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An Approach to Statistical Spatial-Temporal Modeling of Meteorological Fields

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Abstract

In this paper we develop a random field model for the mean temperature over the region in the northern United States covering eastern Montana through the Dakotas and northern Nebraska up to the Canadian border. The readings are the temperatures at the stations in the United States historical climatological network reported in Quinlan, Karl & Williams (1987).

The stochastic structure is modeled by a stationary spatial-temporal Gaussian random field. For this region, we find little evidence of temporal dependence while the spatial structure is temporally stable. The approach strives to incorporate the uncertainty in estimating the covariance structure into the predictive distributions and the final inference.

As an application of the model we derive posterior distributions of the areal mean over time. A posterior distribution for the static areal mean is presented as a basis for calibrating temperature shifts by the historical record. For this region and season, it indicates that under the scenario of a gradual increase of $5^\circ F$ over 50 years, it will take 20-30 years of data before the change will be discernible from the natural variation in temperatures.

Key words: Gaussian random fields; Bayesian statistics; Climatic change.

1. INTRODUCTION

There has been much interest recently in climatic change and potential global warming. Of central focus is the phenomenon popularly called the "greenhouse effect": the heating of the earth via the entrapment, by certain gases, of long-wave radiation emitted from the earth's surface. This effect produces a global mean temperature of about $59^\circ F$ rather than an estimated $-6^\circ F$ in the absence of atmosphere (Mitchell (1989)). Increasing concentrations of the gases thought to contribute to

this effect have led to concern in the scientific community about increases in temperature and the resulting climatic effects.

There appears to be no clear cut consensus on the extent of global warming over the last century. Most estimates run from $0.5^\circ F$ to $1.0^\circ F$. The difficulty is the lack of good long-term data over large regions. The global temperature constantly changes on time scales of tens of thousands of years. In fact there have been times in the past millennium when it has been much warmer than the majority of global warming scenarios. The question here is a rapid change over the next century that will have enormous impact on the environment.

Much of the evidence for a global warming effect has been based on large-scale Global Circulation Models (GCMs). These use multi-level mathematical representations of the atmosphere for weather prediction. Given the complexity of the environment and the relative simplicity of the models there is much controversy concerning their validity. Results from the four most widely cited GCMs (1) the National Center for Atmospheric Research (NCAR), (2) Geophysical Fluid Dynamics Laboratory (GFDL) of the National Oceanographic and Atmospheric Administration, (3) the Goddard Institute of Space Studies (GISS) and, (4) the Hadley Center for Climate Prediction and Research at Bracknell, England, are still far from being in agreement, although all models predict higher winter temperatures at the higher northern altitudes as a function of increasing greenhouse gases.

Significant global warming would have an enormous effect on the environment and the world economy. Altering the economies of the world to reduce the production of the gases suspected of increasing the greenhouse effect would be very costly and/or drastically alter our way of life. If the political decision is to be postponed until the empirical evidence is in, this paper sheds light on how long must we wait to detect a global warming with high confidence.

In this paper we develop spatial-temporal models for

temperature fields over a region in the northern United States covering eastern Montana through the Dakotas ($90^\circ - 107^\circ$ in longitude) and northern Nebraska up to the Canadian border ($41^\circ - 49^\circ$ in latitude). We choose the winter months and this region as our study area because GCM predictions of climatic change ($4^\circ F - 10^\circ F$) induced by increased greenhouse gases are expected to be at maximum for high latitudes during the winter months (Mitchell (1989), IPCC (1990)). The observed lack of temporal trend for this region and period was somewhat of a surprise. However, this finding has been confirmed using different statistical approaches (Lettenmaier, Wood and Wallis (1992)). It is also evident from this later study that, had other regions or periods been picked, the results would have been quite different. In addition the relatively stable and simple topography of the region help to ensure homogeneity and the minimization of localized effects. Data from the United States historical climatological network reported in Quinlan, Karl & Williams (1987) is being used to explore long term changes and potential effects of increased concentrations of the greenhouse gases.

There is much interest in empirical studies of climatic change. Jones, *et al* (1986) considers station data to investigate long-term variation in the surface temperature of the northern hemisphere. Karl (1984, 1985) considers climate variation and change in North America. These studies emphasize the dynamic nature of the climate system, and the existence of abnormal winter temperatures within the climate system. Other empirical work is reported in Diaz & Quayle (1978, 1980). A hindrance to these and earlier studies has been the dearth of quality data with both spatial and temporal extent.

Karl, Heim and Quayle (1991) consider a similar region with the objective of identifying greenhouse effects. They construct a pure time-series model for the averages of the stations enclosed in the region and do not address the spatial aspects of the temperature field. This paper develops a comprehensive model for the spatial dimension in conjunction with the temporal component. In addition the model is for the meteorological field as a whole, rather than just a particular characteristic. This is important as it facilitates direct comparison with GCMs and prediction of derived quantities throughout the region and over time. In particular, it allows the prediction of the meteorological field at each location (e.g. city or county) with an associated assessment of the quality of prediction.

The traditional best linear unbiased prediction procedure ("Kriging") is used in this paper for inference, but within a Bayesian framework. Particular attention is paid to the treatment of parameters in the covariance

structure and their effect on the quality, both real and perceived, of the prediction.

Our approach is to use posterior distributions for the static areal quantities as a basis for calibrating temperature shifts by the historical record. In particular, the objective is to understand how soon gradual increases in temperature over this region would be discernible from the year-to-year variation.

1.1 United States Historical Climatology Network

Recently the U. S. Carbon Dioxide Information Analysis Center established a network of 1219 stations (the HCN network) for the contiguous United States "with the objective of compiling a data-set suitable for the detection of climatic change" (Quinlan, Karl & Williams (1987)). The network record includes maximum, minimum, mean monthly temperatures and total monthly precipitation since 1890s.

As part of a study to improve the land surface parameterization of the GFDL-GCM (Geophysical Fluid Dynamics Laboratory - Global Climate Model), a 41 year daily value database was prepared and made available in CDROM format. (Wallis, Lettenmaier and Wood (1991)). The study used 1036 of the original HCN stations, with missing days flagged and then estimated by correlation to nearby stations. There are differences between the unadjusted monthly mean values reported by Quinlan *et al* and those reported by Wallis *et al*. Some of these differences can be accounted for by a difference in the treatment of missing days; Quinlan *et al* summed the observed days in any given month and divided by their number to calculate the average monthly value; missing days were not estimated. However, Wallis *et al* encountered many cases where even the number of missing days in a given month did not agree between the daily and monthly NCDC data bases. Sites used in our study were chosen so as to minimize the effect of these data peculiarities. If the National Climate Data Center ever prepares a cleaned-up data base, then the calculations reported here could easily be seen. In the interim, we do not believe that data errors for the stations and periods used in this study are large enough to invalidate any of our results or conclusions.

2. ANALYSIS OF SPATIAL STRUCTURE

In this section we discuss a spatial model appropriate for a meteorological field over a single time period. The field discussed here is the average winter temperature. The daily average temperature at a location is defined to be the mean of the daily maximum and the daily minimum at that location. The average winter temperature

is defined to be the average daily average temperature over the months December, January and February. The generalization to include a temporal component is the subject of the following section.

Figure 1 represents the region under study. The locations of the 88 U. S. historical climatological network stations in the region are marked with '+'. The elevation of the stations, in feet, are represented by the overlaying gray-scale image.

FIGURE 1: Locations and elevations of the U. S. historical climatological network stations. The gray-level image is for the elevation above sea-level over the region.

Note that the elevation increases from east to west and also from north to south. As expected, the elevation of the station has a marked impact on temperature and it is essential that the model reflect this relationship. Figure 2 is a gray-level image of the mean winter temperatures for the winter of 1983-84. The winter of 1983-84 is the basis for the examples in this section.

Suppose $Z(x)$ is a real-valued stationary Gaussian random field on the region under study (R) with mean

$$E\{Z(x)\} = f'(x)\beta,$$

where $f(x) = \{f_1(x), \dots, f_q(x)\}'$ is a known vector function, and β is a vector of unknown regression coefficients. Furthermore, the covariance function is represented by

$$\text{cov}\{Z(x), Z(y)\} = \alpha K_\theta(x, y) \quad \text{for } x, y \in R$$

where $\alpha > 0$ is a scale parameter, $\theta \in \Theta$ is a $q \times 1$ vector of structural parameters and Θ is an open set in $\mathbb{R}^p$. The division is purely formal as θ may also determine aspects of scale. This formulation is standard for meteorological networks (Gandin (1963)).

FIGURE 2: Graylevel image of the mean winter temperatures for the winter of 1983-84. The temperature range, in Fahrenheit, is given in the legend.

We observe, from a single realization of the field, $\{Z(x_1), \dots, Z(x_n)\}' = Z$ where $x_1, \dots, x_n$ are the spatial locations of the stations in the network. We will focus on the prediction of $Z(x_0)$, where x_0 is a new location in the region of interest.

The Kriging predictor is the best linear unbiased predictor of the form $\hat{Z}_\theta(x_0) = \lambda'(\theta)Z$, that is, the unbiased linear combination of the observations that minimizes the variance of the prediction error. The quality of the prediction is determined by the distribution of the prediction error, $e(x_0) = Z(x_0) - \hat{Z}_\theta(x_0)$. Note that the underlying Kriging procedure is motivated by sampling considerations, producing point predictions and associated measures of uncertainty for those predictions both based on sampling distributions unconditional on the observed Z . However it is well known that Kriging, when the mean is of known regression form, can be given a Bayesian interpretation (see e.g., Omre & Halvorsen (1989), Handcock & Stein (1989), Hastie and Tibshirani (1990)).

The implementation of this model requires the specification of the regression function $f(x)$ and the spatial covariance structure $K_\theta(\cdot, \cdot)$. The regression function is composed of easily measurable spatial characteristics of the station such a latitude, longitude, elevation and distance to the closest urban area. We shall explore these choices in §3. The specification of the covariance structure is dealt with in the following section.

2.1 Spatial Correlation Structure

In this section we describe a general class of isotropic and homogeneous covariance functions that we feel provides a sound foundation for the parametric modeling of Gaussian random fields. An isotropic and homogeneous covariance function can be represented as $K(x, y) \equiv K(|x - y|)$, $\forall x, y \in R$, so that the class is usually written as a function of a single scalar variable $K(x)$, $x \in \mathbb{R}$. The class is motivated by the smooth nature of the spectral density, the wide range of behaviors covered and the interpretability of the parameters (See Handcock & Stein (1989), Matérn (1986)). The isotropic covariance functions have the form

$$K_{\theta}(x) = \frac{1}{2^{\theta_2-1}\Gamma(\theta_2)} \cdot \left(\frac{2\sqrt{\theta_2} \cdot x}{\theta_1}\right)^{\theta_2} \mathcal{K}_{\theta_2}\left(\frac{2\sqrt{\theta_2} \cdot x}{\theta_1}\right)$$

where $\theta_1 > 0$ is a spatial scale parameter controlling the range of correlation and $\theta_2 > 0$ is the parameter controlling the smoothness of the field. A field with this covariance function is $\lceil \theta_2 - 1$ times (mean-square) differentiable (Cramér & Leadbetter (1967)). Here Γ is the gamma function, $\lceil$ is the integer ceiling function and $\mathcal{K}_{\theta_2}$ is the modified Bessel function of the third kind and order θ_2 discussed in Abramowitz & Stegun (1964), §9.

The well known “Exponential” class corresponds to the sub-class with smoothness parameter $\theta_2 = 1/2$, that is

$$K_E(x) = \theta_1 \exp(-x/\theta_1).$$

We call this class the Matérn class because of the general treatment given in the seminal work of Matérn (1986).

The application in this paper has only two spatial dimensions ($d = 2$) leading to some simplification.

2.2 Model Development and Validation

In traditional Kriging, one estimates α and θ by either likelihood methods or various *ad hoc* approaches. The likelihood approach to the estimation of the covariance structure was first applied in the hydrological and geological fields following Kitanidis (1983), Kitanidis & Lane (1985) and Hoeksema & Kitanidis (1985). See also Mardia & Marshall (1984), Handcock & Stein (1989), and Handcock (1989). Usually the predictor and the behavior of the prediction error are themselves estimated by ‘plugging-in’ the estimates to the expressions for known α and θ .

The values of α , θ and β that maximize this log-likelihood are denoted by, $\hat{\alpha}$, $\hat{\theta}$ and $\hat{\beta}$, respectively.

3. ANALYSIS OF TEMPORAL STRUCTURE

In this section we consider the temporal component of the model, generalizing the random field to $Z_t(x)$ where $t = 1937, \dots$ represents the winter of observation. We consider the time-series of data from each station, independent of the spatial information.

The time-series of a single site exhibits little short-term or long-term dependence. Figure 3 presents the time-series and empirical autocorrelation functions for four spatially separate stations. Individually the time-series are quite variable overtime. The right hand side figures are the sample autocorrelation functions corresponding to the time-series. The dashed boundaries represent approximate 95% confidence limits. Note the similar patterns in the series over time and the lack of first lag autocorrelation. The median cross-correlation is 0.78 with quantiles 0.64 and 0.87, indicating a tendency to move in conjunction with each other over time and reflecting the influence of the spatial correlation modeled in the previous section.

FIGURE 3: Time series and empirical autocorrelation functions for four typical sites.

Our analysis indicates that there is little evidence of either short-memory or long-memory dependency in the time-series of the stations. There is little evidence that the spatial structure changes over the time scale considered here. The substance of the analysis is given in Handcock & Wallis (1990).

4. MEASURING AREAL MEAN TEMPERATURE

In the previous sections we found a complex spatial structure to the mean winter temperatures, little temporal dependence structure and little evidence for changing spatial structure over time. In this section we focus interest in a measure of the areal mean temperature over the region of interest. The time-series of areal mean temperatures is defined by:

$$\bar{Z}_t = \frac{1}{|R|} \int_R Z_t(x) dx \quad t = 1937, \dots, 1986, \dots$$

where $|R|$ is the area of the region R . Thus at each point in time, $\bar{Z}_t$ represents the average temperature over the region and is a function of the field $Z(x)$. $\bar{Z}_t$ provides a natural measure for the detection of changing climatic patterns over the region. As the region is devoid of gross topographic features, it provides a convenient measure of overall temperature during the winter. It is important to note that $\bar{Z}_t$ is a characteristic of the temperature field itself, and not a characteristic of the stations in the network. The behavior of the areal mean temperature will provide an indication of the overall changes in climate over the region independent of the individual stations.

Based on our model, we can summarize the available information for $\bar{Z}_t$ from the predictive density $P(\bar{Z}_t \mid Z_{1937}, Z_{1938}, \dots, Z_{1986})$, that is, the posterior density of $\bar{Z}_t$ given the complete spatial-temporal information available.

How can we further summarize the areal mean temperature? The distributions are symmetric and have a similar t-like distributional shape. The ratio of largest to smallest variance is 2.6. To further explore the temporal changes in $\bar{Z}_t$ we will consider the time-series of maximum *a posteriori* (MAP) values. While this clearly represents a reduction in information relative to the full distribution, it facilitates examination.

Figure 4 represents the MAP values for the last half century. Note the lack of a clear trend over time. Some interesting years have been indicated. The last year for which data are available (1986) also has the highest temperature.

A reasonable model for the mean areal temperature over the last half century is

$$\bar{Z}_t = \mu_t + \epsilon_t \quad t = 1937, \dots, 1986, \dots \quad (4.1)$$

where $\{\epsilon_t\}_{t=1937}^{1986}$ is an independent, and identically distributed Gaussian sequence with zero mean and variance σ^2 . The sequence $\{\mu_t\}_{t=1937}^{1986}$ represents the mean level. The motivation is the absence of strong temporal dependence (§4.1) and the approximate constant variances.

The base line model is that the means are temporally stable: $\mu_t \equiv \mu$. Here μ will be called the static areal mean temperature. The details of the model are given in Handcock & Wallis (1991).

FIGURE 4: This is the time-series of (the MAP estimates for the) areal mean temperature. There is little short or long term persistence. The least square line, which has a non-significant slope, is marked.

5. CALIBRATING CHANGES IN MEAN AREAL TEMPERATURE

The primary motivation for developing these models is as a tool to calibrate changes in the mean areal temperature. The model in the previous section allows this to be done. Many scenarios have been proposed for future global warming. Typically their basis is mathematical rather than empirical.

Consider the distributions in Figure 5. The solid line is the posterior distribution for the static areal mean temperature from 1937-86, this should be compared to Figure 6. It summarizes our uncertainty about the static areal mean temperature by a distribution with mean about $19.7^\circ F$ and a standard deviation of $0.51^\circ F$.

How soon could a $5^\circ F$ gradual increase in static areal mean temperature in this region over half a century be discernible?

Suppose we collected data from the network in this region for the next ten years, as the underlying static temperature gradually increases. Suppose that, besides the creep in level, the stochastic structure remained stable, as is supported by the analysis in §4. The dashed posterior (marked by '10') represents a hypothetical posterior for the static areal mean temperature based on

that 10 years of data. Note that there is still marked overlap with the summary from the last 50 years (solid line). This indicates that it would be extremely difficult to discern such a gradual increase after only ten years.

Also plotted are the hypothetical posteriors after collecting 20, 30 and 50 years of information. Only after about 20-30 years is the gradual level increase discernible with high probability. If additional years of data were incorporated, the scale of the posterior based on the historical record will decrease. That is, the hypothetical experiment conducted here is conservative.

FIGURE 5: The predictive distribution for the static areal mean temperature for 1937-86. The dashed distributions represent hypothetical posterior knowledge for the static areal mean temperature based on additional years of data.

6. CONCLUSION

This approach facilitates the calibration of changes in the areal mean temperature against the historical record. For the scenario of a gradual increase of 5°F over 50 years, it will take 20-30 years of data before the change will be discernible from the natural variation in temperatures. By the time the necessary information had accumulated an increase of between $2 - 3^{\circ}\text{F}$ would already have occurred. The application of the model to alternative global warming scenarios is straight forward. There is no indication that the areal mean temperature for this time of the year in this region has changed over the last half century.

Similar models investigating summer temperatures and precipitation for this region will be reported elsewhere. These facilitate the measurement of runoff and

hence address the central issue of the evaluation of water resources.

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