

**Optimal Pricing Policies for
Temporary Storage at Ports**

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**The University of California
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ricing schemes for temporary storage facilities (sheds) at ports examined in this paper. It is recognized that shippers respond to pricing changes by choosing storage times that maximize their profits. Two types of strategies are considered. Nondiscriminatory strategies set the shed storage charges as a function of shipment volume and time in storage alone (the same for all shippers); they do not require much knowledge about the shippers' behavior and are found easily. Discriminatory strategies have the potential for improved efficiency but require more information. In some cases, identified in this report, nondiscriminatory strategies are just as efficient as their discriminatory counterparts. If the demand is steady and there is no alternative storage site, we find that shed prices should increase linearly with time, at a rate that prevents overflows without causing undue hardship to users. If demand is heavy, then the shed should be close to capacity at the end of the time. There is no need for discrimination. Stochastic variations in demand complicate matters slightly because they make it worthwhile to increase shed prices at an increasing rate with time and to discriminate across shippers. If overflow is prevented by sending cargo to a remotely located warehouse, there is more flexibility and the pricing strategies are almost as simple. Two problems are examined in this paper: finding the optimal shed prices given warehouse price and finding both sets of prices jointly. A computer spreadsheet can be used to find the best pricing policies.

The operation of temporary storage facilities can be improved by the adoption of rational pricing schemes. This introduction section examines current pricing practices for port sheds. The body of the paper presents more refined policies that take into account the user's response to pricing changes. Transit sheds are buildings located within ports—usually alongside cargo berths—used for receiving, storing, and handling various types of in-transit cargo. They provide safe and efficient storage while freight waits for such administrative formalities as customs clearance and the processing of ship documents. Transit sheds also act as buffer zones between fast ship-shore flow and the slower shore-inland goods movement. Within the sheds, import cargo is broken down into small shipments for easy access when the overland shippers come individually to claim it. Conversely, export loads for a specific commodity are consolidated in the shed as they arrive, ensuring that they can be retrieved in the order prescribed by the shipper's plan. Warehouses perform a somewhat different function. Remotely located warehouses are subject to much less severity of capacity constraints than the sheds but require additional cargo

handling; this makes them attractive for longer term storage only.

Shed management directly affects overall port performance. When sheds are congested, they cannot perform their function as buffers for the flow of goods, and this hampers the efficient loading of vessels and increases their turnaround times. Shortages in storage space may also increase costs as a result of additional cargo handling, insurance premiums paid for deteriorated or damaged goods, and shippers' failure to meet delivery dates. Finally, shed congestion may force shippers to use warehouses to store relatively fast-moving cargo, increasing traffic between port, warehouses, and land transportation terminals.

Clearly, adequate pricing policies must avoid congestion by controlling the average cargo stay in the sheds. The importance of this principle is recognized in practice. According to a 1987 United Nations Conference on Trade and Development (UNCTAD) report,

In an Asian port, the demurrage rates for transit sheds were quadrupled to make it unprofitable for consignees to use the transit sheds for warehousing. The result was that congestion was considerably reduced. (1)

Modern container terminals, prevalent in industrialized countries, also need temporary storage areas within the terminal to serve as buffer zones between containerships and trucks or trains. The need to avoid abusive use of these areas is also clear and can be illustrated in practice. For example, at the TransBay Terminal in Oakland, California, a fee is imposed on containers that arrive more than ten days before their scheduled departure date (2).

Of course, if shippers are encouraged to reduce their transit time so much that the storage facility is underutilized, the result—wasted capacity and shipper inconvenience—is also undesirable. How efficient pricing schemes can be developed for a variety of situations is demonstrated in this report.

In an UNCTAD study, which analyzed more than 50 ports (3), it was determined that most current pricing policies for transit sheds exhibit the following features:

1. A fixed time period of free storage, which starts when the goods are deposited in the shed.
2. Storage fees that are proportional to either the storage area occupied, the cargo weight, or the cargo volume, depending on the commodity. (The discussion here will be phrased in terms of volume, but no generality is lost if most of the commodities are priced on the same basis.) The storage fee per unit volume will be called price from now on.
3. Price per unit volume increases with the excess transit time after the free period. Tariffs—defined here as the stor-

age charge per unit volume and unit time—are either constant (in about 20 percent of the cases) or increase with time. Storage times are normally measured in days.

Imakita (4) describes a simple model in which storage time varies across shippers but is insensitive to price and in which a remote warehouse accommodates the shippers that find the shed too expensive.

Because storage times change across shippers, a shed tariff increase does not affect all the shippers equally. If some decide to switch from the shed to the warehouse, the volume stored in the shed will change. The relationship between pricing policy and various measures of performance (shed accumulation, shed revenue, warehouse flow, etc.) is now introduced as a prelude to the elastic demand models object of this paper. The following variables are used:

- q = port's cargo flow (in volume units per unit time);
- q_s = flow through the shed;
- q_w = flow through the warehouse ($q = q_s + q_w$);

C = static shed capacity; that is, the maximum cargo volume that can be stored in the shed at any given time (warehouses are assumed to have infinite capacity);

$F_T(t)$ = proportion of the port's cargo flow that is stored for no more than t time units, assumed to be independent of pricing and storage locale [this function can be viewed as a cumulative probability distribution function for the time in storage T of a randomly chosen flow unit; the corresponding probability density function is denoted $f_T(t)$];

$p_w(t)$ and $p_s(t)$ = warehouse and shed prices (in dollars per unit volume) as functions of time in storage; and

t^0 = indifference time in storage: $p_w(t^0) = p_s(t^0)$.

If shed prices are less than warehouse prices for short stays but escalate faster with time (logically, the shed's marginal tariff should be higher) then the indifference time, if it exists, will be unique. Cost-conscious shippers will choose the shed if $T < t^0$, and the warehouse if $T > t^0$ (see Figure 1).

The flow through the shed is then

$$q_s = q F_T(t^0) \tag{1}$$

and the revenue is $p_s(t^0) q F_T(t^0)$. If for a given t^0 the shed capacity is never exceeded, the average volume in storage can be viewed as the average queue length in a multiserver queueing system with an infinite number of parallel channels. The average volume V_{avg} in storage is therefore

$$V_{avg} = q E(T | T < t^0) = q \int_0^{t^0} f_T(t) t dt \tag{2}$$

If stochastic fluctuations in V can be ignored, the shed will not overflow if $V_{avg} \leq C$. Therefore, we can view V_{avg} as the shed capacity C_{req} required to avoid overflow. With stochastic

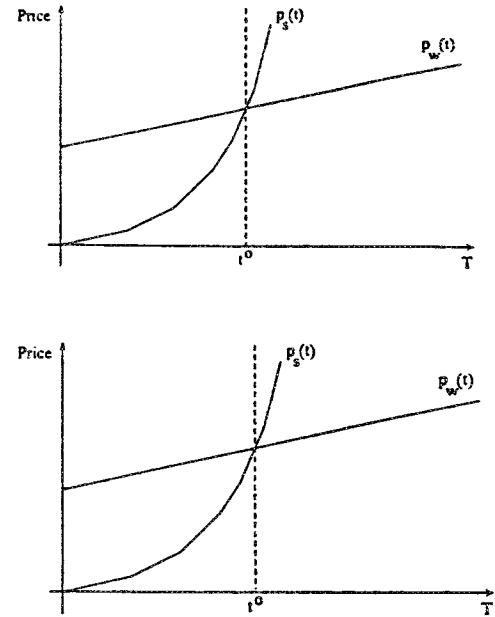


FIGURE 1 Typical shed and warehouse price functions.

fluctuations, considered later, C_{req} must be appreciably larger than V_{avg} if overflow is to be unlikely.

Interest here is in the case where C is not sufficient to accommodate all the traffic: $q E(T) > C$. Operations will then be most efficient if the shed operates near capacity. Definitely, this is to the advantage of the shippers because as much flow as possible then avoids the warehouse. Maximizing utilization does not necessarily correspond to maximizing shed revenue, but this is likely to be a secondary objective for the terminal operator; minimizing the operating cost added by traffic to the warehouse is likely to be of greater importance, especially if there is competition from other ports.

Because V_{avg} increases with t^0 , full shed utilization without overflow is achieved if the shed price function's indifference point t^0 satisfies Equation 2: t^0 can be found numerically for any given $f_T(t)$.

Any shed price function $p_s(t)$ that intersects $p_w(t)$ at such a t^0 (and such that $p_s(t) < p_w(t)$ for $t < t^0$, and $p_s(t) > p_w(t)$ for $t > t^0$) will result in full shed utilization and no overflow. Thus, there is an infinite number of shed price functions that satisfy the optimality condition. Although cargo flow patterns and storage utilization are fixed if t^0 is given, the form of $p_s(t)$ in the interval $[0, t^0]$ does influence the cash flow among the warehouse, the shed, and the shipper. Figure 2 depicts two price functions with identical shed utilization: $p_s^1(t)$ favors the shippers, with low fees, and $p_s^2(t)$ maximizes shed revenue.

An in-between linear price function would seem adequate in this case. Although constant tariffs have their advocates (5), nonlinear price functions (with increasing tariffs for longer stays) can be effective in some of the scenarios about to be examined.

The model just described assumes that flow and length of stay are independent of storage prices. Although it is reasonable to assume that the volume shipped is independent of storage prices—after all, these represent a relatively small fraction of the total transportation costs incurred by the

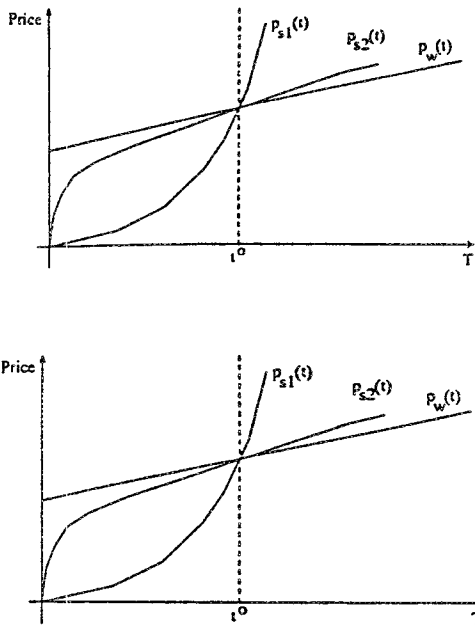


FIGURE 2 "Equivalent" shed price functions.

shipper—the same cannot be said for the time in storage. More likely, as storage prices increase, shippers will try to reduce the time in storage and $F_T(t)$ will shift towards shorter stays.

If shed tariffs were increased to eliminate overflows as recommended, both the indifference time and the average shed storage time would decline. As a result, even with constant throughput, the average shed accumulation would be less than predicted and some shed space would be wasted. Clearly, if storage times depend on price, the method suggested underestimates the effect of price changes. Thus in this paper, total cargo throughput is considered given, but its accumulation is assumed to depend on storage prices.

Attempts are made to overcome the limitations of this model in the remainder of this paper. The next section introduces a model of shipper behavior that attempts to explain how shippers choose their storage time. The following section examines situations without a warehouse, under both deterministic and stochastic demand, and the final section adds the warehouse. The amount of information needed to implement each policy is discussed, as well as the policies themselves. Both discriminating strategies (which offer different tariffs to different customers) and nondiscriminating strategies are considered. The calculations can be easily automated in spreadsheet form and numerical examples are presented.

SHIPPER BEHAVIOR

Shipper costs can be classified as moving expenses (including transportation and handling) and holding costs (capital tied up in inventory and storage rent costs) (6). Moving costs tend to decrease with time in storage, t , as cargo can then be consolidated into more efficient shipments. Holding costs, on the contrary, increase with time in storage, t . It has already

been shown that the rent costs—represented by the price functions $p_s(t)$ and $p_w(t)$ —usually increase with t .

Here interest is in examining the behavior of a cost-minimizing shipper when the storage rent price functions are changed. The sum of all the logistics costs, not including the port storage charges, is called external costs. They typically decrease with t when t is small, eventually reaching a minimum and then increasing. (For t close to zero, shippers would have to retrieve items from the shed on short notice, which would be expensive. As t increases the external costs decrease, because items can then be carried in larger batches, which reduces moving costs—inventory costs are a negligible part of the external costs for small t . If t continues to increase, the moving cost economies of scale eventually disappear, but inventory costs continue to increase; as a result, the external cost must eventually increase.)

The external savings function, $s(t)$, represents the shipper's external cost savings (per unit volume) if the freight is stored near the port for an average of t days rather than being collected on the first day. By definition, the savings should vanish for small t ; in most cases $s(t)$ should be concave with a single maximum. In our examples, $s(t)$ will be approximated by a quadratic function. [In reality, $s(t)$ should be determined from observed data. The quadratic form is used for the examples because it is likely to be a good approximation and because it yields simple and intuitive mathematical results.]

Presented with a storage price function $p(t)$, the shipper is assumed to choose the length of stay t^* that maximizes its actual (net) savings: $s(t) - p(t)$. This is represented by the vertical separation between the two curves in Figure 3a. For the optimal t^* , the marginal savings obtained by using the storage must equal its marginal cost. This can be written as

$$s'(t^*) = p'(t^*) \quad (3)$$

In practice, it is easier to estimate $s'(t)$ than $s(t)$. Because $s'(t)$ suffices to determine t^* (see Figure 3b), the marginal savings and storage price curves $s'(t)$ and $p'(t)$ are often worked with.

Additional measures of performance obtained from the marginal curves include the shed/warehouse revenue per unit flow:

$$p(t^*) = p(0) + \int_0^{t^*} p'(t) dt \quad (4)$$

The shipper's total savings per unit flow, equal to the area between $s'(t)$ and $p'(t)$ in the interval $[0, t^*]$ (see Figure 3b):

$$s(t^*) - p(t^*) = s(0) - p(0) + \int_0^{t^*} (s'(t) - p'(t)) dt \quad (5)$$

The sum of the storage revenue and the shipper's net savings, corrected by the cost of operating the storage facility per unit of flow, $h(t)$, is a measure of total benefit per unit flow (or "system benefit") generated by the operation of the facility, w . Because $h(t)$ should be nearly independent of t if the storage facility is below capacity, it is assumed that it is constant, that is, $h(t) = h$. Thus, the system benefit is

$$w = s(t^*) - h = + \int_0^{t^*} s'(t) dt, \quad (6)$$

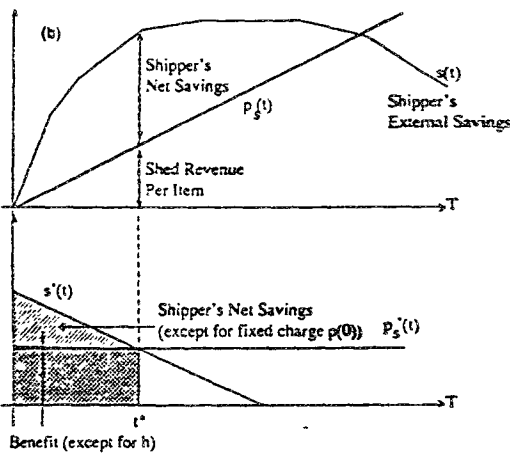
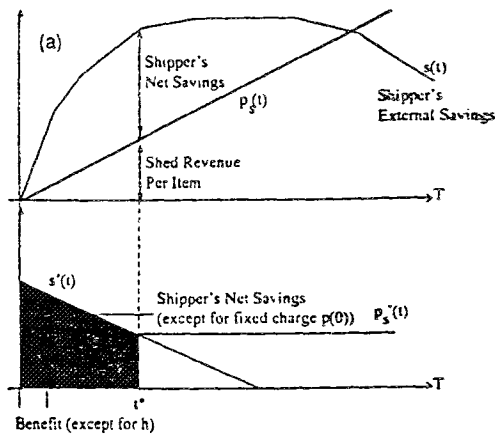


FIGURE 3 (a) Savings and price functions for one shipper. (b) Marginal savings and price functions for one shipper.

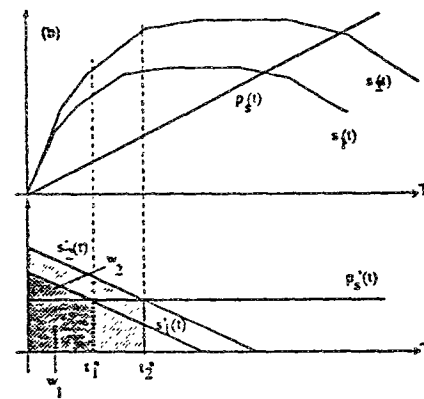
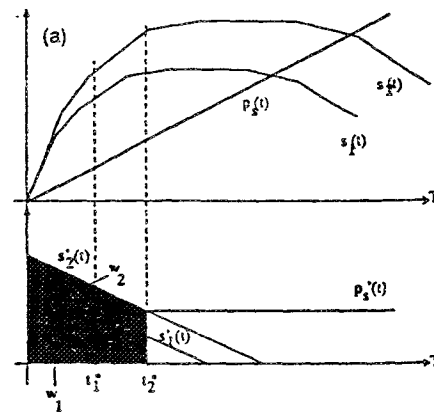


FIGURE 4 (a) Savings and price functions for two shippers. (b) Marginal savings and price functions for two shippers.

which, except for the constant h , is the shipper savings; that is, the area below $s'(t)$ in the interval $[0, t^*]$ as depicted in Figure 3b. Because system benefit is independent of $p(t)$ for a given t^* , any two price functions yielding the same t^* also yield the same system benefit.

Although the storage/retrieval cost is assumed to be fixed for a given storage facility, this cost can be quite different for different facilities. If a warehouse is remotely located, then its fixed storage/retrieval cost, h_w , will be much greater than the equivalent cost for a shed, h . This will become important when systems with two storage facilities are considered, as total system benefit will be used for comparing strategies.

In later sections, differences across shippers will be captured by differences in their external savings functions. These differences will be the result of the shippers' inland locations, the value of their freight, and so on. Pricing strategies that differentiate across commodities can also be easily constructed. They are discussed in the conclusion. Figure 4 shows the external savings functions for two shippers and a price function; it also depicts the marginal savings functions, the tariff (marginal storage price) function, the desired storage times, and the system benefit per unit flow for the two shippers.

One can see at a glance that if the tariffs, $p'(t)$, were to be increased, t_1^* and t_2^* would decrease, and so would the total system benefit. Thus, one would like to lower $p'(t)$ as much as possible subject to the storage capacity limitations. This point will be addressed in the next section.

The relationship between system benefit and tariffs can also be captured analytically. If the marginal price $p'(t)$ and marginal external savings $s'(t)$ functions are linear

$$p'(t) = \alpha + \beta t \quad \text{and} \quad (7)$$

$$s'(t) = a - b t \quad (8)$$

Then, assuming that $a > \alpha$, t^* and w are given by

$$t^* = (a - \alpha)/(b + \beta) \quad (9)$$

$$w = \frac{b(a^2 - \alpha^2) + 2a\beta(a - \alpha)}{2(b + \beta)^2} - h \quad (10)$$

As expected, t^* and w decrease if the tariff coefficients (α and β) increase.

NO WAREHOUSES

Deterministic Demand

In this section, a situation where many shippers must utilize a single storage facility is investigated. Each shipper sends or receives through the port q_i volume units of freight per unit time, at a steady rate ($\sum q_i = q$). The external savings function for shipper i is denoted $s_i(t)$.

A pricing policy that maximizes system benefit while ensuring that the shed capacity is not exceeded is sought. A discriminatory pricing policy would allow different price functions for different customers; in the most general case, each customer could be offered a different price function $p_{si}(t)$. A nondiscriminatory pricing policy would assume that all shippers are treated equally, with the same shed price function $p_s(t)$ for all. Nondiscriminatory policies are more common, but they may also be less efficient since they embody additional restrictions.

The remainder of this subsection shows that for the current situation—with no warehouse and steady demand—discriminatory and nondiscriminatory strategies are equivalent; in fact, a constant tariff is optimal: $p_{si}(t) = p_s^*$.

Because total system benefit per day, W , only depends on the price policy through the equilibrium times t_i^* for each shipper

$$W = \sum_{i=1}^n q_i [s_i(t_i^*) - h] \quad (11)$$

and because the total freight accumulation in the shed at any time is also a function of these variables

$$V_{avg} = \sum_{i=1}^n q_i t_i^* \quad (12)$$

only the optimal t_i^* need to be found. Any price functions yielding these t_i^* will be optimal. The optimal times maximize W , subject to $V_{avg} \leq C$ and $t_i \geq 0$ (for all i).

If the maximization of system benefit without the capacity constraint yields a V_{avg} strictly smaller than C , then the resulting times are optimal. These are the times that maximize the individual s_i curves, which are obtained for a pricing policy with zero tariff. Thus, if shed space is plentiful, then allowing free storage maximizes system benefit. System benefit is also maximized if the port charges a fixed price per unit volume independent of length of stay, provided the charge is so low that no shippers are discouraged from using the shed.

If, as is more likely, shed space is at a premium, the capacity constraint will hold as an equality. Consideration reveals that any positive t_i^* must satisfy for optimality

$$s_i'(t_i^*) = \alpha \quad (13)$$

where α is the Lagrange multiplier for the capacity constraint. To achieve this result the discriminatory pricing functions must satisfy

$$p_{si}'(t_i^*) = s_{si}'(t_i^*) = \alpha \quad (14)$$

Note that α can be viewed as the optimal tariff at each t_i^* .

Because it is the same for all i , discrimination is clearly unnecessary. A nondiscriminatory pricing function with constant tariff should satisfy the above condition. Simply let $p_{si}(t) = \alpha t$, for all i , and increase or decrease α until the average volume in the shed closely matches its capacity.

This simple policy maximizes system benefit without any knowledge of individual shipper behavior.

Example

Let us consider a simple case where the $s_i(t)$ are quadratic functions $s_i(t) = a_i - b_i t$.

For a given α , the condition $\{s_i'(t_i^*) = \alpha; \text{ if } t_i^* > 0\}$ yields

$$t_i^* = \max \left\{ 0, \frac{a_i - \alpha}{b_i} \right\} \quad (15)$$

This expression recognizes that the shipper can only benefit from storage if $\alpha < a_i$. If $\alpha > a_i$, then the tariffs increase too rapidly for the shed to be of use to shipper i . (In Figure 3a, the pricing curve would be steeper than the external savings curve near the origin, and thus $t_i^* = 0$.)

As α is increased, thus, shippers with the smallest a_i are excluded from the shed—or are forced to use it for a minimal amount of time. If α is optimal, the remaining shippers must use up the shed's capacity; that is

$$V_{avg} = \sum q_i t_i^* = \sum q_i a_i / b_i - \alpha \sum q_i / b_i \quad (16)$$

where the summations are only taken for i such that $a_i > \alpha$. A simple expression for α is obtained if all the a_i are large, so that no shippers are excluded. The summations in Equation 16 then are independent of α , and

$$\alpha^* = \frac{\sum q_i a_i / b_i - C}{\sum q_i / b_i} \quad (17)$$

A simple computer spreadsheet was developed using these expressions. The spreadsheet can be used to test different price functions when the shipper data are given; the shed price functions and the external savings functions are assumed to be quadratic. In addition to system benefit, other performance measures, such as shed revenue and percent of occupied capacity, are calculated.

In this example, the optimal shed price function for a situation where five shippers must utilize the shed is calculated. The data set is as follows:

Shipper	1	2	3	4	5
a_i	10	11	12	13	14
b_i	0.5	0.5	0.5	0.5	0.5
q_i	500	500	500	500	500

The static shed capacity, C , is 2,000 units.

Expression 7 predicts $\alpha^* = 8$. The spreadsheet confirms that nonlinear price functions are inferior and that the best pricing policy is indeed to charge a flat rate of \$8.00 per unit of cargo per day. The resulting system benefit table, showing benefit values in thousands of dollars per day for different α and β , is partially reproduced in the following (negative system benefit values indicate shed overflow):

α	β		
	0.0	0.1	0.2
7.00	-235.00	117.20	186.99
7.25	-141.97	203.19	179.38
7.50	-38.89	194.70	171.62
7.75	76.03	186.00	163.70
8.00	205.00	177.08	155.61
8.25	194.84	167.95	147.37
8.50	184.38	158.59	138.97
8.75	173.59	149.02	130.40
9.00	162.50	139.24	121.68

Stochastic Demand

More realistically, it is now assumed that the volumes shipped change from day to day, without any seasonal trend. Then, the volume from shipper i arriving on any given day can be viewed as the outcome of a random variable Q_i with time-independent mean and variance:

$$E[Q_i] = q_i \quad \text{and} \quad (18)$$

$$\text{var}[Q_i] = I_i q_i \quad (19)$$

where I_i is a coefficient with volume units.

The volume in the shed, V_i , can also be viewed as a random variable changing from day to day. Because the system is ergodic, Little's formula holds and $E(V_i) = q_i t_i$, where t_i is the average time in storage for items i .

The variance of V_i depends on the behavior of shippers, but the expression

$$\text{var}[V_i] = q_i t_i I_i \quad (20)$$

will be used for illustrative purposes. This expression holds if shipper i sends (receives) constant size shipments so infrequently that two of its shipments are almost never in storage simultaneously. (In that case, the constant I_i can be shown to represent the size of a shipment.) The expression also holds for frequent and variable size shipments, provided that all the shipments remain in storage for a fixed time t_i .

If shippers act independently, then the total volume in storage $V = \sum V_i$ must satisfy

$$E[V] = \sum q_i t_i \quad \text{and} \quad (21)$$

$$\text{var}[V] = \sum q_i t_i I_i \quad (22)$$

Without a warehouse, overflow must be avoided. Thus, the capacity constraint is modified as follows:

$$\sum q_i t_i + K \left(\sum q_i t_i I_i \right)^{1/2} \leq C, \quad (23)$$

where K is a number of standard deviations (comparable with 3) that will ensure that random fluctuations in the shed's accumulation are unlikely to reach its capacity.

If the coefficients of variation I_i are different from zero, the Lagrangian optimality condition no longer implies that all $s_i(t_i)$ should be equal, as was the case in the deterministic problem. It is now

$$s_i(t_i) = \alpha \left[1 + K I_i / 2 \left(\sum q_i t_i I_i \right)^{-1/2} \right]. \quad (24)$$

This indicates that discriminating pricing functions, which allow shippers with small I_i to stay longer, may be desirable. The same system benefit level can be achieved with a non-discriminatory price function satisfying $p_i'(t_i) = s_i'(t_i)$; this function will exist if all the t_i are different.

Because the best nondiscriminatory function is likely to be awkwardly shaped, in practice one may want to select the best candidate from a family of acceptable price functions, even if the resulting system benefit is lower. This problem can be solved easily. One would express t_i as a function of the parameters in the price function, which would then become the decision variables of the optimization problem: maximizing W , subject to the stochastic capacity constraint. Because a reasonable family of functions would only include a few parameters (e.g., 3 at most), the optimization problem can be solved easily within the scope of a computer spreadsheet.

Example

An example with only two shippers is used because the optimal solution can then be easily obtained analytically, for comparison with the numerical spreadsheet solution.

The data are as follows:

Shipper	1	2
a_i	10	12
b_i	0.5	0.5
q_i	500	600
I_i	400	1000

The safety coefficient, K , is 2, and the shed capacity is still 2,000 units.

The analytical solution, obtained using Equation 25, is $\alpha = 4.075$ and $\beta = 0.204$.

If this price functions were adopted, cargo from Shippers 1 and 2 would spend 8.414 and 11.254 days in storage, respectively, and the average shed accumulation would be 10,955 units ($\sum q_i t_i$), with 9,045 units of storage to spare as a buffer. The total system benefit would be \$115,928 per day.

The spreadsheet finds $\alpha = 5.25$ and $\beta = 0.1$ as the optimal coefficients, yielding a system benefit of \$114,447 per day:

α	β			
	0.00	0.10	0.20	0.30
4.25	-858.28	-872.93	114.41	103.71
4.50	-861.13	-875.92	111.47	100.89
4.75	-864.13	-879.02	108.44	98.00
5.00	-867.30	-882.24	105.34	95.05
5.25	-870.63	114.44	102.15	92.04
5.50	-874.13	111.00	98.87	88.97
5.75	-877.78	107.44	95.52	85.83
6.00	-881.60	103.78	92.08	82.63
6.25	114.42	100.00	88.56	79.36

Although the coefficients α and β are different from the analytical ones, for t in the range of optimality (8 to 12), the two $p_i'(t)$ and the corresponding times in storage are very close. For the new set of parameters, the times would be 7.92 and 11.25 days (as opposed to 8.414 and 11.254). The total system benefit in both cases is also similar: \$114,437/day versus \$115,928/day.

Although the solution obtained using the spreadsheet is marginally worse than the one obtained analytically, the

spreadsheet method can be applied to cases in which there are many shippers. The spreadsheet also supplies at-a-glance information on other measures of performance and can accommodate simple constraints easily.

SHEDS AND WAREHOUSES

In this section, a case in which cargo can be stored either at the shed or at one or more remotely located warehouses is analyzed. Although shed capacity is limited, it is assumed that enough warehousing space is made available to accommodate demand; that is, there is no capacity restriction at the warehouse. Because shed overflows can now be routed to the warehouse without serious disruptions to port operation, stochastic phenomena need not be considered as explicitly as in the previous section. Focus here is on a deterministic model and stochastic effects are discussed qualitatively.

For the maximization of system benefit, it is assumed that the cost of sending one unit of flow through the warehouse is given by an increasing function of the time in storage $t_w: h_w(t_w)$. Paid by the port, the warehouse or the public (but not by the shipper who is charged a fee $p_w(t_w)$ for the service), this cost accounts for handling inside the warehouse, transportation between the port and warehouse, the provision of secure storage space, as well as noise and congestion in the surrounding area. In most cases, $h_w(0)$ is considerably greater than the handling cost through the shed h_s .

Two related questions are examined: For a given warehouse price function $p_w(t)$ outside the port's control, how should the shed price function be chosen? If $p_w(t)$ is under the port's control, how should the two price functions be chosen jointly? The answer to the first question will help with the second.

Fixed Warehouse Price Function

Given shed and warehouse price functions, it is assumed that shippers choose the most cost-effective duration and form of storage. As before, pricing strategies will be compared on the basis of their contribution to system benefit (i.e., joint benefit to port and shippers). It is assumed that the given warehouse price function is nondiscriminatory. Therefore, the following quantities associated with shipper i are fixed as follows:

- t_{wi} = shipper's chosen storage time at the warehouse, as explained previously;
- s_{wi} = shipper's external savings per unit volume if the warehouse is used; that is, $s_i(t_{wi})$;
- h_{wi} = cost generated by the shipment of said volume unit: $h_w(t_{wi})$;
- w_{wi} = system benefit generated by the same volume unit: $s_{wi} - h_{wi}$.

In addition to these constants, the total system benefit generated per day is only a function of the fraction of flow sent by each shipper through the shed x_i , and the associated time in storage t_{si} . The total system benefit is

$$W = \sum q_i \{x_i [s_i(t_{si}) - h_s] + (1 - x_i) w_{wi}\}. \quad (25)$$

If the system benefit obtained when all the flow is routed

through the warehouse (a constant, $\sum q_i w_{wi}$) is subtracted from this expression, an equivalent objective function W' is obtained:

$$W' = \sum q_i \{x_i [s_i(t_{si}) - h_s - w_{wi}]\}. \quad (26)$$

This expression can be interpreted as the shed's contribution to system benefit. We seek the $0 \leq x_i \leq 1$ and $t_{si} \geq 0$ that maximize W' while satisfying the shed capacity constraint

$$\sum q_i x_i t_{si} \leq C. \quad (27)$$

As occurred in the previous section, if two shippers use the shed— $(x_i, t_{si}, t_{sj}) > 0$ —then their marginal external savings must be equal: $s'_i(t_{si}) = s'_j(t_{sj})$. The argument is simple. If $s'_i > s'_j$, then increasing the time in the shed by a small amount $\epsilon/(q_i x_i)$ for shipper i , and decreasing it by $\epsilon/(q_j x_j)$ for shipper j , satisfies all the constraints and increases system benefit by $\epsilon (s'_i - s'_j) > 0$.

As a result, if the x_i are given, the positive t_{si} in the optimal solution must satisfy $s'_i(t_{si}) = \alpha$ for some α . It is not difficult to see along the same arguments that if one shipper j does not use the shed, the $s'_j \leq \alpha$. Clearly, α represents a tariff; if α was known the t_{si} could be identified as per the construction of Figure 3a, with a price function $p_s(t) = \alpha t$. The problem thus reduces to finding α and $\{x_i\}$.

Because the t_{si} are fixed conditional on α , for a given α the optimal $\{x_i\}$ are the solution to a knapsack maximization problem with W' as the objective function and $\sum q_i x_i t_{si} \leq C$ as the constraint. The optimal solution, thus, satisfies

$$\begin{aligned} x_i &= 0, \text{ if } [w_{wi} - w_{wi}]/t_{si} < \tau, \\ 0 \leq x_i \leq 1, & \text{ if } [w_{wi} - w_{wi}]/t_{si} = \tau, \text{ and} \\ x_i &= 1, \text{ if } [w_{wi} - w_{wi}]/t_{si} > \tau \end{aligned} \quad (28)$$

for a constant τ that ensures the capacity constraint is met as an equality. The resulting system benefit $W'(\alpha)$ should then be compared with the system benefit for other tariffs; the largest can be chosen.

Note that the optimal tariff should be the same for all shippers, as happened in the previous section. The optimal splits $\{x_i\}$ can be obtained with discriminatory shed price functions (with the right ordinates at the optimal t_{si} to ensure that the shipper's choice is as desired); also as before, this would require information on the individual $s_i(t)$ functions.

Nondiscriminatory Policies

In the absence of this information—or if price functions must be kept fair and simple—we may wish to choose a nondiscriminatory price function with constant tariff, $p_s(t) = \alpha t$, and let each shipper choose its split and storage times.

The construction of Figure 3a reveals that t_{si} and $s_i(t_{si})$ are decreasing functions of α . Because the attractiveness of the shed to shipper i (as measured by $s_i(t_{si}) - \alpha t_{si}$) decreases with α , x_i also decreases with α . Consequently, both W' and the left side of the shed's capacity constraint decrease with α .

Obviously, thus if one wishes to accommodate the resulting shed volumes without overflow (e.g., to avoid disgruntled

customers), the smallest tariff consistent with the shed's capacity must be optimal. No information is needed to reach this decision.

If the demand varies unpredictably from day to day and overflows are to be avoided, the tariff should be a little larger. The average accumulation in the shed will then be a little smaller than its capacity, allowing the accumulation fluctuations to be absorbed. The desired tariff would satisfy

$$W' = \sum q_i \{x_i [s_i(t_{si}) - h_s - w_{wi}]\} \tag{29}$$

$$\sum q_i x_i t_{si} + K (q_i x_i t_{si} I_i)^{1/2} = C \tag{30}$$

(Note that the left side of this equality still decreases with α .)

If overflows are acceptable, then it may be optimal to set a tariff so low that systematic overflows ensue even in the deterministic case. But detailed information on the $s_i(t)$ is needed to determine the precise tariff and the value of $W'(\alpha)$. If this information is available, one might want to choose the price function from a larger family of curves (e.g., quadratic).

For a given $p_i(t)$ shipper i 's decisions (x_i and t_{si}) are known. These can be used to determine the proportion of shed traffic that is not diverted to the warehouse, y

$$y = \min \left\{ 1, \frac{C}{\sum q_i x_i t_{si}} \right\} \tag{31}$$

In the deterministic case, if all the shippers have the same probability of being routed to the warehouse (against their wishes), then it is a simple matter to calculate $W'(\alpha)$

$$W' = \sum q_i \{y x_i [s_i(t_{si}) - h_s - w_{wi}]\} \tag{32}$$

For stochastic demands, the expression for W' is identical, but the overflow will be somewhat greater than y . The appropriate queuing expression (e.g., for a multichannel queue without a buffer, as would apply to telephone systems) should be used.

The best price function can be found by testing the members of the price function family using a spreadsheet. In all cases though, if some traffic is flowing to the warehouse the shed must be fully used.

Example

In this example, five shippers may use a shed or a warehouse for temporary storage. The table below summarizes the data for the problem:

Shipper	1	2	3	4	5
a_i	10	11	12	13	14
b_i	0.5	0.5	0.5	0.5	0.5
q_i	500	500	500	500	500

The capacity of the shed is 20,000 units, and the warehouse is assumed to have unlimited capacity. The handling cost associated with shed usage, h_s , is 5 \$/unit, and use of the warehouse costs $h_w(t) = 40 + t$ \$/unit. The price of warehouse storage to the shipper is $p_w(t) = 50 + 2t$ \$/unit.

Initially, let us determine a nondiscriminatory policy with a constant tariff such that all shed volume can be accom-

modated without overflow. As discussed earlier, in this case the optimal policy is to charge the smallest tariff consistent with the capacity of the shed.

In practice, the desired result could be achieved by starting with a very high tariff and decreasing it until the shed reached its capacity, or by starting with a low tariff and increasing it until no more shed overflow were observed. For the data presented in the preceding, the spreadsheet indicates that the lowest no-overflow tariff would be 4.6 \$/day/unit.

If this tariff were adopted, Shippers 1, 2, and 3 would use the shed, storing their cargo for about 11, 13, and 15 days, respectively. Shippers 4 and 5 would choose to use the warehouse for 22 and 24 days. The shed would be almost fully utilized, with no overflow, and the total system benefit generated would be approximately \$259,000/day.

It will now be assumed that all the preceding information is available to the shed authority, and pricing policies that create systematic overflows are considered acceptable. The objective is simply to maximize system benefit, which can be accomplished by setting up a system benefit table analogous to the ones in the previous examples as follows.

α	β		
	0.00	0.10	0.20
2.00	260.00	259.60	254.34
2.25	260.32	259.07	253.07
2.50	260.53	258.39	251.64
2.75	260.61	257.57	253.54
3.00	260.56	256.59	251.82
3.25	260.36	255.44	245.63
3.50	260.00	256.91	242.66

As this table shows, it is possible to increase system benefit by reducing the shed tariff to 2.75 \$/day/unit. This tariff would cause approximately 57 percent of the traffic to be routed to the warehouse because of shed overflow, but the total system benefit would increase to approximately 261,000 \$/day. In this example, the availability of additional information would represent an additional system benefit of about 2,000 \$/day.

Variable Warehouse Price Function

The x_i and t_{si} that maximize W' remain the same whether $p_w(t)$ can be changed or not. We have already seen that for a given warehousing price function, there is a discriminating set of shed price functions that can achieve the optimum. The question now is whether the optimum can be achieved without discrimination.

We now show that the optimal system benefit is achieved if $p_w(t) = h_w(t)$, the cost of sending a unit of flow through the warehouse when the storage time is t , and $p_s(t) = h_s + \alpha t$, in which the α is the lowest shed tariff that avoids overflow.

With these price functions, the shed times only depend on α and are denoted by $t_{si}(\alpha)$. The shed will be chosen, $x_i = 1$, if

$$s_i(t_{si}(\alpha)) - [h_s + \alpha t_{si}(\alpha)] > s_{wi} - h_w(t) \text{ or}$$

$$\frac{w_{wi} - w_{wi}}{t_{si}(\alpha)} > \alpha \tag{33}$$

If this inequality is reversed, the shipper prefers the warehouse, $x_i = 0$; if the relationship is a pure equality the shipper is indifferent about the form of storage. If α is chosen equal to τ (as small as possible without creating overflow), then these conditions are identical to the knapsack condition for $\{x_i\}$, specified in the previous subsection. Therefore, the solution is optimal.

The conclusion is simple: system benefit is maximized if the storage facilities are priced at cost and a constant tariff is added to the fixed capacity shed to prevent overflows.

CONCLUSIONS

Temporary storage facilities and regular warehouses accomplish distinct functions and should therefore be analyzed and managed differently. Establishing shed pricing policies using procedures developed for regular storage facilities or by trial and error will usually lead to sub-optimal utilization of the facility.

Efficient use of temporary storage facilities at transportation terminals, not just ports, can be achieved through the adoption of rational pricing policies. To determine such policies, management must define the operational objectives of the facility, taking into account the consequences of overflow.

Optimal shed pricing policies are affected by the capacity of the sheds, by the characteristics of its users, and by the availability of warehouses. With this information, the shed pricing strategy that maximizes a given objective (e.g., system benefit, shed revenue, a combination of these, etc.) can be found using a computer spreadsheet, as demonstrated in the body of this report. If system benefit is the objective, the best shed pricing policy often is very simple and can be identified analytically.

Data requirements for the optimization are modest. Even in situations in which the $s_i(t)$ are needed, the quadratic approximations for the savings functions $s_i(t)$ should be adequate in most practical cases. That being the case, the coefficients a_i and b_i should be easily estimable from shippers' responses to past rate changes and/or from shipper surveys. An empirical determination of the best functional form for the $s_i(t)$ is beyond the scope of this paper, however, as it would require before and after data.

The results of this paper can be used to develop pricing schemes that discriminate across both shipper and commodity

type. Shippers that transport more than one commodity can be simply viewed as an aggregation of single-commodity shippers. If one wishes to discriminate across commodities only, all shippers transporting the same commodity would be viewed as a single shipper.

The results of this paper apply to terminals other than bulk and container ports, since nothing in the derivations was port specific. The model applies, for example, to the pricing of short-term and long-term airport parking services—if as a first approximation we ignore that h_w and h_s may depend on the traveler i . If both parking rates are determined by the airport commission then to maximize system benefit these services should be priced at cost, with a short-term parking surcharge proportional to time. The surcharge, perhaps changing seasonally, should be low enough to ensure that the short term lot is not underutilized.

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