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Product Line Rivalry with Brand Differentiation

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Key words: product differentiation, entry, oligopoly

Abstract

Competition with product rivalry is examined in a model where products are differentiated by both quality and brand name. With no commitments, symmetric equilibria exist where firms produce all products and the mark-ups over marginal cost and profits are identical to the mark-ups and profits with single product firms. If product innovation is costly, there is a prisoners'dilemma in product introduction. When firms can commit to restrict their product offerings, firms will specialize in different products if the degree of brand-specific differentiation is small, but will produce all products if the degree of brand-specific differentiation is sufficiently large.

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Product Line Rivalry with Brand Differentiation

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I. Introduction

This paper examines competition in an industry where firms may choose to produce one or more products that are differentiated according to both brand and product-specific characteristics. For example, General Motors and Ford each sell a line of automobiles that cover the spectrum from economy compacts to luxury sedans. Within each category, both companies have similar offerings, but consumers are typically not indifferent between GM and Ford products. The particular form of differentiation we consider assumes that firms are characterized by a single parameter (e.g. distance to the consumer) and products are arrayed vertically according to quality. Consumers differ according to their valuation of quality. Given this structure of tastes we contrast competition in this market to the outcome of competition in markets with only a single source of product differentiation and we examine the consequences of alternative product portfolio choices by rival firms.

We assume that production costs favor the introduction of more than one product by each firm. There are strong economies of scope as defined by Baumol, Panzar and Willig [1982]. Yet strategic considerations also play a role in the determination of optimal product choices. An objective of this paper is to study how the introduction of vertical product

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differentiation in a model of spatial competition alters the incentives for multiproduct production in a market with an incumbent firm facing potential entry. Judd [1985] has argued that with constant marginal costs of production a multiproduct firm is particularly vulnerable to entry. A potential entrant will choose to locate at a point in the space of product characteristics already occupied by one of the incumbent firm's products. Although this will induce maximum price competition, the rivalry will adversely impact the incumbent firm's neighboring product(s), inducing the firm to abandon its challenged product. Sunk costs associated with the challenged product do not alter Judd's conclusions. The incumbent firm may have low marginal production costs for the challenged product as a consequence of sunk investment, but this does not change Judd's observation that the incumbent can increase the profitability of its neighboring products by abandoning the challenged product, and it should do so if rivalry at the challenged product is sufficiently intense.

Judd's results (and also findings by Shaked and Sutton [1981]) contrast with the suggestion by Schmalensee [1978] and others that incumbent firms will proliferate products in order to deter potential competitors.² Their argument is that firms can crowd a product space sufficiently to make entry unattractive, and thereby enjoy the benefits of a protected monopoly. Brander and Eaton [1984] apply this line of reasoning to the case of competition in a product line, and find conditions under which firms might choose to offer a cluster of close substitutes or might prefer to interleave their products with those of a competitor.

Crucial to the literature on product proliferation is the ability, or lack thereof, of an established firm to commit to production of a set of products. A neglected, but equally

² Contributors to the argument that established firms can crowd a product space in order to deter entry include Hay [1976], Prescott and Visscher [1977], Eaton and Lipsey [1979], and Lane [1980].

important commitment problem is the ability of a firm to credibly withhold a product from the market. We find that in a model with both horizontal and vertical product differentiation, the Judd result holds in the limit as the degree of horizontal (firm-specific) product differentiation goes to zero, provided that firms can commit to not compete in their rivals' markets and can signal this commitment to their competitors. Such a commitment can be difficult to maintain, as it requires that firms take credible actions to foreclose production opportunities. If firms can commit in this way, Judd's result need not hold when rival firms are unable to replicate the characteristics of the incumbent firm, even if they can replicate exactly the characteristics of the incumbent firm's products. When firms cannot commit to withhold products from the market, a Nash equilibrium, if it exists, entails the production of a full product line by rival firms.

Section II describes the basic model and derives prices and profits under the assumption that duopolists compete in a one-stage game in which they choose prices for all possible goods simultaneously. In a symmetric equilibrium of the one-stage game, profits are independent of the number of products that are produced. Yet neither firm would choose to restrict its product slate. Thus there can be a prisoners' dilemma in the one-stage game, with profits dissipated by the costs of new product introduction. Section III examines a three-stage game, in which firms can make credible commitments to not produce one or more products. Specialization occurs in the limit as the degree of firm-specific product differentiation goes to zero, but firms produce all products when the degree of firm-specific differentiation is sufficiently large. The credibility of product proliferation to deter entry is discussed in Section IV, along with the welfare implications of multiproduct competition.

II. Firm and Product-Specific Differentiation: The Symmetric Case

There are two goods, "basic" (good 0) and "premium" (good 1). A total of N consumers are located along a line of length L . Consumers are identical except for their taste for quality, which is indexed by a parameter y . Each consumer has a demand for one unit of either the basic or the premium good. The reservation price for good $j=0,1$ of a consumer with taste y is independent of that consumer's location and is given by:

$$(1) \quad R_j(y) = v + v_j y$$

with $v > \bar{v}$ ($\bar{v}$ sufficiently large to ensure that every customer has positive surplus from the basic good), $v_0 = 0$ and $v_1 > 0$. Without loss of generality, let $v_1 = 1$.

At each location, t , there is a distribution of consumers according to taste for quality. For simplicity assume that the taste distribution is independent of location. Let $f(y)$ be the density of consumers with taste y and $F(y)$ the proportion of consumers with a taste parameter less than or equal to y .³

Products can be sold by two firms $i = a, b$. Firm a is located at $t = 0$, firm b is located at $t = L$. There is a unit transportation cost. Let t^i be the transportation charge for a consumer at t that buys from firm i ($t^b = L - t^a$) and let P_j^i be firm i 's price for product j . A consumer at t with taste y buys product j from firm i if

$$(i,j) = \arg \max_{i,s} [R_s(y) - (P_s^i + t^i)]$$

³ The model in this paper can be extended in a straightforward matter to the case in which customers are located on either side of firms a and b with no change in the main conclusions.

provided the maximum is greater than zero. Each consumer buys at most one product from one supplier and chooses the product and the supplier that yield the highest consumer surplus, provided this surplus is positive. In the event of a tie, the consumer patronizes a firm at random.⁴

Assume for now that each firm sells only the basic good. The technology is described by the cost function $C^i(x^i) = C x^i + F$. Customers purchase the basic good from firm a if they are located at

$$(2) \quad t \leq \frac{P_0^b - P_0^a + L}{2}$$

Firm i 's profit assuming $0 \leq t \leq L$ is:

$$(3) \quad \Pi^i(P_0^i, P_0^j) = \frac{N}{L} (P_0^i - C) \left(\frac{P_0^j - P_0^i + L}{2} \right)$$

and the corresponding Nash reaction function is

$$(4) \quad P_0^i = \frac{L + C + P_0^j}{2}$$

The Nash equilibrium price is $P_0^* = L + C$; the mark-up of price over marginal cost is equal to the distance separating the two firms. The equilibrium profit of each firm is $\frac{N L}{2}$.

⁴ In a contemporaneous paper, Neven and Thisse [1988] consider conditions for existence of equilibria in a duopoly where firms choose the location and quality of a single product.

Turning to the case of competition with both the premium and the basic goods, let $(i',j')^{t,y} > (i,j)^{t,y}$ denote that a consumer at location t with taste y prefers product i' from firm j' to product i from firm j . Define:

$$(5) \quad t_j = \frac{P_j^b - P_j^a + L}{2},$$

the location of the customer who is indifferent between buying good j from firm a or firm b , and

$$(6) \quad y^i = P_1^i - P_0^i$$

the taste parameter of the customer who is indifferent between buying good 1 or good 0 from firm i .

A consumer located at t with taste y will purchase the basic good from firm a if and only if

- (i) $(0,a) > (0,b)$ which requires $t \leq t_0$
- (ii) $(0,a) > (1,b)$ which requires $y \leq y^b + 2(t_0 - t)$
- (iii) $(0,a) > (1,a)$ which requires $y \leq y^a = y^b - 2(t_1 - t_0)$

In condition (i), firm a wins out in a competition with its rival firm for sales of the same product. In (ii), the rivalry is between different firms producing different products. In case (iii), firm a is competing with itself, and sales of the basic good are at the expense of sales of

the premium good.

Taken together the above conditions imply that a customer at (t,y) will purchase the basic good from firm a if it yields a positive surplus and

$$t \leq t_0; \quad y \leq \min \{y^a, y^b + 2(t_0 - t)\}$$

Similarly, a consumer at (t,y) will purchase the premium good from firm a if it yields a positive surplus and

$$t \leq t_1; \quad y > \max \{y^a, y^b - 2(t_1 - t)\}$$

The sales regions for each good are shown in Figure 1 for the case in which $t_0 < t_1$ under the assumption that both firms sell positive amounts of both goods.

Consider the single stage game in which firms a and b simultaneously choose the prices to charge for both the basic and the premium goods. Each firm has the cost function

$$(7) \quad C^i(x_0^i, x_1^i) = C_0 x_0^i + C_1 x_1^i + F$$

This cost function, which is assumed in the remainder of this paper, exhibits strict economies of scope as defined by Baumol, Panzar and Willig [1982]. In an efficient equilibrium, both products would be produced provided $D \equiv C_1 - C_0$ is less than the maximum utility premium that customers assign to the higher quality good. In a market equilibrium in which firms are

unable to commit to a restricted product slate, a good will be produced unless the price is so high that no customers would choose to buy at the firm's asking price. As the firms are positioned symmetrically in this game, the logical candidate for a Nash equilibrium is one in which both firms choose the same prices.

Proposition 1. In the one-stage game with two goods and constant marginal costs, a symmetric Nash Equilibrium in pure strategies, if it exists, has equal mark-ups for both goods. Equilibrium profits and mark-ups are identical to the profits and mark-ups with only one good. A sufficient condition for the existence of a symmetric Nash equilibrium when y is uniformly distributed with upper support $\bar{y}$ is, with L normalized to one, $D > 2$ and $\bar{y} > D+3$.

Let m_j^a be firm a 's mark-up of price over marginal cost for product j . If $t_0 > t_1$, firm a 's profit can be written as

$$(8) \quad \Pi^a = \frac{N}{L} [m_0^a t_0 F(y^a) + m_1^a t_1 (1-F(y^a)) + O(t_0 - t_1)^2]$$

The last term in (8) is of order $(t_0 - t_1)^2$ and represents the triangle in Figure 1. If a symmetric equilibrium exists, this term can be ignored in the derivation of equilibrium prices.⁵ The first order conditions when $t_0 = t_1$ are:

$$(9a) \quad \partial \Pi / \partial m_0^a = \frac{N}{L} [F(y^a) (t_0 - m_0^a/2) + t_0 f(y^a) (m_1^a - m_0^a)]$$

$$(9b) \quad \partial \Pi / \partial m_1^a = \frac{N}{L} [(1-F(y^a)) (t_0 - m_1^a/2) + t_0 f(y^a) (m_0^a - m_1^a)]$$

⁵ In a symmetric equilibrium, $t_0 = t_1$ and this term is zero. Furthermore, its derivative evaluated at equilibrium prices is also zero, and hence this term has no effect on equilibrium strategies.

The first order conditions are satisfied at

$$\begin{aligned}m_0^a &= m_0^b = L \\m_1^a &= m_1^b = L\end{aligned}$$

Firm profits in this differentiated product market are not globally concave and additional conditions are required for existence of an equilibrium. Appendix A shows that if a symmetric equilibrium exists, it must have $m_j^i = L$ and derives conditions that are sufficient for existence of an equilibrium.

In a symmetric Nash equilibrium the mark-up of price over marginal cost is the same for each product and the same as the mark-up in the benchmark case where each firm sells only the basic good. Moreover, the equilibrium mark-ups are independent of customers' tastes for quality. Ignoring the fixed costs associated with a new product, equilibrium firm profits are the same whether both firms produce only one identical product (either basic or premium) or whether both firms produce both products. In either case, firm profits equal $\frac{N L}{2}$.

A price increase, say by firm a for good 1, causes consumers to substitute the basic good from the same firm and both the basic and the premium goods from the rival firm. With equal mark-ups on both goods, the firm is indifferent to substitution of the basic for the premium good. Moreover, near a symmetric equilibrium, substitution of the basic good from the rival firm for the premium good is small and can be ignored. This leaves only the positive effect of a higher margin for firm a and the negative effect of substitution by consumers of the rival firm's premium good. The marginal impacts of these two effects is identical to the marginal impacts of a price increase when the two firms sell only one good. The only

difference is that the impacts are limited to exactly one-half of the total market, but this scaling factor does not affect the pricing incentives. Thus the mark-ups with two goods are the same as in the case of a single good.

A corollary of Proposition 1 is that even though firms are no better off when they produce both goods compared to the situations in which they produce only the basic or only the premium good, in the single stage game it does not pay for either firm to withhold a product.

Corollary 1.1 In the single stage game, if $0 < D < \bar{y}$, where $\bar{y}$ is the upper support of the quality taste parameter distribution, if a Nash Equilibrium in pure strategies exists, it must entail production of both goods by both firms.

Proof:

Consider the situation where both firms sell the basic good. Given P_0^a and P_0^b , if firm a introduces the premium good with $P_1^a < P_0^a + \bar{y}$, sales increase by the area A in Figure 2.

Suppose firm a sets a price for the second good such that the mark-ups on the two goods are the same (this is possible if $D < \bar{y}$). Then any profits lost from reduced sales of good 0 will be offset by sales of good 1, and the firm will make additional profits from the sales to customers in region A. Therefore it will be profitable for firm a to produce both goods.

Suppose both firms produce only the premium good. Firm a can increase sales by introducing the basic good at a price $P_0^a < P_1^a$. The cost is the lost margin on sales of the premium good that are replaced by the basic good. But if $D > 0$, firm a can choose a P_0^a such that total sales increase and the profit margin on sales of the basic good exceeds the margin on

sales of the premium good and therefore sale of the basic good is profitable. Note that if $D = 0$, firm a cannot increase profits by introducing the basic good.

Now suppose firm b produces both goods. Should firm a specialize, or produce both goods? Assume firm a produces the basic good. By setting a price for the premium good equal to $P_1^a = P_1^b + L - \epsilon$, firm a gains sales equal to the dotted rectangle in Figure 3(a). Firm a's lost sales of the basic good are proportional to ϵ^2 , and for some ϵ sufficiently small, the net gain is positive. Figure 3(b) shows a different situation in that firm a already captures all of the customers in its immediate neighborhood. This can occur if the upper support of the taste distribution is sufficiently small. Firm a can set a price for good 1 that generates sales of the premium good of order ϵ^2 and substitutes premium for basic sales in the amount of the dotted rectangle as shown in Figure 3(b). There is a net gain only if the margin on premium good is at least as large as the margin on the basic good. We will show that this must be the case.

A consumer located at $(s, \bar{y})$ in Figure 3(b) is indifferent between purchasing good 1 from firm b or good 0 from firm a. Therefore,

$$(10) \quad v - (P_0^a + s) = v + \bar{y} - (P_1^b + L - s)$$

Suppose firm a sets a price P_1^a for good 1 so that, given P_1^b , $t_1 = s + \epsilon$ for some ϵ arbitrarily small. This implies that $m_1^b = m_1^a - L + 2(s + \epsilon)$. Then firm a's sales of good 1 will be determined by the dotted rectangle in Figure 3(b). Substituting m_1^b in (10) gives

$$m_1^a - m_0^a = \bar{y} - D - 2\epsilon,$$

which must be positive for sufficiently small ε if $D < \bar{y}$. Thus, taking prices as given, firm a can make positive sales of the premium good at a mark-up that exceeds its mark-up for the basic good, provided only that someone would want to buy the premium good if it had a zero price.

A similar argument shows that with Nash price expectations, if firm b sells both goods and firm a sells only the premium good, firm a would choose to introduce the basic good provided $D > 0$.⁶ Moreover, neither firm would elect to retire a product, given that its rival firm produces both goods. Thus we conclude that if firms act as price-takers with no ability to pre-commit to their product offerings, a stable equilibrium must have both firms producing both products. Yet in equilibrium when both firms produce both products, they are no better off than they would have been with a restricted product line. If the introduction of new products entails any overhead costs, the firms in the industry are strictly worse off in the symmetric equilibrium with product proliferation. $\square$

The model can be extended in a straightforward way to allow for more than two products. Define the reservation price

$$(11) \quad R_j = v + v_j y \quad \text{for } j = 0, \dots, J$$

with $v_0 = 0$ and $v_{j+1} > v_j > 0$. Let

⁶ Campbell [1986] describes a two dimensional location model with features similar to the model here and argues that by changing a single product characteristic a firm can increase its market share and simultaneously inflict relatively higher losses on its rival. This corresponds, for example, to Figure 3(a) in which firm a gains almost entirely at the expense of firm b. While Campbell argues that this may be a basis for a theory of rational predation, the next section shows that with commitment the incentives to expand market share depend on the equilibrium response of the industry to new product introduction.

$$(12) \quad y_j^i = \frac{P_{j+1}^i - P_j^i}{v_{j+1}^i - v_j^i}$$

Lemma 1: A necessary condition for consumers to purchase all $j = 0, \dots, J$ products from firm i is that $y_{j+1}^i > y_j^i$ for all $j = 0, \dots, J-1$.

Proof:

A consumer with taste parameter y_j^i is indifferent between purchasing good j and good $j+1$. It must be the case that this consumer strictly prefers good $j+1$ to good $j+2$ and strictly prefers good j to good $j-1$. The first condition requires

$$(13) \quad R_{j+1} - (P_{j+1}^i + t) > R_{j+2} - (P_{j+2}^i + t)$$

for the consumer with taste parameter y_j^i . Substituting y_j^i in the reservation values yields the equivalent condition $y_{j+1}^i > y_j^i$. The second condition yields the inequality $y_j^i > y_{j-1}^i$.

The logic of this condition is clear. If prices for higher quality goods did not increase in the way dictated by inequality (13), a consumer with taste y could move up to a higher quality good at an increment in price that is less than the increment in value received. This would lead consumers to skip one or more products.

Proposition 2: Assume there exist $J+1$ products labeled $j = 0, \dots, J$. Suppose that for both firms $i = a, b$, the critical taste parameter values y_j^i defined in (12) have the property that at the prices P_j^i , $y_{j+1}^i > y_j^i$ for all $j = 0, \dots, J-1$ and $i = a, b$. Then, if a symmetric Nash equilibrium

exists, it will have the property that

$$m_j^i \equiv P_j^i - C_j = L$$

for all $j = 0, \dots, J$ and $i=a,b$.

Proof: See Appendix B.

Welfare

In this model price-taking firms that cannot pre-commit to a restricted product line may dissipate profits by introducing too many products in an attempt to attract new customers. The result is excessive product differentiation for the industry, although not necessarily from a social point of view. In a first-best equilibrium, product j should be produced if and only if for some consumer with taste $y \in [0, \bar{y}]$, $R_j(y) \geq C_j$ and if the total surplus generated by the product exceeds the fixed cost of introducing the product. In the one-stage game, $R_j(t,y) \geq C_j$ is necessary and sufficient for product introduction by the firms, but there is no guarantee that the firms will break even on their products if there are overhead costs. Thus the set of products produced in the one-stage game is the surplus maximizing set of products if the first best calls for production of both goods by both firms. Otherwise, there are too many goods (or brands) produced in the one-stage game. With the cost function in (7) it is efficient for firms to produce both goods, but if F is sufficiently large, only one firm should operate in a social optimum.

III. Sequential Product Choice

A single-stage game can be criticized as a model of product innovation. If the innovation decision involves a specific commitment of resources to a production program, a manager can choose whether or not to obligate resources that would permit the introduction of a new product and the decision should be based on the prices that would be expected to result in equilibrium after the product is introduced.

Consider a production technology which exhibits strong economies of scope. Each firm has the cost function $C^i(x^i) = C_0 x_0^i + C_1 x_1^i + F$ where F is sunk once production of either product begins. Production of one of the products enables a firm to produce the other product with no set-up cost. We assume for now that both firms co-exist in the market. In the next section we assign a first-mover advantage to firm a and consider the possibilities for entry deterrence through product choice. In stage 1 of the game, firm a can choose, at negligible cost, to eliminate one or both products from its feasible set. This could be accomplished, for example, by neglecting to place the order for parts and supplies that would be necessary for production. Firm b has full information about firm a's actions in stage 1 and in stage 2 can choose to eliminate one or more products from its feasible set. In stage 3 both firms compete as Nash competitors conditional on their feasible product sets. If a firm does not eliminate a product when it has the opportunity to do, it will produce the product in the Nash equilibrium of the final stage of the game (see Section II). The sequential game is illustrated in Figure 4. In computing its optimal product choice in stage 1, firm a "folds back" the equilibrium outcomes at the end of the game tree to determine its best action. Thus outcomes of the three-stage game are restricted to Nash equilibria that are subgame perfect.

In what follows we characterize conditions under which firm a will choose both

products and contrast them with conditions under which specialized production will occur. The following proposition shows that under some conditions product proliferation is not profitable. This result parallels that in Judd [1985].

Proposition 3: Assume $0 < D < \bar{y}$. In the limit as $L \rightarrow 0$, a subgame perfect equilibrium of the three-stage game involves specialization by the rival firms.

The proof make use of the following lemma.

Lemma 2: Suppose firm a produces both products and firm b produces only good j, where j is either the basic or the premium good. Then as $L \rightarrow 0$, $m_j^a \rightarrow 0$ and $m_j^b \rightarrow 0$. In the limit as $L \rightarrow 0$, neither firms earns profits on sales of good j.

Proof:

Suppose firm b produces only the basic good. Firm b earns, excluding sunk costs

$$(14) \quad \Pi^b_{[0,1;0]} = \frac{N}{L} [m_0^b \int_{t_0}^L F(y(t)) dt]$$

where

$$(15) \quad y(t) = m_1^a - m_0^b + D - L + 2t.$$

Firm b's reaction function is

$$(16) \quad m_0^b = 2 \frac{\int_{t_0}^L F(y(t)) dt}{F(y(t_0))}$$

and as $L \rightarrow 0$, both m_0^b and $\Pi^b \rightarrow 0$.

As $m_0^b \rightarrow 0$ in the limit, either $m_0^a \rightarrow 0$ or firm a makes no sales of the basic good. Section II shows that a firm with two products will sell both if a Nash equilibrium exists, so $m_0^a \rightarrow 0$. A similar argument holds if firm b specializes in the sales of the premium good. $\square$

Let α, β be the product choices of firms a, b. Given firm a's product choice α as determined by its actions in the first stage, firm b will choose $\beta^*(\alpha) = \arg \max_{\beta} \Pi^b[\alpha; \beta]$. Anticipating firm b's actions, firm a will choose $\alpha^* = \arg \max_{\alpha} \Pi^a[\alpha; \beta^*(\alpha)]$. Suppose firm a does not restrict its product set in stage 1. If firm b does not restrict its product set in stage 2, both firms earn $NL/2$, which tends to zero with L . If firm b restricts its product offerings so that it can produce only the basic good, firm a would earn in the limit as $L \rightarrow 0$, using Lemma 1

$$(17) \quad \Pi^a[0,1;0] = \frac{N}{L} m_1^a [L(1-F(y^a))]$$

In the limit as $m_0^a \rightarrow 0$, m_1^a must be bounded above by $\bar{y} - D$ (the maximum willingness to pay for the purchase of good 1 from firm a) if firm a is to make any sales of the premium good. Thus it is easily shown that Π^a is bounded above by $N(\bar{y}-D)^2/4\bar{y}$. Similarly, if firm b produces only the premium good, in the limit as $L \rightarrow 0$ firm a's profits are

$$(18) \quad \Pi^a[0,1;1] = \frac{N}{L} m_0^a [LF(y^a)]$$

In the limit as $m_1^a \rightarrow 0$, m_0^a must be bounded above by D if firm a is to make any sales of the basic good. Thus Π^a is bounded above by $ND^2/4\bar{y}$. Therefore, by choosing not to restrict its profit set, firm a would earn

$$\Pi^a[0,1;\beta^*(0,1)] \leq \max [N(\bar{y}-D)^2/4\bar{y}, ND^2/4\bar{y}]$$

Suppose instead that firm a restricts its product set so that it can produce only the premium good. If firm b does not specialize, by symmetry and the same argument applied to firm a, $\Pi^b[1;0,1] \leq ND^2/4\bar{y}$. But Appendix C shows that by specializing in the basic good firm b can earn

$$\Pi^b[1;0] = N(\bar{y}+D)^2/9\bar{y}$$

and this strictly exceeds $\Pi^b[1;0,1]$. Similarly, if firm a specializes in the basic good, by specializing in the premium good firm b can earn

$$\Pi^b[0;1] = N(2\bar{y}-D)^2/4\bar{y}$$

and this strictly exceeds $\Pi^b[0;0,1]$. Thus in the limit as $L \rightarrow 0$, firm a will choose to specialize in either the basic or premium good and firm b will specialize in the other good. Neither firm a nor firm b can do better by producing both goods. $\square$

There are two possible subgame Nash equilibria of the sequential product choice game in the limit as $L \rightarrow 0$: either $[1;0]$ or $[0;1]$. The profits of firms a and b in each equilibrium depend on the distribution of tastes for quality and on the cost penalty for the premium good. Because the sequential game assigns a first-mover advantage to firm a, this firm can select its preferred equilibrium by restricting its product slate in the first stage to the product that yields maximum profits when the firms specialize. Firm a would choose to specialize in the premium (basic) good if $D \leq \bar{y}/2$ ($D > \bar{y}/2$). The first-mover advantage is arbitrary and for

either equilibrium it is the case that as $L \rightarrow 0$ the firms will choose to specialize their production choices.

The specialization result is dependent on the extent of firm-specific differentiation, L . If L is very small, the firms benefit by establishing separate market niches, and these niche strategies are non-cooperative equilibria of the sequential game. However, as the next proposition shows, specialization does not occur if firm-specific differentiation is large.

Proposition 4: In the limit as $\frac{L}{\bar{y}} \rightarrow \infty$, both firms will choose to produce both products.

Proof:

Assume that as $L/\bar{y} \rightarrow \infty$, every customer continues to purchase one unit of either the basic or premium good. (If transport costs are so high that firms in the middle of the market would not patronize either firm, then each firm has a natural monopoly and, with the assumed economies of scope, would offer both products.) Suppose firm a does not restrict its product set. If firm b produces both products, profits equal $NL/2$, and for $L/\bar{y}$ sufficiently large this exceeds the profits from specialization. We will show that producing both products is a dominating strategy for each firm. Suppose firm b restricts its production to either the basic or premium good. Taking firm b's product choice *and price* as given, it is optimal for firm a to produce both products (see the proof of Corollary 1.1 in Section II). We need to show that firm a is not worse off at the new equilibrium prices, and this is done in Appendix D. Thus product proliferation is optimal for firm a if firm b restricts its product set. Moreover, firm a would not be disadvantaged if firm b responded by producing both products; profits are the same in any equilibrium where firms' product offerings are matched. Appendix D also shows that firm a would not wish to restrict its product offering when firm b produces both products.

Thus, by symmetry, both firms will produce both products when $L/\bar{y} \rightarrow \infty$. $\square$

IV. Credible Preemption

Judd [1985] argued that a multiproduct firm is particularly vulnerable to entry. Price competition from an entrant cuts into the earnings of all of the multiproduct firm's offerings. By abandoning the brand closest to the location of the entrant, the multiproduct firm escapes the consequences of vigorous price competition on its other brands. Knowing this, a potential entrant would be inclined to enter as close as possible to the brand of an existing multiproduct firm. Unless the established firm faces substantial exit costs, it will abandon the brand that was challenged by the entrant. Shaked and Sutton [1981,1983] demonstrated this result in a model of vertical product differentiation.

We will argue that while the Judd argument is correct, its application to multiproduct competition and entry deterrence depends on the ability of firms to pre-commit to a restricted product set and on the way in which the firm's products are differentiated.

Proposition 5: If an established firm *cannot* commit to exclude a product from production, entry is deterred if $F > NL/2$.

Proof:

Section II shows that in a Nash equilibrium, with the assumed technology a firm will produce both the basic and the premium good (see the proof of Corollary 1.1 in Section II). Thus a potential entrant should anticipate that an established firm will produce both products unless the incumbent has a credible action it can take to not produce a product, and can signal this

commitment to the potential entrant. In the absence of such a commitment, the best an entrant can do is to match the established firm and produce both products. The entrant's gross profit from entry would be $NL/2$, and net profit would be negative if $F > NL/2$. $\square$

If the firms can commit to restrict their product lines (and if they can convince rivals that they have made this commitment), the opportunity for entry deterrence will depend on the extent of horizontal and vertical product differentiation. Suppose we interpret the spatial component of product differentiation (L) as a factor that is unique to the firm (e.g. a reputation factor, as in Katz [1984]). If this reputation factor cannot be replicated by an entrant, it is appropriate to put a lower bound on the distance separating the firms. The entrant can compete head-on with an incumbent by duplicating the products that it offers, but it cannot duplicate its reputation. For example, Chevrolet can build a car that is functionally and even aesthetically identical to a Ford Taurus, but it is still a Chevrolet.⁷ We will show that in our example of vertical product differentiation, head-on entry does not imply that a multiproduct firm will abandon a challenged brand, even if it can commit to doing so.

In this section we assume that consumers are distributed uniformly according to taste in the interval $[0, \bar{y}]$. The sequential game of credible preemption adds a stage to the game in Section III. Prior to the beginning of the game (Stage 0), there exists an established firm with the technology described by the cost function in (7). In the first stage firm b chooses whether to enter the market with the same cost function or to stay out. If it enters it incurs the sunk cost F . The second, third and fourth stages are identical to the three stages in Section III. In

⁷ The force of brand-specific product differentiation in the automobile market is illustrated by the example of automobile divisions that produce identical twins which consumers view as very close, but not perfect, substitutes. While the cloning behavior of automobile manufacturers may be motivated by circumstances outside of our model (such as dealer market power), it suggests limitations on the ability to replicate competitors' products.

the second stage firm a can choose to restrict its product slate, and firm b can do the same in the third stage. In stage four the firms compete conditional on their product offerings.

Section III considered the situation in which both firms were operating in the market and would produce both products unless they chose to restrict their product offerings. In this section firm b can choose not to enter the market. If it does enter, it is on an equal footing with firm a unless the firms choose to differentiate themselves by restricting their product offerings. As in the previous section, each firm anticipates the consequences of its decisions on equilibrium prices and profits.

Proposition 6: For $0 < D < \bar{y}$, in the limit as $L \rightarrow 0$ entry will occur if $F < \min [N(2\bar{y}-D)^2/9\bar{y}, N(\bar{y}+D)^2/9\bar{y}]$.

Proof:

From Proposition 3 in Section III, we know that as $L \rightarrow 0$, if entry occurs the outcome will be product specialization and firm a will specialize in the product that results in the higher profit. From Appendix C, $\Pi^a[1;0] = N(2\bar{y}-D)^2/9\bar{y}$ and $\Pi^a[0;1] = N(\bar{y}+D)^2/9\bar{y}$. By symmetry, the profits of firm b will be $\min \{ \Pi^a[1;0], \Pi^a[0;1] \}$, and entry will occur if this exceeds fixed costs. $\square$

Proposition 7: As $\frac{L}{\bar{y}} \rightarrow \infty$, entry will occur if $F < NL/2$.

Proof:

From Proposition 4 in Section III we know that as $\frac{L}{\bar{y}} \rightarrow \infty$ the subgame perfect equilibrium of the sequential game calls for both firms to produce both goods. Thus a potential entrant should expect multiproduct production by the incumbent, and would do the same if it should

enter. Entry is profitable only if the sunk cost of entry is less than $NL/2$, the profits when both firms produce both products. $\square$

It follows immediately from the proof of Proposition 7 that the specialization result in Judd [1985] does not apply to the multiproduct market when the degree of firm-specific product differentiation is sufficiently large.

Corollary 7.1: Assume a uniform taste distribution with $y \in [0, \bar{y}]$ and suppose that incumbent faced with the threat of entry can commit to any combination of goods 0 and 1. If $0 < D < \bar{y}$ and if L is sufficiently large, firm a will commit to the production of both goods.

Proof: follows from continuity and the optimality of multiproduct production by both firms as $L/\bar{y} \rightarrow \infty$.

Figure 5 shows the results of numerical analysis of the first order conditions for both firms when $\bar{y} = 10$ and $D = 5$. The solid line is the profit when both firms produce both goods ($NL/2$). The upper dashed line is the profit of firm a when it produces both goods and firm b produces only the basic good. The lower dashed line is the profit to firm b in this situation. (The problem is symmetric to the identity of the firms and the products. For example, figure 5 would not change if firm b produced only the premium good.) The dotted line is the profit when the firms specialize. If L is sufficiently small, firm a is better off specializing in the production of either good 0 or 1. However, for L larger than about 4, firm a is better off with a full product line when firm b produces only the basic good. Figure 5 shows that specialization is a desirable strategy only when the rival firm responds by specialization in the other good. If firm a produces both goods, firm b is always better off producing both goods as

well. Specialization makes firm b strictly worse off if firm a chooses to be a multiproduct firm.

Qualitatively similar results obtain for any $0 < D < \bar{y}$, although the symmetry is obviously lost when $D \neq \bar{y}/2$. If $D=0$ or $D=\bar{y}$, there are no benefits from a complete product line. Specialization has the advantage that it reduces rivalry when the firms are horizontally close. Its disadvantage is that it eliminates the possibility of gaining additional sales by introducing a product that appeals to customers who are patronizing the rival firm. Each consumer has a weak preference for the premium good, so the basic good must have a lower price to generate any sales. If $D=0$, this means that the mark-up on the basic good must be less than the mark-up on the premium good, and gaining market share by introducing the basic good is costly to the firm. If $D=\bar{y}$, the mark-up on the premium good must be less than the basic good's mark-up to generate sales, so introducing the premium good is costly to the firm.

Figure 6 is a close-up of the profits under different product assumptions. For $L < 4$, specialization dominates any other product combination. For $L > 5$, both firm a and firm b would be better off producing both products. If entry occurs when $L > 5$, both the entrant and the incumbent would choose to be multiproduct firms. When $4 < L < 5$, firm a is better off being a multiproduct firm when firm b specializes in the production of both goods. Knowing that this is true, firm b would not choose to specialize. If firm b entered the market, it would enter as a multiproduct firm. But note that both firms would be better off specializing when $4 < L < 5$. Thus in this region *firm a can deter entry even if the cost of entry were less than the profits in a specialization equilibrium, and even though the specialization equilibrium results in the largest profit for both firms*. The reason is that specialization is not a subgame

perfect equilibrium when $4 < L < 5$. In this interval product proliferation is a dominant strategy for firm a and, conditional on entry, for firm b.

Welfare

The assumed cost function $C = C_0x_0 + C_1x_1 + F$ exhibits strict economies of scope so that if it is optimal to produce both goods, production should be joint within a firm. The only issue is whether there should be more than one firm. It is not difficult to show that if $F \leq NL/4$, the optimal product set is for both firms to produce both goods, and if $F \geq NL/4$, the optimal product set calls for either firm a or firm b, but not both, to produce both goods.

What is the outcome of the market game? Referring to Figure 5, if F is less than $\Pi[0,1;0,1] = NL/2$, both firms can coexist in the industry and if L is large, both firms will produce both goods. Thus if $NL/4 \leq F \leq NL/2$, and if L is large, there is an excessive number of products (brands). If L is sufficiently small, the firms will specialize in the production of either the premium or the basic good. Thus for small brand-specific differentiation the market set of products is not efficient. Either there are too few products (if $F \leq NL/4$ and L is small) or the number of products is correct but they are produced by too many firms.

If F is sufficiently large, firm a may be able to blockade entry when it produces both goods, even though both firms should produce both goods in an optimal industry configuration. If both L and F are sufficiently small, firm a accommodates entry by specializing in the production of either the basic or the premium good, and if L is large but F is small, firm a produces both goods and entry occurs. This contrasts with the one-dimensional differentiation case, where the firms specialize whenever entry occurs. In the region corresponding to

$4 < L < 5$ the only equilibrium outcome is multiproduct production. If $F < NL/2$, entry will occur with both firms producing both products, and if $NL/4 < F < NL/2$, the result is too many brands. Entry should not occur if F exceeds $NL/2$ but is less than the profits with specialization. The inability to commit to a specialization equilibrium means that if entry were to occur, both firms would be worse off. But, this inability to commit is what makes entry deterrence feasible in this region, and it results in both higher profits for the incumbent firm and the correct choice of products.

Extensions

The sequential game with vertical as well as horizontal product differentiation shows conditions under which an incumbent firm will choose to produce a range of products in order to defend itself from competition by rival firms or to make entry by rivals more difficult. However, the range of parameters for which this product proliferation strategy is profitable, compared to a strategy in which the firm specializes in a single good, is rather limited. Firm-specific product differentiation must be large relative to the perceived differences between the products that each firm might offer to make proliferation a profitable strategy.

The prevalence of multiproduct firms suggests that firms cannot commit to exclude products from production or they are myopic in their product innovation decisions (consistent with the one-stage Nash game in Section II), or that there are other reasons for product proliferation outside the scope of our model (such as reputation effects). One possibility relates to the assumed consumption and search behavior of the customers. We assume that each customer buys only one good and has perfect information about product prices. In many industries, customers or their agents buy a range of products. For example, a lighting

distributor stocks a complete line of light bulbs. The distributor could shop for different bulbs from different suppliers, but that could involve additional transaction costs that would be avoided by purchasing the entire product line from a single supplier.

The basic model can be extended to consider the implications for competitive behavior of customers that demand the entire product line. Suppose there are M customers that wish to buy both goods. Their reservation price for the bundle is v_b . Suppose firm a sells both goods and firm b sells only good 0. A bundle customer located at t will buy from firm a if $P_0^a + P_1^a + t < P_0^b + P_1^a + L$, or if $P_0^a < P_0^b + L - t$. Bundle customers located closer to firm a are inframarginal consumers of firm a's products. For the parameters we have examined, in the equilibrium where firm a sells both goods and firm b sells only the basic good, all bundle customers are inframarginal to firm a except those located very close to firm b ($t \approx .9L$ for $D=5$ and $\bar{y}=10$). For simplicity, we ignore these customers. This is consistent with the assumption that the bundle customers have no brand preference, so they can be considered to be located at the center of the market ($t=.5L$).

The inclusion of bundle customers alters the incentives for product choice if their purchases are large enough to change the profitability of the multiproduct firm. Of course this depends on the number of multiproduct customers relative to those that specialize. With less firm-specific product differentiation, it takes more bundle consumers to justify multiproduct production, and if firm-specific differentiation is small, bundle customers do not provide adequate incentive for multiproduct production even if they comprise the entire market. Figure 7 shows the share of sales that must go to bundle customers to make it profitable for the incumbent firm to choose both products, under the conditions corresponding to Figure 5. For L less than about 1.5, the incumbent is better off specializing if bundle consumers account

for nearly all sales (this assumes, however, that bundle consumers remain inframarginal and so do not affect equilibrium prices).

V. Concluding Remarks

We have explored how the potential for vertical product differentiation affects the incentives for product choice in a model where there is also brand-specific differentiation. Assuming constant marginal costs, symmetric equilibria in this technological environment have the property that the mark-ups on goods of differing qualities are identical and, ignoring overhead expenses of product introduction, profits are independent of the number of goods produced. In a one-stage game of product choice and pricing with no means to commit to product choices, firms introduce the maximum number of product varieties although they do not benefit from the production of more goods and are worse off relative to a minimum product slate if there are product-specific overhead costs. When product innovation is costly and firms cannot commit to exclude products from production, there is a "prisoners' dilemma", with the result that firms produce too many product varieties. As in models studied by Dixit and Stiglitz [1977] and others, competitive equilibria may involve too many or too few products relative to efficient production, however, if the first-best equilibrium calls for the production of a full product slate and if there are no product-specific overhead costs, then the market equilibrium in the absence of product commitment is first-best.

With product commitment, which we model as a three-stage game, the subgame perfect Nash equilibria in a model with established firms may involve any combination of products produced by either firm. As in Judd [1985], specialization occurs if the degree of firm-specific differentiation is sufficiently small. Moreover, this specialization occurs even if

there are economies of scope in the production of goods of different qualities. Unlike Judd [1985], specialization need not occur if the degree of brand-specific differentiation is sufficiently large. When the game is extended to allow for the possibility of entry into an industry with an established firm, the results again depend on the extent of firm-specific differentiation. The ability to commit to specialized production puts a lower bound on the profit an entrant can expect to earn, and thus makes entry more likely. There is, however, scope for entry deterrence. Specifically, an incumbent firm may choose to produce a full product slate even if it would be better off specializing in the event of entry. This can be a subgame perfect equilibrium for intermediate values of firm-specific differentiation. The full product slate allows the firm to deter entry in situations where entry would occur with a restricted product slate.

The model of vertical and horizontal product differentiation studied in this paper casts a somewhat different light on the incentives for product production relative to the conclusions from a model with one dimension of product differentiation. The model with both vertical and horizontal differential permits a richer set of strategic variables for the rival firms and suggests how incentives for price discrimination may justify the choice of a full product line.

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FIGURE 1

FIGURE 2

FIGURE 3a

FIGURE 3b

SEQUENCE OF MOVES

SEQUENTIAL GAME WITH ESTABLISHED DUOPOLY

STAGE 1

FIRM A CHOOSES TO DROP PRODUCT $\{0\}$, $\{1\}$, $\{0,1\}$ OR $\{\text{NONE}\}$

STAGE 2

FIRM B CHOOSES TO DROP PRODUCT $\{0\}$, $\{1\}$, $\{0,1\}$ OR $\{\text{NONE}\}$

STAGE 3

FIRMS A AND B COMPETE NASH IN PRICE
CONDITIONAL ON REMAINING PRODUCTS

Profits when A sells $\{0,1\}$; B sells $\{0\}$
 Cost Difference = 5, $Y_{\max} = 10$

FIGURE 5

Profits when A sells $\{0,1\}$; B sells $\{0\}$
 Cost Difference = 5, $Y_{\max} = 10$

FIGURE 6

Bundle Customers Required for Multiproduct Production

Max Income = 10; Cost Difference = 5

FIGURE 7

Appendix A

Proof of Proposition 1

The conditions governing the market for each of the products of the two firms are summarized in inequalities (i) – (vi) in Section II of the paper. Two cases must be considered depending on the location of the indifferent customers for each good.

Case 1: $t_1 \geq t_0$

Provided $y^a > 0$ and $0 < t_0, t_1 < L$, firm a's profit function is:

$$\Pi^a = N \{ m_0^a t_0 F(y^a) + m_1^a [t_1 [1 - F(y^a)] - \frac{1}{2} (t_1 - t_0) (F(y^a) - F(y^b))] \}$$

First-order conditions for an interior solution are

$$(A.1) \quad \frac{1}{N} \frac{\partial \Pi^a}{\partial m_0^a} = F(y^a) (t_0 - m_0^a / 2) + f(y^a) (m_1^a t_1 - m_0^a t_0) - m_1^a / 2 [(t_1 - t_0) f(y^a) + \frac{1}{2} (F(y^b) - F(y^a))] = 0$$

$$(A.2) \quad \frac{1}{N} \frac{\partial \Pi^a}{\partial m_1^a} = (1 - F(y^a)) (t_1 - m_1^a / 2) - f(y^a) (m_1^a t_1 - m_0^a t_0) + m_1^a / 2 [(t_1 - t_0) f(y^a) + \frac{1}{2} (F(y^b) - F(y^a))] - \frac{1}{2} (t_1 - t_0) (F(y^b) - F(y^a)) = 0$$

Imposing symmetry, (A.1) and (A.2) have an interior solution if and only if $m_j^i = L$ for $i = a, b$ and $j = 0, 1$. Thus, if a symmetric equilibrium exists, mark-ups equal the distance separating the firms.

To prove existence, consider the case of a uniform distribution, with $y \in [0, \bar{y}]$ and $F(y) = \frac{y}{\bar{y}}$. Assume $m_0^b = m_1^b = L$. Consider the Hessian (with the superscripts for firm a

suppressed)

$$H^a = \begin{vmatrix} \partial^2 \Pi / \partial m_0^2 & \partial^2 \Pi / \partial m_0 \partial m_1 \\ \partial^2 \Pi / \partial m_0 \partial m_1 & \partial^2 \Pi / \partial m_1^2 \end{vmatrix}$$

where

$$(A.3) \quad \partial^2 \Pi / \partial m_0^2 = -\frac{1}{\bar{y}} [2L + D - 3m_0^a + \frac{3}{2}m_1^a]$$

$$(A.4) \quad \partial^2 \Pi / \partial m_0 \partial m_1 = -\frac{1}{\bar{y}} [\frac{3}{2}m_0^a - 2L]$$

$$(A.5) \quad \partial^2 \Pi / \partial m_1^2 = -\frac{1}{\bar{y}} [\bar{y} + 2L - D - \frac{3}{2}m_1^a]$$

$m_j^a = L$ is a best response to $m_j^b = L$ for $j=0,1$, when firm a sells both goods if the Hessian is negative definite for all feasible values of m_j^a . Because $m_0^b = m_1^b = L$ (by assumption), firm a makes positive sales of both goods only if $m_0^a, m_1^a < 2L$. The firm would not choose a negative mark-up, so the feasible set for the production of both goods is $0 < m_0^a, m_1^a < 2L$.

The Hessian is negative definite if $\partial^2 \Pi / \partial m_0^2 < 0$, $\partial^2 \Pi / \partial m_1^2 < 0$, and $(\partial^2 \Pi / \partial m_0^2)(\partial^2 \Pi / \partial m_1^2) - (\partial^2 \Pi / \partial m_0 \partial m_1)^2 > 0$. The first condition requires $D > 4$ if it is to be satisfied for all feasible values of m_j^a . The second condition requires $\bar{y} > L + D$. The third condition is satisfied if $m_0^a > m_1^a$, which is implied by the assumption that $t_1 > t_0$.

Case 2, in which $t_0 > t_1$, proceeds similarly and imposes no new constraints.

The proof requires the further demonstration that a corner condition cannot be optimal for firm a and that the second order conditions are at least locally satisfied at the symmetric equilibrium. Several alternatives need to be examined. (1) Holding $P_1^a = P_1^{a*}$, P_0^a could be changed to attract all customers who had purchased (1.i) the basic good from firm b, or (1.ii)

the premium good from firm b. (2) Holding $P_0^a = P_0^{a*}$, P_1^a could be changed to attract all customers who had purchased (2.i) the premium good from firm b, or (2.ii) the basic good from firm b. (3) P_0^a and P_1^a could be changed simultaneously so that firm a would serve the entire market.

Alternative (1.i) requires $P_0^a \leq P_0^b - L = C_0$. Firm a would lose on customers that substitute the basic for the premium good and total profits would be lower. (1.ii) requires $P_0^a < C_0$, which is clearly unprofitable. Alternative (2.i) requires $P_1^a = C_1$ and is unprofitable for the same reason as (1.i). (2.ii) requires

$$P_1^a + t - y \leq P_0^b \quad \forall t, y$$

Thus

$$P_1^a < P_0^b - L$$

which implies

$$P_1^a < C_0 < C_1,$$

and this is clearly unprofitable. Alternative (3) is a combination of (1.i) and (2.i) and again is unprofitable.

Summarizing, if a symmetric equilibrium exists in this market, it has the property that $m_j^i = L$ for $i=a,b$ and $j=0,1$. Furthermore, under the assumption of a uniform income distribution, the symmetric equilibrium exists provided $D > 4$ and $\bar{y} > L + D$. These conditions are sufficient for existence of a symmetric equilibrium with a uniform income distribution. They need not be necessary.

Because P_j^a affects only Π_j^a , Π_{j+1}^a , and Π_{j-1}^a ,

$$\partial \Pi^a / \partial P_j^a = \partial \Pi_j^a / \partial P_j^a + \partial \Pi_{j+1}^a / \partial P_j^a + \partial \Pi_{j-1}^a / \partial P_j^a$$

and

$$\partial \Pi_{j+1}^a / \partial P_j^a = m_{j+1}^a \sum_{k=\underline{k}-1}^{\bar{k}-1} t_{kj+1} f(y_j^a) / (v_{j+1} - v_j) +$$

$$m_{j+1}^a \sum_{k=\underline{k}-1}^{\bar{k}+1} (v_{k+1} - v_{j+1}) \partial E(y_j^a) / \partial P_j^a$$

$$\partial \Pi_{j-1}^a / \partial P_j^a = m_{j-1}^a \sum_{k=\underline{k}-1}^{\bar{k}-1} t_{kj-1} f(y_{j-1}^a) / (v_j - v_{j-1}) +$$

$$m_{j-1}^a \sum_{k=\underline{k}-1}^{\bar{k}+1} (v_{k-1} - v_{j-1}) \partial E(y_{j-1}^a) / \partial P_j^a$$

Under symmetry,

$$\partial \Pi_{j+1}^a / \partial P_j^a = m_{j+1}^a / 2 [f(y_j^a) / (v_{j+1} - v_j)]$$

$$\partial \Pi_{j-1}^a / \partial P_j^a = m_{j-1}^a / 2 [f(y_{j-1}^a) / (v_j - v_{j-1})]$$

We now verify that if $m_j^a = L$ for all j , then $\partial \Pi^a / \partial P_j^a = 0$ when

$$(B.9) \quad s_j^a = [F(y_j^a) - F(y_{j-1}^a)] / 2$$

But (B.9) must hold in a symmetric equilibrium. The quantity in brackets in (B.9) is the total proportion of consumers that purchase good j , i.e. those whose taste parameters are in the interval $y_{j-1}^a < y < y_j^a$, and firm a has a 50% share of these consumers.

Appendix C

*Profits with firm a selling only the premium good
and firm b selling only the basic good*

Customers buy good 1 from firm a only if

$$v+y-(P_1^a+t) > v-(P_0^b+L-t)$$

or

$$y > P_1^a - P_0^b + 2t - L \equiv y(t)$$

Assume that $y(0) > 0$ and $y(L) < \bar{y}$. These conditions require

$$(B.1) \quad m_0^b + L - D < m_1^a < m_0^b - L - D + \bar{y}$$

from which it follows that $\bar{y} > 2L$. (These assumptions will be verified after solving for equilibrium prices). With these assumptions

$$(B.2) \quad \Pi^a = N/L m_1^a \{L(1-F(P_1^a - P_0^b + L)) + (1/2)L(F(P_1^a - P_0^b + L) - F(P_1^a - P_0^b - L))\}$$

Assuming a uniform income distribution with $y \in [0, \bar{y}]$,

$$\Pi^a = N m_1^a \{1 - (P_1^a - P_0^b + L)/\bar{y} + (1/2)((P_1^a - P_0^b + L)/\bar{y} - (P_1^a - P_0^b - L)/\bar{y})\}$$

or

$$(B.3) \quad \Pi^a = N m_1^a (1 - (P_1^a - P_0^b)/\bar{y})$$

Firm a's reaction function is:

$$(B.4) \quad P_1^a = 1/2 (C_1 + P_0^b + \bar{y})$$

Similarly for firm b:

$$(B.5) \quad \Pi^b = N/L m_0^b \{L F(P_1^a - P_0^b - L)) + (1/2)L(F(P_1^a - P_0^b + L) - F(P_1^a - P_0^b - L))\}$$

With a uniform income distribution this reduces to

$$(B.6) \quad \Pi^b = N m_0^b (P_1^a - P_0^b)/\bar{y}$$

and firm b's reaction function is

$$(B.7) \quad P_0^b = 1/2 (C_0 + P_1^a)$$

The equilibrium prices are:

$$(B.8a) \quad \hat{P}_1^a = \frac{1}{3} (2 C_1 + C_0 + 2\bar{y})$$

$$(B.8b) \quad \hat{P}_0^b = \frac{1}{3} (2C_0 + C_1 + \bar{y})$$

and if $D = C_1 - C_0$, the equilibrium mark-ups are:

$$(B.9a) \quad \hat{m}_1^a = \frac{1}{3} (2\bar{y} - D)$$

$$(B.9b) \quad \hat{m}_0^b = \frac{1}{3} (\bar{y} + D)$$

The corresponding equilibrium profits are:

$$(B.10a) \quad \hat{\Pi}^a = \frac{N(2\bar{y}-D)^2}{9\bar{y}}$$

$$(B.10b) \quad \hat{\Pi}^b = \frac{N(\bar{y}+D)^2}{9\bar{y}}$$

Note that profits are independent of the distance L separating the firms. If L is sufficiently small, both firms earn higher equilibrium profits when they specialize in the production of either the basic or the premium good. Furthermore, if $D < \bar{y}/2$, the firm that specializes in the production of the premium good earns strictly higher profits than the firm that specializes in the basic good.

Appendix D

Proof that firm a earns lower profits by specializing

when firm b sells both goods and $\frac{L}{\bar{y}} \rightarrow \infty$

The proof assumes that firm a produces only the premium good. The argument is similar if firm a specializes in the basic good. The proof is for the case of a uniform quality taste parameter distribution.

When $L/\bar{y}$ is large, firm a's profit is:

$$(D.1) \quad \Pi^a[1;1,0] = \frac{N}{L} \left\{ m_1^a \left(t_1 - \frac{(y^b)^2}{4\bar{y}} \right) \right\}$$

and the first-order condition is:

$$(D.2) \quad \partial \Pi^a / \partial m_1^a = \frac{N}{L} \left\{ t_1 - m_1^a / 2 - (y^b)^2 / 4\bar{y} \right\}$$

Firm b's profit is:

$$(D.3) \quad \Pi^b[1;0,1] = \frac{N}{L} \left\{ m_0^b \left((L-t_1) \frac{y^b}{\bar{y}} + \frac{(y^b)^2}{4\bar{y}} \right) + m_1^a (L-t_1) \left(1 - \frac{y^b}{\bar{y}} \right) \right\}$$

and firm b's first order conditions are:

$$(D.4) \quad \partial \Pi^b / \partial m_0^b = \frac{N}{L} \left\{ \left(\frac{L-t_1}{\bar{y}} \right) (m_1^b - m_0^b + y^b) - m_0^b \frac{y^b}{2\bar{y}} + \frac{(y^b)^2}{4\bar{y}} \right\}$$

$$(D.5) \quad \partial \Pi^b / \partial m_1^b = \frac{N}{L} \left\{ (L-t_1) \left(1 - y^b / \bar{y} \right) - m_1^b \left((1 - y^b / \bar{y}) / 2 + (L-t_1) / \bar{y} \right) + \right.$$

$$m_0^b((L-t_1)/\bar{y} - y^b/2\bar{y}) + \frac{y^b}{2\bar{y}} \}$$

As $L/\bar{y} \rightarrow \infty$, m_1^a , m_0^b , m_1^b , and t_1 are of order L . Therefore, we may approximate (D.2), (D.4) and (D.5) by

$$(D.2') \quad \partial \Pi^a / \partial m_1^a = \frac{N}{L} \{t_1 - m_1^a/2\}$$

$$(D.4') \quad \partial \Pi^b / \partial m_0^b = \frac{N}{L} \{(\frac{L-t_1}{\bar{y}})(m_1^b - m_0^b + y^b) - m_0^b \frac{y^b}{2\bar{y}}\}$$

$$(D.5') \quad \partial \Pi^b / \partial m_1^b = \frac{N}{L} \{(L-t_1)(1-y^b/\bar{y}) - m_1^b((1-y^b/\bar{y})/2 + (L-t_1)/\bar{y}) + m_0^b((L-t_1)/\bar{y} - y^b/2\bar{y})\}$$

Looking for an interior solution, (D.4') yields (for large L)

$$(D.6) \quad m_0^b \approx m_1^b + y^b$$

Substituting (D.6) in (D.5') and approximating for large L gives

$$(D.7) \quad m_0^b \approx 2(L-t_1) + y^b(1-y^b/\bar{y})$$

Combining (D.2'), (D.6) and (D.7) gives

$$(D.8) \quad m_1^a \approx L - (y^b)^2/3\bar{y} < L$$

and

$$(D.9) \quad t_1 \approx m_1^a/2 < L/2$$

Firm a could earn more than $NL/2$ (the profit it could earn by producing both goods) only if either its mark-up exceeds L or its market share exceeds $1/2$. But (D.8) and (D.9) show that neither is the case and by producing only one good, firm a earns profits that are strictly less than the profit it could earn by producing both goods.

These calculations are also sufficient to show that if firm b specialized in the premium good, firm a would maximize profits by producing both goods when $L/\bar{y}$ is large. Solving (D.2'), (D.4') and (D.5') for m_0^b and m_1^b , and substituting the results, along with (D.9) for t_1 , into the expression for firm b's profits (D.3) shows that firm b earns profits that strictly exceed $NL/2$ when $L/\bar{y}$ is large. By symmetry, this means that each firm would strictly prefer to produce both products if its rival specialized, when $L/\bar{y}$ is large.

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