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Microgenetic learning analysis: A methodology for studying knowledge in transition

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Abstract: This paper introduces and exemplifies a qualitative method for studying learning, *microgenetic learning analysis* (MLA), which is aimed jointly at developing theory and at establishing useful empirical results. Among modern methodologies, the focus on theory is somewhat distinctive. We use two strategies to describe MLA. First, we develop a framework for comparing the focus and means of different methods, particularly qualitative methods, aimed at studying learning. Using the framework, we compare and contrast MLA with two better-known methods, microgenetic analysis and grounded theory. Second, we aim to schematize elements of MLA—from large-scale patterns of work to detailed analytical strategies—and to exemplify some of them in a case study using data collected some years prior to this elaboration of MLA.

Running head: Microgenetic learning analysis

Keywords: cognitive processes, conceptual change, learning theory, theory, knowledge, microgenesis, qualitative research, research methods.

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Introduction

Goals and Means

The aim of this paper is to contribute to qualitative methodology for researching learning. We focus on qualitative methodology for two reasons. First, we are interested centrally with theory development concerning learning. While quantitative methods can be extremely valuable in testing theories, we believe they are comparatively weak in helping us generate well-grounded and adequate theories. A focus on theory in learning represents, also, a judgment that theories of learning are currently less adequate than we need (diSessa, 1991). The second reason to focus on qualitative methods is that they are generally less well understood (why and when must one perform them in particular ways), for example, in comparison to quantitative methods.

This paper introduces and exemplifies a qualitative method for studying learning called *microgenetic learning analysis* (MLA) and is aimed jointly at developing theory and at establishing useful and particular empirical results. We use two strategies to explain MLA. First, we provide a rough framework of dimensions that characterize qualitative methods of studying learning. The framework will allow us to specify the form of qualitative study presented here, will highlight what might be new and different in the method as well as what is common with other methods, and will provide a better sense of the “why” of various characteristics of the method and how they work synergistically. In this regard, we believe the framework itself may be more generally useful for thinking about methods of studying learning, although it is not our purpose here to argue this possibility.

The second strategy to explain MLA is a case study of its use. Methodological case studies are appropriate for several reasons. First, they help ground and exemplify the general principles of the method. Learning and truly understanding any method, particularly a qualitative method, founders on finding and managing the general forms of analysis (which one finds in abstract presentations of methods) in the specifics of particular study. Case studies also help to *dig out*, *schematize*, and *subject to critical examination* the wisdom of practices that grow up in the field on a more informal basis. For example, Alan Newell (1982) claimed that his construct of “the knowledge level” was precisely a digging out and articulating of the wisdom that had developed tacitly in the artificial intelligence community concerning the nature of knowledge.

More importantly, case studies are appropriate when the scope and character of the relevant method are not well understood. We are, as yet, in no position to understand all of the hows, whens, and whys of MLA. Case studies are intended to draw out the generalities of a method, but also display incidental characteristics and even activities that may be important, but, as yet, resist rational reconstruction.

Microgenetic Learning Analysis

In this section we briefly introduce “theory-focused microgenetic learning analysis,” microgenetic learning analysis (MLA) for short. MLA has the following characteristics:

1. Theory-focused: One of the primary aims of MLA is to generate and improve theories concerning learning. In this, the relevant theories have an *epistemological* component: studying the form and content of knowledge is extremely important. More particularly, in MLA it is not assumed that we necessarily know the relevant forms of knowledge in

advance. “Concepts,” “ideas,” “theories” may be familiar knowledge terms from everyday experience, including experience in the practice of science. Yet, our position is that more scientifically accountable knowledge categories—what we call *knowledge ontology*—are important to develop. We retain a skeptical attitude even toward supposedly technical terms, such as schema, and wish to see assumed properties validated in the details of learning events. Beyond knowledge ontology, processes and mechanisms are central concerns.

2. Fine-grained: The gold standard within the MLA perspective is having a moment-by-moment explanatory account of learning in particular contexts. Toward this goal, typical studies of learning that involve treatments and before-and-after assessments—for example, “race horse” studies of different instructional techniques—are largely irrelevant. Similarly, cross-sectional studies are unlikely to produce detailed, relevant information. Constraints on learning processes (e.g., limited working memory) and conditions for better learning (e.g., people often learn better in groups) are sometimes helpful, and they might be all that can be achieved in some cases. But these are secondary goals. The processes of change and, indeed, the nature of changing elements are best seen at the smallest observable time-scales, where, for example, an important noticing event or connection between ideas might be made by a learner.

“Fine-grained” applies both to time-scale and “conceptual resolution.” Conceptual resolution means that very fine distinctions in meaning are relevant and therefore must be tracked. At this grain size in time and meaning, a high degree of individual and contextual variation is expected.

3. Open consideration of relevant aspects of data: Because it seeks to uncover, more than demonstrate, features of knowledge, of context, individual particularities, and learning processes, any feature of a learning event might potentially be relevant. So, for example, verbal language is important, but gesture (Goldin-Meadow, Alibali, & Church, 1993) can be equally illuminating of meanings that students may have available. Posture and prosody can display confidence or suggest non-verbal or tacit knowledge. Eye gaze can show what aspects of the material situation are relevant and being considered. For these purposes, video recording is a minimal requirement, and analysis typically uses the video rather than early-on data reduction methods (such as coding in pre-determined categories, field notes, or even transcription of verbal data).

While we do not take it to be definitive, MLA frequently takes the form of case studies. This is natural, given the need for extended, open, and careful consideration of data. In addition, we believe the discovery of new theoretical categories out of data works best during open and extended consideration of data. As many researchers (e.g., Siegler, 2006) have observed, uncovering the processes of learning is a prize well worth the effort of looking carefully at data. Studies that analyze large amounts of data may achieve the *apparent generality* of many cases, but their constructs may lack insight or sufficient warrants as natural kinds, as opposed to mere coding categories. In partial compensation for their limits, case studies can often be drawn from more transparently ecologically valid sources, such as real-world and even classroom learning, which are too diverse and uncontrolled for the methods used in many laboratory studies.

Examples in the Family of MLA

While there is not a vast literature that fits the MLA model, it is growing and readers may be familiar with research that we take to have family resemblances to MLA. The following examples were chosen to illustrate a reasonable range of theoretical and topical foci, some variation in intent and in the nature of outcomes, while simultaneously illustrating core and typical characteristics.

Schoenfeld, Smith and Arcavi (1993) produced an extended, microgenetic study of one student's learning about algebra, graphs, and functions in the context of interaction with a software system designed to support such learning. Seven hours of video were analyzed by a research group over several months with the aim of meticulously documenting exactly the knowledge that the student had at various times, its context sensitivity, and development. Results included that the student had some procedural and conceptual knowledge, but almost none of the general background that made the procedures justifiable and the concepts sensible and connected with other concepts. Given those conditions, the student showed striking context sensitivity with respect to basic concepts and procedures. Although the goal of the analysis was explicitly to uncover mechanisms of learning, in the end "learning events" were rarely if ever directly observed. Instead of mechanisms, a number of important constraints on the nature of learning were obtained. For example, learning was highly non-linear, with the student back-tracking and side-slipping on many occasions.

In contrast to the case study of MLA below, Schoenfeld et al. did not develop new or lean heavily on existing technical knowledge terms (ontology). Similarly, processes and mechanisms were not explicitly named, but important constraints, sufficient in the authors' judgment to rule out certain learning models, were obtained.

Izsak (2000) consisted of a microgenetic case study of two students working together to figure out how to represent the behavior of a physical device using algebra. The device, called a "winch," involved a crank-handle connected to two common-axis spools of different diameters, around which threads were wound. Cranking the spools led to coordinated movements of weights hanging at the end of the threads. Algebraically, the heights of each of the weights can be represented by a linear equation, $h = mx + b$, where m is the height gain from one crank of the relevant spool, x is the number of crank turns, and b is the starting height of the weight. However, the students started without the competence to produce any such equation and took a full half-hour to work through to a valid, if unconventional equation.

In many ways, Izsák's work is prototypical of MLA, and the method is (not by name, or in full generality) articulately defended. In particular, Izsák starts by criticizing past treatments of similar learning for their theoretical inability to account for the kind of data he gathered. If any data at all is brought to bear on prior models of learning that addressed his conceptual topic, it is cross-sectional or diachronic (with a coarse grain size). Thus, prior studies do not—and Izsák argues cannot—address his fine-grained data, which shows a very complex, highly contextualized processes.

Izsák is also explicitly concerned both with knowledge ontology and mechanisms of change. He focuses on one theoretically well-developed knowledge type, called a *form*, which was previously identified and studied by Sherin (2001). The main result of Izsák's analysis is

two learning mechanisms (*notation variation* and *mapping variation*) that are repeatedly documented in his video corpus to partially account for learning. All in all, Izsák's study exemplifies all the main features of MLA: It is theory-focused, including attention to both ontology and mechanism, it is very fine-grained (explicitly in contrast to a particular body of other studies), and it grew out of an open consideration of data.

Levrini and diSessa (2008), unlike any of the above studies, is a case study drawn from classroom data. It documents and explains students' learning while they struggle with the relativistic concept of "proper time." The driving event in the study occurs when all the members of the class misunderstand an example that seems to be a relatively simple application of the idea of proper time, with which they had previously showed significant competence. The subsequent analysis aims to show that their prior apparent competence had been on the basis of several intuitive glosses that worked effectively in the situations students had previously encountered. The means by which the students improved their knowledge are described, consistent with a previously identified model of concepts and concept learning, but also with reference to many details relevant to the situation at issue. Those particular considerations led to extensions of the in-coming model of concept learning, which the authors argue are "natural" extensions of the theory, not refutations or even revisions of the core of the theory.

This study has two salient commonalities with this paper's methodological case study. First, both study particular forms of knowledge in significant detail. Secondly, both studies deal explicitly with improving and developing an existing theory (coordination class theory, described below). MLA's focus on theory is often in the form of a constant testing for the purpose of extending and improving, rather than merely establishing an assumed-to-be-valid perspective once-and-for-all, or making a perfunctory or ideological "up front" choice in an empirical study. If new theory is not created, then old theory is tested and refined. This attitude toward theory is described as an orientation toward "humble theory" in diSessa and Cobb (2004).

A Dimensional Framework for Classifying and Understanding Qualitative Methodologies¹

The Framework

In this section we develop a set of dimensions along which to compare and contrast different methodologies. This is not intended to be a complete and universal list, but it is instead chosen to highlight what we consider important commonalities and differences among methodologies "neighboring" MLA. "Dimensions" are not quantifiable parameters, but rather foci that beg particular descriptions of features of different methods. Not any aspect of each of these dimensions may make a difference: Which aspects are consequential for what purposes, and which are not, needs to be specified.

After introducing the dimensions, we use them somewhat systematically to compare and contrast MLA with a pair of related methodologies: microgenetic analysis and grounded theory. We present our list in a hierarchical form, and illustrate with representative questions.

¹ We leave the range of qualitative methodologies illuminated by this framework somewhat open. Methods focused on learning are prototypical, but other kinds of methods, such as grounded theory, also seem amenable to this analysis.

1. **Theoretical and metatheoretical commitments.** Given the central role that theory plays in MLA, theoretical or metatheoretical orientations are salient. In contrast, many methods, especially quantitative methods, are (at least avowedly) independent of theory. We do not presume to say, in advance, whether theoretical point of view actually makes a difference in the conduct of an MLA study, but we feel a relatively wide range of theoretical orientations can be accommodated within MLA.
 - a. **Status and nature of relevant theory.** Is the method theory-specific, or not? Are there variations in the methods induced by important theoretical differences?
 - b. **Source of the theory.** Does the method require or expect the theory to be specified in advance of empirical study? Is the theory generated from data, or, alternatively, from analytical or other general considerations?
 - c. **Time-profile of theory development.** What is the expected trajectory of theory development? (E.g., are theories developed “once and for all” or are they under constant scrutiny and development?) How does theory development interact, if it does, with empirical studies?
2. **Intellectual regime and focal content.** Is the method limited to, or more relevant to one or another “intellectual regimes,” such as problem solving, skill acquisition, or learning?
3. **Empirical design.** Are there characteristic features of the range of study designs associated with the methodology?
4. **Data analysis.** Are there characteristic data analysis strategies associated with the methodology?

Reference “Contrast” Methodologies: Microgenetic Analysis and Grounded Theory

Rather than sampling a wide range of methods to compare with MLA, we have chosen two that are particularly relevant, both in commonalities and in differences. The first method is microgenetic analysis. Microgenetic studies have come to greater prominence in the literature in the last decade or so, particularly in developmental studies of learning. Generally speaking, microgenetic analyses involve: (a) a high density of “data points” during intellectual change, and (b) open attention to performance of subjects so as to induce both the relevant intellectual structures and processes of change (Siegler, 2006).²

Microgenetic studies have been used extensively to track the discovery and use of different strategies by children in solving simple arithmetic problems (Siegler & Crowley, 1991; Siegler, 2006). Typical results include that (a) strategy change happens slowly, (b) it involves “overlapping waves” of use of a diversity of strategies, some of which are waxing and some of which are waning in frequency of use, and (c) it involves surprisingly high levels of diversity across individuals and situations.

² Siegler also includes a third characteristic, that change is rapid during the period of study.

Given both the name and description, it is not obvious, at first blush, that MLA is anything other than the use of general microgenetic methods applied to an area (conceptual learning) that has been somewhat neglected in the microgenetic literature. However, differences between MLA and microgenetic studies in general will be important to consider.

Our second contrast methodology is grounded theory. Grounded theory arose in sociology and is explicitly intended as an empirical technique to develop theory that is tightly related to and practically applicable to particular “substantive domains.” For example, one might want to develop a substantive theory concerning the treatment of end-of-life patients in hospitals (Glaser & Strauss, 1967). Grounded theory is explicitly opposed to “formal theory,” which transcends substantive domains and is often developed from a priori considerations, and thus, according to Glaser and Strauss, difficult to apply. These authors, in fact, are highly suspicious of formal theory and assert that it likely can only be effectively achieved by building on and extending substantive theories.

Grounded theory is associated with one very particular and extensively described method, *constant comparison*. In this, the researcher considers his/her data, which often consist of notes on observations in the field, to develop a collection of categories at as fine a level of distinction as possible. Merging categories for the purposes of stating a general theory comes later. “Constant comparison” refers specifically to the practice of comparing occasions that were previously coded in the same way with the current one to think out similarities and differences. Categories are developed, revised, and used to code further data using constant comparison until the categories are “saturated,” that is, until no further categories are needed to cover a sufficient corpus of data. Grounded theories, then, are general relations among the categories, for example, that workers attend to *particular features* to distinguish *high status* patients, which are then given *distinctive care* toward the end of their lives.

The important and central common characteristic of MLA and grounded theory is the focus on empirical means of creating and improving theory. In fact, grounded theory is one of very few extensively documented methods aimed at theory building. This characteristic makes it quite distinct from most microgenetic analysis, as we shall elaborate later.

In what follows, we use mainly one authoritative source for each contrast methodology: Siegler (2006) for microgenetic analysis; Glaser and Strauss (1967) for grounded theory (But see also, for example, Siegler & Crowley, 1991; Chinn, 2006; Strauss & Corbin, 1990).³ Since MLA is nascent, we pretend, for the time being, that our own views are authoritative.

Systematic Comparison

Table 1 provides a summary of the arguments we make below.

³ Disputes about the essential nature of grounded theory have arisen. For our purposes, providing benchmarks against which to compare MLA, we do not need to adjudicate what is “real” grounded theory, nor would completeness with respect to branches provide significant additional value.

Table 1. Summary comparison of methodologies. Substantial commonalities across methods are marked with shading.

Framework Element	Sub-Element	MICROGENETIC LEARNING ANALYSIS (MLA)	MICROGENETIC ANALYSIS	GROUNDED THEORY
THEORY	STATUS OF THEORY	In development	Given, agnostic	In development
	NATURE OF THEORY	“Humble”	Grand	Substantive (opposed to grand)
	ORIGIN OF THEORY	Exogenous or endogenous	Exogenous	Endogenous
	TIME PROFILE	Extended	Unconstrained (owing to exogenous nature)	Small number of studies
INTELLECTUAL REGIME		Learning, especially conceptual change	Strategy change (prototypically)	Established cultural categories
EMPIRICAL DESIGN AND ANALYSIS	QUALITY	Naturalistic or lab, case studies (prototypically)	Lab, controlled (prototypically)	Naturalistic
	FOCUS	Intellectual change	Intellectual change	Stable social functioning
	UNIT	Extended learning encounter	Strategy choice and application (prototypically)	Ethnographic sampling
	TEMPORAL AND CONCEPTUAL GRAIN-SIZE	Very fine	Fine	Coarse or unspecified
	PROTOTYPICAL ANALYSIS FORM	Extended interpretation, complex argumentation	Classifying, counting, and time-sequencing strategies	Developing and applying coding categories

Status and nature of relevant theory. Microgenetic analysis is explicitly agnostic about its theoretical bases. Siegler, for example, mentions that microgenetic analysis has been used within several theories, such as information processing theory, Piagetian theory, neo-Piagetian theory, and dynamical systems theory. It seems clear that “theory,” within this framework, would be something like “grand theory” (in contrast to “humble theory,” as explained in diSessa & Cobb, 2004) or “formal theory” (in contrast to substantive theory within grounded theory). Glaser and Strauss (were they to study learning or development) would probably consider these theories exactly what they are trying to avoid: Highly abstract theories, which are hard to understand by practitioners and of uncertain usefulness in particular substantive domains. It is

difficult to teach teachers Piagetian theory or information processing theory, except in watered down form, and even then it is hard to know what to do with such theories in classrooms. (In this regard, see Metz, 1997, contesting “easy application” of Piaget to classrooms.) Siegler’s synthesis of microgenetic analysis includes essentially no description of the role of the method in developing new theories or in revising old ones.

It is sensible to believe that MLA practitioners have particular meta-theoretical commitments, perhaps aligning with the grand theories listed by Siegler. Whether this makes a difference in how MLA is carried out, however, is uncertain. Probably it is fairer to think of the proximal goal of MLA to be “humble theory” which is not necessarily strongly constrained by a particular grand theory.

While the theoretical concerns of MLA are more like those of substantive theory—to use data to bootstrap theory—MLA is much less prejudiced against abstract theory and a priori theorizing. Glaser and Strauss, in fact, recommend leaving all theoretical predilections at the door before entering a grounded theory study. We do not think that is possible, much less advisable. The reference humble theory of our case study, coordination class theory, was originally developed without data, on the basis of a theoretical critique of elements of information processing psychology (diSessa, 1994). Nonetheless, MLA explicitly attends to theory creation or refinement far beyond what appears evident in the microgenetic literature.

Grounded theory, as originally proposed by Glaser and Strauss, claimed that theory’s being easily recognized by practitioners and useful in their everyday work was critical both for validating the theory as well as using it in practical cases. MLA makes no such presumptions.

Source of the theory. Theory sources are largely exogenous to microgenetic analysis. Dynamical systems theory and information processing theory were not developed, in the first instance, on the basis of meticulous empirical study. Piagetian theory was not developed on the basis of microgenetic studies; those came much later. Whether a successor to information processing or Piagetian theory will be found is not particularly the concern of microgenetic analysis. There is no systematic concern or expectation concerning the process of developing a competitor theory within the microgenetic perspective. In contrast, MLA is primarily aimed at theory development.

Grounded theory has a strong, if not absolute data-based and inductive orientation. Glaser and Strauss, in fact, go so far as to assert that categories are easy to come by and almost always converge (become “saturated”) rapidly. MLA is much more eclectic, and we do not at all presume that the development of relevant categories will be easy or transparently empirical.

Time profile of theory development. As mentioned, microgenetic analysis treats theory as exogenous, so the time-profile of theory development is unconstrained. MLA considers that humble theory might originate in an MLA study, or such a study might refine and further develop an existing theory.⁴ In general, MLA treats theory development—certainly grand theory development—as an extended process to which many MLA studies will gradually contribute.

⁴ Of course, refutation of a theory is also a possibility. However, it is much less highlighted as a function for an experiment in MLA than, for example, in a Popperian view of science (Popper, 1963).

Grounded theory has very specific expectations for theory building. The aim of the method is to develop fairly complete substantive theories, each in one or a few grounded theory studies. In addition, domain-transcending theories are expected to be induced on the basis of several grounded theory developments across different domains. MLA makes no principled distinction between substantive and formal theory, although, as suggested above, we project that Glaser and Strauss (as of 1967) would consider its goals more formal than substantive, particularly since they felt that easy comprehension by practitioners is nearly *sine qua non*.

Intellectual regime and topic. Whether the topic of a learning study makes a substantial difference to the methods or theories created by MLA is, for us, unsettled. We intentionally mentioned studies that focused on physics, mathematics, and the relation between them. This may not be a very wide scope, and it is hard to know whether the method would be useful, for example, in studying learning sociology.

Microgenetic analyses have been conducted across a fairly wide range of topics. Siegler does not discuss any topic-dependence with respect to the conduct of microgenetic analyses.

In contrast, grounded theory was originally completely unconcerned with learning, so the question of “topic” does not naturally arise. We are unaware of any extensive literature using grounded theory in learning, as opposed, for example, to the sociology of educational practice, befitting its sociological roots. We do not here pursue the question of whether this is an in-principle limitation, or an accidental one.

Intellectual regime is a different matter. By this term, we mean to refer to qualitatively different general patterns of intellectual functioning, such as problem solving, skill deployment, skill learning, or concept learning. Of course, whether such regimes are coherently definable is a matter of debate. At various times, researchers have taken strong positions, such as early information processing views that claimed that almost all human activity can productively be seen as problem solving (Newell & Simon, 1972). Later, many topics of learning that are construed by many as strongly conceptual (e.g., algebra and programming) were treated as “skill acquisition” (e.g., Anderson, Conrad, & Corbett, 1989; compare with the Schoenfeld et al., 1993, account of learning about algebra). Our own position is driven by the informal observation that the study of skill acquisition seems to have made almost no contribution at all to one of our central concerns, conceptual change (for a compendium of recent work, see Vosniadou, 2008). Conceptual change, in fact, is a prototypical focus for MLA, and there are no studies of which we are aware using MLA to study skill acquisition. Siegler, in contrast, discusses the relative lack of microgenetic studies of learning mainly as a historical anomaly or oversight, which might need attention, but he marks no systematic distinction. By far the greatest number of microgenetic studies are of skill acquisition. So, MLA might belong to the generic category of microgenetic analyses as described by Siegler, but it is probably better treated as a related family involving major adaptations to apply to concept learning. Our predilection to think of it in this way might be related to the more local and context-adapted sense of theory that we take on, hence conceptual change and skill acquisition, for example, probably require different theories. In any case, it seems clear that MLA’s focus on theory development means that strategies directly related to theory development must be part of its repertoire, and these are, as yet, invisible in the main body of microgenetic literature.

Grounded theory is not specific about intellectual regimes. We imagine that the very idea of intellectual regimes is a rather cognitive one, not on the radar of sociologists. On the other hand, it seems evident that the focus of theorizing is different from microgenetic analysis or MLA: (a) Grounded theory has been concerned mainly with everyday cognition of particular groups and cultures, not so much how these develop or how new members might become enculturated; (b) Its prototypical theoretical statement is probably that a category of context is associated with a tendency for stimulating a category of action. Following the example introduced before (from Glaser & Strauss, 1967), when a high-status individual (rich, upper class, influential) is involved, healthcare staff give preferential treatment. This is far in both form and substance from deep theories of learning or conceptual change, or even skill acquisition. Glaser and Strauss's categories are likely virtually conscious in the community, or at least relatively easy to evoke. A recent practitioner and proponent of grounded theory, Shkedi (2005) says that, "During the interview, researchers help the informants make the unconscious more conscious." (pp. 131-132) These aims and conditions of grounded theory would seem responsible for Glaser and Strauss's claim that generating and even saturating categories is easy, and that it should be easy for practitioners to understand and use grounded theory.⁵ In contrast, since the waning of behaviorism, few believe that mechanisms of learning are evident to casual inspection, and they are not close to consciously accessible. Furthermore, if cognitive processes are imputed in grounded theory, the time-scale and meaning-resolution are large and indefinite compared to microgenetic analyses or MLA. On the whole, then, the focus of theorizing for grounded theory seems to be far from the structures and processes of learning or cognitive development so that application of the method to the foci of microgenetic analysis or MLA seems uncertain, even unlikely.

Empirical design. Siegler allows a wide range of empirical setups within the purview of microgenetic analysis, including case studies. However, the prototypical study that he considers in his review of 150 microgenetic papers is (a) highly controlled and laboratory based, and (b) implemented on extensive iteration of simple tasks, such as repeatedly asking subjects to add a pair of numbers. In contrast, conceptual learning tasks are almost always fairly extended and there is no evident sense in which one can ask subjects to repeat a learning exploration in order to validate their current "way of learning" and to gradually see them learn or perform exactly the same task in a different manner. In our terminology, we believe that the development of routine strategies is a different intellectual regime compared to learning complex concepts, and that has consequences in empirical design.

As briefly described above, MLA is committed (a) to the idea that concepts are complex, (b) that they are usually not evident in (but can be determined from) behavior, and (c) that they must be studied with respect to their development and deployment in a good number of contexts. Learning a concept simply cannot happen in a single try, as a new strategy usually appears, even if strategies take some time to become dominant. In contrast, Siegler explicitly notes that the mode for microgenetic study has been established in part because strategies are easy to observe and subjects can even pretty reliably self-report on their own use. Concepts, on the other hand,

⁵ Once again, the "virtually conscious" and narrative-based conditions that seem to be an essential focus for some versions of grounded theory may not be part of all branches of grounded theory. In that case, the considerations here may define a substantial fault line between different versions of grounded theory.

simply cannot be directly reported by subjects,⁶ and even their identification by researchers takes multiple contexts to triangulate reliably. See our prior discussion of apparent normative use of the concept of proper time, followed by a particular contextual case that revealed substantial non-normative properties (Levrini & diSessa, 2008).⁷

Concepts do appear in Siegler's (2006) review, but almost exclusively as constraints on, or facilitators of, strategy change. For example, conceptual competence with addition allows students to rule out some new strategies without trying them. In sum, then, we think that there is almost certainly an intellectual regime difference between the focus of MLA analyses and at least prototypical microgenetic analyses, and this makes a difference in empirical setup. Siegler, in contrast, sees concept learning as only incidentally different from strategy development. Siegler ends his synthetic paper by asserting that, as far as one can see now, long-term development (in which we would implicate the development of concepts) works on the basis of the same set of principles and mechanisms as short-term developments, such as strategy change.

Returning from theory to the proximal focus, empirical setup, MLA has "preference" for extended and complex engagements, hence a preference for case studies over simple, iterative performance. This is so for multiple reasons, which may be taken as assumptions of the approach: (a) a substantial conceptual learning event necessarily takes a certain amount of time; (b) even triangulating on the nature of students' concepts at any particular time requires extended consideration; and (c) learning events, themselves, are complex and compound events that are highly diverse, based on individual and contextual differences.⁸ MLA tends toward students' relatively autonomous learning. However, this certainly does not leave out more clinical, interactive studies. Because diversity is expected and because it is expected to be explained by *including* contextual elements, rather than *controlling for* them, even messy classroom studies are not particularly marginalized.

Anticipating later discussion of our case, we can illustrate the above methodological considerations. The choice made in our case study was to observe pairs of students. We believe this is in line with typical MLA empirical design. With respect to the polarity of "experimental" vs. "naturalistic and manifestly ecologically valid," this work tends toward the naturalistic end. Although some might argue that a researcher observing and interacting (albeit infrequently and minimally) with a pair of students is a bit far from the gold standard of classroom instruction, we note that real-world learning often happens in small groups, sometimes even with tutorial oversight. The experimenter took pains to make sure the students felt this to be a context in which they were in control, although she did intervene by answering questions for clarification and other purposes, and occasionally encouraged students to pursue avenues they might otherwise "prematurely" terminate. These interventions served to enrich both the exploration and

⁶ If this point needs further argument, we would add that concepts cannot be reported because they are very complex constructions of many elements that have unfamiliar ontological properties. Subjects simply do not have the theoretical vocabulary for reporting their own concepts. In contrast, strategies are behavioral and involve reporting "what was done" with easy-to-reference elements, such as numbers, fingers, pointing and counting.

⁷ It is possible that the difference between strategies and concepts is empirical accessibility, rather than intellectual regime. That is, perhaps strategies are as complex as concepts, but all we can see are the "macro" changes, shifts in strategy. This is not our preferred interpretation.

⁸ If they turn out to be compounded of strategy learning events, the assumptions behind Siegler's position may well be validated.

to help ensure adequate data on the students' knowledge state. In analysis, the general attitude is, to *take into account*, rather than *control for*, effects of the experimenter. So, for example, one would not count researcher-instigated episodes so as to assess inclination toward reflection. Similarly, episodes of learning during such a reflective response would not be assessed in terms of its bare likelihood, but with respect to the *possibility* that learning might happen in a particular way, *given the instigated reflection*.

Grounded theory, at least in its original form, is very far from engaging the tension between “experimental” vs. “naturalistic.” It just is naturalistic. Its metatheoretical position does not even require microgenetic study; its theoretical categories are (relatively) “easy to see” in the field, so laboratory setup is unnecessary. Both time resolution and conceptual resolution do not require video recording.

Data analysis. We will be brief with data analysis because the task of synthesizing and rationalizing analytical techniques appropriate for MLA is difficult and the case study, below, will introduce, tentatively, most of what we feel we have to say. However, we repeat the claim that a prototypical MLA analysis is in the form of an interpretation of learning, the elements of which are established by extended argumentation, using constantly-improved and empirically tested theoretical entities, and using extensive triangulation based on open use of data. Quantitative analyses have multiple roles in validation and so on, but they are not *sine qua non* and usually not the top-level form of analysis.

In contrast, Chinn (2006) explicitly rules out the possibility of microgenetic studies that are not directly quantitative and he also rules out those that do not involve control. He says, “Each participant typically encounters similar tasks and measures repeatedly (in randomized or counterbalanced orders) so as to permit systematic comparison across and within individuals.” (p. 441) Hence, by his criteria, none of the exemplars of MLA discussed above are microgenetic. Siegler also generally excludes studies that do not include quantitative data analysis from his survey of microgenetic analyses. However, he does include studies if they explicitly claim to be microgenetic, and he does not list any quantitative specification in the core principles of the method.

Analytical methods of grounded theory are highly focused on a synergistic process of coding, developing coding schemes, and looking for general connections among categories. There is almost no overlap with what Siegler discusses concerning microgenetic analysis, and because of the different kinds of theories pursued in MLA and grounded theory, there is unlikely to be much overlap there, either.

Summary of Comparative Analysis

Each of the primary comparison methodologies represents a central principle behind MLA. Microgenetic analysis represents a full-on and open attention to all aspects of performance during learning, at the smallest time-scale of change that is tractable. Grounded theory explicitly focuses on theory development, which is backgrounded in most microgenetic studies.

In terms of theoretical and metatheoretical orientation, MLA is positioned between the two comparison methodologies. Grounded theory, in common with MLA, focuses on more local theoretical developments, for example, those concerning one intellectual regime, such as

conceptual change. However, the prototype in grounded theory is more granular; roughly, one theory is developed in one set of empirical analyses. We would say that MLA aims for “deeper” theory than grounded theory, although such statements are bound to be contested. “Theory” for microgenetic analysis is typically taken to refer to “grand” theories that cover very large swaths of intellectual activity. MLA does not avoid such theories (unlike grounded theory), but concentrates on more local development, fitting particular contexts or regimes.

In terms of the focus of theorizing, MLA shares with microgenetic analysis a central concern with the structures and processes behind learning and change. However, MLA has a particular epistemological focus, which problematizes the very forms of knowledge in terms of which learning descriptions are made (knowledge “ontology”), as well as the processes and mechanisms of learning. Grounded theory, with its sociological roots, has not been particularly concerned with forms of knowledge or skill, nor with processes of change.

Synthesizers of microgenetic analysis (Siegler, 2006; Chinn, 2006) emphasize quantitative methods, or even exclude studies that are not controlled and do not use quantitative techniques. Since MLA is very theory focused, qualitative analyses are emphasized. However, quantitative methods, such as “coding and counting” to reveal significant differences, certainly are relevant to MLA studies in several ways, including validation of hypotheses. Video recording does not seem necessary for grounded theory. It may be useful for microgenetic analysis, and it is probably indispensable for MLA.

While microgenetic analysis aims to enfold both laboratory and, when possible, naturalistic observation, in practice it has been applied far most extensively in controlled laboratory situations. MLA often or usually cedes control in trade for the ecological validity of complex learning situations. Instead of controlling for factors, the aim is to include and explain their impact. The prototypical study for grounded theory is sociological, involving observations of everyday, “real world” activity, emphasizing different populations and contexts. Some such situations that are dense in learning might be useful for MLA.

A Methodological Case Study of MLA

As introduced earlier, our second strategy to explain MLA is to focus on a case study that exemplifies the general principles of the method with specific strategies, within a particular context, and with particular results.

Parnafes (2007) examines the development of students' conceptual understanding of “simple harmonic motion”,⁹ particularly with the mediation of dynamic and interactive representations (computer-based simulations). The analysis concerns two questions: (1) How can we describe conceptual change when it happens in this context? (2) How do the changes observed relate to the use of the representations? The data for this research were produced from

⁹ Simple harmonic motion is the motion within a canonical model in physics, the simple harmonic oscillator. A prototypical example of a simple harmonic oscillator is a mass attached to a “perfect” spring that is anchored at its opposite end. The motion of such a system is periodic and sinusoidal. The motion is characterized by its amplitude, its period (the time for a single oscillation)—or, equivalently, its frequency (the number of cycles per unit time)—and its phase (the starting time of the sine wave with respect to some standard).

observations of eight pairs of high-school students engaged in explorations of simple harmonic motion using both physical oscillating devices and computational representations.

The case study exemplifies the key characteristics of MLA:

- (1) It is theory-focused. Students' developing understandings through the use of representations was analyzed from a "knowledge in pieces" perspective (diSessa, 1983; 1993). The analysis uses an existing theory of concepts—coordination class theory—and improves it by (a) adapting it to a new context—the use of computer-based representations—and (b) elaborating four new constructs, mechanisms of learning, while still retaining core elements of the original theory.
- (2) It is fine-grained. The data consists of "only" eight sessions of one and a half hours each, a total of 12 hours of video, of which a small number of hours were analyzed in detail. However, the close examination of short segments of two or three minutes contained huge amounts of relevant detail to be accounted for, and provided opportunities for important schematizations of learning mechanisms.
- (3) It involves open consideration of data. In studying the interaction of representations with learning, students' verbal expression was central. However, gestures over representations, eye gaze, and manipulation of the representations were also particularly important (see examples later on).

MLA High Level Process: The Appropriation and Evolution of a Theoretical Framework

Developing even a modest bit of a learning theory from data involves inventiveness and insight, and that is far from a routine. Nonetheless, we attempt to describe some elements of the process.

We schematize the process of data analysis as it is carried out through the negotiation of an existing theory with the data at hand, which results in the appropriation of this theory to the specific research agenda and context. This process is comprised of three phases:

- 1. The incubation phase** – This phase is characterized by an open exploration, relatively free of any specific theory or model. The researcher is embedded in the data, looking for emerging patterns for more systematic examination.
- 2. The theory/data negotiation phase** – The patterns and ideas collected in the incubation phase are now examined against evolving or existing theoretical constructs. This phase may involve "shopping around" for existing theories that may be revealing for the particular questions and issues.
- 3. The theory appropriation phase** – In this phase, the source theory's constructs are applied to the data in view of the particular concerns of the research. Particular constructs of the theory may get modified or elaborated, and some new theoretical constructs may be developed. Characteristically, this is an intertwined process of theory application and theory development.

Even in the incubation phase, we do not assume that data analysis is completely theory-free. Data collection and interpretations rely on implicit and explicit meta-theoretical commitments. In particular, our two main research questions are based on the assumption that it is sensible to talk about knowledge structures and to infer them from observable behavior. The “knowledge in pieces” framework was taken as orienting our approach to conceptual change. However, that was only an orientation and did not specify terms and specific analyses. In fact, as reported below, a specific theoretical framework outside the knowledge in pieces was explored because it had some particular affinity with our general concerns and with some early data analysis. It turned out that a more specific theory *within* the knowledge in pieces perspective, coordination class theory, was eventually chosen to pursue detailed analyses. However, even that theory was adjusted and expanded to meet the needs of analysis and specific research questions. Thus, coordination class theory was treated as a humble theory, developed through multiple studies, of which this study is one.

Before describing the three phases in detail, we need a short description of coordination class theory to illustrate how the theory works in the case at issue, and also to show changes and extensions in the theory that were made during the study.

Coordination class theory

Coordination class theory (diSessa & Sherin, 1998; diSessa & Wagner, 2005) provides a means to describe and explain some properties of well-developed (e.g., normative) knowledge, in contrast to the nature of intuitive knowledge, which was the focus of earlier work within the knowledge in pieces framework (e.g., diSessa, 1993). It also aims to give an account of changes necessary in coming to a well-developed understanding.

A coordination class is a somewhat detailed model of a certain class of concepts, which appear to be important in learning school science. The model stipulates the concept as a set of mental structures that enact the process of getting a certain class of information from the world. “The fundamental assumption behind the idea of coordination classes is that information is not transparently available in the world. Instead, we have to learn how to access different kinds. Indeed, in different circumstances, we may need to use very different means to determine the same kind of information.” (diSessa, 2002, p. 43) A well-formed coordination class guarantees that the same type of information would be inferred across different situations. This property is called alignment.

The “relevant information” is that which defines the concept at issue. For example, knowing a force is tantamount to knowing its magnitude, direction, and location. Consider the following situations: a person pushing a block along a surface; an object falling down; and a book lying on a table. A first observation is that force is not directly perceptually available—one needs to learn how to “see” forces in the world. Second, force appears in different guises in the above situations, and retrieving information about forces varies from one situation to another. For example, the person pushing a block can feel the relevant force, but that is not possible in the other situations. Because one reasons differently in different situations, alignment (cross-contextual consistency) is threatened. It turns out novices frequently fail at producing aligned determinations; they think they are “seeing” force in different situations, but actually do not. Novices seem to understand the basic idea behind alignment; at least, they are disturbed when different attempts to determine the same information lead to different results. This phenomenon played an important role in the case developed here.

A coordination class includes two parts: a perceptual part and a causal part. The perceptual part includes the strategies of getting *any* relevant type of information from the world by means of directing attention to specific aspects. These are called *readout strategies*. One such readout strategy is proprioceptive sensation, for example, the pressure one feels in one's hand while pushing an object. The causal part includes the body of knowledge and reasoning strategies that determine the concept-characteristic information from the other information that is read out. This component is called the *causal net*. To exemplify, physicists know that proprioceptive sensations often directly implicate a force (its location, direction, and rough magnitude). Surprisingly, novices may not assume this.

The better developed a coordination class is, the more extensive and complex it is likely to be. Such complexity allows for breadth of use (called *span*) while maintaining cross-contextual consistency (alignment). Conversely, an intuitive conceptual system is fragmented and its knowledge elements are activated in different contexts not necessarily in a coordinated way, possibly leading to inconsistency (from a normative point of view) of inferences across contexts. Throughout the learning process, intuitive elements will often get slowly reorganized to be activated in relevant contexts in ways more and more similar to those in an expert's knowledge system.

Coordination class theory was chosen among competitors, as described below, as a starting basis for answering our general research questions. Through its use, it was adapted and refined to suit the data and the emergent, more particular research questions. In the following, we elaborate and exemplify the three-phase model of appropriation and evolution—the incubation, the theory-data negotiation, and the theory appropriation phases—which we propose to be typical, but not definitive, of MLA.

The incubation phase

Data analysis began with an extensive viewing of the videos from the studies,¹⁰ writing down notes and interpretations for segments that seemed interesting,¹¹ given our research focus. This was a broad exploration, looking for ways into the data, common patterns of learning or failing to learn, or puzzles that seemed to require explication. At this stage, no specific model or theory drove attention or filtered noticed phenomena.

Still, we can point to some structure in this early phase. The process can be sketched as an iteration of three practices: observation, schematization, and systematization (diSessa, 2008). We call this the OSS model. The first practice involves an open *observation* of an intellectual situation and identifying some notable occurrences. This usually involves identifying something that looks interesting, perhaps familiar from other contexts, and naming it for reference. For example, a possible observation is of students watching the simulation and expressing surprise that it is not behaving in line with their expectations. The second practice involves the

¹⁰ Obviously developing a data set that is rich in relevant data is a critical requirement, which we do not discuss here.

¹¹ The data was analyzed with the support of video analysis software – Transana (Wisconsin center for education research, <http://www.transana.org>). The software was developed specifically for educational research, and it enables uploading video movies, transcribing them, writing notes and interpretations, and creating small clips for easier access and examination.

schematization of the observed occurrences, which means iteratively generating descriptions of them, looking for data supporting or undermining the validity of that description, thus gradually improving the description. Schematization aims to produce explicit categories or types, descriptions of their essential properties, and how they might relate to other types.

The third practice involves the *systematization* and formalization of the characteristics of the schematized phenomena. This is done by the application of the schemas developed earlier on a broader range of episodes while strengthening and formalizing the framework throughout this process.

The OSS process is highly bootstrapped, and in fact, none of the practices can get very far without the others. Cycles may be long or short, and phases may not follow in the cyclic order portrayed. This prototypical process could lead to developing a theory (diSessa, 2008), but in the current case study, it built on the foundation of an existing theory, adapting and extending it to a new context.

The following two examples demonstrate issues that were generated at the incubation phase of the research. The two examples were selected because they left important traces through the entire analysis. Other ideas and directions proved less productive.

Example 1: *Breakdowns and their role in the process of developing understanding through the use of representations.* In one kind of observed occurrence, repeated several times with different examples, students' explorations drew them into perplexity.

These moments of perplexity were initially called *breakdowns*. They were *schematized* as moments involving: puzzlement, confusion, discomfiture, hesitation, and so on, but they all also involved various degrees of persistence in resolving the perplexity. A range of processes that lead to breakdowns was identified and characterized. For example, two students watched the simulation of harmonic oscillation (see Appendix A) with the expectation that, as the motion dies out, both the velocity and the frequency will decrease. However, they were surprised that, although the velocity decreased, the frequency remained constant. Another kind of breakdown followed from the difficulty of trying to map an interesting behavior in the representations to *any* corresponding behavior in the physical world. These schematizations facilitated the recognition of relations between the use of representations and the processes that led to breakdowns, and they also illuminated processes by which students manage breakdowns.

Systematization was not endemic in this phase since ideas were not pursued far, for example, to the point where they were tested systematically on various segments of data. Systematization happened more (and, in general, should be expected to happen more) when a starter-theory got appropriated. Still, some of the issues were pursued even at this phase to the point where some systematization took place, as in the following example. Traces of the breakdowns can still be seen in one of the core results of the study, that is, in one of the mechanisms (called “challenging”) of the developed model.

Example 2: *Different meanings of the word “fast.”* So-called *simple* harmonic motion, turns out not to be that simple for students (Parnafes, 2010). Early on, in one of the first iterations, it was noticed that the students used the word “fast” to mean different things (e.g.,

average velocity, instantaneous velocity, frequency, and more). This observation raised the jointly theoretical and empirical questions: How many meanings do the students use for the word “fast”? Do different students use the word “fast” differently? Is there a development through the session of the use of the word “fast”? What are the implications of the use of “fast” for the conceptual understanding of oscillatory motion, and on understanding the relation of velocity, time, and distance? Students’ different meanings for “fast” was then schematized and systematized into a rubric that included four different meanings, with attendant issues for each, including assumed relations (if any) among time, distance, and velocity. This rubric was systematically applied on several of the sessions for the examination of word use evolutions as well as cross-session variation in word use to see if patterns in word use evolution were an indication of conceptual development. The issue was pursued further and became one of the central foci of analysis. In later phases, the various meanings of “fast” were explicated theoretically and their differences were explained using coordination class theory.

These two example phenomena were among many that were collected and explored in the incubation phase. Other ideas led to dead ends (e.g., no coherent schematization could be conceived), or they were less attractive candidates for analysis owing to, for example, scarcity of data. Others seemed promising but did not fit well with the evolving cluster of issues. Incubation involves preliminary attempts to get to know the data intimately, to explore emergent issues, and to get a feel for various patterns in the data. Incubation produces seeds of subsequent theoretical constructs, as they become better articulated, more precise, and systematic.

The theory-data negotiation phase

While pursuing the incubation phase, a few specific theoretical frameworks were examined. Once a framework is selected for examination, it enters a phase of negotiation with the issues generated in the exploratory interpretation of the data. The candidate framework is applied on a range of segments of data, especially those that seem most central and promising, to examine the ways it supports interpretation and what insight it brings. We present the two main frameworks that were considered in this research. The first framework initially looked attractive but was abandoned for some critical failures. The second framework was eventually chosen for this research.

Framework 1: One of the theoretical frameworks examined was the semiotic model of Roth & Bowen (2001). This model describes the activity of sign interpretation and as such was promising in a context that highlights students’ representational activity. The model describes the activity of sign interpretation as containing two component activities: structuring and grounding. The structuring component refers to the identification of relevant signs from the symbol system. Specifically, when signs or representations are less familiar, interpretation requires a structuring activity in which one has to identify some features as signs that are intended to convey meaning. For example, a student can identify the space between two bars in the oscillation simulation (see appendix A) as a sign that has a specific meaning. The grounding component is based on the relationships among a sign S, a referent R, and an interpretant, I. An interpretant is another sign that stands for or elaborates the relation of sign to referent. For example, “ ‘A’ can stand for the sound ‘ah.’ ” Interpretants are the output of the processes in which learners construct new signs, which they link to the original sign in order to enhance their grasp of the content.

The framework initially looked appealing because many of its constructs were easily applied to the data. However, a substantial part of the representational activity occurring in the data is not captured by the model, which originated in learning about static representations. A few extensions needed to be applied to the framework in order to incorporate a context where representations are dynamic and interactive. These included: (a) the important activity of changing parameters in the simulation, and (b) the experimental activity—including prediction, testing, and reformulation—central to computer-based representation. Making these extensions and adaptations seemed both possible and fruitful. Still, the model was abandoned for a fundamental reason: The semiotic model did not take into consideration the conceptual activity of the student. Instead, it described the representational situation solely as observed external behavior. In short, the model yielded no purchase on how difficult concepts develop and could not help answer one of our main questions: How do students develop their conceptual understanding of science through the use of representations?

Framework 2: A later attempt to match and adjust a framework to the data and the questions at hand was using coordination class theory (diSessa & Sherin, 1998). Coordination class theory has some features that made it a good candidate for our purposes. First, our research aimed to describe how a conceptual system gets systematized and reorganized through a micro-analytical (element-by-element) analysis. Coordination classes already demonstrated the capacity to serve as a good micro-analytical model for describing students' conceptual knowledge (diSessa, 1991; diSessa, 1994; diSessa & Sherin, 1998; Thaden-Koch, 2003; Dufresne et al., 2005; Wagner, 2003; diSessa & Wagner, 2005; Wagner, 2006; 2010). Second, since coordination class theory highlights perceptual processes (a rather-neglected characteristic of concepts), it seemed likely to be particularly suitable in the context of using external representations. How does the (sometimes) easy observation of elements and phenomena in a computer-based representational system bootstrap seeing unnoticed or improperly coordinated features of the real world?

The following example illustrates the kind of insight that coordination class theory brought to the analysis, both in illuminating the phenomenon of students' multiple meanings of the word "fast" (see the second example in the incubation phase), and in providing clues concerning the representations' role in disentangling the clutter of meanings of "fast."

The case of Debbie and Rachel, using coordination class theory. This example has two parts. The first part demonstrates students' reasoning with physical oscillations and the tangle of meanings that were evoked. The second part demonstrates students' reasoning with the representations and suggests the role that representations serve in the process of coordination class development. In both parts, we highlight the role of the coordination class constructs to illuminate the issues.

Debbie and Rachel were asked to describe the differences and similarities between various oscillators: pendulums, springy rods, and springs. At one point they compared a short-string pendulum and a long-string pendulum and concluded that the long-string pendulum is faster than the short-string pendulum. Immediately after, they argued that a short springy rod (clamped by hand near the edge of a desk), when plucked, is faster than a long one. From a normative perspective, they arrived at opposite conclusions in two situations that, in fact, work according to the same principles. Their explanations are illuminated using coordination class theory: In each

context, the students are attuned to specific affordances and read out specific information, and therefore come up with different inferences to the critical determination of “fast”¹²:

Rachel: *(Pushes the ball again)* You see? It's faster... and then, when you hold it here *(shorten the string and holds the string closer to the ball)*, when you hold it close, then it *(the shorter pendulum)* seems a little slower, I don't know, it seems a little slower... and if you hold it at the top...

OP: Can you show me how it seems slower? If you hold it like that... *(referring to holding the string closer to the ball)* Why do you say it's slower?

Rachel: And it has less space to move, and this one is like more space to move.

Rachel explains that the long-string pendulum is faster than the short-string one because it has “more space to move.” In this specific context, Rachel reads the salient difference in distance, and infers “faster” (linear velocity) following the causal relation: the more distance it travels, the faster it has to go. The inference is non-normative in that it does not take into account duration of movement, but such unqualified inferences about speed have extensive documentation in the literature, going back to Piaget (Piaget, 1969; Levin & Gardosh, 1993). In this case, the conclusion is correct provided that “faster” means either average or instantaneous speed at corresponding moments.¹³

While distance was very salient in a pendulum, a different aspect of the motion draws their attention in the case of a springy rod:

Rachel: If you hold it, like, if you hold it from here *(clamps the springy rod so that only a short part oscillates, and pulls it; the high frequency tapping of the rod on the edge of the desk is heard)*, like, from close, then it's, it moves faster. And when you hold it up here *(extends a longer part of the springy rod, and pulls the end; the tapping of the rod is heard with a lower frequency)* it goes slower...

Rachel now argues that the short springy rod is faster because she hears the tapping of the rod on the desk, and it sounds “faster.” In this context, Rachel reads out a quality of the sound of the beats (“frequency,” as rendered by a physicist), and infers (implicitly) “faster.” In both contexts, Rachel infers from attending to a particular facet of the situation that the oscillator moves “fast” (or slow), even though, scientifically, frequency and linear speed are different concepts. This is a canonical case of lack of alignment; while Rachel believes she is determining the same kind of information in both situations, “fast or slow,” she is actually determining two different kinds of information. The development of the separate coordination classes of frequency and velocity—so as to direct attention (readout strategies) appropriately and infer

¹² Although in these excerpts only Rachel is speaking, both girls are active and Debbie complements Rachel’s arguments with similar ones. For brevity, we provide an excerpt with only Rachel’s statements.

¹³ This is true provided the pendulums are started with roughly equal angular displacements.

(using the causal net) the properly associated information—will mark a development in the students’ understanding of oscillatory motion. In preparation for later discussion, we note that period is not easy to see in the pendulum, and linear speed is almost impossible to determine perceptually in the springy rod. This is exactly the kind of situation coordination class theory was designed to deal with: Different situations draw attention to different feature, but they are used to “see” the same thing—here, speed.

Coordination class theory produced insights into the distinctive nature of students’ learning through use of representations. An excerpt from a later part of this session demonstrates this potential. As the students try to make sense of the representations, they also begin to distinguish the two meanings of “fast.” They read out particular information in the representations, and this information is mapped to aspects in the real-world phenomenon that may have been overlooked. This mapping between evident aspects of the situation and corresponding attributes in the world usually sets off a process of modest changes in the coordination classes.

Debbie and Rachel started by making correspondences between the movement of the ball (the red dot in large upper box) and the velocity-versus-time graph (bottom box). They note that the graph’s up and down path corresponds to the left and right motion of the ball:

Debbie: So, this would be the left and right, I guess (*pointing to the upper and the lower parts of the graph*)

Rachel: Yeah, this should be moving from here to here...and it does again the same thing

They read out some information that is particularly salient in this representation (the up and down path of the graph), and mapped it into another representation (the ball display), which mimics “real-world” harmonic motion. Then, Debbie moved to the middle representation, where the parameter is labeled “period.” They tried to make sense of period:

Rachel: Wait, so what’s the period?

Debbie: Maybe it tells you like... [Rachel: back and forth. I mean...] Oh, how long it takes it to move from one side to another...

Rachel: And then back and then forth... see... if you...

Debbie: Every time it reaches one side...

The information directly read out from the period display, a distance, is perceptually simple and salient. (What should be inferred from that, using the causal net, is not so obvious.) A short vertical bar is drawn periodically. The students immediately notice that the bar is drawn exactly when the ball begins a new cycle. This leads them to infer that the period corresponds to how long it takes for the ball to move from one side to another and back. Recall that the students did not attend to the information about the period when they looked at the pendulum in the first excerpt. Now that one of the representations highlights this aspect, however, they begin to attend to and talk about this aspect of the oscillation; that is, the period representation bootstrapped their attention to the period in the ball display.

After a few more exchanges, the students changed some parameters; they added friction so that the oscillation died out after a while. This led to the discovery of an important new relationship.

Rachel: And then it goes like shorter, and see, it becomes smaller

Debbie: Oh, and like the distance (*Debbie points to the distance between the bars.*), like the time it takes it to move from one side to another (*Debbie points to the animating ball.*) stays the same, so the only reason it really goes like less (*less distance*) is because it goes so slow.

Rachel reads out two pieces of information: (1) The ball travels a shorter distance every cycle; (2) the graph declines. Debbie reads out the information from the period representation: The distance between the bars stays the same. Immediately after, she says something about the relation between going less (in distance) and going slow (linear velocity). It appears that she is using a compensation causal relation: The reason that the ball goes less distance is not because of period, which is explicitly marked as constant, but only because it is moving more slowly. A slower speed can compensate for a smaller distance. This is a more sophisticated and more appropriate causal relation than the one Rachel has used before when she analyzed the motion of the pendulum: The more distance it travels, the faster it goes, without considering period.

This example demonstrates how the perceptual aspect of coordination class theory plays out in this analysis. Students read out salient information from the representations, which corresponds to essential information in the physical world; this draws attention to information that may have been ignored before (i.e., it seeds relevant readouts), and provides grounding for more careful and normative inferences (using the causal net). However, what drove this process and how it happened in detail needed to be explored and developed theoretically. Coordination

class theory provided a model of how concepts relate to attention, and it provided typical things that go wrong during construction of a scientific concept (misalignment). But the specific roles that representations could play in this process were previously unarticulated. This is precisely the starting point of the next phase, constructing a taxonomy and model of representation-based mechanisms that drive more adequate conceptualization.

To sum up this section, selecting a specific theory for answering specific questions using a specific data set must take into account both the fit with the researcher's theoretical commitments, the research questions, and the data set itself. We saw here two candidates for starting theories. Both theories were relevant in some obvious ways to the research questions. Both theories needed extension, and it was unclear before-the-fact whether the extension could be managed. In this case, Roth and Bowen's framework failed our goals in that it provided no guidance at all concerning conceptual development, a central aim of the work. After the fact, this seems to be a systematic and core commitment of Roth and Bowen's framework. It intentionally regards representational activity as establishing relations between external elements, but it does not illuminate the microstructure of coming to see new things in the world (e.g., a normative version of "fast"). In the case of coordination class theory, the representational component of the theory was not explicit in the original theory. However, this absence turned out to be an opportunity for a creative extension of the theory. These adaptations are explained in the next section.

Appropriating and developing the theoretical framework

Even though coordination class theory already existed, it was not used as a grand theory in the sense implied by Glaser and Strauss (1967). Rather, it was used as a theory in development: The applicability of a theory to a particular context is usually an empirical question, and it should not be surprising that modifications or extensions are necessary. In this case, the specific context of analysis was original in two ways: This was the first time coordination class theory was applied to the use of representations in supporting learning, and this was also only the second time the theory was used for tracking conceptual change in progress (the precedent being Wagner, 2006).

Theory appropriation takes place during continuing application of aspects of the theory on segments of data. Issues and insights from the incubation phase feed in to enrich and refine the theory. The idea is to start from "obvious" examples on which the framework applies quite naturally, and then, to extend the application to segments that are less likely to be explained by the framework, adapting as necessary. The application on less natural segments is important for understanding the limits of the framework, and extending it, if possible. As the range of phenomena broadens, the framework becomes firmer.

In our example, the theory was amended in various ways. We will mention briefly two types of elaborations, and then go into greater detail concerning a third type.

(1) Undeveloped constructs were developed through their application on the data. diSessa and Wagner (2005) mention, somewhat speculatively, that several coordination classes with close connections might participate in a mutual bootstrapping process—as one class becomes better defined and "stronger," related classes become more easily refined. Coordination classes that mutually bootstrap each other were called *coordination clusters*. However, this study was

the first time that coordination clusters were documented and specific connections marked out. In particular, the subject matter of harmonic motion revealed a clear relation between two coordination classes: velocity and frequency. The analysis described intricate relations between them during learning. The general theoretical move here might be described as “filling in a placeholder,” where a conjectured but undocumented element of a theory gets validated, exemplified, and, possibly, refined in later work.

(2) Newly developed constructs. “Coordination class” is a structural description of knowledge, and it does not specify whether a concept is normative or not. So, the theory does not rule out that naïve or intuitive ideas might be coordination classes. Indeed, the intuitive idea of “fast,” as examined in this data, seems potentially cogent and coherent—something like “how much of a certain kind of thing happens in a given time,” without specifying exactly what is happening (e.g. either distance or cycles might be “accumulating”). The intuitive idea of “fast” certainly has a range of contexts in which it can be effectively employed, and it is not clear that everyday inferential use of the idea stands much chance of uncovering misalignment. For example, the idea that “to go faster, you need to try harder” works for a runner both with respect to frequency (of steps) and with respect to linear speed.

Scientific coordination classes entail a very wide range of possible alignment checks. Similarly, the inferential structure of scientific ideas is quantitatively precise and, again, very extensive. But if we take into account the fact that a concept might be used only in a restricted range of contexts, and that the range and “resolution” (compare quantitatively precise to rough characterizations like “fast”) of inferences might be similarly limited, then coordination class theory might be insightful of the properties and development of everyday concepts. This line of thinking, supported by data in the case of the intuitive idea of “fast,” resulted in the development of the idea of *intuitive coordination class* (Parnafes, 2007).

(3) Relevant parts of the theory were used in new ways to generate insights into the current context. The central innovation of this research was generating a model that describes specific and fine-grained mechanisms that drive the process of developing understanding through the use of representations.

Coordination class theory claims that one type of change in a coordination class may be seen as an extension of the span of a coordination class. This means that a new context is drawn into the range of performance of the coordination class so that the coordination class now includes readout strategies and causal relations apt for determining the relevant information in this new context. For example, students can get information about velocity reliably in many everyday contexts, but they could not get reliable information about velocity in particular types of oscillatory motion. They would sometimes get information about frequency, sometimes about instantaneous velocity, and at other times about average velocity. Extending the span of the velocity coordination class means that they learned how to get reliable information concerning velocity in this new context of oscillatory motion. So, the focal question arose: How do computer-based representations contribute to increasing the span of a coordination class?

The answer to this question came in the form of four interrelated and repeated patterns of development, which we describe as *mechanisms of learning*. Each is described in coordination class terms, but, as detailed below, each extends coordination class theory specifically to include

the role of representations, and the mechanisms are also accountable to the specific conceptual accomplishments students made in this topic domain.

1. *The pattern detection mechanism:* When looking at dynamic and interactive representations, students may notice various patterns, either in the behavior of one representation, or the relations among various representations. In this mechanism the students act at the perceptual-representational level: Well-designed representations provide stable and reliable readouts of specific kinds of information, which are not usually read out of world situations. In this mechanism the patterns are detected mainly in the representations (e.g., the height of the graph declines) and not necessarily with any reference to the corresponding meaning in the real world (e.g., velocity declines).

2. *The anchoring mechanism:* This mechanism consists of a mapping between representational aspects and corresponding aspects in the physical phenomenon. The mapping of trusted, precise information in the representation onto information in the world that was hitherto vague and elusive promotes the elaboration of the coordination class specifically with respect to what one can see and how one directs attention to see it. In short, the students begin to develop reliable readout strategies to access correspondent information in the physical world. We describe this as “anchoring” or stabilizing readout strategies, and it happens with specific types of information that are privileged visually by the representation. In the example given in the second phase, recall how the students’ recognition of period in the physical world followed the observation of period in the representation.

3. *The challenging mechanism:* A conceptual challenge may arise when students recognize an inconsistency or a puzzle as they try to apply a causal relation. This is usually followed by a self-conscious conceptual effort aimed at resolving the challenge, which, if successful, may drive a reorganization or refinement of their causal net. For example, students expect that the distance between the bars in the representation will change as the motion dies out, but the distance remains constant. This representational behavior is surprising when applying a causal relation that relates a change in velocity directly to a change in time. Thus, the expectation of a specific behavior is not consistent with the representational behavior, leading to a conceptual challenge.

4. *The re-application mechanism:* In the re-application mechanism students resolve a conceptual challenge specifically by applying an existing causal relation that already works in other familiar situations. In this way the causal net of the related coordination class is extended to apply in a new context. This is a canonical case of increasing the span of the coordination class. We saw a limited version of this mechanism in the example of Debbie and Rachel. A previously used causal relation between two variables: “the more distance it travels, the faster it goes” transitioned to a more sophisticated causal relation among three variables: “the time it takes it to move from one side to another can be constant because distance and velocity compensate.” In this case, the transition was provoked by a conscious search for a way to resolve a conflict generated by the representation: The students saw that the period was the same regardless of amplitude (greater distance travelled). How could this be? The solution was to consider the joint effect of speed and distance, which the students did find salient in other contexts.

In the description of the mechanisms, one can see both the manifestation of the coordination class theory, and also some traces of ideas originated from the incubation phase and more advanced phases. For example, the realization of breakdowns and their role in the development of understanding is expressed in the third mechanism. A conceptual challenge has an important role in the process. It is caused by an inappropriate use of a causal relation, and is resolved by extending the range of causal relations that the students show in other contexts, but not initially in the context at hand.

It is worth emphasizing that the mechanisms emerged out of a selected set of events out of those considered in earlier phases. They were events in which learning was definite, in which representations were implicated (a choice for this research), and in which there was evident information about how the learning took place. Afterward, the mechanisms were applied to more problematic cases.

Reflections on More Detailed Practices of Data Interpretation in Conjunction with a Theoretical Framework

The foregoing section looked at “big picture” issues in terms of theory appropriation and development. This section reflects on some details concerning the interaction of exploratory observations and theoretical framework. We provide a set of principles for maintaining a sound approach to data analysis for the purposes of developing and revising theoretical frameworks. The practices described here cut across all three phases described in the previous section and are followed by examples as reflected in one particular piece of data analysis from the case study.

Keeping close to the data: We emphasize pitting interpretations directly against what the students say and do, and finding in their actions the locus of categories and processes. The researcher should be open to notice things that put his interpretation at risk. Such an approach keeps the researcher very close to the data; it avoids letting the interpretation or the theory “run the show.” This approach constantly problematizes interpretations, focuses systematically on specifically *what in the data* suggests or validates the interpretation, and strenuously avoids treating interpretations as evident.

Two levels of interpretations: We advocate a particular discipline that assists cogent interpretation: explicitly maintain two levels of interpretation in the data analysis. The first level attempts to be almost literal, and it includes many contextual details of the situation and avoids committing to particular theoretical terms. The second level restates the literal interpretation in relevant theoretical terms. Concerning this second level, it is critical to ensure that the theoretical level fits the literal level of interpretation. Do the theoretical terms really match the properties of what they purport to re-schematize from the literal level? Sometimes there are presumptions and preconditions in the theory that are not evident in the data; they might need to be ferreted out, or, failing that, recognized as a weakness in the interpretation.

Alternative interpretations: Conscientiously building alternative interpretations contributes to the rigor of the data analysis. This relates to the method of *competitive argumentation* (Schoenfeld, 1992; Schoenfeld et al., 1993, after VanLehn, Brown & Greeno, 1982), which involves constructing alternative coherent stories of the observed activity and pitting them against one another. Researchers who develop a rich common sense about how and when learning happens are at a particular advantage in developing alternative interpretations. Although

such interpretations may seem less rigorous than interpretations that are explicitly couched in theory, they are often difficult to displace convincingly. While competitor theories might also be a source of alternative interpretations, this might happen less often than one might think: Different theories tend to interpret different phenomena, hence they may not intersect in analysis of data.

Whether generated by the focal theoretical frame, by competing frames, or by relatively commonsense generalities, a consistent interpretation across multiple data segments can provide reinforcing or contradictory evidence. A consistent global frame is more difficult to maintain than interpreting different segments with different frameworks. In addition, searching out reasons that particular segments should *not* be interpreted in a particular way is a good discipline. This contrasts with simply removing a segment from the data base of interpretable segment because it does not seem consistently interpretable.

Before exemplifying the above practices, we briefly note that all the examples below are from the “literal” level of analysis. We restrict ourselves to this level because work at the higher, more theoretical level is less approachable. It is less familiar, more subtle, and innovating theoretical categories is, simply and by itself, difficult. As this is the first time that MLA has been explicitly named and described as a method, we expect later papers to tackle the harder problem of theoretical interpretation and bootstrapping. Many researchers new to methods like MLA will be surprised at how complex, but important, even “literal” interpretation can be. Developing data requires more than usually subtle interpretations of student actions. It requires us to know a lot about where people are focusing their attention and what they “see” early in their interpretive process. Attention and *readout* (in the coordination class sense) are often invisible or, at best, implicit in talk. Indeed, it is often true that students are just not aware of these lower-level processes, so they likely could not even respond reliably to questions, let alone automatically provide relevant verbal commentary. Other means are usually required to reach the requisite focus and level of detail.

Our example of the above-described practices and disciplines comes from the work of Debbie and Rachel, described before. During the exploration of the simulation, Debbie and Rachel changed parameters and increased the friction. They analyzed what happened in this situation:

Rachel: And then it goes shorter, and see, it becomes smaller, and...

Debbie: Oh, and like the distance (*Debbie points to the distance between the bars*), like the time it takes it to move from one side to another (*Debbie points to the animating ball*) stays the same, so the only reason it really goes like less is because it goes so slow.

In this segment, Rachel looked at all the representations and provided a description: “And then it goes shorter, and see, it becomes smaller, and...” This statement seems very simple, but it has many ambiguities. First, what does “it” refer to? There are three different representations, and it is not clear to which she is attending. Indeed, within each representation there are multiple features to attend to. Second, she uses two words, shorter and smaller. Do these words refer to two different features, or are they synonymous?

Without explicitly tracking these ambiguities, one could get attached to the first interpretation that comes to mind as something evident in the data. The first interpretation made by the researcher in this case was “obvious.” Rachel is looking at the graph representation, in which a distinctive and characteristic behavior is salient (at least it was salient to the researcher!), and she describes the behavior of the graph as getting shorter and smaller: synonymous words to describe one aspect. This was a theoretically relevant interpretation, apparently clean and simple. One feature (height of the graph) can be mapped to another one (velocity; the graph is labeled velocity!) in the physical world. However, a careful examination gave rise to an alternative interpretation. Rachel used two different words, shorter and smaller, and she might well be referring to two different features in the graph representation: One is that the peaks of the graph are getting shorter, and the second is that the graph as a whole is now smaller (horizontally; how long it takes for motion to decay), because it “ends” sooner. When transcripts fail to provide enough information, going back to the video may provide subtle contextual information that is necessary for precise interpretation. It could also provide hints as to where exactly on the screen the students are attending. Voice intonation could also be revealing in many cases, timing of prosodic element in coordination with on-screen action, and of course, gestures can disambiguate (for example, one versus two gestures for “shorter and smaller,” or horizontal versus vertical gestures).

Examining the video again brought about yet a third interpretation. It turned out that Rachel ended her description before the graph terminates (becomes “flat”). Therefore it is not reasonable that she is describing the length of the entire graph. (However, other students did attend to duration of decay, so this is not a priori an unrealistic hypothesis.) A small gesture gave rise to a new interpretation. As Rachel says, “and see, it becomes smaller...” she makes a very short gesture to point at the graph representation, as if to say, “See? The graph becomes smaller.” This gesture suggests that, before she made it, she might have attended to something else when she said: “and then it goes shorter.” The hypothesized interpretation at this point (yet to be tested by competitive argumentation) is that she attended to the ball going back and forth, realizing that the distance it travels becomes shorter; only afterward did she attend to the graph and note that it also becomes smaller.

This interpretation coheres with what comes right after, when Debbie uses Rachel’s observations to posit some relationships. Debbie’s statement is less ambiguous because she is using the mouse. As she talks, she points exactly to what she is referring:

Oh, and like the distance (*Debbie points with the mouse to the distance between the bars*), like the time it takes it to move from one side to another (*Debbie points with the mouse to the animating ball*) stays the same, so the only reason it really goes like less is because it goes so slow.

Debbie realizes that the distance between the bars, mapped by her to the period, stays the same. She explains this by talking about two compensating aspects—exactly those aspects that Rachel (apparently) referred to: the shorter distance covered by the ball, and the decline of the graph. Rachel’s acquiescence helps, assuming the girls are aligned with each other’s talk, which

is something that can be supported by the general quality of their interactions. But, another part of the interpretation is still unsettled. Does Debbie really interpret the graph as a velocity versus time graph? She might as well infer that the velocity declines from the animating ball representation. There are two pieces of evidence that this is probably what she does. First, she used the mouse to indicate where she is looking, and when she talks about velocity, the mouse is not active; it stays near the animated ball. Second, using competitive argumentation, when later episodes are examined, both Rachel and Debbie have difficulties interpreting the graph representation. When they use it, they usually interpret it as a distance-versus-time representation (which is a common difficulty, noted in the literature).

This example concerned the first, “literal” level of interpretation. Although it is a critical level, providing a foundation for the “real” analysis, it is often obscured or completely left out of analyses in papers. It highlights the sophistication and care needed in order to understand even the literal level of students' activity of sense making. Note that this level of analysis did not explicitly include the constructs coordination class, readout strategies, or causal relations, nor did it show detail in the bootstrapping of changes in constructs, much less developing new constructs. The higher level of interpretation consists mainly of an (extensively argued) overlay of theoretical terms on top of the foundational level of interpretation.

Discussion

In this paper, we introduced a method called microgenetic learning analysis (MLA), a qualitative method for studying knowledge in transition. The method has three main characteristics: (1) *Theoretical* - Its primary aim is to generate and improve theories about learning; (2) *Fine grain size* - It entails looking at small time-scales and detailed nuances of meaning, where processes of change are best seen; (3) *Open consideration* - The focus of attention is fundamentally unconstrained by preconceptions concerning, or pre-filtering, what counts as relevant data.

We developed a framework for comparing methods that may be used for studying learning along four main dimensions: (a) theoretical and metatheoretical commitments, (b) intellectual regime and topic, (c) empirical design, and (d) data analysis. We then provided an elaborated comparison between MLA and two well-known methodologies—microgenetic analysis and grounded theory—summarized in Table 1. The top level observation is that MLA aligns with grounded theory exclusively with respect to item 1, above (theoretically oriented), but it aligns with microgenetic analysis with respect to items 2 and 3, grain size and open attention to data. More generally, we hope the dimensions of analysis might be of use in understanding a range of qualitative methodologies, not just those of interest here, and their affinities, dissimilarities, appropriate and inappropriate uses.

We exemplified MLA using one case study. Through this example, we highlighted and schematized a few methodological issues and practices in order to provide some sense of the “how to” of the method. In particular, we described a process, typical to MLA research, of selecting, appropriating, and further developing a theoretical framework. We schematized this process as involving three phases: (1) The incubation phase, where ideas for analysis are seeded; (2) The theory-data negotiation phase, where a theory is selected and/or developed for answering the research question; (3) The theory appropriation phase, where the theory is more systematically applied to the specific context of research. In addition, we described a general

practice (the OSS model: observe, schematize, systematize) that may be applied at many points in the above three-phase model. Finally, we provided some guiding principles and listed some “good practices” for maintaining a sound approach to data analysis within the MLA method. These included competitive argumentation and maintaining a more literal level of interpretation at least somewhat independent of theoretical interpretation.

Many additional topics need to be addressed in future work. For example, (1) we did not discuss the difficult task of developing a rich data source; (2) we barely introduced the complex strategies and judgments relevant to the details of theory bootstrapping; and (3) we said little about the delicate task of properly overlaying theoretical interpretation on a “literal” level of analysis. Concerning the general process of iterative development of theory, one would like to know something about when that can succeed, and when it might degenerate into ever more “epicycles” of complexity. Finally, as we mentioned at the beginning, a case study—as well as having strengths in getting down to particulars and showing more of what actually is done and with what results—has clear limitations. We encourage further case studies and reflective consideration of them in order to develop more complete, more secure, more practical, and more broadly applicable descriptions of MLA and its variants.

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Appendix A – The simulation of harmonic oscillation

The simulation of simple harmonic oscillation used in this research, includes three linked representations (**Error! Reference source not found.**): (1) An animation of an oscillating object that goes back and forth and leaves traces behind; (2) “The bar representation” is produced by the depiction of one bar on a time line, every time the oscillating object begins a new cycle—representing the periods and the frequency of the oscillator; (3) A velocity versus time graph.

Students can change the amount of displacement, the friction and the spring constant of the oscillator, and thus explore some of the causality in oscillatory motion.

[Insert figure 1 about here]