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THE MEAN LIFE OF THE NEUTRAL π MESON

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UCRL-10118
UC-34 Physics
TID-4500 (17th Ed.)

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

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Maung Hla Shwe
(Ph. D. Thesis)

June 1962

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ABSTRACT

The proper mean life of the neutral pion was measured by a new method utilizing a large relativistic time dilation. Negative pions of 3.5 Bev/c were allowed to make interactions in Ilford K.5 nuclear research emulsions. Among 3600 interactions we found 103 neutral pions decaying by the Dalitz mode $\pi^0 \rightarrow (e^+ + e^-) + \gamma$. The π^0 path lengths could be accurately measured, and were automatically calculated by specially developed microscope equipment.

We assumed the π^0 momentum spectrum to be the same as that of the secondary charged pions found by multiple scattering measurements. The mean transverse momentum of secondary charged pions at all angles is 274 ± 10 Mev/c.

The opening-angle distribution of the electron pairs was measured and compared with extensive calculations of the expected laboratory distribution, in order to estimate the number of unobserved events.

The mean life (τ_0) of the π^0 was evaluated statistically. The maximum-likelihood π^0 estimate is $\tau_0 = 2.0^{+0.5}_{-0.3} \times 10^{-6}$ sec. Systematic errors were carefully eliminated and their residual effect is thought to be small.

The significance of the measured π^0 mean life for various theories is reviewed.

I. INTRODUCTION

The neutral pion was first experimentally observed by Bjorklund, Crandall, Moyer, and York.¹ Using an independent method for the production of the π^0 , Steinberger, Panofsky, and Steller verified the observations of Bjorklund et al., and established the normal decay mode $\pi^0 \rightarrow \gamma + \gamma$; $\gamma \rightarrow e^+ + e^-$.² In cosmic-ray studies of large nuclear "stars" in emulsion, Carlson, Hooper, and King showed that the production of π^0 accompanies that of $\pi^\pm$.³ From the energy spectrum of γ rays they were able to verify the normal 2γ -decay mode of the π^0 . They also showed that the energy spectrum of the π^0 is similar to that of $\pi^\pm$. Knowing that the average pair conversion length of a γ ray is about 4 to 5 cm in nuclear emulsion, Dalitz⁴ noted the abundance of (e^+, e^-) pairs in the vicinity of the star centers in the experiment of Carlson et al.³ He then predicted an alternative mode of decay for the neutral pion, $\pi^0 \rightarrow \gamma + (e^+ + e^-)$. In this process, the π^0 is transformed first into a real and a virtual photon, the latter then directly produces an (e^+, e^-) pair. He estimated the branching ratio to be

$$\frac{\pi^0 \rightarrow (e^+ + e^-) + \gamma}{\pi^0 \rightarrow \gamma + \gamma} \approx (2 [\ln(m_{\pi^0}/m_e)] - 3.5) \frac{2e^2}{3\pi}$$

$$(\hbar = c = 1)$$

$$\approx 0.01188.$$

More refined theoretical calculations of this branching ratio have been done by Kroll and Wada,⁵ Joseph,⁶ and Kerimov, Mukhtarov, and Gadzhiev.⁷ The latest experimental value is 0.01166 ± 0.00047 , as reported by Samios.⁸ Derrick et al.⁹ and Badagov et al.¹⁰ also obtained consistent values for the branching ratio. Kroll and Wada⁵ also calculated the branching ratio

$$\frac{\pi^0 \rightarrow (e^+, e^-) + (e^+, e^-)}{\pi^0 \rightarrow \gamma + \gamma} \text{ to be } 3.47 \times 10^{-5} (\approx 1/29000).$$

Drell,¹¹ and Berman and Geffen,¹² have estimated the ratio

$$\frac{\pi^0 \rightarrow (e^+, e^-)}{\pi^0 \rightarrow \gamma + \gamma}$$

to be of the order of 10^{-7} .

Since the discovery of the π^0 , experiments have been performed to determine its intrinsic properties; its mass,¹³ spin,¹⁴ parity,¹⁵ and its branching ratio of various decay modes.^{8,9,10,15} Many attempts have also been made to measure the π^0 lifetime. Early estimates, as summarized by Anand in 1953,¹⁶ have since proven too high. The values ranged from $>10^{-15}$ to $<10^{-11}$ sec. By using 450-Mev π^- to produce π^0 through charge exchange in nuclear emulsion, Schein et al. set an upper limit of 4.8×10^{-15} sec for the π^0 lifetime.¹⁷ The first attempt to determine the π^0 mean life using $K_{\pi 2}$ decay, $K^+ \rightarrow \pi^+ + \pi^0$, $\pi^0 \rightarrow \gamma + e^+ + e^-$, was done by Harris, Orear, and Taylor.¹⁸ With 12 events in Ilford G.5 nuclear emulsion (developed grain diameter $\approx 0.6 \mu$), they were able to establish an upper limit of $\approx 7 \times 10^{-16}$ sec for the lifetime.

The main difficulty encountered in experiments in the past was the small path lengths of the π^0 . In the case of π^0 from the decay of $K_{\pi 2}$, the relativistic flight-path dilation factor η is about 1.5. (η is the ratio of the pion momentum in Mev/c to its rest energy in Mev. It is proportional to $\beta(1-\beta^2)^{-1/2}$.) The small path length, about 1/6 of the wavelength of light used for the measurement, coupled with the uncertainty in the point of production of the π^0 , resulted in large errors in the path-length measurements. In the cosmic-ray experiments, there was insufficient precision in determining the points of production and decay of the π^0 .

An alternative method¹⁹⁻²³ for determining the π^0 mean life, first suggested by Primakoff, is to use the inverse process of its 2γ decay, which is the photoproduction of π^0 by the Coulomb field of a nucleus. With this method, Davidson obtained a value of $1.5^{+10}_{-1.0} \times 10^{-17}$ sec for the π^0 mean life,²⁴ and Tollestrup et al. reported a preliminary value of $\geq 5 \times 10^{-17}$ sec.²⁵ Recently Sona²⁶ suggested another method similar to Primakoff's, the only difference being that the Coulomb field of Sona's method is provided by an electron.

In regard to the Primakoff method, it has been pointed out that in addition to the lifetime effect there are other possible causes of deviation from theoretical predictions at small angles.²⁴ These additional unknowns occur even at small values of momentum transfer to the nucleus where the lifetime effect is greatest. An obvious process contributing to these deviations is the inelastic production at forward angles. Other

contributing processes may very well be the production of multipion states. One other difficulty with the Primakoff method is the low intensity of the high-energy incident photons.

Our experiment was designed so that it would be capable of measuring the π^0 mean life to an order of magnitude smaller than the presently found value. This required the development of rather elaborate measurement and data-reduction equipment. We also introduced a principle that lowers almost indefinitely the theoretical minimum observable mean life. The proper time, τ , in the rest frame of the pion varies with the flight path s according to the relationship $s = \tau\beta c(1 - \beta^2)^{-1/2}$. The factor $\beta(1 - \beta^2)^{-1/2}$ produces a relativistic flight dilation that can be made 10 or 100 by observing neutral pions of the appropriate velocities. A "digitized" precision microscope was used for the measurements. Data recorded automatically on IBM cards were then fed into the IBM 650 computer for the necessary calculations.

A reliable value of the π^0 mean life will enable us to check on the theoretical predictions utilizing this parameter. Up to the present, some of the predictions themselves have been used as methods for determining the π^0 mean life. These are discussed below and at the end of this report.

Various theoretical estimates of the π^0 mean life have been made, before and since its experimental discovery by Bjorklund et al. in 1950. Finkelstein, in 1947, used the then known meson theory and quantum theory to calculate the γ instability of the various types of mesons.²⁷ For a pseudoscalar meson having a mass close to that of π^0 and decaying into 2γ , he estimated the lifetime to be $\approx 10^{-16}$ sec. Using the method of subtraction fields in meson perturbation theory, Steinberger, in 1949, estimated the mean life for the π^0 to be $\approx 5 \times 10^{-17}$ sec.²⁸ More recent calculations using dispersion relations have been done by Frautschi,²⁹ and by Jacob and Mathews.³⁰ In the proton Compton scattering, they considered the dominant contribution to be from the process in which a π^0 is exchanged between the γ and the proton: in other words, the process $\gamma + p \rightarrow \gamma + p$ is broken up into two parts, namely $p \rightarrow p + \pi^0$ and $\pi^0 \rightarrow \gamma + \gamma$. With the experimental data available to them at the time, Jacob and Mathews concluded that the values of the π^0 mean life between the limits of 5×10^{-19} and 10^{-16} sec are consistent with experiments

on $\gamma + p \rightarrow \gamma + p$.³⁰

Very recently Gell-Mann et al.³¹ have proposed the decay of the (ω_η) meson³² $\left[(\omega_\eta) \rightarrow \pi^+ + \pi^- + \pi^0 \right]$ to be dominated by the process $(\omega_\eta) \rightarrow \rho + \pi$ followed by $\rho \rightarrow 2\pi$. They used a dispersion theory type of calculation and treated the ρ meson as a nearly stable particle. They made an application to the decay of the π^0 . Assuming that the decay of π^0 is dominated by the process $\pi^0 \rightarrow \rho^0 + \omega^0$ followed by $\rho^0 \rightarrow \gamma$, $\omega^0 \rightarrow \gamma$, they calculated the π^0 mean life as $\approx 6 \times 10^{-18}$ sec.

II. EXPERIMENTAL PROCEDURE

A. Exposure and Scanning

A stack of 600- μ thick, 2.5-by-7.5-cm Ilford K.5 nuclear emulsion was exposed to the 3.5-Bev/c negative pions of the Bevatron. The total intensity was about $3 \times 10^5 \text{ cm}^{-2}$. The fractional momentum spread was ± 0.15 (full width at half-maximum). The experimental arrangement is shown in Fig. 1. To obtain more reliable multiple-scattering measurements, the emulsion was exposed in the form of plates with glass backing. For easy relocation of each event, grid coordinates were printed at the back of each plate. An enlarged photograph of the type of grid coordinates used in the Barkas physics research group is shown in Fig. 2. We used the standard processing technique for 600- μ -thick Ilford K.5 emulsion, with some variations. The types of chemicals and the durations for all the stages of processing are tabulated in Appendix A.

Primary π^- interactions with emulsion nuclei were found by area scanning. Each interaction was recorded on a keysort Data Card, reproduced in Fig. 3. Dip angles and projected angles were recorded on the same card.

B. Measurements and Data Reduction

In our experiment (see Figs. 4, 5, and 6) the point P of decay of the π^0 is taken to be the vertex of the (e^+, e^-) pair [Dalitz decay mode: $\pi^0 \rightarrow (e^+ + e^-) + \gamma$]. The origin O of the π^0 is found from the intersection of the trajectories of near-minimum-ionizing particles produced at the

MU-26155

Fig. 1. Experimental arrangement of the stack exposure to 3.5-Bev/c π^- at the Bevatron.

(a)

(b)

ZN-3044

Fig. 2. (a) Enlarged photograph of grid coordinates. Each square in the grid is 1 mm on a side. (b) Actual view when microscope is focused at emulsion-glass surface.

100 RD = 132.6 μ

OBSERVER H.S. TERMINUS: PLATE V-5 PARTICLE 123

PICK UP: PLATE + SCATTER: PLATE 3 1
1 2

CHECKED: ✓ 7 D.P.

COMMENTS: *small angle*

$\theta_1 = 359.6$ $\Delta z_1 = 0$
 $\theta_2 = 94.8$ $\Delta z_2 = \frac{45.3}{50}$
 $\theta_3 = 160.0$ $\Delta z_3 = -7.2$
 $\theta_4 = 231.5$ $\Delta z_4 = 0$
 $\theta_5 = 253.2$ $\Delta z_5 = +2.9$
 $\theta_6 = 278.0$ $\Delta z_6 = +10.2$

PRONG # _____
OBSERVER _____
PLATE _____
CHECKED _____

PRONG # _____
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PLATE _____
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PRONG # _____
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Mo Year

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Fig. 3. Reproduction of Keysort card used for recording.

MUR-772

Fig. 4. An event in which a π^0 , produced at point 0, decays in the Dalitz mode, $\pi^0 \rightarrow e^+ + e^- + \gamma$, at point P. Points are centers of grains; straight lines are least-squares fits. Tracks 2 and 5 are near-minimum; tracks 3 and 4 are the (e^+, e^-) pair.

Fig. 5. An event similar to that in Fig. 4. Tracks 4 and 5 are near-minimum; tracks 2 and 3 are the (e^+ , e^-) pair.

MUB-916

Fig. 6. A typical event in which there is no π^0 Dalitz decay.

primary π^- interaction point, plus the primary itself.

We also made two other exposures. One was with Ilford L4 nuclear emulsion hypersensitized with triethanolamine. The other was an exposure to 16-Bev negative pions at CERN, Geneva. The higher grain density obtained with hypersensitization in this experiment turned out to be not as important as the freedom from a background of single grains. Because some of the pair angles become smaller, the star center more poorly defined, and the multiple scatterings of the secondary charged particles more difficult, the higher incident energy is not necessarily better for our experiment.

There are, however, obvious advantages in having higher incident π^- energy and thus producing pions of higher velocities. We expect (a) larger gaps due to relativistic flight-path dilation, (b) smaller error in fitting least-squares straight lines through the grains of the tracks, and (c) containment of the pions in a smaller forward cone. In the emulsion we used for this experiment the grain density of a minimum-ionizing track is 20 grains per 100 μ . The developed mean grain diameter is about 0.5 μ .

All interaction stars having two "minimum"-ionizing secondaries plus one or more minimum or near-minimum secondaries were analyzed. The primary and at least one secondary were needed to define the point 0 (Figs. 4, 5, and 6). For the measurements we used a precision microscope (Koristka MS 2) with the stage modified in such a way that it could be rotated from one position stop to another at 90 degrees. The filar micrometer eyepiece readout shaft was coupled to an analog-to-digital converter. This "digitizer" fed through an electronic translator assembly into an IBM card-punch machine. We determined the relative x and y coordinates of the centers of the six grains (see Appendix B) closest to the star center for each fast track, including that of the primary π^- . (See Appendix C for further description of the digitized microscope and its components.) We made a drawing like that shown in Fig. 7 for each event to be measured at the Koristka. An IBM 650 computer was programmed to calculate the least-squares best straight-line fit to each track. It also calculated the intersections of pairs of

lines and their errors. (See Appendix D for derivations of formulae for these calculations.)

When the star center was found from the intersections of more than two pairs of tracks, the average distance between these points was $0.06 \pm 0.01 \mu$. The distribution of this distance is shown in Fig. 8. Typically, then, the location of the star center (point 0 in Figs. 4, 5, and 6) was known to better than 0.03μ . An (e^+, e^-) pair was suspected whenever a pair of minimum-ionizing tracks did not intersect in the small region where all the others did (including nonminimum tracks and the primary track). Each event was measured by at least two different trained observers, often on different days. The agreement was compatible with quoted statistical errors. The tracks of the suspected electrons were then followed until they left the plate. In most cases when one or both of the tracks stayed in the emulsion for 5 mm or more, one could show that they were electrons. In no case was a suspected electron found not to be one.

Occasionally a background track was mistaken to have originated from the interaction point. In some other rare occasions a wrong primary π^- was selected. These were detected later when the intersections of pairs of tracks were calculated by the IBM computer. The events were then re-examined on the microscope and the necessary eliminations of the wrong tracks were made.

The momentum spectrum of the secondary charged pions was found by measuring the multiple scattering of the tracks having dip angles of less than 6 deg and correcting the spectrum for the bias thus introduced. Limiting the dip angle to 6 deg was necessary to avoid the effect of "spurious scattering," which increases with dip angle. More discussion of the multiple scattering measurements is given in Appendix E.

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Fig. 7. A typical drawing made for measuring the coordinates of the grains.

Fig. 8. Distribution of the distances between the points of intersections of pairs of tracks.

III. EXPERIMENTAL RESULTS

A. Data

By careful scanning, 3600 π^- stars in nuclear emulsion were found. As mentioned before, precision measurements with the automated "digitized" microscope yielded 103 associated (Dalitz) electron pairs. Only the projection of the path of the π^0 on the plane of the emulsion could be determined with close accuracy. For the mean-life measurement, we selected π^0 trajectories making projected angles of ≤ 60 deg with the path of the primary π^- . Fig. 9a shows the distribution of the projected angles of the neutral pions. Although it is possible that a π^0 meson outside of a 60-deg cone could be included by our method of selection, the error involved is negligible, partly because a bias exists against observing an event with a dip angle near 90 deg, and partly because a large dip angle would have been measureable in the longer of the flight paths. The projected-angle distribution of the charged secondary pions is shown in Fig. 9b. The similarity between the projected angle distributions of the charged and neutral pions is satisfactory.

The projected path length of the π^0 in no case exceeded 2.31μ . The average γ -ray pair conversion length is about 4 cm in nuclear emulsion. Therefore we estimated the contamination from ordinary π^0 γ -ray pair conversion to be one event. Table I gives projected path lengths of all measurable neutral pions and projected angles of emission relative to the primary negative pion. The opening angles of the electron pairs in the laboratory system are also listed. At high pion energies, some angles are increased and others decreased from their c.m. values. For a gap observation to be considered a measurement, the gap length had to exceed the error in its measurement by a factor of 3.5.

Among the events that were missed or unmeasured were the following types:

(a) Events with only two secondary minimum-ionizing tracks. Another secondary is needed to define the star center; therefore, no measurement could be made. We observed ten stars of this type.

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Fig. 9. Projected angular distributions of (a) secondary neutral pions, and (b) charged pions. In (a), only the measured events are included (see footnotes to Table I).

TABLE I

List of Observed Dalitz Pairs
(in order of increasing opening-angle)

Event No.	Projected π^0 path length (microns) s	Projected angle of π^0 path relative to incident π^- (deg)	Opening angle of pair in laboratory (deg) α
V4-213	1.86 ± 1.69 ^b	d	1.3
V8-101	0.98 ± 1.42 ^b	d	1.5
V6-212	1.05 ± 0.98 ^b	d	1.6
V10-3	0.80 ± 1.33 ^b	d	1.9
V6-38	1.02 ± 0.96 ^b	d	2.0
V4-104	0.32 ± 0.56 ^b	d	2.3
V2-103	^c	d	2.3
V2-144	0.49 ± 0.62 ^b	d	2.4
V11-186	^c	d	2.4
V4-82	0.52 ± 0.61 ^b	d	2.8
V3-91	0.63 ± 0.70 ^b	d	2.8
V9-109	0.58 ± 0.72 ^b	d	2.8
V6-352	2.31 ± 0.34	20.9	2.9
V6-105	0.39 ± 0.60 ^b	d	3.1
V6-41	0.57 ± 0.53 ^b	d	3.2
V4-364	0.35 ± 0.42 ^b	d	3.5
V7-237	0.54 ± 0.55 ^b	d	3.5
V7-184	^c	d	3.5
V3-24	0.40 ± 0.67 ^b	d	4.1
V2-79	^c	d	4.1
V6-253	0.49 ± 0.32 ^b	d	4.2
V3-3	0.80 ± 0.31	18.5	4.3
V4-322	1.37 ± 0.27	25.1	4.4
V3-83	1.25 ± 0.19	35.1	4.7
V2-124	1.44 ± 0.25	5.2	4.7
V9-132	^c	d	5.5

TABLE I (cont.)

Event No.	Projected π^0 path length (microns) s	Projected angle of π^0 path relative to incident π^- (deg)	Opening angle of pair in laboratory (deg) α
V11-23	0.50 ± 0.42 ^b	d	5.6
V10-41	^c	d	6.0
V3-41	1.05 ± 0.17	14.1	6.0
V6-572	1.99 ± 0.03	114.9	6.0
V3-102	0.38 ± 0.36 ^b	d	6.4
V2-84	1.06 ± 0.16	159.6	6.9
V6-86	1.03 ± 0.14	142.3	7.3
V11-203	0.93 ± 0.16	3.1	7.7
V8-102	1.87 ± 0.18	21.7	7.8
V3-202	^c	d	8.0
V3-9	^c	d	8.2
V6-298	0.59 ± 0.11	16.5	8.3
V6-274	2.16 ± 0.10	29.8	8.4
V7-148	0.77 ± 0.07	8.6	8.8
V6-184	0.68 ± 0.14	18.5	8.9
V4-312	^c	d	9.2
V10-84	^c	d	9.4
V10-14	0.51 ± 0.12	4.1	9.4
V10-177	1.01 ± 0.05	25.5	9.5
V6-228	0.98 ± 0.08	9.3	9.6
V2-114	1.30 ± 0.15	84.0	10.2
V6-248	0.64 ± 0.11	26.2	10.3
V10-39	0.37 ± 0.10	16.3	10.5
V7-18	0.55 ± 0.04	23.7	10.9
V10-104	0.34 ± 0.09	50.6	11.2
V12-111	0.40 ± 0.14	22.6	11.3
V11-143	0.81 ± 0.03	2.8	11.8
V2-244	0.37 ± 0.05	61.9	12.9
V3-72	0.51 ± 0.09	38.1	13.1

TABLE I (cont.)

Event No.	Projected π^0 path length (microns) s	Projected angle of π^0 path relative to incident π^- (deg)	Opening angle of pair in laboratory (deg) α
V6-94	0.49 ± 0.11	19.2	13.2
V4-149	0.72 ± 0.06	13.3	14.5
V11-168	0.52 ± 0.10	20.1	14.9
V6-140	0.95 ± 0.05	152.6	15.0
V2-110	0.98 ± 0.10	2.4	15.3
V9-44	0.41 ± 0.06	31.4	16.5
V10-329	0.37 ± 0.05	171.6	16.7
V6-189	0.55 ± 0.04	9.4	16.9
V8-32	0.51 ± 0.06	5.2	17.4
V6-75	1.53 ± 0.06	78.7	17.6
V2-66	0.34 ± 0.06	3.3	17.8
V2-113	0.51 ± 0.06	12.8	18.3
V6-137	0.71 ± 0.05	8.1	18.5
V12-136	0.63 ± 0.07	32.9	19.1
V4-479	0.90 ± 0.05	70.3	19.4
V6-47	0.42 ± 0.09	7.3	19.8
V4-209	0.25 ± 0.07	10.7	20.5
V4-436	0.24 ± 0.05	30.5	21.3
V4-207	0.49 ± 0.05	18.8	21.7
V6-211	0.53 ± 0.08	3.7	22.3
V9-23	1.50 ± 0.05	177.8	22.8
V11-210	0.75 ± 0.04	100.6	24.6
V2-209	0.20 ± 0.05	68.2	24.9
V6-488	0.88 ± 0.04	17.8	27.0
V6-533	0.28 ± 0.03	68.5	29.2
V11-32	0.54 ± 0.06	69.8	29.3
V11-35	0.51 ± 0.02	25.6	30.0
V4-140	0.19 ± 0.02	16.2	30.8
V6-37	0.27 ± 0.03	38.0	31.8

TABLE I (cont.)

Event No.	Projected π^0 path length (microns) s	Projected angle of π^0 path relative to incident π^- (deg)	Opening angle of pair in laboratory (deg) α
V9-34	0.47 ± 0.03	10.7	32.6
V4-137	0.39 ± 0.03	39.8	34.0
V6-50	0.93 ± 0.04	24.1	34.4
V6-235	0.25 ± 0.03	27.3	37.0
V10-12	0.17 ± 0.04	3.7	39.1
V6-410	0.35 ± 0.04	11.1	39.9
V4-93	0.25 ± 0.03	32.3	41.0
V2-97	0.25 ± 0.02	169.6	41.5
V11-97	0.14 ± 0.03	83.5	41.5
V6-62	1.18 ± 0.04	3.8	42.6
V10-333	0.16 ± 0.03	25.1	43.2
V11-156	0.51 ± 0.04	23.4	46.7
V10-30	0.31 ± 0.03	10.5	47.1
V6-111	0.30 ± 0.02	60.4	49.8
V11-205	0.24 ± 0.02	81.1	54.8
V6-433	0.35 ± 0.02	40.0	55.8
V11-47	0.43 ± 0.04	112.0	57.0
V10-48	0.38 ± 0.02	32.9	61.0
V6-391	0.73 ± 0.02	4.3	72.2

(Footnotes to Table I)

- ^a The opening angles of all observed events were measurable. The error in the opening-angle measurement is of the order of one degree. Therefore, small angles are measured with rather large percentage errors, and their distribution is unreliable. The π^0 path lengths could be measured accurately (the path length exceeding the error of its measurement by a factor > 3.5) when $\alpha > 4 \mu\text{-deg}$. As seen in Fig. 14, however, the pairs could not be found reliably unless the angle α is less than approximately 50-70 deg.
- ^b These distances, labeled events of type (b) are not considered measurements because the errors are comparable to the gap lengths. They were used only indirectly in the π^0 mean-life evaluation. Correction for this bias has been made. See text for more discussion.
- ^c Labeled type (a) in the text, these π^0 path lengths were not measurable because there was no way of determining the star center.
- ^d The path of the neutral pion could not be defined.

(b) Events with a very close (e^+ , e^-) pair. When the product of the angle between the tracks and the gap becomes less than $3.44 \mu\text{-deg}$, the error is too large for accurate measurement of the gap. We observed 18 events of this type.

(c) Pairs of large opening angle but with very small gaps which lie within the "circle of confusion" around the star center.

We measured 75 π^0 events at all angles relative to the incident π^- . An additional 28 events were observed which were of types (a) and (b) above. The opening angles for all these events are listed in Table I. The measured π^0 path lengths for type (b) events are also listed. Within the 60-deg cone, the corresponding numbers were 56 (measured) and 19 (additional observed). The number of events of type (c) (and which were missed) will be estimated later from the theoretical distribution in the laboratory system of the opening angles of the Dalitz pairs.

Figure 10 shows the opening angles (in the laboratory system) of all observed Dalitz pairs. The smooth curve is the calculated distribution from the assumed momentum spectrum of the π^0 and the opening angle distribution in the rest system of the π^0 as calculated by Dalitz.^{4, 10} (Appendix G gives the derivation of the formulae and the procedure followed in obtaining the expected opening-angle distribution in the laboratory system.) The points in the figure are the actual values obtained in the calculations. The smooth curve is the best fit through these points. The theoretical points are normalized to the number of events with opening angles between 10 and 50 deg; in this interval it is judged that few events were missed.

Figure 11 shows the distribution of the angle between the plane containing the (e^+ , e^-) pair and the emulsion plane. The angle is measured from the emulsion (horizontal) plane (see Appendix F). The distribution is expected to be isotropic from 0 to 90 deg. From the figure we see that there may be a small deficiency of angles between 60 and 90 deg. This amounts to approximately 6% of the total number observed, but is not statistically significant.

In principle the π^0 momentum is measurable, and in fact it was measured for a number of individual events. We chose a statistical

MU-26157

Fig. 10. Laboratory-system opening-angle distribution of all observed Dalitz pairs from π^0 decay. The smooth curve is the theoretical distribution. More description is given in the text.

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Fig. 11. Distribution of the angle between the plane containing the Dalitz pair and the emulsion (horizontal) plane.

approach, however, to evaluate the mean life of the π^0 . In this way we could use most of our π^0 -gap data. We have assumed the π^0 momentum spectrum to be the same as that of the charged pions (which we found by multiple-scattering measurements). Justification for this assumption is given below.

The probability $P(s)$ that the π^0 path exceeds a distance s (cm) before decay is given by

$$P(s) = \int_0^{\infty} [\exp(-s/c\eta\tau)] f(\eta) d\eta, \quad (1)$$

where τ is the proper mean life of the π^0 in seconds, η is the ratio of the pion momentum in Mev/c to its rest energy in Mev, and c is the velocity of light in cm/sec. The normalized momentum spectrum $f(\eta)d\eta$ was found as described above.

We can also construct a likelihood function as follows:

The probability $\phi(s)ds$ that a π^0 path length be between s and $s + ds$ is

$$\phi(s) ds = \frac{ds}{c\tau} \int_0^{\infty} [\exp(-s/c\tau\eta)] \frac{f(\eta)}{\eta} d\eta,$$

where the symbols are as defined for Eq. 1.

For the i th event, the π^0 path is measured to be s_i . Therefore the likelihood function is

$$P(\tau) = (c\tau)^{-n} \prod_{i=1}^n \int_0^{\infty} [\exp(-s_i/c\tau\eta)] [f(\eta)/\eta] d\eta, \quad (2)$$

where n is the total number of π^0 paths measured.

Equations (1) and (2) apply to the case in which every π^0 path length is seen and measured. We discuss below how we eliminated the biases introduced because some events could not be measured.

Figure 12 shows the momentum spectrum of secondary charged pions within a cone of half-angle 60 deg relative to the primary negative pion. The transverse momentum distribution for the secondary charged pions emitted at all angles is shown in Fig. 13. The smooth curve is normalized to the total number of secondaries measured. It is the best fit to the measured distribution with a function of the form $f(p_T) = Kp_T \exp(-p_T/p_0)$. The mean transverse momentum is

MU-25592

Fig. 12. Momentum distribution of secondary charged pions within 60-deg forward cone.

MU-25590

Fig. 13. Transverse momentum distribution of secondary charged pions at all angles.

274 ± 10 Mev/c.

The total number of events we have observed (measured and unmeasured) plus the number missed of type (c) can be compared with that expected. The mean multiplicity of the secondary charged pions is 4.6. We assume that approximately half as many secondary neutral pions were produced as the charged.^{3, 31-34} Therefore, with the experimental branching ratio of 0.01166 ± 0.00047 for the Dalitz mode,⁸ we would expect to find a total of 97 ± 15 events at all angles, and 73 ± 9 events within the 60-deg cone.

The comparison is as follows:

	<u>All angles</u>	<u>Within 60-deg cone</u>
Measured	75	56
Additional observed	28	19
Type (c) correction	<u>6</u>	<u>4</u>
Total	109	79
Expected	97 ± 15	73 ± 9

(It should be noted that the events of the type (c) correction above were estimated by considering the area shown in Fig. 10 between the toe of the calculated curve and the observed histogram.)

We see that the expected numbers agree rather satisfactorily with those observed (after correction). This helps to justify our assumption that the momentum spectrum of the secondary charged pions is similar to that of the neutral pions. Further discussion on this point is given later.

B. Evaluation of the Neutral Pion Mean Life

Equations (1) and (2) are for the case in which all the events are seen and measured without any bias caused by the three types of events listed on page 15.

To take account of the selection bias in the measured π^0 events, we used the following procedure for the mean-life evaluation.

A Dalitz pair from the decay of a π^0 with η in the interval $d\eta$ has the probability $q(\eta, \alpha)d\alpha$ that its opening angle lies in the interval $d\alpha$. The pion has the probability $\exp(-s/c\tau\eta)$ of going a distance exceeding s , and the probability $(c\tau\eta)^{-1}[\exp(-s/c\tau\eta)] ds$ of decaying in the interval ds . When the pion path length is s and the opening angle is α , the

probability of being recognized as a Dalitz pair and being measured is $W(s, a) = w(sa)$.

The justification for taking $W(s, a)$ to be $w(sa)$ is as follows: We see that all the observed but unmeasurable π^0 events have small opening angles. The error in the intersection of the Dalitz pair (π^0 decay point) becomes too large for the gap determination when the angle is small. For small opening angles, the error in the intersection varies inversely as the angle. There is a certain value of sa above which we can measure the path length. In our experiment this number was found to be $3.44 \mu\text{-deg}$ for the events within the 60-deg half-angle forward cone. Therefore, to take account of the bias due to the unmeasured events with small opening angles, we take $W(s, a) = w(sa)$.

From the distribution of (s) vs (sa) [Fig. 14], we select the following limits for $w(sa)$:

$$\text{for } 4.0 \leq sa \leq 72.2s, \quad w(sa) = 1;$$

$$\text{for } 4.0 > sa > 72.2s, \quad w(sa) = 0.$$

The straight line $(sa) = 72.2s$ goes through the point (π^0 event) of the largest measured opening angle.

The lower limit is required because for $(sa) > 4.0 \mu\text{-deg}$ the error in the gap measurement becomes comparable to the gap length. For $(sa) > 72.2s$, some events of large opening angle are probably lost.

Withing the 60-deg half-angle forward cone, we have 53 measured π^0 events for nonzero $w(sa)$. These events are used in the evaluation of the π^0 mean life by methods (1) and (2) discussed below.

We believe that the additional ten events observed of type (a) do not introduce any bias in the mean life evaluation. Their path-length distribution is very similar to a typical sample of all the events observed.

The probability that η lies in the interval $d\eta$ is $f(\eta)d\eta$. Therefore, the combined probability $\lambda(\eta, a, s)d\eta dads$ that a π^0 decaying by the Dalitz mode by observed and measured is

$$\lambda(\eta, a, s)d\eta dads = (c\tau\eta)^{-1} [\exp(-s/c\tau\eta)] f(\eta)q(\eta, a)w(sa)d\eta dads.$$

The probability $p(s)ds$ that a π^0 path length in the interval ds will be measured is, therefore,

MU-26159

Fig. 14. Distribution of (s) vs $(s\alpha)$ for all measured π^0 events within the 60-deg half-angle cone. In the region $4.0 > s\alpha > 72.2$ s, the events are inaccessible to measurement. They are shown in the diagram, and correction has been made for this bias in the statistical treatment of the data.

$$p(s)ds = (c\tau)^{-1} ds \int_0^\infty d\eta f(\eta) \eta^{-1} [\exp(-s/c\tau\eta)] \int_0^\pi q(\eta, a) w(sa) da.$$

The function $q(0, a)$ has been given by Dalitz.^{4, 10} In Appendix G the effect of the motion of the π^0 mesons on the distribution of opening angles is determined by calculation. It is found that in the entire region of interest the distribution of opening angles is expressed well by the function

$$q(\eta, a) da = \frac{a^\epsilon}{\Gamma(\epsilon)} [\exp(-a/a)] a^{-(1+\epsilon)} da,$$

where ϵ and a depend on η as given in the table in Appendix G.

The above function has been normalized for a ranging from 0 to ∞ , but of course its physical upper limit is π . No appreciable error is thereby introduced, since only about 1% of the area under the distribution lies above $a = 100$ deg.

Hence we have

$$\begin{aligned} p(s) ds &= (c\tau)^{-1} ds \int_0^\infty d\eta \eta^{-1} f(\eta) \\ &\quad \times [\exp(-s/c\tau\eta)] \frac{(as)^\epsilon}{\Gamma(\epsilon)} \int_{\mu_0}^{72.2 s} [\exp(-\frac{as}{as})] (as)^{-(1+\epsilon)} d(as) \\ &= (c\tau)^{-1} ds \int_0^\infty d\eta \eta^{-1} f(\eta) \\ &\quad \times [\exp(-s/c\tau\eta)] \frac{(as)^\epsilon}{\Gamma(\epsilon)} \int_{1/72.2 s}^{1/\mu_0} [\exp(-asx)] x^{\epsilon-1} dx, \quad (3) \end{aligned}$$

where we have put $(1/x) = as$, and $\mu_0 = 4.0 \mu\text{-deg}$.

For the i th event, the π^0 path is measured to be s_i . Therefore, the likelihood function is

$$\begin{aligned} P(\tau) &= (c\tau)^{-n} \prod_{i=1}^n \int_0^\infty d\eta \eta^{-1} f(\eta) \\ &\quad \times [\exp(-s_i/c\tau\eta)] \frac{(as_i)^\epsilon}{\Gamma(\epsilon)} \int_{1/72.2 s_i}^{1/\mu_0} [\exp(-as_i x)] x^{\epsilon-1} dx. \end{aligned}$$

(4)

The probability $P(s)$ that a π^0 will have a path length exceeding s and be measured is

$$P(s) = \int_s^{\infty} p(s) ds, \quad (5)$$

where $p(s) ds$ is given by Eq. (3).

Thus, we have the following two methods for the evaluation of the neutral pion mean life:

1. Curve Fitting to Integral Gap Length Distribution

We used for this method the integral distribution of all the measured π^0 projected path lengths within the 60-deg half-angle forward cone. The experimental distribution was fitted with the theoretical, which was obtained by using Eq. (5) and the measured momentum spectrum. The value of τ was varied until we got the best χ^2 fit to the experimental. Curve fitting was restricted to the region of $s > 0.16 \mu$, since this is the smallest π^0 path measured (within the 60-deg half-angle forward cone and for nonzero $w(s)$). Figure 15 shows the experimental points and the theoretical curve with the most probable τ value. Figure 16 is the χ^2 distribution for different assumed values of τ . From this distribution we obtained an uncorrected mean life of $\tau = 2.0^{+0.6}_{-0.4} \times 10^{-16}$ sec.

The confidence interval was determined as described under the following maximum-likelihood method.

If the primary track (π^-) is considered to be the x axis, the z component-perpendicular to the emulsion surface-of the π^0 path was not measured directly. Its root-mean-square value was estimated from the mean-square value of the y component, and τ was corrected to

$$\tau = 2.1^{+0.6}_{-0.4} \times 10^{-16} \text{ sec.}$$

2. Maximum-Likelihood Method

For this method we used the likelihood function given by Eq. (4). The likelihood function for each τ was calculated by using the measured momentum spectrum. Figure 17 is a plot of this function for various assumed values of τ , whose most likely value is

$$\tau = 1.9^{+0.5}_{-0.3} \times 10^{-16} \text{ sec.}$$

MU-26160

Fig. 15. Integral distribution of projected π^0 path lengths. Dotted points are experimental, and the solid curve is plotted from Eq. (5) with the most probable τ and the measured momentum spectrum (Method 1).

MU-26161

Fig. 16. χ^2 distribution for different values of τ .

MU-26162

Fig. 17. Relative-likelihood plot for various values of τ (Method 2).

With the correction due to z-projection of the π^0 paths (as mentioned in method 1), we have the most likely value of τ to be

$$\tau = 2.0^{+0.5}_{-0.3} \times 10^{-16} \text{ sec.}$$

For both methods the confidence interval corresponds approximately to ± 1 standard deviation. It encompasses two-thirds of the area under the curve of the distribution function, one-third above the mode and one-third below.

The agreement on the value of the π^0 mean life between the two methods is satisfying.

It has been proved that the maximum-likelihood method makes the best use of the data.³⁷ Here we see that method (1) gives a τ value consistent with that of method (2).

Thus we have the following value for the mean life of the π^0 :

$$\tau_{\pi^0} = 2.0^{+0.5}_{-0.3} \times 10^{-16} \text{ sec.}$$

The confidence interval is purely statistical.

Taking the limits of integration over (as) (in Eq. 3) to be from 3.44 to ∞ , we obtained, for the mean life of the π^0 ,

$$\tau_{\pi^0} = 1.7^{+0.6}_{-0.5} \times 10^{-16} \text{ sec, by method (1);}$$

$$\tau_{\pi^0} = 1.7^{+0.5}_{-0.4} \times 10^{-16} \text{ sec, by method (2).}$$

These results are interpreted to signify that the mean-life estimate is not sensitive to the choice of the upper limit of (as) considered in the integration. It is nevertheless apparent that the integration should not be extended to infinity.

C. Treatment of Systematic Errors

We now list some important sources of systematic errors, their effects on the mean life evaluation, and some corrections for them.

1. Momentum Spectrum of the Secondary Charged Pions

As mentioned in Appendix E, the individual tracks were measured with error $\leq 20\%$. Checks were also made by having two operators measure different segments of the same track. Each track must satisfy

all three conditions listed in the appendix. For error estimation, we measured some 100 primary π^- tracks.

We assume that the true distribution of the sagitta (mean second difference D ; see Appendix E) is distorted by a Gaussian-like distribution with variance σ^2 . We can then calculate σ from the relation

$$2\sigma^2 = \frac{1}{n} \sum_{i=1}^n (D_1^2 - D_2^2)^2,$$

where D_1 and D_2 are the values of the sagitta obtained by two operators for different segments of the same track. We get $\sigma = 0.006 \mu$.

The distribution of the measured values of D for the secondary charged pions can be approximated by two Gaussian-like distributions; for each, the standard deviation is about 0.3μ .

It can be shown that the product of two Gaussian functions, with variances σ^2 and σ'^2 , is another Gaussian function with variance Σ^2 given by $\Sigma^2 = \sigma^2 + \sigma'^2$.

To see the effect of this fluctuation (in the measured D distribution) on the mean life evaluation, we take Eq. (1) for illustration.

For constant cell length t , we can write $\eta \approx K_1/D$. K_1 is a slowly varying function of the velocity β and the cell length t (see Appendix E). For the measured relativistic pions with β ranging from 0.95 to 1.0, we can assume that K_1 is approximately constant for fixed t .

Consider one of the two Gaussian distributions (which approximate the true D distribution) to be

$$\exp \left[\frac{-(D - \langle D \rangle)^2}{2\sigma'^2} \right].$$

Therefore, for this part of the D distribution, Eq. (1) becomes

$$P(s) \approx \int \left[\exp \left(\frac{sD}{c\tau K_1} \right) \right] \left[\exp \frac{-(D - \langle D \rangle)^2}{2\sigma'^2} \right] D^{-2} dD.$$

We assume that the true D distribution is distorted by a Gaussian-like distribution with variance σ^2 .

Therefore, because of fluctuations in the measurements of D ,

σ' in the above equation has to be replaced by Σ , where

$$\Sigma \approx \sigma' \left(1 + \frac{1}{5000} \right).$$

We see, therefore, that the fluctuations in the multiple-scattering measurements introduce insignificant effects on the mean-life evaluation.

2. Assumption that the Momentum Spectrum of the Secondary Charged Pions is the Same as that of the Neutral Pion

Figure 9 shows that the projected angle of the secondary charged pions is very similar to that of the neutral pions. This gives us the first piece of evidence that there is great similarity in the production of $\pi^{\pm}$ and π^0 . In the experiments on multiple pion production by negative pions incident on emulsion nuclei, it has been found that the ratio of the secondary charged to neutral pions is very close to 2.0.³⁶ This ratio was also obtained in several cosmic-ray experiments.^{3, 33-35} As mentioned before, the number of π^0 events expected agrees satisfactorily with the number we observed.

We, therefore, believe that no appreciable error is introduced by our assumption regarding the momentum spectrum.

3. General Measurement Errors

These include errors due to individual operators, and variation in the room temperature and humidity. In order to eliminate these effects the emulsion plate was set in temperature and humidity equilibrium with the surroundings before any measurement began. Each event was measured by two operators, often on different days. It was required that the two results agree within quoted errors. In the case in which a π^0 event was found, each value of the π^0 path was weighted by the error and the two values combined to give the resulting π^0 path and its combined error.

4. Measurement Precision and Scanning Efficiency

Out of 3600 interaction stars, the first two measurements were in disagreement or ambiguous in about 80 cases. These events required measurement for the third and fourth time (by different operators). In all cases, we were able to come to some conclusion after the second set of measurements.

A third measurement was also made on each of a sample of five

randomly picked π^0 events. Each of these measurements was found to agree well with the first two.

To check on the efficiency of the scanners in observing secondary minimum-ionizing tracks, a fair sample of about 200 stars was rescanned. Some 15 additional secondary "minimums" were found, with not more than one additional for each star. None of these additional "minimums" affect the near 100% efficiency in the observation of π^0 events. Pairs are usually more conspicuous than lone minimum tracks.

5. Contamination of (e^+, e^-) Pairs from γ Conversion

The average γ -conversion length in emulsion is about 4 cm. The mean multiplicity for secondary charged pions is 4.6. We assume that approximately half as many secondary neutral pions were produced as charged.^{3, 33-36} The largest π^0 path length measured was 2.31 μ . Therefore, the number of γ -conversion pairs within 2.31 μ of the interaction points would be $(4.6 \times 3600) \times 2.31 \times (40,000)^{-1} \approx 1$. This is $< 1\%$ of the total number of Dalitz-decay π^0 events observed.

6. Unmeasured Events of Type (a)

The total number of unmeasured stars of type (a) [Section III-A] is 10. These events were not measurable because we had no way to determine the point of production of the π^0 . We believe that the π^0 path-length distribution for these events must be very similar to a typical sample of all the events observed. This effect therefore does not introduce appreciable error in our mean-life evaluation.

Combining all the errors discussed above (1 through 6), we see that the overall systematic error is small compared with the statistical error we have quoted for the value of the π^0 mean life.

Therefore the value of the π^0 mean life, as determined in this experiment, is

$$\tau = 2.0^{+0.5}_{-0.3} \times 10^{-16} \text{ sec.}$$

IV. DISCUSSION

The mean life of the π^0 as found in this experiment ($2.0^{+0.5}_{-0.3} \times 10^{-16}$ sec) is in good agreement with the values published recently. They are

Blackie et al. ³⁸	$\tau_{\pi^0} = 3.2 \pm 1.0 \times 10^{-16}$ sec,
Glasser et al. ³⁹	$\tau_{\pi^0} = 1.9 \pm 0.5 \times 10^{-16}$ sec,
Tollestrup et al. ²⁵	$\tau_{\pi^0} > 5 \times 10^{-17}$ sec; ($\approx 10^{-16}$ sec).

The first two experiments were carried out by using neutral pions from the $K_{\pi 2}$ decay ($K^+ \rightarrow \pi^+ + \pi^0$), which produce path lengths of the π^0 just at the limit of measurability. The third experiment (Tollestrup et al.) used the Primakoff effect of producing π^0 by one incident γ and another γ from the Coulomb field of a nucleus. A rough attempt by Tollestrup et al. to unfold the data yielded $\tau \approx 10^{-16}$ sec.

There is other evidence recently published indicating that the value of τ_{π^0} lies near our measured value. Wong has calculated the decay of the π^0 neutral pion by extending the process $\gamma + \pi \rightarrow \pi + \pi$ to $\gamma + \pi \rightarrow \pi + \pi \rightarrow \gamma$.⁴⁰ He treated the dominant intermediate state as that of 2π , rather than $N-\bar{N}$ pairs only—as considered by Goldberger and Treiman.⁴¹ With the parameter $\Lambda = 1.8$ obtained by J. S. Ball (in the paper on the application of the Mandelstam representation to photo-production of pions from nucleons),⁴² and using Wong's calculation, we get $\tau_{\pi^0} \approx 2.2 \times 10^{-16}$ sec.

On the basis of a semiclassical model of the Bohr-Sommerfeld type, Sternglass has investigated the relativistic electron-pair system in the limit of high velocities.⁴³ He showed that a lowest state exists that possesses an energy approximately equal to the π^0 rest energy. The calculated π^0 rest energy is 134.4 Mev, compared to the measured value of 135.0 Mev.⁴⁴ Next, he showed that the lifetime of the system against annihilation into two photons is 2.06×10^{-16} sec, which is in good agreement with the value we have found.

Our experimental value for the π^0 mean life disagrees, however, with that calculated³¹ on the assumption that the π^0 decay is dominated

by the process $\pi^0 \rightarrow \rho^0 + \omega^0$ ³² followed by $\rho^0 \rightarrow \gamma$, $\omega^0 \rightarrow \gamma$. The calculated value of τ_0 for this process is $\approx 6 \times 10^{-18}$ sec. According to this scheme, the branching ratio $(\rho \rightarrow \eta\pi)/(\rho \rightarrow 2\pi)$ is expected to be $\approx 1/4$, whereas the experimental value is $< 1\%$ ⁴⁵ and is consistent with zero.

In the recent experimental data of King⁴⁶ there seems to be some deficiency of Dalitz pairs in the vicinity of the star centers. Within 10μ from the star centers, he observed only three Dalitz pairs, compared with the expected number of 8 pairs. The probability that this disagreement is due to purely statistical fluctuation is about 4%. We believe that the deficiency in the number of observed pairs is mainly owing to the scanning (or measurement) efficiency.

Without determining the points of intersection of all pairs of relativistic particles by a technique such as ours, a large fraction of the pairs will certainly be missed. A bias exists especially against finding pairs of large opening angle.

ACKNOWLEDGMENTS

I am deeply grateful to my teacher and friend, Dr. Walter H. Barkas, for having suggested this problem and for his kind guidance through all phases of my work. I acknowledge with gratitude the interest and encouragement of my former teacher, Professor Robert L. Thornton. My deepest gratitude also goes to Frances M. Smith, who taught me various techniques in my work.

My sincerest thanks go to all the scientific data analysts of the Barkas Physics Research Group for their excellent work in taking and analyzing these data, particularly to Ernestine Beale, Douglas Greiner, Thelma Monzon, Walter Nelson, Arthur Toor, and Hester Yee.

Mr. James Hodges and Mr. Roger Sherman were most helpful for necessary modifications to our precision microscopes and for their maintenance. My sincere appreciation also goes to Mr. Maurilio Antuna and his group for maintenance of the electronic equipment.

I sincerely thank the members of the Barkas Physics Research Group for their cooperation in various stages of this project and for their valuable discussions and advice.

I wish to acknowledge the State scholarship granted to me by the government of the Union of Burma.

This work was done under the auspices of the U. S. Atomic Energy Commission.

APPENDICES

A. Emulsion-Processing Technique

The chemicals and processing technique for the processing of the nuclear emulsion used in this experiment are described in the following table.

Table II. Processing technique used for the 600- μ -thick Ilford K.5 nuclear emulsion. (Prepared by Carl N. Cole.)

Bristol Developer

Distilled water	1000	cc
Sodium sulfite (anhydrous)	7.2	g
Sodium bisulfite	1.0	g
Potassium bromide (10% solution)	8.7	cc
Amidol	3.25	g

Short Stop

Distilled water	1000	cc
Acetic acid (glacial)	2	cc

Fixer

Distilled water	1000	cc
Sodium thiosulfate	300	g
Sodium bisulfite	22.5	g

Processing Times and Temperatures

Presoak	2.5 hours	5°C
Cold developer soak	2.5 hours	5°C
Hot stage	1.0 hour	23 to 25°C
Shortstop	2.5 hours	10°C
Fixer	3 days ^a	10°C
Washing	3 days ^b	10°C

^a Fixing time is approximately time of clearing plus 50%

^b Washing is carried out until a permanganate test shows no hypo present in the last few drops to drain from a plate.

B. Determination of the Optimum Number of Grains for Each Track

We determined the optimum number of grains for each track as follows:

We divided a portion of the track nearest to the interaction point into two equal segments of about 10 to 12 grains each. The grains of each segment are numbered as shown in Fig. 18. With the method described in the text, and using the same "digitized" precision microscope, we did least-squares straight-line fitting to each segment having a certain number of grains in each. The angle α goes through a minimum value for a particular (optimum) number of grains taken for each segment. Figure 19 shows some typical variations of α with the number of grains in each segment. We picked a fair sample of tracks consisting of all types that we measured in our main experiment. The mean optimum number of grains for the sample is six.

C. Digitized Microscope and Components

As described in the text, we modified the stage of the Koristka MS2 precision microscope so that we could measure the relative x as well as y coordinates of the grains by rotating the stage through 90 deg. The analog-to-digital converter (digitizer) is Datex type, yielding 1000 counts per revolution for 100 revolutions. With total magnification of about 3000 $\times$, each count corresponds to 0.0026μ ($1 \mu = 10^{-4} \text{cm}$). Figures 20, 21, and 22 show the microscope with its various components. The associated electronic equipment performs a variety of functions, illustrated by the simple flow diagram in Fig. 23. Provision has been made to use only one translator, one punch-control chassis, and one card punch even when more than one microscope are in operation. More detailed discussions on the whole assembly of the digitized microscopes used in the Barkas physics research group may be found in a thesis by Conrad J. Mason,⁴⁷ and also in a report by Walter H. Barkas.⁴⁸ It takes, on the average, about 10 minutes to measure the coordinates of all the grains for each event in our experiment.

MU-26163

Fig. 18. Drawing to illustrate how optimum number of grains is determined for least-squares straight-line fit to each track.

MU-26164

Fig. 19. Plots of $\tan \alpha$ vs number of grains. See Fig. 18 and text for more discussion.

ZN-3041

Fig. 20. Four-unit assembly of the microscope coordinate readout system. The operator sits at the microscope which is equipped with the "digitizer." There is a foot pedal for punching the IBM cards automatically.

ZN-3042

Fig. 21. "Digitized" precision microscope (Koristka MS2 type).
The encoder assembly is mounted at the top.

ZN-3043

Fig. 22. Indicative information panel. The items of information registered on this panel, as well as the coordinates of a point, are punched on each IBM data card.

MU-26165

Fig. 23. Information flow diagram. The paths of the indicative information and of the coordinate information are represented by the dotted and solid lines, respectively.

D. Derivation of Formulae for the Least-Squares Straight-Line Fit

We consider the least-squares best fit of a straight line to n observed points (x_i, y_i) where both x_i and y_i have errors.

The equation of a straight line through the point (x_t, y_t) is

$$y_t = a + bx_t,$$

where a is the y intercept, b is the slope, and (x_t, y_t) are related to the measured (x_i, y_i) as follows:

$$y_t = y_i + \epsilon_{y_i},$$

$$x_t = x_i + \epsilon_{x_i},$$

where x_i and y_i have errors ϵ_{x_i} and ϵ_{y_i} respectively. We also have (see Fig. 24)

$$\epsilon_i^2 = \epsilon_{x_i}^2 + \epsilon_{y_i}^2, \quad (D-1)$$

$$(\epsilon_{x_i} / \epsilon_{y_i}) = -b, \quad (D-2)$$

and

$$\epsilon_{y_i} = a + bx_i - y_i + b\epsilon_{x_i}. \quad (D-3)$$

From Eqs. (D-1) and (D-2), we obtain

$$\epsilon_{y_i}^2 = \epsilon_i^2 / (1 + b^2). \quad (D-4)$$

and from Eqs. (D-2) and (D-3), we get

$$\epsilon_{y_i} = (a + bx_i - y_i) / (1 + b^2). \quad (D-5)$$

Therefore, Eqs. (D-4) and (D-5) give us

$$\epsilon_i^2 = (a + bx_i - y_i)^2 / (1 + b^2).$$

Summing the above expression over all the n points, we get

$$\sum \epsilon_i^2 = (1 + b^2)^{-1} \left[na^2 - 2a \sum y_i + \sum (y_i)^2 + 2b \left(a \sum x_i - \sum x_i y_i \right) + b^2 \sum (x_i)^2 \right], \quad (D-6)$$

where the summations are from $i = 1$ to $i = n$.

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Fig. 24. Least-squares straight-line fit to a set of points. Only one point (x_t, y_t) in the figure.

We now minimize the expression in (D-6) with respect to the two parameters a and b :

$$(i) \quad \frac{\partial}{\partial a} \left(\sum \epsilon_i^2 \right) = 0$$

gives

$$b = \left(\sum y_i - na \right) / \left(\sum x_i \right), \quad (D-7)$$

and

$$(ii) \quad \frac{\partial}{\partial b} \left(\sum \epsilon_i^2 \right) = 0$$

gives, after using Eq. (D-7) and rearranging terms,

$$Aa^2 + Ba + C = 0,$$

where

$$A = n \sum x_i \sum y_i - n^2 \sum x_i y_i,$$

$$B = 2n \sum y_i \sum x_i y_i + n \sum x_i \left(\sum x_i^2 - \sum y_i^2 \right) - \sum x_i \left[\left(\sum y_i \right)^2 + \left(\sum x_i \right)^2 \right],$$

and

$$C = \sum x_i y_i \left[\left(\sum x_i \right)^2 - \left(\sum y_i \right)^2 \right] + \sum y_i \sum x_i \left(\sum y_i^2 - \sum x_i^2 \right).$$

Therefore, we obtain

$$a = \left[-B \pm (B^2 - 4AC)^{1/2} \right] / 2A. \quad (D-8)$$

From Eq. (D-7), we get

$$b = \left(\sum y_i - na \right) / \sum x_i = (\bar{y} - a) / \bar{x}. \quad (D-9)$$

Since a and b are double valued—the extrema—we must decide which sign to take for the radical in Eq. (D-8), i.e., which sign gives the best fit. We do this by taking one of the observed points (x_i, y_i) , which is away from the mean values, and finding which sign agrees.

Error Calculations

(a) y Intercept and slope. Since x and y are the only independently measured quantities, we have

$$\sigma_a^2 = \sigma_{\bar{y}}^2 + b^2 \sigma_{\bar{x}}^2, \quad (D-10)$$

and

$$\sigma_b^2 = \sigma_a^2 / (\bar{x})^2, \quad (D-11)$$

where

$$\sigma_{\bar{y}}^2 = (1 + b^2)^{-2} \left(\sum \left[y_i - (a + bx_i) \right]^2 \right) \left(n(n-1) \right)^{-1},$$

and

$$\sigma_{\bar{x}}^2 = b^2 \sigma_{\bar{y}}^2.$$

(b) Coordinates of the point of intersection. Subscripts i, j now refer to two straight lines intersecting at (x_{ij}, y_{ij}) , where

$$x_{ij} = \frac{a_j - a_i}{b_i - b_j}, \quad (D-12)$$

and

$$y_{ij} = a_i + b_i x_{ij} = \frac{a_j b_i - a_i b_j}{b_i - b_j}. \quad (D-13)$$

We have

$$\sigma_{x_{ij}}^2 = \left(\frac{\partial x_{ij}}{\partial \bar{x}_i} \right)^2 \sigma_{\bar{x}_i}^2 + \left(\frac{\partial x_{ij}}{\partial \bar{y}_i} \right)^2 \sigma_{\bar{y}_i}^2 + \left(\frac{\partial x_{ij}}{\partial \bar{x}_j} \right)^2 \sigma_{\bar{x}_j}^2 + \left(\frac{\partial x_{ij}}{\partial \bar{y}_j} \right)^2 \sigma_{\bar{y}_j}^2 \quad (D-14)$$

and a similar expression for $\sigma_{y_{ij}}^2$, with y_{ij} replacing x_{ij} in Eq. (D-14).

If we have a function $F(a_i, a_j, b_i, b_j)$, we know that

$$\frac{\partial F}{\partial \bar{x}_i} = \frac{\partial F}{\partial a_i} \cdot \frac{\partial a_i}{\partial \bar{x}_i} + \frac{\partial F}{\partial b_i} \cdot \frac{\partial b_i}{\partial \bar{x}_i}, \quad (D-15)$$

and that similar expressions apply for $\partial F / \partial \bar{y}_i$.

Taking F to be (x_{ij}, y_{ij}) , we get (by Eq. D-15) expressions for $(\partial x_{ij} / \partial \bar{x}_i)$, $(\partial x_{ij} / \partial \bar{y}_i)$, $(\partial y_{ij} / \partial \bar{x}_i)$, and $(\partial y_{ij} / \partial \bar{y}_i)$. Then, using these expressions and Eq. (D-14), we get

$$\sigma_{x_{ij}}^2 = (b_i - b_j)^{-2} [1 + (x_{ij} / \bar{x}_i)]^2 \sigma_{a_i}^2 + (b_i - b_j)^{-2} [1 + (x_{ij} / \bar{x}_j)]^2 \sigma_{a_j}^2,$$

and

$$\sigma_{y_{ij}}^2 = b_j^2 (b_i - b_j)^{-2} [1 + (x_{ij} / \bar{x}_i)]^2 \sigma_{a_i}^2 + b_i^2 (b_i - b_j)^{-2} [1 + (x_{ij} / \bar{x}_j)]^2 \sigma_{a_j}^2,$$

where σ_a^2 and σ_b^2 are given by Eqs. (D-10) and (D-11).

E. Multiple-Scattering Measurement

The method we used for the measurement of multiple scattering of a track is that of the sagitta, the track being aligned approximately parallel to the x axis. The y coordinates are read at regular intervals (cells) of length t . From these readings we obtained the second differences, the mean of which is denoted by $\langle \Delta'' \rangle$, called the sagitta.

The relation between the momentum-velocity ($p\beta$), and the sagitta D is given (for singly charged particles) by

$$p\beta = \frac{K_{co} t^{3/2}}{573 D_{co}}, \quad (E-1)$$

where

$p\beta$ is in Mev/c,

t is the cell length in μ ,

D_{co} is the noise-eliminated sagitta with a particular cutoff procedure applied to eliminate large discrete scatterings,

K_{co} is called the scattering constant; it is slightly dependent on the velocity β and the cell length t . It also depends on the choice of cutoff procedure applied. Some workers preferred to replace all second differences $> 4 \langle \Delta'' \rangle$ by $4 \langle \Delta'' \rangle$, rather than reject all those $> 4 \langle \Delta'' \rangle$. We have used the latter procedure.

Extensive discussions on multiple scattering are given in Chapter 8 of the book by Barkas.⁴⁹

We can eliminate noise with measurements at two different cell lengths. For constant noise with cell length, we have

$$D_{co}'' = \left[\frac{\langle 2t \Delta'' \rangle^2 - \langle \Delta'' \rangle^2}{2^{3.13} - 1} \right]^{1/2}.$$

General distortions may be eliminated by the use of third differences with two cell lengths. When noise is constant with cell, we have

$$D_{co}'' = \frac{\langle 2s \Delta''' \rangle^2 - \langle s \Delta''' \rangle^2}{1.5(2^{3.13} - 1)}^{1/2} \frac{1}{F}; \quad F = 1.06.$$

The value of K_{co} is also dependent on the particular type of cutoff procedure applied to the second differences. We have used the procedure whereby all second differences $> 4 \langle \Delta_{ns}'' \rangle$ are rejected. The variation of K_{co} with β and t is then given by the relations⁴⁷

$$\text{Equivalent cell length } t_1 = t / (0.23 + 0.77\beta^2),$$

and

$$K_{co}^2 = 675 \left[0.090 + 0.272 \log_{10}(5t_1) \right].$$

Conditions set on each track that was measured by multiple scattering were:

- (a) In order that "spurious scattering" effects would not be appreciable, only tracks with dip angles ≤ 6 deg were measured. The momentum spectrum was then corrected for the bias thus introduced.
- (b) The ratio of signal to noise must be at least 2. That is,

$$\langle \Delta'' \rangle_{\text{signal}} \geq 2 \langle \Delta'' \rangle_{\text{noise}}.$$
- (c) The minimum number of cells for each track, satisfying (a) and (b) above, must be 25.

Most of our measurements were made with the nuclear research microscope built by Cooke, Throughton, and Simens, and modified at this laboratory for precision multiple-scattering measurements. Data were recorded with an automatic tape recorder and then punched on IBM cards. An IBM 650 computer was programmed to do all calculations leading to the momentum p of each particle. Checks were often made by having a particular track measured by two trained operators. Some checks were also made by using the Koristka MS2 microscope. Agreements were found to be satisfactory.

F. Angle Measurements

1. Measurement of Space Angles in Nuclear Emulsion

In order to find the space angle between two tracks in nuclear emulsion, we measure (a) the projection of the space angle in the horizontal plane (the plane of the emulsion), and (b) the inclination of each of the two tracks with respect to the horizontal, i. e., the dip angles. The projected angle is found with the aid of a hairline reticle mounted in the focal plane of the microscope ocular, which in turn is clamped to a goniometer affixed to the eyetube. The dip angle of a track is obtained by measuring its vertical displacement after it has traversed a known horizontal distance (usually 100 reticle units, corresponding to about 132μ). The true vertical displacement is obtained when we multiply the measured vertical displacement by a factor called "shrinkage factor." This factor takes into account the fact that contraction occurs after the emulsion is processed. The shrinkage factor, then, as the name suggests, is the ratio of the thickness of the unprocessed emulsion to that of the emulsion after processing. For precision measurements, the shrinkage factor must be determined for each region and plate of the emulsion stack. It varies with the relative humidity, too.

All the above angle measurements were performed on microscopes equipped with $10\times$ oculars and $100\times$ oil-immersion objectives.

The space angle ϕ_{12} between tracks 1 and 2 with projected angle θ_{12} between them, and dip angles δ_1 and δ_2 , is given by the relation

$$\cos \phi_{12} = \cos \delta_1 \cos \delta_2 (\tan \delta_1 \tan \delta_2 + \cos \theta_{12});$$

where

$$\tan \delta_i = \frac{(\Delta Z)_i S_i}{L_i};$$

$(\Delta Z)_i$ being the measured vertical displacement for projected path length L_i , the shrinkage factor for the region being S_i .

2. Angle Between the Plane of the Dalitz Pair and the Emulsion Plane

First we find the angle between the normal to the (e^+ , e^-) plane and the emulsion (horizontal) plane. This angle is called the dip angle of the normal. We refer to the e^+ and e^- by subscripts 1 and 2 respectively, and to the normal by n . We denote the projected (on the emulsion) angles between two lines by θ_{ij} , and the space angle by ϕ_{ij} . We then have

$$\cos \phi_{1n} = 0 = \cos \delta_1 \cos \delta_n (\tan \delta_1 \tan \delta_n + \cos \theta_{1n}), \quad (F-1)$$

$$\cos \phi_{2n} = 0 = \cos \delta_2 \cos \delta_n (\tan \delta_2 \tan \delta_n + \cos \theta_{2n}), \quad (F-2)$$

and

$$\cos \theta_{2n} = \cos \left[\pm(\theta_{12} - \theta_{1n}) \right] = \cos \theta_{12} \cos \theta_{1n} + \sin \theta_{12} \sin \theta_{1n}. \quad (F-3)$$

From Eqs. (F-1) and (F-2), solving for $(-\tan \delta_n)$, we get

$$-\tan \delta_n = \cos \theta_{1n} \cdot \tan^{-1} \delta_1 = \cos \theta_{2n} \cdot \tan^{-1} \delta_2. \quad (F-4)$$

From Eqs. (F-3) and (F-4), we solve for $\cos \theta_{1n}$ in terms of δ_1 , δ_2 , and θ_{12} . We get

$$\cos \theta_{1n} = \sin \theta_{12} \left[(\tan \delta_2 \tan^{-1} \delta_1 - \cos \theta_{12})^2 + \sin^2 \theta_{12} \right]^{-1/2}. \quad (F-5)$$

Term δ_n is then given by Eq. (F-4):

$$\tan \delta_n = \cos \theta_{1n} \tan^{-1} \delta_1.$$

The angle between the plane of the (e^+ , e^-) pair and the emulsion plane is $90 \text{ deg } \delta_n$.

G. Calculation of the Laboratory-System Opening Angle Distribution of Dalitz Pairs

(For all measured values of η)

Figure 25 shows the orientation of the Dalitz decay (e^+ , e^-) in the rest frame of the π^0 (called c.m. below)—all quantities relative to this reference frame have asterisks. Quantities without asterisks refer to the laboratory frame.

We assume that the e^+ in the c.m. system and the π^0 motion in the laboratory system (along x^* axis) lie in the x^*y^* plane. Therefore

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Fig. 25. Orientations of the Dalitz (e^+ , e^-) in the rest frame of the π^0 .

we see that u can have any value between 0 and 2π with equal probability, and $\cos \theta_+^*$ can have any value between -1 and +1 with equal probability.

The relation between $\theta_{\pm}$ and $\theta_{\pm}^*$ is given by

$$\tan \theta = \frac{\sin \theta^*}{\gamma_0 \left[\cos \theta^* + (\beta_0/\beta^*) \right]} \quad (G-1)$$

Since the electron rest energy of approx 1/2 Mev is always much smaller than its total energy in the c.m. system, we can take $\beta^* \approx 1$. We have $\beta_0 \gamma_0 = \eta = p/m$, where p and m are momentum (Mev/c) in laboratory-system and rest energy (Mev) of the π^0 .

Considering the scalar product of the two unit vectors in the laboratory-system along the e^+ and e^- , we have

$$\cos \alpha = \cos \theta_+ \cos \theta_- + \sin \theta_+ \sin \theta_- \cos \phi, \quad (G-2)$$

where $\phi = \phi_- - \phi_+$. We also have, referring to Fig. 25,

$$\sin \theta_-^* \sin \phi_-^* = \sin \alpha^* \sin u^*. \quad (G-3)$$

With $\phi_+ = \phi_+^* = 0$, we have

$$\phi_- = \phi_-^* = \phi = \phi^*. \quad (G-4)$$

Therefore, from Eqs. (G-2), (G-3), and (G-4), we get

$$\cos \alpha = \cos \theta_+ \cos \theta_- \pm \sin \theta_+ \sin \theta_- \left[1 - \frac{\sin^2 \alpha^* \sin^2 u^*}{\sin^2 \theta_-^*} \right]^{1/2}. \quad (G-5)$$

Also, we have

$$\cos \alpha^* = \cos \theta_+^* \cos \theta_-^* + \sin \theta_+^* \sin \theta_-^* \left[1 - \frac{\sin^2 \alpha^* \sin^2 u^*}{\sin^2 \theta_-^*} \right]^{1/2}. \quad (G-6)$$

From Eq. (G-6), we get

$$\cos \theta_-^* = \cos \alpha^* \cos \theta_+^* \pm \cos u^* \sin \theta_+^* \sin \alpha^*. \quad (G-7)$$

For simplicity, let us call $\cos \theta_-^* = a$. Then, from Eqs. (G-1), (G-5), and (G-7), we get

$$\cos \alpha = \pm \left[(1+f)(1+g) \right]^{-1/2} \pm \left[1 - (1+f)^{-1} \right]^{1/2} \left[1 - (1+g)^{-1} \right]^{1/2} \\ \times \left[1 - (1-a^2)^{-1} (\sin^2 \alpha^* \sin^2 u^*) \right]^{1/2}, \quad (G-8)$$

where

$$f = \sin^2 \theta_+^* \left[(1+\eta^2)^{1/2} \cos \theta_+^* + \eta \right]^{-2},$$

and

$$g = (1-a^2) \left[(1+\eta^2)^{1/2} a + \eta \right]^{-2}.$$

To find the theoretical distribution of α , we proceed as follows:

As mentioned above, the distributions in $\cos \theta_+^*$ and u^* are isotropic from -1 to +1 and from 0 to 2π , respectively. The weighting factor for each α corresponding to each set of α^* , u^* , η , and $\cos \theta_+^*$ is simply

$$f(\eta) \exp(-\alpha^*/a^*) a^{*-1},$$

where $f(\eta)$ is the measured momentum spectrum described in the text. The $a^{*-1} \exp(-\alpha^*/a^*)$ is an empirical fit to the theoretical opening-angle distribution of the (e^+, e^-) pair, as calculated by Dalitz.^{4, 10} In the text this distribution is denoted by $q(0, \alpha)$. According to D. W. Joseph (quoted in reference 9), we have taken the distribution to approach zero linearly from $\alpha^* \approx 100$ to 180 deg. With 18.1 deg for the median angle,¹⁰ we obtained $a^* \approx 1.8$ deg.

In our calculation, we first picked eight equal intervals for each of the four variables α^* , u^* , η and $\cos \theta_+^*$. To improve the calculation we next picked, for each, another set of intervals that lie halfway between the previous set.

Physically, the $\pm$ signs mean that the pair can be in any of the octants. We know experimentally that the majority of the opening angles have $\cos \alpha \geq 0$.

In our calculation, we took all possible combinations of the + and - signs, and let the four variables take all possible values at equal intervals (as described above).

We have seen that the theoretical distribution of laboratory-system opening angles (for all values of η) is not much different from that in the rest frame of the π^0 . This fact enables us to assume that the distribution

of laboratory-system opening angles falls as $a^{-(1+\epsilon)}$ for large angles, where ϵ is a function of η . Using the calculations above, we can determine (as a first approximation) ϵ and $a_{1/2}$ as functions of η . The $a_{1/2}$ is the median laboratory-system opening angle for each η . Next, to take care of the distribution at small angles (for the limited region of η in our calculation), we assume the laboratory-system distribution of opening angles to be

$$q(a, \eta) = (\exp -a/a) a^{-(1+\epsilon)} da,$$

where a is a function of η . This distribution reduces to the distribution

$$(\exp -a^*/a^*) a^{*-1} da^*$$

for $\eta = \epsilon = 0$.

With the previously obtained values of $a_{1/2}$ and ϵ as functions of η , we can find the values of a (as a function of η also) from the relation

$$\int_{a_{1/2}}^{a_{\max}} \left[\exp(-a/a) \right] a^{-(1+\epsilon)} da = \int_0^{a_{1/2}} \left[\exp(-a/a) \right] a^{-(1+\epsilon)} da.$$

We took $a_{\max} \approx 100$ deg. Beyond this angle we have experimentally observed none. Any deviations from the results for $a_{\max} = 180$ deg will be just second- or third-order corrections. Moreover, we are using the opening-angle distribution for the correction of bias on those events with small opening angles.

Iterated processes then give us the best values of ϵ and a as functions of η . Their computed values for various values of η are given in Table III.

Table III. Computed values of ϵ and a for various values of η .		
η	ϵ	$a(\text{deg})$
0.0	0	1.7
3.0	0.006	1.0
6.0	0.012	0.6
9.0	0.018	0.4
12.0	0.024	0.22
15.0	0.030	0.13
18.0	0.036	0.08
21.0	0.042	0.05
$(\epsilon \approx 0.002 \eta) \quad a \approx 1.7 \exp(-0.17\eta)$		

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