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How Does the Bond Market Perceive Macroeconomic Risks under Zero Lower Bound?

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How Does the Bond Market Perceive Macroeconomic Risks under Zero Lower Bound?

A dissertation submitted in partial satisfaction
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by

Yuji Sakurai

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Abstract of the Dissertation

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by

Yuji Sakurai

Doctor of Philosophy in Management

University of California, Los Angeles, 2016

Professor Mikhail Chernov, Co-chair

Professor Francis A. Longstaff, Co-chair

I present a joint model of yield curves and macroeconomic variables with an explicit effective zero lower bound by employing the concept of shadow interest rates. Bond yields are derived by assuming no arbitrage opportunities. However, they are not affine due to the zero lower bound. I thus develop a new approximate bond pricing formula that is correct up to a second order. To describe macroeconomic dynamics, I employ a standard New Keynesian macroeconomic model and estimate the model parameters for the US and Japan.

In the first chapter, I conduct three different types of counterfactual analyses of monetary policy. First, I evaluate a counterfactual analysis of raising the target inflation level. For both the US and Japan, I find that a higher inflation target steepens the yield curve when the current policy interest rate is not constrained by the zero lower bound. On the other hand, a higher inflation target increases long-term nominal yields while keeping short-term nominal yields unchanged when the current policy interest rate is constrained by the zero lower bound. Second, I study the effect of suddenly ending the zero interest rate policy. Third, I examine the impact of introducing a negative interest rate on the bond markets and the macro economy.

In the second chapter, I investigate whether the empirical findings documented before the zero lower bound period holds during the zero lower bound period. For example, I study how macroeconomic risks impact the shape of yield curves by looking at their decompositions and their factor loadings.

In the third chapter, I conduct two additional exercises. First, I incorporate a Markov regime switching feature into a New Keynesian macro finance model with the zero lower bound for nominal bond pricing. Second, I study the excess sensitivity of long-distant real forward interest rates to changes in the short-term nominal interest rate using a dataset of Japanese fixed income investors.

The dissertation of Yuji Sakurai is approved.

Andrew Granger Atkeson

Eduardo S. Schwartz

Roger E. Farmer

Francis A. Longstaff, Committee Co-chair

Mikhail Chernov, Committee Co-chair

University of California, Los Angeles

2016

To my wife and my parents

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Sakurai, Yuji, Yoshihiko Uchida, 2014. "Rehypothecation dilemma: Impact of collateral rehypothecation on derivative prices under bilateral counterparty credit Risk," *Journal of Banking and Finance*, 48, 361-373.

Baba, Naohiko, Yuji Sakurai, "When and how US dollar shortages evolved into a full crisis: Evidence from the cross-currency swap market," *Journal of Banking and Finance*, 35, 1450-1463.

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CHAPTER 1

New Keynesian macro finance model with the zero lower bound

1.1 Introduction

Central banks in developed countries have been confronting the zero lower bound and inflation that is below their targets over several years. A central bank can end the zero interest rate policy any time it wants. Yet, such a sudden interest rate hike could be contractionary to the economy. As an alternative, some prominent policy makers have proposed that a central bank should raise the inflation target level to generate a higher expected inflation and thus increase the future nominal interest rate.[1] They argue that higher expected nominal interest rates ease the zero lower bound problem.[2] This chapter evaluates the merit of this approach.

A common approach in describing interactions between various macroeconomic variables and how they are impacted by policy shocks is to estimate Vector Autoregressions (VARs). However, when an economy is constrained by the zero lower bound, such a method is not applicable for two reasons. First, a policy interest rate is constant and therefore difficult to directly incorporate into VARs. Second, evaluating a policy that has never been implemented requires a counterfactual

[1] See Blanchard et al. (2010), McCallum (2011), Ball and Mazumder (2011), Ball (2013), Williams (2014), Krugman (2014), Aruoba and Schorfheide (2015) for reference.

[2] For instance, Krugman (2014) argues "Escaping from this feedback loop appears to require more radical economic policies than are likely to be forthcoming. As a result, a relatively high inflation target in normal times can be regarded as a crucial form of insurance, a way of foreclosing the possibility of very bad outcomes."

analysis, which needs a structural model.

With respect to the first issue, the term structure of interest rates contains the information about the future policy interest rate that helps us describe the relationship between interest rates and macroeconomic variables even under the zero lower bound. I extract a *shadow* policy interest rate that is equal to the nominal policy interest rate as long as that rate is above the zero lower bound. The concept of a shadow interest rate was developed by Black (1995). In this research, I generalize the original definition of the shadow interest rate to allow for a non-zero lower bound and estimate it using data. The approach of a shadow interest rate is a parsimonious way to allow a short-term nominal rate to stay at zero for a certain period of time. The shadow interest rate can be easily included as a proxy for an actual policy interest rate in specifying the joint dynamics of interest rates and macroeconomic variables.

To resolve the second issue, I assume that macroeconomic variables are described by a standard New Keynesian model that is widely used among policy makers. I take the zero lower bound into account in the New Keynesian model. This feature allows us to conduct a counterfactual analysis such as raising the inflation target and introducing a negative interest rate on central bank's reserve. It also naturally captures a possible change in macroeconomic dynamics when an economy is constrained by the zero lower bound.

Specifically, I develop and estimate a joint model of yield curves and macroeconomic variables with an explicit modeling of the zero lower bound both in nominal bond pricing and macroeconomic dynamics. There are four state variables: real GDP, potential GDP, inflation and the shadow interest rates. These variables follow a VAR (1) system with time-varying coefficient matrices.

I employ a no-arbitrage term structure model of interest rates that is flexible enough to capture the cross-section of nominal bond yields before and during the zero interest rate policy. Since the model is non-affine to the state variables due to the zero lower bound, I develop a second order approximation for pricing bonds. The approximation reflects the optionality arising from the zero lower bound. The no-arbitrage restrictions allow us to transform the private sector's expectations about macroeconomic variables into the bond market's expectations contained in yield curves while keeping the joint model internally consistent. Once the no-arbitrage term structure model is estimated with the cross-section of nominal bond yields, one can decompose nominal bond yields into the path of expected nominal interest rates and the term premium. The expected nominal interest rate is a key input in a standard New Keynesian model as I will explain below. For simplicity, I assume a linear market price of risk.[3]

Four equations describe the standard New Keynesian model. The first one is the Investment-Saving (IS) equation that determines the relationship between expected nominal interest rates, expected inflation and real output. The IS equation is derived from an inter-temporal consumption Euler equation. The zero lower bound is explicitly modeled in the IS equation. The other three equations follow a standard New-Keynesian model. First, the shadow interest rate mean-reverts to a policy target level that is specified by a Taylor rule. Second, current inflation depends on lagged inflation, expected inflation and the output gap. Third, potential GDP is assumed to be exogenously driven and mean-reverting. These structural assumptions are widely used in macroeconomic studies and made for conducting counterfactual analyses. They also facilitate the interpretation and estimation of the parameters in the VAR (1) system.

I apply my model of yield curves and macroeconomic variables for the US and Japan. I jointly estimate parameters of a standard New Keynesian model and market price of risk parameters. Using market price of risk parameters, I

[3]In general, it is difficult to match nominal bond yields generated from a standard macroeconomic model to actual data because of undervaluation of term premium. To fix this issue, one needs more sophisticated utility functions such as Epstein-Zin preferences documented by Rudebusch and Swanson (2012). Yet, these more sophisticated utility functions are then difficult to incorporate into the term structure model with the zero lower bound. Thus, I employ a simple no-arbitrage term structure model with the linear market price of risk.

compute nominal bond prices with the zero lower bound and fit model-implied nominal bond yields to actual yields with some measurement errors. The parameters of a standard New Keynesian model are estimated to capture the dynamics of shadow interest rates and other macroeconomic variables in the physical measure.

After estimating the model parameters, I conduct different types of counterfactual analyses of the interaction between interest rates and macroeconomic variables given a change in monetary policy. The estimated term structure model also provides some interesting results such as long-run real interest rates and an evolution of the shadow interest rates during the zero lower bound period.

Not surprisingly, extracted shadow interest rates for the US show that the effective (zero) lower bound became binding in 2009. It is approaching the bound recently, again. In the case of Japan, the shadow interest rates have been negative since the fourth quarter of 2008 and became more negative since 2014 as the long-term nominal bond yields are further lowered by the Bank of Japan's QQE (Quantitative and Qualitative Easing). When the long-run real interest rate is computed as a mean-reverting level of the real shadow interest rate with zero output and inflation gaps, it is equal to 1.02% in the US during the 1991-2015 period and it is -2.60% in Japan during the 2004-2015 period.

I conduct a counterfactual analysis of raising the target inflation level with negative shadow interest rates for the initial time period. Recall that a negative current shadow interest rate means that the zero lower bound is currently binding. In both the US and Japan, a higher inflation target increases long-term nominal bond yields while keeping short-term yields unchanged. For comparison, I also study the case when the zero interest rate policy is suddenly abandoned. Given the sudden termination of the zero interest rate policy, the long-term nominal bond yields do not increase as much as those in the case of higher inflation target for the US. Plus, I examine the response of real output and inflation given a higher inflation target and suddenly ending the zero interest rate policy. I find that a sudden ending of the zero interest rate policy is contractionary while raising the inflation target is expansionary. In this respect, raising the inflation target is preferred as it is expansionary.

It is also possible to conduct other types of counterfactual analyses. For example, I evaluate an introduction of a negative lower bound. I consider the case when a current shadow interest rate is negative. I document that it steepens the nominal yield curve by allowing the short-term nominal yields to be negative and by increasing the long-term nominal yields.

The rest of the first chapter is structured as follows. Section 1.2 provides a literature review. Section 1.3 describes a joint model of macroeconomic and term structures dynamics with an explicit zero lower bound. Section 1.4 discusses the datasets used and the estimation methodology employed in this study. Section 1.5 shows the main results. Section 1.6 concludes.

1.2 Literature review

The two strands of the literature are closely related to the model I developed in this paper. I briefly review relevant papers in each strand in this section. The list of the papers is not exhaustive but chosen based on relevance.

1.2.1 Shadow interest rate models

One theoretical contribution of this research is modeling yield curves under zero lower bound. After the Federal Reserve lowered its policy rate to zero, there have been many studies on modeling the zero lower bound in the term structure of interest rates. As an early study, Longstaff (1992) examines the CIR term structure model and discusses a sticky boundary behavior of interest rates. Black (1995) interprets the nominal rate as an option on a hypothetical interest rate called a shadow interest rate.[4] Gorovoi and Linetsky (2004) revisit Black (1995)'s work and obtain analytical expressions for yield curves when a shadow interest rate follows Vasicek process or CIR process. Equipped with this analytical formula, Ueno, Baba and Sakurai (2006) calibrate the term structure model of shadow interest rates to the Japanese government bond markets and find that it fits better than the conventional Vasicek model. Oda and Ueda (2007) is an early study of macro-finance term structure models using a concept of a shadow interest rate. As in this paper, they assume that a shadow interest rate is specified by the Taylor rule and find that the Bank of Japan's unconventional monetary policy functioned through the zero interest rate commitment. More recently, Kim and Singleton (2012) develop a two-factor model of shadow interest rates and estimate their model parameters using extended Kalman filtering. They show that the model outperforms conventional affine counterparts and quadratic Gaussian models. Christensen and Rudebusch (2015) and Ichiue and Ueno (2013) estimate two-factor shadow interest rate models based on an approximation developed by Krippner (2013a,b). Bauer and Rudebusch (2013) employ a simulation-based formula for bond pricing. Wu and Xia (2014) approximate forward interest rates with zero lower bound in a discrete-time farmework. Priebsch (2013) develops a cumulant-based technique to approximate bond yields under Gaussian shadow interest rate models. [5] Lombardi and Feng (2014) estimate a dynamic factor model using a dataset up until the time when the Federal Reserve started the zero interest rate policy and extrapolate a shadow policy rate during the zero lower bound period. Yet, none of those studies develop multi-factor shadow interest rate models with an explicit modeling of the effect of the zero lower bound

[4]Bomfim (2003) estimates the probability that the federal funds rate hits the zero lower bound. Yet, he employs a conventional affine model for that.

[5]Imakubo and Nakajima (2015) also employ a shadow interest rate model to decompose a nominal yield into an expected real interest rate, real rate risk premium, expected inflation and the inflation risk premium but they use first order approximation and abstract the convexity effect.

on macroeconomic dynamics.[6] My approximation formula is related to Krippner (2013a,b) in which he applies an option-based approximation for nominal bond prices. A technical contribution of this research is that it develops a second-order approximation for bond pricing with the zero lower bound by solving a system of partial differential equations. The approximation is interpreted as the convexity adjustment based on the delta of the option arising from zero lower bound. I provide a formal proof as well as an intuitive derivation in the Appendix of this chapter.

1.2.2 New Keynesian macro finance models

The empirical part of this research contributes to testing term structure models that explicitly incorporate structural macroeconomic dynamics. In these models, forward-looking agents optimize their behavior. An evolution of real output is described by the Investment-Saving (IS) equation derived from an intertemporal consumption Euler equation. An evolution of inflation is determined by the Aggregate-Supply (AS) equation. To the best of my knowledge, Hördhal, Tristani and Vestin (2006) and Wu (2006) pioneer the literature by incorporating a New Keynesian macro framework into a no-arbitrage affine term structure model. Hördhal, Tristani and Vestin (2006) find that forecasting performance of their model is superior to affine counterparts without structural macroeconomic dynamics. In a similar framework, Wu (2006) reports that the slope factor of the yield curve is driven by monetary policy shocks while the level factor is explained by technology shocks. Bakaert, Cho and Moreno (2010) further extend the model

[6] There are several other approaches to impose the zero lower bound on nominal interest rates. The first one is directly modeling a diffusion process of a discount bond or pricing kernel as it is done by Jin and Glasserman (2001). The second one is using CAR process developed by Gourieroux *et al.* (2014) and Monfort *et al.* (2015). The third one is modeling the zero interest rates as one specific state triggered by some stochastic process as in Kabanov, Kijima and Rinaz (2006). These different approaches have their own advantages and disadvantages. Yet, the approach of the shadow interest rate is easy to integrate into a structural macroeconomic model.

in Hördhal, Tristani and Vestin (2006) by modifying its pricing kernel to be consistent with the IS equation. In this respect, the model estimated in this research is very close to theirs. However, they assume constant risk premia. Thus, it is difficult to understand the effect of monetary policy on the expected path of nominal policy interest rates and the term premium separately. The other closely related papers are Bikbov and Chernov (2013), Dew-Becker (2014) and Kung (2015). Bikbov and Chernov (2013) incorporate a log-linearized New Keynesian model with regime switching into a no-arbitrage affine term structure model. Dew-Becker (2014) and Kung (2015) also construct term structure models with New Keynesian macroeconomic dynamics and solve their forward-looking macroeconomic models using higher-order approximations. One difference from these three papers is an explicit consideration of zero lower bound that is important to analyze a recent behavior of interest rates. Campbell, Pflueger and Viceira (2015) study the impacts of monetary policy rules and macroeconomic shocks on nominal bond risks by employing New Keynesian macroeconomic term structure models. They conduct a counterfactual analysis and find that nominal bond risk increases after 1977 due to a more anti-inflationary stance. In this paper, I primarily focus on implications of the zero lower bound for term structure and macroeconomic dynamics. To do so, I employ a textbook-style New Keynesian model since solving a large-scale forward-looking macroeconomic model with the zero lower bound is computationally difficult.

1.3 Model

I develop a joint model of yield curves and macroeconomic variables with the zero lower bound in this section. First, I explain the building blocks of a textbook-style New Keynesian macroeconomic model. I show that the New Keynesian model has a VAR (1) representation. For expository simplicity, I first explain the model without the zero lower bound and then discuss the case with an explicit zero lower bound.

Second, I explain a no-arbitrage term structure model of interest rates with an explicit modeling of the zero lower bound. The dynamics of state variables in the physical measure are modeled as a VAR(1). The coefficient matrices of VAR (1) is determined by the New Keynesian model. Given the physical measure process, I need to specify the market price of risk for pricing nominal bonds. I employ a linear market price of risk. I then discuss the approximation to price the bonds with the zero lower bound.

1.3.1 Structural New Keynesian macroeconomic dynamics

I rely on a simple New Keynesian macroeconomic model. Variants of this model are widely used for macroeconomic analyses. The model has micro foundations in the sense that it can be derived by assuming forward-looking households and profit-maximizing firms. Thus, it is suitable for conducting counterfactual analyses of monetary policy.

The main reason to adopt a structural model is to conduct a counterfactual analysis. Yet, there are some other advantages. First, a stylized structural macroeconomic model helps us interpret estimated parameters. Second, it reduces the number of model parameters to be estimated.[7] I admit that the model may be too simple to describe a complicated behavior of an economy. However, this is the first step towards a joint modeling of term structure and structural macroeconomic dynamics with the zero lower bound. The model can be easily extended if one is interested in some other aspects.

In what follows, I explain each building block of the model in detail. There

[7] There are $2 \times 4 \times 4 = 32$ parameters in the VAR coefficient F and the matrix H. The vector G has 4 parameters. In total there are 36 parameters. When we allow the time to exit from the zero lower bound affects VAR coefficients, F, G, H, in macroeconomic dynamics, these VAR coefficients become time-varying so that the number of model parameters increase dramatically and makes their direct estimation difficult.

are four key equations: an inflation equation, policy rule equation, and two output equations for real and potential output. I show how the structural model is represented as a reduced VAR form.

First, let us describe how inflation evolves over time. As in textbook-style New Keynesian models, current inflation is determined by three components: an expected inflation $\mathrm{E}_t[\pi_{t+1}]$, lagged inflation π_{t-1} as well as the output gap $\Delta y_t = y_t - y_t^n$.

$$\pi_t \;=\; \mu_\pi \mathrm{E}_t[\pi_{t+1}|I_t] + (1-\mu_\pi)\pi_{t-1} + \kappa(y_t - y_t^n) + \epsilon_{AS,t}, \tag{1.1}$$

where $\epsilon_{AS,t}$ is sampled from a normal distribution $N(0, \sigma_{AS})$. This equation is often called the Aggregate-Supply (AS) equation or as the New Keynesian Phillips curve. The information set I_t is defined as $I_t = \{y_t, \pi_t, x_t, y_t^n\}$. Real output is denoted with y_t. Potential output is denoted with y_t^n. In my empirical analysis, I use real GDP and potential GDP as a proxy for y_t and y_t^n, respectively.

This inflation equation (1.1) is derived as a first-order condition of the monopolistically competitive firms' optimal price setting. In the right hand side of the equation, output gap $y_t - y_t^n$ measures real marginal costs for the firms. The dependence of current inflation on the lagged inflation is motivated by empirical studies of the dynamics of inflation. It captures the degree of backward-looking behavior of the firms or nominal price indexation.

Second, I assume that a central bank determines its policy target by following a Taylor rule with current real output y_t, potential output y_t^n and inflation π_t.

$$x_t^{target} \;=\; i_t^* + \gamma_y(y_t - y_t^n) + \gamma_\pi(\pi_t - \bar{\pi}), \tag{1.2}$$

where i_t^* is a policy-neutral nominal interest rate. The sum of the second and the third terms reflects the central bank's adjustment of its target interest rate. I assume that the policy-neutral nominal interest rate i_t^* is decomposed into two components.

$$i_t^* = r^* + \pi_t, \tag{1.3}$$

where r^* is constant. I interpret r^* as the long-run real interest rate or an equilibrium real interest rate. I set a target inflation rate $\bar{\pi} = 0.02$ in my empirical analysis since there are two constant terms, r^* and $\gamma_\pi \bar{\pi}$. 2% inflation target is realistic in both the US and Japan. [8]

A shadow interest rate mean-reverts to the target shadow interest rate x_t^{target}.

$$x_t = \mu_x x_t^{target} + (1 - \mu_x) x_{t-1} + \epsilon_{x,t}, \tag{1.4}$$

where $\epsilon_{x,t}$ is sampled from a normal distribution $N(0, \sigma_x)$. The speed of mean reversion μ_x is smaller than one and captures the fact that a central bank gradually changes its policy interest rate to a desired target interest rate. Empirical studies on the US Treasury bonds often document μ_x to be very close to zero and thus a policy interest rate is close to a random walk.[9]

Third, potential output y_t^n follows AR(1) process.

$$y_t^n = \mu_{y^n} \bar{y^n} + (1 - \mu_{y^n}) y_{t-1}^n + \epsilon_{y^n,t}, \tag{1.5}$$

where $\epsilon_{y^n,t}$ is sampled from a normal distribution $N(0, \sigma_{yn})$. Notice that this specification allows both (1) a very persistent process as μ_{y^n} goes to zero and (2) a very fast mean-reverting process as μ_{y^n} goes to 1. The former case is relevant to US while the latter is relevant to Japan. The equation (1.5) is also employed by Bakaert, Cho and Moreno (2010).

Finally, I close the model by introducing one more equation for real output and a real interest rate. The equation is called Investment-Saving (IS) equation. I consider two different specifications. The first one is

$$[\textit{IS-ZLB}] \quad y_t = \alpha_{IS} + \mu_y^+ \mathrm{E}_t[y_{t+1}|I_t] + (1 - \mu_y^-) y_{t-1} - \phi(i_t - \mathrm{E}_t[\pi_{t+1}|I_t]) + \epsilon_{IS,t}, \tag{1.6}$$

[8]The Bank of Japan announced that they will strictly target 2% inflation in April of 2013. Before that, the target inflation was not explicitly stated but Masaaki Shirakawa, the former governor of the Bank of Japan, stated that the inflation consitent with the price stability is in a positive range of 2 percent or lower. See Shirakawa (2012).

[9]See Clarida, Galí and Gertler (2000) for reference.

where $\epsilon_{IS,t}$ is sampled from a normal distribution $N(0, \sigma_{IS})$. [10] Notice that a current real output depends on both the expected real output and the lagged real output. This type of the IS equation is derived as a first-order condition of a utility-maximizing representative agent with external habit formation. The parameter μ_y^- measures the degree of the agent's external habit. I call (1.6) IS-ZLB. The second one is given by

$$[\textit{IS-SR}] \quad y_t = \alpha_{IS} + \mu_y^+ \mathrm{E}_t[y_{t+1}|I_t] + (1 - \mu_y^-)y_{t-1} - \phi(x_t - \mathrm{E}_t[\pi_{t+1}]) + \epsilon_{IS,t}. \tag{1.7}$$

I call the specification above IS-SR. [11] The only difference between IS-ZLB and IS-SR is that a nominal policy interest rate i_t is replaced with a shadow policy interest rate x_t in IS-SR.

The reason why I consider these two specifications is as follows. Recall that a nominal interest rate (policy interest rate) is a non-linear function of the shadow policy interest rate.

$$i_t = \max(x_t, \bar{i}), \tag{1.8}$$

where the max operator arises from the effective (zero) lower bound on nominal interest rates. Substituting (1.8) for (1.6), one obtains a nonlinear forward-backward-looking equation.

$$y_t = \alpha_{IS} + \mu_y^+ \mathrm{E}_t[y_{t+1}|I_t] + (1 - \mu_y^-)y_{t-1} - \phi(\max(x_t, \bar{i}) - \mathrm{E}_t[\pi_{t+1}|I_t]) + \epsilon_{IS,t}. \tag{1.9}$$

A non-linearity arising from the zero lower bound makes it difficult to apply a conventional method to solve forward-backward equations. In this respect, the IS-SR case is more tractable than the IS-ZLB case. I first explain how a VAR (1) form is obtained from the structural New Keynesian macroeconomic dynamics in

[10] ZLB is the abbreviation for zero lower bound.
[11] SR is the abbreviation for a shadow interest rate.

the IS-SR case for expository simplicity. I will discuss the IS-ZLB case in the next subsection. I focus on the IS-ZLB case in my empirical analysis. Notice that the zero lower bound in nominal bond pricing is not abstracted in the IS-SR case even if the max operator in (1.9) is dropped.

There can be another reason to consider the IS-SR case in addition to its tractability. Suppose that a shadow interest rate x_t is below the effective lower bound $\bar{i}$ at time t. In the IS-ZLB case, (1.6) is reduced to

$$y_t = \alpha_{IS} + \mu_y^+ \mathrm{E}_t[y_{t+1}|I_t] + (1-\mu_y^-)y_{t-1} - \phi(\bar{i} - \mathrm{E}_t[\pi_{t+1}]) + \epsilon_{IS,t}. \quad (1.10)$$

There is no term involving a current shadow interest rate x_t in (1.10). Thus, a current shadow interest rate does not have any *direct* impact on current real output although it may have an indirect impact on future real output ($\mathrm{E}_t[y_{t+1}]$) by changing the future path of the nominal policy rate. In IS-SR case, a current shadow interest rate has a *direct* impact on current real output. Recall that a more negative shadow interest rate leads to lower long-term nominal bond yields. One can interpret the IS-SR case as a parsimonious way to capture a possible relationship between *current* long-term interest rates and current real output when the zero lower bound is binding.

Combining these four equations of macroeconomic dynamics, an entire system is described as follows. For notational simplicity, I write $\mathrm{E}_t[y_{t+1}|I_t]$ and $\mathrm{E}_t[\pi_{t+1}|I_t]$ as $\mathrm{E}_t[y_{t+1}]$ and $\mathrm{E}_t[\pi_{t+1}]$, respectively.

$$y_t = \alpha_{IS} + \mu_y^+ \mathrm{E}_t[y_{t+1}] + (1-\mu_y^-)y_{t-1} - \phi(x_t - \mathrm{E}_t[\pi_{t+1}]) + \epsilon_{IS,t}, \quad (1.11)$$

$$\pi_t = \mu_\pi \mathrm{E}_t[\pi_{t+1}] + (1-\mu_\pi)\pi_{t-1} + \kappa(y_t - y_t^n) + \epsilon_{AS,t}, \quad (1.12)$$

$$x_t = \mu_x\left(r^* + \pi_t + \gamma_y(y_t - y_t^n) + \gamma_\pi(\pi_t - \bar{\pi})\right) + (1-\mu_x)x_{t-1} + \epsilon_{x,t}, \quad (1.13)$$

$$y_t^n = \mu_{y^n}\bar{y^n} + (1-\mu_{y^n})y_{t-1}^n + \epsilon_{y^n,t}, \quad (1.14)$$

Recall that IS-SR (1.11) is replaced with IS-ZLB (1.6) when the zero lower bound is explicitly modeled in the New Keynesian macroeconomic dynamics. The system of these four equations above is standard in the New Keynesian literature and almost the same as the one employed in Gürkaynak, Sack and Swanson (2005) and Hördhal, Tristani and Vestin (2006) except that (1) they abstract the zero lower bound and (2) they do not have the equation (1.14) and model output gap $\Delta y_t (= y_t - y_t^n)$ directly.[12] In this paper, I separately treat the two equations, (1.11) and (1.14), to make it clear that the zero lower bound arises in the IS equation for real output, not for the output gap in the IS-ZLB case. [13] The other closely related papers such as Bakaert, Cho and Moreno (2010) and Campbell, Pflueger and Viceira (2015) also abstract the zero lower bound. Precisely speaking, Bakaert, Cho and Moreno (2010) have one additional equation to associate the long-run expected inflation with the inflation target. Similarly, Campbell, Pflueger and Viceira (2015) assume that the inflation target follows a random walk and describe the macroeconomic dynamics by the four equations. To keep my macroeconomic model as simple as possible, I employ the four equations described above.

The IS equation (1.11) and the AS equation (1.12) are rewritten as

$$-\mu_y^+ \mathrm{E}_t[y_{t+1}] - \phi \mathrm{E}[\pi_{t+1}] = -y_t - \phi x_t + (1-\mu_y^-) y_{t-1} + \alpha_{IS} + \epsilon_{IS,t}, \tag{1.15}$$

$$-\mu_\pi \mathrm{E}_t[\pi_{t+1}] = -\pi_t + (1-\mu_\pi)\pi_{t-1} + \kappa(y_t - y_t^n) + \epsilon_{AS,t}. \tag{1.16}$$

The policy rule equation (1.13) and the potential output equation (1.14) are also rearranged as

$$0 = \mu_x \gamma_y y_t + \mu_x (1+\gamma_\pi)\pi_t - x_t - \mu_x \gamma_y y_t^n + (1-\mu_x) x_{t-1} + \mu_x r^* - \mu_x \gamma_\pi \bar{\pi} + \epsilon_{x,t}, \tag{1.17}$$

$$0 = -y_t^n + \mu_{y^n} \bar{y^n} + (1-\mu_{y^n}) y_{t-1}^n + \epsilon_{y^n,t}. \tag{1.18}$$

The four equations, (1.15), (1.16), (1.17) and (1.18) are represented as a VAR (1) system:

$$A E_t[X_{t+1}] = B X_t + C X_{t-1} + D + \epsilon_t, \tag{1.19}$$

[12]A textbook-style New Keynesian model is often represented by the first three equations: the IS equation for output gap, the AS equation and a policy rule equation.

[13]Recall that the potential output y_t^n follows AR(1) process in this research.

where $X = (y_t, \pi_t, x_t, y_t^n)^T$ and $\epsilon_t = (\epsilon_{IS,t}, \epsilon_{AS,t}, \epsilon_{x,t}, \epsilon_{y^n,t})^T$. Specifically, coefficient matrices A and B are given by

$$A = \begin{pmatrix} -\mu_y^+ & -\phi & 0 & 0 \\ 0 & -\mu_\pi & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 & -\phi & 0 \\ \kappa & -1 & 0 & -\kappa \\ \mu_x\gamma_y & \mu_x(1+\gamma_\pi) & -1 & -\mu_x\gamma_y \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{1.20}$$

Matrix C is specified as

$$C = \begin{pmatrix} 1-\mu_y^- & 0 & 0 & 0 \\ 0 & 1-\mu_\pi & 0 & 0 \\ 0 & 0 & 1-\mu_x & 0 \\ 0 & 0 & 0 & 1-\mu_{y_n} \end{pmatrix}. \tag{1.21}$$

Vector D is given by

$$D \;=\; (\alpha_{IS}, 0, \mu_x(r^* - \gamma_\pi\bar{\pi}), \mu_y\bar{y^n})^T. \tag{1.22}$$

In the literature of a structural New Keynesian macroeconomics, the techniques to solve the VAR system are widely known. So I briefly review how to solve (1.19). Let us assume the solution of (1.19) is represented as

$$X_{t+1} = FX_t + G + H\epsilon_t. \tag{1.23}$$

Substituting (1.23) for (1.19) yields

$$X_t = (AF - B)^{-1}(CX_{t-1} + D - AG + \epsilon_t). \tag{1.24}$$

Applying the method of undetermined coefficients, one obtains

$$F = (AF - B)^{-1}C, \tag{1.25}$$

$$G = (AF - B)^{-1}(D - AG), \tag{1.26}$$

$$H = (AF - B)^{-1}. \tag{1.27}$$

(1.25) and (1.26) are simplified as

$$AF^2 - BF - C = 0, \tag{1.28}$$

$$G = (AF - B + A)^{-1}D. \tag{1.29}$$

It is straightforward to solve the quadratic equation (1.28) for the matrix F. Once F is obtained, substituting F for (1.29) and (1.27) yields G and H. One technical issue is the existence of multiple solutions when the quadratic equation (1.28) is numerically solved. I select a solution that is not explosive. [14]

Once (1.23) is obtained, one can forecast the state variables $X = (y_t, \pi_t, x_t, y_t^n)^T$. The forecasts of k-period-ahead state variables $\mathrm{E}_t[X_{t+k}]$ are given by

$$\mathrm{E}_t[X_{t+k}] = F^k X_t + (I + F + F^2 + \cdots + F^{k-1})G, \tag{1.30}$$

where I is a 4×4 identity matrix. The forecasts of the time average of state variables over n periods are given by

$$\mathrm{E}_t[\bar{X}_{t+n}] = \sum_{k=1}^{n} \mathrm{E}_t[X_{t+k}]/n = \bar{F}_n X_t + \bar{G}_n, \tag{1.31}$$

where $\bar{F}_n = \sum_{k=1}^{n} F^k/n$ and $\bar{G}_n = \sum_{k=1}^{n}\sum_{j=1}^{k} F^{j-1}G/n$. The second element of $\mathrm{E}_t[\bar{X}_{t+k}]$ is the forecast of the average inflation over n periods at time t, denoted as $\mathrm{E}_t[\bar{\pi}_{t+k}]$. In estimating the model, I use survey-based forecasts of the average inflation. I denote the survey-based forecasts of the average inflation over n periods at time t with s_t^n. I assume that s_t^n is determined by

$$s_t^n = \mathrm{E}_t[\bar{\pi}_{t+k}] + \alpha_s^n + \epsilon_t^{s^n}, \tag{1.32}$$

where α_s^n captures a bias of the survey-based forecast of inflation. $\epsilon_t^{s^n}$ is sampled from $N(0, \sigma_{s^n})$. Notice that s_t^n is a linear function of all state variables X_t because $\mathrm{E}_t[\bar{\pi}_{t+k}]$ is a linear function of X_t. Therefore, one can rewrite (1.32) as

$$s_t^n = \bar{F}_{n,2} X_t + \bar{G}_{n,2} + \alpha_s^n + \epsilon_t^{s^n}, \tag{1.33}$$

where $\bar{F}_{n,2}$ is the second row of $\bar{F}_n$ and $\bar{G}_{n,2}$ is the second element of $\bar{G}_n$.

[14] It is worth mentioning that $F_{44} = 1 - \mu_{y_n}$ and $G_4 = \mu_{y_n}\bar{y_n}$.

1.3.2 Zero lower bound in macroeconomic dynamics

In the previous subsection, I replace a nominal policy interest rate with a shadow (nominal) policy interest rate and focused on the IS-SR case (1.7). In the following subsection, I discuss the IS-ZLB case (1.6) where the zero lower bound is explicitly considered in a structural New Keynesian macroeconomic model.

The system of four equations with zero lower bound is given by

$$
\begin{aligned}
y_t &= \alpha_{IS} + \mu_y^{+}\mathrm{E}_t[y_{t+1}] + (1-\mu_y^{-})y_{t-1} - \phi(i_t - \mathrm{E}_t[\pi_{t+1}]) + \epsilon_{IS,t}, && (1.34)\\
\pi_t &= \mu_\pi \mathrm{E}_t[\pi_{t+1}] + (1-\mu_\pi)\pi_{t-1} + \kappa(y_t - y_t^n) + \epsilon_{AS,t}, && (1.35)\\
x_t &= \mu_x\left(r^* + \pi_t + \gamma_y(y_t - y_t^n) + \gamma_\pi(\pi_t - \bar{\pi})\right) + (1-\mu_x)x_{t-1} + \epsilon_{x,t}, && (1.36)\\
y_t^n &= \mu_{y^n}\bar{y^n} + (1-\mu_{y^n})y_{t-1}^n + \epsilon_{y^n,t}, && (1.37)
\end{aligned}
$$

where $i_t = \max(x_t, \bar{i})$. Notice that a shadow interest rate x_t cannot have any impact on current real output y_t if the zero lower bound is binding ($x_t < 0$) in (1.34). Yet, a shadow interest rate could have an impact on current real output by changing the expected real output.[15] This effect depends on how long a central bank keeps its policy rate at zero percent. As a result, coefficients F and G are time-dependent when the shadow interest rate is negative.

The four equations above can be expressed in the form of two matrix equations.

$$
\begin{aligned}
A^* E_t[X_{t+1}] &= B^* X_t + C^* X_{t-1} + D^* + \epsilon_t \quad \textit{if} \quad x_t \le \bar{i}, && (1.38)\\
A E_t[X_{t+1}] &= B X_t + C X_{t-1} + D + \epsilon_t \quad \textit{if} \quad x_t > \bar{i}, && (1.39)
\end{aligned}
$$

where A, B, C and D are already defined in the previous subsection. It is easy to confirm the coefficient matrix $A^* = A$ that is defined by (1.20) and a diagonal

[15] To see this, consider a simplified IS equation below:

$$
\begin{aligned}
y_t &= \mathrm{E}_t[y_{t+1}] - \phi(i_t - \mathrm{E}_t[\pi_{t+1}])\\
&= -\phi \mathrm{E}_t\left[\sum_{k=1}^{\infty}(i_{t+k} - \pi_{t+k+1})\right] - \phi(i_t - \mathrm{E}_t[\pi_{t+1}]).
\end{aligned}
$$

From the first term in the equation above, one can see a theoretical possibility that a real output can be increased if the future nominal interest rate is lowered even when a current policy interest rate i_t is at zero percent in the second term.

matrix $C^* = C$ in (1.21) because the zero lower bound does not appear in these matrices. The coefficient matrix B^* and the vector D^* are different from their counterparts, B and D in the IS-SR case.

$$B^* \quad = \quad \begin{pmatrix} -1 & 0 & 0 & 0 \\ \kappa & -1 & 0 & -\kappa \\ \mu_x\gamma_y & \mu_x(1+\gamma_\pi) & -1 & -\mu_x\gamma_y \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{1.40}$$

Vector D^* is given by

$$D^* = (\alpha_{IS} - \phi\bar{i}, 0, \mu_x(r^* - \gamma_\pi\bar{\pi}), \mu_y\bar{y^n})^T. \tag{1.41}$$

To solve a set of the two forward-backward looking matrix equations, (1.38) and (1.39), I employ the method developed by Guerrieri and Iacoviello (2015) with a little modification. Loosely speaking, I extrapolate a solution obtained in the region without the zero lower bound to the region with the zero lower bound by taking into account the ending time of the zero interest rate policy.

The key assumption of their method is that a nominal policy interest rate reverts to a positive level and the economy will not be constrained by the zero lower bound after a certain time period τ. This assumption allows us to have a boundary condition and dramatically reduces a complexity of (1.34)-(1.37). My modification of their method is that (1) I maintain shocks ϵ_t in (1.38) while they drop them. (2) I restrict the case when τ is the first exit time from the zero lower bound while they allow regimes to switch from one to another before $t < \tau$.

I denote the first expected ending time of zero interest rate policy with τ that is mathematically defined as

$$\tau = \min(s; \mathrm{E}_t[x_s] > \bar{i}, \mathrm{E}_t[x_{s-1}] \leq \bar{i}). \tag{1.42}$$

When a shadow interest rate $x_t < \bar{i}$ at time $t = 1$, the solution at the time period $t = 1$ takes the following form.

$$X_{t+1} \quad = \quad F_t^* X_t + G_t^* + H_t^* \epsilon_t \quad \textit{if} \quad 1 \leq t < \tau, \tag{1.43}$$

$$X_{t+1} = FX_t + G + H\epsilon_t \quad if \quad t \geq \tau, \tag{1.44}$$

where F, G and H are VAR coefficients without the zero lower bound and numerically computed by solving (1.19). F_t^*, G_t^* and H_t^* are solved recursively from $t = \tau$ to $t = 1$, given F, G and H. My additional assumption is that the expected shadow interest rate hits to zero and gets above the zero lower bound only when $t = \tau$. Notice that the expected exit time of zero interest rate policy τ becomes more distant from now when a current shadow interest rate x_t is more negative. Thus, the expected exit time τ must be consistent with the current shadow interest rate x_t when solved. The numerical solution is postponed to the Appendix 1.7.1.

1.3.3 No-arbitrage term structure model

When the zero lower bound is binding, a policy interest rate is kept constant and thus a shadow policy interest rate x_t is not directly observable. I use the information contained in the cross section of nominal bond yields to extract a shadow interest rate x_t. In doing so, I construct a term structure model of interest rates with the zero lower bound. Since the model is non-linear and thus a closed-form solution is not available, I develop an approximate bond pricing formula that is correct up to second order.

I incorporate the structural macroeconomic model as parameter restrictions on the dynamics of state variables into a no-arbitrage term structure model.[16] Recall that I have shown that forward-looking equations in a New Keynesian macroeconomic model is reduced to a VAR (1). That VAR (1) describes the dynamics of the state variables in the physical measure when modeling term structure.

As in empirical studies of no-arbitrage term structure models, the model en-

[16] There are $2 \times 4 \times 4 = 32$ parameters in the VAR coefficient F and the matrix H. The vector G has 4 parameters. In total there are 36 parameters. When we allow the time to exit from the zero lower bound affects VAR coefficients, F, G, H, in macroeconomic dynamics, these VAR coefficients become time-varying so that the number of model parameters increase dramatically and makes their estimation difficult.

ables us to decompose nominal bond yields into the expected real rates, the real interest rate risk premium, the expected inflation, and the inflation risk premium. Furthermore, one can investigate how factor loadings of yield curves change before and during the zero lower bound period without treating these two periods separately.

Following Black (1995), I employ the concept of a shadow interest rate to model the zero lower bound on nominal interest rates. A shadow interest rate x_t can take a negative value and a positive part of the shadow interest rate is set equal to an observed nominal policy interest rate i_t. The mathematical definition is

$$i_t = \max(x_t, \bar{i}). \tag{1.45}$$

In (1.45), I slightly generalize the original definition of shadow interest rates by introducing an additional parameter $\bar{i}$. I call $\bar{i}$ effective lower bound. The effective lower bound $\bar{i}$ can be understood as interest rate on reserve (IOR). It arises from the fact that banks cannot keep cash physically and they are required to hold reserves at a central bank. I estimate $\bar{i}$ for actual data.

Next, let us introduce a vector of stochastic factors $X_t = (y_t, \pi_t, x_t, y_t^n)^T$ that drive a shadow interest rate at the next time step, x_{t+1}. There are three other macroeconomic variables: real output y_t, inflation π_t, potential output y_t^n.

I assume that the vector of these stochastic factors X_t follows a VAR(1).

$$X_t = FX_{t-1} + G + H\epsilon_t, \tag{1.46}$$

where F and H are 4×4 matrix and G is 4×1 vector. ϵ_t is a 4×1 vector of fundamental shocks. The i-th element of ϵ_t is sampled from a normal distribution $N(0, \sigma_i)$. These innovations $\epsilon_{i,t}$ are not correlated with each other. Correlations between them are captured by H.

If we look at the dynamics of the shadow interest rate x_t, it is given by

$$x_t = F_{31}y_{t-1} + F_{32}\pi_{t-1} + F_{33}x_{t-1} + F_{34}y_{t-1}^n + G_3 + H_3\epsilon_t, \tag{1.47}$$

where F_{ij} is the (i, j) element of the matrix F. H_3 are the third-row of each matrix. G_3 is the third element of the vector G. The equation (1.47) tells us that the H_3 mixes different fundamental shocks on a shadow interest rate x_t.

One potential concern for (1.46) is that a shadow interest rate may not be able to have any impact on real output when it is negative and a nominal rate is equal to zero. In other words, F_{13} may depend on the level of x_t. To avoid this issue, I have explicitly modeled the dependence of macroeconomic dynamics on a shadow interest rate using the IS-ZLB specification in Section 1.3.2.

As it is often used in the literature, I assume that a market price of risk λ_t is a linear function the state variables X_t in order to model the dynamics of the vector X_t under the risk neutral measure.

$$\lambda_t = \lambda^0 + \lambda^1 X_t, \tag{1.48}$$

where $\lambda^0 = [\lambda^0_y, \lambda^0_\pi, \lambda^0_x, \lambda^0_{y^n}]^T$ and λ^1 is 4×4 upper triangle matrix except modification that allows $\lambda^1_{2,1}$ to be non-zero.[17]

Nominal bond price P^n_t with maturity τ at time t is recursively computed. Suppose that we are pricing bonds under the time frequency Δt such that $\tau = n\Delta t$. The pricing formula for a nominal bond is given by

$$P^n_t(X) = \mathrm{E}^Q_t[e^{-\sum_{k=0}^{n} i_{t+k\Delta t}} | X_t = X], \tag{1.49}$$

where the expectations are computed under the risk neutral measure. Henceforth, E^Q_t is the expectations under the risk-neutral measure. X is an initial value of macroeconomic variables including a shadow interest rate ($X_t = X$).

The dynamics of the four stochastic factors under the risk-neutral measure are given by

$$X_t = F^Q X_{t-1} + G^Q + H\epsilon_t, \tag{1.50}$$

[17]If $\lambda^1_{2,1}$ is non-zero, the dependence of inflation on previous real output is different between the risk neutral measure and the physical measure.

where F^Q an G^Q are given by

$$F^Q = F - H\mu^1, \tag{1.51}$$

$$G^Q = G - H\mu^0. \tag{1.52}$$

Similarly, real bond price D_t^n with maturity τ at time t is also recursively computed. The formula for real bond pricing is given by

$$D_t^n(X) = \mathrm{E}_t^Q[e^{-\sum_{k=0}^n r_{t+k\Delta t}}|X_t = X], \tag{1.53}$$

where r_t is a real interest rate $r_t = i_t - \pi_t = \max(x_t, \bar{i}) - \pi_t$. Nominal bond yields i_t^n with maturity τ at time t are defined as

$$i_t^n(X) = -\log(P_t^n(X))/\tau. \tag{1.54}$$

Similarly, real bond yields r_t^n with maturity τ at time t are defined as

$$r_t^n(X) = -\log(D_t^n(X))/\tau. \tag{1.55}$$

In estimating the model for actual nominal bond yields, I assume that there is an observation noise for bond yields. The observation noise w_t is sampled from the normal distribution $N(0, \sigma_R)$ with the standard deviation σ_R.

$$i_t^{n,data}(X) = i_t^{n,model}(X) + w_t. \tag{1.56}$$

I assume that the observation noise w_t is independent of shocks to other macroeconomic variables ϵ_t. I also assume that w_t for the maturity τ is independent from w_t for other maturity $\tau'(\neq \tau)$.

When estimating a joint model of yield curves and macroeconomic variables, bond yields are computed many times given different parameters. Thus, it is important to obtain either a closed-form solution or an approximate solution for computational feasibility. One cannot use a well-known affine bond pricing formula here even though stochastic factors X_t follow VAR(1) system since stochastic factors X_t are non-linearly related to a policy interest rate i_t.

To overcome this issue, I develop an approximate formula for nominal bond prices when there exists an effective (zero) lower bound on nominal interest rates. Recall the definition of the shadow interest rate and notice that it can be represented as

$$i_t = \max(x_t, \bar{i}) = x_t + \max(\bar{i} - x_t, 0). \tag{1.57}$$

The second term is analogous to a put option with the strike $\bar{i}$ on the shadow interest rate x_t. The nominal bond yield is

$$i_t^n(X) = i_t^{n,affine}(X) + P_t^n(X, \bar{i}). \tag{1.58}$$

The value of the put option P_t^n depends on (1) the volatility of the underlying shadow interest rate and (2) to what extent the shadow interest rate is negative. For example, the option value arising from the zero lower bound is ignorable if the shadow interest rate is strongly positive and its volatility is low. Loosely speaking, the approximation takes the volatility effect into account up to a second order. The second effect is captured as delta of the option.[18] I provide an intuitive derivation in Appendix 1.7.2. A more formal proof is postponed to the Appendix 1.7.3.[19]

1.4 Data and estimation methodology

1.4.1 Data

I study government bond yields and macroeconomic variables in the US and Japan. The frequency of the data used in my empirical analysis is quarterly. For the US case, I study the period from October of 1991 (4th quarter) to October of 2015 (4th

[18]Delta is a terminology widely used in the option pricing literature. It refers to as the first-order derivative of the option value with respect to the underlying asset. Delta captures moneyness of the put option. In other words, delta naturally takes into account to what extent the shadow interest rate is negative.

[19]Section 1.7.7 shows detailed comparisons of bond prices based on the approximate formula with the bond prices based on Monte Carlo simulation.

quarter). I obtain the US Treasury bond yields from the website of the Federal Reserve Board of Governors. The data is constructed based on Gürkaynak, Sack and Wright (2007).[20] GDP growth and CPI inflation are downloaded from FRED at the website of the Federal Reserve Bank of St. Louis. I also use 10-year CPI forecasts from the Survey of Professional Forecasters. The starting date is determined due to availability of 10-year CPI forecasts. I use nominal bond yields data constructed by Gürkaynak, Sack and Wright (2007). Figure 1.1 shows a time series plot of the nominal bond yields used in estimating the model for US. It is clear that the zero lower bound has been binding in the US since December of 2008.

For the case of Japan, I focus on the period from July (3rd Quarter) of 2004 to October (4th quarter) of 2015 because fixed income investors' inflation forecasts are available only from July of 2004. Survey-based inflation forecasts are from Quick. For computing the average of CPI forecasts, I adjust a consumption tax hike in 2013 April. I obtain the Japanese government bond yields from the website of Ministry of Finance.[21] GDP growth and CPI inflation are downloaded from Cabinet Office and Statistics Bureau. Figure 1.2 shows the historical data of nominal bond yields used in estimating the model for Japan. One can see that 10-year bond yield is very low and even became lower than 50 bps in 2015.

I use demeaned real GDP and potential GDP as a proxy for real output y_t and potential output y_t^n, respectively. I employ CPI inflation as a proxy for inflation π_t.

Table 1.1 shows the average nominal yields of the US Treasury bonds and the Japanese government bonds, respectively. The US Treasury bond yields are constrained by the zero lower bound after the Federal Reserve employed the zero interest rate policy. To formally test this argument, I estimate the effective lower bound $\bar{i}$ in the next section. The Japanese government bond yields have been

[20]http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html
[21]http://www.mof.go.jp/english/jgbs/reference/interest_rate/index.htm

very low during the entire sample period. After QQE (Quantitative Qualitative Easing), they are further lowered.

Table 1.2 shows summary statistics of survey-based CPI inflation forecasts. In the US case, the historical average of realized CPI inflation was around 2.32% during 1991-2015 period. The historical average of 10-year-average CPI inflation forecast is 0.31% higher than that. The difference seems not large. This is in sharp contrast to Japan. The historical average of realized CPI inflation was 0.03% during 2004-2015 period in Japan. The historical average of 10-year-average CPI inflation forecast is 1.10%.

1.4.2 Estimation methodology

As described in the previous sections, my joint model of yield curves and macroeconomic variables is described by (1.23), (1.33) and (1.56). To clarify what I am going to estimate, I reproduce those equations below:

$$i_t^{n,data} = i_t^{n,model}(X_t, \Theta, \lambda_0, \lambda_1) + w_t^n, \tag{1.59}$$

$$X_t = F(\Theta)X_{t-1} + G(\Theta) + H(\Theta)\epsilon_t, , \tag{1.60}$$

$$s_t^n = \bar{F}_{n,2}(\Theta)X_t + \bar{G}_{n,2}(\Theta) + \alpha_s^n + \epsilon_t^{s^n}, \tag{1.61}$$

where $X_t = (y_t, \pi_t, x_t, y_t^n)^T$. s_t^n is n-quarter-average inflation forecast at time t. I set $n = 40$ and use the 10-year-average CPI forecasts as a proxy of the long-term inflation forecast. Measurement error w_t^n is sampled from a normal distribution $N(0, \sigma_M)$. λ_0 and λ_1 are parameters of market price risk as discussed in the previous section. Θ is a vector of the New Keynesian model parameters.

$$\Theta = \left\{\alpha_{IS}, \mu_y^+, \mu_y^-, \phi, \mu_\pi, \kappa, \mu_x, r^*, \gamma_y, \gamma_\pi, \bar{\pi}, \bar{y^n}, \mu_{y_n}, \bar{i}\right\}. \tag{1.62}$$

Notice that volatilities of four macroeconomic shocks $\sigma = \{\sigma_{IS}, \sigma_{AS}, \sigma_x, \sigma_{y_n}\}$ are standard deviations of a vector of these shocks ϵ_t. When IS-ZLB is employed, (1.60) should be replaced with (1.43) and (1.44). I estimate the model under the specification of IS-ZLB unless otherwise mentioned. Recall that the shadow interest rate x_t is latent when the zero lower bound is binding.

I estimate the model by two methods. In the first method, I extract a shadow interest rate x_t at time t by fitting the model-implied nominal yield curve $i_t^{n,model}$ to cross-sectional nominal bond yields $i_t^{n,data}$. This approach is widely used in empirical studies of affine term structure models. Unlike affine term structure models, one cannot analytically solve the state variables as a function of observable bond yields.[22] Thus, I numerically solve the state variable x_t.

$$\hat{x}_t = \text{argmin} \sum_{n \in N} \left(i_t^{n,data} - i_t^{n,model}(X_t, \Theta, \lambda_0, \lambda_1) \right)^2, \tag{1.63}$$

where a vector of maturities $N = \{0, 1, 2, 3, 4, 5, 7, 10\}$. The nominal interest rate with $N = 0$ means the policy interest rate. the Rather than assuming that nominal bond yields with specific maturities can be observed without noise, I use all cross-sectional bond yields to extract the shadow interest rate.[23] Then, I employ maximum likelihood estimation for the VARs (1) ((1.60) and (1.61)) where the four state variables are given by $\hat{X}_t = (y_t, \pi_t, \hat{x}_t, y_t^n)^T$.

To reduce the computational time for estimation, I estimate model parameters $\bar{y}$, μ_{y^n}, σ_{y^n} in the equation of potential output (potential GDP), (1.14) (or equivalently (1.37)) separately because the potential output is completely exogenous. I also fix Taylor rule coefficients $\gamma_y = 0.5$ and $\gamma_\pi = 0.5$ for both the US and Japan, as in the original Taylor rule. This assumption is not necessary but helps us obtain an estimate of the long-run real interest rate r^* in the shadow policy

[22] In M-factor affine term structure models, a vector of M number of yields, i_t^M is given by

$$i_t^M = A + BX_t,$$

where A is a $M \times 1$ vector. B is $M \times M$ matrix. Then one obtains

$$\hat{X}_t = B^{-1}(i_t^M - A).$$

The above explicit function of $\hat{X}_t$ cannot be obtained in non-linear term structure models.

[23] I have also estimated the model by assuming no observable noise for nominal bond yield for one specific maturity. Yet, it makes solving a nonlinear equation unstable due to non-linearity of the model, especially when the bond yield is approaching zero.

rule equation (1.13) (or (1.36)).

In the second method, I treat shadow interest rates as a latent factor and thus employ unscented Kalman Filtering. Since bond yields in my term structure model are not linear to latent variables, I cannot use conventional Kalman filtering to compute likelihood. Unscented Kalman Filtering (UKF) developed by Julier and Uhlmann (2004) is applicable to non-linear models. It is possible to estimate the model parameters by extended Kalman filtering based by linearizing the model. Yet, Christoffersen *et al.* (2014) report that unscented Kalman filter performs better than extended Kalman Filtering.

The second method is just for a robustness check. All of results are based on the first method.

1.5 Results

1.5.1 Estimated parameters and pricing errors

Table 1.3 shows absolute pricing errors of the model. The average errors across all maturities are 24bps for US and 7bps for Japan. These numbers are reasonably low, considering that the model has some restrictions on the dynamics of the state variables X_t under the physical measure.

Tables 1.4 and 1.5 present estimates of structural New Keynesian model parameters as well as market price of risk parameters for the two countries. We observe the following: First, the effective lower bound $\bar{i}$ is slightly positive for both cases. For US, $\bar{i}$ is 17bps while it is 10bps for Japan. This indicates that specifying exactly zero percent as the zero lower bound in modeling might give misleading results.[24]

Second, the long-run real rate is negative for Japan with $r^* = -2.60\%$. The

[24] The Bank of Japan has been keeping an interest rate on reserve (IOR) to 10bps since 2008 October.

Bank of Japan is currently targeting for a 2% inflation. If the Japanese inflation π_t converges to the official target inflation of 2%, the long-run (policy-neutral) nominal interest rate $i_t^*(= r^* + \pi_t)$ is equal to -0.60%. Yet, it is still lower than the effective (zero) lower bound $\bar{i}(= 0.10\%)$. It indicates that the Bank of Japan might need to have a higher inflation ($> 2.70\%$) in order to exit from the (effective) zero lower bound. In the case of the US, the long-run real rate is equal to 1.02%. Thus, policy-neutral nominal interest rate should be above the effective lower bound if the inflation is higher than -0.85(=0.17-1.02) %.

Third, the bias of inflation survey α_s^{40} is 0.36% for the US while it is around -1.00% for Japan. Survey-based inflation forecasts are biased compared to rational expectations of inflation, especially for the case of Japan.

Figure 1.3 shows the evolution of the shadow interest rate extracted from the yield curves and the federal funds rate in the US. The shadow interest rate hit the zero lower bound in early 2009 and became very negative in 2012. It is approaching the bound recently, again. In the case of Japan, although I do not report here, the shadow interest rates have been negative over years and became more negative since 2014 as the long-term nominal yield is further lowered by Quantitative and Qualitative Easing (QQE).

1.5.2 Counterfactual analyses

Equipped with the estimated New Keynesian macroeconomic model, I conduct a counterfactual analyses of monetary policy. Figures 1.4 and 1.5 show the nominal bond yield curve given a higher inflation target level for the US.[25] In Figure 1.4, the zero lower bound is not binding at the current time period ($x_t = 1.00\%$). Thus, a higher inflation target lowers a policy interest rate in the short run, but it increases the long-run expected inflation. Thus, long-term nominal bond yields

[25] I assume that an initial value of the shadow interest rate is lowered by 0.50% due to the 1.00% increase in the target inflation rate because the target shadow rate is lowered by 0.50%.

increase.

In Figure 1.5, the zero lower bound is binding ($x_t = -1.00\%$). As a result, a higher inflation target cannot further lower short-term nominal yields. Figure 1.5 also shows the yield curve when the zero interest rate policy is suddenly abandoned. The short-term yields increase more than those in the case of a higher inflation target, but the long-term yields do not. Furthermore, Figure 1.6 shows that real output increases when a higher inflation target is adopted. By contrast, Figure 1.7 shows that real output decreases when the zero interest rate policy suddenly ends. These results indicate that raising the inflation target is better than suddenly ending the zero interest rate policy in stimulating the macro economy and generating higher long-run nominal interest rates.

Figure 1.8 documents that the increase in the nominal yields caused by the higher inflation target is explained by the increase in the expected nominal interest rate as well as the increase in the term premium. That indicates that a higher inflation target generates higher expected inflation and thus increases the expected nominal interest rates. Not surprisingly, the term premium is also impacted by a higher target inflation.

Figures 1.9 and 1.10 show the nominal bond yield curves given a higher inflation target level for Japan. In Figure 1.9, the zero lower bound is not binding at the current time period ($x_t = 1.00\%$). In Figure 1.10, the zero lower bound is binding ($x_t = -1.00\%$). The result is similar to the US case although long-term yields are less impacted in the case of Japan.

Figures 1.11 and 1.12 show the nominal bond yield curves given an introduction of a negative lower bound for the US and Japan. The negative lower bound steepens the nominal yield curve by allowing short-term nominal yields to be negative and by increasing the long-term nominal yields. Figure 1.13 shows that real output and inflation increase given an introduction of a negative lower bound in the US case. The long-term nominal yields increase as these macroeconomic variables increase.

1.6 Conclusion

In this paper, I present a joint model of term structure and macroeconomic variables with an explicit zero lower bound. I employ the concept of a shadow interest rate to model the zero lower bound in bond pricing. Bond yields satisfy no-arbitrage condition, but not affine to the state variables due the zero lower bound. A well-known closed form solution for bond prices under an affine term structure model is not available. Thus, I develop a new approximate bond pricing formula that is correct up to a second order of shadow interest rate volatility. The formula is intuitive and compatible with other features such as regime switching. I assume that the market price of risk is a linear function of the state variables.

To conduct a counterfactual analysis of monetary policy, I assume that macroeconomic dynamics are described by a standard New Keynesian model with the zero lower bound. The New Keynesian model has a VAR (1) representation with time-varying coefficient matrices that depend on the level of the shadow interest rate. I include the shadow policy interest rate as a state variable in the VAR (1). The other macroeconomic variables are real output, inflation, and potential output.

The yield curve contains the information about the expected policy interest rate that is a key input in the IS equation of the New Keynesian model. Thus, I extract a shadow policy interest rate by using yield curves.

I apply the model for the US and Japanese economy. I jointly estimate the New Keynesian model parameters and the linear market price of risk parameters using yield curves, survey-based inflation forecasts and macroeconomic variables.

I find that the long-run real interest rate is equal to 1.02% in the US during the 1991-2015 period and it is -2.60% in Japan during the 2004-2015 period. I also document that the effective (lower) bound on nominal interest rates is slightly positive for both the US and Japan.

I conduct different types of counterfactual analyses. As the main feature of my model, I conduct a counterfactual analysis of raising the target inflation level. In both the US and Japan, a higher inflation target steepens the yield curve when the zero lower bound is not binding. On the other hand, a higher inflation target increases long-term yields while keeping short-term yields unchanged under the zero lower bound. For comparison, I also conduct a counterfactual analysis of suddenly ending the zero interest rate policy. Given the sudden termination of the zero interest rate policy, the long-term nominal yields do not increase as much as those in the case of a higher inflation target for the US. When I look at the effects of these policies on real output, raising the inflation target is expansionary while ending the zero interest rate policy is contractionary. In this respect, raising the inflation target is more appropriate.

One methodological contribution of this research is the use of information contained in the yield curves for estimating structural macroeconomic models with a zero lower bound. The structural macroeconomic model employed in this research is so stylized that many aspects of a real economy are not captured. I assume a textbook-style New Keynesian macroeconomic dynamics in this research to keep the key aspects of my joint model clear. More empirical investigations about macro-finance models with a zero lower bound should be conducted to facilitate a robust monetary policy.

1.7 Appendix

1.7.1 Solving a log-linearized New Keynesian model with the zero lower bound

The numerical procedure is as follows.

During the time period when the zero lower bound is binding, VAR coefficients F_t^*, G_t^* and H_t^* are computed given F, G and H. First, guess the expected exit time τ. From (1.44), the evolution of X_t at time $t = \tau$ is determined by

$$X_{\tau+1} \quad = \quad FX_\tau + G + H\epsilon_\tau. \tag{1.64}$$

Substituting (1.64) for (1.38) at time $t = \tau$, one obtains

$$A^*(FX_\tau + G) = B^*X_\tau + C^*X_{\tau-1} + D^* + \epsilon_t.$$
$$\Leftrightarrow X_\tau = (A^*F - B^*)^{-1}(C^*X_{\tau-1} + D^* - A^*G + \epsilon_t). \tag{1.65}$$

Comparing (1.65) with (1.43) at time $t = \tau - 1$, we have

$$F_{\tau-1}^* \quad = \quad (A^*F - B^*)^{-1}C^*, \quad G_{\tau-1} = (A^*F - B^*)^{-1}(D^* - A^*G), \tag{1.66}$$
$$H_{\tau-1}^* \quad = \quad (A^*F - B^*)^{-1}. \tag{1.67}$$

Given $F_{\tau-1}$, $G_{\tau-1}$, $H_{\tau-1}$, one can recursively compute the previous coefficients F_t, G_t, H_t $(1 \leq t \leq \tau - 1)$.

$$F_t^* \quad = \quad (A^*F_{t+1}^* - B^*)^{-1}C^*, \quad G_t = (A^*F_{t+1}^* - B^*)^{-1}(D^* - A^*G_{t+1}), \tag{1.68}$$
$$H_t^* \quad = \quad (A^*F_{t+1}^* - B^*)^{-1}. \tag{1.69}$$

Equipped with F_t, G_t, one can compute the expected exit time τ'. If $\tau' < \tau$, lower τ. Otherwise raise τ. Repeating this procedure, one should obtain τ that is consistent with x_t at time $t = 1$.

The algorithm is summarized as outlined below.

- Solve VAR coefficient F, G and H without the zero lower bound.

Step 1 Guess the ending time of the zero interest rate policy τ.

Step 2 Compute VAR coefficient F_t^* G_t^*, and H^* during the time period of zero interest rate policy (from $t = \tau - 1, \cdots, 1$) using (1.68) and (1.69).

Step 3 Generate the expected path of shadow interest rates $E_1[x_t]$ for $t = 1, \cdots, \tau - 1$ using (1.43) and compute τ' as the first hitting time of the shadow interest rate to the effective (zero) lower bound $\bar{i}$. $E_1[x_t]$ is obtained as one element of the vector $E_1[X_t]$.

Step 4 compare whether τ' is equal to τ or not. If not, update τ.

- Repeat the Step 1 to Step 4 until convergence.

In practice, I set the upper bound for the hitting time τ equal to 40 (10 years) since I extract the information about expected interest rate up to 10 years.

1.7.2 Approximate bond pricing formula (single-factor case): An intuitive derivation

In this section, I give an intuitive derivation of the approximate bond pricing formula with zero lower bound. A formal proof is in the Appendix 1.7.3.

In a continuous-time framework, the approximate formula is given by

$$i_t^n(X) = \frac{1}{\tau}\int_t^{t+\tau} V(X,t,s)ds - \frac{1}{\tau}\int_t^{t+\tau} \mathrm{E}_t^Q\left[\frac{\sigma_x^2}{2}\left(\int_s^{t+\tau}\Delta(X_s,s,u)du\right)^2\right]ds. \tag{1.70}$$

In a discrete-time framework, the approximate formula is represented as

$$i_t^n(X) = \frac{1}{n}\sum_{k=0}^{n} V(X,t,t+k\delta t) - \frac{1}{n}\sum_{k=0}^{n}\mathrm{E}_t^Q\left[\frac{\sigma_x^2}{2}\sum_{j=k}^{n}\Delta(X_{t+j\delta t},t+j\delta t,T)^2\right], \tag{1.71}$$

where I define a call option on shadow interest rates with $\bar{i}$ strike $V(x,\tau)$ and its first-order derivative $\Delta(x,\tau)$ as

$$V(X,t,T) = \mathrm{E}_t^Q[\max(x_T,\bar{i})|X_t = X], \qquad \Delta(X,t,T) = \frac{\partial V(X,t,T)}{\partial x}, \quad (1.72)$$

where x is the value of a shadow interest rate at time $t(x_t = x)$.

In the following, I briefly explain the intuition behind the approximate formula. First, consider the the following identity.

$$\log \mathrm{E}_0\left[e^{y_\tau}\right] = \mathrm{E}_0\left[y_\tau\right] + \log \mathrm{E}_0\left[e^{y_\tau - E_0[y_\tau]}\right], \quad (1.73)$$

where y_t is some stochastic variable. One can approximate the second term in the equation (1.73).

$$\begin{aligned} \log \mathrm{E}_0\left[e^{y_\tau}\right] &= \mathrm{E}_0\left[y_\tau\right] + \log\left[1 + \mathrm{E}_0[y_\tau - E_0\left[y_\tau\right]] + \frac{1}{2}\mathrm{E}_0[(y_\tau - E_0\left[y_\tau\right])^2] + \cdots\right] \\ &\approx \mathrm{E}_0\left[y_\tau\right] + \frac{1}{2}\mathrm{Var}_0[y_\tau]. \end{aligned} \quad (1.74)$$

Consider a special case when y_t is given by

$$y_\tau = -\int_0^\tau \max(x_s,\bar{i})ds, \quad (1.75)$$

where x_t is a shadow interest rate at time t. Substituting (1.75) for (1.73), we obtain

$$\log \mathrm{E}_0\left[e^{-\int_0^\tau \max(x_s,\bar{i})ds}\right] \approx -\int_0^\tau \mathrm{E}_0\left[\max(x_s,\bar{i})\right] ds + \frac{1}{2}\mathrm{Var}_0[\int_0^\tau \max(x_s,\bar{i})ds]. \quad (1.76)$$

The first term in the right hand side of the equation (1.76) corresponds to $V(X,t,T)$ in (1.70). The second term is difficult to approximate directly. A more rigorous proof of the approximate formula (1.70) is provided in the Appendix 1.7.3. So suppose that one can rewrite (1.76) as

$$\log \mathrm{E}_0\left[e^{-\int_0^\tau \max(x_s,\bar{i})ds}\right] \approx -\int_0^\tau \mathrm{E}_0\left[\max(x_s,\bar{i})\right] ds + \frac{\left(\int_0^\tau \Delta(x_0,0,s)ds\right)^2}{2}\mathrm{Var}_0[x_\tau], \quad (1.77)$$

where $\Delta(t)$ is defined as

$$\Delta(x_t, t, T) = \frac{\partial}{\partial x_t} \mathrm{E}_t \left[\max(x_s, \bar{i}) \right]. \tag{1.78}$$

Finally, one obtains

$$i_0^\tau \approx \frac{1}{\tau} \int_0^\tau \mathrm{E}_0 \left[\max(x_s, \bar{i}) \right] ds - \frac{\left(\int_0^\tau \Delta(x_0, 0, s) ds \right)^2}{2\tau} \mathrm{Var}_0[x_\tau]. \tag{1.79}$$

The second term in (1.79) is now easily associated with the second term in (1.70). Also notice that (1.79) is represented as

$$i_0^\tau \approx \frac{1}{\tau} \int_0^\tau \mathrm{E}_0 \left[x_s \right] ds + \frac{1}{\tau} \int_0^\tau \mathrm{E}_0 \left[\max(\bar{i} - x_s, 0) \right] ds - \frac{\left(\int_0^\tau \Delta(x_0, 0, s) ds \right)^2}{2\tau} \mathrm{Var}_0[x_\tau]. \tag{1.80}$$

In (1.80), the first term can be associated with a nominal bond yield without the zero lower bound. The second and the third term are analogous to a put option arising from the zero lower bound. As discussed in the main text, the value of this put option naturally reflects (1) the volatility of the underlying shadow interest rate and (2) to what extent the shadow interest rate is negative. Yet, as I mentioned above, it is not easy to justify the derivation from (1.76) to (1.77). In the next appendix, I derive (1.70) by solving a system of partial differential equations.

1.7.3 Approximate bond pricing formula (single-factor case): A more formal proof

In this section, I provide a formal proof of the approximate bond pricing formula with the zero lower bound. First, I drive an approximate pricing formula for nominal bonds under zero lower bound using an asymptotic expansion. The first step is to obtain the approximate formula in a continuous time and then discretize it. Consider that a shadow interest rate follows the stochastic differential equation below.

$$dx_t = \mu(x_t) dt + \sigma(x_t) dW_t, \tag{1.81}$$

where W_t is a Brownian motion. Let us denote $x_t = x$. The Feynman-Kac formula provides a link between a stochastic process and its partial differential equation. It tells us that the price of a nominal discount bond $P(x,t,T)$ with expiry T at time t is a solution of the following partial differential equation.

$$\frac{\partial P}{\partial t} + \mu(x)\frac{\partial P}{\partial x} + \frac{\sigma(x)^2}{2}\frac{\partial^2 P}{\partial^2 x^2} = x^+ P, \tag{1.82}$$

where x^+ is defined as $\max(x, \bar{i})$. Notice that this definition is slightly different from the conventional notation in which $x^+ = \max(x_t, 0)$. Suppose that the bond price is represented as $P(x,t,T) = e^{-f(x,\tau)}$ where $\tau = T - t$. Substituting this for (1.82), one obtains

$$\frac{\partial f}{\partial \tau} = \mu(x)\frac{\partial f}{\partial x} + \frac{\sigma(x)^2}{2}\frac{\partial^2 f}{\partial^2 x^2} + x^+ - \frac{\sigma(x)^2}{2}\left(\frac{\partial f}{\partial x}\right)^2. \tag{1.83}$$

Suppose that the last term in the right-hand side of (1.83) is replaced with $\epsilon\frac{\sigma(x)^2}{2}\left(\frac{\partial f}{\partial x}\right)^2$ where $0 < \epsilon \leq 1$. Also, consider that $f(x,\tau)$ has an asymptotic expansion.

$$f = f_0 + \epsilon f_1 + \epsilon^2 f_2 + \cdots. \tag{1.84}$$

Substituting (1.84) for (1.83), one obtains a system of PDEs.

$$\begin{aligned}
\frac{\partial f_0}{\partial \tau} &= \mu(x)\frac{\partial f_0}{\partial x} + \frac{\sigma(x)^2}{2}\frac{\partial^2 f_0}{\partial^2 x^2} + x^+, && (1.85)\\
\frac{\partial f_1}{\partial \tau} &= \mu(x)\frac{\partial f_1}{\partial x} + \frac{\sigma(x)^2}{2}\frac{\partial^2 f_1}{\partial^2 x^2} - \frac{\sigma(x)^2}{2}\left(\frac{\partial f_0}{\partial x}\right)^2, && (1.86)\\
\frac{\partial f_2}{\partial \tau} &= \mu(x)\frac{\partial f_2}{\partial x} + \frac{\sigma(x)^2}{2}\frac{\partial^2 f_2}{\partial^2 x^2} - \sigma(x)^2\left(\frac{\partial f_0}{\partial x}\right)\left(\frac{\partial f_1}{\partial x}\right). && (1.87)\\
&\vdots
\end{aligned}$$

The first term f_0 in the asymptotic expansion is given by

$$f_0(x,t,T) = \int_t^T \mathrm{E}_t[x_s^+]ds = \int_t^T \int_{-\infty}^{+\infty} x_s^+ \phi(x_s, s|x_t, t)ds, \tag{1.88}$$

where $\phi(x_s, s|x_t, t)$ is a transition density of a shadow interest rate x_s at time s from $x_t = x$ at time t. This transition density $\phi(x_s, s|x_t, t)$ becomes a normal distribution, when x_t follows Vasicek process (AR(1) process).

Substituting (1.88) for (1.86) leads to

$$f_1(x,t,T) = \int_t^T \mathrm{E}_t\left[-\frac{\sigma(x_s)^2}{2}\left(\frac{\partial f_0(x_s,s,T)}{\partial x_s}\right)^2\right] ds. \tag{1.89}$$

The equation above clearly shows that a convexity effect depends on the volatility of a shadow interest rate and moneyness of a call option arising from the zero lower bound.

An asymptotic expansion for a nominal bond price $P(x,t,T)$ is

$$\begin{aligned} f(x,\tau) &= -\log P(x,t,T) \\ &= \int_t^T \mathrm{E}_t[x_s^+]ds + \epsilon \int_t^T \mathrm{E}_t\left[-\frac{\sigma(x_s)^2}{2}\left(\frac{\partial f_0(x_s,s,T)}{\partial x_s}\right)^2\right] ds + O(\epsilon^2). \end{aligned} \tag{1.90}$$

Let us denote a nominal bond yield with maturity $\tau = n\delta t$ at time t with $y_t^n(x)$. As in the main text, I define a call option $V(x,t,T)$ arising from the zero lower bound and its first-order derivative $\Delta(x,t,T)$ as

$$V(x,t,s) = \mathrm{E}_t[\max(x_s,0)], \qquad \Delta(x,t,s) = \frac{\partial V(x,t,s)}{\partial x}. \tag{1.91}$$

It is easy to confirm that

$$\int_t^T V(x,t,s)ds = f_0(x,t,T), \qquad \int_s^T \Delta(x_s,s,u)du = \frac{\partial f_0(x_s,s,T)}{\partial x_s}. \tag{1.92}$$

One can obtain an approximation for a nominal bond yield by the order of ϵ is computed as

$$\begin{aligned} i_t^n(x) &= -\log P(x,t,T)/\tau \\ &= \frac{1}{\tau}\int_t^{t+\tau} V(x,t,s)ds - \frac{1}{\tau}\int_t^{t+\tau} \mathrm{E}_t\left[\frac{\sigma(x_s)^2}{2}\left(\int_s^{t+\tau}\Delta(x_s,s,u)du\right)^2\right] ds. \end{aligned} \tag{1.93}$$

In a discrete-time framework, the pricing formula above is written as

$$i_t^n(x) \approx \frac{1}{n}\sum_{k=0}^n V(x,t,t+k\delta t) - \frac{1}{n}\sum_{k=0}^n \mathrm{E}_t\left[\frac{\sigma(x_{t+j\delta t})^2}{2}\sum_{j=k}^n \Delta(x_{t+j\delta t},t+j\delta t,T)^2\right]. \tag{1.94}$$

Notice that in both (1.93) and (1.94), it suffices to have a transition density $\phi(x_s, s|x_t, t)$ of a shadow interest rate to compute i_t^n.

1.7.4 Example: Random walk

Consider that a shadow interest rate follows a random walk.

$$dx_t = \sigma_x dW_t^x. \tag{1.95}$$

Suppose that the effective (zero) lower bound $\bar{i} = 0$. Applying the approximate formula developed in the previous subsection, one obtains:

$$V(x,t,s) = x\left(1 - N\left(\frac{-x}{\sigma_x\sqrt{s-t}}\right)\right) + \frac{\sigma_x\sqrt{s-t}}{\sqrt{2\pi}}e^{-\frac{x^2}{2\sigma_x^2(s-t)}}, \tag{1.96}$$

$$\Delta(x,t,s) = 1 - N\left(\frac{-x}{\sigma_x\sqrt{s-t}}\right). \tag{1.97}$$

If $\frac{x}{\sigma\sqrt{s-t}}$ is large, $V(x,t,s) \approx x$ and $\Delta(x,t,s) \approx 1$. Substituting these for (1.93), one obtains:

$$\begin{aligned} i_t^n(x) &\approx \frac{1}{\tau}\int_t^{t+\tau} x ds - \frac{1}{\tau}\int_t^{t+\tau}\frac{\sigma_x^2}{2}\left(\int_s^{t+\tau} 1du\right)^2 ds \\ &= x - \frac{1}{\tau}\int_t^{t+\tau}\frac{\sigma_x^2}{2}(t+\tau-s)^2 ds = x - \frac{\sigma_x^2}{6}\tau. \end{aligned} \tag{1.98}$$

This is consistent with a theoretical bond price under a random walk without the zero lower bound. The bond price computed from the approximate formula approaches the theoretical price without the zero lower bound as $\frac{x}{\sigma\sqrt{s-t}}$ is large.

1.7.5 Approximate formula for nominal bond yields (multi-factor case)

Next, suppose that a shadow interest is driven by multiple stochastic factors.

$$dx_t = \mu_x(x_t, Z_t)dt + \sigma_x(x_t, Z_t)dW_t^x, \tag{1.99}$$

where the vector $Z_t = [z_t^1, z_t^2, \cdots, z_t^N]$ and a stochastic process for each element of stochastic factor z_t^i is given by

$$dz_t^i = \mu_i(x_t, Z_t)dt + \sigma_i(x_t, Z_t)dW_t^i. \tag{1.100}$$

W_t^x and W_t^i are Brownian motions with correlation ρ_i. The correlations between W_t^i and W_t^j are ρ_{ij}. I denote $x_t = x$ and $z_t^i = z^i$. The Feynman-Kac formula shows the price of a nominal discount bond $P(x, Z, t, T)$ with expiry T at time t as a solution of the following partial differential equation.

$$\begin{aligned} x^+ P &= \frac{\partial P}{\partial t} + \mu_x \frac{\partial P}{\partial x} + \frac{\sigma_x^2}{2} \frac{\partial^2 P}{\partial^2 x^2} + \sum_{i=1}^{N} \left(\mu_i \frac{\partial P}{\partial z_i} + \frac{\sigma_i^2}{2} \frac{\partial^2 P}{\partial^2 z_i^2} + \rho_i \sigma_x \sigma_i \frac{\partial^2 P}{\partial z_i \partial x} \right) \\ &\quad + \sum_{i=1, j>i}^{N} \rho_{ij} \sigma_i \sigma_j \frac{\partial^2 P}{\partial z^i \partial z^j}, \end{aligned} \tag{1.101}$$

where I drop the subscripts of $\sigma_i(x_t, Z_t)$ for notation simplicity.

Suppose that the nominal bond price is represented as $P(x, Z, t, T) = e^{-f(x, Z, \tau)}$ where $\tau = T - t$. Substituting this for (1.101), one obtains

$$\begin{aligned} \frac{\partial f}{\partial \tau} &= x^+ + \mu_x \frac{\partial f}{\partial x} + \frac{\sigma_x^2}{2} \frac{\partial^2 f}{\partial^2 x^2} + \sum_{i=1}^{N} \left(\mu_i \frac{\partial f}{\partial z_i} + \frac{\sigma_i^2}{2} \frac{\partial^2 f}{\partial^2 z_i^2} + \rho_i \sigma_x \sigma_i \frac{\partial^2 f}{\partial z_i \partial x} \right) \\ &\quad + \sum_{i=1, j>i}^{N} \rho_{ij} \sigma_i \sigma_j \frac{\partial^2 f}{\partial z^i \partial z^j} - \frac{\sigma_x^2}{2} \left(\frac{\partial f}{\partial x} \right)^2 - \sum_{i=1}^{N} \left\{ \frac{\sigma_i^2}{2} \left(\frac{\partial f}{\partial z_i} \right)^2 + \rho_i \sigma_x \sigma_i \left(\frac{\partial f}{\partial z_i} \right) \left(\frac{\partial f}{\partial x} \right) \right\} \\ &\quad - \sum_{i=1, j>i}^{N} \rho_{ij} \sigma_i \sigma_j \left(\frac{\partial f}{\partial z_i} \right) \left(\frac{\partial f}{\partial z_j} \right). \end{aligned} \tag{1.102}$$

Suppose that the following nonlinear terms in the right-hand side of (1.102) are multiplied by ϵ where $0 < \epsilon \leq 1$.

$$-\frac{\sigma_x^2}{2} \left(\frac{\partial f}{\partial x} \right)^2 - \sum_{i=1}^{N} \left\{ \frac{\sigma_i^2}{2} \left(\frac{\partial f}{\partial z_i} \right)^2 + \rho_i \sigma_x \sigma_i \left(\frac{\partial f}{\partial z_i} \right) \left(\frac{\partial f}{\partial x} \right) \right\} - \sum_{i=1, j>i}^{N} \rho_{ij} \sigma_i \sigma_j \left(\frac{\partial f}{\partial z_i} \right) \left(\frac{\partial f}{\partial z_j} \right). \tag{1.103}$$

Also, consider that $f(x, Z, \tau)$ has an asymptotic expansion.

$$f = f_0 + \epsilon f_1 + \epsilon^2 f_2 + \cdots. \tag{1.104}$$

Repeating the same analysis as we did in the previous subsection, the first term f_0 is given by

$$f_0(x, \pi, t, T) = \int_t^T \mathrm{E}_t[x_s^+] ds. \tag{1.105}$$

The second term f_1 is given by

$$f_1(x,Z,t,T) = -\int_t^T \mathrm{E}_t \left[\frac{\sigma_x^2}{2}\left(\frac{\partial f}{\partial x}\right)^2 + \sum_{i=1}^N \left\{ \frac{\sigma_i^2}{2}\left(\frac{\partial f}{\partial z_i}\right)^2 + \rho_i\sigma_x\sigma_i\left(\frac{\partial f}{\partial z_i}\right)\left(\frac{\partial f}{\partial x}\right) \right\} + \sum_{i=1,j>i}^N \rho_{ij}\sigma_i\sigma_j\left(\frac{\partial f}{\partial z_i}\right)\left(\frac{\partial f}{\partial z_j}\right) \right] ds. \tag{1.106}$$

(1.105) and (1.106) indicate that we only need to compute the transition density of a shadow interest rate x, even when a shadow rate is driven by more than one stochastic factors. Especially, if we know that a shadow interest rate follows a Gaussian process (even though a nominal interest rate does not), what we need to compute is only its mean and variance at time $t = s$.

Although there are many terms in (1.106), several terms are very small. For example, the volatility of potential output is often very smaller than those of output, inflation and shadow interest rates. Furthermore, the sensitivities of the zero lower bound option V with respect to y, π, y_n are small compared to x in empirical applications since a (shadow) interest rate is usually persistent. Thus, the largest sensitivity is the derivative with respect to the shadow interest rate x_t. Thus, we can effectively reduce the number of terms in (1.106).

When a regime switching feature is introduced, expectations should be calculated not only over shadow interest rates x but also across regimes s_t.

1.7.6 Approximate formula for real bond yields

For real bond yields, we can apply a similar method. Let us consider that stochastic processes for inflation and a shadow interest rate are given by

$$dx_t = \mu_x(x_t)dt + \sigma_x(x_t)dW_t^x, \tag{1.107}$$

$$d\pi_t = \mu_\pi(\pi_t)dt + \sigma_\pi(\pi_t)dW_t^\pi, \tag{1.108}$$

where W_t^x and W_t^π are Brownian motions with correlation ρ. Let us denote $x_t = x$ and $\pi_t = \pi$. The Feynman-Kac formula gives us the price of a real discount bond

$D(x,t,T)$ with expiry T at time t as a solution of the following partial differential equation.

$$\frac{\partial D}{\partial t} + \mu_x \frac{\partial D}{\partial x} + \mu_\pi \frac{\partial D}{\partial \pi} + \frac{\sigma_x^2}{2}\frac{\partial^2 D}{\partial^2 x^2} + \frac{\sigma_\pi^2}{2}\frac{\partial^2 D}{\partial^2 \pi^2} + \rho\sigma_x\sigma_\pi \frac{\partial^2 D}{\partial \pi \partial x} = (x^+ - \pi)D. \tag{1.109}$$

Suppose that the real bond price is represented as $D(x,t,T) = e^{-f(x,\pi,\tau)}$ where $\tau = T - t$. Substituting this for (1.82), one obtains

$$\begin{aligned} \frac{\partial f}{\partial \tau} &= \mu_x \frac{\partial f}{\partial x} + \mu_\pi \frac{\partial f}{\partial \pi} + \frac{\sigma_x^2}{2}\frac{\partial^2 f}{\partial^2 x^2} + \frac{\sigma_\pi^2}{2}\frac{\partial^2 f}{\partial^2 \pi^2} + \rho\sigma_x\sigma_\pi \frac{\partial^2 f}{\partial \pi \partial x} + x^+ - \pi \\ &\quad + \frac{\sigma_x^2}{2}\left(\frac{\partial f}{\partial x}\right)^2 + \frac{\sigma_\pi^2}{2}\left(\frac{\partial f}{\partial \pi}\right)^2 + \rho\sigma_x\sigma_\pi \frac{\partial f}{\partial x}\frac{\partial f}{\partial \pi}. \end{aligned} \tag{1.110}$$

Suppose that the last three terms in the right-hand side of (1.110) are multiplied by ϵ where $0 < \epsilon \leq 1$. Also, consider that $f(x,\tau)$ has an asymptotic expansion.

$$f = f_0 + \epsilon f_1 + \epsilon^2 f_2 + \cdots. \tag{1.111}$$

Repeating the same argument for nominal bonds, the first term f_0 is given by

$$f_0(x,\pi,t,T) = \int_t^T \mathrm{E}_t[x_s^+]ds - \int_t^T \mathrm{E}_t[\pi_s]ds. \tag{1.112}$$

The second term f_1 is given by

$$\begin{aligned} f_1(x,t,T) &= \int_t^T \mathrm{E}_t\left[-\frac{\sigma_{x_s}^2}{2}\left(\frac{\partial f_0(x_s,\pi_s,s,T)}{\partial x_s}\right)^2 - \frac{\sigma_{\pi_s}^2}{2}\left(\frac{\partial f_0(x_s,\pi_s,s,T)}{\partial \pi_s}\right)^2 \right. \\ &\qquad \left. -\rho\sigma_{x_s}\sigma_{\pi_s}\frac{\partial f_0(x_s,\pi_s,s,T)}{\partial x_s}\frac{\partial f_0(x_s,\pi_s,s,T)}{\partial \pi_s}\right] ds. \end{aligned} \tag{1.113}$$

Discretizing (1.113), one obtains an approximate real bond pricing formula. Let us denote a real bond yield with maturity $\tau = n\delta t$ at time t with $r_t^n(x)$. We obtain

$$\begin{aligned} r_t^n(x) &\approx \frac{1}{n}\sum_{k=0}^{n}(V_x(x,t,t+k\delta t) - V_\pi(\pi,t,t+k\delta t)) \\ &\quad -\frac{1}{n}\sum_{k=0}^{n}\mathrm{E}_t\left[\frac{\sigma_{x_{t+j\delta t}}^2}{2}\sum_{j=k}^{n}\Delta^x(x_{t+j\delta t},t+j\delta t,T)^2 + \frac{\sigma_{\pi_{t+j\delta t}}^2}{2}\sum_{j=k}^{n}\Delta^\pi(\pi_{t+j\delta t},t+j\delta t,T)^2\right] \\ &\quad -\frac{1}{n}\sum_{k=0}^{n}\sum_{j=k+1}^{n}\mathrm{E}_t\left[\rho\sigma_{x_{t+j\delta t}}\sigma_{\pi_{t+j\delta t}}\Delta^x(x_{t+j\delta t},t+j\delta t,T)\Delta^\pi(\pi_{t+j\delta t},t+j\delta t,T)\right], \end{aligned} \tag{1.114}$$

where V^x, V^π, Δ^x and Δ^π are defined as

$$V^x(x,t,s) = \mathrm{E}_t[\max(x_s,0)], \qquad \Delta^x(x,t,s) = \frac{\partial V(x,t,s)}{\partial x}, \tag{1.115}$$

$$V^\pi(\pi,t,s) = \mathrm{E}_t[\pi_s], \qquad \Delta^\pi(\pi,t,s) = \frac{\partial V^\pi(\pi,t,s)}{\partial \pi}. \tag{1.116}$$

1.7.7 Performance of the approximations

From Table 1.6 to Table 1.11, I show the comparisons between approximate nominal bond yields and the benchmark yields computed based on Monte Carlo simulation with different model parameters for a two-factor Vasicek model. The top panel of each table shows the comparison between the first-order approximation and the benchmark. The bottom panel shows the comparison between the second-order approximation with the benchmark yields. One can see that the second-order approximation outperforms the first-order approximation in almost all cases. Both approximations perform more poorly as shadow rate volatility and maturity increase.

Table 1.1: Summary statistics of nominal bond yields

The table shows the average nominal bond yields for the US Treasury bonds from October of 1991 to October of 2015 and the Japanese government bonds from July of 2004 to October of 2015. "Before ZIRP" denotes the sub-sample period starting on October of 1991 and ending on October of 2008, which is two and a half months before the Federal Reserve announced the zero interest rate policy (ZIRP). "After ZIRP" is the sub-sample period during January of 2009 to October of 2015. "FFR" is the Federal Funds Rates. "Before QQE" is the sub-sample period that starts on July of 2004 and ends on January of 2013, which is one quarter before the Bank of Japan announced Quantitative Qualitative Easing (QQE). "After QQE" is the sub-sample period from April of 2013 to October of 2015. "ON" stands for overnight uncollateralized interest rate.

Maturity (years)	FFR/ON	1	2	3	4	5	7	10
US								
Full Sample	3.02%	3.00%	3.27%	3.52%	3.76%	3.97%	4.35%	4.77%
Before ZIRP	4.19%	4.10%	4.37%	4.58%	4.77%	4.94%	5.24%	5.58%
AfterZIRP	0.14%	0.29%	0.57%	0.91%	1.26%	1.59%	2.15%	2.75%
Japan								
Full Sample	0.15%	0.22%	0.29%	0.38%	0.49%	0.59%	0.81%	1.17%
Before QQE	0.18%	0.27%	0.36%	0.47%	0.60%	0.72%	0.97%	1.36%
After QQE	0.05%	0.05%	0.06%	0.07%	0.12%	0.16%	0.29%	0.55%

Table 1.2: Summary statistics of CPI inflation forecasts

The table shows summary statistics of survey-based CPI forecasts for the US and Japan. For the US case, the data is from Survey of Professional Forecasters. For the Japanese case, the data is from Quick Monthly Market Survey of Bond. The average CPI inflation forecasts are computed after adjusting a consumption tax hike in April of 2013. "Realized" means realized inflation during the corresponding time period. "Avg" is the average forecast number of CPI for each horizon. "Std" denotes the dispersion of opinions that is computed as the cross-sectional standard deviation of CPI forecasts. "Before ZIRP" denotes the sub-sample period from October of 1991 to October of 2008, which is two and a half months before the Federal Reserve announced the zero interest rate policy (ZIRP). "After ZIRP" is the sub-sample period from January of 2009 to October of 2015. "Before QQE" means that the sub-sample period that starts on July of 2004 and ends on January of 2013, which is one quarter before the Bank of Japan announced Quantitative Qualitative Easing (QQE). "After QQE" is the sub-sample period from April of 2013 to October of 2015.

	Realized	1y-Avg	2y-Avg	10y-Avg	1y-Std	2y-Std	10y-Std
US							
Entire Sample	2.32%	2.42%	-	2.63%	0.98%	-	0.52%
Before ZIRP	2.70%	2.62%	-	2.77%	0.88%	-	0.51%
After ZIRP	1.38%	1.94%	-	2.30%	1.21%	-	0.54%
Japan							
Entire Sample	0.03%	0.19%	0.52%	1.10%	0.36%	0.41%	0.60%
Before QQE	-0.15%	0.07%	0.36%	1.04%	0.27%	0.35%	0.59%
After QQE	0.60%	0.54%	1.02%	1.27%	0.67%	0.60%	0.63%

Table 1.3: Average absolute pricing errors

The table shows summary statistics of the absolute pricing error for each maturity in both the US and Japanese cases. Absolute errors are computed as the difference between model-implied bond yields and actual bond yields. "Before ZIRP" denotes the sub-sample period starting on October of 1991 and ending on October of 2008, which is two and a half months before the Federal Reserve announced zero interest rate policy (ZIRP). "After ZIRP" is the sub-sample period from January of 2009 to October of 2015. "Before QQE" means the sub-sample period that starts on July of 2004 and ends on January of 2013, which is one quarter before the Bank of Japan announced Quantitative Qualitative Easing (QQE). "After QQE" is the sub-sample period from April of 2013 to October of 2015. "Avg" is the average of absolute pricing error across all maturities.

Maturity (years)	1	2	3	4	5	7	10	Avg
US								
Full Sample	0.30%	0.17%	0.15%	0.17%	0.22%	0.30%	0.40%	0.24%
Before ZIRP	0.36%	0.16%	0.14%	0.19%	0.26%	0.35%	0.42%	0.27%
After ZIRP	0.17%	0.18%	0.15%	0.12%	0.13%	0.16%	0.36%	0.18%
Japan								
Full Sample	0.06%	0.05%	0.05%	0.06%	0.07%	0.07%	0.07%	0.07%
Before QQE	0.07%	0.05%	0.05%	0.06%	0.07%	0.08%	0.08%	0.07%
After QQE	0.06%	0.06%	0.05%	0.05%	0.06%	0.04%	0.04%	0.05%

Table 1.4: Estimates of the New Keynesian model parameters

This table provides the estimated parameters of the structural New Keynesian macro finance model for the US and Japan. Asymptotic standard errors are computed as the estimate of the Fisher information matrix. For each parameter, $*$ denotes statistical significance at the 5% level. † means that the number is given by the assumptions explained in the main text.

Description of parameters	Notation	US	Japan
Sensitivity of real output to real interest rate	ϕ	0.0096^*	0.0095^*
Sensitivity of inflation to output gap	κ	0.0106^*	0.0250^*
Mean-reverting level of potential output	$\bar{y_n}$	$0.00^\dagger$	$0.00^\dagger$
Sensitivity of policy rate to output gap	γ_y	$0.50^\dagger$	$0.50^\dagger$
Sensitivity of policy rate to inflation gap	γ_π	$0.50^\dagger$	$0.50^\dagger$
Constant term in IS equation	α_{IS}	0.00	0.00
Equilibrium real interest rate	i^*	$1.02^*\%$	$-2.60^*\%$
Effective lower bound	$\bar{i}$	$0.17^*\%$	$0.10^*\%$
Bias of survey-based inflation forecast	α_s^{40}	$0.36^*\%$	$-1.00^*\%$
Dependence on lagged real output	μ_y^-	0.543^*	0.505^*
Dependence on expected real output	μ_y^+	0.540^*	0.493^*
Dependence on expected inflation	μ_π	0.914^*	0.692^*
Dependence on lagged shadow rate	μ_x	0.072^*	0.094^*
Dependence on lagged potential output	μ_{yn}	0.001^*	0.541^*
IS shock volatility	σ_{IS}	0.0036^*	0.0031^*
AS shock volatility	σ_{AS}	0.0030^*	0.0030^*
Shadow interest rate volatility	σ_x	0.0093^*	0.0080^*
Potential output volatility	σ_{yn}	0.0028^*	0.0027^*
Inflation survey noise	σ_s	0.0087^*	0.0100^*
Observation noise	σ_{ob}	0.0012^*	0.0010^*

Table 1.5: Estimates of the market price of risk

This table provides the estimated parameters of the market price of risk for the US and Japan. λ^0 is a constant term. λ^1 is the sensitivity with respect to the state variables $X_t = (y_t, \pi_t, x_t, y_t^n)^T$. For example, λ^1_{i3} is the third column of λ^1. Recall that λ^1_{41}, λ^1_{42}, λ^1_{43} are equal to zero by assumption. Asymptotic standard errors are computed as the estimate of the Fisher information matrix. For each parameter, $*$ and $**$ denote statistical significance at the 5% level and at the 1% level, respectively.

	λ^0	λ^1_{i1}	λ^1_{i2}	λ^1_{i3}	λ^1_{i4}
US					
	0.90^{**}	-22.10^{**}	-0.59^{**}	-28.66^{**}	21.51^{**}
	6.25^{**}	24.21^{**}	-256.34^{**}	-26.82^{**}	-31.52^{**}
	-0.12^{**}	4.18^{**}	8.53^{**}	-4.98^{**}	-3.40^{**}
	0.00	0.00	0.00	0.00	3.02^{**}
Japan					
	-0.48^{**}	-25.99^{**}	1.23^{**}	-12.11^{**}	0.27^{**}
	-0.89^{**}	46.43^{**}	-122.62^{**}	-21.09^{**}	-13.43^{**}
	-0.57^{**}	5.95^{**}	17.61^{**}	-9.56^{**}	-5.22^{**}
	0.00	0.00	0.00	0.00	-206.19^{**}

Table 1.6: Performance of approximation-based nominal bond yields given different maturities and parameters L_0

This table shows comparison of approximation-based nominal bond yields with a bench mark based on Monte Carlo simulation for different maturities and different settings of L_0. The numbers in the table show the difference between the two prices. The absolute error is reported for each maturity. A stochastic process for a shadow interest rate is given by $x_t = L_t + S_t$ where $dL_t = \sigma_L dW_t$ and $dS_t = \kappa(\theta_S - S_t)dt + \sigma_S dW_t$ where $\sigma_L = 0.005$, $\theta_S = 0.01$, $\kappa = 1.0$, $\sigma_S = 0.005$, $\rho dt = dW_L dW_S = 0$. The initial value for $L_0 = 0.01$ and $S_0 = -0.05$. The number of simulation paths is 10000 for Monte Carlo simulation.

L_0/years	1	2	3	4	5	7	10
First-order approximation							
-3.0%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
-1.0%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.02%
1.0%	0.00%	0.01%	0.01%	0.01%	0.00%	0.01%	0.03%
3.0%	0.04%	0.01%	0.01%	0.01%	0.02%	0.04%	0.06%
5.0%	0.13%	0.07%	0.05%	0.04%	0.05%	0.06%	0.08%

L_0/years	1	2	3	4	5	7	10
Second-order approximation							
-3.0%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	0.06%
-1.0%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%
1.0%	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
3.0%	0.03%	0.01%	0.00%	0.00%	0.01%	0.02%	0.02%
5.0%	0.13%	0.13%	0.04%	0.04%	0.04%	0.04%	0.04%

Table 1.7: Performance of approximation-based nominal bond yields given different maturities and parameters σ_L

This table shows comparison of approximation-based nominal bond yields with a bench mark based on Monte Carlo simulation for different maturities and different settings of σ_L. The numbers in the table show the difference between the two prices. The absolute error is reported for each maturity. A stochastic process for a shadow interest rate is given by $x_t = L_t + S_t$ where $dL_t = \sigma_L dW_t$ and $dS_t = \kappa(\theta_S - S_t)dt + \sigma_S dW_t$ where $\sigma_L = 0.005$, $\theta_S = 0.01$, $\kappa = 1.0$, $\sigma_S = 0.005$, $\rho dt = dW_L dW_S = 0$. The initial value for $L_0 = 0.01$ and $S_0 = -0.05$. The number of simulation paths is 10000 for Monte Carlo simulation.

σ_L/years	1	2	3	4	5	7	10
First-order approximation							
0.4%	0.00%	0.00%	0.01%	0.01%	0.00%	0.01%	0.03%
0.8%	0.01%	0.00%	0.00%	0.01%	0.01%	0.03%	0.07%
1.2%	0.01%	0.00%	0.00%	0.01%	0.03%	0.07%	0.14%
1.6%	0.01%	0.00%	0.00%	0.01%	0.03%	0.09%	0.19%
2.0%	0.02%	0.03%	0.05%	0.08%	0.11%	0.18%	0.32%

σ_L/years	1	2	3	4	5	7	10
Second-order approximation							
0.4%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.01%
0.8%	0.01%	0.00%	0.00%	0.01%	0.01%	0.00%	0.00%
1.2%	0.01%	0.00%	0.01%	0.01%	0.00%	0.00%	0.01%
1.6%	0.01%	0.00%	0.01%	0.02%	0.02%	0.02%	0.02%
2.0%	0.02%	0.02%	0.02%	0.03%	0.03%	0.03%	0.01%

Table 1.8: Performance of approximation-based nominal bond yields given different maturities and parameters S_0

This table shows comparison of approximation-based nominal bond yields with a bench mark based on Monte Carlo simulation for different maturities and different settings of S_0. The numbers in the table show the difference between the two prices. The absolute error is reported for each maturity. A stochastic process for a shadow interest rate is given by $x_t = L_t + S_t$ where $dL_t = \sigma_L dW_t$ and $dS_t = \kappa(\theta_S - S_t)dt + \sigma_S dW_t$ where $\sigma_L = 0.005$, $\theta_S = 0.01$, $\kappa = 1.0$, $\sigma_S = 0.005$, $\rho dt = dW_L dW_S = 0$. The initial value for $L_0 = 0.01$ and $S_0 = -0.05$. The number of simulation paths is 10000 for Monte Carlo simulation.

S_0/years	1	2	3	4	5	7	10
First-order approximation							
-3.0%	0.01%	0.01%	0.01%	0.01%	0.01%	0.02%	0.05%
-1.0%	0.04%	0.02%	0.02%	0.02%	0.02%	0.03%	0.05%
1.0%	0.00%	0.00%	0.00%	0.01%	0.01%	0.03%	0.05%
3.0%	0.04%	0.02%	0.01%	0.00%	0.00%	0.01%	0.03%
5.0%	0.09%	0.04%	0.02%	0.01%	0.00%	0.01%	0.03%

S_0/years	1	2	3	4	5	7	10
Second-order approximation							
-3.0%	0.01%	0.01%	0.01%	0.00%	0.00%	0.01%	0.01%
-1.0%	0.04%	0.02%	0.01%	0.01%	0.01%	0.01%	0.02%
1.0%	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%
3.0%	0.04%	0.02%	0.01%	0.01%	0.01%	0.00%	0.01%
5.0%	0.09%	0.04%	0.02%	0.02%	0.01%	0.01%	0.04%

Table 1.9: Performance of approximation-based nominal bond yields given different maturities and parameters θ_S

This table shows comparison of approximation-based nominal bond yields with a bench mark based on Monte Carlo simulation for different maturities and different settings of θ_S. The numbers in the table show the difference between the two prices. The absolute error is reported for each maturity. A stochastic process for a shadow interest rate is given by $x_t = L_t + S_t$ where $dL_t = \sigma_L dW_t$ and $dS_t = \kappa(\theta_S - S_t)dt + \sigma_S dW_t$ where $\sigma_L = 0.005$, $\theta_S = 0.01$, $\kappa = 1.0$, $\sigma_S = 0.005$, $\rho dt = dW_L dW_S = 0$. The initial value for $L_0 = 0.01$ and $S_0 = -0.05$. The number of simulation paths is 10000 for Monte Carlo simulation.

θ_S/years	1	2	3	4	5	7	10
First-order approximation							
0.6%	0.00%	0.00%	0.00%	0.00%	0.00%	0.02%	0.04%
1.2%	0.00%	0.00%	0.01%	0.01%	0.00%	0.01%	0.03%
1.8%	0.01%	0.01%	0.01%	0.00%	0.00%	0.02%	0.04%
2.4%	0.01%	0.00%	0.01%	0.00%	0.00%	0.02%	0.05%
3.0%	0.02%	0.00%	0.01%	0.00%	0.00%	0.02%	0.05%

θ_S/years	1	2	3	4	5	7	10
Second-order approximation							
0.6%	0.00%	0.00%	0.01%	0.00%	0.00%	0.00%	0.01%
1.2%	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%
1.8%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%
2.4%	0.01%	0.00%	0.01%	0.01%	0.01%	0.00%	0.01%
3.0%	0.02%	0.00%	0.01%	0.01%	0.01%	0.00%	0.01%

Table 1.10: Performance of approximation-based nominal bond yields given different maturities and parameters κ_S

This table shows comparison of approximation-based nominal bond yields with a bench mark based on Monte Carlo simulation for different maturities and different settings of κ_S. The numbers in the table show the difference between the two prices. The absolute error is reported for each maturity. A stochastic process for a shadow interest rate is given by $x_t = L_t + S_t$ where $dL_t = \sigma_L dW_t$ and $dS_t = \kappa(\theta_S - S_t)dt + \sigma_S dW_t$ where $\sigma_L = 0.005$, $\theta_S = 0.01$, $\kappa = 1.0$, $\sigma_S = 0.005$, $\rho dt = dW_L dW_S = 0$. The initial value for $L_0 = 0.01$ and $S_0 = -0.05$. The number of simulation paths is 10000 for Monte Carlo simulation.

κ_S/years	1	2	3	4	5	7	10
First-order approximation							
0.4	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.03%
0.8	0.00%	0.00%	0.00%	0.00%	0.00%	0.01%	0.04%
1.2	0.00%	0.01%	0.01%	0.00%	0.01%	0.02%	0.05%
1.6	0.00%	0.02%	0.02%	0.01%	0.00%	0.02%	0.04%
2.0	0.02%	0.04%	0.03%	0.02%	0.01%	0.00%	0.02%

κ_S/years	1	2	3	4	5	7	10
Second-order approximation							
0.4	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
0.8	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%	0.00%
1.2	0.00%	0.01%	0.01%	0.00%	0.00%	0.01%	0.02%
1.6	0.00%	0.03%	0.02%	0.01%	0.01%	0.00%	0.00%
2.0	0.02%	0.04%	0.03%	0.02%	0.02%	0.02%	0.02%

Table 1.11: Performance of approximation-based nominal bond yields given different maturities and parameters σ_S

This table shows comparison of approximation-based nominal bond yields with a bench mark based on Monte Carlo simulation for different maturities and different settings of σ_S. The numbers in the table show the difference between the two prices. The absolute error is reported for each maturity. A stochastic process for a shadow interest rate is given by $x_t = L_t + S_t$ where $dL_t = \sigma_L dW_t$ and $dS_t = \kappa(\theta_S - S_t)dt + \sigma_S dW_t$ where $\sigma_L = 0.005$, $\theta_S = 0.01$, $\kappa = 1.0$, $\sigma_S = 0.005$, $\rho dt = dW_L dW_S = 0$. The initial value for $L_0 = 0.01$ and $S_0 = -0.05$. The number of simulation paths is 10000 for Monte Carlo simulation.

σ_S/years	1	2	3	4	5	7	10
First-order approximation							
0.4%	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.03%
0.8%	0.00%	0.01%	0.02%	0.02%	0.01%	0.01%	0.02%
1.2%	0.00%	0.00%	0.00%	0.00%	0.01%	0.01%	0.04%
1.6%	0.00%	0.01%	0.01%	0.00%	0.01%	0.03%	0.05%
2.0%	0.01%	0.01%	0.01%	0.00%	0.01%	0.02%	0.04%

σ_S/years	1	2	3	4	5	7	10
Second-order approximation							
0.4%	0.00%	0.01%	0.01%	0.02%	0.02%	0.01%	0.00%
0.8%	0.00%	0.01%	0.02%	0.02%	0.02%	0.01%	0.01%
1.2%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
1.6%	0.00%	0.01%	0.01%	0.01%	0.00%	0.01%	0.01%
2.0%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%

Figure 1.1: Evolution of the US Treasury bond yields

This figure provides a time series plot of US Treasury bond yields during the period from October of 1991 to October of 2015. The data is obtained from the website of the Federal Reserve Board of Governors. FFR is the federal funds rate. The details of computation of the nominal bond yields are found in Gürkaynak, Sack and Wright (2007).

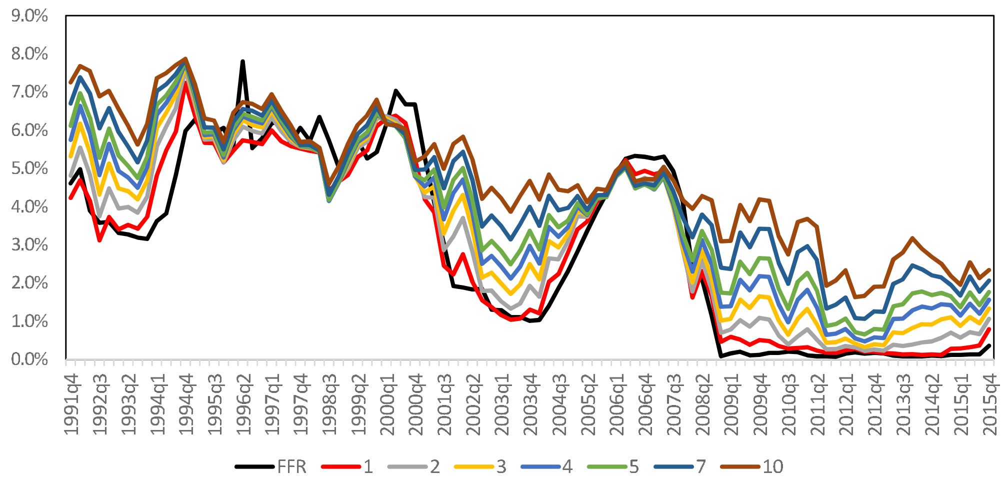


Figure 1.2: Evolution of the Japanese government bond yields

This figure provides a time series plot of the Japanese government bond yields during the period from July of 2004 to October of 2015. The data is obtained from the Ministry of Finance in Japan.

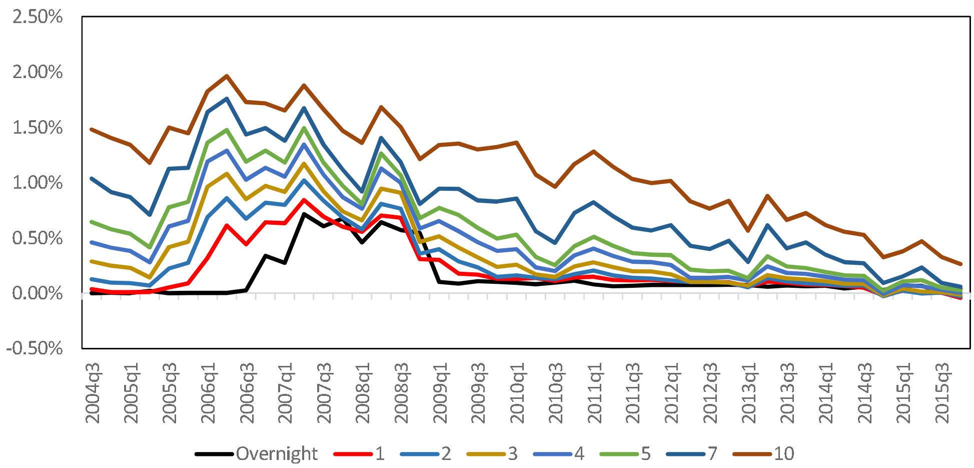


Figure 1.3: Evolution of the US shadow interest rates

This figure provides a time series plot of the shadow interest rate and the federal funds rate in the US during the period from October 1991 to October 2015. Estimates of the model parameters are in Tables 1.4 and 1.5.

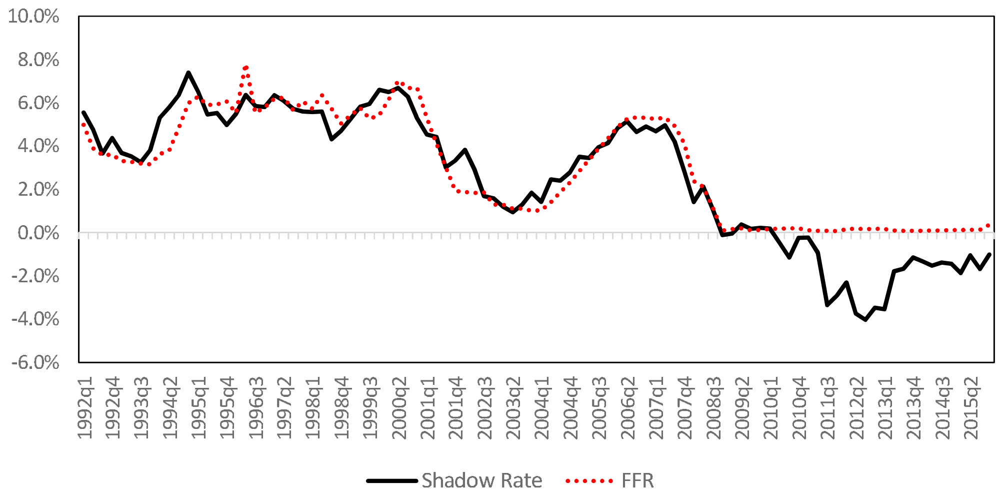


Figure 1.4: Raising the inflation target in the US during the normal period

This figure provides a counterfactual analysis of raising the inflation target for the US when the zero lower bound is not binding at the initial time period. Estimates of the model parameters are in Tables 1.4 and 1.5. The shadow interest rate x_t is equal to $+1\%(> 0)$. I call this setting normal period. Inflation $\pi_t = 2\%$ at the initial time period. Real and potential output are $y_t = y_t^n = 0\%$.

Figure 1.5: Raising the inflation target in the US during the ZLB period

This figure provides counterfactual analyses of (A) raising the inflation target and (B) suddenly ending the zero interest rate policy (ZIRP) for the US when the zero lower bound is binding at the initial time period. Estimates of the model parameters are in Tables 1.4 and 1.5. In benchmark case, I set $x_t = -1\%(< 0)$. I call this setting ZLB(zero lower bound) period. The benchmark case is colored black. For (A), I set the inflation target to 3% (red line). For (B), I set the shadow interest rate $x_t = 0\%$ (blue line).

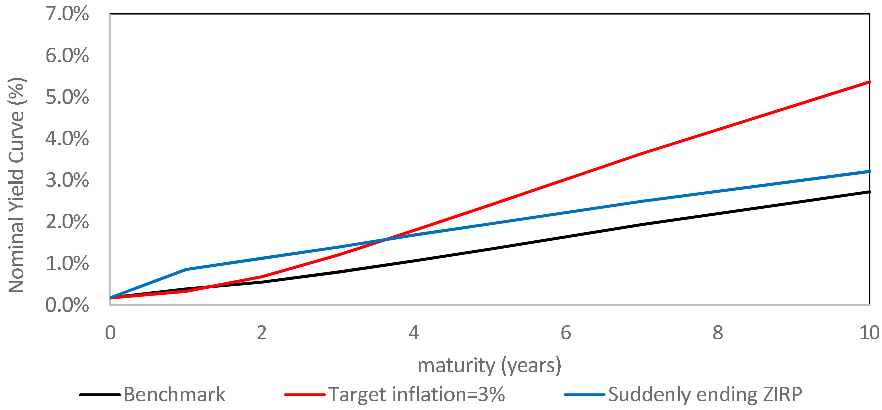


Figure 1.6: The response of macroeconomic variables given the counterfactual analysis of raising the inflation target in the US during the ZLB period

This figure shows the changes in three macroeconomic variables when the inflation target is increased from 2% to 3% during the ZLB period. Estimates of the model parameters are in Tables 1.4 and 1.5. The evolution of potential output is not shown since it is exogenously driven and thus does not change.

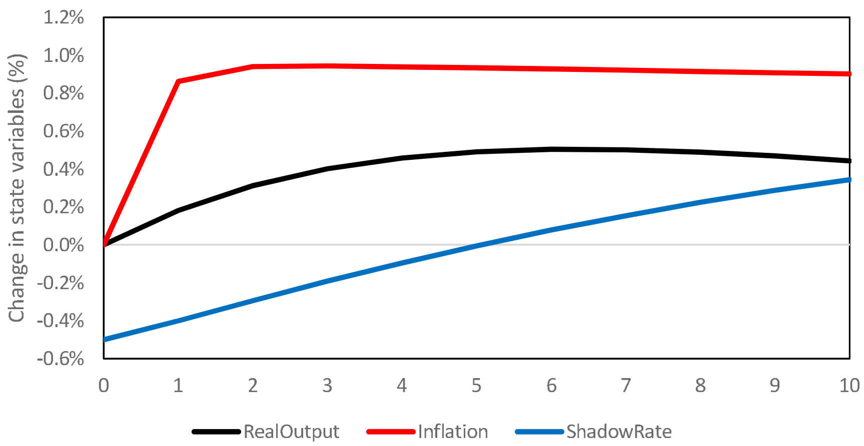


Figure 1.7: The response of macroeconomic variables given the counterfactual analysis of suddenly ending the zero interest rate policy in US

This figure shows the changes in three macroeconomic variables when a central bank suddenly abandons the zero interest rate policy. The shadow interest rate increases from -1% to 0%. Estimates of the model parameters are in Tables 1.4 and 1.5. The evolution of potential output is not shown since it is exogenously driven and thus does not change.

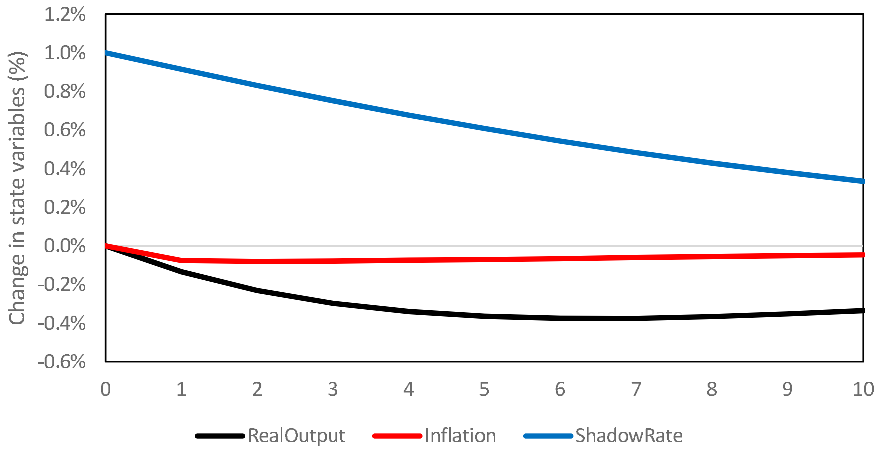


Figure 1.8: Decomposition of nominal bond yield changes given an increased inflation target during the ZLB period in the US

This figure shows the changes in expected (nominal) interest rates and term premium if the inflation target is hypothetically risen for the US when the zero lower bound is binding at the initial time period. Estimates of the model parameters are in Tables 1.4 and 1.5. The shadow interest rate x_t is equal to -1%. I call this setting ZLB(zero lower bound) period. Inflation $\pi_t = 2\%$ at the initial time period. Real and potential output are $y_t = y_t^n = 0\%$. The target is changed from 2% to 3%.

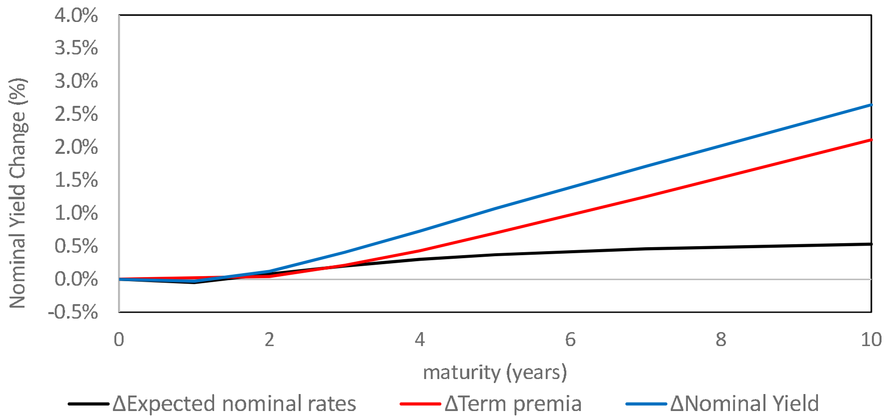


Figure 1.9: Raising the inflation target in Japan during the normal period

This figure provides a counterfactual analysis of raising the inflation target for Japan when the zero lower bound is not binding at the initial time period. Estimates of the model parameters are in Tables 1.4 and 1.5. The shadow interest rate x_t is equal to $+1\%(> 0)$. I call this setting normal period. Inflation $\pi_t = 2\%$ at the current time period. Real and potential output are $y_t = y_t^n = 0\%$.

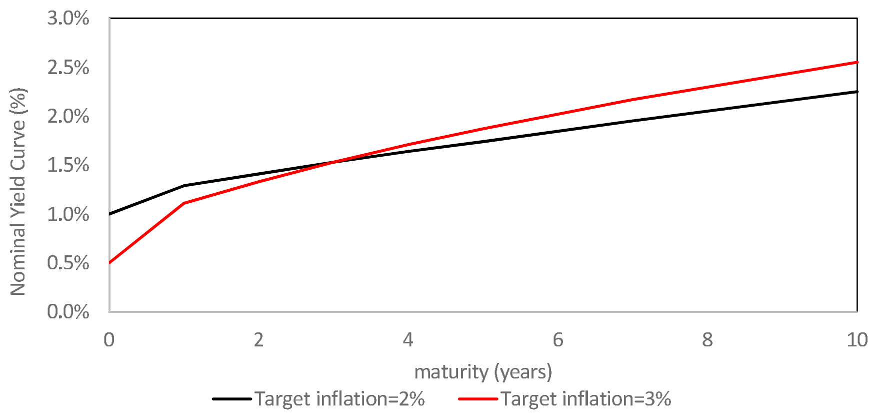


Figure 1.10: Raising the inflation target in Japan during the ZLB period

This figure provides counterfactual analyses of (A) raising the inflation target and (B) suddenly ending the zero interest rate policy (ZIRP) for Japan when the zero lower bound is binding at the initial time period. Estimates of the model parameters are in Tables 1.4 and 1.5. In benchmark case, I set $x_t = -1\%(< 0)$. I call this setting ZLB(zero lower bound) period. The benchmark case is colored black. For (A), I set the inflation target to 3% (red line). For (B), I set the shadow interest rate $x_t = 0\%$ (blue line).

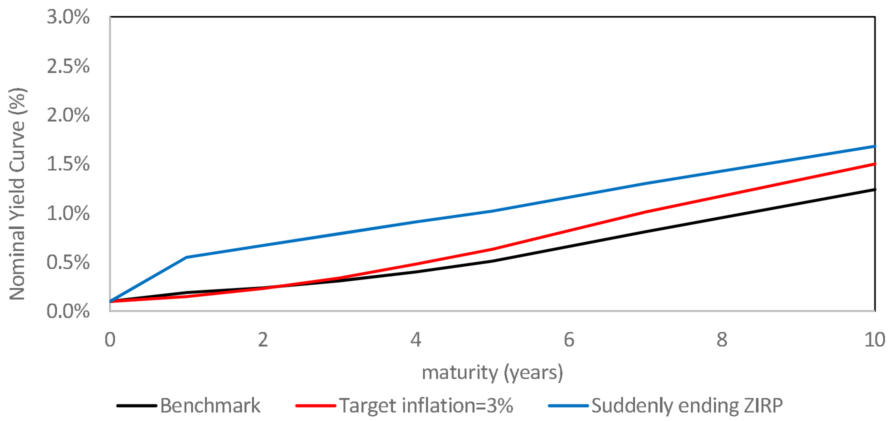


Figure 1.11: Introducing the negative lower bound of nominal interest rates in the US

This figure provides a counterfactual analysis of introducing the negative lower bound for the US when the shadow interest rate is negative. The effective lower bound $\bar{i}$ is set to -0.5%. Estimates of the model parameters are in Tables 1.4 and 1.5. Shadow interest rate x_t is equal to -1.0%. Inflation π_t is 2.0%. Real and potential output are $y_t = y_t^n = 0\%$.

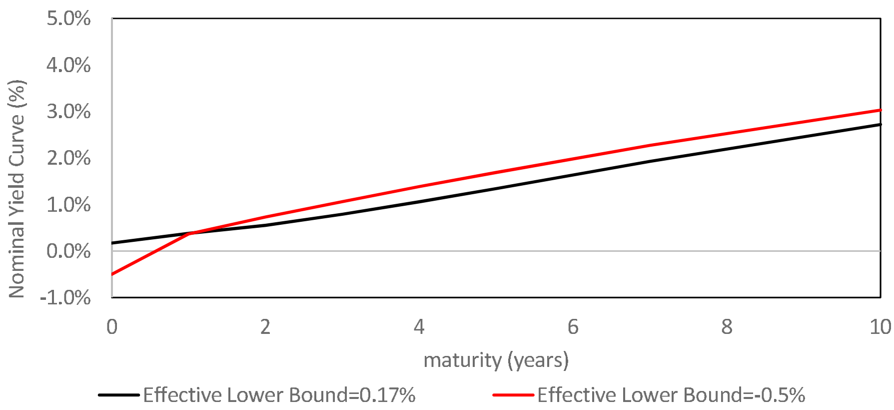


Figure 1.12: Introducing the negative lower bound of nominal interest rates in Japan

This figure provides a counterfactual analysis of introducing the negative lower bound for Japan when the shadow interest rate is negative. The effective lower bound $\bar{i}$ is set to -0.5%. Estimates of the model parameters are in Tables 1.4 and 1.5. Shadow interest rate x_t is equal to -1.0%. Inflation π_t is 2%. Real and potential output are $y_t = y_t^n = 0\%$.

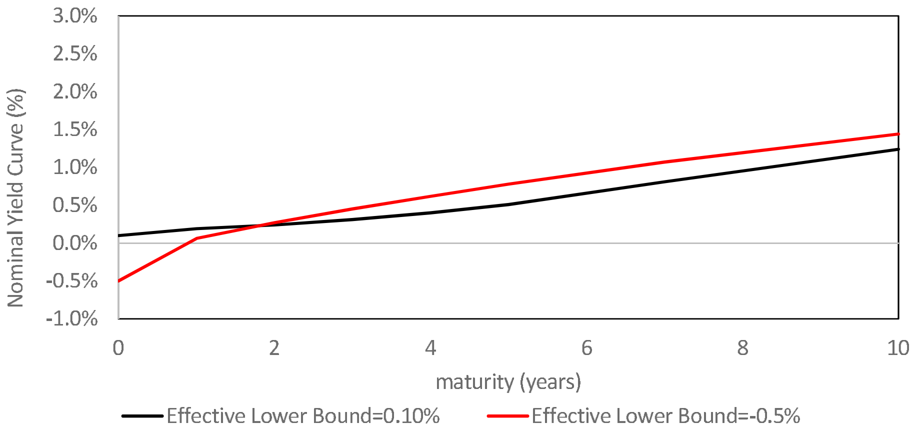
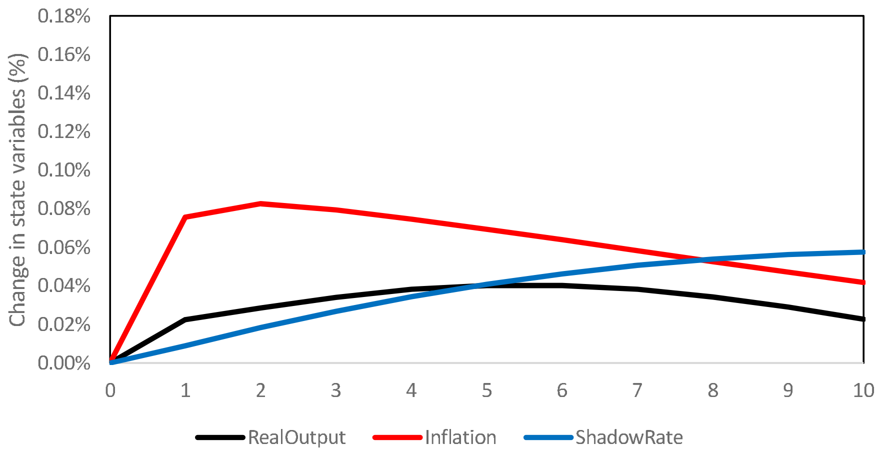


Figure 1.13: The response of macroeconomic variables given the counterfactual analysis of introducing the negative lower bound in the US

This figure shows the changes in three macroeconomic variables when a central bank introduces the negative lower bound for US. when the shadow interest rate is negative. The effective lower bound $\bar{i}$ is set to -0.5%. Estimates of the model parameters are in Tables 1.4 and 1.5. Shadow interest rate x_t is equal to -1.0%. Inflation π_t is 2.0%. Real and potential output are $y_t = y_t^n = 0\%$.



CHAPTER 2

Revisiting empirical findings during the zero lower bound period

2.1 Introduction

In this chapter, I investigate whether empirical findings documented before the ZLB period hold during the ZLB period. As in the previous studies, I investigate how macroeconomic risks are priced in the yield curves by looking at decompositions of the yield curves and the factor loading of the shape of the yield curves. There are several notable findings.

First, I decompose the nominal bond yields into the expected interest rate and the term premium. For both the US and Japan, the term premium is larger for longer maturities. This finding is consistent with previous studies. In the case of Japan, the nominal bond yields are almost entirely explained by the term premium. This is not surprising since the Bank of Japan has been employing the zero interest rate policy during the 2004-2015 period studied in this paper.[1]

Second, I decompose nominal bond yields into expected real interest rates, real interest rate risk premium, expected inflation, and inflation risk premium. I show that the nominal bond yield curve is upward sloping mostly due to the real interest rate risk premium in the US during the 1991-2015 period. Similarly, an upward-sloping nominal bond yield is mostly explained by the real interest rate risk premium in the case of Japan. The expected inflation implied by the nominal

[1]Precisely speaking, the Bank of Japan hiked its policy interest rate to the positive level during the period between July of 2006 and December of 2008.

bond yield is also contributing to an upward-sloping nominal bond yield. Yet, it is negative or close to zero for all maturities. This is reasonable because the historical average of realized inflation is almost zero as confirmed in Table 1.2.

Third, following Wright (2011), I investigate whether a larger cross-sectional dispersion in survey-based inflation forecasts explains a larger inflation risk premium. Wright (2011) conducts the same analysis for ten industrialized countries during the 1990-2010 period. He finds a statistically significant positive relationship between the cross-sectional dispersion and the inflation premium. Yet, he employs a conventional affine term structure model to obtain the inflation premium. I extend his analysis by modeling the zero lower bound explicitly. I find that the signs of the coefficients are positive and thus consistent with his findings in almost all maturities in the case of Japan, although the coefficients are not statistically significant. In the US case, only the sign of the coefficient for 10-year bond is positive. All of the coefficients are not statistically significant for the US. Thus, I find limited support for Wright (2011)'s findings during the zero lower bound period.

Fourth, the model also provides a unified understanding of the macroeconomic drivers of yield curves. It is well known that (1) variations in yield curves are explained by level, slope and curvature factors and (2) level and slope factors are often associated with inflation and real economic activity. Yet, once the zero lower bound is binding, there is no clear level factor.[2] Without a macro-finance term structure model that incorporates a zero lower bound, it is difficult to have a consistent explanation of macroeconomic drivers of yield curves. The model in this research offers a better understanding of macroeconomic effects on term structure of interest rates both before and after the zero interest rate policy is employed. For example, in the case of the US, the results show that shadow interest rate functions as a level factor during normal times but it works as a slope factor

[2]Figures 2.1 and 2.2 show the results of the principal component analysis before and during the zero lower bound period. In Figure 2.2, there is no clear level factor.

during the zero lower bound time period.

The rest of the second chapter is organized as follows. Section 2.2 briefly discusses the literature. Section 2.3 explains the methodology and the dataset. Section 2.4 presents main results. Section 2.5 concludes the chapter.

2.2 Literature review

There have been many studies on decomposing the yield curve into an expected real interest rate, real rate risk premium, expected inflation and the inflation risk premium. For example, Wright (2011) estimates affine term structure models and documents that the term premium has been declining since the early 1990 for both the US and Japan. Abrahams et al. (2015) find that the US nominal bond risk premium is mostly driven by the real interest risk premium rather than the inflation and the liquidity risk premium.

With respect to variance decompositions of nominal bond yields, Ang, Bakaert and Wei (2008) document that the variations in expected inflation and the inflation risk premium explain the most of the variations in nominal bond yields for long maturities.

The relationship between the shape of yield curves and macroeconomic variables has been also well studied. Wu (2006) estimates a New Keynesian macro finance model for the US economy and documents that the monetary policy shocks affect the shape of yield curves mostly by changing its slope. On the other hand, technology shocks affect the shape of yield curves by shifting its level. Hördhal, Tristani and Vestin (2006) also report that a monetary policy shock functions as a slope factor. Bakaert, Cho and Moreno (2010) document that inflation target shocks function as a level factor and monetary policy shocks are associated with both slope and curvature factors. By contrast, Moench (2013) estimates a reduced-form term structure model for the US economy and finds that the curvature factor is correlated with economic slowdowns.

Compared to the strands of the literature discussed above, there are a few number of studies on the relationship between inflation risk premium and a cross-sectional dispersion of inflation forecasts. Wright (2011) uses the cross-sectional dispersion of inflation forecasts as a proxy for inflation uncertainty and finds that the inflation risk premium is positively associated with the dispersion of inflation forecasts for the US. Buraschi and Whelan (2012) document that disagreement about the real economy predicts excess bond returns but disagreement about inflation does not forecast excess bond returns strongly in the case of the US.

2.3 Data and estimation methodology

I employ the same model described in the first chapter and use the same parameters reported in Tables 1.4 and 1.5. Cross-sectional dispersions in survey-based inflation forecasts are computed as cross-sectional standard deviations of survey-based inflation forecasts. The data of cross-sectional dispersions in survey-based inflation forecasts is from FRED at the website of the Federal Reserve Bank of St. Louis. For Japan, the data is from QUICK Corp.

2.4 Results

2.4.1 Decompositions of yields

Table 2.1 shows decompositions of nominal bond yields into the expected interest rate and the term premium for the US and Japan. The numbers are the average values during the entire sample period. For both countries, the term premium is larger for longer maturities. This finding is consistent with previous studies. In the case of Japan, the nominal bond yields are almost entirely explained by the term premium. This is not surprising since the Bank of Japan has been employing the zero interest rate policy during the sample period except the period between July of 2006 and December of 2008.

Table 2.2 shows decompositions of nominal bond yields into the expected real rates, the real interest rate risk premium, the expected inflation, and the inflation risk premium for the US and Japan.

In the US case, one can see that an upward-sloping nominal bond yield curve arises mostly from the real interest rate risk premium. The expected real interest rates and the inflation risk premium are also contributing to an upward-sloping nominal bond yield curve. Expected inflation is almost constant across maturities.

Similarly, an upward-sloping nominal bond yield is mostly explained by the real interest rate risk premium in the case of Japan. The real interest rate risk premium is negative for 1-year maturity but it is positive for longer maturities. The expected inflation is contributing to an upward-sloping nominal bond yield curve as it is higher for longer maturities. Yet, the expected inflation is negative or close to zero for all maturities. This is natural because the historical average of realized inflation is almost zero as confirmed in Table 1.2. By contrast, the term structures of the expected real interest rates and the inflation risk premium are downward sloping in the case of Japan.

2.4.2 Dispersion of inflation forecasts and inflation risk premium

Following Wright (2011), I conduct regression analyses to investigate whether the inflation risk premium can be explained by disagreement about future inflation as summarized in Table 2.3. Wright (2011) finds that a larger cross-sectional dispersion of inflation forecasts predicts a larger inflation risk premium. Yet, his term structure model abstracts the zero lower bound. I extend his analysis by taking this into account. I find that the signs of coefficients are negative except for the ten year maturity in the US. However, none of these coefficients are statistically significant. By contrast, the signs of coefficients are positive except tor the ten year maturity in Japan. Again, none of these coefficients are statistically significant. [3] Overall, I confirm the very weak positive relationship between the inflation risk premium and the dispersion of inflation forecasts only in the case of Japan. [4]

2.4.3 Variance decompositions

Table 2.4 presents variance decompositions of nominal bond yields into the expected interest rate variance and the term premium variance. Notice that the sum of variances of these two components is not equal to the variance of nominal yields since there is a covariance term. Also notice that the table shows standard deviations, not variance itself.

In the US case, the variance of nominal yields is explained mostly by the expected interest rate variance for the short maturity, but the term premium variance is more dominant for longer maturity.

In the case of Japan, the variance of nominal bond yields arises from the variance of the term premium, not the variance of the expected nominal interest rates. This is realistic because the policy interest rate in Japan has been staying at zero percent during the almost entire sample period.

Table 2.5 presents variance decompositions of nominal bond yields into the variances of the expected real interest rate, the real rate premium, the expected inflation, and the inflation risk premium. Again, notice that the sum of variances

[3] There is a possibility that deflation risk premium (negative inflation risk premium) positively depends on cross-sectional inflation dispersion. Thus, I conduct the following asymmetric regressions:

$$\Delta ip_{t+1}(\tau) \quad = \quad \alpha^{ip} + \beta^{Std+}\Delta Std_t \cdot 1_{\pi_t>0} + \beta^{Std-}\Delta Std_t \cdot 1_{\pi_t<0} + \omega_t^{ip},$$

where $ip_t(\tau)$ is the inflation risk premium of τ-year maturity at time t. α^{ip}, β^{Std+} and β^{Std-} are constants. ω_t^{ip} is distributed from $N(0, \sigma^{ip})$. ΔStd_t is the time change in the cross-sectional standard deviation of CPI forecasts. Yet, the results are not statistically significant.

[4] I also investigate the relationship between the term premium and the dispersion of inflation forecasts. I obtain quantitatively similar results. For both countries, the sign of coefficient for the 10 year maturity term premium is positive but the signs of the other coefficients are negative.

of these two components is not equal to the variance of nominal yields since there are covariance terms. Consistent with the finding in Table 2.4, the contributions of the variances of the real interest rate risk premium and the inflation risk premium are larger for longer maturity for both the US and Japan.

2.4.4 Factor loadings relating yield curves to macroeconomic variables

Figures 2.1 and 2.2 show factor loadings of the first three factors of nominal bond yields in a principal component analysis (PCA) in the US case. In Figure 2.1, I conduct PCA for the nominal bond yields during from October of 1991 to October of 2008. In Figure 2.2, I use the data from January of 2009 to October of 2015 when the Federal Reserve had been employing the zero interest rate policy. In Figure 2.1, it is clear that PCA1, PCA2, PCA3 correspond to level, slope, curvature factors, respectively. In Figure 2.2, such a clear mapping is difficult. Both PCA 1 and PCA 2 behave like slope factors. This means that the principal component analysis cannot provide us a unified understanding of the macroeconomic drivers of yield curves.

Figures 2.3 and 2.4 enable us to understand economic drivers of nominal bond yields with and without the zero interest rate policy in the US case. These two figures show factor loadings to four different macroeconomic variables. In Figure 2.3, the initial shadow interest rate is positive ($x = 1.00\%$) but it is negative ($x = -1.00\%$) in Figure 2.4. Figure 2.3 shows that the shadow interest rate works as a level factor. The other three variables function as slope factors. In Figure 2.4, there is no clear level factor. All four macroeconomic factors function as slope factor but with different magnitudes.

Figures 2.5 and 2.6 show factor loadings of those four variables in the case of Japan. In Figure 2.5, the initial shadow interest rate is positive ($x = 1.00\%$), In Figure 2.6 it is negative ($x = -1.00\%$). The results are almost same as the US case. One difference is that output functions as a curvature factor in Japan rather than a slope factor.

2.5 Conclusion

In this chapter, I revisit empirical findings documented by previous studies conducted before the zero lower bound period. I use the model with an explicit zero lower bound.to include the 2008-2015 period. The model developed in the first chapter allows us to have a consistent understanding of term structure dynamics before and after the zero interest rate policy. Thus, there is no need to divide a full sample period into two subsample periods.

Most of empirical findings documented before the zero lower bound period hold during the zero lower bound period. Yet, I find that a monetary policy factor functions as a slope factor once the zero lower bound is binding. Also, I report that the relationship between the inflation risk premium and a cross-sectional dispersion of inflation forecasts is not statistically significant.

Table 2.1: Decomposition of nominal bond yields into two components

This table provides a decomposition of nominal bond yields into (1) the expected path of the nominal interest rate and (2) the term premium. The details of the computation are as follows: First, I compute nominal bond yields under both the physical measure and the risk-neutral measure. Second, expected interest rate path components are computed as nominal bond yields under the physical measure. Third, term premium components are computed as the difference between the nominal bond yields under the physical and the risk-neutral measure. Nominal yields are estimated model-implied yields. These nominal yields are equal to the sum of expectation component of the nominal interest rate and term premium. The numbers are the time average values for a specific maturity of each component during the sample period.

Maturity (year)	1	2	3	4	5	7	10
US							
Nominal yields	2.91%	3.29%	3.53%	3.74%	3.93%	4.27%	4.72%
Expectation	2.96%	3.03%	3.09%	3.14%	3.18%	3.23%	3.28%
Term premium	-0.05%	0.27%	0.45%	0.60%	0.75%	1.04%	1.44%
Japan							
Nominal yields	0.24%	0.30%	0.36%	0.44%	0.52%	0.77%	1.16%
Expectation	0.11%	0.10%	0.10%	0.10%	0.10%	0.10%	0.10%
Term premium	0.14%	0.19%	0.26%	0.33%	0.41%	0.67%	1.06%

Table 2.2: Decomposition of nominal bond yields into four components

This table provides a decomposition of nominal bond yields into (1) the expected path of the real interest rate, (2) the real risk premium, (3) the expected inflation, and (4) the inflation risk premium. The details of the computation are as follows: First, I obtain nominal and real bond yields under both the physical measure and the risk-neutral measure. Second, expected real yields are computed as those yields under the physical measure. Third, expected inflation is calculated as the difference between nominal yields and real yields under the physical measure. Fourth, the real rate risk premium is computed as the differences between the real bond yields under the physical and the risk-neutral measure. Fifth, the inflation premium is computed as the differences between the nominal bond yields under the physical and the risk-neutral measure and further subtracting the real premium. Nominal yields are estimated model-implied yields. The numbers are the time average values for a specific maturity during the sample period.

Maturity (year)	1	2	3	4	5	7	10
US							
Nominal yields	2.91%	3.29%	3.53%	3.74%	3.93%	4.27%	4.72%
Expected real yields	0.69%	0.77%	0.84%	0.90%	0.95%	1.01%	1.07%
Real risk premium	-0.48%	-0.21%	-0.10%	0.00%	0.11%	0.36%	0.74%
Expected inflation	2.28%	2.26%	2.25%	2.24%	2.23%	2.22%	2.21%
Inflation risk premium	0.42%	0.48%	0.54%	0.60%	0.63%	0.68%	0.70%
Japan							
Nominal yields	0.24%	0.30%	0.36%	0.44%	0.52%	0.77%	1.16%
Expected real yields	0.50%	0.46%	0.42%	0.38%	0.34%	0.30%	0.26%
Real risk premium	-0.20%	0.07%	0.32%	0.58%	0.85%	1.52%	2.54%
Expected inflation	-0.39%	-0.36%	-0.31%	-0.27%	-0.24%	-0.20%	-0.16%
Inflation risk premium	0.34%	0.13%	-0.06%	-0.24%	-0.44%	-0.85%	-1.48%

Table 2.3: Regressions of the inflation risk premium on cross-sectional dispersion of inflation forecasts

This table provides estimates of coefficients in the regressions of quarterly changes in the inflation risk premium for n-year maturity nominal bond on previous quarter changes in the cross-sectional dispersion (ny-Std). For each parameter, $*$ denotes statistical significance at the 5% level based on the t-statistic.

Maturity (year)	1	2	3	4	5	7	10
US							
1y-Std	-0.103	-0.107	-0.098	-0.088	-0.077	-0.059	-
10y-Std	-	-	-	-	-	-	0.056
Japan							
1y-Std	1.621	-	-	-	-	-	-
2y-Std	-	1.576	1.454	1.355	1.280	1.186	-
10y-Std	-	-	-	-	-	-	-0.600

Table 2.4: Variance decomposition of nominal bond yields into two components

This table provides a variance decomposition of τ-year nominal bond yields $y_t^{rn}(\tau)$ into (1) the expected path of the nominal interest rate $y_t^{ph}(\tau)$ and (2) the term premium $p_t(\tau)$. The following equation is used:

$$\mathrm{var}(y_t^{rn}(\tau)) = \mathrm{var}(y_t^{ph}(\tau)) + \mathrm{var}(p_t(\tau)) + 2\mathrm{cov}(y_t^{ph}(\tau), p_t(\tau)).$$

Notice that the numbers in this table are standard deviations.

Maturity (year)	1	2	3	4	5	7	10
US							
Nominal yield	0.49%	0.48%	0.48%	0.47%	0.45%	0.42%	0.37%
Expected yield	0.42%	0.36%	0.31%	0.26%	0.22%	0.17%	0.12%
Term premium	0.12%	0.15%	0.19%	0.22%	0.24%	0.26%	0.26%
Japan							
Nominal yield	0.09%	0.12%	0.15%	0.17%	0.19%	0.22%	0.27%
Expected yield	0.03%	0.03%	0.02%	0.02%	0.02%	0.02%	0.02%
Term premium	0.08%	0.11%	0.13%	0.16%	0.18%	0.21%	0.25%

Table 2.5: Variance decomposition of nominal bond yields into four components

This table provides a variance decomposition of τ-year nominal bond yields $y_t^{rn}(\tau)$ into (1) the expected path of the real interest rate $r_t^{ph}(\tau)$, (2) the real interest rate premium $rp_t(\tau)$, (3) the expected inflation $\pi_t(\tau)$ and (4) the inflation risk premium $ip_t(\tau)$. The following equation is used:

$$\mathrm{var}(y_t^{rn}(\tau)) = \mathrm{var}(r_t^{ph}(\tau)) + \mathrm{var}(rp_t(\tau)) + \mathrm{var}(\pi_t(\tau)) + \mathrm{var}(ip_t(\tau)) + \mathrm{cov}.$$

"cov" denotes the remaining covariance terms. Notice that the numbers in this table are standard deviations.

Maturity (year)	1	2	3	4	5	7	10
US							
Nominal yield	0.49%	0.48%	0.48%	0.47%	0.45%	0.42%	0.37%
Expected real yield	0.47%	0.41%	0.35%	0.30%	0.25%	0.19%	0.14%
Real risk premium	0.53%	0.45%	0.39%	0.36%	0.34%	0.32%	0.30%
Expected inflation	0.06%	0.05%	0.05%	0.04%	0.04%	0.03%	0.02%
Inflation risk premium	0.51%	0.45%	0.40%	0.35%	0.32%	0.26%	0.20%
Japan							
Nominal yield	0.09%	0.12%	0.15%	0.17%	0.19%	0.22%	0.27%
Expected real yield	0.26%	0.20%	0.16%	0.13%	0.11%	0.08%	0.06%
Real risk premium	0.53%	0.48%	0.45%	0.44%	0.43%	0.48%	0.61%
Expected inflation	0.26%	0.19%	0.15%	0.12%	0.10%	0.07%	0.05%
Inflation risk premium	0.54%	0.48%	0.44%	0.41%	0.39%	0.38%	0.43%

Figure 2.1: Principal component analysis of nominal bond yields during the normal period

This figure provides factor loadings of nominal bond yields to the first three factors of nominal bond yields in principal component analysis: I conduct principal component analysis for nominal bond yields from October of 1991 to October of 2008 (normal period) for the US and plot factor loadings. During this time period, the policy rate is positive.

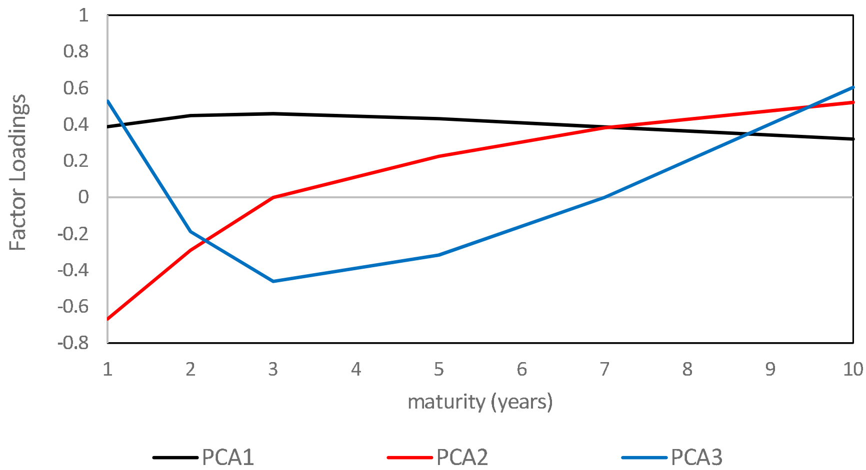


Figure 2.2: Principal component analysis of nominal bond yields during the ZLB period

This figure provides factor loadings of nominal bond yields to the first three factors of nominal bond yields in principal component analysis: I conduct principal component analysis for nominal bond yields from January of 2009 to October of 2015 (ZLB period) for the US and plot factor loadings. During this time period, the Federal Reserve employs the zero interest rate policy.

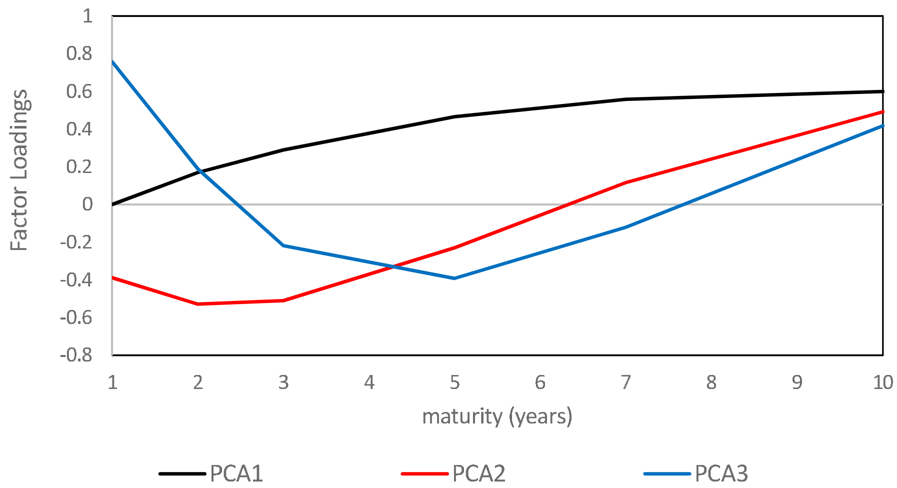


Figure 2.3: Factor loadings in the US during the normal period

This figure provides factor loadings of nominal bond yields to the macroeconomic variables in the case of the US. The shadow interest rate is positive $x_t = 1\%(> 0)$. I call this setting normal period. Inflation $\pi_t = 2\%$. Real and potential output are $y_t = y_t^n = 0\%$. I compute the change in the nominal bond yields given the change in each variable. The numbers shown are calculated relative to the 10-year change.

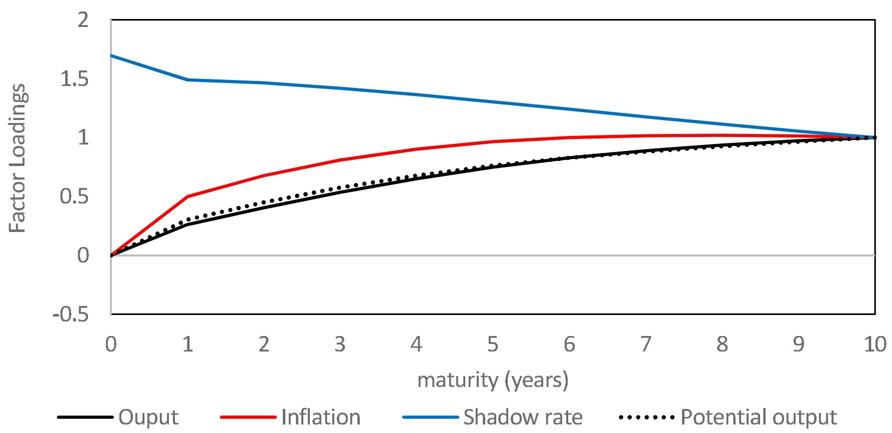


Figure 2.4: Factor loadings in the US during the ZLB period

This figure provides factor loadings of nominal bond yields to the macroeconomic variables in the case of the US. The shadow interest rate is negative $x_t = -1\%(< 0)$. I call this setting ZLB (zero lower bound) period. Inflation $\pi_t = 2\%$. Real and potential output are $y_t = y_t^n = 0\%$. I compute the change in the nominal bond yields given the change in each variable. The numbers shown are calculated relative to the 10-year change.

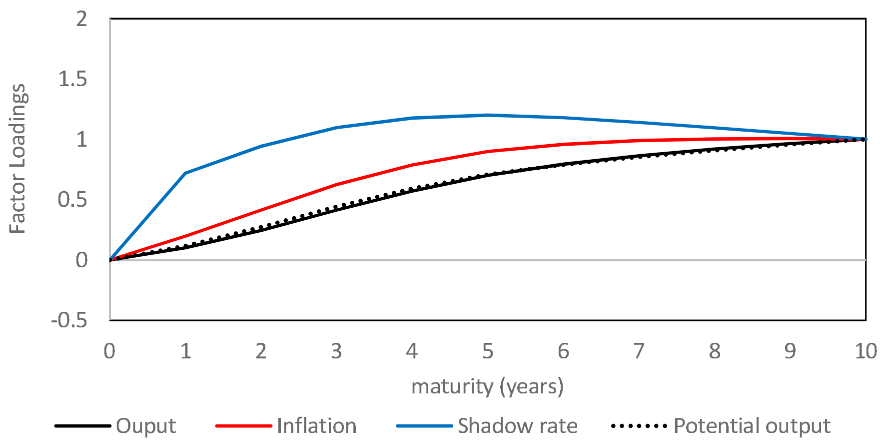


Figure 2.5: Factor loadings in Japan during the normal period

This figure provides factor loadings of nominal bond yields to the macroeconomic variables in the case of Japan. The shadow interest rate is positive $x_t = 1\%$. Inflation $\pi_t = 0\%$. Real and potential output are $y_t = y_t^n = 0\%$. I compute the change in the nominal bond yields given the change in each variable. The numbers shown are relative to the 10-year change.

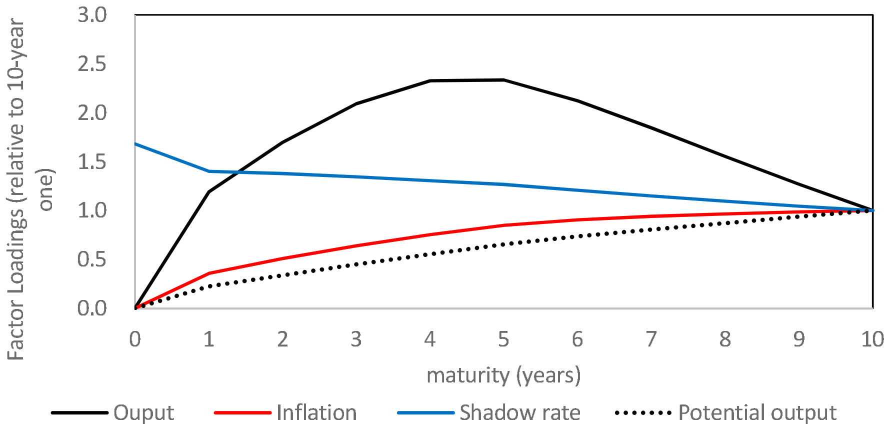
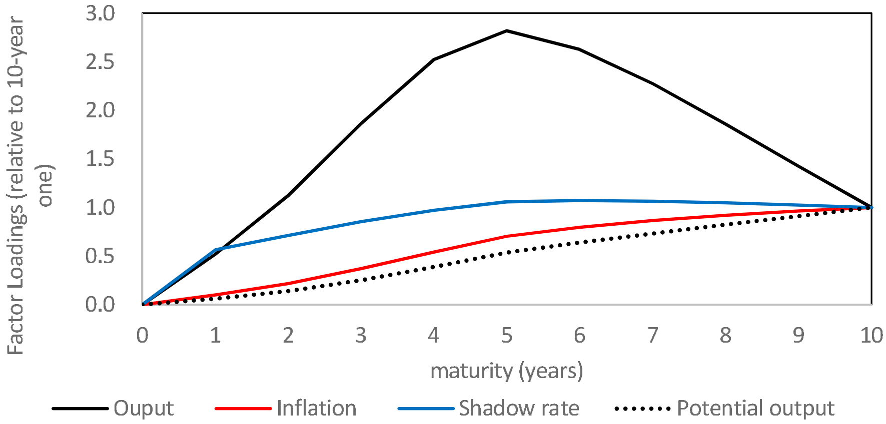


Figure 2.6: Factor loadings in Japan during the ZLB period

This figure provides factor loadings of nominal bond yields to the macroeconomic variables in the case of Japan. The shadow interest rate is negative $x_t = -1\%(< 0)$. I call this setting ZLB (zero lower bound) period. Inflation $\pi_t = 0\%$. Real and potential output are $y_t = y_t^n = 0\%$. I compute the change in the nominal bond yields given the change in each variable. The numbers shown are relative to the 10-year change.



CHAPTER 3

Additional modeling and empirical exercises

3.1 Introduction

In this chapter, I conduct two different types of additional exercises.

As an additional modeling exercise, I incorporate a Markov regime switching feature into the New Keynesian macro finance model. Specifically, I allow a long-run real interest rate to depend on a regime. I assume that there are two regimes: a high long-run real interest regime and a low long-run real interest regime. I estimate the extended model for the US economy. [1] I find that the low long-run real interest rate was dominant during the 2009-2011 period but the high long-run real interest rate regime has been dominant since 2012.

As an additional application of the benchmark model, I study the excess sensitivity of long-distant real forward interest rates to changes in the short-term nominal rate in Japan. This has been studied as a puzzle in the US.[2] I document that there is a similar excess sensitivity in Japan. Utilizing a comprehensive dataset of fixed income investors, I investigate the duration adjustment effect proposed by Hanson and Stein (2015). Their hypothesis is as follows. When short-term interest rates are lowered, fixed income investors re-balance their bond portfolios

[1] Due to the computational difficulty in solving the New Keynesian macroeconomic model with both the zero lower bound and Markov regime switching features, I model the zero lower bound only in bond pricing but not in the New Keynesian macroeconomic dynamics in this additional modeling exercise.

[2] The puzzle is that a short-term nominal interest rate impacts long-distant real forward rate although nominal changes are supposed not to have such a strong effect on long-distant real interest rates in a standard New Keynesian model.

toward longer-term bonds in an effort to keep their portfolio returns unchanged. Such a re-balancing further lowers the longer-term bond yields. This feedback effect is a key mechanism in creating the excess sensitivity in the hypothesis.

One issue in their study is that they use a crude measure of portfolio duration held by commercial banks and primary dealers due to limited availability of data in the US. For commercial banks, as a proxy for duration, they use the average fraction of non-trading account securities with a current remaining maturity or next repricing date of one year or longer. For primary dealers, they use the data from the Federal Reserve Bank of New York that categorizes bonds into four buckets: shorter than 3 years, 3 to 6 years, 6 to 11 years and longer than 11 years.

The dataset of Japanese fixed income investors used in this chapter is more detailed. It allows us to see each institutional investor's duration across several financial sectors such as banking, asset management and insurance. Equipped with this dataset, I investigate the hypothesis of Hanson and Stein (2015). Specifically, I conduct two different types of regressions. In the first regression analysis, I study the impact of each financial sector's change in duration on nominal bond yields.[3] In the second regression analysis, I investigate the reverse causality: whether a change in nominal bond yields induces investor's adjustment of duration. Although the signs of some coefficients are consistent with the hypothesis of Hanson and Stein (2015), I do not find statistically significant results in the first or second regression analyses. Overall, the results do not strongly support the hypothesis of Hanson and Stein (2015).

The rest of the third chapter proceeds as follows. Section 3.2 reviews the literature related to each of the two additional exercises. Section 3.3 explains the additional modeling exercise. Section 3.4 discusses the additional empirical exercise. Section 3.5 concludes this chapter.

[3]Fixed income investors are quite segregated in terms of their bond maturities. For example, banks are active in short-term maturities while insurance companies are more active in long-term bonds. In my regression analyses, I take into account this heterogeneity.

3.2 Literature review

3.2.1 Regime-switching term structure model of interest rates

Motivated by empirical evidence of a structural change in macroeconomic variables or a central bank's policy stance, several papers employ Markov-regime switching term structure models. Ang, Bakaert and Wei (2008) present an affine term structure model with regime switching where a nominal short rate is driven by three factors including expected inflation. They show that unconditional real rate curve is flat and around 1.3% although it is strongly downward sloping in one regime. Li, Li and Yu (2013) study the effect of a central bank's monetary policy stance on the term structure of interest rates using regime switching. They find that the Federal Reserve is more proactive in one regime than in other regime and the stance contributes to Great Moderation. Bikbov and Chernov (2013) formulate a term structure model in which inflation and real GDP are governed by a forward-looking macroeconomic model with regime switching. They allow both monetary policy stance and macroeconomic volatility to be regime-dependent. A new aspect of the model in this research is an explicit consideration of zero lower bound in nominal bond pricing under regime-switching term structure model.

3.2.2 Excess sensitivity of long-term real interest rates

This chapter contributes to empirical studies of excess sensitivity of long-term forward interest rates to short-term nominal interest rates. Gürkaynak, Sack and Swanson (2005) is the first study to document it for the US case during 1990 and 2002. They find that it is driven by a private sectors' change in long-term expected inflation. They also show that the similar pattern is not observable in the case of UK Treasury bond markets. Nakamura and Steinsson (2013) conduct a similar study and show that it is mostly driven by a change in forward real rates. Hanson and Stein (2015) propose a hypothesis that some investors are yield-oriented and those investor's re-balancing of their fixed-income portfolio impacts on long-term real interest rates. I study the excess sensitivity of long-term forward interest rates in the case of Japan with a comprehensive dataset of fixed income investors.

3.3 Additional modeling exercise: Regime-switching macroeconomic dynamics

Recently, there is a debate that an equilibrium real interest rate has been declining during the last decade. For example, Summers (2014) argues that an equilibrium real interest rate in the US has recently declined by citing the estimates based on Laubach and Williams (2003) model and lists several factors explaining such a downward trend of the real interest rates.[4] King and Low (2014) document that weighted real interest rates across G7 countries have been gradually declining since 1985. Motivated by such a debate, I incorporate regime switching feature to a target shadow interest rate. Specifically, I assume that a target shadow interest rate depends on two regimes, $s_t = u, d$.

$$x_t^{target} \quad = \quad r^*(s_t) + \pi_t + \gamma_y(y_t - y_t^n) + \gamma_\pi(\pi_t - \bar{\pi}_t). \tag{3.1}$$

It is tough to allow regime switching feature in the case of IS-ZLB. Thus, I focus on the case of IS-SR when employing (3.1). Given this modification, the four equations are rewritten as the following forward-backward looking equation.

$$AE_t[X_{t+1}] = BX_t + CX_{t-1} + D(s_t) + \epsilon_t, \tag{3.2}$$

where the coefficient matrices A, B, C are same as those in the case of IS-SR. The only vector D becomes regime-dependent.

$$D(s_t) = (\alpha_{IS}, 0, \mu_x(r^*(s_t) - \gamma_\pi\bar{\pi}), \mu_y\bar{y^n})^T. \tag{3.3}$$

[4]See also Teulings and Baldwin (2014) for reference.

There are several techniques to solve regime-dependent forward-backward equation.[5] In this special case, one can apply a similar method used to solve a forward and backward looking equation without a regime-switching feature. Let us guess the solution takes the following form.

$$X_{t+1} = FX_t + G(s_t) + H\epsilon_t, \tag{3.4}$$

where F, G, H solve

$$AF^2 - BF - C = 0, \quad G(s_t) = (AF - B + A)^{-1}D(s_t), \quad H = (AF - B)^{-1}. \tag{3.5}$$

It is easy to see that F does not depend on regime s_t. Thus, one can otain F first and then compute G and H with already obtained F.

In pricing nominal bonds, we need to specify a function of market price of risk. For simplicity, I assume that factor-dependent market price of risk is independent of regimes.

$$\mu_t = \mu^0(s_t) + \lambda^1 X_t. \tag{3.6}$$

The constant market price of risk is allowed to depend on the regime s_t in general. Yet, I assume that μ^0 is independent of the regime s_t in my empirical analysis below. The evolution of the four stochastic factors under the risk-neutral measure is given by

$$X_{t+1} = F^Q X_t + G^Q(s_t) + H\epsilon_t, \tag{3.7}$$

where F^Q and $G^Q(s_t)$ are given by

$$F^Q = F - H\mu^1, \tag{3.8}$$

$$G^Q(s_t) = G - H\mu^0(s_t). \tag{3.9}$$

The approximate formula for nominal bond yields under regime switching is briefly discussed in the Appendix 1.7.5. The approach to obtain the formula is same as the one without regime switching.

[5] Davig and Leeper (2007), Farmer, Waggoner and Zha (2009, 2011) and Cho (2009) define and discuss the concept of solutions based on minimum state variable. Svensson and Williams (2005) also proposed a simple algorithm to solve this type of equations.

3.3.1 Data and estimation methodology

When estimating shadow interest rate models with regime switching, I simply apply a standard methodology to estimate regime switching model for VAR system developed by Hamilton (1989).

In addition to that, I develop a regime-switching unscented Kalman filtering as an additional robustness check. I integrate Kim(1994)'s method for regime-switching dynamic linear model with unscented Kalman filtering.[6] The second approach is just for a robustness check. All of results are based on the first approach.

3.3.2 Results

Table 3.1 shows estimates of the parameters of an extended joint model with Markov regime switching. I estimate only regime-dependent parameters and observation noise and keep other parameters fixed as in Tables 1.4 and 1.5. Thus, I report only the estimates of the regime-dependent long-run real interest rates $r^*(s_t)$ in this table. The long-run real rate $r^*(u)$ is equal to 1.17% in the high regime and $r^*(d)$ is equal to 0.91% in the low regime. This estimate is in line with the estimated range of the long-run real interest rates reported in Laubach and Williams (2015). One can see that the long-run real interest rate $r^*(d)$ in the low regime is still positive.

Figure 3.1 shows that the low regime was dominant during 2009-2011 period. Yet, the high regime has been more dominant since 2012. This indicates that the long-run real interest rate for the US temporarily declined during the Great Recession but the US economy re-entered the high long-run real rate regime in 2012.

[6] Kim (1994) developed algorithm to estimate state space models with Markov regime switching. The detail is Appendix 3.6.

3.4 Additional empirical exercise: Impact of yield-oriented investors on bond yields

3.4.1 Data and estimation methodology

Hanson and Stein (2015) argue that investors' adjustment of their fixed-income portfolio can explain excess sensitivities of long-term real interest rates to short-term nominal interest rates. Motivated by their study, I conduct a simple statistical analysis to see whether duration adjustments by fixed income investors are impacting interest rates and, reversely, that interest rates changes incentive to the fixed income investors to adjust their durations (average maturity). One big difference in my research from Hanson and Stein (2015) is that the dataset in this chapter allows me to see each fixed income investor's duration (average duration of their bonds).

A comprehensive survey of Japanese fixed income investors provided by QUICK Corp allows us to see the average duration (maturity) of each fixed income investor. Utilizing that data, I study how medium and long-term bond yields are lowered by fixed income investors when they extend their average maturity given lower bond yields of maturities to which they are exposed. For brevity, I call this impact the duration adjustment effect.

The duration adjustment effect is related to a portfolio re-balancing effect often used to justify the real economic effect of large-scale asset purchases. A portfolio re-balancing effect is that investors increase the position in risky assets when the returns from those assets currently held by them are lowered.

To quantify the duration adjustment, I consider the two way effects between financial sector's duration and bond yields in the following econometric framework.

First, I formulate the impact of duration on bond yields.

$$\Delta y_t^{actual}(\tau) \quad = \quad \alpha^{D-y} + \Delta y_t^{model}(\tau) + \beta^{D-y}\Delta D_{t-1} + \epsilon_t^{D-y}, \qquad (3.10)$$

where D_t is a vector of durations for financial sectors. In the following analysis, I define $D_t = (D_t^{Bank}, D_t^{AM}, D_t^{Insurance})^T$ where D_t^{Bank}, D_t^{AM}, and $D_t^{Insurance}$ are the average duration of banking sector, asset management sector and insurance sector, respectively. $y_t^{actual}(\tau)$ and $y_t^{model}(\tau)$ are a $N \times 1$ vector of actual and model-implied τ-year bond yields. N is the number of bond yields used for the estimation. Δ is a time lag operator. α^{D-y} is a $N \times 1$ vector and β^{D-y} is a $N \times 3$ matrix. The distribution of error term ϵ_t^{D-y} follows a multi-variate normal distribution with zero mean and covariance matrix Σ^{D-y}.

Next, I model the impact of bond yields on duration.

$$\Delta D_t = \alpha^{y-D} + \beta^{y-D} \Delta y_{t-1}^{actual}(\tau) + \epsilon_t^{y-D}, \tag{3.11}$$

where α^{y-D} is a 3×1 vector and β^{y-D} is a $3 \times N$ matrix. The distribution of error term ϵ_t^{y-D} follows a multi-variate normal distribution with zero mean and covariance matrix Σ^{y-D}.

Figure 3.2 shows the model-implied sensitivity of real forward rates to 2-year yield change given a shadow rate policy shock under the physical measure as well as the actual sensitivity.[7] The actual sensitivity is computed using yield changes before and after monetary policy meetings between April of 2006 and December of 2008 during which the policy rate was above zero percent. It tells us that model-implied sensitivity is not so persistent compared to the actual one. This is in line with previous studies in the US.

Table 3.2 is the summary statistics of this data. It shows the average durations across the banking, asset management, and insurance sectors. Notice that the banking sector's duration ranges from 3.48 to 4.4 years while the asset management sector's duration ranges from 5.01 to 8.32 years. The insurance sector's duration is distributed from 5.66 to 9.99 years. Figure 3.3 shows a time series plot of those average durations. The average duration of the banking sector is stable.

[7] When the sensitivities are computed under the risk-neutral measure, they are almost flat and around one since shadow interest rates function as a level factor.

The average durations of the asset management sector and the insurance sector have been increasing over time.

3.4.2 Results

Table 3.3 shows the interaction between the average durations and nominal bond yields. The top and the bottom of the table shows estimated coefficients of regressions, (3.10) and (3.11), respectively. In these regressions, I take into account the range of each financial sector's duration. For example, I do not regress residual components of 10-year nominal bond yields on the banking sector's duration since the banking sector's duration ranges from 3 to 5 years. It is difficult to imagine that this could impact on 10-year nominal yield directly.

The top of the table shows that changes in the average durations have little effect on changes in the residual components of nominal yields. Some coefficients are negative but not statistically significant. The bottom of the table indicates that the changes in yields have little effect on the average durations. For example, the average maturities of the bonds held by the asset management sector become longer if nominal bond yields with 5 years are lowered. The average maturities of the bonds held by the insurance sector become longer if nominal bond yields with 7 and 10 years are lowered. However, these coefficients are not statistically significant. Also, the coefficients of the banking sector's duration are not negative. Given these two results, it is difficult to argue that there exists a strong effect of the fixed-income investors' bond portfolio adjustment on bond yields.[8]

[8]I also conduct the regression (3.11) at individual investor level. The average coefficient is negative and thus supports the hypothesis of re-balancing but not statistically significant.

3.5 Conclusion

In this chapter, I conduct two different types of additional exercises based on the model developed in the first chapter. First, I incorporate a Markov regime switching feature into the New Keynesian macro finance model with the zero lower bound for nominal bond pricing. Specifically, I allow a long-run real interest rate to depend on a regime. I find that the US economy was staying at the low regime during 2009-2011 period but re-entered the high regime in 2012. Second, I study the excess sensitivity of long-distant real forward interest rates to changes in the short-term nominal rate using a dataset of Japanese fixed income investors. The dataset allows us to observe each investor's duration. I investigate whether a portfolio-re-balancing effect helps us understand the excess sensitivity of long-distant real forward interest rates. I cannot find any strong evidence.

3.6 Appendix: Regime-switching unscented Kalman filtering

I explain the algorithm of regime-switching unscented Kalman filtering that combines unscented Kalman filtering developed by Julier and Uhlmann (2004) and Kalman filtering for regime-switching state space model proposed by Kim (1994). In unscented Kalman filtering, I match the first and second moments of the distribution of observable variables using several sequences of latent variables. In fact, Unscented Kalman Filtering can be understood as the special case of Quasi-Monte Carlo filtering where the distribution is approximated by the quasi-random sequences.

Let us denote the vector of yields with different maturities at time t with $y_t = (y(t,\tau_1), y(t,\tau_2), \cdots, y(t,\tau_N))$. Here, τ_n denotes n-th maturity. I also define $h(x_t)$ as the function of the yield vector y_t with respect to the vector of latent variables. x_t denotes the state variables. The number of regimes is L.

Similar to linear Kalman filtering, unscented Kalman filtering consists of two operations: prediction and filtering. First, I describe the prediction algorithm. Second, I explain filtering algorithm. Both of these algorithms are done given the current and previous regimes. Thus, in the final part, I show how to take into account regime changes given transition matrix and per-period log likelihood.

3.6.1 Prediction

I introduce σ points that are the collection of the vectors $\{\hat{x}^i_{t|t-1}(m)\}$ $(m = 1, \cdots, 2d+1)$. Here, d is the number of latent variables. i denotes the i-th regime. In this paper, the model has $d = 4$ factors. Each σ point is calculated as follows.

$$\hat{x}^i_{t|t}(0) = \hat{x}^i_{t|t-1}, \tag{3.12}$$

$$\hat{x}^i_{t|t}(m) = \hat{x}^i_{t|t-1} + \left(\sqrt{(d+\lambda)P^i_{t|t-1}}\right)_m, \quad (3.13)$$

$$\hat{x}^i_{t|t}(m+d) = \hat{x}^i_{t|t-1} - \left(\sqrt{(d+\lambda)P^i_{t|t-1}}\right)_m. \quad (3.14)$$

To step forward in time, I calculate the vector of latent variables $\hat{x}$ at time $t+1$ for each σ point.

$$\hat{x}^{i,j}_{t+1|t}(m) = f^j(\hat{x}^i_{t|t}(m)), \quad (3.15)$$

where the function $f^j(x_t)$ governs the time evolution of latent variables. In this paper, given linear market price of risk and the VAR(1) dynamics of macroeconomic variables, the function $f(\cdot)$ is specified as

$$x^{i,j}_{t+1|t}(m) = F^j x^i_{t|t}(m) + G^j, \quad (3.16)$$

where F^j and G^j are defined in (1.46).

I then take an average of the $2d$ vectors $\hat{x}^i_{t+1|t}$ as the predicted value.

$$\hat{x}^{i,j}_{t+1|t} = \sum_{m=0}^{2d} W^{(m)} \hat{x}^{i,j}_{t+1|t}(m). \quad (3.17)$$

I also calculate the covariance matrix $P^{i,j}_{t+1|t}$.

$$P^{i,j}_{t+1|t} = \sum_{m=0}^{2d} W^{(m)} [\hat{x}^{i,j}_{t+1|t}(m) - \hat{x}^{i,j}_{t+1|t}][\hat{x}^{i,j}_{t+1|t}(m) - \hat{x}^{i,j}_{t+1|t}]' + Q^i, \quad (3.18)$$

where Q^i is the covariance matrix of system error and it is given by $H\Sigma H'$ where the matrix $\Sigma = \text{diag}([\sigma^2_y, \sigma^2_\pi, \sigma^2_x, \sigma^2_{yn}])$.

3.6.2 Filtering

Second, I explain the filtering algorithm. I compute σ points as I have done in the prediction algorithm.

$$\hat{x}^{i,j}_{t|t-1}(0) = \hat{x}^{i,j}_{t|t-1}, \quad (3.19)$$

$$\hat{x}^{i,j}_{t|t-1}(m) = \hat{x}^{i,j}_{t|t-1} + \left(\sqrt{(d+\lambda)P^{i,j}_{t|t-1}}\right)_m, \quad (3.20)$$

$$\hat{x}^{i,j}_{t|t-1}(m+d) = \hat{x}^{i,j}_{t|t-1} - \left(\sqrt{(d+\lambda)P^{i,j}_{t|t-1}}\right)_m, \quad (3.21)$$

where λ is a free parameter and need to be adjusted for each specific case.

I calculate the yield vector y_t for each σ point.

$$\hat{y}^{i,j}_{t|t-1}(m) = h(\hat{x}^{i,j}_{t|t-1}(m)), \tag{3.22}$$

where $h(\cdot)$ is a non-linear function. In this paper, $h(\cdot)$ is a function of bond yields with respect to stochastic factors $X_t = [y_t, \pi_t, x_t, y^n_t]$. The weighted prediction is given by

$$\hat{y}^{i,j}_{t|t-1} = \sum_{m=0}^{2d} W_h(m)\hat{y}^{i,j}_{t|t-1}(m), \tag{3.23}$$

where the weight $W(m)$ is defined as

$$\begin{aligned} W(0) &= \frac{\lambda}{d+\lambda}, & (3.24)\\ W(m) &= \frac{\lambda}{2(d+\lambda)}, & (3.25)\\ W(m+d) &= \frac{\lambda}{2(d+\lambda)}. & (3.26) \end{aligned}$$

Conditional covariance matrices are given by

$$\begin{aligned} V^{i,j}_{t|t-1} &= \Sigma^{2n}_{m=0} W_h(m)[\hat{y}^{i,j}_{t|t-1}(m) - \hat{y}^{i,j}_{t|t-1}][\hat{y}^{i,j}_{t|t-1}(m) - \hat{y}^{i,j}_{t|t-1}]' + R, & (3.27)\\ U^{i,j}_{t|t-1} &= \Sigma^{2n}_{m=0} W_h(m)[\hat{x}^{i,j}_{t|t-1}(m) - \hat{x}^{i,j}_{t|t-1}][\hat{y}^{i,j}_{t|t-1}(m) - \hat{y}^{i,j}_{t|t-1}]', & (3.28) \end{aligned}$$

where R is the covariance matrix of measurement errors, $\sigma^2_{ob} I_N$ where I_N is a $N \times N$ identity matrix. Kalman gain is given by

$$K^{i,j}_t = U^{i,j}_{t|t-1}(V^{i,j}_{t|t-1})^{-1}. \tag{3.29}$$

Filtered latent variables and covariance matrix are given by

$$\begin{aligned} \hat{x}^{i,j}_{t|t} &= \hat{x}^{i,j}_{t|t-1} + K^{i,j}_t[y_t - \hat{y}^{i,j}_{t|t-1}], & (3.30)\\ P^{i,j}_{t|t} &= P^{i,j}_{t|t-1} - U^{i,j}_{t|t-1}(V^{i,j}_{t|t-1})^{-1}(U^{i,j}_{t|t-1})^T. & (3.31) \end{aligned}$$

Equipped with the time series of $y_{t|t-1}$ and $V_{t|t-1}$, I can calculate the log likelihood as I have done in linear Kalman filtering.

3.6.3 How to eliminate lagged regime dependence

What is remaining is to compute lagged-regime-dependent latent factors, $x^{i}_{t|t}$ given $x^{i,j}_{t|t}$. In order to eliminate the dependence of lagged regime dependence, one can take expectations over regime transitions.

$$x^{j}_{t|t} = \frac{\sum_{i=1}^{L} \Pr[S_{t-1} = j, S_t = j|\phi_t]\hat{x}^{i,j}_{t|t}}{\Pr[S_t = j|\phi_t]}, \tag{3.32}$$

$$P^{j}_{t|t} = \frac{\sum_{i=1}^{L} \Pr[S_{t-1} = j, S_t = j|\phi_t]\left(P^{i,j}_{t|t} + (x^{j}_{t|t} - \hat{x}^{i,j}_{t|t})(x^{j}_{t|t} - \hat{x}^{i,j}_{t|t})^T\right)}{\Pr[S_t = j|\phi_t]}. \tag{3.33}$$

The next problem is how to compute the probabilities of having two specific regimes in current and previous states. That can be done in the following way:

Step 1 Predicting the probabilities of having two specific regimes in current and previous states from the previous one using a given transition matrix.

$$\Pr[S_{t-1} = j, S_t = j|\phi_t] = \Pr[S_t = j|S_{t-1} = i] \times \sum_{i=1}^{L} \Pr[S_{t-2} = i', S_{t-1} = i|\phi_{t-1}]. \tag{3.34}$$

Step 2 Compute the joint density of observable variables y_t, the current regime $S_{t-1}$and the previous regime S_{t-1} .

$$f(y_t, S_{t-1}, S_t = j|\phi_{t-1}) = f(y_t, S_{t-1} = i, S_t = j, \phi_{t-1}) \times \Pr[S_{t-1} = i,_S t = j|\phi_{t-1}], \tag{3.35}$$

where $f(y_t, S_{t-1}, S_t = j|\phi_{t-1})$ is a density function given current and previous regimes that is per-period log likelihood.

Step 3 "Filtering" the probabilities of having two specific regimes in current and previous states from the previous one using per-period likelihood.

$$\Pr[S_{t-1} = i, S_t = j|\phi_t] = \frac{f(y_t, S_{t-1}, S_t = j|\phi_{t-1})}{f(y_t|\phi_{t-1})}, \tag{3.36}$$

where $f(y_t|\phi_{t-1})$ is given by

$$f(y_t|\phi_{t-1}) = \sum_{j=1}^{L}\sum_{i=1}^{L} f(y_t, S_{t-1} = i, S_t = j|\phi_{t-1}). \tag{3.37}$$

Step 4 This part is not mandate but needed when one wants to see the probability of being in a specific regime.

$$\Pr(S_t = j|\phi_t) = \sum_{j=1}^{L}\sum_{i=1}^{L} \Pr[S_{t-1} = i, S_t = j|\phi_{t-1}]. \tag{3.38}$$

Table 3.1: Estimates of regime-dependent long-run real interest rates

This table provides the estimates of regime-dependent long-run real interest rates and the transition probabilities for a structural New Keynesian term structure for the US case. I denote the low regime with $s_t = d$ and the high regime $s_t = u$. Asymptotic standard errors are computed as the estimate of the Fisher information matrix. For each parameter, $*$ and $**$ denote statistical significance at the 5% and 1% level, respectively.

	Notation	Value
Long-run real interest rate (low)	$r^*(d)$	$0.91\%^{**}$
Long-run real interest rate (high)	$r^*(u)$	$1.17\%^{**}$
Transition probability (low regime to low regime)	$Pr(s_t = d \vert s_{t-1} = d)$	0.968^{**}
Transition probability (high regime to high regime)	$Pr(s_t = u \vert s_{t-1} = u)$	0.950^{**}
Observation noise	σ_{ob}	0.010^{**}

Table 3.2: Summary statistics of the average durations across different financial sectors

The table shows summary statistics of the average durations for the banking, asset management, and insurance sectors. In banking, there are commercial banks as well as regional banks, but no investment banks. AM sector denotes asset management sector. Insurance sector includes both life insurance firms and non-life insurance firms. For each category, I compute the average duration across the same-type financial firms in each month. The original data source is Quick Monthly Market Survey of Bond.

	$Duration_t^{Bank}$	$Duration_t^{AM}$	$Duration_t^{Insurance}$
number of sample (average)	39.8	41.7	18.0
mean (year)	3.48	6.54	8.00
min (year)	3.03	5.01	5.66
max (year)	4.44	8.32	9.99
trend in monthly change	0.02	0.07	0.08
volatility of monthly change	0.15	0.21	0.39

Table 3.3: Interaction between nominal bond yields and the average durations of different financial sectors

This table shows the results of two different types of regressions. The top part of the table (A) provides estimates of coefficients in the regressions of quarterly changes in the residual nominal bond yield for each maturity on previous quarter changes in the average duration of different financial sectors. The residual nominal bond yield is computed by subtracting the model-implied bond yield from the actual nominal bond yield for each maturity. The bottom part (B) provides estimates of coefficients in the regressions of changes in the average duration of different financial sectors on previous quarter changes in the actual nominal bond yield for each maturity. For each parameter, $*$ and $**$ denote statistical significance at the 5% and 1% level, respectively. $\Delta y_t(\tau)^{resi}$ is defined as $\Delta y_t^{actual}(\tau) - \Delta y_t^{model}(\tau)$.

(A)	$\Delta y_t(3)^{resi}$	$\Delta y_t(4)^{resi}$	$\Delta y_t(5)^{resi}$	$\Delta y_t(7)^{resi}$	$\Delta y_t(10)^{resi}$
$\Delta Duration_{t-1}^{Bank}$	-0.02	0.02	0.03	-	-
$\Delta Duration_{t-1}^{AM}$	-	-	0.01	-0.02	-
$\Delta Duration_{t-1}^{Insurance}$	-	-	-0.02	0.01	0.02
(B)	$\Delta y_{t-1}(3)$	$\Delta y_{t-1}(4)$	$\Delta y_{t-1}(5)$	$\Delta y_{t-1}(7)$	$\Delta y_t(10)$
$\Delta Duration_t^{Bank}$	0.10	0.07	0.07	-	-
$\Delta Duration_t^{AM}$	-	-	-0.05	0.05	-
$\Delta Duration_t^{Insurance}$	-	-	0.10	-0.02	-0.21

Figure 3.1: Evolution of the probability of being at each regime

This figure shows a time series plot of the probability of being at each regime: The high regime ($s_t = u$) is defined as the regime where the long-run real interest rate is equal to $r^*(u)(r^*(u) > r^*(d))$. The low regime ($s_t = d$) is defined as the regime where the long-run real interest rate takes $r^*(d)(r^*(d) < r^*(u))$.

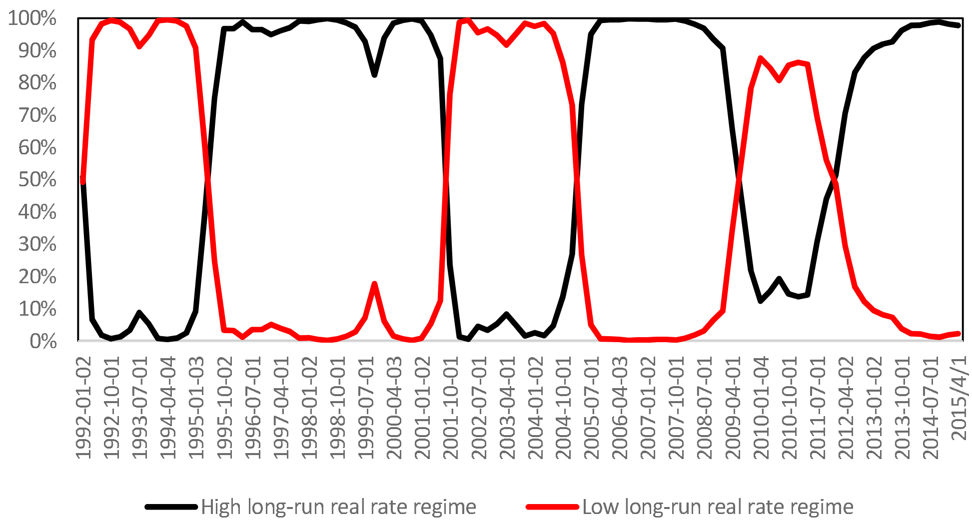


Figure 3.2: Sensitivity of real forward interest rates to the policy interest rate in Japan

This figure shows actual and model-implied sensitivities of the real forward interest rates to the policy interest rate when the zero lower bound is not binding in the case of Japan. In computing the model-implied sensitivity, I calculate the sensitivity of real forward interest rates to the change in the shadow interest rate x_t. I then divide it by the 2-year sensitivity for standardization. The sensitivities are computed under the physical measure.

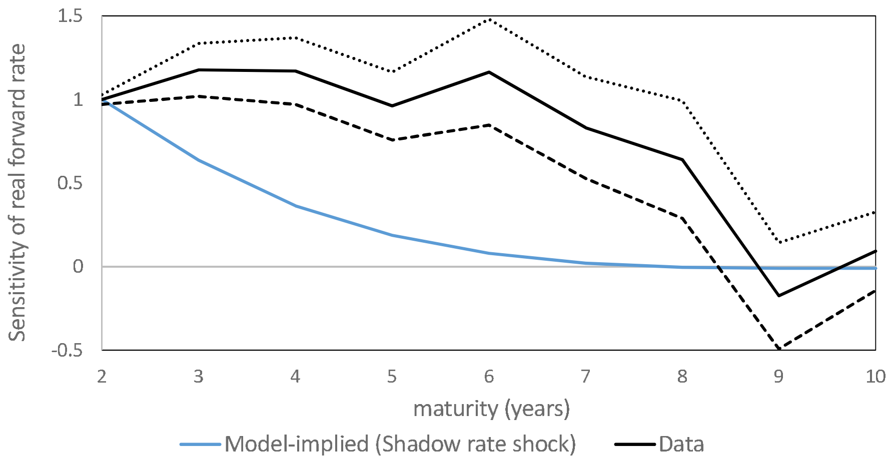
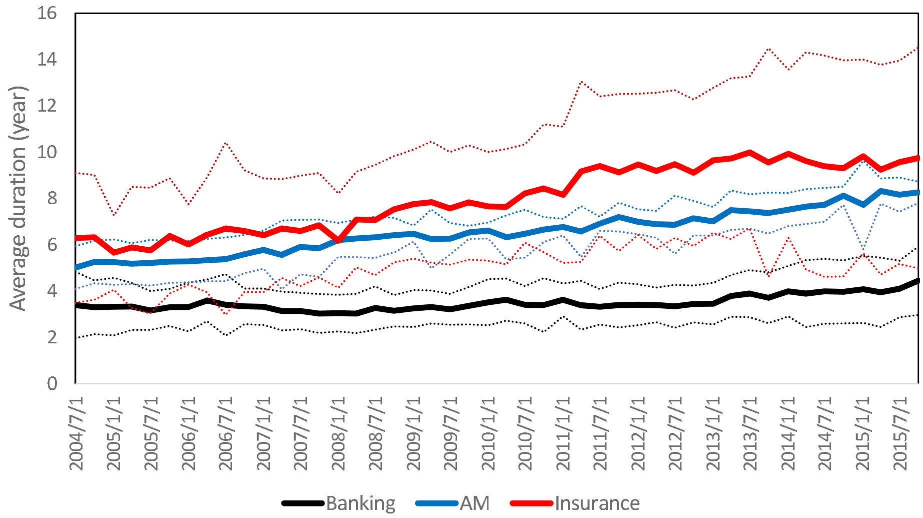


Figure 3.3: Evolution of durations across three different financial sectors

This figure shows a time series plot of durations across three financial sectors from July of 2004 to October of 2015: banking, asset management, and insurance sectors. For each category, the solid line shows the average duration across the same-type financial firms. The dotted line corresponds to a cross-sectional one standard deviation.



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