For International Conference on
Meson Spectroscopy, Philadelphia, Pa.,
May 1-2, 1970

UCRL-19826
Preprint

CENTRIFUGAL BARRIER EFFECTS IN MESON DECAYS

C. Quigg and F. von Hippel

May 1970

AEC Contract No. W-7405-eng-48

LAWRENCE RADIATION LABORATORY
UNIVERSITY of CALIFORNIA BERKELEY

Centrifugal Barrier Effects in Meson Decays

C. Quigg* and F. von Hippel†

Lawrence Radiation Laboratory
University of California
Berkeley, California 94720

I. INTRODUCTION

The recent observations of fine structure in the meson spectrum[1] suggest that the hadron spectrum may have a richness and complexity comparable to the level structures of heavy nuclei. This in turn suggests that in probing the intricacies of hadron spectroscopy we may profitably borrow from the wisdom of our colleagues in nuclear spectroscopy. To this end, we have resurrected the centrifugal barrier penetration factors and used them to investigate the kinematics of meson decays. We neglect all dynamical effects: our aim is to provide a framework in which the dynamics may be isolated and studied experimentally.

To our knowledge no systematic application of the effects of centrifugal barrier factors to elementary particle decays

---

*Address after Sept. 15, 1970: Institute for Theoretical Physics, State University of New York, Stony Brook, Long Island, New York 11790.
†Alfred P. Sloan Foundation Fellow; address after Sept. 1, 1970: Argonne National Laboratory, Argonne, Illinois 60439.

has yet been given. As meson resonances of moderate spin $(J = 3, 4, 5, \cdots)$ become accessible to experimental study, it will be appealing to seek dynamical regularities among the resonances lying on a Regge trajectory. A first step in the study of decay dynamics is the identification of dynamical effects, i.e., the separation of dynamics from kinematics. To achieve this separation in a plausible way, we have adapted the prescription developed by Feshbach et al.[2] for single particle decays of nuclear resonances. To make the problem tractable, we shall assume throughout that all decays are into two body or quasi-two body final states. This seems a reasonable assumption for nonstrange mesons considering the large number of open two-body channels and is consistent with present experiments.

In the next Section we remind the reader of the classical treatment of centrifugal barrier factors. We make there some brief remarks about the shapes of resonances and about the possibility that different decay products of a single resonance may appear as different line shapes in the mass spectrum. In Section III we discuss a few kinematical predictions for the branching ratios of states on the leading Regge trajectory. We also remark on the rather different features which characterize decays from the lower trajectories.

II. RESONANCE SHAPES

APOLOGY. We realize that most of what follows is known to many of our readers. We happily acknowledge our debt to Professors Blatt and Weisskopf[3] who knew twenty-five years ago what we restate below, and to those good men and true whose lot it has been to recycle from time to time these simple and useful ideas.

RECIPE FOR CENTRIFUGAL BARRIER FACTORS. In the discussion of Ref. 2, it is assumed that the logarithmic derivative of the external wave function in the entrance channel at a radius R just outside the range of the nuclear force (u_ℓ) is a linear function of momentum (for mesons we take momentum squared), in an interval centered at the resonance position s_R, which interval is large compared to the resonance width $[2\Gamma\sqrt{s_R}]$. Thus we would have

$$\rho u_\ell'(\rho)/u_\ell(\rho) = D \equiv D_0 + D_1(s_R - s) , \qquad (1)$$

where $\rho = kR$, k is the external wave number of a decay product in the cm frame, ℓ is the orbital angular momentum, and s is the square of the total cm energy. The parameters D_0 and D_1 are adjusted to give the resonance mass, width, elasticity, and background.

The assumption of linearity cannot be expected to hold in the domain of particle resonances where, in contrast to nuclear resonances, momenta in external channels are of the same order of magnitude as the momenta inside the potential wells which best approximate the resonance positions and shapes. Consequently we do not expect that the forms of both the resonant and the background terms obtained by Feshbach <u>et al</u>. will carry over to the high energy regime. Our hope and operating assumption will be that the resonance term itself (unlike the background term) is sensitive primarily to the existence of the resonance and to the centrifugal barrier effects in the several decay channels.

The resonant amplitude in the absence of background is then

$$T_{fi} = \frac{[f_i\ B_i\ f_f\ B_f]^{\frac{1}{2}}}{\frac{2(s_R - s)}{\Gamma\sqrt{s_R}} - i \sum_n f_n B_n} , \qquad (2)$$

where $\rho_j = k_j R$, $b_j = \rho_j |h_{\ell_j}^{(1)}(\rho_j)|^{-2}$, $B_j = b_j/[b_j]_{s=s_R}$, and $h_\ell^{(1)}$ is the Hankel function of the first kind of order ℓ.

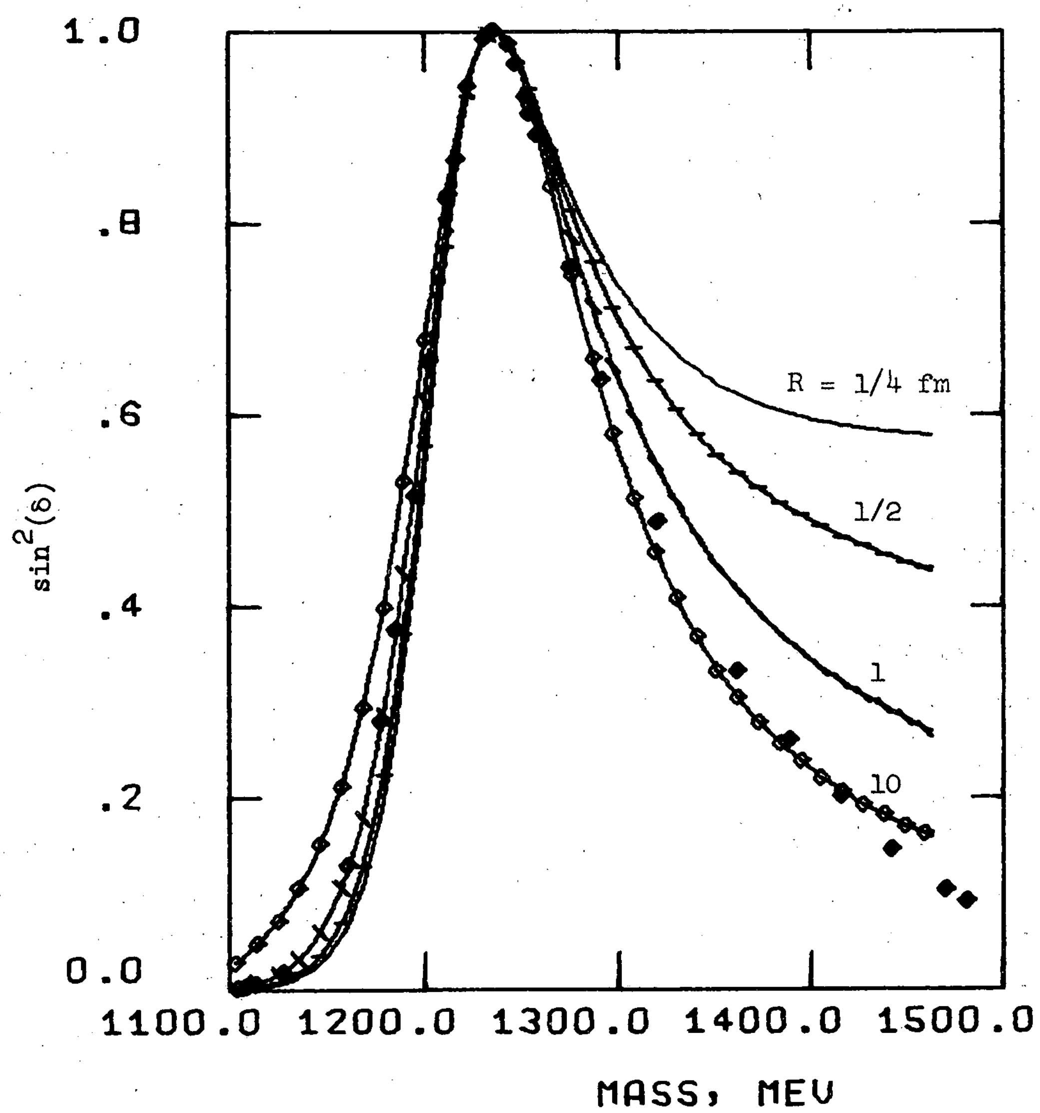

Figure 1 Comparison of the predictions of Eq. (2) with the $\Delta(1236)$ phase shifts, for various radii R. Contrary to folklore, one does not obtain a perfect fit with R = 1 fm. We assume the Δ is elastic.

The f_j are branching fractions and Γ is the total width.

We have looked for experimental tests of this form and found only one case for which the experimental amplitude is known well enough (from formation experiments) away from the resonance peak to provide a stringent test: the $\Delta_{3,3}$ phase shift. In Fig. 1 we compare the predictions of Eq. (2) with the phase shifts.[4] There is some sensitivity to R, and little background is required except at the highest energies ($2\frac{1}{2}\,\Gamma$ above resonance) where it seems impossible to obtain a sufficiently small partial cross section. [This was already noticed by Galtieri.[5] For the $\Lambda(1520)$ the high energy tail predicted is likewise much larger than is observed.[6]]

APPLICATIONS TO MESON RESONANCES. High statistics, high resolution data on boson resonances with masses greater than 1 GeV/c^2 are being accumulated at a rapid rate. These states pose new problems which argue for more sophisticated analyses of resonance shapes. A fundamental question is the interplay between resonant states and background. Certainly benign neglect is not the answer, but we have no solution to offer at this time. We would remark, however, on the power of the S-matrix unitarity equation to constrain phenomenological parameters. An easier question, and the one to which we address ourselves here, is what to do when a resonant state has several significant decay modes.

Typically, experimenters fit each decay mode independently to a Breit-Wigner formula with the requisite centrifugal barrier factor. It is not uncommon to find a different mass and width quoted for each decay mode of a meson. This is chaotic! We now wish to show how simultaneous study of all important decay modes of a resonant state by means of Eq. (2) can bring some order to the meson spectrum.

As it stands, Eq. (2) provides a complete description of the scattering amplitude in the neighborhood of a resonance.

However, since we are dealing with production experiments, the entrance channel is essentially unknown. We therefore assume the entrance channel to be the sum over all open channels. Also to restrict the number of parameters in the discussion we assume that the radius R is the same for all decay modes, and in each channel ℓ is minimized. The effect we wish to demonstrate is that decay modes may appear to have different line shapes, just because of the peculiar kinematics of their decays. That this can happen is obvious if the orbital angular momentum ℓ is different for one decay than for another, but even for the same value of ℓ alterations may be caused by the different decay momenta.

It is well to illustrate these ideas with a simple example. Let us consider an idealized A_2, uncomplicated by splitting, which we take to have a mass of 1310 MeV and a width of 90 MeV. The A_2 decays principally into three channels, which we list in Table I, with idealized branching fractions. All the decays

Table I

Decay Modes of the Schematic A_2 Meson

Channel	$\ell_{minimum}$	K (MeV)	Branching Fraction (%)
$\pi\rho$	2	415	70
$\pi\eta$	2	428	20
$K\bar{K}$	2	529	10

are d-wave so the effects which we shall see are due to the different decay momenta. Momentarily we shall discuss a case in which the ℓ values are not all equal. In Fig. 2 we plot the line shapes for the three principal decay modes, as listed in Table I, and for the total A_2 "missing mass," for three values of the radius R. Notice that the three decay channels do not manifest themselves as identical line shapes. In

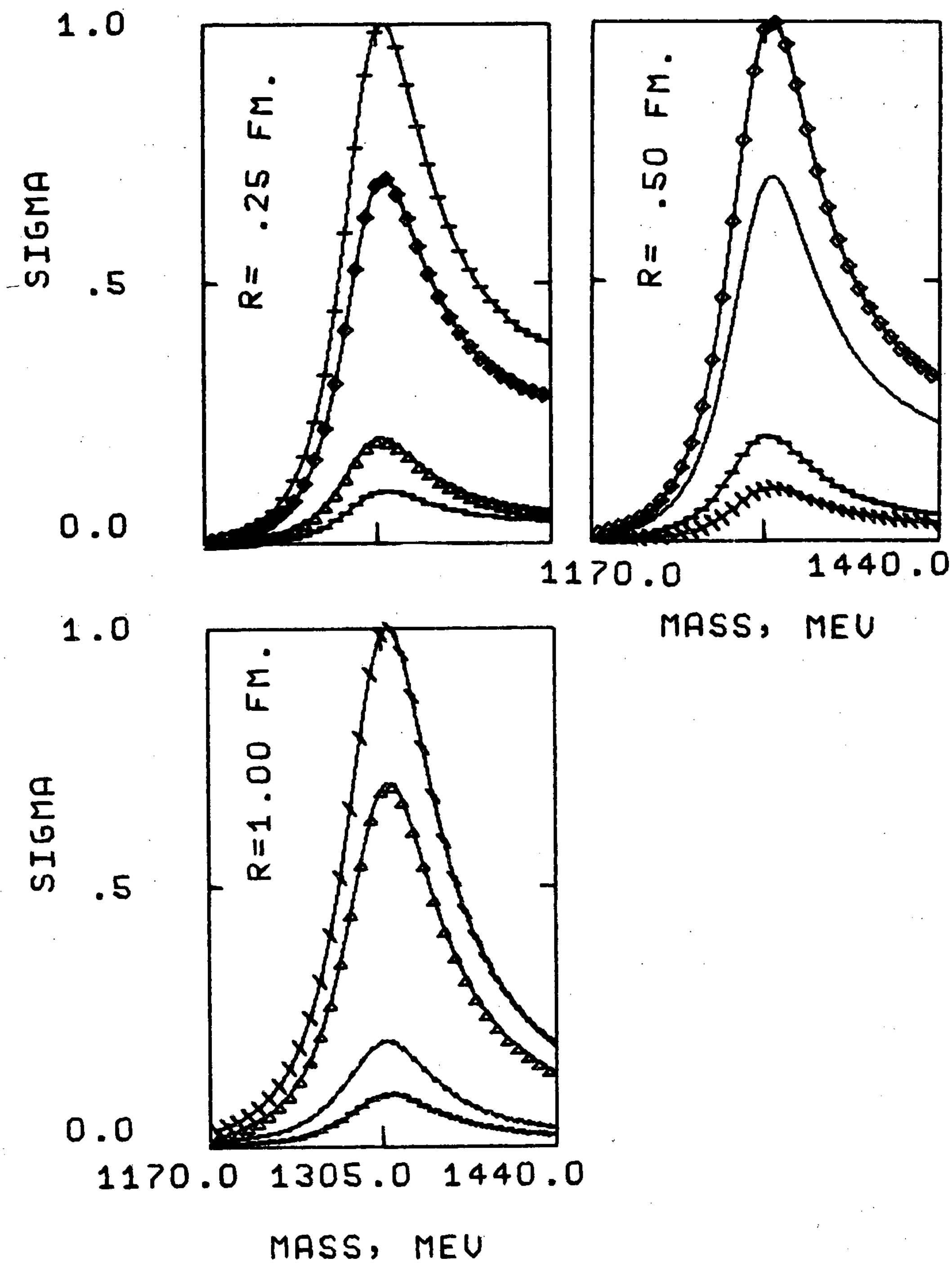

Figure 2 Line shapes for the schematic A_2 described in Table I. The three graphs are for $R = \frac{1}{4}, \frac{1}{2}, 1$ fm. On each graph is shown the "missing mass" shape, and the line shapes for the three decay modes $\pi\rho$, $\pi\eta$, and $K\bar{K}$, normalized to the assumed branching ratios. The quantity plotted is $|T|^2$.

particular the $\pi\eta$ mode and the $K\bar{K}$ mode are skewed more than the dominant $\pi\rho$ mode. Thus we should expect that if the three modes were fitted independently with conventional, single-channel Breit-Wigner formulae, the widths determined for the $\pi\eta$ and $K\bar{K}$ modes should be larger than the width determined for $\pi\rho$. In this example the difference between the single-channel and multiple-channel Breit-Wigner widths is on the ten percent level.

In the situation we have just discussed, in which all the decay modes are known, and the identification of those decay channels with the resonance is quite certain, the more sophisticated analysis we propose adds some neatness to the experimental picture. We next discuss a case in which the multiple-channel Breit-Wigner formalism may assist in deciding whether several observed decay modes are in fact different incarnations of the same object. Again we proceed by example, fabricating a $J^P = 3^-$ g(1670) with $\Gamma = 110$ MeV to illustrate. Its properties are listed in Table II. Our contrived meson is consistent

Table II

Decay Modes of the Schematic g Meson

Channel	$\ell_{minimum}$	K (MeV)	Branching Fraction (%)
$\pi\pi$	3	830	46
$K\bar{K}$	3	675	4
πA_2	2	330	25
$\pi\omega$	3	655	25

with the observations of the g(1660) and the $\rho_N(1710)$ listed in the wallet cards.[7] In Fig. 3 we plot the shapes of the "missing mass," of the two body states ($\pi\pi$ and $K\bar{K}$), and of the four body states (πA_2 and $\pi\omega$), for three reasonable values of the radius R.

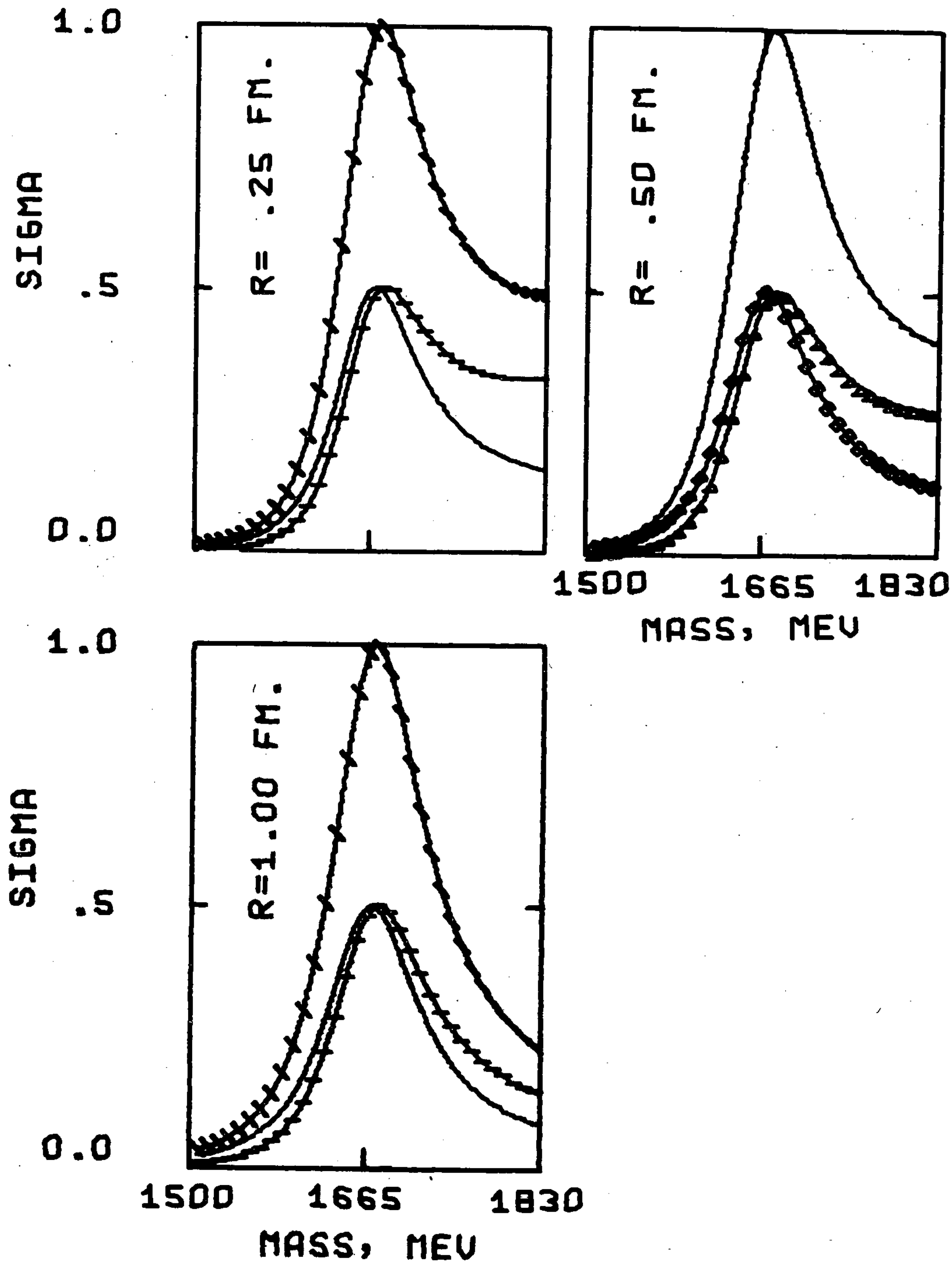

Figure 3 Line shapes for the idealized g(1670) [Γ = 110 MeV] described in Table II. The three graphs are for R = $\frac{1}{4}$, $\frac{1}{2}$, 1 fm. On each graph is shown the "missing mass" shape, and the line shapes for two body and four body decays. Notice that the four body line is shifted toward higher masses. The quantity plotted is $|T|^2$.

The most striking feature of the plots is the shift toward higher masses of the four body decay modes. The lower edge of the four body line shape is shifted by 10 to 15 MeV. The center of the four body line is shifted by a like amount, the value of which depends upon the way in which experimental background might be drawn. This effect lends weight to our prejudice that the distinction between the g and the $\rho_N(1710)$ may be artificial.

III. SOME KINEMATICAL PREDICTIONS

INTROIT. We invite the reader to join us on a brief flight of fancy, during which we shall investigate the predictions of Eq. (2) under the assumption that centrifugal barrier effects determine

$$f_i = b_i \Big|_{s=s_R} \Big/ \sum_n [b_n]_{s=s_R} . \qquad (3)$$

We concentrate our attention on the $I = 1$, $Y = 0$ mesons in the hope of raising some questions which can be answered experimentally by the time of the next Meson Conference. We are assuming here that all coupling constants are equal. Evidently our hypotheses have no air of theoretical finality: conscious of this we shall try to pose questions which are interesting in and of themselves, independent of the simple model we use to discover the questions.

FISSION ON THE LEADING TRAJECTORY. It is expected that for very high masses (and therefore spins) the states on the leading $(\rho, A_2, \cdots)$ trajectory will decay principally by the cascade mode.[8] That is, the state of spin J will decay into a pion and a slightly lower spin member of the leading trajectory. This expectation follows from the same kind of reasoning we wish to apply here, namely that kinematics determines all, and we agree that asymptotically in mass and spin it is reasonable. However we find that for the states of moderate spin ($J = 3, 4, 5, 6, \cdots$) on the leading trajectory the cascade behavior is not expected. Instead, the kinematically preferred mode of decay is fission, or decay into two approximately equal (in mass and spin) fragments, each with approximately half the mass and spin of the decaying state. To proceed with some concrete examples, we have calculated the branching fractions from kinematics, for $R = \frac{1}{4}, \frac{1}{2}$ fm. For $R = \frac{1}{4}$ fm., the total widths of ρ, A_2, g are in roughly the correct ratio, but for $R = \frac{1}{2}$ fm. the A_2 and g are too fat. The branching fractions are however less sensitive to the radius and specifically the order of importance of the various decay channels is quite stable. In Table III are listed the decay modes of the g(1670)

Table III

Predicted Decay Modes of g(1670), $J^P = 3^-$ in Order of Importance

Mode	Rank	Comments	ℓ
$\rho\rho$	1	"fission"	1
$\pi\pi$, πA_1	2		3, 2
$K\bar{K}$	4		3
$\pi\omega$	5		3
$\epsilon\rho$	6		2
πA_2	7	"cascade"	2

in order of their predicted importance. Now, the $\rho\rho$ mode is not known experimentally to be important in the g region. For the moment this may be blamed on the difficulty of identifying two rhos (because the rho is so broad). We encourage experimenters to look carefully in their high statistics samples for this fission mode, and stress that its absence will teach us as much as its presence. If $g \to \rho\rho$ is strongly supressed, that is a piece of dynamical information of the kind we hope to uncover through this kinematical approach.

Let us quickly give two more examples which bear on states we expect to be identified soon. In Table IV we list the

Table IV

Predicted Decay Modes of $4^+(1975)$ in Order of Their Importance

Mode	Rank	Comments	ℓ
$\rho\omega$	1	"fission"	2
$K^*\bar{K}^*$	2	"strange fission"	2
Many π+ something, including πg	3	"cascade," other modes	2 etc.

expected decay modes of the spin 4 recurrence of the A_2. For this state the fission mode $(\rho\omega)$ is so strong that it dominates over the sum of all other channels. In this case it is the (approximately) equal mass kinematics of the fission mode which selects it as the most important. Notice that the trajectory function, $J \approx \frac{1}{2} + M^2$ requires that the 4^+ object decay into $1^- 1^-$, rather than $2^+ 2^+$. Ultimately (for $J \sim 10$) this quadratic dependence upon the mass will suppress the fission mode.

As a last example of fission dominance we consider the spin 5 recurrence of the rho and g, $5^-(2240)$ for which the three fission modes again are highly favored. The $\rho\rho$, ρf, and ωA_2 channels dominate over all others, including the $\pi 4^+(1975)$ cascade channel.

BEHAVIOR OF THE LOWER TRAJECTORIES. Similar considerations to those used above can be applied to the lower trajectories, among which are the π, B, and A_1. In general we find that the first few recurrences are likely to be rather broad and perhaps, therefore invisible. (An exception is the 2^- recurrence of the pion, which may have a width equal to the B or A_1 widths.) There are so many open channels for these particles to decay into that it is hard to select a dominant channel. The general trend, not surprisingly, is to emit a pion and jump to spin J or J-1 on the leading trajectory.

THE ELUSIVE $\rho'(1300)$. A good deal of effort has been expended in the past year in searching for the ρ' suggested by the Veneziano model of Shapiro and Yellin.[9] If it is a good approximation that all coupling constants are equal, then the kinematics of centrifugal barrier factors tells us that the ρ' might easily be only 30% elastic. The reader may make of this what he will. In general we find that raising the mass of a meson by a few hundred MeV would increase its kinematically-predicted width by an order of magnitude.

IV. SUMMARY

We have suggested that an approximation to the multi-channel usual Breit-Wigner resonance formula with centrifugal barriers will permit a more sophisticated and more organized analysis of mesons which decay into many channels. This amplitude describes the kinematics of the decay processes, and the kinematical influence of one decay mode upon another.

We also propose to use the multi-channel resonant amplitude as a theoretical tool, to discover what the properties of meson resonances would be if dynamics were negligible. A specific suggestion is that the mesons of spin 3, 4, 5 on the leading trajectory should fission unless dynamical effects are important.

* * * * * *

This work was supported in part by the United States Atomic Energy Commission.

REFERENCES

1. W. Kienzle, Combined Evidence for the Splitting of the A_2 Meson, in Meson Spectroscopy, C. Baltay and A. H. Rosenfeld, Eds. (W. A. Benjamin, Inc., New York, 1968), p. 265.
2. H. Feshbach, D. C. Peaslee, and V. F. Weisskopf, Phys. Rev. 71, 145 (1947).
3. J. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics (John Wiley and Sons, New York, 1952).
4. A. Donnachie, R. G. Kirsopp, and C. Lovelace, Phys. Letters 26B, 161 (1968). See also the collection of D. J. Herndon, A. Barbaro-Galtieri, and A. H. Rosenfeld, Lawrence Radiation Laboratory Report UCRL-20030πN, 1970.
5. A. Barbaro-Galtieri, Baryon Resonances, in Advances in Particle Physics, R. Cool and R. Marshak, Eds. (Academic Press, New York, 1968), v. II.
6. R. D. Tripp (Lawrence Radiation Laboratory), personal communication.
7. Particle Data Group, Rev. Mod Phys. 42, 87 (1970).
8. H. Goldberg, Phys. Rev. Letters 21, 778 (1968).
9. J. Shapiro and J. Yellin, Lawrence Radiation Laboratory Report UCRL-18500 (unpublished) and J. A. Shapiro, Phys. Rev. 179, 1349 (1969).