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IN SIGNALLING FIRM VALUE

November, 1983
Revised, April, 1984

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Abstract

This paper analyzes the case of an entrepreneur who is the sole initial owner of a firm and who has private information as to the firm's true value. It is shown here that it is possible for the entrepreneur's chosen level of capital expenditures for the firm's production process to perfectly signal his private information, with the market's assessment of his firm's value a positive function of the expenditure level. The assessment is also demonstrated to be affected by the number of shares that the entrepreneur retains in the firm. In this sense it can be said that the investment and entrepreneurial shareholding level jointly signal the value of the firm, with investment serving as the "true" signal. Among other results it is shown, in contrast to a conclusion of a study by Leland and Pyle, that in this equilibrium, with shareholdings and investment jointly serving as signals, the number of shares retained by the entrepreneur need not be positively correlated with the favorableness of his information.

Introduction

When an entrepreneur first publicly sells shares in a wholly owned investment project he is likely to have better information about the project's future return than do potential investors. The major purpose of this paper is to show that in such a situation the entrepreneur's information may be perfectly revealed through the level of capital expenditures he chooses for the investment project; the higher the input level, the more favorable the information. This follows because input is costly to the entrepreneur; the more he incurs for a given level of shares retained in the firm, the higher must be the return that he expects from the project. The resulting signalling equilibrium has the property that, given the entrepreneur's shareholding level, there will be a greater level of capital expenditures in the firm's production process than if the expenditures did not serve as a signal. Such overinvestment in the signal is a typical result in signalling theory and follows in this case because a higher input level has the added benefit of increasing investors' assessment of the value of the firm.

The investment level, however, is not the only decision variable that can signal the entrepreneur's information. As Leland and Pyle (1) (henceforth LP) show for the case of a fixed input level, the level of shareholdings retained by a risk averse entrepreneur in his firm may serve as a signal; the more shares that the entrepreneur holds, the more favorable is his information. This result depends on the entrepreneur choosing his shareholdings so as to maximize the utility from the return on his portfolio holdings. Then, entrepreneurs who are willing to retain larger positions in their firms, and therefore hold less diversified portfolios, will be those whose projects generate higher returns. But, in reality other factors may constrain the entrepreneur's shareholding decision. For example, wealth constraints may set an upper bound on his shareholdings. In this case shareholdings cannot serve as

a signal; however, as is demonstrated, the investment level can still function as one. Its signalling ability is independent of whether or not shareholdings are constrained.

A further interesting question, one that cannot be addressed in the context of LP where the production level is fixed, arises when both the shareholding and production decisions of the entrepreneur are unconstrained—namely, which one will actually signal the entrepreneur's information? Given that either variable can serve as the signal, the entrepreneur will choose that one which results in the lower signalling cost to him. This cost will depend on the entrepreneur's level of risk aversion along with the form of the firm's production function. However, as is demonstrated here, when both variables are unconstrained they really jointly serve to signal the value of the firm. The variable that acts as the "true" signal is the one from which the valuation schedule is derived. But, that schedule is affected by the chosen level of the remaining variable. In the context here, with investment serving as a signal, it is shown that the shareholding level of the entrepreneur still affects the firm valuation schedule. For any given investment level the greater the number of shares retained by the entrepreneur the greater the valuation attached to the firm. Alternatively stated, the more shares that the entrepreneur is willing to hold, the lower will be the input level required to signal the firm's value. The more that the entrepreneur shows to the market his confidence in the firm's final output through his shareholdings, the less he need do so through the input level. The two variables, then, work together in signalling the entrepreneur's information.

The fact that both shareholdings and the production level jointly signal firm value has important empirical implications. Chief among them is the relation between the favorableness of the entrepreneur's information and the

level of shareholdings he retains in the firm. With the investment level assumed fixed, LP show that the better the entrepreneur's information, and therefore the greater the value of the firm, the larger the number of shares held by him. In the model presented here, as the favorableness of the information improves, the production level increases. However, because of the tradeoff between investment and shareholdings in signalling firm value, the shareholding level of the entrepreneur may actually decrease. A negative relation may exist between firm value and entrepreneurial shareholdings. Whether or not this negative correlation holds is shown to depend on the form of the firm's production function.

Although this analysis is done principally from the viewpoint of an entrepreneur first publicly selling shares, many of the results may be extended to a firm that is already trading publicly. In such a case private information held by the firm's manager about a new project's return can be perfectly signalled by the level of investment in the project as long as the manager is simultaneously a net seller of shares, on personal account, in the firm. This latter requirement will often be satisfied because capital expenditures are frequently financed by a sale of new shares, resulting in a dilution in the manager's shareholdings position (unless he buys at least a pro-rata portion of the new issue). Then, a higher input level will imply that the manager holds more favorable information.

Empirical support consistent with the theoretical model presented here is given by McConnell and Muscarella (2) who examine the effect of changes in firms' capital expenditure plans on their stock prices. They find that, in general, announcements of increases (decreases) in capital expenditures are followed by higher (lower) stock prices.

The analysis here points to an additional, but previously undiscussed, role for the firm's auditor. To see this note that the input level can serve

as a signal of firm value only if the announced level can be verified as the one actually chosen. The auditor is the one to provide such verification. Just by monitoring the accuracy of historical information, and without directly giving any additional information about the future returns of the firm, the auditor thereby serves a valuable role in investors' learning more about the firm's value.

1. Economic Setting

The economy consists of one good and two dates, 0 and 1, with investors maximizing their utility of date 1 consumption. Two assets are available for investment at date 0, one a risky firm initially owned entirely by an entrepreneur, and the other a riskless asset, assumed, for simplicity, to have a zero yield. The production function of the risky firm is assumed to be of the Diamond type, so that if state s occurs at date 1 and q_0 is the date 0 input, then the output at date 1 is $k_s f(q_0)$, where $f(q_0)$ is strictly increasing and concave and $f(0) = 0$. The date 1 output is normally distributed with mean $\bar{k}f(q_0)$, where \bar{k} is the mean of k_s , and with variance of $\text{var}(\tilde{k})f^2(q_0)$. (The tilde will be dropped in the analysis below for notational convenience.) It is assumed that all investors know $\text{var}(k)$ but that only the entrepreneur knows \bar{k} . All investors, apart from the entrepreneur, are risk neutral, while the entrepreneur is either risk neutral or risk averse with a negative exponential utility function.¹ Perfect competition is assumed to the extent that all investors, aside from the entrepreneur, believe that their shareholding decisions do not affect prices in the economy. However, the entrepreneur's shareholding and input decisions do affect prices since they cause investors' perceptions of the value of the firm to change.

2. The Signalling Equilibrium

Given that the risky asset's return is normally distributed, the entrepreneur's expected utility can be written as:

$$E(U) = E(W_{1s}) - \frac{1}{2} a \text{ var}(W_{1s}) \quad (1)$$

where:

$$W_{1s} = W_0 + (1-\alpha)v(q_0) - q_0 + \alpha k_s f(q_0) \quad (2)$$

is the entrepreneur's date 1 wealth in state s , $E(W_{1s})$ is its expectation over all states, and $\text{var}(W_{1s})$ is its variance;

W_0 = the entrepreneur's date 0 endowment of the riskless asset;
 α = fractional shareholdings retained by the entrepreneur in the firm;

$v(q_0)$ = date 0 market value of the firm;

a = entrepreneur's risk aversion parameter.

' a ' is greater than zero if the entrepreneur is risk averse and equal to zero if the entrepreneur is risk neutral.

Given that investors are risk neutral the firm will be valued at:

$$v(q_0) = \bar{k}(q_0)f(q_0) \quad (3)$$

where $\bar{k}(q_0)$ is investors' estimate of \bar{k} based on the entrepreneurs' chosen input level, q_0 . If the q_0 chosen perfectly signals the entrepreneur's information, then, at that input level:

$$\bar{k}(q_0) = \bar{k} \quad (4)$$

Using (3), the derivative of the entrepreneur's utility with respect to the investment level, q_0 , given a shareholding level, α , is:

$$\frac{\partial U}{\partial q_0} = (1-\alpha) \frac{\partial(k(q_0)\bar{f}(q_0))}{\partial q_0} - 1 + \alpha\bar{k}f'(q_0) - a\alpha^2\text{var}(k)f(q_0)f'(q_0) \quad (5)$$

$$= 0 \quad (6)$$

at the entrepreneur's optimum. Using the signalling condition (4), (6) can be rewritten as:

$$(1-\alpha) \frac{\partial\bar{k}(q_0)}{\partial q_0} f(q_0) - 1 + \bar{k}(q_0)f'(q_0) - a\alpha^2\text{var}(k)f(q_0)f'(q_0) = 0 \quad (7)$$

Solving this differential equation gives:^{2,3}

$$\bar{k}(q_0) = \frac{\int_0^{q_0} [f(x)]^{\frac{\alpha}{1-\alpha}} dx}{(1-\alpha)[f(q_0)]^{\frac{1}{1-\alpha}}} + \frac{a\alpha^2\text{var}(k)f(q_0)}{2-\alpha} \quad (8)$$

For ease of notation the function $f(q_0)$ will be restricted in the analysis below to be of the form eq_0^b , a power function, with $0 < b < 1$. This does not affect any of the subsequent results; they have all been proven using a general $f(q_0)$. With the restriction on $f(q_0)$, the signalling schedule (8) can be rewritten as:

$$\bar{k}(q_0) = \frac{q_0^{1-b}}{e[\alpha(b-1)+1]} + \frac{ae\alpha^2\text{var}(k)q_0^b}{2-\alpha} \quad (9)$$

Two important properties of the valuation schedule can readily be verified. The first is that $\bar{k}(q_0)$ is an increasing function of q_0 . This can be demonstrated by differentiating $\bar{k}(q_0)$ with respect to q_0 :

$$\frac{\partial\bar{k}(q_0)}{\partial q_0} = \frac{(1-b)q_0^{-b}}{e[\alpha(b-1)+1]} + \frac{abe\alpha^2\text{var}(k)q_0^{b-1}}{2-\alpha} \quad (10)$$

which is positive. The greater the input level, the greater the value attached to the entrepreneur's information. A second property, which immediately follows from this, is that the effect on firm value of a small increase in input, $\frac{\partial(\bar{k}(q_0)f(q_0))}{\partial q_0}$, is greater than $\bar{k}f'(q_0)$, the amount by which the value of the firm would change if investor beliefs were unaffected by the level of q_0 . This inequality holds because an increase in q_0 not only increases output but also raises investors' perceptions of the true value of the firm.

It is reasonable to expect that the investment level in the firm's production process can serve as a perfect signal. Given the entrepreneur's shareholding level, an increase in investment has a greater marginal benefit to him the more favorable his information (that is, the higher is \bar{k}). He is therefore willing to invest more (incur a larger signalling cost) the better his information. Looked at this way, it is easy to see that Spence's (3,4) requirement for existence of a signalling equilibrium is satisfied in this context. He showed, in a labor market setting, that for a signalling equilibrium to exist the marginal cost of signalling must be lower the more productive the worker. Here, although the marginal signalling cost is the same for all entrepreneurs, the marginal benefit is higher the better the entrepreneur's information.

For the remainder of the analysis it will be more convenient to work with the schedule $v(q_0)$ rather than $\bar{k}(q_0)$. Given (3) and (9):

$$v(q_0) = \frac{q_0}{\alpha(b-1)+1} + \frac{ae^{2\alpha} \text{var}(k)q_0^{2b}}{2-\alpha} \quad (11)$$

The signalling equilibrium is characterized by the following:

Proposition 1: Given the entrepreneur's level of final shareholdings, there will be a larger investment in the firm's production process when private information is being signalled as compared to when no information is being signalled.

Proof: If the entrepreneur has no private information (so that q_0 does not serve as a signal and all investors agree on the value \bar{k}), then the optimality condition (6) reduces to:

$$(1-\alpha)\bar{k}f'(q_0) - 1 + \alpha\bar{k}f'(q_0) - \alpha\alpha^2\text{var}(k)f(q_0)f'(q_0) = 0 \quad (12)$$

At the q_0 satisfying (12), the derivative of utility with respect to input when there is signalling (equation (5)) is:

$$\frac{\partial E(U)}{\partial q_0} = (1-\alpha) \left(\frac{\partial(\bar{k}(q_0)f(q_0))}{\partial q_0} - \bar{k}f'(q_0) \right) \quad (13)$$

With $\frac{\partial(\bar{k}(q_0)f(q_0))}{\partial q_0} > \bar{k}f'(q_0)$, the derivative is positive at that point. The entrepreneur therefore sets a higher input level in the case where he has private information.

Q.E.D.

Given the entrepreneur's final shareholdings, an increase in the input level has a greater positive impact on his utility when it acts as a signal since it raises investors' expectations. He therefore has a motivation to increase the input level above that which he would choose if he did not have private information. This is a typical result in signalling theory, that there will be overinvestment in the signal.

In order to derive further implications of the signalling equilibrium, note that the valuation schedule is a function of the shareholdings retained by the entrepreneur. Two cases need to be distinguished. The first is where the entrepreneur's shareholding decision is constrained and the second is where it is unconstrained.

2.1 The Case Where Shareholdings are Constrained

This will be the case, for example, if there is an upper bound on the entrepreneur's shareholdings due to financial constraints. Since the analysis in the preceding section did not specify how the entrepreneur's shareholdings were chosen, the signalling schedule (11) holds when shareholdings are constrained. In such a case the tradeoff between shareholdings and investment as signals of firm value can be most clearly seen.

Proposition 2: Given the entrepreneur's private information, an increase in his shareholding level in the firm will lower the input level required to signal his information and consequently will lower the value of the firm.

Proof: This follows by totally differentiating the signalling condition (4) with respect to q_0 and α :

$$\frac{\partial \bar{k}(q_0)}{\partial \alpha} d\alpha + \frac{\partial \bar{k}(q_0)}{\partial q_0} dq_0 = 0 \quad (14)$$

With $\frac{\partial \bar{k}(q_0)}{\partial q_0} > 0$, $\frac{dq_0}{d\alpha}$ has the opposite sign of $\frac{\partial \bar{k}(q_0)}{\partial \alpha}$.

$$\frac{\partial \bar{k}(q_0)}{\partial \alpha} = \frac{q_0^{1-b}(1-b)}{e[\alpha(b-1)+1]^2} + \frac{2ae\alpha \text{var}(k)q_0^b}{2-\alpha} + \frac{ae\alpha^2 \text{var}(k)q_0^b}{(2-\alpha)^2} \quad (15)$$

is positive since all terms on the right hand side of (15) are positive.

Therefore $\frac{dq_0}{d\alpha} < 0$. With \bar{k} unchanged, this also means that $\frac{dv(q_0)}{d\alpha} < 0$.

Q.E.D.

With all other parameters remaining fixed, if the entrepreneur becomes more heavily invested in the firm on his personal account, his utility will be more dependent on the output of the firm and less dependent on the price he receives for the shares he sells. There is therefore less of an incentive for any entrepreneur to signal falsely by raising the input level of the firm in order to convince the market that his information is more favorable than is actually the case. A lower level of investment will therefore be required by any entrepreneur to successfully distinguish himself from those with less favorable information. Looked at another way, the more the entrepreneur shows his confidence in the firm's output through his shareholdings, the less he need do so through the investment level. There is a tradeoff between shareholdings and investment in the signalling of firm value. As a consequence, with \bar{k} fixed the value of the firm is negatively correlated with the entrepreneur's shareholding level.

2.2 The Case Where Shareholdings are Unconstrained

Consider now that the entrepreneur's shareholding decision is unconstrained. The entrepreneur's optimal shareholdings are found by differentiating (1) with respect to α , giving:

$$\frac{\partial E(U)}{\partial \alpha} = (1-\alpha) \frac{\partial v(q_0)}{\partial \alpha} - v(q_0) + \bar{k}f(q_0) - \alpha \text{var}(k)f^2(q_0) \quad (16)$$

$$= 0 \quad (17)$$

at the entrepreneur's optimum. Given that $v(q_0) = \bar{k}f(q_0)$ at the signalling equilibrium and using the valuation schedule (11), (17) can be simplified to:

$$(1-\alpha) \left[\frac{q_0(1-b)}{(\alpha(b-1)+1)^2} + \frac{2ae^2\alpha\text{var}(k)q_0^{2b}}{2-\alpha} + \frac{ae^2\alpha^2\text{var}(k)q_0^{2b}}{(2-\alpha)^2} \right] - ae^2\alpha\text{var}(k)q_0^{2b} = 0 \quad (18)$$

or:

$$(1-\alpha) \frac{q_0(1-b)}{(\alpha(b-1)+1)^2} - \frac{ae^2\alpha^2\text{var}(k)q_0^{2b}}{(2-\alpha)^2} = 0 \quad (19)$$

Note from (16) that the optimal α is equal to one if the entrepreneur is risk neutral ($a=0$) since, in that case, $\frac{\partial E(U)}{\partial \alpha}$ is positive for all $\alpha < 1$. In contrast, when the entrepreneur is risk averse ($a > 0$), the optimal α must be less than one since $\frac{\partial E(U)}{\partial \alpha}$ is negative at that point. But, since at $\alpha=0$ the derivative is positive, he will hold some shares.

It is to be expected that the optimal α is one for a risk neutral entrepreneur, that is, that he should not sell shares in the firm if he does not face financial constraints. This is because, first, he does not gain utility through a more diversified portfolio. Second, by not selling any shares he can invest so as to maximize his personal valuation of the firm, $\bar{k}f(q_0) - q_0$, given his private information. He would set $\bar{k}f'(q_0) = 1$. However, if the entrepreneur sells shares, he must choose that input level which accurately signals his firm's true value, consequently investing more than the level which maximizes his personal valuation of the firm. (See equation (6). With $\alpha < 1$ and $a=0$, the chosen q_0 is such that $\bar{k}f'(q_0) < 1$.) The increased input level is a cost to him while there are no compensating benefits. However, if the entrepreneur is risk averse, he gains by diversification. He will, therefore, be willing to sell at least some of his shares. However, he will also

hold some shares. This is because at $\alpha = 0$ the entrepreneur's portfolio is riskless and so the entrepreneur is, at the margin, risk neutral. It is not costly for him to hold a small fraction of the firm's shares. Further, by holding some shares he benefits from the higher firm value that will result (given that $\frac{\partial \bar{k}(q_0)}{\partial \alpha}$ and therefore $\frac{\partial v(q_0)}{\partial \alpha} > 0$).

In this setting, the effect of changes in the entrepreneur's risk aversion level and of changes in the favorableness of his information can be profitably analyzed.

2.2.1 The Effect of Risk Aversion

It might be expected that as the entrepreneur becomes less risk averse he will increase the level of investment in the firm's production process. Given his shareholding level, the value to him of increased output is greater because the increase in the variance of output has less of an adverse effect on him. Alternatively stated, because of the greater benefit of increasing investment, it takes a larger investment level for the entrepreneur to distinguish himself from those with less favorable information. This reasoning is correct if the entrepreneur's shareholding level remains fixed. However, as risk aversion decreases, optimal shareholdings increase. This introduces an additional effect, as demonstrated in Proposition 2. That is, with investment and shareholdings acting as joint signals, the increase in shareholdings reduces the investment level required to signal the value of the firm, everything else the same. Because of these two opposing effects, then, it is unclear how the investment level will change as a result of a change in the entrepreneur's risk aversion level. As shown in the following proposition, the sign of the relation depends on the form of the firm's production function.

Proposition 3: As the entrepreneur's risk aversion decreases, α increases, while for $b < \frac{1}{2}$, q_0 and consequently $v(q_0)$ decrease, and for $b > \frac{1}{2}$, q_0 and $v(q_0)$ increase.

Proof: See Appendix A.1.

It follows from this proposition that as risk aversion decreases toward $a=0$ and optimal shareholdings consequently increase toward $\alpha=1$, the input level approaches that chosen by a risk neutral entrepreneur who does not sell any of his shares in the firm. As discussed before, this is the q_0 that satisfies the condition $\bar{k}f'(q_0) = 1$. From Proposition 3, if $b < \frac{1}{2}$, the input level chosen approaches this q_0 from above and if $b > \frac{1}{2}$ it approaches it from below. This proposition also indicates that, when shareholdings are chosen so as to maximize the entrepreneur's utility, the value of the firm will be negatively correlated with his shareholding level, as risk aversion changes, if the production function is concave enough.

2.2.2 The Effect of a Change in the Entrepreneur's Information

An important empirical prediction of the model concerns the relation between the favorableness of the entrepreneur's information and his chosen shareholding level. With the investment level assumed fixed LP show that there will be a positive relation between the favorableness of the information, or firm value, and the entrepreneur's shareholding level. As the following proposition states, this conclusion does not necessarily follow if the investment level can also be chosen optimally by the entrepreneur:

Proposition 4: As the favorableness of the entrepreneur's information, \bar{k} , improves, q_0 and consequently $v(q_0)$ increase, while α increases if $b < \frac{1}{2}$ and decreases if $b > \frac{1}{2}$.

Proof: See Appendix A.2.

The reason for the ambiguous direction of change in shareholdings is that as the favorableness of the entrepreneur's information increases there are two partially offsetting effects. The first effect is an increase in shareholdings required to have firm value properly signalled, keeping investment unchanged. (This is the cause for LP's result.) The second effect comes again from the fact that investment and shareholdings act together to signal firm value. As the favorableness of the information increases, optimal investment increases, thereby decreasing the shareholding level required for the value of the firm to be properly signalled. If the production function is concave enough ($b < \frac{1}{2}$), the first effect dominates the second and the optimal α increases. But if the production function is less concave ($b > \frac{1}{2}$), the second effect dominates the first and the optimal α decreases.

In contrast to the result of LP, if the production function is not too concave, then, for any given entrepreneur (and risk aversion level), as \bar{k} increases, the entrepreneur's shareholding level will be negatively correlated with the true value of the firm. The more favorable information causes the entrepreneur to expand his input level, increasing the value of the firm, while at the same time allowing him to reduce his optimal shareholdings. For these production functions it is optimal for the entrepreneur to trade off increased input against lower shareholdings.

3. Extension to a Firm Already Trading Publicly

While the previous results were shown for the case of a firm first going public, they may be extended to a firm already publicly trading. To see this, assume, similar to the preceding analysis, that the firm is faced with a new

investment opportunity and that the firm's manager knows its expected return while outside investors do not. For simplicity, assume also that the firm has no other assets. If the manager has control over the investment decision, he will choose the input level, q_0 , satisfying the first order condition (analogous to equation (6)) of:

$$(\bar{\alpha} - \alpha) \frac{(\partial \bar{k}(q_0) f(q_0))}{\partial q_0} - \bar{\alpha} + \alpha \bar{k}(q_0) f'(q_0) - a\alpha^2 \text{var}(k) f(q_0) f'(q_0) = 0 \quad (20)$$

where $\bar{\alpha} < 1$ is the manager's endowed fractional shareholding level in the firm. Solving this differential equation for $f(q_0) = eq_0^b$ gives the valuation schedule:

$$\bar{k}(q_0) = \frac{\bar{\alpha} q_0^{1-b}}{e[\alpha(b-1) + \bar{\alpha}]} + \frac{ae\alpha^2 \text{var}(k) q_0^b}{2\bar{\alpha} - \alpha} \quad (21)$$

It can easily be verified that a sufficient condition for $\bar{k}(q_0)$ to be an increasing function of q_0 is that $\bar{\alpha} > \alpha$; that is, that the manager be a net seller of shares on personal account.^{4,5} This is not surprising since $\bar{k}(q_0)$ has already been shown to be increasing in q_0 for an entrepreneur who is a net seller of shares. This condition is likely to be satisfied in many cases since the public sale of shares to finance the investment dilutes the manager's position in the firm (unless he buys at least a pro-rata share of the new stock issue). In these cases the level of investment will then, again, act as a signal of the manager's private information, with a higher investment level associated with a higher firm value.

Empirical evidence consistent with this conclusion is provided by McConnell and Muscarella (3) who study the effect of changes in capital expenditures on firm value. They find that an announcement by a firm of an increase (decrease) in its capital expenditures is generally accompanied by an increase (decrease) in its stock price.

4. Summary and Conclusions

It was demonstrated here that the amount of investment that an entrepreneur makes in his firm's production process can be a perfect signal of the true value of the firm. The greater the amount that the entrepreneur invests, the greater will be the value assigned to the firm by investors. In addition, although not acting as the "true" signal, the more shares the entrepreneur retains in the firm, the higher will be the value assigned given any investment level. Investment and shareholdings then essentially jointly act to signal firm value. Because of this, contrary to a result of LP, as the favorableness of the entrepreneur's information changes there need not be a positive correlation between the entrepreneur's shareholdings and the market value of his firm. Finally it was shown that the investment level may also serve as a signal for firms that are already publicly traded, a result consistent with empirical evidence.

It must be emphasized here that investment can only serve as a signal if the level undertaken can be verified. Verification of such historical information is one of the tasks of auditors. This study, then, highlights an unacknowledged benefit of the auditing function. Even though it is dealing only with historical information it allows investors to more accurately value the firm by permitting the investment level to serve as a signal.

AppendixA.1 Proof of Proposition 3

To simplify the proof the following notation is employed:

$$X \equiv \frac{q_0}{\alpha(b-1)+1}$$

and

$$Y \equiv \frac{ae^{2\alpha} \text{var}(k) q_0^{2b}}{2-\alpha}$$

Then the signalling condition that $v(q_0) = \bar{k}eq_0^b$ at the chosen q_0 can be re-written as:

$$X + Y - \bar{k}eq_0^b = 0 \tag{A1}$$

while the optimality condition for shareholdings, (19), reduces to:

$$(1-\alpha)\frac{\partial X}{\partial \alpha} - \frac{Y}{2-\alpha} = 0 \tag{A2}$$

To analyze how α and q_0 vary with 'a', (A1) and (A2) must be totally differentiated with respect to these variables as 'a' changes. Doing so gives:

$$\left(\frac{\partial X}{\partial q_0} + \frac{\partial Y}{\partial q_0} - b\bar{k}eq_0^{b-1}\right)dq_0 + \left(\frac{\partial X}{\partial \alpha} + \frac{\partial Y}{\partial \alpha}\right)d\alpha + \frac{\partial Y}{\partial a} da = 0 \tag{A3}$$

and:

$$\left((1-\alpha)\frac{\partial^2 X}{\partial \alpha \partial q_0} - \frac{\partial Y}{\partial q_0}\right)dq_0 + \left((1-\alpha)\frac{\partial^2 X}{\partial \alpha^2} - \frac{\partial X}{\partial \alpha} - \frac{\partial Y}{\partial \alpha} - \frac{Y}{(2-\alpha)^2}\right)d\alpha - \frac{\partial Y}{\partial a} da = 0 \tag{A4}$$

Combining (A3) and (A4) gives:

$$\begin{aligned} & \left[\left(\frac{\partial X}{\partial q_0} + \frac{\partial Y}{\partial q_0} - b \bar{k} e q_0^{b-1} \right) + (2-\alpha)((1-\alpha) \frac{\partial^2 X}{\partial \alpha \partial q_0} - \frac{\partial Y}{\partial q_0}) \right] \frac{dq_0}{da} \\ + & \left[\frac{\partial X}{\partial \alpha} + \frac{\partial Y}{\partial \alpha} + (2-\alpha)((1-\alpha) \frac{\partial^2 X}{\partial \alpha^2} - \frac{\partial X}{\partial \alpha} - \frac{\partial Y}{\partial \alpha} - \frac{Y}{(2-\alpha)^2}) \right] \frac{d\alpha}{da} = 0 \end{aligned} \quad (A5)$$

Denote by A the first bracketed term and by B the second bracketed term.

Then (A5) becomes:

$$A \frac{dq_0}{da} = - B \frac{d\alpha}{da} \quad (A6)$$

Substituting this in (A4) and rearranging gives:

$$\frac{\left[(2-\alpha)((1-\alpha) \frac{\partial^2 X}{\partial \alpha \partial q_0} - \frac{\partial Y}{\partial q_0}) B - (2-\alpha)((1-\alpha) \frac{\partial^2 X}{\partial \alpha^2} - \frac{\partial X}{\partial \alpha} - \frac{\partial Y}{\partial \alpha} - \frac{Y}{(2-\alpha)^2}) A \right] \frac{dq_0}{da}}{B} = \frac{\partial Y}{\partial a} \quad (A7)$$

With $\frac{\partial Y}{\partial a} > 0$, $\frac{dq_0}{da}$ has the same sign as its coefficient. Given (A2), B can be simplified to:

$$B = -2(1-\alpha) \frac{\partial X}{\partial \alpha} + (2-\alpha)(1-\alpha) \frac{\partial^2 X}{\partial \alpha^2} \quad (A8)$$

With $f(q_0) = eq_0^b$:

$$\frac{\partial^2 \chi}{\partial \alpha^2} = \frac{2(1-b)}{(\alpha(b-1)+1)} \frac{\partial \chi}{\partial \alpha} \quad (\text{A9})$$

B then further simplifies to:

$$B = [-2(1-\alpha) + \frac{2(2-\alpha)(1-\alpha)(1-b)}{\alpha(b-1)+1}] \frac{\partial \chi}{\partial \alpha} \quad (\text{A10})$$

which is positive if $b < \frac{1}{2}$ and negative if $b > \frac{1}{2}$. To simplify the numerator of the coefficient of $\frac{dq_0}{da}$ in (A7) note that it can be written as:

$$A_2(B_1+B_2) - B_2(A_1+A_2) \quad (\text{A11})$$

or:

$$A_2B_1 - B_2A_1 \quad (\text{A12})$$

where:

$$A_1 \equiv \frac{\partial \chi}{\partial q_0} + \frac{\partial Y}{\partial q_0} - b\bar{k}eq_0^{b-1} \quad (\text{A13})$$

$$A_2 \equiv (2-\alpha)((1-\alpha) \frac{\partial^2 \chi}{\partial \alpha \partial q_0} - \frac{\partial Y}{\partial q_0}) \quad (\text{A14})$$

$$B_1 \equiv \frac{\partial \chi}{\partial \alpha} + \frac{\partial Y}{\partial \alpha} \quad (\text{A15})$$

$$B_2 \equiv (2-\alpha)((1-\alpha) \frac{\partial^2 \chi}{\partial \alpha^2} - \frac{\partial \chi}{\partial \alpha} - \frac{\partial Y}{\partial \alpha} - \frac{Y}{(2-\alpha)^2}) \quad (\text{A16})$$

A_1 is positive, as shown previously. B_2 is just a positive multiple of the second order condition for a utility maximum with respect to shareholdings, which, at the entrepreneur's optimum, is negative. Therefore, $-B_2A_1$ is positive. With tedious calculation it can also be shown that A_2 and B_1 are both positive for $b < \frac{1}{2}$ and both negative for $b > \frac{1}{2}$. Therefore (A12) is positive. This implies that $\frac{dq_0}{da}$ has the same sign as B : positive for $b < \frac{1}{2}$ and negative for $b > \frac{1}{2}$. By further calculation it can be shown that A is positive. Then, from (A6), $\frac{d\alpha}{da}$ is negative.

A.2 Proof of Proposition 4

Differentiating (A1) and (A2) with respect to q_0 and α as \bar{k} changes give, respectively:

$$\left(\frac{\partial X}{\partial q_0} + \frac{\partial Y}{\partial q_0} - b\bar{k}eq_0^{b-1}\right) dq_0 + \left(\frac{\partial X}{\partial \alpha} + \frac{\partial Y}{\partial \alpha}\right) d\alpha - eq_0^b d\bar{k} = 0 \quad (A17)$$

and:

$$\left((1-\alpha) \frac{\partial^2 X}{\partial \alpha \partial q_0} - \frac{\partial Y}{\partial q_0} \frac{\partial Y}{2-\alpha}\right) dq_0 + \left((1-\alpha) \frac{\partial^2 X}{\partial \alpha^2} - \frac{\partial X}{\partial \alpha} - \frac{\partial Y}{2-\alpha} - \frac{Y}{(2-\alpha)^2}\right) d\alpha = 0 \quad (A18)$$

Note that the coefficient of $d\alpha$ in (A18) is just the second order condition for optimal shareholdings, which is negative at the shareholding optimum. Further, the coefficient of dq_0 in (A18) has been shown in the proof to Proposition 3 to be positive for $b < \frac{1}{2}$ and negative for $b > \frac{1}{2}$. Therefore, from (A18), for $b < \frac{1}{2}$, $\frac{dq_0}{d\bar{k}}$ and $\frac{d\alpha}{d\bar{k}}$ must have the same sign. But because the coefficients of dq_0 and $d\alpha$ in (A17) have already been shown to be positive, condition (A17) can only be satisfied if both $\frac{dq_0}{d\bar{k}}$ and $\frac{d\alpha}{d\bar{k}}$ are positive. Therefore, for $b < \frac{1}{2}$, shareholdings, input level, and consequently firm value, increase with \bar{k} .

From (A18), for $b > \frac{1}{2}$, $\frac{dq_0}{d\bar{k}}$ and $\frac{d\alpha}{d\bar{k}}$ are of opposite sign. However, it must be true for those b that $\frac{dq_0}{d\bar{k}}$ is positive. To see this, note first that, from simple calculations, $\frac{d\alpha}{d\bar{k}} = 0$ and $\frac{dq_0}{d\bar{k}} > 0$ at $b = \frac{1}{2}$. With $\frac{dq_0}{d\bar{k}}$ a continuous function of b , it is not possible for $\frac{dq_0}{d\bar{k}}$ to ever become negative as b increased from $\frac{1}{2}$ toward 1 since it would have to be equal to zero at some level of b . But, for (A18) to be satisfied at that b , $\frac{d\alpha}{d\bar{k}}$ must also be zero. But then (A17) would not be satisfied. $\frac{dq_0}{d\bar{k}}$ could then never equal zero. Therefore, for all b between $\frac{1}{2}$ and 1, $\frac{dq_0}{d\bar{k}}$ must be positive, and consequently $\frac{d\alpha}{d\bar{k}}$ must be negative. For $b > \frac{1}{2}$, as \bar{k} increases, the input level, and consequently the firm value, increase, while shareholdings decrease.

Footnotes

1. The assumption that all investors are risk neutral (except possibly for the entrepreneur) is not a restrictive one. It is similar in its implications to the assumption that all investors are risk averse but that an additional investment, the market portfolio, is available. The latter is the setting of LP.
2. To solve the differential equation use was made of the boundary condition $f(0) = 0$. This accounts for the absence of an arbitrary constant in the solution.
3. It is easy to verify that, given this signalling schedule, the q_0 solving (6) represents a utility maximum.
4. This is also sufficient for the second order condition for a utility maximum to be satisfied.
5. If the manager were a net buyer a signalling equilibrium need not exist in which a higher q_0 signals a higher \bar{k} . To see why, assume that such an equilibrium did exist. In this equilibrium a manager with more favorable information might have an incentive to set a lower q_0 in order for the market value of the firm to be low so that he could make his share purchases more cheaply. But this action is in contradiction to the supposition that a higher q_0 signals a higher \bar{k} , showing that such an equilibrium need not exist.

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