Pulsar kicks from a dark-matter sterile neutrino

George M. Fuller,[1,5] Alexander Kusenko,[2,3,5] Irina Mocioiu,[4,5] and Silvia Pascoli[2,5]
[1]*Department of Physics, UCSD, La Jolla, California 92093-0319, USA*
[2]*Department of Physics and Astronomy, UCLA, Los Angeles, California 90095-1547, USA*
[3]*RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, USA*
[4]*Department of Physics, University of Arizona, Tucson, Arizona 85721, USA*
[5]*KITP, UCSB, Santa Barbara, California 93106, USA*
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We show that a sterile neutrino with a mass in the 1–20 keV range and a small mixing with the electron neutrino can simultaneously explain the origin of the pulsar motions and the dark matter in the Universe. An asymmetric neutrino emission from a hot nascent neutron star can be the explanation of the observed pulsar velocities. In addition to the pulsar kick mechanism based on resonant neutrino transitions, we point out a new possibility: an asymmetric off-resonant emission of sterile neutrinos. The two cases correspond to different values of the masses and mixing angles. In both cases we identify the ranges of parameters consistent with the pulsar kick, as well as cosmological constraints.

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Pulsars are known to have large velocities ranging from 100 to 1600 $\mathrm{km\,s^{-1}}$ [1]. In contrast, the average velocity of an ordinary star in our galaxy is 30 $\mathrm{km\,s^{-1}}$. Born in a supernova explosion of an ordinary star, the pulsar must, therefore, receive a substantial "kick" at birth. The high angular velocities of pulsars may also be explained by the same kick [2]. The origin of the pulsar kicks remains an intriguing outstanding puzzle.

According to numerical simulations of supernova explosions [3], the asymmetries in the core collapse could not account for a kick velocity of more than 300–600 $\mathrm{km\,s^{-1}}$. Although the average pulsar velocity is in this range [1], there is a substantial population of pulsars with velocities in excess of 700 km s, including some with speeds over 1000 $\mathrm{km\,s^{-1}}$. These velocities appear to be too large to arise from an asymmetry in convection. Evolution of close binary systems has also been considered as a source of a kick velocity [4]. Alternatively, it was argued [5] that the pulsar may be accelerated during the first few months after the supernova explosion by its electromagnetic radiation, the asymmetry resulting from the magnetic dipole moment being inclined to the rotation axis and offset from the center of the star. Both of these mechanisms, however, have difficulties explaining the observed magnitudes of the pulsar kick velocities.

In this paper we point out a new way in which the pulsar kicks could be generated from off-resonant active-sterile neutrino conversion in the neutron star core. In addition, we identify the parameters for which the resonant conversions can take place. Both cases are consistent with the sterile neutrino being the dark matter.

The neutrinos emitted from a nascent hot neutron star carry a total energy of 10^{53} erg. Only a 1% asymmetry in the distribution of these neutrinos could produce a kick consistent with the observed magnitudes of pulsar velocities. Despite negligible magnetic moments, neutrinos are affected by the strong magnetic field of the neutron star through their interactions with the polarized electrons in the medium. In particular, resonant neutrino oscillations occur at different densities depending on the direction of the magnetic field relative to the neutrino momentum [6,7]. Oscillations of active neutrinos could explain the observed velocities of pulsars if the resonant conversion $\nu_{\mu,\tau} \leftrightarrow \nu_e$ took place between two different neutrinospheres [8–12]. However, the neutrino masses required for the resonant transition between the two neutrinospheres, at density $10^{11}-10^{12}\ \mathrm{g\,cm^{-3}}$, are inconsistent with the present limits on the masses of standard electroweak neutrinos.

These limits do not apply, however, to sterile neutrinos that may have only a small mixing with the ordinary neutrinos. Resonant oscillations of active neutrinos into sterile neutrinos have been proposed as a possible explanation of the pulsar kicks [13]. It was recently pointed out that conversions of trapped neutrinos in the core of a hot neutron star can make the effective mixing angle close to that in vacuum [14]. This has important implications for the pulsar kick mechanism because, in the absence of matter suppression of the mixing, the nonresonant production of sterile neutrinos can dominate. This is asymmetric, and it can explain the pulsar velocities as well. The same sterile neutrino in the keV mass range can be the dark matter in the universe [14,15]. Sterile neutrino with the requisite masses and mixing appear in a number of theoretical models [16], for example, in models with broken mirror parity [17].

We will assume that neutrinos have negligible magnetic moments and that they have only the standard interactions with matter. Unusually large magnetic moments of neutrinos [18], as well as other kinds of new physics [19] have also been considered as the origin of the pulsar kicks. We will further assume that only ν_e has a significant mixing with ν_s, characterized by $\sin^2\theta \sim 10^{-11}-10^{-7}$, while the oscillations between $\nu_{\mu,\tau}$ and ν_s are suppressed by small mixing angle and/or matter effects.

For a sufficiently small mixing angle between ν_e and ν_s, only one of the two mass eigenstates, ν_1, is trapped. The orthogonal state, $|\nu_2\rangle = \cos\theta_m|\nu_s\rangle + \sin\theta_m|\nu_e\rangle$, escapes from the star freely. This state is produced in the same basic urca reactions ($\nu_e+n\rightleftharpoons p+e^-$ and $\overline{\nu}_e+p\rightleftharpoons n+e^+$) with the effective Lagrangian coupling equal the weak coupling times $\sin\theta_m$.

Urca processes are affected by the magnetic field, and so the active neutrinos are produced asymmetrically depending on the direction of their momenta relative to the magnetic field. Chugai [20] and Dorofeev *et al.* [21] have proposed that this asymmetry might explain the pulsar kick velocities. However, the asymmetry in the *production* amplitudes does not lead to an appreciable asymmetry in the *emission* of neutrinos because this asymmetry is washed out by scattering [22]. Therefore, the mechanism considered in Refs. [20,21] does not work.

However, since $|\nu_2\rangle$ is not trapped anywhere in the star, the asymmetry in its emission is exactly equal to the production asymmetry. Depending on the parameters, this asymmetry can be as large as 25%, just as in the case of the active neutrinos [21]. If the sterile neutrinos carry away about 10% of the thermal energy, the resulting overall asymmetry can reach the required few percent, which is what one needs to explain the pulsar kick velocities.

The production cross section of ν_2 depends on the effective mixing angle in matter θ_m, which, in general, is different from the vacuum mixing angle θ:

$$\sin^2 2\theta_m=\frac{(\Delta m^2/2p)^2\sin^2 2\theta}{(\Delta m^2/2p)^2\sin^2 2\theta+(\Delta m^2/2p\cos 2\theta-V_m)^2}, \tag{1}$$

where the matter potential V_m is positive (negative) for $\nu(\overline{\nu})$, respectively; p is the momentum. For the case of ν_e, which is relevant for our discussion, V_m reads

$$V_m=\frac{G_F\rho}{\sqrt{2}m_n}(3Y_e-1+4Y_{\nu_e}+2Y_{\nu_\mu}+2Y_{\nu_\tau}). \tag{2}$$

In a core collapse supernova, the initial value of this matter potential is $V_m\simeq(-0.2\ldots+0.5)V_0$, where $V_0=G_F\rho/\sqrt{2}m_n\simeq 3.8\ \mathrm{eV}(\rho/10^{14}\ \mathrm{g\ cm^{-3}})$. The average probability of $\nu_e\to\nu_s$ conversion in presence of matter is

$$\langle P_m\rangle=\frac{1}{2}\left[1+\left(\frac{\lambda_{\rm osc}}{2\lambda_{\rm s}}\right)^2\right]^{-1}\sin^2 2\theta_m, \tag{3}$$

where we have included a possible suppression due to quantum damping [23], which depends on the oscillation length $\lambda_{\rm osc}$ and scattering length $\lambda_{\rm s}$.

It was pointed out in Ref. [14] that, in the presence of sterile neutrinos, rapid conversions can take place between different neutrino flavors. This, in turn, can drive the effective potential to its stable equilibrium fixed point $V_m\to 0$. This equilibration takes place on a very short time scale and results in the destruction of the initial asymmetry of ν_e over $\overline{\nu}_e$. (The initial electron lepton number asymmetry in the supernova core depends on the details of the prior stellar collapse history and, especially, the electron capture on heavy nuclei and neutrino transport in the in-fall epoch.) This, in turn, leaves the system out of β-equilibrium. On a time scale $\sim$ms, electron capture reactions in the core will return the system to beta equilibrium and the ν_e so produced will be converted. These processes will continue until both beta equilibrium and a steady state equilibration are achieved. Once the equilibrium is achieved, the effective mixing angle in matter is close to that in vacuum. In Ref. [13], this effect was not taken into account, and only the resonant emission from a thin shell was considered. A larger mixing angle makes a big difference in that the emission off resonance becomes possible from the entire volume of the neutron star core.

The equilibration mechanism relies on the fact that, in general, the probability of $\nu_e\to\nu_s$ oscillations is different for neutrinos and antineutrinos due to the opposite sign of V_m in Eq. (1) [32]. The resulting change in the amounts of ν_e and $\overline{\nu}_e$ drives V_m to zero. Following Ref. [14], we estimate the time scale for this process:

$$\tau_V\simeq\frac{V_m^{(0)}m_n}{\sqrt{2}G_F\rho}\left(\int d\Pi\,\frac{\sigma_\nu^{\rm urca}}{e^{(\epsilon_\nu-\mu_\nu)/T}+1}\langle P_m(\nu_e\to\nu_s)\rangle-\int d\Pi\,\frac{\sigma_{\overline{\nu}}^{\rm urca}}{e^{(\epsilon_{\overline{\nu}}-\mu_{\overline{\nu}})/T}+1}\langle P_m(\overline{\nu}_e\to\overline{\nu}_s)\rangle\right)^{-1}, \tag{4}$$

where $d\Pi=(2\pi^2)^{-1}\epsilon_\nu^2 d\epsilon_\nu$, and $V_m^{(0)}$ is the initial value of the matter potential V_m.

Since the neutrino emission depends on the value of $\langle P_m\rangle$, the time scales for resonant and off-resonant conversions differ. In the vacuum case, $\Delta m^2/(2\langle E\rangle)>|V_m|$, this mechanism is not relevant, because V_m is negligible from the start. In the case of resonance, $\Delta m^2/(2\langle E\rangle)\sim V_m$, the conversions of ν_e's are strongly enhanced over the $\overline{\nu}_e$'s. Therefore the equilibration mechanism is very efficient. Using Eq. (4), we estimate the time:

$$\tau_V^{\text{on-res}}\simeq\frac{2^5\sqrt{2}\pi^2 m_n}{G_F^3\rho}\,\frac{(V_m^{(0)})^6}{(\Delta m^2)^5\sin 2\theta}\left(e^{\frac{\Delta m^2/2V_m^{(0)}-\mu}{T}}+1\right)$$
$$\sim\frac{2\times10^{-9}\ \mathrm{s}}{\sin 2\theta}\,\frac{10^{14}\ \frac{\mathrm{g}}{\mathrm{cm}^3}}{\rho}\left(\frac{20\ \mathrm{MeV}}{T}\right)^6\frac{\Delta m^2}{10\ \mathrm{keV}^2}. \tag{5}$$

This holds as long as the resonance condition $V_m^{(0)}\simeq\Delta m^2/(2\langle E\rangle)$ is satisfied. The back-reaction effects on V_m due to the change in Y_{ν_e} may slow down the equilibration. Their detailed analysis is beyond the scope of this paper; we do not expect the back reaction to alter the time scales significantly.

If neutrinos are converted off-resonance, it is necessary to evaluate the integral in Eq. (4) taking into account that both the chemical potential and the reaction rates for neutrinos and antineutrinos are different. An approximate evaluation of the time scale yields

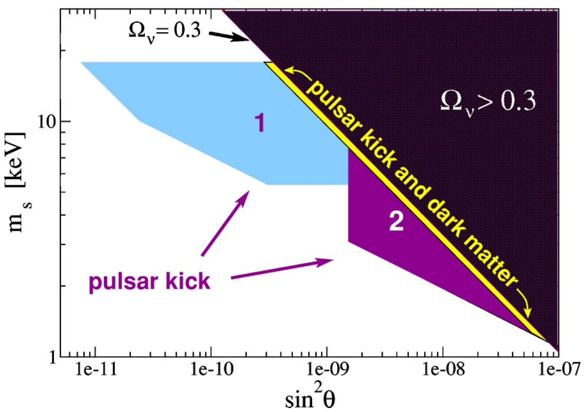

FIG. 1. Neutrino conversions can explain the pulsar kicks for values of parameters either in region 1 (Ref. [13]), or in region 2 [present work]. In a band near the $\Omega_\nu = 0.3$ line, the sterile neutrino can be the dark matter.

$$\tau_V^{\text{off-res}} \simeq \frac{4\sqrt{2}\pi^2 m_n}{G_F^3 \rho} \frac{(V_m^{(0)})^3}{(\Delta m^2)^2 \sin^2 2\theta} \frac{1}{\mu^3}$$

$$\sim \frac{6\times 10^{-9}\ \text{s}}{\sin^2 2\theta}\left(\frac{V_m^{(0)}}{0.1\ \text{eV}}\right)^3\left(\frac{50\ \text{MeV}}{\mu}\right)^3$$

$$\times\left(\frac{10\ \text{keV}^2}{\Delta m^2}\right)^2. \tag{6}$$

Requiring that the off-resonance equilibration takes place during the first second, imposes a combined lower bound on Δm^2 and $\sin^2 2\theta$. In Fig. 1 this bound, for $V_m^{(0)} = 0.1$ eV, $\mu = 50$ MeV, $T = 20$ MeV, limits region 2 from below. ($T = 20$ MeV is plausible for the immediate post core bounce conditions, where models suggest a core temperature between tens of MeV and about 60 MeV [24].)

The probability of oscillation depends on the values of Δm^2, $\sin\theta$, and V_m. One can identify three different regimes. (i) the "vacuum" case: $\Delta m^2/(2\langle E\rangle) \gtrsim |V_m|$; (ii) the resonance regime, $\Delta m^2/(2\langle E\rangle) \approx V_m$, in which the conversion probability is strongly enhanced; (iii) the "matter-suppressed" oscillations, for $\Delta m^2/(2\langle E\rangle) \ll V_m$, which have a vanishing probability.

Whether or not a resonance occurs anywhere in the core depends on the mass of the sterile neutrino and its mixing angle with ν_e. The resonance occurs at some point in the core for parameters in the range marked "1" in Fig. 1. The lower boundary of this region comes from enforcing the adiabaticity and the weak damping conditions. The vertical span reflects the range of densities in the outer core of a neutron star.

The case of resonant emission. If the resonance condition is satisfied (region 1 in Fig. 1), the emissivity is strongly enhanced in the region of resonant density, while it is suppressed elsewhere. The resonant region is a thin layer, whose deviation from a spherical shape is due to the electron polarization in the magnetic field [6–8,13]. For small mixing angles (region 1), the time $\tau_V^{\text{off-res}}$ is longer than a few seconds for some reasonable initial conditions. Hence, the matter potential is not affected outside the resonant region. Therefore, in region 1 the pulsar kick mechanism works as described in Ref. [13], where the equilibration was not considered [33].

Off-resonance emission after $\mathbf{V}_m$ has equilibrated to zero. To get the pulsar kick, we require that a substantial fraction of energy be emitted in sterile neutrinos. The dominant process of neutrino production in the core is urca. One can estimate the emissivity in sterile neutrinos following Ref. [14]:

$$\mathcal{E} \sim \int d\Pi\, \epsilon_\nu \frac{\sigma_{\text{urca}}}{e^{(\epsilon_\nu - \mu)/T} + 1} \frac{1}{2}\langle P_m\rangle. \tag{7}$$

The emissivity depends on the value of $\langle P_m\rangle$. In the "vacuum" regime the emissivity is proportional to $\sin^2 2\theta$ and is independent of Δm^2. The total energy emitted per unit mass is simply the emissivity multiplied by the time of emission, which is 5–10 s. For given values of μ and T, the requirement of emitting a certain fraction of the energy in sterile neutrinos translates into a lower bound on the allowed values of $\sin^2 2\theta$, as shown in Fig. 1. Such a bound is independent of Δm^2, as long as the $V_m \simeq 0$ condition is satisfied. If the equilibration process is efficient, $V_m \approx 0$ and the matter effects are negligible. Then the oscillations $\nu_e \to \nu_s$ are controlled by the vacuum mixing angle $\sin^2 2\theta$.

As discussed by Dorofeev *et al.* [21], the urca processes result in an asymmetric neutrino production if the electrons are polarized. This asymmetry arises from the spin-multiplicity factor. Only the electrons in the lowest Landau level ($n=0$) contribute to this asymmetry. The number of neutrinos dN emitted into a solid angle $d\Omega$ can be written as $dN/d\Omega = N_0(1+\epsilon\cos\Theta_\nu)$, where Θ_ν is the angle between the direction of the magnetic field and the neutrino momentum, and N_0 is some normalization factor. The asymmetry parameter ϵ is equal

$$\epsilon = \frac{g_V^2 - g_A^2}{g_V^2 + 3g_A^2} k_0 \left(\frac{\mathcal{E}_s}{\mathcal{E}_{\text{tot}}}\right), \tag{8}$$

where g_V and g_A are the axial and vector couplings, $\mathcal{E}_{\text{tot}}$ and $\mathcal{E}_s$ are the total neutrino luminosity and that in sterile neutrinos, respectively. Following Ref. [21], we obtain

$$k_0 = \frac{F(0)}{F(0) + 2\sum_{n=1}^{\infty} F(n)}, \tag{9}$$

where

$$F(n) = \int_0^\infty dp \frac{(m_n - m_p - \sqrt{p^2 + (m_e^*)^2 + 2Bn})^2}{\exp\left\{\frac{\sqrt{p^2 + (m_e^*)^2 + 2Bn} - \mu}{T}\right\} + 1}. \tag{10}$$

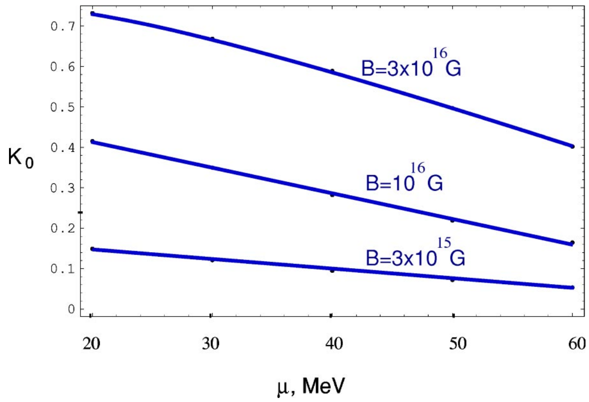

FIG. 2. The fraction of electrons in the lowest Landau level as a function of μ for $T=20$ MeV. The value of the magnetic field in the core of a neutron star is shown next to each curve.

Here n is the electron Landau level, and we have assumed that protons and neutrons are non-relativistic, non-degenerate, and that their polarization can be neglected. The effective electron mass at high density, m_e^*, remains below 10–11 MeV everywhere in the star [24,25]. Its exact value does not affect the expression for $F(n)$ in Eq. (10) by more than a few percent. If only the lowest Landau level is populated, then $k_0=1$. The weaker the magnetic field, the more levels are populated, the smaller is the fraction k_0. In Fig. 2 we show k_0 as a function of chemical potential μ for different values of the magnetic field B. We took the temperature to be $T=20$ MeV.

The ratio $(\mathcal{E}_s/\mathcal{E}_{tot})=\mathcal{E}_s/(\mathcal{E}_s+\mathcal{E}_\nu)$ can be estimated from the following:

$$\left(\frac{\mathcal{E}_s}{\mathcal{E}_\nu}\right)\sim\sin^2\theta\left(\frac{T_{\rm core}}{T_{\nu\text{-sphere}}}\right)^6 f_{\rm M}\, f_{\rm d.o.f.}, \qquad (11)$$

where $f_{\rm M}<1$ is the fraction of enclosed mass of the core from which the emission of sterile neutrinos is efficient, and $f_{\rm d.o.f.}\geqslant 1$ is an enhancement due to a possible increase in the effective degrees of freedom at high density [24]. We take the core temperature $T_{\rm core}\approx(20–50)$ MeV, while the neutrinosphere temperature is $T_{\nu\text{-sphere}}\approx(2–10)$ MeV. Hence, for $\sin^2\theta\sim10^{-8}$ the ratio $(\mathcal{E}_s/\mathcal{E}_{tot})$ can be in the range

$$r_{\mathcal{E}}=\left(\frac{\mathcal{E}_s}{\mathcal{E}_{\rm tot}}\right)\sim 0.05–0.7. \qquad (12)$$

A higher ratio is in conflict with the observation of neutrinos from SN1987A, while a lower value does not give a sufficient pulsar kick. Our point here is that, for reasonable values of parameters, the range in Eq. (12) is *possible*, even if not necessary.

Finally, the magnitude of the asymmetry is

$$\epsilon\sim 0.02\left(\frac{k_0}{0.3}\right)\left(\frac{r_{\mathcal{E}}}{0.5}\right), \qquad (13)$$

which can be of the order of the requisite few percent for magnetic fields 10^{15}–10^{16} G, as can be seen from Fig. 2.

The magnetic field at the surface of an average pulsar is estimated to be of the order of 10^{12}–10^{13} G [26]. However, the magnetic field inside a neutron star may be as high as 10^{16} G [26–29]. The existence of such a strong magnetic field is suggested by the dynamics of formation of the neutron stars, as well as by the stability of the poloidal magnetic field outside the pulsar [29]. Moreover, the discovery of soft gamma repeaters and their identification as magnetars [28], i.e., neutron stars with *surface* magnetic fields as large as 10^{15} G, gives one a strong reason to believe that the interiors of many neutron stars may have magnetic fields as large as 10^{15}–10^{16} G and that only in some cases this large magnetic field breaks out to the surface. There is also a plausible physical mechanism by means of which such a large magnetic field can form inside a neutron star through the so called $\alpha-\Omega$ dynamo effect [29,30]. Nuclear matter inside a cooling neutron star is expected to have a sufficient amount of convection to generate magnetic fields as large as 10^{16} G [29].

The ratio of luminosities $r_{\mathcal{E}}$ translates into an upper bound on the mixing angle. The allowed region for the masses and mixing angles is marked "2" in Fig. 1.

Gravity waves from a pulsar kick due to neutrino conversions may be observable [31] and can provide a way to test this mechanism.

To summarize, we have analyzed the pulsar kick resulting from active to sterile neutrino conversions. We have shown that, in addition to resonant transitions [13], the off-resonant neutrino oscillations can also result in a pulsar kick consistent with observations. The two mechanisms require different masses and mixing angles. We have identified the range of parameters for which each of the two mechanisms can explain the observed velocities of pulsars. Part of this range is consistent with the sterile neutrino being the dark matter with $\Omega_\nu^{\rm WDM}\approx0.27$.

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