Lawrence Berkeley National Laboratory

Recent Work

Title

POLARIZATION OF ELASTICALLY SCATTERED NUCLEONS FROM NUCLEI IN THE BORN APPROXIMATION

Permalink

<https://escholarship.org/uc/item/1n95q8p6>

Author

Heckrotte, Warren.

Publication Date

1954-04-06

UCRL 2538

UNCLASSIFIED

UNIVERSITY OF
CALIFORNIA

*Radiation
Laboratory*

TWO-WEEK LOAN COPY

*This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545*

BERKELEY, CALIFORNIA

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

UNIVERSITY OF CALIFORNIA

Radiation Laboratory

Contract No. W-7405-eng-48

POLARIZATION OF ELASTICALLY SCATTERED NUCLEONS FROM NUCLEI
IN THE BORN APPROXIMATION

Warren Heckrotte

April 6, 1954

Berkeley, California

POLARIZATION OF ELASTICALLY SCATTERED NUCLEONS FROM NUCLEI
IN THE BORN APPROXIMATION

Warren Heckrotte

Radiation Laboratory, Department of Physics
University of California, Berkeley, California

April 6, 1954

Recent experiments on the polarization of high energy nucleons elastically scattered from nuclei¹ have led to the interpretation of these results in terms of an effective spin-orbit nuclear potential² in addition to the usual central nuclear potential used to explain³ the total and experimental cross sections of the high energy nuclear scattering. A small spin orbit potential is sufficient to account for the large polarization observed.

At these high energies (~ 300 Mev) and at least for lighter nuclei, it has been pointed out that the Born approximation or equivalent simple arguments² yield a qualitative estimate of the amount and character of the polarization phenomena. The fact that the first Born approximation yields a polarization is a consequence of the use of a complex central potential. The polarization is a result of the selective absorption of nucleons scattered with spins up ($j = \ell + \frac{1}{2}$) and with spins down ($j = \ell - \frac{1}{2}$).

It is the purpose of this note to point out that the Born approximation result for the polarization is independent of the shape of the nuclear potential if the spin-orbit potential is taken to be proportional to the gradient of the central nuclear potential⁴. The central potential is taken to be $V_c(r) = (u_0 + iw_0) \rho(r)$ in Mev, where $\rho(r)$ represents the radial dependence of the potential. The spin orbit potential is taken to be

-3-

$V_g = -U_g(r) \vec{\sigma} \cdot \vec{L} / \hbar$ where $U_g(r) = \mu a^2 \frac{1}{r} \frac{\partial}{\partial r} \rho(r)$. Roughly speaking,

μ can be considered as the depth of an equivalent square well of radius a .

In Born approximation, the scattered amplitude, $f(\theta)$, is given by

$$\frac{2m}{\hbar^2} f(\theta) = A(\theta) + i \vec{\sigma} \cdot \vec{n} \sin \theta B(\theta)$$

where

$$A(\theta) = \int j_0(gr) V_g(r) r^2 dr$$

$$B(\theta) = -\frac{k^2}{g} \int j_1(gr) U_g(r) r^3 dr.$$

$\vec{n}$ is the unit vector normal to the plane of scattering, k is the incident wave number, g is the momentum transfer, and j_0 and j_1 are spherical Bessel functions⁵. Using the fact that $x^2 j_0(x) = \frac{\partial}{\partial x} (x^2 j_1(x))$ and performing a partial integration on $A(\theta)$, one can show that

$$A(\theta) = -\frac{1}{g} \int j_1(gr) \left(\frac{1}{r} \frac{\partial}{\partial r} V_g \right) r^3 dr.$$

Accordingly, $A(\theta)$ and $B(\theta)$ have the same functional form and the scattered amplitude can be written as

$$\frac{2m}{\hbar^2} f(\theta) = \left(1 - i \vec{\sigma} \cdot \vec{n} \sin \theta \frac{k^2 a^2 \mu}{u_0 + i w_0} \right) A(\theta).$$

-4-

The polarization is given then by

$$P = \frac{-2k^2 a^2 \mu w_0 / (u_0^2 + w_0^2)}{1 + (k^4 a^4 \mu^2 \sin^2 \theta) / (u_0^2 + w_0^2)} \sin \theta .$$

The polarization is thus independent of the shape of the nuclear potential, except that u_0 , w_0 and μ must be adjusted to yield the total and absorption cross sections for a particular choice of the radial dependence, $\rho(r)$. The result is also independent of the target nucleus unless such a dependence is introduced into μa^2 (say, for example, $a \sim A^{1/3}$).

If we take, for example, a square well, the constants obtained in fitting the total nuclear cross sections for 300 Mev neutrons are $u_0 \simeq 0$ and $w_0 = 18 \text{ Mev}$ ⁽³⁾. Using $\mu a^2 = 5 \times 10^{-26} \text{ Mev cm}^2$ which corresponds to $\mu = 1/2 \text{ Mev}$ when a is equal to the radius of carbon, the polarization is given by

$$P = \frac{-8.5}{1 + 18.1 \sin^2 \theta} \sin \theta .$$

The polarization obtained in the Born approximation can be expected to hold only for forward scattering angles. For larger angles the angular dependence of $B(\theta)$ relative to $A(\theta)$ changes sufficiently to introduce large corrections to the Born approximation result, particularly in the region of the diffraction minima. Thus, it is principally in these latter regions that model dependent features of the polarization can be expected to appear. In addition, if non-square well shapes are taken, it has been found

~~5~~

that u_0 and w_0 must be increased over their square well values in order to fit the observed cross sections. In some cases the increase is sufficient to invalidate the Born approximation. The results of more accurate calculations for neutrons and protons and for a number of well shapes is in the course of preparation.

I wish to thank Dr. Burton Fried for a discussion during which the observation on the polarization in Born approximation was made. This work was performed under the auspices of the Atomic Energy Commission.

-6-
REFERENCES

1. C. L. Oxley, W. F. Cartwright, and J. Rouvina, Phys. Rev. 93, 806 (1954); and O. Chamberlain, E. Segre, R. Tripp, C. Wiegand, and T. Ypsilantis, (unpublished results).
2. E. Fermi, Nuovo Cimento (to be published); B. Malenka, to be reported on at the American Physical Society Meeting, Washington, 1954; and W. Heckrotte and J. V. Lepore, Phys. Rev. (to be published).
3. S. Fernbach, R. Serber, and T. B. Taylor, Phys. Rev. 75, 1352 (1949); and S. Fernbach, Thesis, University of California Radiation Laboratory Report 1382.
4. W. Heisenberg, Theorie Des Atomkernes (Göttingen, 1951), p. 22.
5. L. Schiff, Quantum Mechanics (McGraw-Hill Book Co., Inc., New York, 1949), p. 77.