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A bi-objective facility location problem: Coverage and Access

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A bi-objective facility location problem:

Coverage and Access

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To my parents, Gwanghyun Baik and Hakwon Lee,

To my sister, GyeongHee Baik.

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Abstract

A bi-objective facility location problem: Coverage and Access

by

Jiwon Baik

Selecting a good location for an activity or service is fundamentally important. Many different approaches across a range of disciplines have been proposed, developed, and explored to address such strategic decision-making. With better understanding and insight, as well as better geo-spatial data, a number of computational and mathematical advances have been made in recent years. Perhaps the most prominent strategic location problem is attributed to Weber, seeking a site in continuous space that minimizes the sum of weighted distances to multiple destinations. However, increasingly important is accounting for additional concerns, with spatial coverage being particularly critical. This thesis introduces a bi-objective strategic location problem. A mathematical model formulation is derived, and an optimal solution approach is developed. Application findings are reported for several application case studies.

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Chapter 1

Introduction

An important strategic decision involves the selection of a site that provides service or facilitates an activity. There are many examples of an associated activity and/or service facility, including a factory, distribution center, emergency response staging area, waste collection site, cybersecurity data vault, disaster command and control post, etc. Recent work by Murray et al. (2020) detailed the siting of a juice production facility. The COVID-19 pandemic highlights the need for locating a testing and/or vaccination site in a region. Interestingly, such service and activity sites often must consider multiple criteria associated with locational decision-making. Two fundamentally critical geographic attributes of a selected facility site are access and coverage.

Access and accessibility are generally addressed through a measure of proximity, with average distance (or equivalently total weighted distance) being particularly common. The goal then is finding the location for a facility that provides the lowest average distance, thereby making it the site with the best access. Weber (1909) discussed this siting problem in great detail, seeking to locate a factory in continuous space that provides the best access, measured by total weighted distance. The challenge is to site a factory or production plant to minimize associated transportation costs. Accordingly, the transportation costs are viewed as critically important under conditions where land acquisition and labor costs are relatively constant across a region. What has made this fundamental problem of such great interest is that economic efficiency is formalized and that the importance of spatial location is inherently central to notions of efficiency. A network restricted case of finding the best site for a telephone switching center was discussed in Hakimi (1964), with access considered as well as coverage. The intent was to identify a location that minimizes the maximum distance to any demand. The notion of coverage along these lines is an important one as well. Hakimi (1964) explicitly discusses an additional context where a facility is a police station or hospital. The implication is that service response to/from demand is often time-critical, with fast arrival likely thwarting a crime or saving a life. Church and Murray (2018) detail that 4-6 minute response time standards are not uncommon in such contexts. Hakimi (1964) recognizes for the case of siting the facility on a network that addressing both access and coverage was indeed complicated, opting instead to focus on solving for the best facility location with respect to either access or coverage independently.

This thesis seeks to address the facility siting situation that simultaneously considers both access and coverage. The following section reviews related literature. This is followed by a formal mathematical model representing the intended bi-objective spatial optimization problem, addressing both access and coverage. Details on how the problem can be solved are then given, along with proof of associated solution properties. Application results are presented to illustrate problem characteristics. In particular, the nature of decision-making involving multiple objectives is highlighted. The thesis ends with a discussion and concluding comments.

Chapter 2

Background

There has been considerable work oriented toward locating a range of different service facilities. Reviews of the location modeling approach to support this can be found in Francis et al. (1992), Church and Murray (2009), and Laporte et al. (2019), among others. Various spatial features of facilities and services are noted, formalized, and structured in an optimization context. Among these features, access and coverage have been particularly important across many planning, management, and decision-making contexts.

Access and accessibility have been longstanding themes, reflecting broad interest in identifying sites in close proximity to the demand being served. Weber (1909) is generally credited with articulating a challenging single facility (factory) siting problem, with an objective to minimize total transportation costs in locating the facility in a region. One simple form of this problem involves service to a finite set of discrete points, allowing the facility to be anywhere in the plane and distance represented using the Euclidean metric. Algorithmic development of approaches to solving the Weber problem has been substantial, including the Varignon frame (see Weber 1909). A prominent solution technique is that of Weiszfeld (1937), based on partial derivatives used in an iterative process that converges to the optimum, representing the point with the least total weighted distance (see also Miehle

1958, Kuhn and Kuenne 1962, Cooper 1963, Vergin and Rodgers 1967, Weiszfeld and Plastria 2009). A recent review is offered in Murray (2018) with linkages to related single facility problems, as well as various spatial extensions.

There have been many extensions of the classic Weber problem. Hakimi (1964) discussed finding the absolute median on a network consisting of weighted arcs and demand nodes. This work spawned much interest in discrete versions of this problem. Recent work by Church (2019) discusses that while much attention has been given to the classic Weber problem, there are indeed many interesting and important variants that remain to be investigated. Further, solution approaches for most of these extensions have not yet been devised. Along these lines, Murray et al. (2020) examine the case where only a subset of demand is to be allocated to the sited facility, dynamically affecting which demand is considered in the derivation of the distance minimization goal. The Weber problem collectively has been the subject of continuing interest because of its general utility and the fact that it accurately reflects access and accessibility considerations.

Of course, there are other facets to access and accessibility. One is a maximum travel standard known as coverage (see Church and Murray 2018), as noted above. The idea is that there is a maximum distance or travel time beyond which service quality degrades such that it is ill-advised or ineffective. Further, coverage reflects the notion of a range of a good or service reflected in central place theory (Church and Murray 2009). Coverage models have been a critical category of location optimization work, oriented toward enhancing accessibility to facilities and/or services. When demand is within the critical service standard of a sited facility, the demand is considered covered. Hakimi (1964) initially discussed vertex covering problems in the context of policing a highway network. Continuous space variants, such as the location that minimizes the maximum distance from demand (Brady and Ronsenthal, 1980; Mehrez et al., 1983), can also be observed. Subsequent work has followed, suggesting two classic coverage approaches, the location set covering problem (LSCP) and maximal covering location problem (MCLP). The LSCP aims to site the minimum number of facilities needed to ensure complete coverage of all demand, with Toregas et al. (1971) the first to formalize this approach in the context of emergency response. The MCLP, structured in Church and ReVelle (1974) relaxes that all demand must be served, seeking instead to cover the most demand possible given a fixed number of facilities. Extensions are many, with interesting cases explored in Berman et al. (2003) and Kim and Murray (2008).

Subsequent work on coverage has been substantial. Noteworthy in this thesis is the generalization of the MCLP defined in Church (1984), where potential facility locations are not fixed to a finite set but can be anywhere in the plane. To deal with the infinite number of potential facility locations, Church (1984) proved that a discretization of continuous space is possible and it would contain an optimal solution, the so-called circle intersection point set (see also Mehrez and Stulman 1982). Murray and Tong (2007) extended this work to account for situations where demand might be represented as any spatial object, such as points, lines, and polygons. Matisziw and Murray (2009) explored an extension of the MCLP where demand is continuously distributed and a single facility is to be located. They exploited an equivalent line representation of a demand region, known as the medial axis or skeleton, and proved that an optimal solution lies on this line representation.

A challenge is addressing multiple considerations in location siting. In particular, the interest here is to account for access and coverage simultaneously. Doing so suggests a bi-objective optimization context. Multiobjective optimization problems are particularly challenging and remain the focus of much academic research. Cohon (1978) offers an overview of multiobjective optimization. In essence, multiple objectives result in Pareto optimal solutions. Each identified Pareto optimal solution represents a trade-off between objectives, where one cannot improve one objective without degrading the other. Therefore, these so-called non-dominated solutions are of critical importance and represent the best possible choices for a decision-maker. One approach for identifying Pareto solutions in the bi-objective location planning context is the non-inferior set estimation algorithm proposed by Cohon et al. (1979), with subsequent work by Solanki (1991) associated with a mixed-integer programming problem. Medrano and Church (2014) proposed a heuristic algorithm to identify non-dominated solutions to a bi-objective problem involving two different costs associated with the shortest path between the origin and destination, which traverses a given intermediate arc or node. The heuristic algorithm can find a subset of non-dominated solutions in a mixed-integer context within a reasonable computational time.

As noted above, this thesis aims to simultaneously address access and coverage in the continuous space location of a facility, similar to that posed by Weber (1909) but taking into account service coverage as well. One line of work attempts to incorporate both through the constraint on the maximum distance a demand could be from the sited facility. Schaefer and Hurter (1974) referred to this as a Weber problem with metric constraints on the plane, suggesting that it was related to the LSCP. Subsequent work by Hansen et al. (1982) and

Watson-Gandy (1985), as well as others, focused on the solution of this distance-constrained problem. In a discrete space environment, where demand and potential facility sites are finite in number and defined at given locations, somewhat related work is that of Pirkul and Schilling (1991) and Yao et al. (2019). Pirkul and Schilling (1991) extended the capacitated MCLP applied to fixed demand and potential facility location, including another objective to minimize the total travel distance of demand beyond the coverage standard, while Yao et al. (2019) extended the LSCP to account for maintaining a prespecified number of existing facilities and optimizing access. In contrast, in this thesis potential facility sites are not identified in advance but rather are infinite in number since a facility may be located anywhere in a region. Beyond this, there have been a number of developments in a network setting worth mentioning. Church and Meadows (1977) suggested a solution method to a coverage problem where the facilities could be sited anywhere on the network, at nodes or along arcs, proposing the network intersect point set in the context of the distance constrained p-median problem. The solution method identifies a single solution as coverage is a constraint. Church (1980) details a heuristic for a tri-objective problem to locate a number of interstate solid waste recycling centers. The approach minimizes the weighted distance to all demand locations while maximizing coverage within a given standard as well as maximizing the number of centers that meet a minimum threshold of service demand. Other heuristics for solving p-median problems with coverage constraints on a network are noted in Saez-Aguado and Trandafir (2012). Daskin (1995) introduces an algorithm to identify the tradeoff solutions of coverage and weighted distance for a discrete location problem in a network setting, providing accompanying software and datasets.

Based on the above review, it is clear that no existing research has dealt with access and coverage simultaneously in a continuous space context. Given the known challenges of multiobjective (bi-objective) optimization combined with the continuous space siting orientation, this thesis addresses an important planning problem that involves locating a single facility anywhere in a region in order to maximize demand coverage and minimize average distance.

Chapter 3

Mathematical Model

As highlighted previously, access and coverage objectives are important planning criteria that guide locational decision-making. Simultaneously addressing these concerns offers great potential for better supporting analysis, planning, management, and policy. In this section, the precise mathematical model is formalized, enabling it to be better understood and contrasted with previous research. Consider the following notation:

i = index of demand to be served

a_i = total activity at demand i

(x_i, y_i) = geographic coordinates of demand i

S = service coverage standard

M = a large number

Given this notation, the supporting decision variables are as follows:

(X, Y) = location of the facility

$$Z_i = \begin{cases} 1 \text{ if demand } i \text{ within } S \text{ of a facility at } (X, Y) \\ 0 \text{ otherwise} \end{cases}$$

Thus, what is sought is the best location of the facility, (X, Y), in accordance with the goals that the facility be the most accessible as well as provide the greatest service coverage. A model integrating these decision variables to address access and coverage goals is the following:

Minimize	$\sum_i a_i\sqrt{(x_i - X)^2 + (y_i - Y)^2}$	(1)
Maximize	$\sum_i a_i Z_i$	(2)
Subject to	$S + M * (1 - Z_i) \geq \sqrt{(x_i - X)^2 + (y_i - Y)^2} \quad \forall i$	(3)
	X, Y unrestricted in sign	(4)
	$Z_i = \{0,1\} \quad \forall i$	(5)

The facility location decision variables, (X, Y), are intuitive, representing the coordinate reference. The Z_i decision variables are introduced to account for whether coverage is provided to a given demand i. The bi-objective model includes one objective, (1), seeking to minimize the total weighted distance from the facility to all demand points and a second objective, (2), to maximize the amount of demand covered by the sited facility. Constraints (3) defines whether or not suitable coverage is provided to each demand based upon the stipulated service distance, S. Constraints (4) and (5) note conditions on decision variables.

An essential distinction of problem formulation (1)-(5) is that objective (1) is nothing other than the total weighted distance measure reflecting the intent of the classic Weber problem (see Wesolowsky 1993, Yao and Murray 2014, Murray 2018, Church 2019, Murray et al. 2020). Minimizing total weighted distance is equivalent to minimizing average distance and hence reflects the notion of access and accessibility. Of course, Equations (1)-(5) goes beyond the classic Weber problem's intention to include coverage as well.

Further discussion and details are therefore necessary for objective (2) and constraints (3). It is perhaps most straightforward to focus on constraints (3) initially. Two fundamental conditions must be tracked, whether the sited facility covers a demand or if it does not. This suggests two cases for a given demand i in constraints (3). Consider first the case where demand i is not covered. Given the corresponding decision variable, this means that $Z_i = 0$. Accordingly, constraint (3) takes the following form of when $Z_i = 0$:

$$S + M * (1 - 0) \geq \sqrt{(x_i - X)^2 + (y_i - Y)^2} \quad (6a)$$

This simplifies to:

$$S + M \geq \sqrt{(x_i - X)^2 + (y_i - Y)^2} \quad (6b)$$

As the left-hand side is a large number, $S + M$, this will be larger than any possible distance in the region. Therefore, there is no impact on the distance measure recorded on the right-hand side of equation (6b). Further, as intended, since the demand is not covered, the contribution to objective (2) is zero. Viewed in an alternative manner, since the orientation of objective (2) is to maximize, then the associated distance must be preventing the decision variable Z_i from being anything greater than zero. This is the intended outcome.

The second case is when demand i is covered, $Z_i = 1$, with constraint (3) taking the following form:

$$S + M * (1 - 1) \geq \sqrt{(x_i - X)^2 + (y_i - Y)^2} \quad (7a)$$

This simplifies to:

$$S \geq \sqrt{(x_i - X)^2 + (y_i - Y)^2} \quad (7b)$$

Thus, the distance from the demand to the sited facility on the right-hand side of equation (7b) must be less than the coverage standard, S, on the left-hand side. As a result, objective (2) would correctly account for demand coverage in this situation, as intended. Clearly, problem formulated by Equations (1)-(5) represents a unique and important extension of the classic Weber problem, offering the capability to address both access and coverage issues simultaneously. Unfortunately, there is no existing method for solving this single facility location problem. The bi-objective nature of the problem, along with non-linear objectives and constraints, makes it exceedingly complicated to solve.

Chapter 4

Exact Solution Algorithm

An exact algorithm is proposed and implemented to solve (1)-(5) presented in chapter 3. The approach is based on a strategic partitioning of the geographic region, R, enabling evaluation of each partition in order to find the location that simultaneously optimizes both access and coverage objectives. This means that continuous space consisting of an infinite number of potential facility locations can be reduced to a finite set of partitions, where a more focused search for the best location within the partition can be carried out. In essence, the coverage objective enables such a partitioning to be done, and then the solution process reduces to finding the best access location within the partition. The significance is that the search can be enumerative, focusing on each individual partition. Further, such an approach facilitates the identification of all non-dominated solutions.

Assume that a valid discrete and finite partition of the region is possible. Subsequent discussion will detail how this can be done along with proof of partition validity. Given this, an exact solution algorithm is offered in Figure 1, guaranteed to identify all non-dominated solutions. The solution approach outlined in Figure 1 is only the beginning, as theoretical details remain regarding specification and proof of optimality. The initial focus is on the coverage aspects of Equations (1)-(5) (see chapter 3). The following definition of demand coverage represents a starting point for details to come:

Definition (demand coverage): If demand i is within the distance/travel time standard S of the sited facility (X, Y), then the corresponding expected activity a_i at i is covered (or served).

Input: I, S
Output: Ψ

 # derive regional Weber point

$$(\hat{X}, \hat{Y}) \leftarrow \min_{(\hat{X},\hat{Y}) \subset R} \sum_{\boldsymbol{i} \in \boldsymbol{I}} \boldsymbol{a_i} \sqrt{(\hat{X} - \boldsymbol{x_i})^2 + (\hat{Y} - \boldsymbol{y_i})^2}$$

 # create buffer for each demand point
for each $i \in I$ **do**
$\mu_i \leftarrow \{(x, y) \subset R \mid d_{(x,y)(x_i,y_i)} \leq S\}$
end-for

 # face overlay
$O \leftarrow \{\mu_i \mid \forall\, i \in I\}$
$\mathrm{K} \leftarrow \{\nu | \delta \in \wp\,(O)$ where $\forall\, \upsilon, \hat{\upsilon} \subset \delta\;\; \upsilon \cap \hat{\upsilon} = \emptyset \wedge \bigcup_{\nu} \nu = R\}$

 # derive face Weber points
for each $\kappa \in K$ **do**

$$(X_\kappa, Y_\kappa) \leftarrow \min_{(X_\kappa,Y_\kappa) \subset \nu_\kappa} \sqrt{(X_\kappa - \hat{X})^2 + (Y_\kappa - \hat{Y})^2}$$

end-for

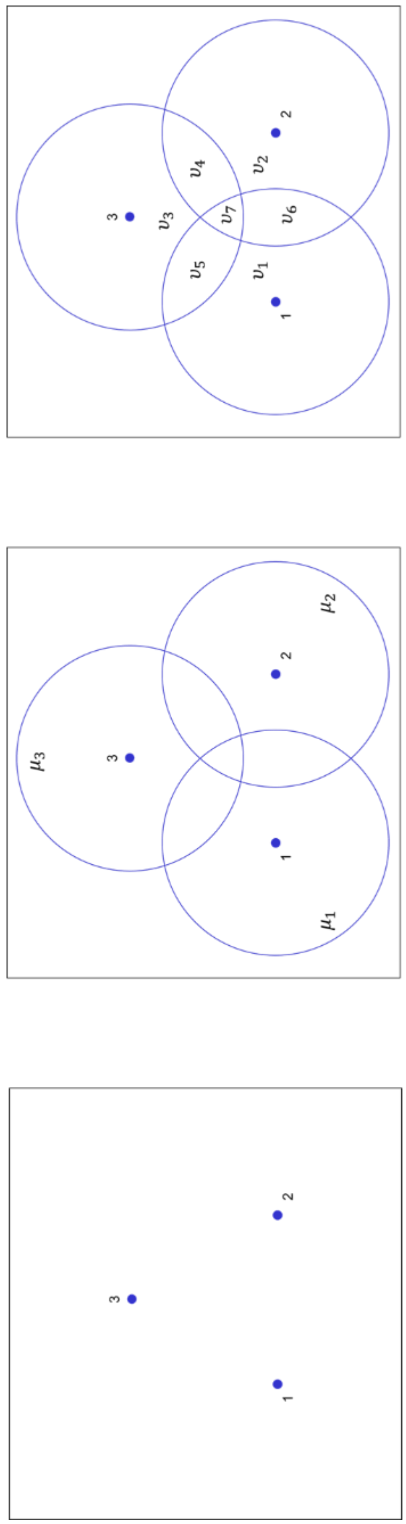
 # filter non-dominated solutions
$\boldsymbol{\Psi} \leftarrow \{\}$
for each $\kappa \in K$ **do**
if $\boldsymbol{non - dominated}$ **then** $\boldsymbol{\Psi} \leftarrow \boldsymbol{\kappa}$
end-for

Figure 1. Exact solution algorithm for identifying all non-dominated solutions.

Given demand i defined as a point with reference (x_i, y_i), then it is possible to derive the area in the study region R within which the sited facility would be capable of providing coverage. Formally, for demand i this is:

$$\mu_i \leftarrow \left\{(x, y) \subset R \middle| d_{(x,y)(x_i,y_i)} \leq S\right\} \tag{8}$$

where (x, y) is a location and $d_{(x,y)(x_i,y_i)}$ is the distance from the location to demand i. As indicated in (1) and (3), the Euclidean distance metric is assumed, suggesting that μ_i is a circle of radius S since the demand object is a point. In general, however, this need not be the case as any distance or travel time measure could be relied upon. As noted in Murray (2021), the set theoretic specification in (8) formalizes the buffer operation in GIS (geographic information systems) and computational geometry. Without loss of generality, since demand consists of the set of I points, assume that $R = \bigcup_{i \in I} \mu_i$. If $(X, Y) \subset \mu_i$ (the facility is sited somewhere in the area object), then demand i would satisfy the demand coverage definition, providing service to a_i total activity. Thus, area μ_i represents a discrete object in the continuous space region R for which coverage potential could be assessed. Specifically, $(X, Y) \subset \mu_i$ indicates that demand i is covered and $(X, Y) \notin \mu_i$ implies that demand i is not covered. To illustrate coverage along these lines, three demand points are given in Figure 2a. The associated areas of coverage based on S are shown in Figure 2b, namely μ_1, μ_2 and μ_3. Thus, $(X, Y) \subset \mu_1$ means that demand 1 is covered, $(X, Y) \subset \mu_2$ results in demand 2 being covered and $(X, Y) \subset \mu_3$ indicates that demand 3 is covered.



(a) Demand points in a region

(b) Coverage area for each demand point

(c) Faces that result from overlay

Figure 2. Three demand points and corresponding areas of coverage.

Consider the case of two demand locations, i and i', with demand coverage areas μ_i and $\mu_{i'}$, respectively. Assuming that $\mu_i \cap \mu_{i'} \neq \emptyset$, then the two coverage areas overlap. Thus, $(X, Y) \subset \mu_i \cap \mu_{i'}$ implies that the sited facility covers i and i', with total demand coverage of $a_i + a_{i'}$. This can be contrasted to $(X, Y) \subset \mu_i \backslash \mu_{i'}$ (μ_i less $\mu_{i'}$) only capable of covering demand i (a_i total demand) or $(X, Y) \subset \mu_{i'} \backslash \mu_i$ only capable of covering demand i' ($a_{i'}$ total demand). As a result, three discrete partitions emerge for the continuous space region in this case, enabling coverage potential to be assessed with respect to facility placement. When the entire set of demand locations I is considered, then the set of demand coverage areas can be denoted $O = \{\mu_i \mid \forall\, i \in I\}$. In order to identify all possible discrete sub-areas or partitions of R, the overlay operator is useful:

$$\mathrm{K} = \{\nu | \delta \in \wp\,(O) \text{ where } \forall\, \upsilon, \hat{\upsilon}\ \subset \delta\ \ \upsilon \cap \hat{\upsilon} = \emptyset\ \wedge\ \cup_{\nu}\, \nu = R\ \} \quad (9)$$

where $\wp\,(O)$ is the power set operator that enumerates potential combinations of objects in the set O. Therefore, the resulting set K in (9) represents all unique overlapping objects or faces (see Murray 2021). The face is nothing other than a partitioning of continuous space based on demand coverage areas. Each face can be further defined and/or specified using the following additional notation:

κ = index of faces (|K| in number)

ν_κ = face κ

$\Omega_\kappa =$ set of demand coverage areas that define the face ν_κ

Definition (face): A face is the area formed by the intersection of a set of overlapping demand coverage areas, (8), maintaining conditions (9) where $\nu_\kappa = \bigcap_{i \in \Omega_\kappa} \mu_i$.

The set of faces identified in (9) is comprised of discrete areas, finite in number, essentially partitioning the region based on demand coverage. The faces that result from overlay of the coverage areas in Figure 2b are shown in Figure 2c, namely ν_1, ν_2, ν_3, ν_4, ν_5, ν_6 and ν_7. These faces reflect the definition, and are significant in the following manner.

Theorem 1. The optimal solution with respect to coverage, (2), only is to site the facility in one of the faces ν_κ.

Proof: The region R is comprised of faces ν_κ such that $\bigcup_\kappa \nu_\kappa = R$ in (10). Suppose that $(X, Y) \notin R$. Thus, no coverage of demand is possible since $(x_i, y_i) \subset R \; \forall i$ and $\bigcup_i \mu_i = R$. However, $(X, Y) \subset R$ implies that there exists at least one $\nu_\kappa \in \mathrm{K}$ with $(X, Y) \subset \nu_\kappa$ where $\sum_{i \in \Omega_\kappa} a_i > 0$, contradicting the proposition that $(X, Y) \notin R$. Further, since $|\mathrm{K}|$ is finite, there must exist κ such that $\sum_{i \in \Omega_\kappa} a_i \geq \sum_{i' \in \Omega_{\kappa'}} a_{i'}$ for all other $\kappa' \in |\mathrm{K}|$, meaning that an optimal solution with respect to (2) is contained within a face.

Theorem 1 is significant because a face reflects total demand covered by siting the facility anywhere in the face. Specifically, $(X, Y) \subset \nu_\kappa$ implies a known level of coverage, equal to $\sum_{i \in \Omega_\kappa} a_i$. Without loss of generality, denote this facility location within a face as (X_κ, Y_κ). Note that related simplifications of continuous space to a discrete space representation can be found in Mehrez and Stulman (1982), Church (1984) and Murray and Tong (2007) (see also Church and Murray 2018).

Of course, Equations (1)-(5) (see chapter 3) is a bi-objective spatial optimization problem, so coverage is but one aspect of locational siting criteria. However, Theorem 1 implies that identifying non-dominated solutions essentially involves a search of each face in order to find the location that offers the best access, which corresponds to the location within each face that minimizes total weighted distance, (1). Formally, this is the following:

$$\min_{(X_\kappa, Y_\kappa) \subset \nu_\kappa} \sum_i a_i \sqrt{(x_i - X_\kappa)^2 + (y_i - Y_\kappa)^2} \tag{10}$$

This means that the goal is to find (X_κ, Y_κ) for each face $\nu_\kappa \in \mathrm{K}$ according to (10), with a known level of demand coverage $\sum_{i \in \Omega_\kappa} a_i$. The remaining challenge is optimally solving (10) for each $\nu_\kappa \in \mathrm{K}$.

If only objective (1) is considered, then the optimal solution, $(\hat{X}, \hat{Y})$, can be found using the iterative approach of Weiszfeld (1937) (see also Kuhn and Kuenne 1962, Weiszfeld and Plastria 2009, Plastria 2011, Church 2019). $(\hat{X}, \hat{Y})$ can be thought of as the regional Weber point, in contrast to the face Weber point, (X_κ, Y_κ). Since (1) is convex (see Kuhn and Kuenne 1962, Plastria 2011), the following can be inferred.

Theorem 2. The point (X_κ, Y_κ) with the lowest total weighted distance in each face ν_κ, (10), is that which is closest to $(\hat{X}, \hat{Y})$.

Proof: The contention is that $\min_{(X_\kappa,Y_\kappa)\subset\nu_\kappa}\sqrt{\left(X_\kappa-\hat{X}\right)^2+\left(Y_\kappa-\hat{Y}\right)^2}$ is an optimum for (10). Suppose there exists $(X,Y)\subset\nu_\kappa$ such that $\min_{(X,Y)\subset\nu_\kappa}\sum_i a_i\sqrt{(x_i-X)^2+(y_i-Y)^2}<\min_{(X_\kappa,Y_\kappa)\subset\nu_\kappa}\sum_i a_i\sqrt{(x_i-X_\kappa)^2+(y_i-Y_\kappa)^2}$. If this were true, then (1) would not be convex However, (1) is convex (see Kuhn and Kuenne 1962, Plastria 2011 for formal proofs), so $\min_{(X_\kappa,Y_\kappa)\subset\nu_\kappa}\sqrt{\left(X_\kappa-\hat{X}\right)^2+\left(Y_\kappa-\hat{Y}\right)^2}$ must be an optimum for (10). Worth noting is that proofs in terms of visibility are offered in Schaefer and Hurter (1974), Hansen et al. (1982) and Watson-Gandy (1985) and represent an alternative approach for Theorem 2.

Given the above definitions and theorems, the algorithm detailed in Figure 1 is now explained with supporting justification and proof. The next section demonstrates implementation and application feasibility, highlighting the utility of this approach along with showing computational efficiency.

Chapter 5

Application Results

The algorithm outlined in Figure 1 to solve the model (1)-(5) (see chapter 3) was implemented in Python using a Jupyter notebook, relying on ArcPy for needed GIS functionality. Reported computational details are for a Windows 10 AMD Ryzen CPU 3600 with 32GB RAM desktop computer. Two planning applications are examined. The first is utilized in Church and Baez (2020), known as the Swain data, consisting of 55 locations reporting a total demand for service of 640 in the Washington, D.C. region. The second study involves 85 batted balls during college baseball games at UCSB during the 2018-2019 season, with a total value (or total demand) of 280.9886.

In the case of emergency response, the goal is to site the facility (e.g., fire station and/or ambulance) in order to respond to anticipated calls for service, both maximizing demand that can be served within the S distance/time standard as well as minimizing average distance/time to respond to all demand (within and beyond the S standard). The demand for service is shown in Figure 3. The relied upon service coverage standard is $S = 10$ miles.

Demand coverage areas (8) for each of the 55 demand locations result in 1,557 faces when the overlay operation, (9), is applied. Again, the faces are significant (Theorem 1) because they represent unique coverage potential of the area. This means that total coverage

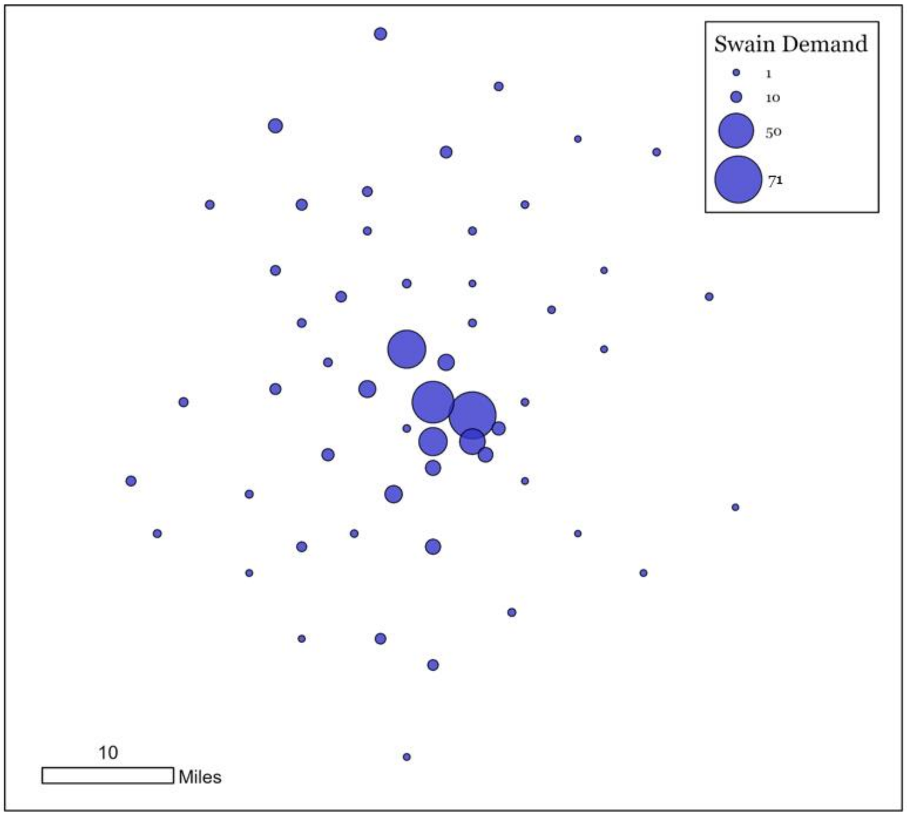


Figure 3. Demand intensity and location (Swain 55 demand locations).

is known for any location within the face, leaving the location within the face that has the lowest total weighted distance to be found. This can be accomplished using Theorem 2. Thus, coverage and minimum total weighted distance can be derived for each face. Total processing and solution time in this case is 15 seconds. Figure 4 summarizes all 1,557 solutions by average distance (x-axis) and coverage (y-axis). This is a standardization of objectives (1) and (2) by total demand (640 in this case) in order to simplify interpretation. As can be seen, most of the solutions are dominated, with seven identified non-dominated (Pareto optimal) solutions superior to all others with respect to both coverage maximization and total weighted distance minimization. Note that Figure 4 is provided for illustration purposes as the only solutions of significance are those that are non-dominated based on optimizing the model (1)-(5) (see chapter 3). Accordingly, Figure 5 shows the non-dominated solutions only, enabling associated trade-offs to be more clearly observed. As can be seen in Figure 5, the leftmost solution reflects an emphasis on minimizing total weighted distance, resulting in an average distance of 9.03 miles and coverage of 64.5% demand within 10 miles, whereas the rightmost solution offers the most coverage possible (68.28%) within the standard and average distance increasing to 10.67 miles. An interesting trade-off solution is highlighted in Figure 6, showing the spatial location selected, offering coverage of 67.97% and an average distance of 9.39 miles.

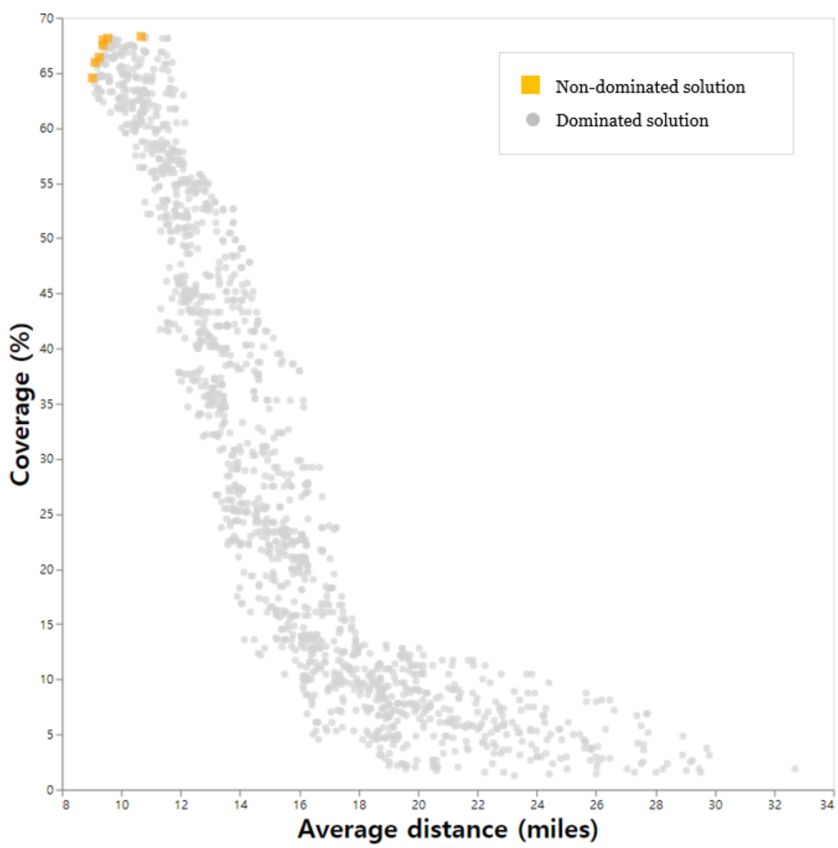


Figure 4. Objective space summary of identified solutions (Swain 55 demand locations in Washington, D.C.).

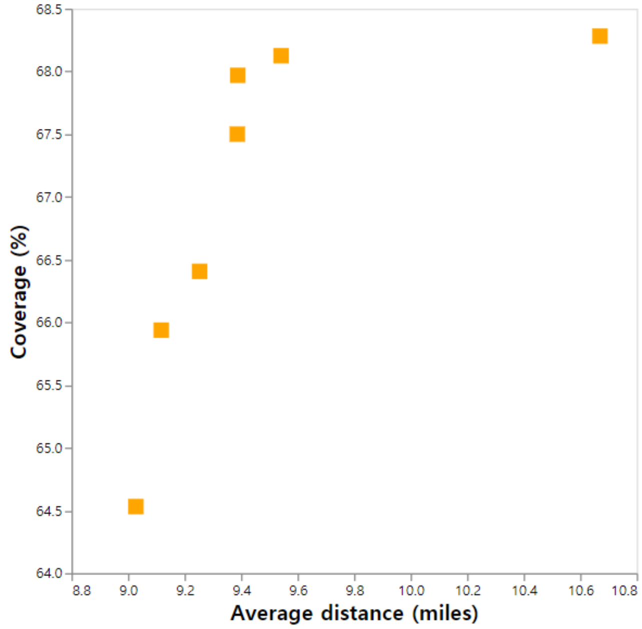


Figure 5. Non-dominated solutions
(Swain 55 demand locations in Washington, D.C.).

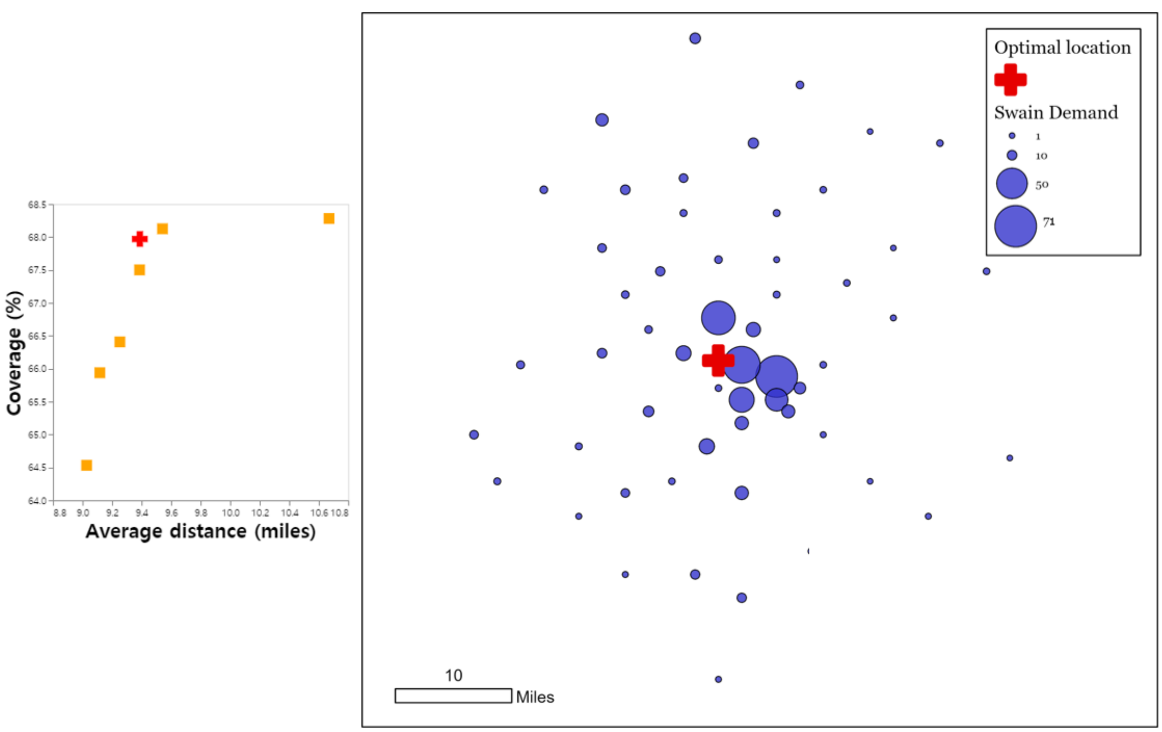


Figure 6. Select a non-dominated solution location
(Swain 55 demand locations in Washington, D.C.).

The baseball setting suggests a goal to position a player (facility) in order to respond to an anticipated hit, both maximizing total demand that can be served within the S distance/time standard as well as minimizing average distance/time to respond to all demand (within and beyond the S standard). The hit importance (demand) for service is shown in Figure 7, totaling 280.989 across the 85 hits. The relied upon service coverage standard is $S = 90$ feet, reflecting the expected range that a collegiate player could respond to a hit under certain hangtime conditions. Accordingly, the player is expected to cover the anticipated hit by catching it without letting it touch the ground when the ball falls within the S standard.

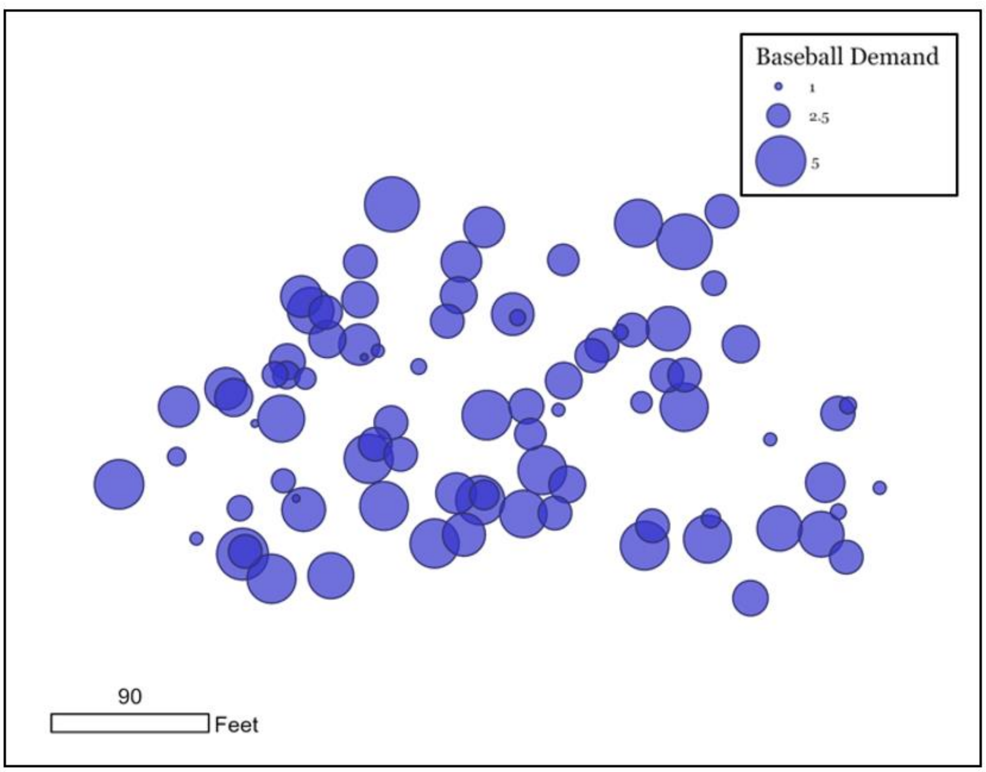


Figure 7. Hit importance and location (UCSB 85 hit locations).

The 85 demand locations result in 4,666 faces for the case of $S = 90$ feet. There are 20 non-dominated solutions identified by the exact solution approach (Figure 1), and these are summarized in Figure 8. The total solution time is 45 seconds. As shown in Figure 8, the leftmost solution reflects an emphasis on minimizing total weighted distance, resulting in an average distance of 112.03 ft. and coverage of 36.76% demand within 90 ft. In contrast, the rightmost solution offers the most coverage possible (47.45%) within the standard and average distance increasing to 115.94 ft. An interesting trade-off solution is highlighted in Figure 9, showing the spatial location selected among the non-dominated solutions, offering coverage of 45.79% and an average distance of 114.43 ft

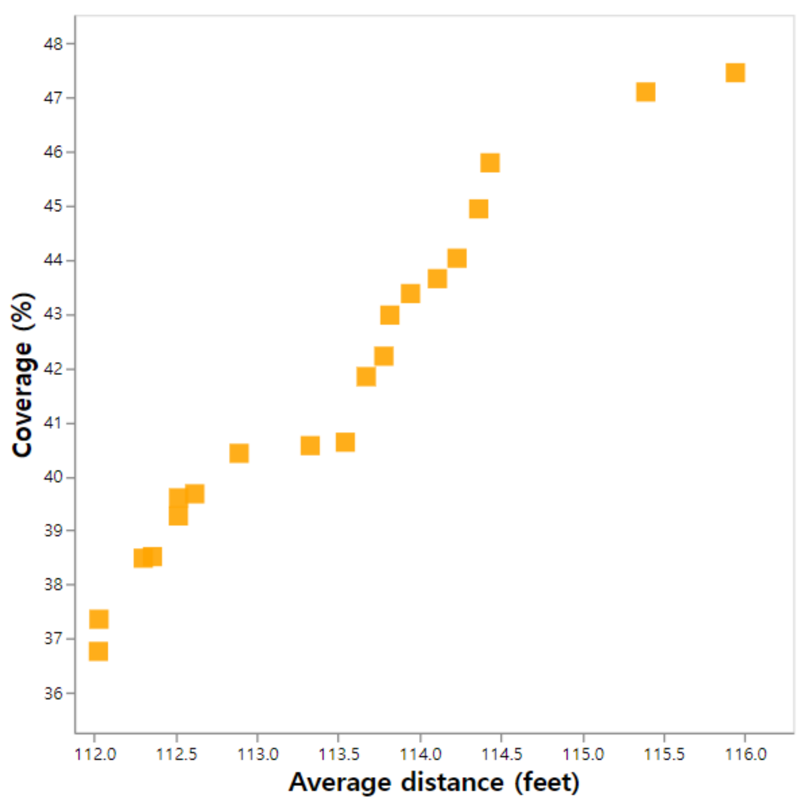


Figure 8. Non-dominated solutions (UCSB 85 hits).

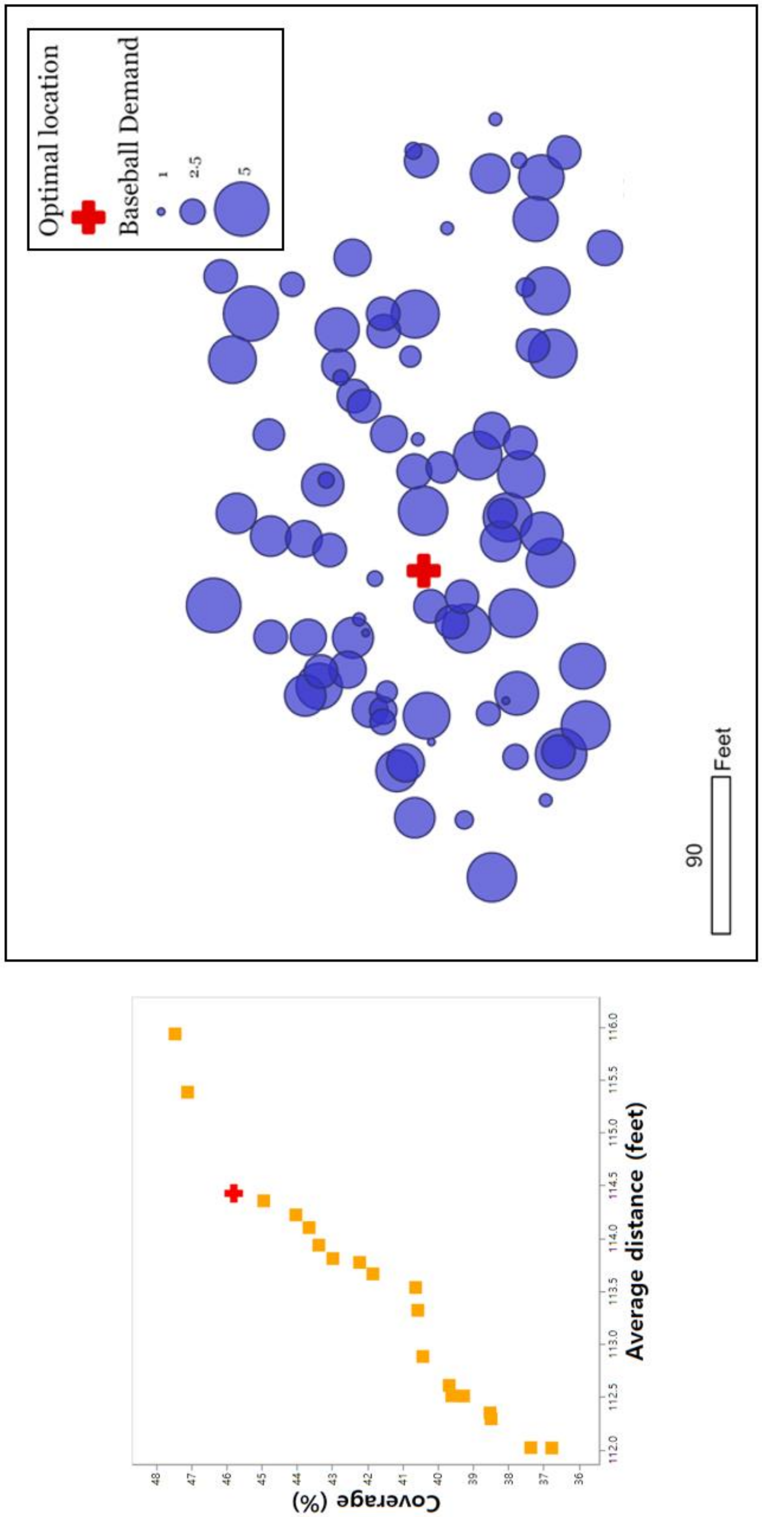


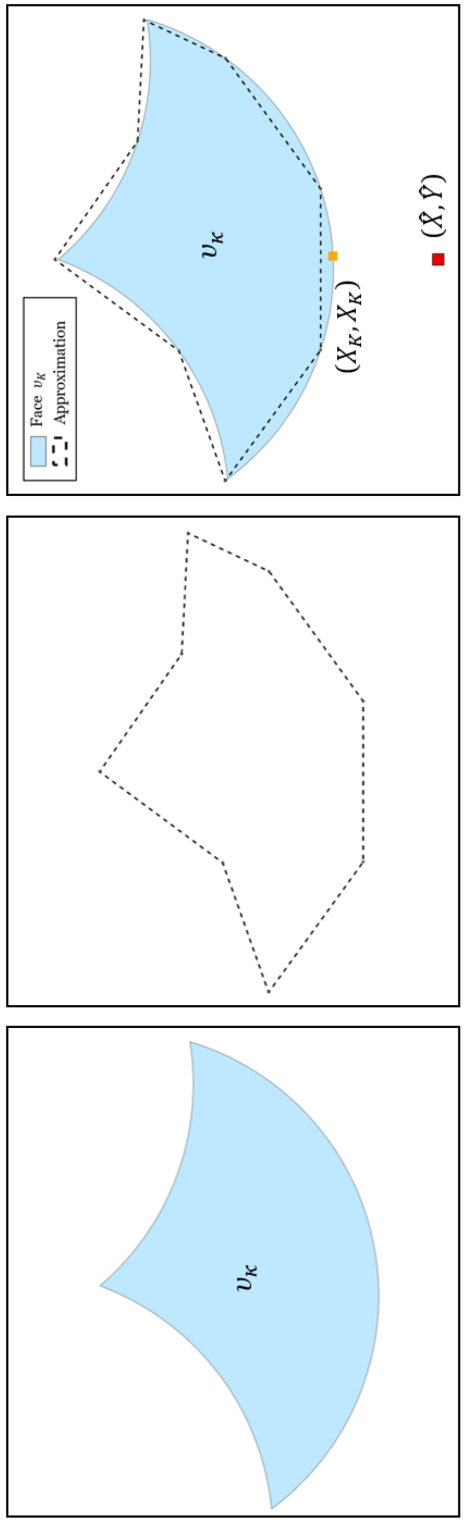
Figure 9. Selected non-dominated solution location (UCSB 85 hits)

Chapter 6

Discussion

There are a number of interesting and important discussion points regarding this research. The first has to do with the impacts of geometry approximation. The second involves the use of commercial solvers capable of dealing with non-linear models. The final is placing the proposed modeling approach in the context of previous research addressing distance constraints.

Precise representation of spatial objects is not trivial. Appropriate geometry generally results in greater computational processing as arcs and other curvatures must be dealt with. One approach that has historically been relied upon to deal with complex geometries is through polyline approximations. This means that circles, curves and other non-linear geometries are represented using a polyline consisting of a number of connected straight-line segments. Commercial and open source GIS software as well as open source GIS libraries (e.g., QGIS, Shapely, matplotlib) often implement linear approximation/simplification of non-linear geometries. Of importance in this research is the likely impact on optimality with respect to Theorem 2. Specifically, Theorem 2 is only valid when the face geometry is accurate. The nearest point from each face, (X_κ, Y_κ), to $(\hat{X}, \hat{Y})$, the location of minimum total weighted distance (e.g., regional Weber point), will be on the



(a) Face v_κ

(b) Approximation of v_κ

(c) Optimal location implications

Figure 10. Face approximation implications.

face boundary. However, if the face boundary is approximated, then the result will be one of four outcomes: outside the face, inside the face, a non-closest boundary point or closest boundary point. The last outcome, the closest boundary point would be virtually impossible since the approximation would consist of a polyline vertex at precisely the optimal location. In other cases, the result is a non-optimal location at best, and at worst a non-feasible location (outside the face).

Figure 10 illustrates the impact of face approximation using polylines. The precise face geometry of υ_{κ} is shown in Figure 10a, with a polyline approximation given in Figure 10b. The implications of such an approximation are offered in Figure 10c, where the regional Weber point, $(\hat{X}, \hat{Y})$, is indicated as a red square and the closest point in face ν_{κ}, (X_{κ}, Y_{κ}) the face Weber point, is also given. The significance is that it is impossible to identify (X_{κ}, Y_{κ}) if the polyline approximation is relied upon. For this reason, this research used ArcPy through ArcGIS in order to ensure correct geometry representation.

Given advances made in solving non-linear optimization problems, supporting commercial software now includes more capabilities than ever before. The natural question is whether such software is capable of solving the model (1)-(5) (see chapter 3), and what are the implications for specialized solution approaches such as that offered in Figure 1. To address this issue, LINGO was considered given its non-linear solution capabilities. A prominent question, no doubt, is cost. The use of a commercial package like LINGO is not necessarily a trivial expense. A single-user license can range from around $500 to $5,000, depending on the number of decision variables encountered. In theory, the solution approach outlined in Figure 1 could be implemented using open source software, but this was not done here given the representation issues encountered in dealing with face arcs. Therefore, the use of ArcPy would require a commercial software license of ArcGIS as well, potentially costing $100 or more. Perhaps of more relevance and significance is solver performance. Consider the Swain 55 locations in Washington, D.C in emergency response context. The weighting method was used to integrate objectives (1) and (2) as a single objective, with the weight $w \in [0,1]$. Various weights were considered, and the model was repeatedly solved. In total, 23 problem instances were solved, requiring 373 seconds. All seven non-dominated solutions were found. In the case of finding an outfielder location for the UCSB 85 baseball hits, LINGO found only 4 of the 20 non-dominated solutions, requiring 1,294 seconds to solve. This is based on a selective set of weights, with 23 total problems solved in the previous case. An alternative would involve the use of the constraint method, requiring 878 seconds to solve 26 problem instances in the Swain 55 case. Irrespective of the method employed, there is a clear benefit of the developed algorithm in Figure 1 using computational geometry as it is much more efficient than the commercial non-linear solver considered here, LINGO. However, it is encouraging to see that LINGO was able to do an outstanding job in solving the non-linear problem instances, generally capable of finding non-dominated solutions, albeit requiring more computational effort significantly.

Schaefer and Hurter (1974) referred to the Weber problem with metric constraints, a special type of LSCP. The problem was navigated by subsequent work by Hansen et al. (1982) and Watson-Gandy (1985). Watson-Gandy (1985) applied their solution algorithm to a dataset with 20 demand points with the same demand. The service standard distance was set as 3 (units). This application is examined using the proposed modeling approach, Equations (1)-(5) in chapter 3, and solved using the exact algorithm outlined in Figure 1. The solution in this case is trvial (Figure 11), however, as the regional Weber point $(\hat{X}, \hat{Y})$ is actually within a face that can cover all demand within the 3 unit standard. Thus, $(\hat{X}, \hat{Y}) = (X_\kappa, Y_\kappa)$ for face κ providing 100% coverage. This is curious because it defeats the purpose of the approach developed in Watson-Gandy (1985) to, in essence, find the closest point on the face to regional Weber point, capable of covering all demand. Further, there are no tradeoffs because all demand can be covered by $(\hat{X}, \hat{Y})$.

In order to provide a compelling situation more reflective of the intent of Watson-Gandy (1985), three demand points were modified with higher demand (see Figure 13). This results

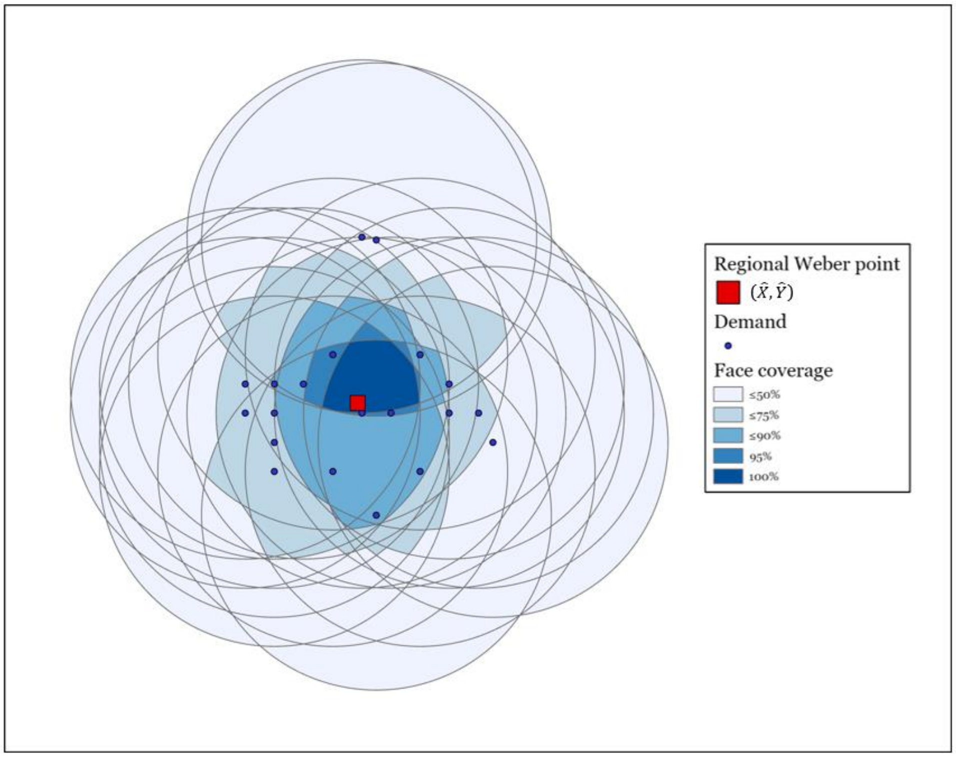


Figure 11. Solution for the Watson-Gandy 20 demand application.

in $\left(\hat{X}, \hat{Y}\right)$ now being outside the face with 100% coverage (Figure 13). Solution of the model formulated by Equations (1)-(5) now identifies multiple non-dominated solutions, summarized in Figure 12. Worth highlighting is that the solution shown in Figure 13a with an average distance of 1.616 and 100% coverage is the only one that would be identified using the approach of Watson-Gandy (1985). In contrast, additional non-dominated solutions emerge when less than 100% coverage is possible. In this case, Figure 12 identifies the non-dominate solution, with coverage of 100%, 96.97% and 93.94% and corresponding average distances of 1.616, 1.603 and 1.474, respectively. Thus, average distance can be reduced by 0.82% and 8.79%, respectively, with a slight loss of demand coverage within the standard. The associated spatial locations for the non-dominated solutions are shown in Figure 13, with Figures 13b and 13c showing the two cases with greater access but slight decreases in coverage.

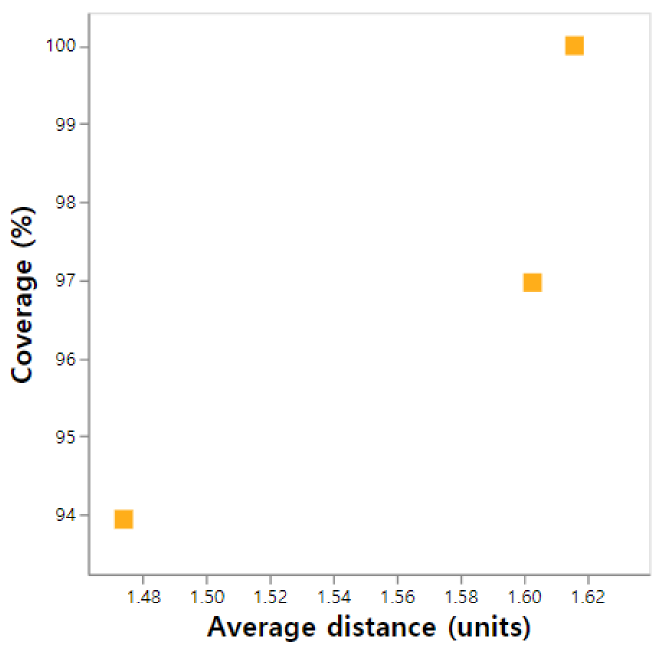
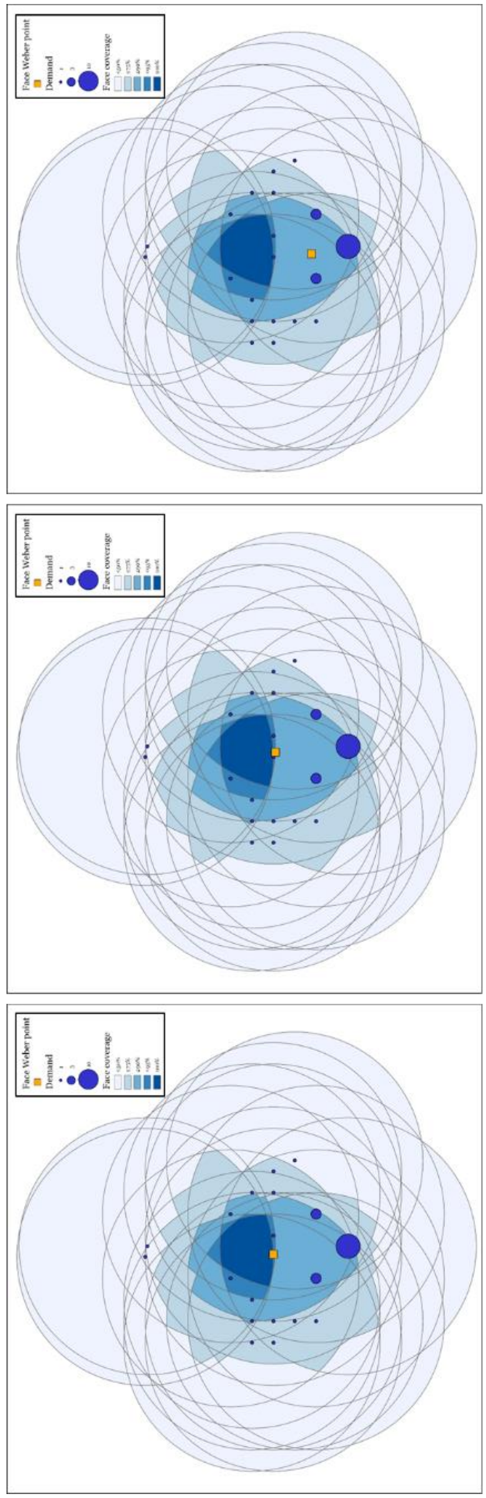


Figure 12. Non-dominated solutions (modified Watson-Gandy 20 points).



(a) Non-dominated solution (1.616, 100%,)

(b) Non-dominated solution (1.603, 97%)

(c) Non-dominated solution (1.474, 94%,)

Figure 13. Spatial location for each non-dominated solution (modified Watson-Gandy 20 points).

Chapter 7

Conclusions

This thesis detailed the use of a spatial analytic approach to a bi-objective extension of the Weber problem, addressing issues of coverage maximization along with access optimization. The mathematical formulation of the problem was presented. Issues of the problem complexity arise due to mixed linearity and non-linearity conditions, necessitating a new and innovative solution approach. An algorithm capable of identifying exact solutions was developed. The algorithm uses GIS-based functionality along with spatial insights to find the entire set of non-dominated solutions. Application results were presented, highlighting the utility and importance of this new modeling approach along with the capabilities of the exact solution algorithm. The potential that GIS and spatial knowledge can contribute to structuring and solving important geographic problem is particularly salient across regional science, but more generally as well.

There are other related problems that may be amenable to the approach presented here. One of these might involve the same general problem, that of locating one facility on a continuous space and minimizing weighted distance and maximizing coverage where distances are measured based upon the Manhattan metric. Another problem of interest could involve a step function of coverage benefits, or even address localized obnoxious elements that are present in some problems (e.g., fire station location).

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