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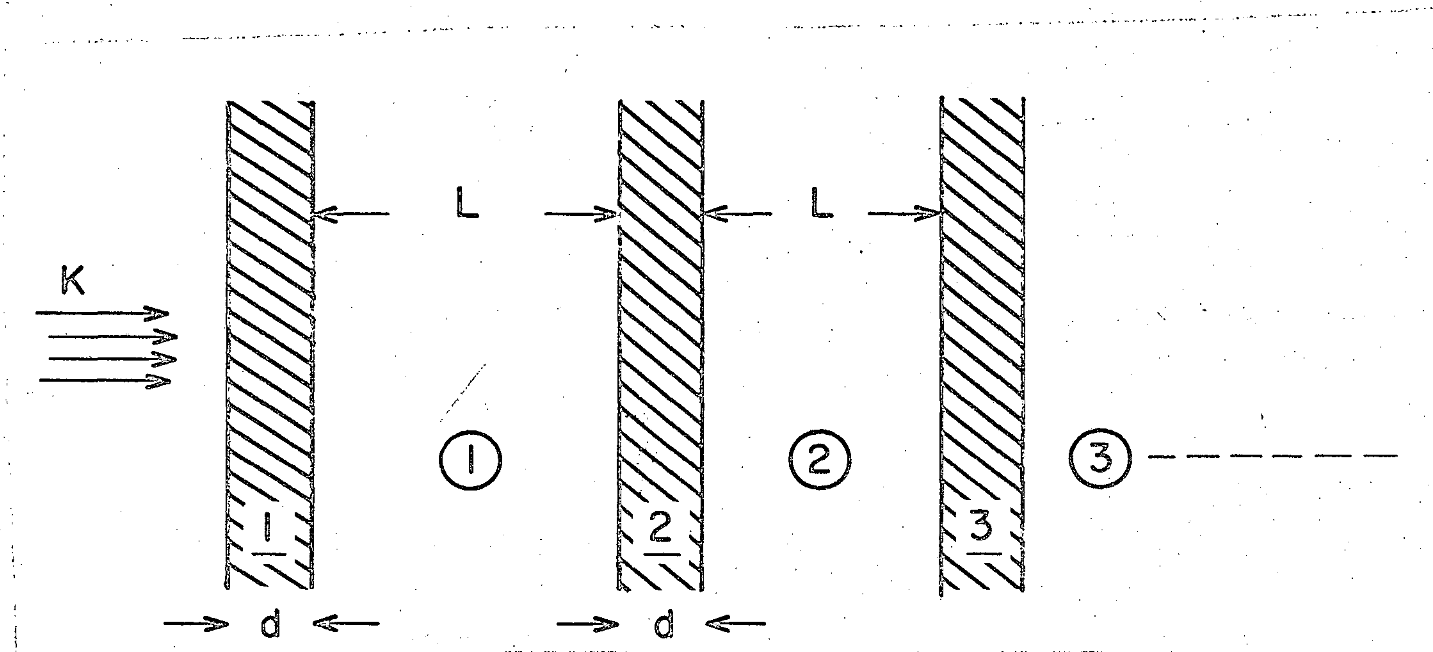
ABSTRACT

Formulae are derived for the propagation of a K^o beam through a series of plates and gaps. Various special cases of practical interest are investigated.

1. INTRODUCTION

We consider the propagation of an arbitrary K_1^o and K_2^o beam through a series of parallel plates, of thickness d, with gap separation L.



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The plates are labeled 1, 2, ··· n.

The gaps are labeled 1, 2, ··· n-1.

We denote by t_x the time, in the laboratory system, for the K^o to cover the distance x. Let τ_1, τ_2 be the lifetimes, in the laboratory system, of the short- and long-lived K^o's respectively.

There are two cases of interest: Case (a), $t_L << \tau_1$; Case (b), $t_L >> \tau_1$.

Since a single plate, as viewed by the K meson, consists mainly of vacuum, it is clear that in Case (a) the set of plates and gaps will, under certain conditions, reproduce the behavior of a single, diffuse plate.

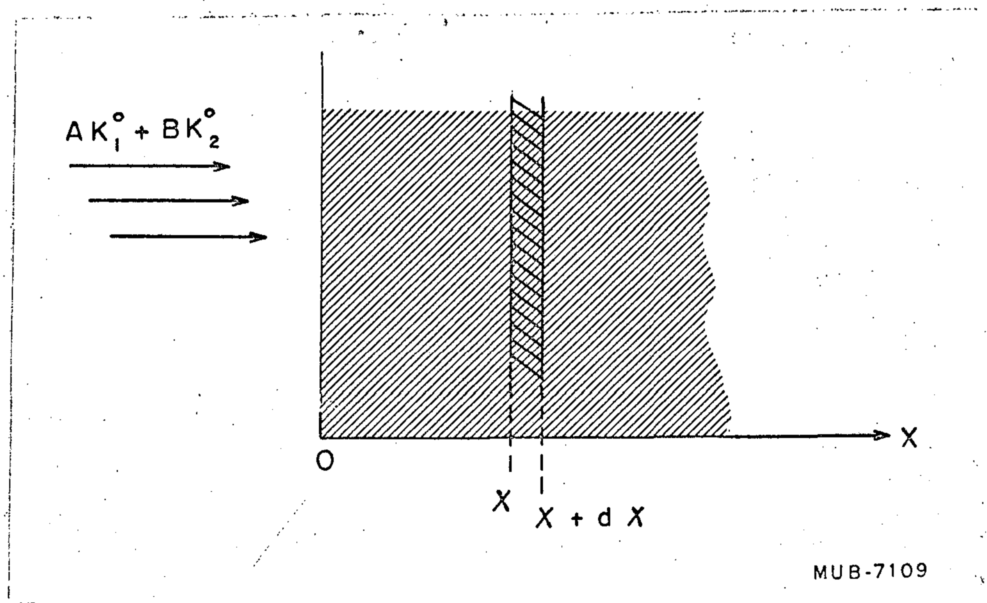
On the other hand, in Case (b) the $K_1^{\,o}$ component dies out completely in each gap, so we expect that the wave function will simply repeat itself in each gap. These intuitive conclusions are borne out in the following calculations.

We shall consider only transmission regeneration. This can be sorted out experimentally from nuclear diffraction regeneration by looking at sufficiently small angles about the forward direction.

The results derived below are not claimed to be new. In fact the calculations are rather trivial, and the results are known to many people. The sole aim of this paper is to collect the formulae and derivations into an easily accessible form, which, it is hoped, will be of some use to the experimentalists.

In Section 2 we consider propagation through a single plate, and in Section 3 propagation through the series of plates and gaps. Section 4 contains a summary of the results.

2. PROPAGATION THROUGH A SINGLE PLATE



In practice the only case of importance is when the wave arriving at $X = 0$,

$$\psi = AK_1 + BK_2 , \tag{2:1}$$

has $|A| << |B|$. It is then permissible to neglect the regeneration of K_2 from K_1.

The K_1 component, at a distance X from the origin, becomes

$$A\, e^{ik_1 X}\, e^{-t_X/2\tau_1}\, e^{-NX\sigma_T/2} , \tag{2:2}$$

where k_1 is the K_1 lab momentum,

N = the number of atoms/cm^3 in the material of the plate, and

$\sigma_T = \sigma_T(K_1) \approx \sigma_T(K_2)$ is the total cross section for K's interacting with the nuclei of the plate.

It is supplemented by the regenerated amplitude

$$\alpha_{21} = \int_0^X d\alpha_{21} \quad , \tag{2:3}$$

where

$$d\alpha_{21} = BiN\lambda f_{21} \exp\,[ik_2 x]\,\exp\,[ik_1(X-x)]\,\exp\left[-\frac{x}{v}\,\frac{1}{2\tau_2}\right]$$

$$\times\left[\exp\; -\frac{X-x}{v}\,\frac{1}{2\tau_1}\right]\exp\,[-NX\sigma_T/2]dx$$

is the K_1 amplitude regenerated in an infinitesmal strip dx.

Here

$$f_{21} = \frac{1}{2}\,[f(o) - \bar{f}(o)] \quad , \tag{2:4}$$

where $f(o)$ and $\bar{f}(o)$ are the forward scattering amplitudes for K^o and $\bar{K}^o$ on the plate nuclei, and λ and v are respectively the wavelength and velocity of the K's (lab).

Putting

$$k_2 - k_1 \approx \frac{m_1\Delta}{k_1} \quad , \tag{2:5}$$

with $\Delta = m_1 - m_2$,

and defining

$$\delta = \frac{\Delta\tau_1^{c.m.}}{\hbar} \quad ; \quad \Lambda = \tau_1 v \quad , \tag{2:6}$$

it is easy to integrate (2:3) and get it into the form

$$\alpha_{21}(X) = Bi \frac{N\lambda f_{21}}{\Lambda(i\delta + 1/2)} e^{ik_1 X} e^{-t_X/2\tau_1} e^{-NX\sigma_T/2}$$

$$\times [e^{it_X\delta/\tau_1} e^{t_X/2\tau_1} e^{-t_X/2\tau_2} - 1] \quad . \tag{2:7}$$

We are neglecting τ_1/τ_2 compared with 1 in the denominator. Lastly, the unconverted K_2 component becomes, at X,

$$Be^{ik_2X} e^{-NX\sigma_T/2} e^{-tX/2\tau_2}$$

$$= B(e^{ik_1X} e^{-t_X/2\tau_1}) (e^{i\delta t_X/2\tau_1} e^{t_X/2\tau_1} e^{-t_X/2\tau_2}) e^{-NX\sigma_T/2} \quad . \tag{2:8}$$

If we now define

$$\gamma(X) = e^{ik_1X} e^{-t_X/2\tau_1}$$

and

$$\beta(x) = \left\{ \exp t_X \left[\frac{i\delta}{\tau_1} + \frac{1}{2\tau_1} - \frac{1}{2\tau_2} \right] \right\} \tag{2:9}$$

then (2:2), (2:6) and (2:18) combine to give, for the total wave at X,

$$\psi(X) = \gamma(X)\, e^{-NX\sigma_T/2} \left\{ \left\langle A + \frac{Bs}{\frac{1}{2} + i\delta} \,[\beta(X) - 1] \right\rangle K_1 + B\beta(X)K_2 \right\} . \tag{2:10}$$

We have introduced the notation

$$s = \frac{i\lambda N f_{21}}{\Lambda} . \tag{2:11}$$

There are two cases of particular interest:

<u>Case (i)</u>, $t_X \ll \tau_1$; $A = 0$ (pure incoming K_2 beam):

$$\psi(x) \approx B \frac{st_X}{\tau_1} K_1 + K_2 , \tag{2:12}$$

i.e., the K_1 component <u>increases linearly with</u> t_X .

<u>Case (ii)</u>, $t_X \gg \tau_1$; $A = 0$:

in this case

$$\gamma(X)[\beta(X) - 1] \approx e^{ik_1 X} \exp\left[t_X \left(\frac{i\delta}{\tau_1} - \frac{1}{2\tau_2} \right) \right] .$$

Thus

$$\psi(X) \approx B \exp\left[ik_1 X - \frac{NX\sigma_T}{2} + t_X\left(\frac{i\delta}{\tau_1} - \frac{1}{2T_2} \right) \right] \left\{ \frac{s}{\frac{1}{2} + i\delta} K_1 + K_2 \right\} , \tag{2:13}$$

i.e., equilibrium is established and the ratio of K_1 to K_2 remains fixed at the value $\frac{s}{\frac{1}{2}+i\delta}$. We note that for a vacuum (2:10) becomes

$$\psi_v(X) = \gamma(X)\,\{AK_1 + B\,\beta(X)\,K_2\}\;, \tag{2:14}$$

and that for the vacuum following the plate on which a pure K_2 beam is impinging,

$$\psi_v(X) = \gamma(d)e^{-N\sigma_T d/2}\cdot\gamma(x)\left\{\frac{s}{\frac{1}{2}+i\delta}\,[\beta(d)-1]K_1 + \beta(x)K_2\right\}, \tag{2:15}$$

where $x = X - d$. For Cases (i) and (ii) above we get

$$\psi_v(X) \approx B\gamma(x)\left[\frac{std}{\tau_1}\,K_1 + \beta(x)K_2\right] \tag{2:16}$$

and

$$\psi_v(X) \approx B\gamma(d)\beta(d)\,e^{-Nd\sigma_T/2}\,\gamma(x)\left[\frac{s}{\frac{1}{2}+i\delta}\,K_1 + \beta(x)K_2\right]. \tag{2:17}$$

3. PROPAGATION THROUGH A SERIES OF PLATES

Consider an incoming pure K_2 beam.

We denote by $\psi_{\underline{n}}(x)$ the wave at a point at distance x inside the $\underline{n}$th plate. Similarly $\psi_{ⓝ}(x)$ is the wave at a distance x inside the $\underline{n}$th gap.

Thus the point x in $\psi_{\underline{n}}(x)$ is actually at

$$X = (n - 1)(L + d) + x \quad 0 \leq x \leq d \quad . \tag{3:1}$$

Similarly, x in $\psi_{ⓝ}(x)$ is the point

$$X = (n - 1)L + nd + x \quad 0 \leq x \leq L \quad . \tag{3:2}$$

Using the fact that $\gamma(x)$, $\beta(x)$ are exponential functions, so that, e.g., $\gamma(x)\,\gamma(y) = \gamma(x + y)$, a repeated application of Eqs. (2:10) and (2:14) yields

$$\begin{aligned}\psi_{\underline{n}}(x) = {} & \gamma\left((n - 1)(L + d) + x\right) \exp[-N\sigma_T/2 \left((n - 1)d + x\right)] \\ & \times \{[s[\beta(d)-1][1 + \beta(L + d) + \beta(2L + 2d) + \cdots + \beta\left((n - 2)(L + d)\right)] \\ & + s[\beta(x)-1]\beta\left((n - 1)(L + d)\right)]K_1 + \beta\left((n - 1)(L + d) + x\right) K_2\} \quad ,\end{aligned}$$

(3:3)

$$\psi_{n}(x) = \gamma\left|nd + (n-1)L + x\right| e^{-N\sigma_T nd/2}\left\{\frac{s}{\frac{1}{2}+j\delta}\left\langle\left|\beta(d) - 1\right| [1 + \beta(L+d)\right.\right.$$

$$\left.\left. + \beta\left|(n-1)(L+d)\right|]\right\rangle K_1 + \beta\left|nd + (n-1)L + x\right| K_2\right\} . \qquad (3:4)$$

The series in square brackets can be summed, and using (3:1) and (3:2), we get finally

$$\psi_{\underline{n}}(x) = \gamma(X) \exp\left[-N\sigma_T[(n-1)d + x]/2\right]\left\{\frac{s}{\frac{1}{2}+i\delta}\left\langle\frac{\beta(d)-1}{\beta(L+d)-1}\cdot\left[\beta\left|(n-1)(L+d)\right| - 1\right]\right.\right.$$

$$\left.\left. + [\beta(x) - 1]\beta\left|(n-1)(L+d)\right|\right\rangle K_1 + \beta(X)K_2\right\} \qquad (3:5)$$

and

$$\psi_{n}(x) = \gamma(X) \exp[-N\sigma_T nd/2]\left\{\frac{s}{\frac{1}{2}+i\delta}\ \frac{\beta(d)-1}{\beta(d+L)-1}\ \left[\beta\left|n(L+d)\right| - 1\right]K_1\right.$$

$$\left. + \beta(X)K_2\right\} . \qquad (3:6)$$

We now make the fundamental assumption that n is so large that

$$\frac{n-1}{n} \approx 1 . \qquad (3:7)$$

Then (3:5) and (3:6) simplify to the form

$$\psi_{\underline{n}}(x) \approx \gamma(X)\ \exp\,[-N\sigma_T nd/2]\left\{\frac{s}{\frac{1}{2}+i\delta}\left\langle\frac{\beta(d)-1}{\beta(L+d)-1}\ [\beta(X)-1]\right.\right.$$

$$\left.\left. +\ [\beta(x)-1]\beta(X)\right\rangle K_1 + \beta(X)K_2\right\} , \tag{3:8}$$

$$\psi_{n}(x) = \gamma(X)\ \exp\,[-N\sigma_T nd/2]\left\{\frac{s}{\frac{1}{2}+i\delta}\left\langle\frac{\beta(d)-1}{\beta(L+d)-1}\,[\beta(X)-1]\right\rangle K_1\right.$$

$$\left. +\ \beta(X)K_2\right\} . \tag{3:9}$$

These are the fundamental equations, valid as long as $n \gg 1$.

We now look at various special cases.

<u>Case (aI)</u>, $t_L \ll \tau_1$; $t_d \approx \tau_1$.

Then

$$\psi_{\underline{n}}(x) \approx \gamma(X)\ \exp\,[-N\sigma_T nd/2]\left\{\frac{s}{\frac{1}{2}+i\delta}\ [\beta(X+x)-1]\ K_1+\beta(X)K_2\right\} , \tag{3:10}$$

$$\psi_{n}(x) \approx \gamma(X)\ \exp\,[-N\sigma_T nd/2]\left\{\frac{s}{\frac{1}{2}+i\delta}\ [\beta(X)-1]K_1 + \beta(X)K_2\right\} . \tag{3:11}$$

Since $n \gg 1$, it follows for Case aI that

$$t_X \gg \tau_1 . \tag{3:12}$$

Then Eqs. (3:10) and (3:11) assume the <u>same form</u>, and we have, using the definitions of γ and β,

$$\psi(X) \approx \exp\left[ik_1X - \frac{NX\sigma_T}{2} + t_X\left(\frac{i\delta}{2} - \frac{1}{2\tau_2}\right)\right]\left\{\frac{s}{\frac{1}{2} + i\delta} \quad K_1 + K_2\right\} \qquad (3:13)$$

for any $t_X >> \tau_1$, whether in a gap or in a plate.

Equation (3:13) is identical with (2:13) for a single thick plate. Thus, as is intuitively clear, a series of thick plates with small gaps behaves like a single thick plate.

<u>Case (aII)</u>, $t_L << \tau_1$; $t_d << \tau_1$.

This is the case of great practical interest.

Equations (3:8) and (3:9) become

$$\psi_{\underline{n}}(x) \approx \gamma(X) \exp\,[-N\sigma_T nd/2]\left\{\frac{s}{\frac{1}{2} + i\delta}\left\langle \frac{d}{d+L}\,[\beta(X)-1] + \frac{t_x}{\tau_1}(i\delta + 1/2)\beta(X)\right\rangle K_1 + \beta(X)K_2\right\}, \qquad (3:14)$$

$$\psi_{n}(x) \approx \gamma(X) \exp\,[-N\sigma_T nd/2]\left\{\frac{s}{\frac{1}{2} + i\delta} \cdot \frac{d}{L+d}\,[\beta(X) - 1]K_1 + \beta(X)K_2\right\}. \qquad (3:15)$$

Now $\frac{t_x}{\tau_1} = \frac{x}{\Lambda}$ $0 \leq x \leq d$ in (3.14). So, provided $[\beta(X) - 1] \not\approx 0$, we can neglect the term $\frac{t_x}{\tau_1}$ in (3:14) .

This will be the case if $t_X \gtrsim \tau_1$. We then get again that there is no difference between $\psi_{\underline{n}}$ and $\psi_{(n)}$, and both are given by

$$\psi(X) = \exp\left[ik_1X - \frac{N\sigma_T nd}{2} + t_X\left(\frac{i\delta}{\tau_1} - \frac{1}{2\tau_2}\right)\right]\left\{\frac{d}{d+L}\frac{s}{\frac{1}{2}+i\delta}K_1 + K_2\right\} . \tag{3:16}$$

If we now define

$$\bar{N} = \frac{d}{d+L} N , \tag{3:17}$$

$$\bar{s} = \frac{i\lambda\bar{N}f_{21}}{\Lambda} ,$$

then (3:16) becomes

$$\psi(X) = \exp\left[ik_1X - \frac{\bar{N}\sigma_T X}{2} + t_X\left(\frac{i\delta}{\tau_1} - \frac{1}{2\tau_2}\right)\right]\left\{\frac{\bar{s}}{\frac{1}{2}+i\delta}K_1 + K_2\right\} , \tag{3:18}$$

which is identical with (2:13) with $N \to \bar{N}$.

Thus the collection of thin plates with small gap lengths behaves for $t_X \gtrsim \tau_1$ like a single plate of diffuse material, having $\bar{N} = \frac{d}{d+L} N$.

Note that in this Case (aII), the condition $t_X \gtrsim \tau_1$ actually implies $n >> 1$, as required for the validity of (3:8) and (3:9)

Case (b), $t_L >> \tau_1$

Here we need not insist on $n >> 1$ to get simple results. Hence we return to Eqs. (3:5) and (3:6), which now simplify to

$$\psi_{\underline{n}}(x) = \gamma(X) \exp[-N\sigma_T[(n-1)d + x]/2] \left\{ \frac{s}{\frac{1}{2} + i\delta} \langle B(-L)[\beta\{(n-1)(L+d)\}-1] + [\beta(X) - 1]\beta\{(n-1)(L+d)\}\rangle K_1 + \beta(X)K_2 \right\}$$

$$\approx \gamma\{(n-1)(L+d)\}\beta\{(n-1)(L+d)\} \exp[-N\sigma_T(n-1)d/2]$$

$$\gamma(x) \exp[-N\sigma_T x/2] \left\{ \frac{s}{\frac{1}{2} + i\delta} [\beta(x) - 1] K_1 + \beta(x)K_2 \right\}, \quad (3:20)$$

$$\psi_{(n)}(x) = \gamma\{(n-1)(L+d)\}\beta\{(n-1)(L+d)\} \exp[-\sigma_T N(n-1)d/2]$$

$$\times \gamma(d)\beta(d) \exp[-N\sigma_T d/2]\gamma(x) \left\{ \frac{s}{\frac{1}{2} + i\delta} [\beta(d) - 1]K_1 + \beta(x)K_2 \right\} . \quad (3:21)$$

In this case the behavior in the plates and gaps does not become identical. However, the wave function in each plate or gap is the same, except for the overall attenuation--phase factor

$$P(n) = \gamma\{(n-1)(L+d)\}\beta\{(n-1)(L+d)\} \exp[-N\sigma_T(n-1)d/2]$$

$$= \exp[i(n-1)k_1(L+d)] \exp\left[(n-1)(t_L+t_d)\left(\frac{i\delta}{\tau_1} - \frac{1}{2\tau_2}\right)\right] \exp[-(n-1)N\sigma_T d/2] . \quad (3:22)$$

Moreover, aside from this attenuation--phase factor, Eqs. (3:20) and (3:21) are identical with (2:10) (for the case A = 0) and (2:15) respectively, i.e., with the single-plate, single-gap formulae. Hence, as expected, large-size gaps lead to results to which the plates act individually as if no others were present, and the pattern simply repeats itself in each plate and gap.

Corresponding to the Cases (i) $t_d \ll \tau_1$ or (ii) $t_d \gtrsim \tau_1$ [analogous to Cases (i) and (ii) in Section 2], we get

$$\psi_{\underline{n}}(x) = P(n) \left\{ \frac{st_x}{\tau_1} K_1 + K_2 \right\} , \tag{3.23}$$

$$\psi_{n}(x) = P(n)\gamma(x) \left\{ \frac{st_d}{\tau_1} K_1 + \beta(x) K_2 \right\} , \quad (t_d \ll \tau_1) , \tag{3:24}$$

and

$$\psi_{\underline{n}}(x) = P(n) \exp \left[ik_1 x - \frac{N\sigma_T x}{2} + t_x \left(\frac{i\delta}{\tau_1} - \frac{1}{2\tau_2} \right) \right] \left\{ \frac{s}{\frac{1}{2} + i\delta} K_1 + K_2 \right\} \tag{3:25}$$

$$\psi_{n}(x) = P(n)\gamma(d)\beta(d) \exp [-N\sigma_T d/2]\gamma(x) \left\{ \frac{s}{\frac{1}{2} + i\delta} K_1 + \beta(x) K_2 \right\} ,$$

$$(t_d \gtrsim \tau_1) . \tag{3:26}$$

4. SUMMARY OF RESULTS

Case (a): For small gap lengths $(t_L \ll \tau_1)$ the set of plates behaves like a single, diffuse thick place of nuclear density $\bar{N} = \frac{Nd}{L+d}$. This behavior sets in when $n \gg 1$ for the case $t_d \gtrsim \tau_1$, and when $t_X \gtrsim \tau_1$, for the case $t_d \ll \tau_1$.

The latter case is especially useful for studying the 2π decay of the K_2^o, since by careful choice of $\bar{N}$ one can arrange for the ratio of K_1 to K_2 to be such that their decay intensities into 2π are of comparable magnitudes.

Case (b): For large gap lengths $(t_L \gg \tau_1)$ the pattern in each plate or gap simply repeats itself except for an overall attenuation--phase factor. The pattern in each plate or gap is the same as it would be for a single plate and gap.