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A NEW REGGE TRAJECTORY IN THE DUAL RESONANCE MODEL*

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ABSTRACT

The existence of a new family of Regge trajectories at the tree diagram level of the Dual Resonance Model is pointed out. The new parent trajectory β is related to the usual trajectory α by $\beta = \frac{1}{2} \alpha - \frac{1}{2}$. It factorizes and has positive signature.

We wish to point out the existence of a new family of Regge trajectories in the Dual Resonance Model (DRM) that until now has not been observed. The parent trajectory β of the new family is related to the standard trajectory α , appearing as an argument of the DRM

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amplitude, by $\beta = \frac{1}{2} \alpha - \frac{1}{2}$. Thus β gives the leading asymptotic behavior for $\alpha < -1$. It has positive signature and is factorizable. Its coupling to two on-shell particles vanishes, which implies that β contributes to high-energy limits only in amplitudes with at least six external particles. The daughter trajectories in the new family are parallel and spaced by $\frac{1}{2}$ unit.

We shall establish the existence and characteristics of the new trajectories by studying high-energy limits of the B_6 function.* In fact the presence of a new contribution can be inferred already from the known expression for α -exchange in the limit of Fig. 1. The α -contribution has a pole structure in momentum transfer which is different from that of the full B_6 , and thus cannot always give the correct asymptotic behavior of the amplitude. Since this is a simple inconsistency argument which may be applicable to more general amplitudes than the DRM, we shall begin by considering it in some detail.

In writing down the contribution $B_6^{(\alpha)}$ of α_t -exchange in Fig. 1, we can take advantage of the fact that Regge residues factorize in the DRM. Thus we only need to know the expressions for the propagators and vertices. If we put the Reggeon-Particle-Particle vertex equal to one, the propagator $D(\alpha_t, \tau_t)$ of a reggeon α_t is obtained directly from the B_4 amplitude,

$$D(\alpha_t, \tau_t) = \Gamma(-\alpha_t) \left(\tau_t + e^{-i\pi\alpha_t} \right) \alpha_s^{\alpha_t} \quad (1)$$

* We use the integral representation of the B_N functions given in ref. 1.

and has nonsense wrong-signature zeros at

$$\alpha_t = -2n + \frac{1}{2}(1 + \tau_t) \quad (n = 1, 2, \dots) \quad (2)$$

The expression for the $(\alpha_a, \alpha_t, 1)$ Reggeon-Reggeon-Particle vertex in Fig. 1 can be obtained from B_5 . It is (2,3,4)

$$V(\alpha_a, \alpha_t; \kappa_1) = \left[\Gamma(-\alpha_t) \left(\tau_t + e^{-i\pi\alpha_t} \right) \right]^{-1} \Gamma(\alpha_a - \alpha_t) \\ \times \left[\tau_a \tau_t + e^{-i\pi(\alpha_t - \alpha_a)} \right] \kappa_1^{-\alpha_a} \Phi(-\alpha_a, 1 + \alpha_t - \alpha_a; -\kappa_1) + (a \leftrightarrow t). \quad (3)$$

Here Φ denotes the ${}_1F_1$ confluent hypergeometric function (5), and we have defined (cf. Fig. 1)

$$\kappa_1 = \alpha_{12}\alpha_s/\alpha_{35}; \quad \kappa_2 = \alpha_{56}\alpha_s/\alpha_{24}. \quad (4)$$

Due to the first factor in Eq. (3), $V(\alpha_a, \alpha_t; \kappa_1)$ has poles at the values of α_t given by Eq. (2).^{*} In the double-Regge limit of B_5 , these poles are cancelled by zeros in the propagator (1), giving a finite result. However, in the expression for the six-point amplitude in the limit of Fig. 1,

$$B_6^{(\alpha)} = D(\alpha_a, \tau_a) V(\alpha_a, \alpha_t; \kappa_1) D(\alpha_t, \tau_t) V(\alpha_t, \alpha_b; \kappa_2) D(\alpha_b, \tau_b) \quad (5)$$

^{*} The existence of these poles has been previously noted by Weis (6), in the context of a study of the factorizability of general amplitudes. He avoids the singularities by imposing additional zeros on the vertex functions. These zeros are not, however, present in the DRM. We are grateful to Y. Eylon for pointing this reference out to us.

there are two singular vertices and still only one vanishing propagator.

Thus $B_6^{(\alpha)}$ has poles at negative integer values of α_t . The residue of the pole at $\alpha_t = -1$ is easily found to be

$$\text{Res}_{\alpha_t=-1} B_6^{(\alpha)} = 2i\pi \tau_a \tau_b (1 + \tau_t) |\alpha_{35}|^{\alpha_a} |\alpha_{24}|^{\alpha_b} |\alpha_s|^{\alpha_t - \alpha_a - \alpha_b} e^{-\kappa_1 - \kappa_2}. \quad (6)$$

Since we know that the exact B_6 amplitude is regular at $\alpha_t = -1$, it immediately follows that $B_6^{(\alpha)}$, given by Eq. (5), cannot represent the correct asymptotic behavior of B_6 at $\alpha_t = -1$. As we show below, there is an additional contribution from the β trajectory that cancels the pole in $B_6^{(\alpha)}$ and restores the proper asymptotic behavior.

The simplest way of seeing how the β -contribution arises is to start from the double helicity pole (HP) limit of B_6 shown in Fig. 2. This limit can be calculated either starting from the general integral representation (1) for B_6 or, more directly, by imposing the HP constraints

$$\alpha_{12}/\alpha_{35} = \alpha_{56}/\alpha_{24} = 0 \quad (7)$$

on the Regge limit of B_6 derived in ref. 4. The result for the (unsignaturized) diagram shown in Fig. 2 is

$$B_6 = (-\alpha_{35})^{\alpha_a} (-\alpha_{24})^{\alpha_b} \Gamma(-\alpha_a) \Gamma(-\alpha_b) \left[(-\kappa)^{-\alpha_a} V_{ab} + (-\kappa)^{-\alpha_b} V_{ba} \right] \quad (8)$$

where $\kappa = \alpha_{24}\alpha_{35}/\alpha_{34}$ and

$$V_{ab} = \frac{\Gamma(\alpha_a - \alpha_b) \Gamma(-\alpha_s) \Gamma(-\alpha_t + \alpha_b)}{\Gamma(-\alpha_b) \Gamma(-\alpha_s - \alpha_t + \alpha_b)} \times {}_2F_2(-\alpha_a, -\alpha_t + \alpha_b; 1 + \alpha_b - \alpha_a, -\alpha_s - \alpha_t + \alpha_b; -\kappa) \quad (9)$$

$$V_{ba} = V_{ab}(a \leftrightarrow b)$$

and ${}_2F_2$ is the confluent hypergeometric function (5)

$${}_2F_2(a, b; c, d; z) = \sum_{n=0}^{\infty} \frac{\Gamma(n+a) \Gamma(n+b)}{\Gamma(a) \Gamma(b)} \frac{\Gamma(c)}{\Gamma(n+c)} \frac{\Gamma(d)}{\Gamma(n+d)} \frac{z^n}{n!} \quad (10)$$

As $|\alpha_s| \rightarrow \infty$ the four-dimensionality constraint implies (4)

$$\eta \equiv \frac{\alpha_{35} \alpha_{24}}{\alpha_{34} \alpha_s} = 1 \quad (11)$$

To calculate the high-energy limit of Fig. 1, we thus need to know the behavior of ${}_2F_2$ in Eq. (10) when $d, z \rightarrow \infty$ with $d/z \rightarrow 1$. This can be derived either using an integral representation of ${}_2F_2$, or directly from the sum (10). We shall only discuss the latter derivation here.*

* A more comprehensive discussion of the novel high-energy behavior of B_6 , together with an investigation of Finite Energy Sum Rules for the reggeon-reggeon amplitude in Fig. 2, will be given elsewhere (7).

For $d \rightarrow \infty$ with $n \leq \mathcal{O}(d^{\frac{1}{2}})$ one may easily verify that

$$\lim_{d \rightarrow \infty} \frac{\Gamma(d)}{\Gamma(n+d)} d^n = \exp\left(-\frac{1}{2} n^2/d\right) \quad (12)$$

while for $n > \mathcal{O}(d^{\frac{1}{2}})$ this factor decreases exponentially with d . In the sum (10) therefore, only values of $n \leq \mathcal{O}(d^{\frac{1}{2}})$ can contribute to the leading asymptotic behavior. For large d , the factor (12) in the sum (10) will be equal to 1 up to large values of n , but eventually provides a (d -dependent) cut-off in n . The remaining factors in the sum behave for large n as

$$\frac{\Gamma(c)}{\Gamma(a) \Gamma(b)} n^{a+b-c-1} \quad (13)$$

Thus, if $a + b - c < 0$, the sum (10) will converge independently of the cut-off factor (12), and the limiting value for large d of the ${}_2F_2$ function is the constant (5) ${}_2F_1(a, b; c; 1)$. On the other hand, if $a + b - c > 0$, the cut-off given by Eq. (12) is necessary for the sum (10) to converge, and this introduces a d -dependence. The leading asymptotic behavior of ${}_2F_2$ can be obtained by noting that it arises from the terms in the sum with large n , so that the approximation (13) can be used, and the sum may be evaluated as an integral. All together we have the result.

$$\lim_{d \rightarrow \infty} {}_2F_2(a, b; c, d; d) = \frac{\Gamma(c) \Gamma(c-a-b)}{\Gamma(c-a) \Gamma(c-b)} + \frac{\Gamma(c) \Gamma\left[\frac{1}{2}(a+b-c)\right]}{2\Gamma(a) \Gamma(b)} (2d)^{\frac{1}{2}(a+b-c)} \quad (14)$$

It is noteworthy that the constraint $z/d \rightarrow 1$, i.e., $\eta = 1$ in Eq. (11), is crucial for obtaining this result. Thus if e.g., we would have $z/d = x < 1$ the sum (10) would always converge regardless of the cut-off factor (12), so that

$$\lim_{d \rightarrow \infty} {}_2F_2(a, b; c, d; xd) = {}_2F_1(a, b; c; x) \quad (0 < x < 1) \quad (15)$$

From Eq. (9) we see that in our case $a + b \leftarrow c = -\alpha_t - 1$. Thus for $\alpha_t > -1$ the first term in Eq. (14) is the leading one. Substituting it back into Eqs. (9) and (8) gives the standard α_t -exchange contribution to B_6 , already given in Eq. (5). For $\alpha_t < -1$ the second term in Eq. (14) is leading, and V_{ab} in Eq. (9) correspondingly behaves for $\alpha_s \rightarrow -\infty$ as

$$V_{ab}^{(\beta)} = \frac{\pi}{\sin \pi(\alpha_a - \alpha_b)} \frac{\Gamma(-\frac{1}{2}\alpha_t - \frac{1}{2})}{\Gamma(-\alpha_a)\Gamma(-\alpha_b)} 2^{-(\alpha_t+3)/2} (-\alpha_s)^{\frac{1}{2}\alpha_t - \frac{1}{2}\alpha_b} \quad (16)$$

This represents the exchange of a new trajectory $\beta_t = \frac{1}{2}\alpha_t - \frac{1}{2}$, which crosses the trajectory α_t at $\alpha_t = -1$.

In considering the contribution of β_t -exchange to the full B_6 amplitude given by Eq. (8), it is important to note that $(-\alpha_s)^{-\alpha_a} V_{ab}$ is antisymmetric under $a \leftrightarrow b$, i.e.,

$$(-\alpha_s)^{-\alpha_a} V_{ab}^{(\beta)} = -(-\alpha_s)^{-\alpha_b} V_{ba}^{(\beta)} \quad (17)$$

According to Eq. (11) we have $\kappa = \eta\alpha_s$ in Eq. (8), so that

$$B_6 = (-\alpha_{35})^{\alpha_a} (-\alpha_{24})^{\alpha_b} \Gamma(-\alpha_a) \Gamma(-\alpha_b) \times \left[\eta^{-\alpha_a} (-\alpha_s)^{-\alpha_a} V_{ab} + \eta^{-\alpha_b} (-\alpha_s)^{-\alpha_b} V_{ba} \right] \quad (18)$$

We can now understand why the β -trajectory has been so elusive in the past. First, as we already noted in Eq. (15), it will show up only if η is exactly equal to 1. But then the two terms in Eq. (18) cancel because of Eq. (17). Thus there will be no contribution $B_6^{(\beta)}$ from β -exchange in a high-energy limit of B_6 where all the asymptotically large variables approach $-\infty$. This does not mean that $B_6^{(\beta)} = 0$ in all physical limits, however. Indeed, among the four diagrams that have to be added to give definite signature to the reggeons α_a, α_b in Fig. 2, there is one for which $\eta = e^{2\pi i}$. This diagram has twists on both the α_a and α_b reggeons, and thus corresponds to the configuration of physical momenta shown in Fig. 3. Since $\alpha_{34}, \alpha_{61} \rightarrow +\infty + i\epsilon$; $\alpha_{24}, \alpha_{35} \rightarrow -\infty$ in this diagram, η rotates through an angle 2π in continuing from the $-\infty$ limit to this physical limit. This phase gives rise to a nonvanishing contribution from β -exchange in Eq. (18)

$$B_6^{(\beta)} = -i\pi \tau_a \tau_b e^{-i\pi(\frac{1}{2}\alpha_t - \frac{1}{2})} \Gamma(-\frac{1}{2}\alpha_t - \frac{1}{2}) 2^{-\frac{1}{2}\alpha_t - \frac{1}{2}} \times (-\alpha_{35})^{\alpha_a} (-\alpha_{24})^{\alpha_b} \alpha_s^{\frac{1}{2}\alpha_t - \frac{1}{2} - \alpha_a - \alpha_b} \quad (19)$$

Here the factor $\tau_a \tau_b$ indicates that only the diagram where both reggeons α_a, α_b are twisted contributes (note that the reggeon β_t has not yet been signaturized). The reason for the neglect of the contribution (19) in previous studies is now apparent. It was

always assumed (3,4) that one first can take a limit of B_6 in the $-\infty$ direction, and then analytically continue the result to the desired physical region. According to the above, however, the power behavior may differ depending on the direction in which the limit is taken. According to the Phragmén-Lindelöf theorem this cannot happen for functions of only one complex variable. Thus this new feature in B_6 is connected to the fact that several variables tend to ∞ .

The above conclusions can be explicitly verified by taking the limit shown in Fig. 3 of B_6 directly, i.e., letting $\alpha_{34}, \alpha_{61} \rightarrow +\infty + i\epsilon$; $\alpha_{24}, \alpha_{35} \rightarrow -\infty$ in the exact amplitude. To do this the usual integral representation of B_6 has to be modified, as it converges only for $\alpha_{34}, \alpha_{61} < 0$. The detailed derivation will be given elsewhere (7), here we only quote the result. In addition to the $B_6^{(\alpha)}$ contribution, which has been derived previously in the usual way in ref. 4, one finds when $\eta = 1$ a new contribution due to β_t -exchange:

$$B_6^{(\beta)} = -i\pi e^{-i\pi\beta_t} 2^{-\beta_t-1} \Gamma(-\beta_t-1) \alpha_{34}^{\beta_t} \times \left[\left(-\frac{\alpha_{24}}{\alpha_{34}} \right)^{\beta_t-\alpha_{23}} \left(1 - \frac{\alpha_{24}}{\alpha_{34}} \right)^{-\beta_t+\alpha_{12}+\alpha_{23}} \right] \times \left[\left(-\frac{\alpha_{35}}{\alpha_{34}} \right)^{\beta_t-\alpha_{45}} \left(1 - \frac{\alpha_{35}}{\alpha_{34}} \right)^{-\beta_t+\alpha_{45}+\alpha_{56}} \right] \quad (20)$$

We shall conclude by mentioning several properties of the new contribution:

(a) Signature. The diagram with a twist on the reggeon line in Fig. 3 can be calculated from Eq. (20) by interchanging the variables $\alpha_{34} \leftrightarrow -\alpha_{35}$, $\alpha_{61} \leftrightarrow -\alpha_{24}$ and $\alpha_{45} \leftrightarrow \alpha_{56}$. It is easy to see that $B_6^{(\beta)}$ is invariant under this operation. Thus the β -trajectory has positive signature, and the signature factor is simply $1 + \tau_t$.

(b) Factorization. Equation (20) is written in an explicitly factorized form. This need not imply, however, that the β -trajectory is singly degenerate, as we have only studied its coupling to three particles.

(c) The residues have no poles in the subvariables α_{12} , α_{23} , etc. Thus the β -trajectory does not couple to two on-shell particles.

(d) Further Regge limits can be obtained by letting some of the cluster subvariables grow large in Eq. (20). In the (HP) limit of Fig. 1 one can verify that the result agrees with Eq. (19), which was derived by taking the limits in a different order.

(e) The poles in β_t appear at $\beta_t = -1, 0, 1, 2, \dots$, which corresponds to $\alpha_t = -1, 1, 3, \dots$. From Eq. (19) one can directly see that the residue of the pole in $B_6^{(\beta)}$ at $\alpha_t = -1$ is such as to cancel exactly the residue in $B_6^{(\alpha)}$ given by Eq. (6) (with $\kappa_1 = \kappa_2 = 0$ due to the HP limit, see Eq. (7); for general values of κ_1, κ_2 this can be seen by calculating the Regge limit from Eq. (20)).

(f) The pole in $B_6^{(\alpha)}$ at $\alpha_t = -2$, which appears in the $\tau_t = -1$ amplitude, must be cancelled by a negative-signatured daughter to β_t . Since $\beta_t = -3/2$ for $\alpha_t = -2$, the daughter must lie $\frac{1}{2}$ unit below β_t . We have explicitly verified that the $\alpha_t = -2$ pole is indeed cancelled in this way. Evidently all the poles in

$B_6^{(\alpha)}$ that appear for the values of α_t given in Eq. (2) can be cancelled in an analogous way by daughters to β_t with spacing $\frac{1}{2}$.

(g) The poles in $B_6^{(\beta)}$ at $\alpha_t = 1, 3, \dots$ do not represent physical resonances. This can be most easily seen from Eq. (20) by noting that the residues of these poles are not polynomials in the dual variables. Since the residues of the full B_6 are polynomials, the $B_6^{(\beta)}$ poles must again be cancelled by poles in $B_6^{(\alpha)}$, this time coming from daughters to the α_t trajectory, which cross β_t at the pole positions. We have verified the cancellation at the $\alpha_t = 1$ pole. Thus in general the pole structure of either the $B_6^{(\alpha)}$ or $B_6^{(\beta)}$ terms is not meaningful separately, the true particle spectrum of B_6 being obtained only from their sum.

These and other questions will be discussed more extensively in ref. 7.

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FIGURE CAPTIONS

- Fig. 1. Multi-Regge limit of the B_6 function. We use the "inclusive" notation common in the DRM, e.g., $\alpha_{13} = \alpha[(p_1 + p_2 + p_3)^2]$, etc. We also use the alternative notation indicated on the figure: $\alpha_a = \alpha_{23}$, $\alpha_b = \alpha_{45}$, $\alpha_t = \alpha_{13}$, $\alpha_s = \alpha_{25} = \alpha_{61}$.
- Fig. 2. A double helicity-pole limit of B_6 .
- Fig. 3. A single Regge limit of B_6 that has a contribution from the new trajectory $\beta_t = \frac{1}{2} \alpha_t - \frac{1}{2}$.

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Fig. 1

Fig. 2

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Fig. 3

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