZ and kSZ terms, and is included to facilitate comparison with other SZ models. We expect both the thermal and the kinetic SZ spectra to vary slowly with angular multipole.

The chains are run for three different assumptions about the kSZ effect: the no kSZ, homogeneous kSZ, and patchy kSZ models described above. In each case, we produce MCMC chains with a fixed kSZ amplitude. The χ^2 of the 15 SPT bandpowers is between 16.3 and 16.4 for the best fits of the three kSZ cases considered; there is essentially no impact on the quality of the fit. The A_{SZ} and power constraints for each case are listed in Table 8.2 and plotted in Figure 8.4.

The bandpower uncertainties in Table 8.1 do not include the sample variance of the tSZ effect, and we must convolve the A_{SZ} distribution in the chains with an estimate of the sample variance to find the true A_{SZ} likelihood function. The sample variance of the tSZ effect is estimated using the simulations of Shaw et al. (2009) rerun with the same intracluster gas model parameters as Sehgal et al. (2010). The simulation consists of 300 map realizations of the size of the SPT sky patch. These are constructed from a base sample of 40 independent maps by separating the components of each map into eight redshift bins (between $0 \leq z \leq 3$) and shuffling these bins between maps to generate a larger sample. We take the spectrum of each realization and find the best-fit A_{SZ} amplitude after weighting ℓ -bins based on the SPT

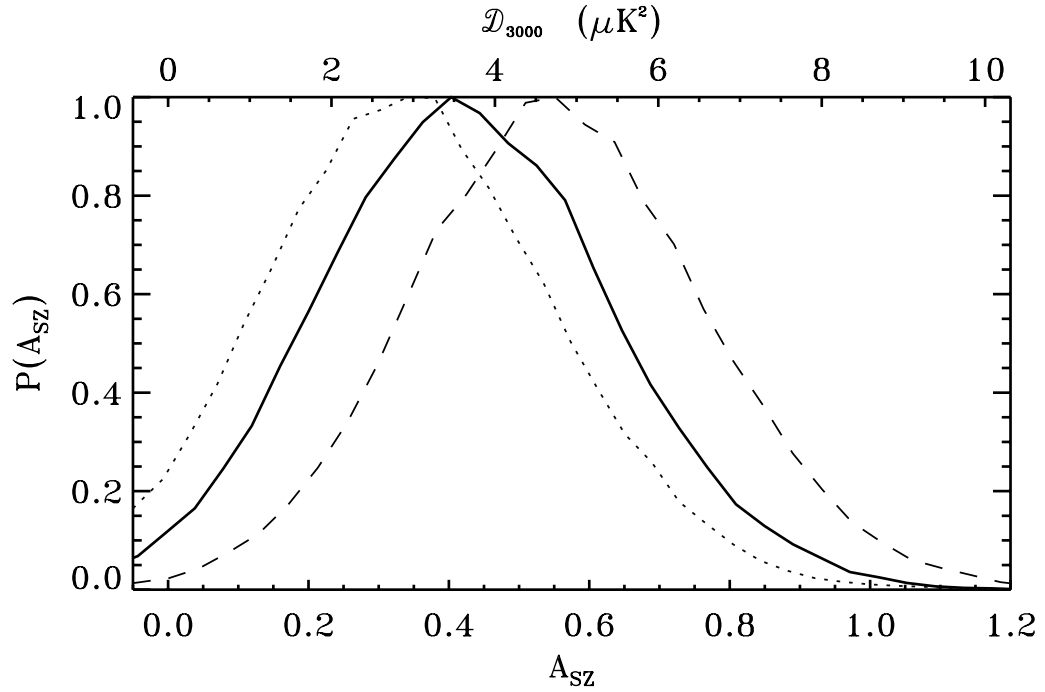


Figure 8.4: The 1D marginalized A_{SZ} constraints from the SPT DSFG-subtracted bandpowers. Three kSZ cases are considered: no kSZ effect (**dashed line**), the homogeneous kSZ model (**solid line**) and the homogeneous model plus a patchy reionization term (**dotted line**). These models are described more fully in Section 8.3.1. *Top axis:* The corresponding tSZ power at $\ell = 3000$ for reference. The no-kSZ curve (dashed line) can be interpreted as a constraint on the sum of $D_{3000}^{\text{tSZ}} + 0.46 \times D_{\ell}^{\text{kSZ}}$.

bandpower uncertainties. We use the distribution of these amplitudes to map out the likelihood function for the tSZ power (see Figure 8.5). Due to the number of independent realizations, we have limited ability to resolve the tail of the likelihood function. Fortunately, the non-Gaussianity is small for such a large sky area and the distribution of $\ln(A_{\text{SZ}})$ is well-fit by a Gaussian with a 12% width. We use this Gaussian fit as an estimate of the full likelihood surface. Small deviations from the true description of the tSZ sample variance will not impact the final results as the sample variance is small compared to both the statistical uncertainties and the assumed 50% model uncertainty. The uncertainties on A_{SZ} are essentially unchanged by the inclusion of the tSZ sample variance. The sample variance and model uncertainty are shown in Figure 8.5.

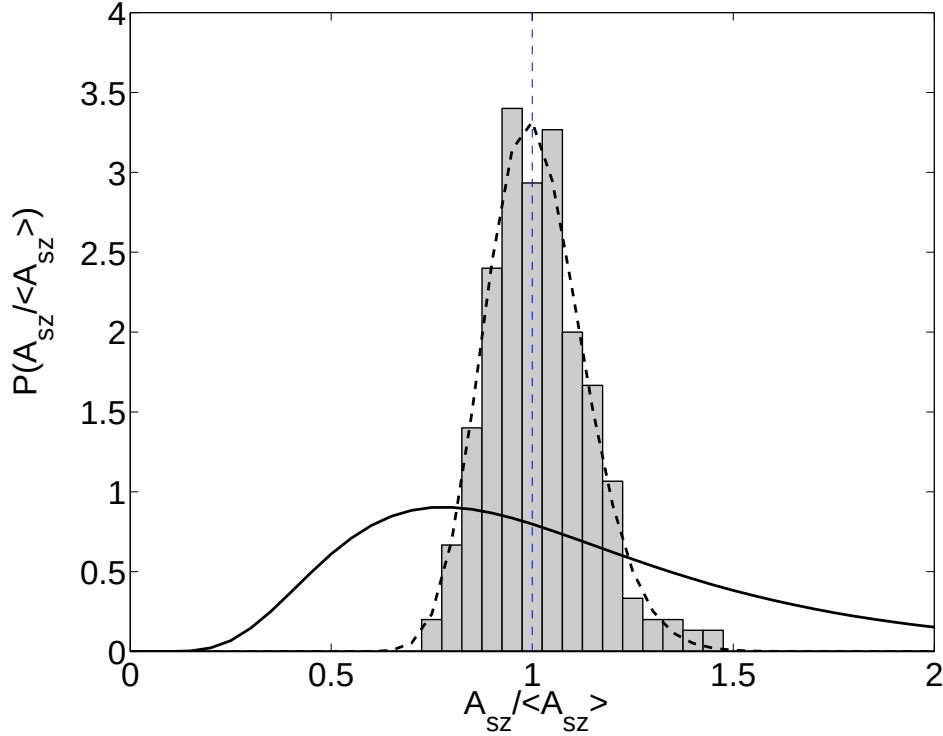


Figure 8.5: Sample variance and assumed theoretical uncertainty on the tSZ amplitude. The histogram shows the sample variance of $A_{\text{SZ}}/\langle A_{\text{SZ}} \rangle$, where $\langle A_{\text{SZ}} \rangle$ is the mean value measured over a sample of 300 simulated maps. The overlying **dashed black line** shows the lognormal fit to the distribution, i.e., a Gaussian fit to $\ln(A_{\text{SZ}})$, with $\sigma_{\ln A} = 0.12$. The **solid black line** is the assumed 50% theoretical uncertainty on A_{SZ} , which we model as a Gaussian distribution in $\ln(A_{\text{SZ}})$.

As discussed earlier, the DSFG-subtracted bandpowers are sensitive to a linear

combination of the tSZ and kSZ effects. We expect analysis of 2009 and later SPT data which include 95 GHz data to be able to separate the two SZ effects. The linear combination is not a simple sum, as the frequency-differencing used to produce the DSFG-subtracted spectrum suppresses the kSZ relative to the tSZ by a factor of $(1 - x)^2 = 0.46$. This factor is uncertain at the 15% level due to the relative calibration uncertainty between the bands. The SPT data detect the combined SZ effect at 2.6σ with $\text{tSZ} + 0.46 \times \text{kSZ} = 4.2 \pm 1.5 \mu\text{K}^2$ at $\ell = 3000$. Using this combined constraint implicitly assumes that the tSZ and kSZ templates are perfectly degenerate, which is a good assumption for the current data quality and templates used in this work.

We can compare the power detected with SPT to that reported by the CBI collaboration (Sievers et al., 2009). We use the best-fit normalization of a Komatsu & Seljak template for WMAP5 parameters to compare directly the results of the two experiments. There are sub-percent differences in the assumed values of σ_8 and $\Omega_b h$ between the template we adopted here and that used in Sievers et al. (2009), which would change the amplitude of the template by $\sim 1\%$ for an assumed scaling of $\sigma_8^7(\Omega_b h)^2$. This effect is negligible. We find the best-fit normalization of the WMAP5 Komatsu & Seljak model to be 0.37 ± 0.17 for the SPT data under the homogeneous kSZ scenario. This is 2.4σ below the best-fit CBI normalization of 3.5 ± 1.3 . The model includes the frequency dependence of the tSZ effect. The smaller SPT band-powers suggest that the CBI excess power may be produced by foregrounds with a frequency dependence falling more steeply than the SZ effect such as radio sources.

8.4 Implications of the A_{SZ} Measurement

The best-fit normalization for the fiducial tSZ spectrum, A_{SZ} , is significantly lower than unity. The cosmological parameters assumed when generating this template may be slightly different than the best-fit models, so we scale the template to match the cosmologies explored by the Markov chain. When these scalings are taken into account, the measured values of A_{SZ} are still low. The tSZ power spectrum depends on the details of how the baryon intracluster gas populates dark matter halos and on cosmology through the number density of these halos. The paucity of tSZ power may reflect an overestimate of the intracluster gas pressure by the fiducial model and so we compare the template to other SZ models. At the same time, this low value of A_{SZ} favors a shift in the derived cosmological parameters, particularly σ_8 .

Even a $\sim 2.5\sigma$ detection of A_{SZ} will produce cosmologically interesting constraints on σ_8 , as we expect A_{SZ} to scale strongly with σ_8 and less strongly with the baryon density. The scaling is approximately $\sigma_8^\gamma(\Omega_b h)^2$ where $7 < \gamma < 9$, depending on the exact cosmology (Komatsu & Kitayama, 1999; Komatsu & Seljak, 2002). We explore the σ_8 dependence under the Press-Schechter halo model and find that this relationship steepens with the currently favored lower values of σ_8 . Sampling cosmological

parameter values from the WMAP5 MCMC chains (which properly treats degeneracies between σ_8 and other parameters), we find the modeled amplitude of the tSZ power spectrum varies approximately as σ_8^{11} . Constraining the amplitude of the tSZ effect offers an independent measurement of σ_8 that can be compared to measurements based on primary CMB anisotropy or large scale structure. Such comparisons test our understanding of the physical processes involved in structure formation.

For each point in the MCMC chain, we calculate the predicted A_{SZ} value from the six basic cosmological parameters. For this calculation, we use the mass function of Jenkins et al. (2001) to determine the abundance of galaxy clusters of a given mass. We then use the mass-concentration relation of Duffy et al. (2008) to determine the dark matter halo properties, and the gas model used in Komatsu & Seljak (2002) to estimate the tSZ signal for each halo according to its mass. In order to convert this analytic tSZ spectrum into an amplitude, we take an ℓ -weighted average designed to match the relative weights each multipole receives in the real tSZ fits. This amplitude is normalized to unity for the cosmological parameters assumed in the fiducial tSZ model. We also allowed the mass-concentration index to vary with cosmology by appropriate scaling of the characteristic mass M_* , but this was found to be a negligible effect within the explored range in parameters.

At each point in the chain, the measured tSZ amplitude is compared to the predicted tSZ amplitude to construct a tSZ scaling factor, $A_{\text{SZ}}/A_{\text{SZ}}^{\text{theory}}$. This procedure will account for any correlations between the measured A_{SZ} parameter and the six Λ CDM parameters in a self-consistent fashion, although we do not see evidence for such correlations in the current data. The distribution of scaling factors vs. σ_8 is illustrated by the black contours in Figure 8.4.

In general, the tSZ scaling factors are less than unity. These low scaling factors suggest either an over-estimate of the tSZ effect or lower values of σ_8 . Models predicting larger kSZ or tSZ effects lead to lower scaling factors. However, the results can not be purely explained by an over-estimate of the kSZ effect since this tension persists in the no-kSZ case. Alternatively, the scaling factors may indicate that the explored range in cosmological parameters is systematically overestimating the RMS of the mass distribution. For instance, as we see in Figure 8.4, points of the chain with lower values of σ_8 , have scaling factors closer to unity.

The first interpretation of the low tSZ scaling factor is that the Sehgal tSZ template overestimates the tSZ power spectrum. There is currently some degree of uncertainty in the expected shape and amplitude of the tSZ power spectrum as predicted by analytic models or hydrodynamical simulations. One reason for this is that cosmological simulations of the intracluster medium have only recently begun to investigate in detail the impact of radiative cooling, non-gravitational heating sources (such as AGN), and possible regulatory mechanisms between them. The computational expense of running hydrodynamical simulations with sufficient resolution to resolve small-scale processes (such as star-formation) while encompassing a large enough volume to adequately sample the halo mass function is prohibitive to a detailed analysis

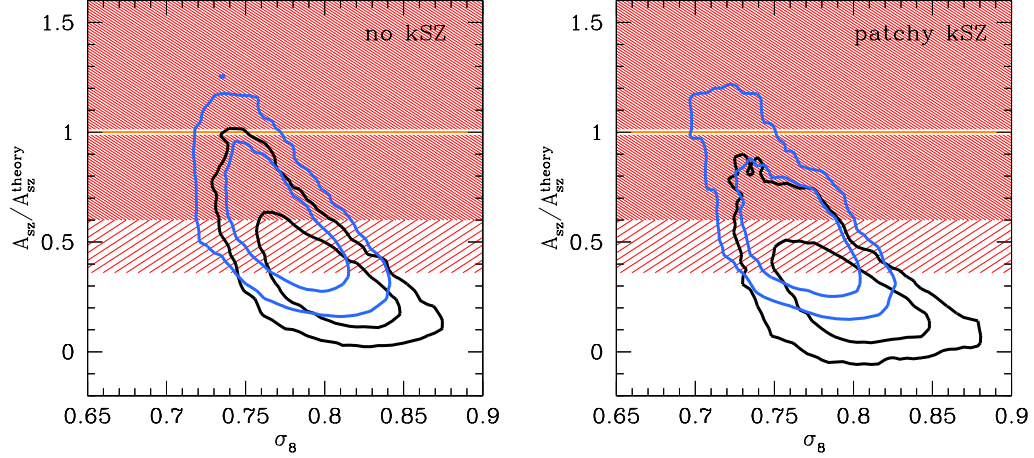


Figure 8.6: Two-dimensional likelihood contours at 68% and 95% confidence for σ_8 versus the tSZ scaling factor, $A_{\text{SZ}}/A_{\text{SZ}}^{\text{theory}}$, derived from the SPT DSFG-subtracted bandpowers. For each point in the Markov chain, the tSZ scaling factor compares the A_{SZ} value fit to the SPT data to the $A_{\text{SZ}}^{\text{theory}}$ value predicted for that point's ΛCDM model parameters (see Section 8.4). The **black contours** show the likelihood surface for the CMBall dataset. We observe no dependence between A_{SZ} and the six parameters of the ΛCDM model. The tilt towards higher scaling factors at lower σ_8 is expected since the predicted $A_{\text{SZ}}^{\text{theory}}$ depends steeply on the value of σ_8 . The black contours also do not account for the cosmic variance of A_{SZ} ; without cosmic variance or uncertainty in modeling the tSZ power, the tSZ scaling factor would be constrained to be exactly unity (**solid orange line**). The **red shaded regions** about unity illustrate the uncertainty we assume for the tSZ scaling factor due to theoretical uncertainty and cosmic variance (also see Fig. 8.5). This uncertainty is modeled as a log-normal distribution. The measured value of the tSZ scaling relation, including the theoretical uncertainty and sample variance in the model, is used to importance sample the Markov chain and obtain the likelihood surface marked by the **blue contours**. *Left panel:* Likelihood surfaces assuming no kSZ contribution. *Right panel:* Likelihood surfaces assuming the patchy kSZ model. The constraints for the homogeneous kSZ model will lie between the results for these two cases.

of the predicted SZ power spectrum. In order to accurately predict the tSZ power spectrum, it is especially important to correctly model the gas temperature and density distribution in low mass ($M < 2 \times 10^{14} h^{-1} M_{\odot}$) and high redshift ($z > 1$) clusters, which contribute significantly to the power spectrum at the angular scales where SPT is most sensitive (Komatsu & Seljak, 2002).

In Figure 8.7, we plot the tSZ power spectrum derived from several different simulations (note that all curves have been normalized to the fiducial cosmology). The thick black solid line shows the base template, obtained from maps generated by Sehgal et al. (2010). The black dot-dashed line shows the power spectrum obtained from the simulations of Shaw et al. (2009), which have a lower energy feedback parameter than the Sehgal et al. (2010) simulations. Increasing the amount of feedback energy has the effect of inflating the gas distribution and suppressing power at small angular scales. The red solid and dot-dashed lines show the power spectrum from maps constructed from a $240 h^{-1}$ Mpc box simulation run using the Eulerian hydrodynamics code CART (Kravtsov et al., 2005, Douglas Rudd, private communication) and from the MareNostrum simulation (Zahn et al., 2010) respectively. Both of these simulations were run in the adiabatic regime, i.e., no cooling, star-formation or feedback. The gas distribution is more centrally concentrated than in the Shaw or Sehgal models, producing tSZ power spectra with significantly more power at small angular scales and less at larger angular scales. The blue solid line shows the analytic template predicted by the Komatsu & Seljak (2002) halo model calculation. The histogram shows the SPT sensitivity to the thermal SZ signal in each ℓ -band (with arbitrary normalization).

The current data are in tension with even the high-feedback simulations for the CMB-derived best-fit cosmological parameter set. Even with the kSZ effect set to zero, the tSZ scaling factor is only 0.55 ± 0.21 of what is predicted for the fiducial WMAP5 cosmology. Meanwhile, the tSZ scaling factors are 0.42 ± 0.21 for the homogeneous kSZ model, and 0.34 ± 0.21 for the patchy kSZ model. The tension grows worse for the medium-feedback simulations by Shaw et al. (2009). Although the Bode et al. (2007) model is calibrated to reproduce observed x-ray scaling relations for high-mass, low-redshift clusters, it may significantly over-estimate the contribution of low-mass or high-redshift clusters (for which there are few direct x-ray observations with which to compare).

As mentioned above, an alternate interpretation of the low tSZ scaling factor is that the CMBall parameter chains without SZ constraints overestimate σ_8 . In order to constrain σ_8 based on the measurement of A_{SZ} , we need to construct the likelihood of observing A_{SZ} for a given cosmological parameter set. At a minimum, this likelihood would reflect the 12% uncertainty due to sample variance of the tSZ effect. However given the large range of tSZ predictions, we add an additional theory uncertainty in quadrature with the sample variance. The grey region in Figure 8.7 shows the 1σ region encompassed by a lognormal distribution of width $\sigma_{A_{SZ}} = 0.5$ around the fiducial model (black solid line). In the range where SPT is most sensitive

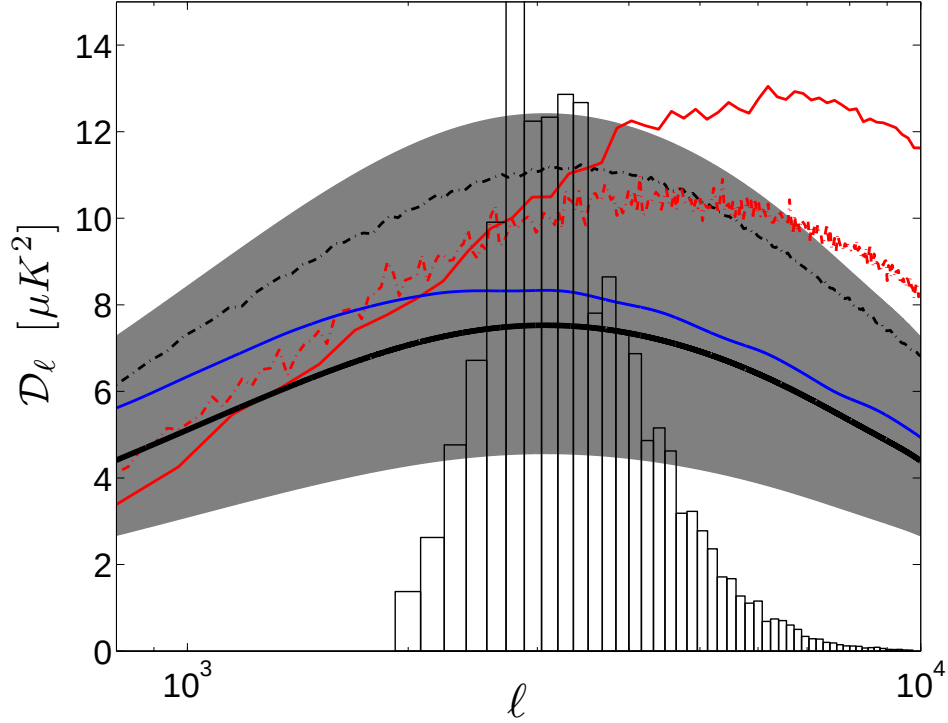


Figure 8.7: Comparison of the tSZ power spectrum (at 153 GHz) as predicted by numerical simulations and halo model calculations. Note that all curves have been re-normalized to the fiducial cosmology. The **thick, black, solid line** shows the base template, obtained from maps generated by Sehgal et al. (2010). The **black, dot-dashed line** shows the modeled spectrum obtained from maps generated by applying the semi-analytic model for intracluster gas of Bode et al. (2007) to the halos identified in an N-body lightcone simulation (as described in Shaw et al., 2009). The **red, solid line** shows the results from maps constructed from an adiabatic simulation produced using the Eulerian hydrodynamical code ART (Kravtsov et al., 2005), while the **red, dot-dashed line** shows results from maps made from the MareNostrum simulations (Zahn et al., 2010). The **blue, solid line** shows the predictions of the Komatsu & Seljak (2002) halo model calculation. The histogram shows the SPT sensitivity to the thermal SZ signal in each ℓ -band (with arbitrary normalization) The **grey band** illustrates the 68% confidence interval of our theoretical uncertainty (see Figure 8.5).

($2000 \leq \ell \leq 6000$), all the predicted curves lie within this region. In order to account for the range of predicted power spectra, we adopt a 50% theory uncertainty on the value of $\ln(A_{sz})$ in the likelihood calculation. The resulting σ_8 constraints are dominated by the theory uncertainty. With this prior, we construct a new chain from the original parameter space through importance sampling. The regions preferred by this prior are shown in Figure 8.4, as are the results of the new Markov chain.

Constraints on σ_8 with and without including the SPT A_{SZ} measurements are shown in Table 8.2 and Figure 8.8. Under the assumption of the homogeneous kSZ model and a 50% theoretical uncertainty in the amplitude of the tSZ powerspectrum, the addition of the SPT data slightly tightens the constraint on σ_8 , while reducing the central value from $\sigma_8 = 0.794 \pm 0.028$ to 0.773 ± 0.025 . The uncertainty in the resulting constraint on σ_8 is dominated by the large theoretical uncertainty in the tSZ amplitude. If instead we assume that the fiducial tSZ model is perfectly accurate and do not account for model uncertainty, the uncertainty on σ_8 is reduced by 30% and the preferred value is significantly reduced to $\sigma_8 = 0.746 \pm 0.017$ for the homogenous kSZ model. Despite the fact that the adopted template is the lowest of the tSZ models shown in Figure 8.5, the Sehgal et al. (2010) template, the value of σ_8 inferred by the SPT data using this model is lower than that favored by WMAP. Additional SPT data will soon determine if the apparent tension between σ_8 inferred from SZ and primordial CMB measurements is robust. In any case, improving our theoretical understanding of both the kSZ and tSZ power spectra is essential for fully realizing the potential of SZ power spectrum measurements to constrain cosmological parameters such as σ_8 .

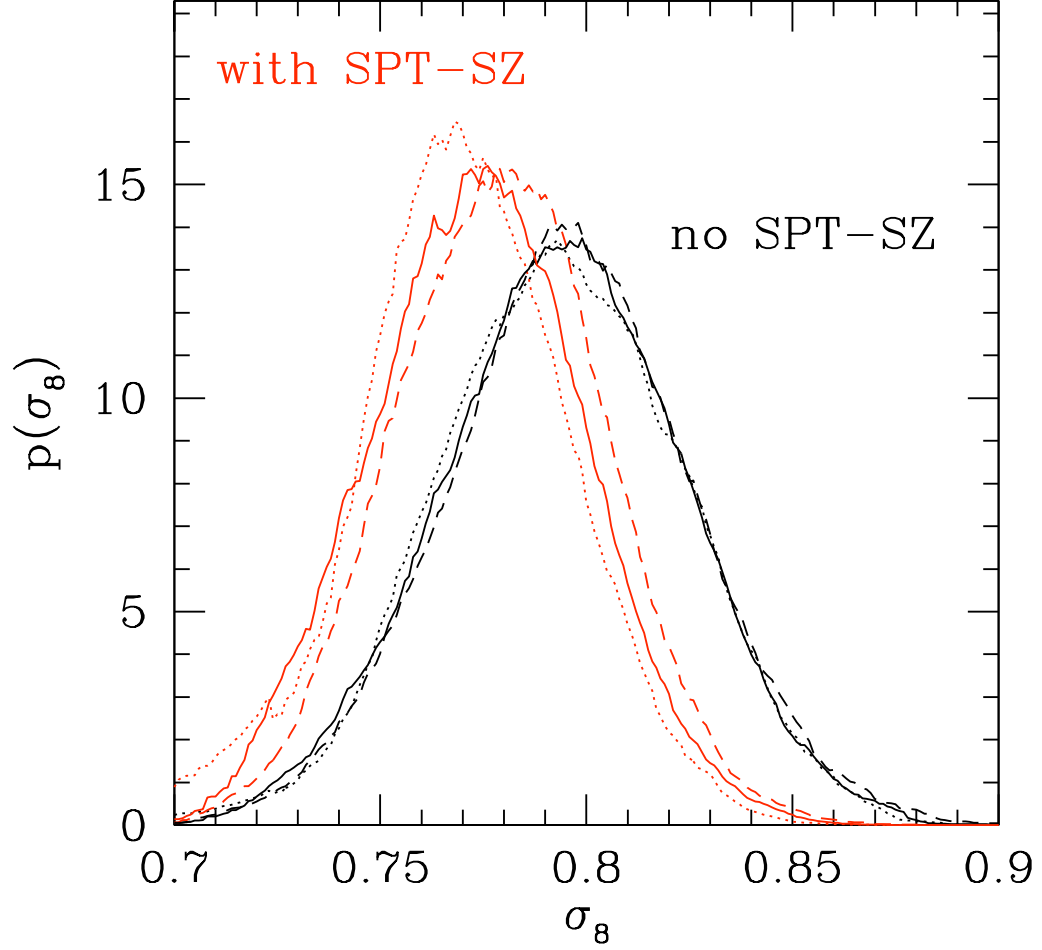


Figure 8.8: The 1D marginalized σ_8 constraints with and without including the SPT DSFG-subtracted bandpowers for three kSZ cases. The **black** lines denote the σ_8 constraints without SPT, while the **red** lines include SPT's bandpowers. Constraints with the patchy kSZ template are shown with a **solid** line. The results when including only the homogeneous kSZ model are shown with the **dashed** lines, and the results for no kSZ effect are shown with **dotted** lines. The SPT data tightens the σ_8 constraint in all three cases.

Table 8.2: Constraints on A_{SZ} and σ_8

	primary CMB	CMB + A_{SZ} :
A_{SZ} :	-	0.55 ± 0.21
A_{SZ} (w homogeneous kSZ):	-	0.42 ± 0.21
A_{SZ} (w patchy kSZ):	-	0.34 ± 0.21
SZ power at $\ell = 3000$ (tSZ + $0.46 \times$ kSZ):	-	$4.2 \pm 1.5 \mu\text{K}^2$
kSZ power at $\ell = 3000$ homogeneous kSZ		$2.0 \mu\text{K}^2$
patchy kSZ		$3.3 \mu\text{K}^2$
σ_8 (no kSZ):	0.795 ± 0.033	0.778 ± 0.024
σ_8 (w homogeneous kSZ):	0.794 ± 0.028	0.773 ± 0.025
σ_8 (w patchy kSZ):	0.788 ± 0.029	0.770 ± 0.024

The 1σ constraints on σ_8 derived from the DSFG-subtracted analysis of the SPT data, when using the simulations in Shaw et al. (2009) to estimate the non-Gaussian cosmic variance of the tSZ power spectrum. The best-fit value for the amplitude of the tSZ power spectrum is also shown, normalized to unity for a WMAP5 cosmology with $\sigma_8 = 0.8$. $A_{\text{SZ}} = 1$ corresponds to a power of $7.5 \mu\text{K}^2$ at $\ell = 3000$. Results are shown with cosmic variance added in quadrature to the statistical uncertainty, however, the A_{SZ} constraints are dominated by statistical uncertainties. Results are shown for no kSZ effect, for a homogeneous model of the kSZ effect (Sehgal et al., 2010) and the homogeneous model with an additional patchy reionization power contribution (Zahn et al., 2005). Finally, the joint constraint on the combined kSZ/tSZ power is shown under the assumption that the two templates are effectively degenerate. For reference, we also quote the power of the two kSZ models considered. The SPT data constrains the combined amplitude of the SZ contributions, which we quote at $\ell = 3000$ where the measurements have the most constraining power. The kSZ and tSZ receive different pre-factors in the frequency-differenced analysis due to their relative spectral dependence.

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Appendix A

Generalized Equations of Motion for TES Devices with Detailed Thermal Structure

Realistic TES devices often do not behave like a single lumped heat capacity. Often the physical absorber is itself a large heat capacity, which is only weakly coupled to the TES. On the other hand, the TES island may be coupled to some extended structure, with significant heat capacity, but relatively low thermal conductivity. Thus we could potentially see gradients in temperature across the bolometer the when we stimulate the TES to measure either the responsivity or the complex impedance. Thermal structures such as these can have profound consequences for the stability of TES sensors, and will make it difficult for us to interpret our measurements of the responsivity or complex impedance. For this reason it is we need to generalize the equations of motion in order to interpret our diagnostic data and to asses the stability of our sensors. Devices with complex thermal structures have been considered elsewhere (e. g. Figueroa Feliciano, 2001). The generalization I present here is scalable to an arbitrary number of lumped elements. I also present conversions between various quantities, such as the relationship between the power-to-temperature responsivity, $s_I(\omega)$ and the power-to-current responsivity $s_T(\omega)$.

We consider a device consisting of N lumped nodes. We assign an index, $1 \leq i \leq N$, to each node, and the TES itself is located at node 1. Each node has a heat capacity, C_i , and is at a temperature, T_i . This constellation of heat capacities is connected each other by a network of thermal conductances. We denote the thermal conductance between nodes i and j as

$$G_{ij} \equiv \left. \frac{\partial P_{ij}}{\partial T_i} \right|_{T_j}, \quad (\text{A.1})$$

where $P_{ij}(T_i, T_j)$ is the power flow from node i to node j . Each node may also be directly connected to a temperature bath via a thermal conductance of G_{ii} . This

scheme allows us to consider any number of possible thermal structures. From this we can generate responsivity matrices, and solve numerically for the complex conductivity $Z(\omega)$, the power-to-current responsivity $s_I(\omega)$, or the power-to-temperature responsivity $s_T(\omega)$.

If there are significant temperature gradients between nodes then the power-temperature relationship will not be strictly linear. The power P_{ij} will typically be of the form:

$$P_{ij} = K_{ij,n}(T_i^{n+1} - T_j^{n+1}) = -P_{ji} \quad (\text{A.2})$$

where $K_{ij,n}$ is a constant which depends on the material properties and geometry of the link. The exponent n is typically 3 for dielectrics or superconductors, where thermal transport is dominated by phonon conduction, while $n = 1$ for normal metals, where thermal transport is dominated by electrons. Based on Equations A.1 and A.2 the thermal conductivity between nodes is then:

$$G_{ij} = nK_{ij,n}T_i^n = \left(\frac{T_i}{T_j}\right)^n G_{ji} \quad (\text{A.3})$$

Thus it is important to note that the thermal conductivity matrix, $\mathbf{G}$, is not necessarily symmetric. That is, if $T_i \neq T_j$ then $G_{ij} \neq G_{ji}$.

We start by considering energy conservation at some node other than the TES node ($i \neq 1$). If we allow for some perturbation in heat to be applied to this node, δP_i , the equation of motion is then:

$$C_i \frac{d}{dt} \delta T_i = \delta P_i - G_{ii} \delta T_i - \sum_{i \neq j} (G_{ij} \delta T_i - G_{ji} \delta T_j) \quad (\text{A.4})$$

$$= -\delta T_i \sum_j G_{ij} + \sum_{i \neq j} G_{ji} \delta T_j + \delta P_i \quad (\text{A.5})$$

The TES node has an extra term of δP_{elec} (Equation 2.2). Thus for the TES (node 0) the energy conservation equation is:

$$C_1 \frac{d}{dt} \delta T_1 = \left(\mathcal{L} G_{\text{eff}} - \sum_j G_{1j} \right) \delta T_1 + \sum_{i \neq j} G_{j1} \delta T_j + (2 + \beta) I_0 R_0 \delta I + \delta P_1 \quad (\text{A.6})$$

For these more sophisticated thermal models we define the loopgain in terms of the effective thermal conductance, $G_{\text{eff}} = \left(\frac{dT_1}{dP_1} \right)^{-1}$:

$$\mathcal{L} \equiv \frac{\alpha P_{\text{elec}}}{G_{\text{eff}} T} \quad (\text{A.7})$$

The effective thermal conductance is equal to the generalized thermal conductance of the TES (Equation 5.4) at zero frequency $G_{\text{eff}} = G(\omega)|_{\omega=0}$. This quantity will depend

on the thermal circuit being considered and should be calculated either analytically or numerically.

The constraint from ohms law is nearly unchanged from Equation 2.7:

$$L \frac{d}{dt} \delta I = -\frac{G_{\text{eff}} \mathcal{L}}{I_0} \delta \Gamma_1 - (1 + \xi + \beta) R \delta I + \delta V \quad (\text{A.8})$$

Analogously to Equation 2.9 we express the equations of motion in matrix form:

$$\frac{d}{dt} \mathbf{v} = \mathbf{A}[\mathcal{L}] \mathbf{v} + \mathbf{p}, \quad (\text{A.9})$$

$$\mathbf{v} \equiv \begin{pmatrix} LI_0 \delta I \\ C_1 \delta \Gamma_1 \\ C_2 \delta \Gamma_2 \\ \vdots \\ C_i \delta \Gamma_i \\ \vdots \\ C_n \delta \Gamma_n \end{pmatrix}, \quad (\text{A.10})$$

$$\mathbf{p} \equiv \begin{pmatrix} I_0 \delta V \\ \delta P_1 \\ \delta P_2 \\ \vdots \\ \delta P_i \\ \vdots \\ \delta P_n \end{pmatrix} \quad (\text{A.11})$$

and lastly:

$$\mathbf{A}[\mathcal{L}] \equiv \begin{pmatrix} -\tau_e^{-1} & -\frac{\mathcal{L} G_{\text{eff}}}{C_1} & 0 & \dots & 0 & \dots & 0 \\ \frac{(2+\beta)}{1+\beta+\xi} \tau_e^{-1} & \frac{\mathcal{L} G_{\text{eff}} - \sum_k G_{1k}}{C_1} & \frac{G_{21}}{C_2} & \dots & \frac{G_{j1}}{C_j} & \dots & \frac{G_{n1}}{C_n} \\ 0 & \frac{G_{12}}{C_1} & -\frac{\sum_k G_{2k}}{C_2} & \dots & \frac{G_{j2}}{C_j} & \dots & \frac{G_{n2}}{C_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \frac{G_{1i}}{C_1} & \frac{G_{2i}}{C_2} & \dots & -\frac{\sum_k G_{jk}}{C_j} & \dots & \frac{G_{ni}}{C_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \frac{G_{1n}}{C_1} & \frac{G_{2n}}{C_2} & \dots & \frac{G_{jn}}{C_n} & \dots & -\frac{\sum_k G_{nk}}{C_n} \end{pmatrix} \quad (\text{A.12})$$

A.1 Solving for $s_T(\omega)$, $G(\omega)$, and $s_I(\omega)$

In this section we write expressions for relating three useful quantities: $s_T(\omega)$, $G(\omega)$, and $s_I(\omega)$.

In order to calculate s_T , we set $\delta P_1(t) = \Delta P(\omega)e^{i\omega t}$, while setting all the other elements of $\mathbf{p}(t) = 0$. We then solve for the change in the TES temperature, T_1 :

$$C_1 \Delta T_1(\omega) = - \left((\mathbf{A}[\mathcal{L}] - i\omega)^{-1} \mathbf{p} \right)_1 \quad (\text{A.13})$$

The the power-to-temperature responsivity $s_T(\omega)$ at loopgain $\mathcal{L}$ is then:

$$s_T(\omega) = \frac{\Delta T_1}{\Delta P_1} = - \left((\mathbf{A}[\mathcal{L}] - i\omega)^{-1} \right)_{11} / C_1 \quad (\text{A.14})$$

The generalized thermal conductance, $G(\omega) = 1/s_T(\omega)_{\mathcal{L}=0} = \frac{P_1(\omega)}{T_1(\omega)}$, is then:

$$G(\omega) = -C_1 / \left((\mathbf{A}[0] - i\omega)^{-1} \right)_{11} \quad (\text{A.15})$$

Since $\mathbf{A}[0]$ is independent of G_{eff} we can solve for G_{eff} in terms of $\mathbf{A}[0]$:

$$G_{\text{eff}} \equiv G(0) = -C_1 / \left(\mathbf{A}[0]^{-1} \right)_{11} \quad (\text{A.16})$$

Lastly one can solve for $s_I(\omega)$ by setting to $\delta P_1(t) = \Delta P(\omega)e^{i\omega t}$ as above, and solving for the change in current:

$$LI_0 \Delta I(\omega) = - \left((\mathbf{A}[\mathcal{L}] - i\omega)^{-1} \mathbf{p} \right)_0, \quad (\text{A.17})$$

and so:

$$s_I(\omega) = - \frac{\left((\mathbf{A}[\mathcal{L}] - i\omega)^{-1} \right)_{01}}{LI_0}, \quad (\text{A.18})$$

A.2 Relating $s_I(\omega)$ to $G(\omega)$

The expressions in the last section are useful for numerically solving for $G(\omega)$ or $s_I(\omega)$, given a particular thermal model. When one is trying to understand the thermal structure of a particular device, one would ideally like to measure $G(\omega)$ directly. However it is more practical to measure $s_I(\omega)$, and then infer $G(\omega)$ from this measurement. In this section we derive the formula to correct a measurement of $s_I(\omega)$, removing the effects of the loopgain thereby transforming it into a measurement of $G(\omega)$.

A.2.1 Expressing the Equations in Terms of $G(\omega)$

Solving for $s_I(\omega)$ and $G(\omega)$ is simplest if we go back to the differential equations.

If all the power in the system (both the stimulus, $\delta P_1(\omega)$, and the TES response, δ_{elec}) enters through the TES, then the temperature each node will be proportional

to the temperature fluctuation of the TES:

$$\begin{pmatrix} \delta T_1 \\ \delta T_2 \\ \delta T_3 \\ \vdots \\ \delta T_n \end{pmatrix} = \begin{pmatrix} 1 \\ g_2(\omega) \\ g_3(\omega) \\ \vdots \\ g_n(\omega) \end{pmatrix} \delta T_1 \quad (\text{A.19})$$

The proportionality coefficients, $g_i(\omega)$, are frequency dependent due to the heat capacities in the network. We will not solve for this for these coefficients here, though one could in principle write down a closed form expression for them, in terms of the elements of $\mathbf{A}$. Rather we will eventually eliminate them all in favor of $G(\omega)$. We apply these proportionalities to Equation A.4:

$$\left(\mathcal{L}G_{\text{eff}} - G_{11} + \sum_{j \neq 1} (G_{j1}g_j(\omega) - G_{1j}) - i\omega C_1 \right) T(\omega) + (2 + \beta)I_0 R_0 I(\omega) + P(\omega) = 0 \quad (\text{A.20})$$

When the $\mathcal{L} = 0$, there is no current response, and so $I(\omega) = 0$ in this case. Therefore we can identify $G(\omega)$ from Equation A.20 to be:

$$G(\omega) \equiv \frac{P(\omega)}{T(\omega)}_{\mathcal{L}=0} = G_{11} + i\omega C_1 + \sum_{j \neq 1} (G_{1j} - G_{j1}g_j(\omega)), \quad (\text{A.21})$$

and we rewrite Equation A.20 in terms of $G(\omega)$:

$$(\mathcal{L}G_{\text{eff}} - G(\omega)) T(\omega) + (2 + \beta)I_0 R_0 I(\omega) + P(\omega) = 0. \quad (\text{A.22})$$

This equation with its apparent simplicity, encapsulates all of the details of the thermal structure. It can be compared directly to Equation 2.8, where for the simple bolometer $G(\omega) = G_0 + i\omega C_0$.

We then solve for either $I(\omega)$ or $T_1(\omega)$ using Equation A.8:

$$R_0 I_0 (1 + \xi + \beta)(1 + i\omega\tau_e) I(\omega) + G_{\text{eff}} \mathcal{L} T_1(\omega) = V(\omega). \quad (\text{A.23})$$

We set $V(\omega) = 0$ in order to solve for $s_T(\omega)$ and $s_I(\omega)$ in terms of $G(\omega)$:

$$s_T(\omega) = \left(G(\omega) + \mathcal{L}G_{\text{eff}} \left(1 - \frac{2 + \beta}{1 + \xi + \beta} \frac{1}{1 + i\omega\tau_e} \right) \right)^{-1} \quad (\text{A.24})$$

$$s_I(\omega) = \frac{\mathcal{L}}{I_0 R_0} \left(\left(\mathcal{L} - \frac{G(\omega)}{G_{\text{eff}}} \right) (1 + \xi + \beta)(1 + i\omega\tau_e) - \mathcal{L}(2 + \beta) \right)^{-1} \quad (\text{A.25})$$

Appendix B

Johnson Noise in AC-biased TESs

In this appendix we calculate the effect that AC-biasing a TES has on the amplitude of the Johnson noise currents and calculate the Johnson noise contribution to the NEP for comparison to the DC biased theory.

B.1 Conventions

Phase rotations are an important consideration for AC-biased TESs. The sinusoidal current flowing through the TES has a particular phase, which may differ from the phase of the voltage bias. We represent these signals in terms of sines and cosines at the TES bias frequency ω_0 :

$$I(t) = I_I(t) \cos \omega_0 t + I_Q(t) \sin \omega_0 t \quad (\text{B.1})$$

$$V(t) = V_I(t) \cos \omega_0 t + V_Q(t) \sin \omega_0 t \quad (\text{B.2})$$

For the purposes of this appendix, the I-phase component, $I_I(t)$, is defined as the component which is in phase with the carrier current, as opposed to the quadrature—or Q-phase—component, $I_Q(t)$. Due to fluctuations in the input power or due to noise, both components may be time varying. Only the I-phase of the current has a non-zero mean, I_0 . The rest of the current is treated as small time dependent perturbations, $\delta I_{I/Q}(t)$, to either phase:

$$I(t) = (I_0 + \delta I_I(t)) \cos \omega_0 t + \delta I_Q \sin \omega_0 t. \quad (\text{B.3})$$

On the other hand since the carrier current may be out of phase of the bias voltage, the I and Q phases of the voltage may both have non-zero means, $V_{0,I}$ and $V_{0,Q}$:

$$V(t) = (V_{0,I} + \delta V_I(t)) \cos \omega_0 t + (V_{0,Q} + \delta V_Q) \sin \omega_0 t. \quad (\text{B.4})$$

As with all of the perturbations in Chapter 2, the voltage and current perturbations can be harmonically expanded:

$$\delta V(t) = \int d\omega e^{i\omega t} [V_I(\omega) \cos \omega_0 t + V_Q(\omega) \sin \omega_0 t] \quad (\text{B.5})$$

$$\delta I(t) = \int d\omega e^{i\omega t} [I_I(\omega) \cos \omega_0 t + I_Q(\omega) \sin \omega_0 t] \quad (\text{B.6})$$

Though the full current oscillates at RF-frequencies, these modulations are relatively slow and occur at audio-frequencies, and without a subscript, ω represents the frequency of the *modulation*. Any complete study of the noise or sensitivity should involve both phases of the current. As we shall see, radio-frequency phase shifts or reactances in series with the TES can rotate signal into the Q-phase.

B.1.1 Power-to-Current Sensitivity, Noise PSDs, and NEPs

In the most general case, radio-frequency power coupling to the bolometer can generate current in both the I and Q phases. Thus the power-to-current sensitivity for an AC-biased sensor is a vector quantity: $\mathbf{s}_I(\omega)$:

$$\mathbf{s}_I(\omega) \equiv \begin{pmatrix} \frac{\delta I_I(\omega)}{\delta P_{\text{ext}}(\omega)} \\ \frac{\delta I_Q(\omega)}{\delta P_{\text{ext}}(\omega)} \end{pmatrix} \quad (\text{B.7})$$

For an external power-fluctuation, $P(\omega)$, the observed signal current is then:

$$\begin{pmatrix} I_I(\omega) \\ I_Q(\omega) \end{pmatrix} = \begin{pmatrix} s_{I,I}(\omega) \\ s_{I,Q}(\omega) \end{pmatrix} P(\omega) \quad (\text{B.8})$$

In general, $s_{I,I}(\omega)$ and $s_{I,Q}(\omega)$ are complex quantities. The phase of either component represents a time delay between the incoming power and the current modulation. For instance, in the case of a spider web absorber the observed current is delayed by a time-constant τ_{opt} , leading to a 90° phase shift between the power $P(t)$ and the modulations $I_I(t)$ or $I_Q(t)$. In general, the phases of these two sensitivity components, $\arg(s_{I,I}(\omega))$ and $\arg(s_{I,Q}(\omega))$, need not be the same, which could in principle make optimal demodulation challenging.

The task of the demodulator is to extract the incoming power which is encoded in the modulations $\delta I_I(t)$ and $\delta I_Q(t)$, and convert it to a voltage, $V_{\text{demod}}(t)$. In the frequency-domain, even the most general demodulator can be represented as an impedance vector, $\mathbf{Z}_{\text{demod}}(\omega)$:

$$V_{\text{demod}}(\omega) = \mathbf{Z}_{\text{demod}}^*(\omega) \cdot \begin{pmatrix} I_I(\omega) \\ I_Q(\omega) \end{pmatrix} \quad (\text{B.9})$$

$$= (\mathbf{Z}_{\text{demod}}^*(\omega) \cdot \mathbf{s}_I(\omega)) P(\omega). \quad (\text{B.10})$$

The optimal demodulator is one where, for some constant C :

$$\mathbf{Z}_{\text{demod}}(\omega) = C \frac{\mathbf{s}_I(\omega)}{|\mathbf{s}_I(\omega)|^2}, \quad (\text{B.11})$$

in which case, by combining Equations B.9 and B.11, we see that $V_{\text{demod}}(\omega) = CP(\omega)$.

The noise in the current I- and Q-phases need not have the same PSD, nor, *a priori*, should they be uncorrelated. Thus the noise variance becomes a noise covariance *matrix*, $\mathbf{S}_I(\omega)$. We represent a given pair of noise realizations by the random variables $i_I(t)$ and $i_Q(t)$, or by their fourier transforms, $i_I(\omega)$ and $i_Q(\omega)$. $\mathbf{S}_I(\omega)$ is then defined by the relation:

$$\mathbf{ii}^\dagger = \begin{pmatrix} \langle i_I(\omega) i_I^*(\omega') \rangle & \langle i_I(\omega) i_Q^*(\omega') \rangle \\ \langle i_Q(\omega) i_I^*(\omega') \rangle & \langle i_Q(\omega) i_Q^*(\omega') \rangle \end{pmatrix} \equiv 2\pi\delta(\omega - \omega') \mathbf{S}_I(\omega). \quad (\text{B.12})$$

Given the complex nature of i_I and i_Q , this matrix is complex and hermitian, meaning that at each frequency, ω , it has two real eigenvalues: $S_{I,(+)}(\omega)$ and $S_{I,(-)}(\omega)$. Broadly speaking, these correspond to the PSDs for two different orthogonal eigenmodes, $\mathbf{I}_{(+)}$ and $\mathbf{I}_{(-)}$. As we shall see, these modes have different noise under electrothermal feedback.

In order to translate this to an NEP, it is easiest to propagate this current to the demodulator output, and then refer it back to a power at the input, as if it were an actual signal. At the output of the demodulator, the noise signal appears as a noise voltage, $v_{\text{demod}}(t)$, with a PSD, $\mathbf{S}_{V,\text{demod}}$. Based on Equations B.9 and B.12 $\mathbf{S}_{V,\text{demod}}$ is:

$$\mathbf{S}_{V,\text{demod}}(\omega) \equiv \frac{\langle v_{\text{demod}}(\omega) v_{\text{demod}}^*(\omega') \rangle}{2\pi\delta(\omega - \omega')} = \mathbf{Z}_{\text{demod}}^\dagger(\omega) \mathbf{S}_I(\omega) \mathbf{Z}_{\text{demod}}(\omega) \quad (\text{B.13})$$

This voltage at the demodulator output, if referred back as a power by Equation B.10 corresponds to an NEP of:

$$\begin{aligned} \text{NEP}^2 &= \frac{\mathbf{S}_{V,\text{demod}}}{|\mathbf{Z}_{\text{demod}}^*(\omega) \cdot \mathbf{s}_I(\omega)|^2} \\ &= \frac{\mathbf{Z}_{\text{demod}}^\dagger(\omega) \mathbf{S}_I(\omega) \mathbf{Z}_{\text{demod}}(\omega)}{|\mathbf{Z}_{\text{demod}}^*(\omega) \cdot \mathbf{s}_I(\omega)|^2} \end{aligned} \quad (\text{B.14})$$

The choice of $\mathbf{Z}_{\text{demod}}$ definitely plays a significant role in determining the NEP. For instance, if $\mathbf{Z}_{\text{demod}}$ is chosen such that it is nearly orthogonal to $\mathbf{s}_I$, then the demodulator gets no signal, but plenty of noise, and so the NEP diverges as $|\mathbf{Z}_{\text{demod}}^* \mathbf{s}_I|^{-2}$. Also as discussed below Equation B.12, there are also two noise eigenmodes for $\mathbf{S}_I$. Depending on how strongly $\mathbf{Z}_{\text{demod}}$ couples to the high-noise mode, this may be an additional noise penalty. Fortunately, when the detectors are biased at the ideal bias frequency, the low noise mode corresponds to the $\mathbf{s}_I(\omega)$, and so in this case the best choice is obviously to set $\mathbf{Z}_{\text{demod}}$ to be parallel to $\mathbf{s}_I$.

B.2 Equations of Motion

In this Section, I turn to the problem of actually calculating $\mathbf{s}_I$ and $\mathbf{S}_I$ for an AC-biased TES. As in Irwin & Hilton (2005), this process starts by writing down the equations of motion for the perturbations $I_I(\omega)$, $I_Q(\omega)$, and $T(\omega)$.

The TES is assumed to be LC-coupled to the voltage bias source by an LC resonator, with a resonant frequency: $\omega_{LC} \equiv 1/\sqrt{LC}$. For this scenario Ohm's law reads:

$$V(t) = I(t)(R_L + R(t)) + L \frac{d}{dt} I(t) + \frac{1}{C} \int dt I(t) \quad (\text{B.15})$$

$$= I(t)(R_L + R(t)) + L \left(\frac{d}{dt} I(t) + \omega_{LC}^2 \int dt I(t) \right) \quad (\text{B.16})$$

As in Chapter 2, R_L is the load resistance which is in series with the bolometer.

The detector is biased at a frequency ω_0 , which in general may be slightly offset from the resonant frequency by $\delta\omega = \omega_0 - \omega_{LC} \ll \omega_0$. We then expand Equation B.16 around ω_0 :

$$V(t) = I(t)(R_L + R(t)) + L \left(\frac{d}{dt} I(t) + \omega_0^2 \left[1 - 2 \frac{\delta\omega}{\omega_0} + \left(\frac{\delta\omega}{\omega_0} \right)^2 \right] \int dt I(t) \right) \quad (\text{B.17})$$

B.2.1 Steady-state solution

Before we can solve for the perturbations, we need to solve for the relationship between the steady-state current, I_0 , and the two phases of the bias voltage, $V_{0,I}$ and $V_{0,Q}$. In the absence of resistance fluctuations or voltage perturbations, the derivative and integral of the current are then trivial to calculate from Equation B.1. We set $I_I(t) = I_{0,I} = I_0$, and $I_Q(t) = I_{0,Q} = 0$ and so:

$$\frac{d}{dt} I(t) = -\omega_0^2 \int dt I(t) = I_{0,I} \cos \omega_0 t - I_{0,Q} \sin \omega_0 t. \quad (\text{B.18})$$

Therefore, to first order in $\delta\omega/\omega_0$, Equation B.17 can be rewritten as:

$$0 = [V_{0,I} - (R_0 + R_L)I_{0,I} - 2L\delta\omega I_{0,Q}] \cos \omega_0 t \quad (\text{B.19})$$

$$+ [V_{0,Q} - (R_0 + R_L)I_{0,Q} + 2L\delta\omega I_{0,I}] \sin \omega_0 t \quad (\text{B.20})$$

This equation can only be satisfied if the coefficients of $\cos \omega_0 t$ and $\sin \omega_0 t$ are set to zero, yielding the solution:

$$\begin{pmatrix} V_{0,I} \\ V_{0,Q} \end{pmatrix} = \begin{pmatrix} R_0 + R_L & 2L\delta\omega \\ -2L\delta\omega & R_0 + R_L \end{pmatrix} \begin{pmatrix} I_{0,I} \\ I_{0,Q} \end{pmatrix} \quad (\text{B.21})$$

$$= \frac{1}{\sqrt{(R_0 + R_L)^2 + (2\delta\omega L)^2}} \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} I_0 \\ 0 \end{pmatrix} \quad (\text{B.22})$$

The off-diagonal terms, $2L \delta\omega$, indicate a relative phase shift, $\phi = \tan^{-1}(\delta\omega \frac{2L}{(R_0+R_b)})$, between the voltage bias and the steady-state current, which is induced by the offset in the tuning frequency.

These rotation matrices will be pretty common in the following analysis, and highlight an important distinction between two different types of phase shifts: RF-band phase shifts, which rotate signals from the I phase to the Q phase, and audio-band phase shifts, which represent temporal delays between incoming power and observed modulations. RF-band phase shifts are induced by reactances in the signal chain and are represented by the relative magnitude of the I and Q components. Meanwhile the complex phase of $I_I(\omega)$ or $I_Q(\omega)$ represents audio-frequency phase shifts, at the *audio-band* frequency ω . These audio-band phase-shifts may likewise be created by reactances in the signal chain, though they can also be introduced by thermal delays in the bolometer.

B.2.2 Perturbations in Ohms Law

As in Chapter 2.1, our goal is to solve for the detector responsivity by perturbing equation B.17:

$$\delta V(t) \approx \delta I(t)(R_L + R_0) + I_0 \delta R(t) + L \left(\frac{d}{dt} \delta I(t) + \omega_0^2 \left[1 - 2 \frac{\delta\omega}{\omega_0} + \left(\frac{\delta\omega}{\omega_0} \right)^2 \right] \int dt \delta I(t) \right) \quad (\text{B.23})$$

The derivative of the current can be written in terms of the Fourier components:

$$\frac{d}{dt} \delta I(t) = \int d\omega e^{i\omega t} [(i\omega I_I(\omega) + \omega_0 I_Q(\omega)) \cos \omega_0 t + (-\omega_0 I_I(\omega) + i\omega I_Q(\omega)) \sin \omega_0 t], \quad (\text{B.24})$$

as can be the anti-derivative of the current:

$$\begin{aligned} \int dt \delta I(t) &= \int d\omega \frac{e^{i\omega t}}{\omega_0^2 - \omega^2} [(i\omega I_I(\omega) - \omega_0 I_Q(\omega)) \cos \omega_0 t + (\omega_0 I_I(\omega) + i\omega I_Q(\omega)) \sin \omega_0 t] \\ &\approx \frac{1}{\omega_0^2} \int d\omega e^{i\omega t} [(i\omega I_I(\omega) - \omega_0 I_Q(\omega)) \cos \omega_0 t + (\omega_0 I_I(\omega) + i\omega I_Q(\omega)) \sin \omega_0 t] \end{aligned} \quad (\text{B.25})$$

We combine Equations B.23, B.5, B.6, B.24, and B.25, neglecting second order terms in $\delta\omega/\omega_0$, ω/ω_0 :

$$\begin{aligned} 0 &= \cos(\omega_0 t) \left[\int dt e^{i\omega t} (I_I(\omega)(R_0 + R_L + 2i\omega L) + 2\delta\omega L I_Q + I_0 R(\omega) - V_I(\omega)) \right] \\ &\quad + \sin(\omega_0 t) \left[\int dt e^{i\omega t} (I_Q(\omega)(R_0 + R_L + 2i\omega L) - 2\delta\omega L I_I(\omega) - V_Q(\omega)) \right] \end{aligned} \quad (\text{B.26})$$

For this equation to be satisfied at all times, the coefficients of $\cos \omega_0 t$ and $\sin \omega_0 t$ must each be equal to zero. Thus Equation B.26 can be rewritten as a pair of linear equations:

$$i\omega 2L \begin{pmatrix} \delta I_X \\ \delta I_Y \end{pmatrix} = - \begin{pmatrix} R_0 + R_L & 2\delta\omega L \\ -2\delta\omega L & R_0 + R_L \end{pmatrix} \begin{pmatrix} \delta I_X \\ \delta I_Y \end{pmatrix} - \delta R \begin{pmatrix} I_0 \\ 0 \end{pmatrix} + \begin{pmatrix} \delta V_X \\ \delta V_Y \end{pmatrix} \quad (\text{B.27})$$

Before we move onto the equation of energy conservation we would like to express the Equation B.27 in terms of the AC-bias loopgain:

$$\mathcal{L} \equiv \frac{\alpha P_{\text{elec}}}{GT_0} = \frac{\alpha I_0^2 R_0}{2GT_0}. \quad (\text{B.28})$$

After some rearrangement, Equation B.27 becomes

$$i\omega 2L \begin{pmatrix} I_0 \delta I_X \\ I_0 \delta I_Y \end{pmatrix} = \begin{pmatrix} -(R_0 + R_L) & -2\delta\omega L \\ 2\delta\omega L & -(R_0 + R_L) \end{pmatrix} \begin{pmatrix} I_0 \delta I_X \\ I_0 \delta I_Y \end{pmatrix} - G\delta T_0 \begin{pmatrix} 2\mathcal{L} \\ 0 \end{pmatrix} + I_0 \begin{pmatrix} \delta V_X \\ \delta V_Y \end{pmatrix}. \quad (\text{B.29})$$

B.2.3 Perturbations in the Conservation of Energy Equation

As in Chapter 2, the conservation of energy equation is:

$$\frac{d}{dt} C\delta T = -G\delta T + \delta P_{\text{elec}} + \delta P_{\text{ext}}, \quad (\text{B.30})$$

where $P_{\text{elec}}(t) = (I_I(t)^2 + I_Q(t)^2) R(t)/2$, and so $P_{\text{elec}}(\omega) = I_0 I_I(\omega) R_0 + I_0^2/2 R(\omega)$. Meanwhile, by Equation B.28: $I_0^2 R(\omega) = 2\mathcal{L}GT(\omega)$. We combine Equation B.29 with Equation B.30 to obtain the three equations of motion for this simple AC-biased TES:

$$i\omega \begin{pmatrix} LI_0 I_I(\omega) \\ LI_0 I_Q(\omega) \\ CT(\omega) \end{pmatrix} = \begin{pmatrix} -\tau_e^{-1} & -\delta\omega & -\mathcal{L}\tau^{-1} \\ \delta\omega & -\tau_e^{-1} & 0 \\ \frac{2}{1+\xi}\tau_e^{-1} & 0 & (\mathcal{L}-1)\tau^{-1} \end{pmatrix} \begin{pmatrix} LI_0 I_I(\omega) \\ LI_0 I_Q(\omega) \\ CT(\omega) \end{pmatrix} + \begin{pmatrix} I_0 V_I(\omega)/2 \\ I_0 V_Q(\omega)/2 \\ P_{\text{ext}}(\omega) \end{pmatrix} \quad (\text{B.31})$$

For an AC-biased detector we define $\tau_e \equiv 2L/(R_0 + R_b)$, and as in Chapter 2, $\xi \equiv R_L/R_0$. These equations can be abbreviated as:

$$A_{\text{ext}} \begin{pmatrix} LI_0 I_I(\omega) \\ LI_0 I_Q(\omega) \\ CT(\omega) \end{pmatrix} = \begin{pmatrix} I_0 V_I(\omega)/2 \\ I_0 V_Q(\omega)/2 \\ P_{\text{ext}}(\omega) \end{pmatrix}, \quad (\text{B.32})$$

where

$$A_{\text{ext}} \equiv \begin{pmatrix} \tau_e^{-1} + i\omega & \delta\omega & \mathcal{L}\tau^{-1} \\ -\delta\omega & \tau_e^{-1} + i\omega & 0 \\ -\frac{2}{1+\xi}\tau_e^{-1} & 0 & (1-\mathcal{L})\tau^{-1} + i\omega \end{pmatrix} \quad (\text{B.33})$$

B.2.4 Voltage Fluctuations Internal to the Bolometer Island

The subscript “ext” in Equation B.32 indicates that these equations are appropriate when the voltage perturbations, $\delta V_I(t)$ and $\delta V_Q(t)$ come from voltage sources external to the bolometer, i.e. power from these fluctuations is not dissipated near the TES.

This is in contrast to an internal voltage source which is located directly on the bolometer. The most important example of an internal voltage source is the Johnson noise generated by the TES itself. The power dissipated by such a source needs to be included in P_{elec} , which is the accounting of all electrical power on the TES island. By Ohm’s law:

$$\begin{pmatrix} V_{I,\text{int}} \\ V_{Q,\text{int}} \end{pmatrix} = \begin{pmatrix} V_{I,\text{ext}} \\ V_{Q,\text{ext}} \end{pmatrix} - R_L \begin{pmatrix} I_I \\ I_Q \end{pmatrix} - 2L \frac{d}{dt} \begin{pmatrix} I_I \\ I_Q \end{pmatrix}. \quad (\text{B.34})$$

Thus, if the bias voltage is the only external voltage source, the power dissipated by internal sources is:

$$P_{\text{elec,int}} = \frac{1}{2} \begin{pmatrix} I_I(t) \\ I_Q(t) \end{pmatrix} \cdot \left[\begin{pmatrix} V_{I,0} \\ V_{Q,0} \end{pmatrix} - R_b \begin{pmatrix} I_I(t) \\ I_Q(t) \end{pmatrix} - 2L \frac{d}{dt} \begin{pmatrix} I_I(t) \\ I_Q(t) \end{pmatrix} \right] \quad (\text{B.35})$$

As before, we expand P_{elec} to first order in δI and δV , and compute the Fourier transform:

$$\delta P_{\text{elec,int}}(\omega) = ((R_0 - R_L)/2 - i\omega L) I_0 I_I(\omega) - L\delta\omega I_0 I_Q(\omega). \quad (\text{B.36})$$

We then substitute δP_{elec} into Equation B.30, and combine with Equation B.29 to get an equation analogous to Equation B.32, though with A_{ext} replaced by:

$$A_{\text{int}}(\omega) = \begin{pmatrix} \tau_e^{-1} + i\omega & \delta\omega & \mathcal{L}\tau^{-1} \\ -\delta\omega & \tau_e^{-1} + i\omega & 0 \\ -\frac{1-\xi}{1+\xi}\tau_e^{-1} + i\omega & \delta\omega & \tau^{-1} + i\omega \end{pmatrix} \quad (\text{B.37})$$

B.2.5 Comparison to DC biased systems

In the limit of perfect frequency tuning ($\delta\omega = 0$), I_Q decouples from I_I and T in both Equations B.31 and B.37. The relationship between I_I , $V_{I,\text{ext}}$, T , and P_{ext} is then controlled by the matrix equation:

$$\begin{pmatrix} \tau_e^{-1} + i\omega & \mathcal{L}\tau^{-1} \\ -\frac{2}{1+\xi}\tau_e^{-1} & (1 - \mathcal{L})\tau^{-1} + i\omega \end{pmatrix} \begin{pmatrix} LI_0 I_I(\omega) \\ CT(\omega) \end{pmatrix} = \begin{pmatrix} I_0 V_I(\omega)/2 \\ P_{\text{ext}}(\omega) \end{pmatrix} \quad (\text{B.38})$$

This equation should be compared to its DC counterpart, Equation 2.11 in Chapter 2. I should note that I have ignored β for the sake of this Appendix. Otherwise, the DC-bias and AC-bias Equations are nearly identical. This indicates the response

to electrothermal feedback, including the effective time-constants, will be nearly the same. This also indicates that AC-biased TESs should exhibit Johnson noise suppression, just as DC-biased devices do. However, the Johnson noise suppression only occurs for the component of the Johnson noise that appears in the I-phase. Q-phase Johnson noise remains at full amplitude.

The one difference is the factor of $I_0/2$ in the voltage term on the right hand side. Though it is not obvious from the way the equation is written here, this does not change the impedance of the detector, nor the amplitude of the Johnson noise when referred to a current. The reason lies in the fact that the TES operating resistance only comes into the equation through τ_e , where it appears as $R_0/2$. It should also be noted that for a detector operating at the same electrical power the amplitude of the bias current is $\sqrt{2}$ higher:

$$I_{0,\text{AC}} = \sqrt{2P_{\text{elec}}R_0} = \sqrt{2}I_{0,\text{DC}}.$$

This may have important consequences for the Johnson Noise NEP. For a given noise voltage $v(t)$ and a power signal $\delta P_{\text{ext}}(t)$, the voltage is deweighted by a factor of $2I_{0,\text{DC}}/I_{0,\text{AC}} = \sqrt{2}$ when the two perturbations become a current, though the power receives the same weight it would in a DC-biased system. This would appear to lead to a de-weighting of Johnson-noise in the system, though this factor is counteracted by the fact that when we expand a wide-band noise source as in Equation B.2, the PSD of the v_I and v_Q modes need to be twice as large to account for all the noise power in $v(t)$. Thus the relative weight between Johnson Noise and external signal is the same between AC-biased and DC-biased bolometers operating at the same resistance and electrical power.

B.3 Noise

For a non-equilibrium system, such as a self-heating TES, calculating the noise from the usual equilibrium arguments can lead to errors. As in Irwin & Hilton (2005), we use the Linear Equilibrium Ansatz (LEA) to calculate the noise observed in an AC biased TES. When applied to a TES, the LEA states the thermodynamic fluctuations in voltage and power in the steady-state are the same as they would be in equilibrium. Thus under the LEA, each resistance in the system is modeled as voltage noise source with a power spectral density of: $S_V(R) = 4kTR$, where T is the temperature of the resistor in question.

In our formulation of the equations of motion for an AC-biased TES, we represent the voltage of a noise source with a steady state resistance R as two stochastic voltage sources, $v_I(t)$ and $v_Q(t)$. The power spectral density for each source is given by $2S_V(R)$, i.e.

$$\langle v_I(\omega)v_I^*(\omega') \rangle = \langle v_Q(\omega)v_Q^*(\omega') \rangle = \frac{1}{2\pi}2S_V(R)\delta(\omega - \omega').$$

Here the factor of 2π arises from our choice of Fourier transform normalization conditions for $v_I(\omega)$ and $v_Q(\omega)$ in Equation B.5. The additional factor of 2 takes into account the fact that power from both sidebands gets folded into one value in this calculation. These sources are independent: $\langle v_I(\omega)v_Q(\omega) \rangle = 0$.

The next step is to calculate the noise in either phase of the current, which can be expressed in terms of the complex impedance matrices $\mathbf{Z}_{\text{int,ext}}$ which can relate either phase of the current to either phase of an interior or exterior voltage perturbation:

$$\begin{pmatrix} \delta V_{I,\text{int}} \\ \delta V_{Q,\text{int}} \end{pmatrix} = \mathbf{Z}_{\text{int}} \begin{pmatrix} \delta I_I \\ \delta I_Q \end{pmatrix} \text{ and } \begin{pmatrix} \delta V_{I,\text{ext}} \\ \delta V_{Q,\text{ext}} \end{pmatrix} = \mathbf{Z}_{\text{ext}} \begin{pmatrix} \delta I_I \\ \delta I_Q \end{pmatrix} \quad (\text{B.39})$$

If we assume no perturbations in the radiative power, δP_{ext} , then we can solve and substitute for the temperature terms in Equation B.32:

$$\mathbf{Z}_{\text{ext}}(\omega) = \begin{pmatrix} \left(1 + \frac{2}{1+\xi} \frac{\mathcal{L}}{(1-\mathcal{L})+i\omega\tau}\right) + i\omega\tau_e & \delta\omega \tau_e \\ -\delta\omega \tau_e & 1 + i\omega\tau_e \end{pmatrix} \times (R_0 + R_L) \quad (\text{B.40})$$

$$= \begin{pmatrix} Z_{\text{TES}}(\omega) + R_L + i2L\omega & 2L\delta\omega \\ -2L\delta\omega & R_0 + R_L + i2L\omega \end{pmatrix} \quad (\text{B.41})$$

Likewise, I do the same for the TES impedance with respect to internal voltage fluctuations using Equation B.37:

$$\mathbf{Z}_{\text{int}} = \begin{pmatrix} \left(1 + \frac{1-\xi}{1+\xi} \frac{\mathcal{L}}{1+i\omega\tau}\right) + i\omega\tau_e \left(1 - \frac{\mathcal{L}}{1+i\omega\tau}\right) & \delta\omega \tau_e \left(1 - \frac{\mathcal{L}}{1+i\omega\tau}\right) \\ -\delta\omega \tau_e & 1 + i\omega\tau_e \end{pmatrix} \times (R_0 + R_L) \quad (\text{B.42})$$

The associated Johnson noise currents can then be solved from the voltage by multiplying the voltages by the admittance matrix $\mathbf{Y}_{\text{int, ext}}(\omega) \equiv \mathbf{Z}_{\text{int, ext}}^{-1}(\omega)$. The noise density matrix for the currents is then:

$$\begin{aligned} \mathbf{S}_{I,\text{int/ext}} &= \frac{\mathbf{i}(\omega)\mathbf{i}^\dagger(\omega')}{2\pi\delta(\omega - \omega')} \\ &= \frac{\mathbf{Y}_{\text{int/ext}}(\omega)\mathbf{v}(\omega)\mathbf{v}^\dagger(\omega')\mathbf{Y}_{\text{int/ext}}^\dagger(\omega')}{2\pi\delta(\omega - \omega')} \\ &= \mathbf{Y}_{\text{int/ext}}(\omega)\mathbf{Y}_{\text{int/ext}}^\dagger(\omega)2S_V(R) \\ &= 8kTR \times (\mathbf{Z}_{\text{int/ext}}^\dagger \mathbf{Z}_{\text{int/ext}})^{-1} \end{aligned} \quad (\text{B.43})$$

In Figures B.2 and B.1, the Johnson Noise amplitude is numerically plotted for the both eigenmodes of $\mathbf{S}_{I,\text{ext}}$ and $\mathbf{S}_{I,\text{int}}$.

B.4 Responsivity and NEP

By setting $V_I(\omega)$ and $V_Q(\omega)$ to zero in Equation B.32, one can also solve for the power-to-current responsivity $s_I(\omega)$, in terms of either $\mathbf{Y}_{\text{ext}}$ or $\mathbf{Y}_{\text{int}}$:

$$\mathbf{s}_I(\omega) = -\frac{2}{I_0} \frac{\mathcal{L}}{1 - \mathcal{L} + i\omega\tau} \mathbf{Y}_{\text{ext}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (\text{B.44})$$

$$= -\frac{2(R_0 + R_L)}{I_0 |\mathbf{Z}_{\text{ext}}|} \frac{\mathcal{L}}{1 - \mathcal{L} + i\omega\tau} \begin{pmatrix} 1 + i\omega\tau_e \\ \delta\omega\tau_e \end{pmatrix} \quad (\text{B.45})$$

$$(\text{B.46})$$

For $\delta\omega = 0$, the Q-phase responsivity is zero, as expected since for perfect frequency tuning, since the Q-phase decouples from the I-phase and from the thermal circuit (Section B.2.5). In this simplest case ($\xi = 0$, $\beta = 0$, and $\delta\omega = 0$), the responsivity becomes:

$$\mathbf{s}_I(\omega) = -\frac{2}{I_0 R_0} \frac{\mathcal{L}}{(1 + \mathcal{L}) + i\omega\tau} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad (\text{B.47})$$

The factor of 2 is due to the fact that the signal current and I_0 are both measured in units of peak amplitude. Some other fMUX documents report I_0 and $\mathbf{s}_I$ in r.m.s. units, in which case this factor of 2 is unneeded.¹

To conclude this Appendix, I calculate the amplitude of the Johnson Noise NEP for low frequency signals for comparison to Equation 2.31. To keep this calculation simple, I will only consider the “internal noise” from the TES itself, and only calculate it at low frequencies, $\omega \approx 0$. I will also only consider the case where $\delta\omega = 0$, $\xi = 0$, and $\beta = 0$. We will use an optimal demodulator (Equation B.11):

$$\mathbf{Z}_{\text{demod}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (\text{B.48})$$

The low frequency Johnson noise PSD, combining Equations B.42 and B.43, is:

$$\mathbf{S}_{i,\text{int}}(0) = \begin{pmatrix} (1 + \mathcal{L})^{-2} & 0 \\ 0 & 1 \end{pmatrix} \frac{8kT}{R_0}. \quad (\text{B.49})$$

Thus combining Equations B.14, B.47, B.48 and B.49 the NEP is

$$\text{NEP} = \frac{I_0^2 R_0}{4} \frac{8kT}{\mathcal{L}^2} = \frac{4kTP_{\text{elec}}}{\mathcal{L}^2}, \quad (\text{B.50})$$

which is the same as for a detector DC-biased at the same NEP, as predicted in Section B.2.5.

¹It should be pointed out that reporting these quantities in r.m.s. units means that almost every other equation in this Appendix would need to change by $\sqrt{2}$. So keeping things in amplitude units has its advantages.

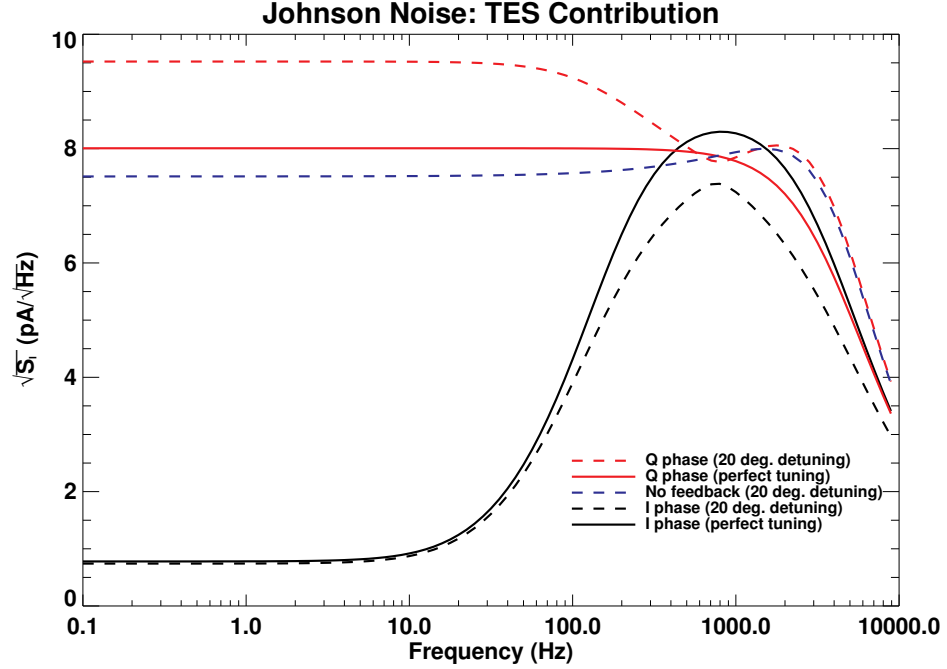


Figure B.1: Comparison of Johnson Noise modes for an active TES. When the TES is perfectly tuned to its resonant frequency, the **black** and **solid red** traces show the High- and low-noise eigenmodes of the noise covariance matrix (Equation B.43). In this calculation the stray load resistor has a value of $30 \text{ m}\Omega$ and is in series with $0.8 \text{ }\Omega$ TES. The temperature of the TES is assumed to be 0.5K , and the loopgain of the TES is $\mathcal{L} = 10$. The **solid** traces represent the noise when the system is perfectly tuned to the resonant frequency. In the perfectly tuned case the low- and high- noise modes correspond to the I- and Q-phase modes respectively. Thus the high-noise eigenmode is exactly the noise one would expect for a passive system ($\mathcal{L} = 0$). Meanwhile the noise is suppressed in the low noise mode. Detuning the system such that the phase shift $\phi = \tan^{-1}(\delta\omega \tau_e) = 20^\circ$ leads to an even more dramatic difference between the two modes, as shown by the **dotted** traces. Note that in this case the high-noise mode is actually higher than the non-feedback noise, illustrated by the **blue** trace.

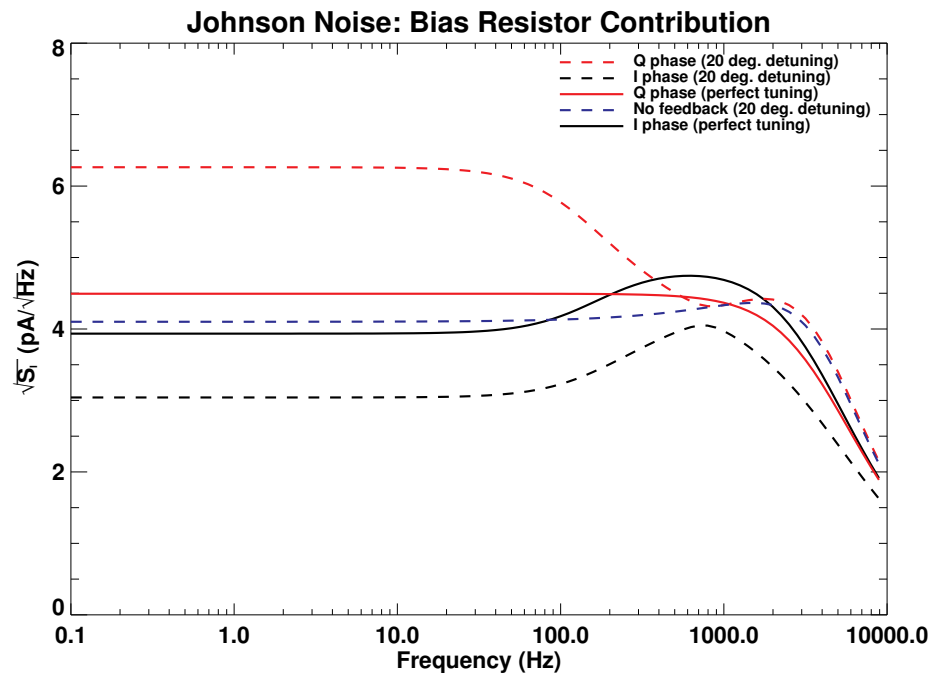


Figure B.2: Comparison of Johnson Noise modes for small load $30 \text{ m}\Omega$ resistor in series with the TES. The legend is the same in this plot as it was in Figure B.1.

Appendix C

Bandpower Covariance Matrix Estimation

The bandpower covariance matrix includes both signal and noise contributions. The signal covariance is calculated from simulations. The noise covariance is estimated from the data. We calculate the variance of the mean power spectrum using the variance of cross-spectra between independent real maps. With 300 independent observations of the same field, these maps are sufficient to generate an accurate estimate of the covariance matrix. Several details of our approach are motivated by the analytical treatments of Tristram et al. (2005) and Polenta et al. (2005), and we first review the analytic estimate of the covariance matrix before discussing the estimator.

C.1 Analytical Considerations

Following Tristram et al. (2005), we represent the expected covariance between two cross spectra as Ξ :

$$\Xi_{\ell\ell'}^{AB,CD} \equiv \left\langle \left(\widehat{D}_\ell^{AB} - \langle \widehat{D}_\ell^{AB} \rangle \right) \left(\widehat{D}_{\ell'}^{CD} - \langle \widehat{D}_{\ell'}^{CD} \rangle \right) \right\rangle \quad (\text{C.1})$$

$$= (K[\mathbf{W}]^{-1})_{b''b} (\langle D_{b''}^{AB} D_{b'''}^{CD} \rangle - \langle D_{b''}^{AB} \rangle \langle D_{b'''}^{CD} \rangle) (K[\mathbf{W}]^{-1})_{b''b'}. \quad (\text{C.2})$$

Our goal is to express this covariance in terms of the noise and signal in the maps, and to compute the magnitude of the diagonal elements, as well as the correlation between bandpowers. As the first step, the central term can be rewritten as,

$$\begin{aligned} \langle D_b^{AB} D_{b'}^{CD} \rangle - \langle D_b^{AB} \rangle \langle D_{b'}^{CD} \rangle &= P_{bk} P_{b'k'} \frac{1}{(2\pi)^2} \int d\theta_k d\theta_{k'} (\langle \tilde{m}_{\mathbf{k}}^A \tilde{m}_{\mathbf{k}}^{B*} \tilde{m}_{\mathbf{k}'}^C \tilde{m}_{\mathbf{k}'}^{D*} \rangle \\ &\quad - \langle \tilde{m}_{\mathbf{k}}^A \tilde{m}_{\mathbf{k}}^{B*} \rangle \langle \tilde{m}_{\mathbf{k}'}^C \tilde{m}_{\mathbf{k}'}^{D*} \rangle) \\ &= P_{bk} P_{b'k'} \frac{1}{(2\pi)^2} \int d\theta_k d\theta_{k'} (\langle \tilde{m}_{\mathbf{k}}^A \tilde{m}_{\mathbf{k}'}^C \rangle \langle \tilde{m}_{\mathbf{k}}^{B*} \tilde{m}_{\mathbf{k}'}^{D*} \rangle \\ &\quad + \langle \tilde{m}_{\mathbf{k}}^A \tilde{m}_{\mathbf{k}'}^{D*} \rangle \langle \tilde{m}_{\mathbf{k}}^{B*} \tilde{m}_{\mathbf{k}'}^C \rangle). \end{aligned} \quad (\text{C.3})$$

To simplify equation C.3, Tristram et al. (2005) and Polenta et al. (2005) make the following assumptions:

1. Fluctuations in the map, i.e., from CMB anisotropies, confusion limited point sources and noise, are well described by a Gaussian random field.
2. The beams and filtering applied to the data are isotropic; $E_{\mathbf{k}} \equiv G_{\mathbf{k}} B_{\mathbf{k}}$ depends only on $|\mathbf{k}|$.
3. The instrumental noise is isotropic.
4. The power spectrum, C_k , is smoothly varying with k , and changes little over scales comparable to the width of the mode coupling matrix.

Using the assumption that C_k and $|E_{\mathbf{k}}|$ do not vary much over small changes in $\mathbf{k}$, these products become:

$$\begin{aligned}
 \langle \tilde{m}_{\mathbf{k}}^A \tilde{m}_{\mathbf{k}'}^{B*} \rangle &= \sum_{\mathbf{k}'' \mathbf{k}'''} \widetilde{\mathbf{W}}_{\mathbf{k}-\mathbf{k}''} \widetilde{\mathbf{W}}_{\mathbf{k}'-\mathbf{k}'''}^* E_{\mathbf{k}''} E_{\mathbf{k}'''}^* \langle a_{\mathbf{k}''}^A a_{\mathbf{k}'''}^{B*} \rangle \\
 &= \sum_{\mathbf{k}''} \widetilde{\mathbf{W}}_{\mathbf{k}-\mathbf{k}''} \widetilde{\mathbf{W}}_{\mathbf{k}'-\mathbf{k}''} |E_{\mathbf{k}''}|^2 C_{\mathbf{k}''}^{AB} \\
 &\approx \widetilde{\mathbf{W}}_{\mathbf{k}-\mathbf{k}'}^2 |E_{\mathbf{k}}|^2 C_k^{AB}
 \end{aligned} \tag{C.4}$$

Here C_k^{AB} is shorthand for $C_k + \frac{N_k^A}{B_k^2} \delta_{AB}$, the expected cross spectrum between two unfiltered, perfectly beam-corrected—though noisy—maps. The additional term is the noise bias that exists in the map auto-spectrum. Assuming isotropic beams and filtering, we can combine equations C.3 and C.4 to obtain a relatively simple expression:

$$\begin{aligned}
 \langle D_b^{AB} D_{b'}^{CD} \rangle &= \langle D_b^{AB} \rangle \langle D_{b'}^{CD} \rangle \\
 &= P_{bk} P_{b'k'} \frac{1}{(2\pi)^2} \int d\theta_k d\theta_{k'} \left| \widetilde{\mathbf{W}}^2 \right|^2 |E_k|^2 |E_{k'}|^2 (C_k^{AB} C_{k'}^{CD} + C_k^{AD} C_{k'}^{BC}) \\
 &= P_{bk} P_{b'k'} E_k^2 E_{k'}^2 M[\mathbf{W}^2]_{kk'} (C_k^{AC} C_{k'}^{BD} + C_k^{AD} C_{k'}^{BC}).
 \end{aligned} \tag{C.5}$$

Combining equations C.3 and C.5 yields:

$$\begin{aligned}
 \Xi_{bb'}^{AB,CD} &= (K[\mathbf{W}]^{-1})_{b(2)b} [P_{b(2)k} P_{b(3)k'} M[\mathbf{W}^2]_{kk'} E_k^2 E_{k'}^2 (C_k^{AC} C_{k'}^{BD} + C_k^{AD} C_{k'}^{BC})] \\
 &\quad \times (K[\mathbf{W}]^{-1})_{b(3)b'}.
 \end{aligned} \tag{C.6}$$

To obtain a simplified expression for the magnitude of the diagonal elements of the covariance matrix, one typically assumes the mode-coupling matrix is nearly diagonal:

$M_{kk'}[\mathbf{W}] \approx w_2 \delta_{kk'}$. In this approximation $(K[\mathbf{W}]^{-1})_{bb'} \approx w_2^{-1} E_b^{-2} \delta_{bb'}$ and $M_{kk'}[\mathbf{W}^2] \approx w_4 \delta_{kk'}$ and thus the covariance of any two cross spectra is:

$$\begin{aligned} \Xi_{bb'}^{AB,CD} &\approx \frac{w_4}{N_b w_2^2} \left(\frac{\ell_{\text{eff},b}(\ell_{\text{eff},b} + 1)}{2\pi} \right)^2 (C_b^{AC} C_b^{BD} + C_b^{AD} C_b^{BC}) \delta_{bb'} \\ &= \frac{\langle \hat{D}_b^{AC} \rangle \langle \hat{D}_b^{BD} \rangle + \langle \hat{D}_b^{AD} \rangle \langle \hat{D}_b^{BC} \rangle}{\nu_b} \delta_{bb'}, \end{aligned} \quad (\text{C.7})$$

where ν_b is the effective number of independent k-modes in each ℓ -band. For isotropic filtering, $\nu_b^{\text{iso}} = \frac{N_b w_2^2}{w_4}$.

One subtlety in estimating the covariance is the fact that although the noise in each map is independent, each map has the same sky coverage. Hence the signal in all maps is correlated. The correct estimator must take this correlation into account. Under the simplifying assumption that all maps are statistically equivalent, the correlation between two cross spectra depends only on whether the spectra have a map in common (e.g. the cross spectrum D_b^{12} is more strongly correlated to the spectrum D_b^{23} than D_b^{34}). Comparing the covariance of two cross-spectra taken from 4 different observations:

$$\Xi_{bb}^{AB,CD} |_{A \neq B \neq C \neq D} = \frac{2}{\nu_b} C_b^2, \quad (\text{C.8})$$

to the covariance of a pair of cross-spectra with a common map,

$$\Xi_{bb}^{AB,BC} |_{A \neq B \neq C} = \frac{1}{\nu_b} \left(2C_b^2 + \frac{N_b}{B_b^2} C_b \right), \quad (\text{C.9})$$

to the covariance of a pair of cross-spectra with two maps in common,

$$\Xi_{bb}^{AB,AB} |_{A \neq B} = \frac{1}{\nu_b} \left(2C_b^2 + 2\frac{N_b}{B_b^2} C_b + \frac{N_b^2}{B_b^4} \right). \quad (\text{C.10})$$

The degree of correlation is also ℓ dependent since it depends on the relative signal vs. noise power in the maps. In the high- ℓ regime, where noise dominates the power in an individual map, all cross spectra are nearly independent. Conversely all cross-spectra are nearly completely correlated at low- ℓ , where the primary CMB anisotropy overwhelms the noise.

Given the assumption of statistical equivalence, we can then compute the expected variance of the mean spectrum based on these variance estimates. This is equal to the correlation between any particular cross-spectrum and the mean,

$$\Xi_{bb}^{\text{mean,mean}} = \Xi_{bb}^{\text{mean},AB} |_{A \neq B} \approx \frac{1}{\nu_b} \left(2C_b^2 + 4C_b \frac{N_b}{n_{\text{obs}} B_b^2} + 2 \frac{N_b^2}{n_{\text{obs}}^2 B_b^4} \right). \quad (\text{C.11})$$

This estimate of the variation of the mean spectrum agrees with the uncertainty estimates given in Polenta et al. (2005).

It should be noted that the noise and filtering of the SPT data are anisotropic. Equation C.7 can therefore be used only as a guideline. In the case of anisotropic filtering or anisotropic beams, the number of independent modes per bin will be typically smaller than ν_b^{iso} , since anisotropic filtering will weight different k-space modes unevenly. For example, the k-space mask completely eliminates all modes with $k_x < 1200$. The variance in each ℓ -bin increases with fewer independent modes. However, even if we account for the effective number of independent modes in an ℓ -bin, equation C.7 does not account for the anisotropic nature of the atmospheric noise contribution.

C.2 The Empirical Covariance Estimator

The existing analytic treatments are not directly applicable to the SPT data due to anisotropies in the noise and filters. By the nature of SPT's scan strategy, atmospheric fluctuations preferentially contaminate low k_x modes. Likewise the filters intended to remove these fluctuations preferentially remove low k_x modes. Instead, we have designed an empirical estimator which reproduces the analytical results when applied to isotropic data, while accurately accounting for the increased uncertainty due to the noise and filtering anisotropies in the actual data.

The noise covariance matrix estimate is divided into two parts, a signal contribution obtained from the Monte Carlo simulations described in §7.3.3 and a noise contribution obtained from real single-observation maps:

$$\mathbf{C}_{bb'} = \mathbf{C}_{bb'}^{\text{MC},s} + \mathbf{C}_{bb'}^{\text{data}}. \quad (\text{C.12})$$

The signal contribution is straightforward to estimate with an approach similar to the MASTER power spectrum error estimator. We use the signal only simulations to obtain an empirical estimate of the sample variance:

$$\mathbf{C}_{bb'}^{\text{MC},s} = \overline{\Delta \hat{D}_b^{\text{MC},s} \Delta \hat{D}_{b'}^{\text{MC},s}}. \quad (\text{C.13})$$

Note that here $\Delta x \equiv x - \bar{x}$ is defined with respect to the sample mean. Since the simulations include only CMB realizations and point sources in the confusion limit, the simulated signals are essentially Gaussian. Therefore we expect the usual sample variance contribution:

$$\langle \mathbf{C}_{bb',(\text{MC},s)} \rangle = \frac{2C_b^{\text{theory}}}{\nu_b}. \quad (\text{C.14})$$

As before, ν_b is the effective number of independent Fourier-modes in each ℓ -band.

The noise contribution is computed from the cross spectra of single-observation maps. We use the following estimator for the noise contribution:

$$\mathbf{C}_{bb'}^{\text{data}} \equiv \frac{2f(n_{\text{obs}})}{n_{\text{obs}}^4} \sum_{\lambda} \sum_{\alpha \neq \lambda} \left(\Delta \hat{D}_b^{\lambda\alpha} \Delta \hat{D}_{b'}^{\lambda\alpha} + 2 \left[\sum_{\beta \neq \lambda, \alpha} \Delta \hat{D}_b^{\lambda\alpha} \Delta \hat{D}_{b'}^{\lambda\beta} \right] \right). \quad (\text{C.15})$$

Here $f(n_{\text{obs}})$ is a correction due to the finite number of realizations. In the limit of many observations this function asymptotes to unity; we use 300 observations so this term can be ignored. The first term can be identified as the sample variance of the cross spectra. The second term accounts for the additional correlations between cross-spectra with a common map.

We can now calculate the expectation value for the noise component of the covariance estimator defined in C.15:

$$\begin{aligned} \langle \mathbf{C}_{bb}^{\text{data}} \rangle &\approx \frac{2}{n_{\text{obs}}^2} \Xi_{bb}^{\lambda\alpha, \lambda\alpha} + \frac{4}{n_{\text{obs}}} \Xi_{bb}^{\lambda\alpha, \lambda\beta} - \left(\frac{4}{n_{\text{obs}}} + \frac{2}{n_{\text{obs}}^2} \right) \Xi^{\text{mean, mean}} \\ &= \frac{1}{\nu_b} \left(4C_b \frac{N_b}{n_{\text{obs}} B_b^2} + 2 \frac{N_b^2}{n_{\text{obs}}^2 B_b^4} \right). \end{aligned} \quad (\text{C.16})$$

This is combined with the signal, or cosmic variance, component in equation C.13 to get the expectation value of the estimator:

$$\langle \mathbf{C}_{bb} \rangle \approx \frac{1}{\nu_b} \left[2C_b^{\text{theory}^2} + 4C_b \frac{N_b}{n_{\text{obs}} B_b^2} + 2 \frac{N_b^2}{n_{\text{obs}}^2 B_b^4} \right]. \quad (\text{C.17})$$

This agrees with the analytic estimate (equation C.11) for the variance of the mean.

C.2.1 Multifrequency Cross Covariances

A multifrequency data set requires an estimate of both the covariance of the each individual set of bandpowers (i.e., both the single-frequency bandpowers, and the cross-frequency bandpowers) and the cross-covariance between these sets. We naturally expect signal correlations between different sets of bandpower due to the fact that all three sets of bandpowers reported here are derived from the same patch of sky. However we also expect *noise* correlations between the 150 GHz $\times$ 220 GHz cross spectrum bandpowers and each set of single-frequency bandpowers since the noise uncertainty in the cross-spectrum is entirely due to noise in the 150 GHz and 220 GHz data. Thus we compute the cross-covariance matrices, $\mathbf{C}_{bb'}^{(i,j)}$, where i and j denote one of the three sets of bandpowers: 150 GHz, 220 GHz and 150 GHz $\times$ 220 GHz:

$$\begin{aligned} \mathbf{C}_{bb'}^{(i,j)} &= \overline{\Delta \hat{D}_b^{\text{MC}, s(i)} \Delta \hat{D}_{b'}^{\text{MC}, s(j)}} \\ &+ \frac{2f(n_{\text{obs}})}{n_m^4} \sum_{\lambda} \sum_{\alpha \neq \lambda} \left(\Delta \hat{D}_b^{\lambda\alpha(i)} \Delta \hat{D}_{b'}^{\lambda\alpha(j)} + 2 \left[\sum_{\beta \neq \lambda, \alpha} \Delta \hat{D}_b^{\lambda\alpha(i)} \Delta \hat{D}_{b'}^{\lambda\beta(j)} \right] \right). \end{aligned} \quad (\text{C.18})$$

C.2.2 Treatment of Off-diagonal Elements

Given the finite number of simulations and data maps, we expect some statistical uncertainty in the covariance estimate, particularly in the off-diagonal elements. Such uncertainty is not unique to the estimation technique described here, rather it is expected for *any* covariance estimate which is computed from a finite number of realizations. We expect the covariance estimates to be Wishart distributed, with $n_{obs} = 300$ degrees of freedom. A given covariance element, $\mathbf{C}_{ij}$ has a statistical variance of:

$$\langle (\mathbf{C}_{ij} - \langle \mathbf{C}_{ij} \rangle)^2 \rangle = \frac{\mathbf{C}_{ij}^2 + \mathbf{C}_{ii}\mathbf{C}_{jj}}{n_{obs}}. \quad (\text{C.19})$$

For diagonal elements we expect a standard deviation of $\sqrt{2/n_{obs}} = 1/\sqrt{150} = 8.1\%$. In addition, there is a statistical uncertainty on the apparent correlation between two bins. If we assume that the true correlation between bins is small, then the standard deviation of the apparent correlation between two bins is $\sqrt{1/n_{obs}}$ or 5.7%. For the choice of bin-size, the statistical error on the correlation of any two bins is much larger than the expected correlation (i.e. the fractional error on the apparent correlation estimates is greater than 100% even for adjacent bins). For bins that are widely separated, these false bin-bin correlations may skew model fitting. Therefore we “condition” the published covariance matrices in order to reduce this statistical uncertainty on the covariance matrices.

From equation C.5, we see that the shape of the correlation matrix (i.e. the relative size of the on-diagonal to off-diagonal covariance elements as a function of bin separation) is determined by the apodization window through the quadratic mode-coupling matrix, $M[\mathbf{W}]_{kk}$. For the ℓ range considered in this work, the off-diagonal elements of this matrix depend only on the distance from the diagonal, $|k - k'|$. Therefore we condition the estimated covariance matrix, $\hat{\mathbf{C}}_{kk'}$ by first computing the corresponding correlation matrix, and then averaging all off-diagonal elements of a fixed separation from the diagonal:

$$\mathbf{C}'_{kk'} = \frac{\sum_{k_1 - k_2 = k - k'} \frac{\hat{\mathbf{C}}_{k_1 k_2}}{\sqrt{\hat{\mathbf{C}}_{k_1 k_1} \hat{\mathbf{C}}_{k_2 k_2}}}}{\sum_{k_1 - k_2 = k - k'} 1}. \quad (\text{C.20})$$

The bandpowers reported in Tables 7.1 and 8.1 are obtained by first computing power spectra and covariance matrices for a bin-width of $\Delta\ell = 100$ with a total of 80 preliminary bins. This covariance matrix was then conditioned according to equation C.20 before averaging the bandpowers and covariance matrix into the final bands.

Equation C.5 is based on the assumption that the filtering is isotropic. In order to test the validity of this equation for the anisotropic filtering, we perform 10000 simple Monte Carlo simulations. In each simulation a white noise realization is subjected to a simplified, though similarly anisotropic, version of the filtering scheme. The variance of the resultant spectra is computed and compared to equation C.5.

Though the apparent correlations between all bins exhibit the expected 1% scatter, the correlations between neighboring bins are consistent with equation C.5.