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Authors

Qiu, Tony Z. Lu, Xiao-Yun Chow, Andy H. F. <u>et al.</u>

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California PATH Working Paper UCB-ITS-PWP-2009-6

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Real-time Density Estimation on Freeway with Loop Detector and Probe Vehicle Data

PATH Working Paper

Tony Z. Qiu, Xiao-Yun Lu, Andy H. F. Chow, Steven Shladover

FHWA-EXARP and Caltrans TO6224

April 20, 2009

Key Words

Density Estimation, Vehicle Infrastructure Integration (VII), Loop Detector, Probe Vehicle

Abstracts

Density, speed and flow are the three critical parameters for traffic analysis. Traffic management and control with high performance require accurate estimation/prediction of distance mean speed and density for large spatial and temporal coverage. Speed, including time mean speed and distance mean speed, and flow estimation are relatively easy to be measured and estimated in the practical site, but accurate density estimation is very difficult. Inductive loop detector systems have been widely deployed, it makes better sense to fully adopt available infrastructure to achieve required traffic measurement. As a new promising technology for transportation system, Vehicle Infrastructure Integration (VII) is developing rapidly with the market penetration of cell phone and GPS systems. This report proposed a method for real-time estimation of density using synchronized loop detector data and VII probe vehicle data. Berkeley Highway Laboratory (BHL) loop detector data and the field collected Probe Vehicle data have been used in the method validation. Density estimated from the vehicle-by-vehicle trajectory tracking in Next Generation Simulation (NGSIM) data has also been used as the second data source for validating the algorithm. Comparison of the two results - that form the loop and VII probe vehicle data and that from NGSIM data, showed that they are very close except a small offset which needs further investigation.

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1 Introduction

Density, speed and flow are the three critical parameters for traffic analysis. Traffic management and control with good performance requires the real-time estimation of distance mean speed and density as input for large spatial and temporal coverage of roadway network. It would be better that density can be measured and estimated lane by lane.

Generally, it is believed that traffic density estimation or prediction is more difficult than the estimation or prediction of distance mean speed and flow. This is partly because the most popularly used sensors are inductive loop detectors which are point sensors while density is a range concept. The disturbances to density estimation include location, weather, and vehicle types, etc. Further, lane changing behavior affects lanewise density significantly. As a matter of fact, both lane-wise distance mean speed and density are very difficult to be measured/estimated accurately in real-time although they are required in Coordinated Ramp Metering (CRM) and Variable Speed Limit (VSL) control.

This report is organized as the following: Section 2 is for literature review. Section 3 lists the traditional method only using loop detector data, and presents the proposed methodology based on available loop detector data and VII probe vehicle data. Section 4 details the results from our field test, and empirical results and their implications on developing traffic control strategies are also discussed. Finally, Section 5 gives some concluding remarks and future study.

1.1 Density Estimation

As an instantaneous and range concept, traffic density is primarily defined by looking into a snapshot photo of the traffic by an aerial camera along a stretch of freeway (Greenshields, 1933; Greenshields, 1935). The density is the number of vehicles divided by the length of the stretch. Average density over lanes is naturally deduced by further dividing the number of lanes. This concept can be described as continuous in space but discrete in time. However, practical traffic network system, particularly freeway network, does not have aerial camera to continuously monitor the traffic in real-time. Although dense point sensor system could approximate continuous measurement in space, the cost is prohibitive in general. In practice, point sensors such as inductive loop detectors are popularly used for traffic detection, which could continuously count vehicle number in traffic stream in real-time at the sensor locations. i.e. practical data provide information continuous in time but discontinuous in space. This is the gap between the state of traffic monitoring and the practical data requirement for real-time traffic control such as CRM and VSL.

Basically, the accuracy of measured flow from existing systems can be reasonably high, while the accuracy of distance mean speed estimation has a large variance. Hence, it is difficult to obtain relatively good estimation for density only based on the flow/speed/density relation with estimated flow and speed data from existing systems. The proposed method derives from the definition of density, and counting the vehicle number within specific segments correctly is the key for this method. With the development of vehicle telematics and other ITS technologies such as GPS (Global Positioning System) and VII (Vehicle-Infrastructure Integration), information from the vehicle itself could be a potentially valuable source to compensate for the deficiency of the point sensors. Reliable flow estimation from loop detector system and accurate vehicle location information from VII probe vehicles make it feasible to count the vehicle numbers correctly within specified segments once the market penetration of vehicles with VII is increasing to some level. This is the basic idea of the proposed methodology.

1.2 Congestion and Density

Why density estimation is important? It is critical to traffic control such as CRM and VSL. To avoid the local undesirable effect of ramp metering, it is necessary to estimate not only the average density, but also spatial-temporal density distribution. Particularly, it is necessary to know when traffic stream with a low or high density is passing the metered onramp. By knowing this in real-time, the ramp metering strategy can be designed so that it not only optimizes the traffic from a macroscopic level which is usually based on average density of each cell (section, or segment), but also injects vehicle into the freeway in right time so that the density distribution is approaching even. It is thus necessary to implement ramp metering strategy in two levels: (a) optimization in a macroscopic level based on average density of each cell along a stretch of freeway, which determines the optimal reference average density; (b) to select appropriate time slot based one density distribution in mainline to avoid imputing vehicles into traffic stream with high density.

1.3 Research Scope

For a stretch of freeway, if inductive loops are installed, vehicle count or volume between two adjacent loop stations can be determined once the onramp and off-ramp flow are known. Based on the vehicle count at initial time instance, vehicle count change and the distance between the two loop stations, an average density can be estimated. However, density variation at each distance point with respect to time cannot be estimated unless the loop stations are dense enough, as this is cost prohibitive for a freeway system. Inductive loop detector systems have been widely deployed, it makes better sense to fully adopt available infrastructure to achieve required traffic measurement. The VII vehicle with GPS provides a continuous measurement of speed and position of the vehicle within a moving window in real-time. This is a very good data source which complementary to the fixed loop data measurement.

This report considers real-time estimation of the density along a stretch of freeways based on (a) inductive loop stations which provide point measurement of traffic speed and vehicle count; (b) certain percentile of probe vehicles which provide moving window over distance for speed and position.

2 Literature Review

It is necessary to develop a ramp metering strategy to control density distribution along a stretch of freeway. If measurement of traffic is not accurate enough and the time for traffic injection through onramp is not proper, the imputed vehicle from onramp could cause congestion. This leads to a new conclusion: ramp metering needs the estimation of density distribution, not just average density or volume in cell (section, or segment). The injection should be at proper time instant – if there is a traffic stream with high density passing by, and ramp meter should not impute any vehicle into the mainline.

A method is discussed for estimating the number of vehicles on a section of a freeway from speed and flow measurements at the entrance and exit points of the section (Gazis 1971). The author proposed one real-time estimation method of traffic densities from time-series of flow and speed data. The method consists of obtaining true estimate of vehicle count at regular intervals, and then filtering random errors of these estimates by means of a sequential correction scheme. Periodically an estimation of vehicle travel times is needed from time-series speed data, and then a rough estimate of vehicle count is obtained from the inventory of car arrivals at the entrance of the section and the travel times of existing cars. The algorithm has been tested using data from three adjoining half-mile sections in the Lincoln Tunnel. This method relies on the accuracy of travel time estimation very much.

Another method bases its estimation on two easily obtained measurements from loop detector data: the flow rate and occupancy (Andrew, 1980). The occupancy of a detector during a particular time interval is the percent of that interval during which the detector signals the presence of vehicles. From measurements at neighboring detector stations, the number of vehicles, and therefore the density, on the segment between stations is tracked. The estimate is calculated using a Kalman filter. The detection of spatially inhomogeneous conditions is done using a Generalized Likelihood Ratio (GLR) event detection scheme. Both the filter and the GLR algorithm are simple discrete-time scalar equations. It is claimed that the unique point of the study is that it estimates the segment density accurately in all types of traffic conditions, and it does this by detecting spatially inhomogeneous traffic conditions and compensating the density estimate appropriately for the adverse effects of the inhomogeneities.

The traffic states were considered as Markov chain as time evolving and used linearized Cell Transmission Model based Kalman filter to provide the estimation of spatiotemporal density by Sun et al (2003). Three methods are evaluated based on traffic data on I-210. The criterion used is the relative error at fixed distance point where the measurement is available. For a stretch of freeways, the real-time estimation of the density at those measurement points was incorporated to enhance the estimation process.

Vehicle Re-identification technique was used to estimate in-lane flow and lane density by Coifman (2003). A method was developed for estimating density in a freeway lane between detector stations and measuring the net number of vehicles to enter or leave the lane. It relies on the accuracy of vehicle re-identification algorithm very much. The author tried to implement lane inflow estimation as well, and it was a good starting point to investigate the feasibility of estimating lane density only using loop station data. There was no dynamic traffic model involved in the algorithm.

Another way to determine traffic flow characteristics, including traffic density, was by definition directly (Ni, 2007). The following criteria need to be met considering to measure traffic flow characteristics: it involves only one type of sensor that is capable of determining density, flow and distance mean speed; the measurement should be accurate; it should preserve the fundamental relationship among density, flow and distance mean speed; and it should be compatible with ITS application. The paper tackled the challenge by formulating a method called the n-t method, the inputs of this method were time-stamped traffic count. Further, it was assumed that a point sensor was able to accurately timestamp the passage of each vehicle at the sensor location. It was feasible to construct cumulative curves of vehicle counts in the cumulative count-time (n-t) domain. The method required that no vehicle was lost or gained within any given region, traffic should be counted and time-stamped accurately, the roadway section under analysis was empty

at the initial time interval; and a mechanism was needed to eliminate error accumulation due to the use of cumulative vehicle counts.

Estimation can also be separated from control if the system state can be directly estimated from sensor data through filtering and/or fusion. In this approach, the sensor measurement and state estimation are independent form the control process. The approach usually does not predict the system state and thus traffic model is not involved in the estimation.

3 Method for Density Estimation

Although, video camera (as range sensor) data are good for density estimation, they are usually unavailable where they are needed. Most of existing traffic detection sensors are inductive loops (as point sensors) which can only generate vehicle count or volume at sensor locations.

Compared with single loop detector, dual loop detector can provide more accurate information which could help in estimating density. Assuming x_i and x_{i+1} are two consecutive loop detector stations which can determine one segment (x_i, x_{i+1}) , dual loop detector can provide spot based speed and vehicle count (equivalent to flow or volume) over the station:

i: detector station index

v(x,k): spot-based speed at location x during time interval k , mph

q(x,k): spot-based flow at location x during time interval k , vphpl

 $v(x_i, x_{i+1}, k)$: segment-based speed at the section between location x_i and x_{i+1} during time interval k, mph

 $q(x_i, x_{i+1}, k)$: average flow at the section between location x_i and x_{i+1} during time interval k, vphpl

 $\rho(x_i, x_{i+1}, k)$: average density at the section between location x_i and x_{i+1} during time interval k, veh/mile/lane

Generally, if the distance between two consecutive loop stations is short, say less than 0.5~1 mile, we may approximate the distance mean speed as the follows:

$$v(x_i, x_{i+1}, k) = \frac{1}{2} (v(x_i, k) + v(x_{i+1}, k))$$

As for average flow along the section (x_i, x_{i+1}) , it can be approximated by:

$$q(x_{i}, x_{i+1}, k) = \frac{1}{2} (q(x_{i}, k) + q(x_{i+1}, k))$$

According to the relation among flow, speed and density, the average density between the section (x_i, x_{i+1}) can be roughly calculated by:

$$\rho(x_{i}, x_{i+1}, k) = \frac{q(x_{i}, x_{i+1}, k)}{v(x_{i}, x_{i+1}, k)}$$

In this paper the above procedure is termed as Method 1 dedicated to using only loop data for density estimation. However, this is not adequate for the estimation of density distribution between two terminal loop stations along a stretch of freeway. It cannot guarantee good results in case there exists on-ramp or off-ramp at the section, or the length of the section (x_i, x_{i+1}) is rather long. Further, the definition of section (x_i, x_{i+1}) is determined by the loop detector locations, and the section definition may be not compatible or appropriate for traffic control.

3.1 Spatiotemporal Density Estimation

With the help of VII probe vehicles, the density distribution estimation between two loop stations or in the section between one loop station and one specific location (the location of VII probe vehicle) at downstream would be feasible. Meanwhile, the definition of section can be more flexible. For example, we can even dynamically define sections based on the need for real-time traffic control.

Existing traffic monitoring systems are based on the vehicle count and occupancy from loop station, which is distributed every 0.5 to 2 miles along freeway. Dual loop station can provide reasonably accurate vehicle speed and vehicle count at one point station. However, this is not adequate for the estimation of density distribution between the two terminal stations along a section of freeway.

With the help of VII vehicles, the density distribution estimation between two loop stations would be feasible. This can be sketched as follows:

Dual loop station provides distance mean speed or time mean speed over the station;
 speed distribution v(x₁,t) and v(x₂,t)

density distribution $\rho(x_1, t)$ and $\rho(x_2, t)$

• Based on the speed and density estimation at the two loop stations and the traffic states (conditions), and the shockwave propagation speed, it is possible to provide an estimation of

```
speed distribution v(x,t)
```

density distribution $\rho(x,t)$

If VII vehicles have certain level of market penetration, the information from those vehicles can be synchronized with loop data for real-time continuously spatial and temporal estimation of speed and density along a stretch of freeways. This can be conducted as follows:

- (1). Estimation of speed and density at point stations;
- (2). Synchronizing/fusing the VII/probe vehicle speed and the loop detector detected speed;
- (3). Compute density between upstream loop detector station and the current location of VII probe vehicle based on vehicle count and distance between VII probe vehicle and upstream loop station;
- (4). Conduct step (3) for each VII probe vehicle, and then interpolate the density around all the VII probe vehicles to obtain the estimation of density continuous in both time and distance.

Since the spatiotemporal characteristics of density is the output of dynamical system with involves both speed and density, and possibly with on-ramp flow. The estimation of the density must involve some dynamic evolving, which means that a model based approach should be taken. With the model as the basis, updated information at the discrete loop stations and the information from probe vehicles should be taken into consideration.

3.2 Assumptions of the Proposed Method

Figure 1 shows the principle of the proposed method with assumptions:

- (1). There is a loop detector at x_i , but not necessarily at x_{i+1} ;
- (2). If there exists on-ramp and/or off-ramp between location x_i and x_{i+1} , detector is assumed to be installed for counting on-ramp and/or off-ramp vehicle;
- (3). The timestamps of loop detector data and GPS from VII vehicle are synchronized;
- (4). If there exist vehicles bypassing VII probe vehicle p during time interval k, the probe vehicle p bypass the same number of vehicles during the same time interval. This is naturally true if lane changing rate is low and can be skipped.

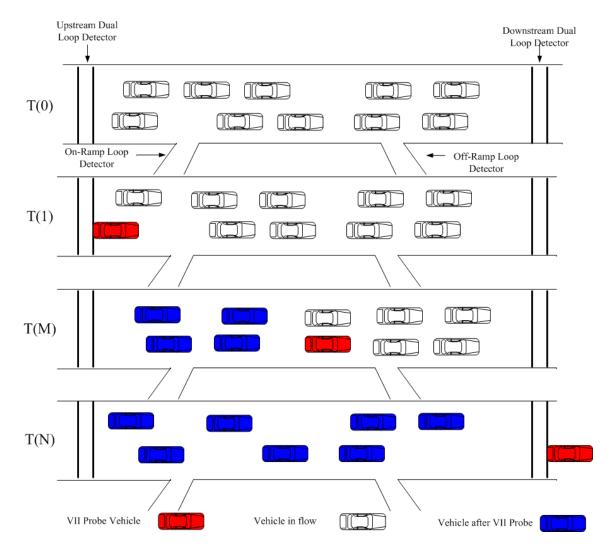


Figure 1. Density Estimation with VII Probe Vehicle and Loop Detector Data

3.3 Notations of the Proposed Method

k: index of time interval, $k = 0, 1, 2, \dots, K$

T: sample time interval for average density estimation, T can be 10 sec, 20 sec, 60 sec, 300 sec, etc., second

 T_u : time interval for VII probe vehicle location update, T_u can be 1 sec, 10 sec, 20 sec, etc., second

j : time index for sampling

 $t = jT_u$: timestamp of vehicle arrival at specified location

L(x, y): distance between location x and y, mile

 $L(x_i, x_{i+1})$: distance between location x_i and x_{i+1} which define a cell, mile

 $L(x_i, p)$: distance between location x_i and VII probe vehicle p, mile

 $LN(x_i, x_{i+1})$: number of lanes between location x_i and x_{i+1}

N(x,t): accumulative vehicle count from loop station at location x from time 0 to t

 $N_r(x_i, x_{i+1}, t)$: accumulative vehicle count from on-ramp loop station between x_i and

 x_{i+1} from time 0 to t

 $N_s(x_i, x_{i+1}, t)$: accumulative vehicle count from off-ramp loop station between x_i and x_{i+1} from time 0 to t

t(x, p): arrival time for VII probe vehicle p at location x

 $\rho(x_i, x_{i+1}, t, p)$: calculated density from vehicle p at the section between location x_i and

 x_{i+1} during time interval k, veh/mile

 $\rho(x_i, x_{i+1}, k)$: average density at the section between location x_i and x_{i+1} during time interval k, veh/mile

 $\hat{\rho}(x_i, x_{i+1}, k)$: estimated density at the section between location x_i and x_{i+1} during time interval k, veh/mile

3.4 Procedures of the Proposed Method

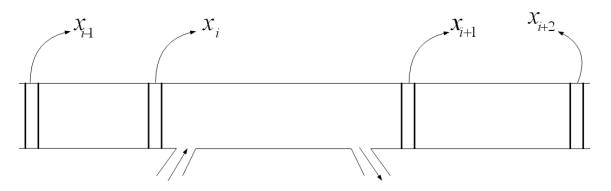


Figure 2. Three consecutive cells

Considering three typical consecutive cells: $[x_{i-1}, x_i), [x_i, x_{i+1})$ and $[x_{i+1}, x_{i+2})$, the general procedure of estimating average density can be formulated as following:

<u>Step 1:</u> Initialization: at the first time interval k = 0, if there exist one valid sample for density estimation, go to step 2 directly; if no, set $\rho(x_i, x_{i+1}, 0)$ as one default density; <u>Step 2:</u> During the time interval $k = 1, 2, \dots, K$, $t \in [(k-1) \times T, kT)$, sampling for every T_u second if one vehicle passed the location x_1 already and is at the location y(y) may be between location x_i and x_{i+1} or may be at the downstream site of location x_{i+1} , the following two inequalities hold $\frac{L(x_i, y)}{L(x_i, x_{i+1})} \ge 80\%$ and $\frac{L(y, x_{i+1})}{L(x_i, x_{i+1})} \le 20\%$, then calculate the measured density at the time instant t from vehicle p at the section between location

 x_i and x_{i+1} as follows.

Main lane vehicle count change:

$$\Delta N = N(x_{i}, t(x_{i+1}, p)) - N(x_{i}, t(x_{i}, p))$$

On-ramp vehicle count change:

$$\Delta N_r = N_r(x_i, x_{i+1}, t(x_{i+1}, p)) - N_r(x_i, x_{i+1}, t(x_i, p))$$

Off-ramp vehicle count change:

$$\Delta N_{s} = N_{s}(x_{i}, x_{i+1}, t(x_{i+1}, p)) - N_{s}(x_{i}, x_{i+1}, t(x_{i}, p))$$

$$\rho(x_{i}, x_{i+1}, t, p) = \frac{\Delta N + \Delta N_{r} - \Delta N_{s}}{L(x_{i}, x_{i+1}) \times LN(x_{i}, x_{i+1})}$$

Otherwise, $\rho(x_i, x_{i+1}, t, p) = 0$

<u>Step 3:</u> Detecting: if there are M_p calculated density from same probe vehicles p during

the time interval k, then $\rho(x_i, x_{i+1}, k, p) = \frac{1}{M_p} \sum \rho(x_i, x_{i+1}, t, p)$ for all $\rho(x_i, x_{i+1}, t, p) \neq 0$

Step 4: At the end of the time interval k, averaging:

a. For one cell determined by location x_1 and x_2 during time interval k, if there are *P* valid VII probe vehicles for sampling, then

$$\rho(x_{i}, x_{i+1}, k) = \frac{1}{P} \sum_{p=1}^{P} \rho(x_{i}, x_{i+1}, k, p)$$

b.

If there is no valid VII probe vehicles for sampling for the cell, then

$$\rho(x_{i}, x_{i+1}, k) = \alpha_{1} \times \rho(x_{i-1}, x_{i}, k-1) + \alpha_{2} \times \rho(x_{i-1}, x_{i}, k) + \alpha_{3} \times \rho(x_{i}, x_{i+1}, k-1) + \alpha_{4} \times \rho(x_{i+1}, x_{i+2}, k-1) + \alpha_{5} \times \rho(x_{i+1}, x_{i+2}, k)$$

3.5 Parameter Identification of the Proposed Method

Ground truth data (exact vehicle count for each time interval) and estimated vehicle count are needed for further parameter identification. Currently, we can apply the following proposed strategy because of insufficient data source.

Case 1: If there exist valid VII probe vehicles from upstream (x_{i-1}, x_i) and there is no valid probe vehicles from downstream (x_{i+1}, x_{i+2})

$$\alpha_1 = 0.3, \alpha_2 = 0.3, \alpha_3 = 0.4, \alpha_4 = 0, \alpha_5 = 0$$

Case 2: If there is no valid VII probe vehicles from upstream (x_{i-1}, x_i) and there exist valid VII probe vehicles from downstream (x_{i+1}, x_{i+2})

$$\alpha_1 = 0.3, \alpha_2 = 0, \alpha_3 = 0.4, \alpha_4 = 0, \alpha_5 = 0.3$$

Case 3: If there is no valid VII probe vehicle from either upstream (x_{i-1}, x_i) or downstream (x_{i+1}, x_{i+2})

$$\alpha_1 = 0.3, \alpha_2 = 0, \alpha_3 = 0.4, \alpha_4 = 0.3, \alpha_5 = 0$$

Case 4: If there exist valid VII probe vehicles from both upstream (x_{i-1}, x_i) and downstream (x_{i+1}, x_{i+2}) $\alpha_1 = 0, \alpha_2 = 0.3, \alpha_3 = 0.4, \alpha_4 = 0, \alpha_5 = 0.3$

In this paper the above procedure is termed as Method 2 using both loop detector and probe vehicle data as input for density estimation.

4 Method Validation

4.1 BHL Data System

To validate the proposed method, the algorithm in Section 3 has been implemented and the practical data from the Berkeley Highway Laboratory (BHL) system have been used. BHL is a test site which covers 2.7 miles of I-80 immediately east of the San Francisco-Oakland Bay Bridge in California, which can be seen from Figure 3. BHL provides event data on individual vehicle actuations, accurate to 1/60th of a second (60 Hz data). Most other loop detector systems collect only aggregated data over periods of 20 seconds or even longer (1 minute, or 5 minutes).

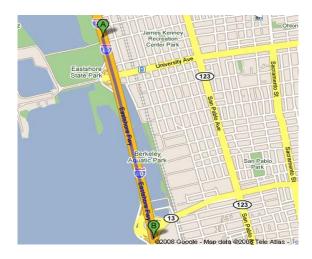


Figure 3. Studied Freeway Stretch – I-80

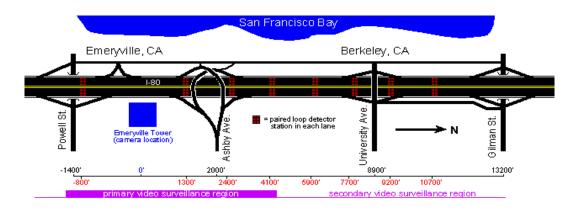


Figure 4. Monitored Freeway Stretch by BHL Data

The format of the raw sorted loop detector data can be seen as the following:

| TimeStamp_1 | TimeStamp_2 | 170 Controller |
|---------------|---------------|--|
| 1218697274000 | 1218697273000 | 27150973F |
| 1218697275000 | 1218697274000 | 271509830 |
| 1218697276000 | 1218697275000 | 271509972C9FEAADACCCFDCDEE0DEAA3F |
| 1218697277000 | 1218697276000 | 271509A0DD26928A5896708F1910A6BB91D68D8B76 |
| 1218697278000 | 1218697277000 | 271509B188EF048 |
| 1218697279000 | 1218697278000 | 271509C2F13105D17C |
| 1218697280000 | 1218697279000 | 271509D4C |
| 1218697281000 | 1218697280000 | 271509E4D |
| 1218697282000 | 1218697281000 | 271509F4E |
| 1218697283000 | 1218697282000 | 27150A02705315365D08168378D1B1739 |
| 1218697284000 | 1218697283000 | 27150A1B34B3EDD4E35E3DE5E40 |
| 1218697285000 | 1218697284000 | 27150A21761D521D47749679743 |
| 1218697286000 | 1218697290000 | 27150A39F4CF5D1479 |
| 1218697287000 | 1218697290000 | 27150A4115D6B41 |
| 1218697288000 | 1218697290000 | 27150A516814B44805 |
| 1218697289000 | 1218697290000 | 27150A646 |
| 1218697290000 | 1218697291000 | 27150A747 |
| 1218697291000 | 1218697291000 | 27150A848 |
| 1218697292000 | 1218697291000 | 27150A926A56954A84943 |
| 1218697293000 | 1218697292000 | 27150AAE2771 |
| 1218697294000 | 1218697293000 | 27150AB1C72204C07EBAE8B0BDC803 |

Table 1 Specification of the raw sorted BHL loop data

The first field is epoch time in milliseconds, and generally is expressed as elapsed time since midnight (beginning of the calendar day) GMT on January 1, 1970. The second field is currently identical to the first field. The third filed represents the raw data from the 170 controller (refer to: <u>http://www.bhl.calccit.org/</u>). The exact location including longitude and latitude of each loop detector is known to us, and it makes it feasible to know the distance between each loop detector and any vehicle of interest and known location.

In the real world, there are no real-time VII probe vehicle data in the studied area. To collect the GPS data to simulate as VII prove vehicle data, we drove two Infinite SUV vehicles five rounds along the studied freeway segment and the data were recorded at 20Hz. In this study, both BHL loop data and the GPS data from test driving are used. We compared two types of density estimation output: estimated density from loop detector data, and estimated density from the proposed method which uses both loop detector data and the GPS. The BHL loop detector data used for this study includes both East-bound

(EB) and West-bound (WB) direction of I-80 on August 14 2008. The format of the GPS data from test driving can be seen as the following:

| Longitude | Latitude | Timestamp |
|---------------------|--------------------|--------------|
| 122.33342650000002 | 37.915342833333334 | 08:56:41.997 |
| 122.33342666666667 | 37.91534266666667 | 08:56:42.047 |
| 122.33342666666667 | 37.91534266666667 | 08:56:42.097 |
| 122.33342666666667 | 37.91534266666667 | 08:56:42.147 |
| 122.33342666666667 | 37.91534266666667 | 08:56:42.197 |
| 122.333427 | 37.91534266666667 | 08:56:42.247 |
| 122.333427 | 37.91534266666667 | 08:56:42.297 |
| 122.333427 | 37.91534266666667 | 08:56:42.347 |
| 122.333427 | 37.91534266666667 | 08:56:42.397 |
| 122.33342733333332 | 37.915342499999994 | 08:56:42.447 |
| 122.33342733333332 | 37.915342499999994 | 08:56:42.497 |
| 122.33342733333332 | 37.915342499999994 | 08:56:42.547 |
| 122.333427333333332 | 37.915342499999994 | 08:56:42.597 |

Table 2 Specification of GPS data from test driving

The first two fields show the longitude and latitude of the corresponding vehicle at each time instance. The third field represents the timestamp in which the record is generated, and the accuracy of timestamp is millisecond.

4.2 Method Validation with BHL and Field Collected Data

The scenarios selected span a wide variety of traffic conditions (free flow, moderate flow), and density has been estimated every 20 seconds. The density estimation results based on two methods are shown in Figure 6 (I-80 WB). The green line is the estimation from two terminal dual loop stations, and the blue spot is from VII probe vehicle and dual loop stations. The length of the section is 334 meters (0.21 mile) with 5 lanes. It is observed from Figure 4 that at time interval around 9:08AM, 9:40AM, and 10:08AM, the estimated results from two methods are quite similar. At 9:22AM, it is still in free flow speed (~ 60 mph), while the flow demand from upstream section is increasing, the density estimation from Method 2 is more sensitive to the change, compared to the Method 1. At 9:53AM, the speed from loop detector has been dropped to around 20 mph for only one time interval for some reason, and it is likely caused by some disturbance rather than actual traffic stream, and then speed went back to moderate flow about 40~50 mph. The estimated density from Method 1 is around 70 veh/mile, while the

one from Method 2 is 28 veh/mile. The distance mean speed estimation in Method 1 has over restricted assumptions which do not hold all the time, and speed estimation cannot guarantee to be reasonable over time. Hence, the density estimation from Method 1 has large errors at times. Because the vehicle count data is relatively good, the estimate from VII probe vehicle and loop detector is able to track the transient behavior of density accurately.

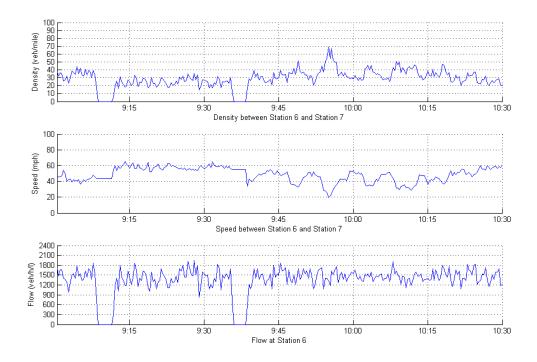


Figure 5. Station 6 – Station 7 @ I-80 West, 08/14/2008

| Lane | Station_6 | Station_7 | St_6_Speed | St_6_Count_1 | St_6_Count_2 | Density |
|------|-----------|-----------|------------|--------------|--------------|---------|
| 5 | 432 | 448 | 38 | 135 | 139 | 19 |
| 4 | 1392 | 1410 | 63 | 459 | 461 | 10 |
| 3 | 2349 | 2364 | 34 | 852 | 856 | 19 |
| 2 | 3258 | 3272 | 44 | 1533 | 1540 | 34 |
| 1 | 4078 | 4092 | 58 | 1919 | 1924 | 24 |
| 5 | 426 | 443 | 39 | 132 | 138 | 29 |
| 4 | 1387 | 1404 | 63 | 456 | 461 | 24 |
| 3 | 2345 | 2360 | 36 | 851 | 854 | 14 |
| 2 | 3244 | 3258 | 49 | 1525 | 1533 | 38 |
| 1 | 4070 | 4084 | 54.3 | 1913 | 1920 | 34 |

Table 3 Calculated density based on Loop Detector and VII Probe Data

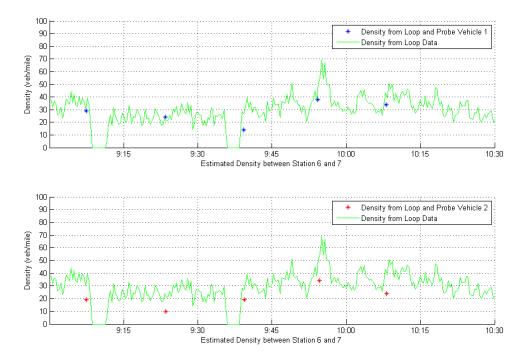


Figure 6. Segment Density Results Comparison between two methods

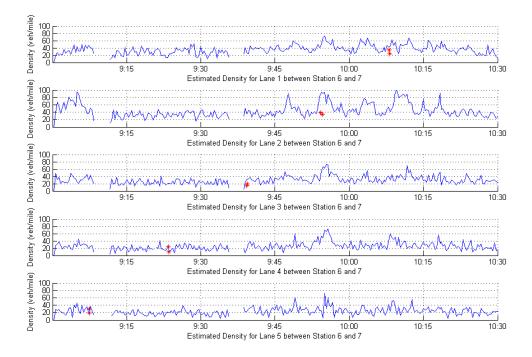


Figure 7. Lane Density Results Comparison between two methods

Figure 7 represents the comparison results of density estimation for each lane individually. The blue line is the estimation from two terminal dual loop stations, and the red spot is from VII probe vehicle and dual loop stations. The five figures represent the five individual lanes from the top to bottom respectively. Similarly, at 9:08AM and 9:20AM the two methods can generate similar results. At 9:53AM, for the same reason, Method 2 is not affected by the accuracy of speed estimation, and is able to track the vehicle count change more accurately. While at 9:40AM and 10:08AM, it is difficult to judge which one has better estimation results, as we don't have ground truth data to compare. In this case, the error of Method 2 comes from the uncertainty of lane change flow. As the length of section is a bit short, the small error of vehicle count can lead to big error of density estimation. This is the limitation of the proposed method applied into lane specific density estimation if the market penetration of VII vehicle is low. If the market penetration is high, there is no such as problem since individual vehicle behavior including lane changing is known. This is the new trend since lane specific density estimation is needed for traffic control such as CRM and VSL control.

4.3 Method Validation with NGSIM data

In 2005, data on individual vehicle trajectories were collected and made available under the Next Generation Simulation (NGSIM) project. The NGSIM data provide a unique opportunity to verify and validate the developing microscopic and macroscopic traffic models. The NGSIM freeway data consists of vehicle trajectories on two test sites: I-80 and US-101 test sections. The I-80 (BHL covered area) test section is chosen for this study, and it is a 0.3125 mile (0.5 km) 6-lane freeway weaving section. The processed data includes 45 minutes of vehicle trajectories in transition period (4:00-4:15PM) and congestion period (5:00-5:30 PM).

In the NGSIM trajectory data, each individual vehicle's position and timestamp were captured every 1/10 second, which can be used as the ground truth of density estimation. In using the NGSIM data, we take each vehicle as virtual VII probe vehicle and take the starting and ending points of the section as virtual loop detector stations. In order to validate the proposed algorithm, with Method 1 (using only loop detector data) and Method 2 (using both virtual loop detector and virtual VII probe vehicle data),

density for the section has been estimated every 20 seconds and every 60 seconds for the same covered time period with NGSIM trajectory data.

The density estimation results based on two methods and ground truth from NGSIM video data are shown in Figure 9 to Figure 14 (I-80 EB). The blue lines in the figures show the density estimation from two terminal dual loop stations, the green lines show the estimated density from virtual loop detector data and virtual VII probe vehicle data. The length of the section is 1650 feet (0.3125 mile, 502.9 meters) with 6 lanes, and three consecutive fifteen minutes data have been analyzed for 04/13/2005. It is transition time period (from free flow to congestion) at 4:00PM-4:15PM, and high congestion period at 5:00PM – 5:30PM.

It is observed from Figure 9 and Figure 10 that the two methods can generate similar estimations from 4:00PM to 4:07PM, while the Method 2 can catch the density dynamic change better than the Method 1 considering the results from 4:07PM to 4:15PM. Similar conclusions can be draw from Figure 11 to Figure 14, the Method 1 has large error than the Method 2 has, whether for 20 second dataset, or for 60 second one. Table 4 detailed the Mean Square Error (MSE) and Root Mean Square Error (RMSE) for different scenarios, and confirmed the conclusions shown from the figures.

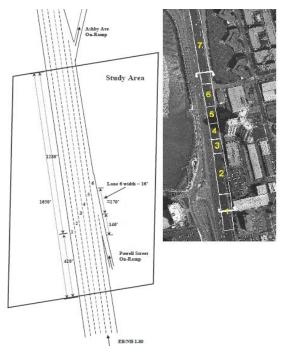


Figure 8. NGSIM Study Area and Camera Coverage

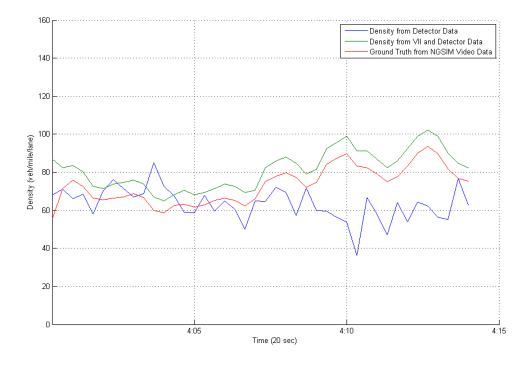


Figure 9. Results Comparison for 20-sec Dataset at 4:00-4:15 PM on 04/13/2005

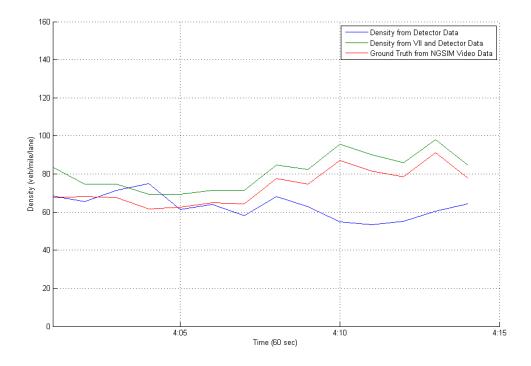


Figure 10. Results Comparison for 60-sec Dataset at 4:00-4:15 PM on 04/13/2005

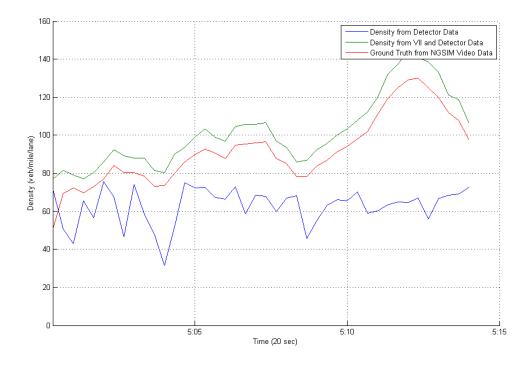


Figure 11. Results Comparison for 20-sec Dataset at 5:00-5:15 PM on 04/13/2005

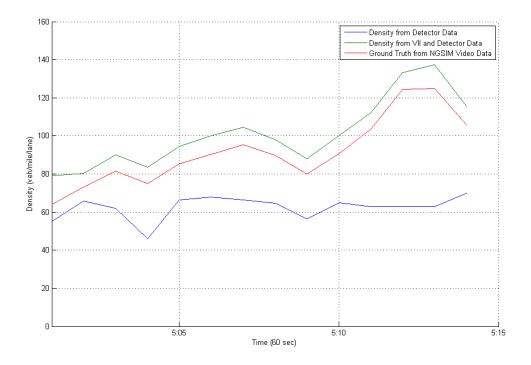


Figure 12. Results Comparison for 60-sec Dataset at 5:00-5:15 PM on 04/13/2005

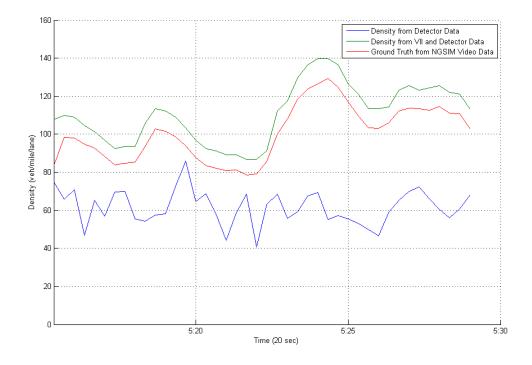


Figure 13. Results Comparison for 20-sec Dataset at 5:15-5:30 PM on 04/13/2005

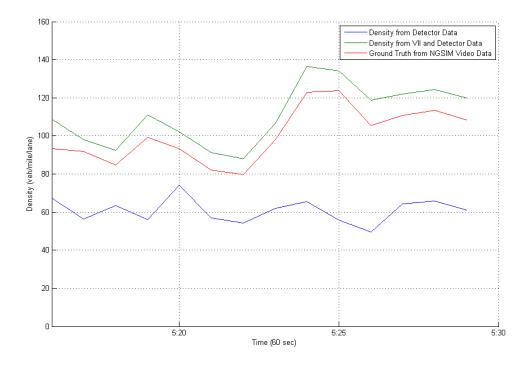


Figure 14. Results Comparison for 60-sec Dataset at 5:15-5:30 PM on 04/13/2005

| | | Time Interval = 20 seconds | | Time Interval = 60 seconds | |
|-------------|------|----------------------------|--------------|----------------------------|--------------|
| | | Detector | VII+Detector | Detector | VII+Detector |
| | | Method | Method | Method | Method |
| 4:00 - 4:15 | MSE | 325.10 | 13.96 | 284.00 | 5.80 |
| | RMSE | 18.03 | 3.74 | 16.85 | 2.41 |
| 5:00 - 5:15 | MSE | 1165.90 | 10.35 | 1107.40 | 5.85 |
| | RMSE | 34.15 | 3.25 | 33.28 | 2.42 |
| 5:15 - 5:30 | MSE | 1791.80 | 5.98 | 1776.80 | 4.17 |
| | RMSE | 42.33 | 2.45 | 42.15 | 2.04 |

Table 4 MSE and RMSE Comparison Between Two Methods

5 Conclusion and Remarks

In this study we tried a new method to integrate the advantages of loop detector and VII probe vehicle information to estimate density for pre-defined freeway segments at a specified time interval in real-time.

More reliable flow estimation from loop detector system and more accurate vehicle location information from VII probe vehicles make it feasible to count the number of vehicles correctly in the time-varying segment bounded by upstream loop station and the probe vehicle. The proposed method derives from the definition of density, and counting the vehicle number within the time-varying freeway segments correctly is the key for this method. With the proposed method applied, segments for traffic network can be defined more freely, and more accurate density estimation can be obtained as well. Experimental results show that the proposed method can generate more reasonable density estimated results – close to the density estimated from NGSIM data.

There is no on ramp and off ramp data currently available in BHL loop detector system. However, the method is still valid once the ramp data are available. In fact, the proposed method should have much higher accuracy if ramp data are available.

Lane specific density estimation may be needed in the practical real-time traffic control system. If the proposed method combining loop detector and VII probe vehicle is applied, the high accuracy of GPS units are needed on vehicle so that VII vehicles can be mapped into each individual lane.

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Appendix

Matlab Code for Density Estimation

```
%% Calculate the mean Occupancy, Speed of segment of interest
%% function Est_State(date_str,up_st_str,dn_st_str)
date_str = '2008-08-14';
st_str = '7';
up_st_str = '6'; % upstream loop station #
dn_st_str = '7'; % downstream loop station #
st dir = 'W';
Start_Hour = 9;
DeltaT_SampleTime = 20; % unit is second
Total_SampleTime = 5400;
No_Interval = Total_SampleTime/DeltaT_SampleTime; % 5400 / 60 = 90, one
hour and half
N state=4;
N dat=100000;
fid_1=0;
  while fid_1<1</pre>
                                 %% Open file for reading
    [fid_1, message]=fopen(['st',st_str,'-',date_str,'-',st_dir,'-
1.dat'],'r');
    if (fopen(fid 1) == -1)
       disp(message);
    end
  end
  [A0,Count]=fscanf(fid_1,'%f %f %f %f %f',[N_state,N_dat]);
  A0=A0';
  fclose(fid_1);
fid 1=0;
  while fid 1<1
                                 %% Open file for reading
    [fid_1, message]=fopen(['st',st_str,'-',date_str,'-',st_dir,'-
2.dat'],'r');
    if (fopen(fid_1) == -1)
       disp(message);
    end
  end
  [A1,Count]=fscanf(fid 1,'%f %f %f %f %f',[N state,N dat]);
  A1=A1';
```

```
fclose(fid_1);
fid_1=0;
  while fid 1<1
                                %% Open file for reading
    [fid_1, message]=fopen(['st',st_str,'-',date_str,'-',st_dir,'-
3.dat'],'r');
    if (fopen(fid_1) == -1)
      disp(message);
    end
  end
  [A2,Count]=fscanf(fid_1,'%f %f %f %f %f',[N_state,N_dat]);
  A2=A2';
  fclose(fid 1);
fid_1=0;
                                %% Open file for reading
  while fid_1<1</pre>
    [fid_1, message]=fopen(['st',st_str,'-',date_str,'-',st_dir,'-
4.dat'],'r');
    if (fopen(fid_1) == -1)
      disp(message);
    end
  end
  [A3,Count]=fscanf(fid 1,'%f %f %f %f %f',[N state,N dat]);
  A3=A3';
  fclose(fid 1);
fid 1=0;
                                %% Open file for reading
  while fid 1<1</pre>
    [fid_1, message]=fopen(['st',st_str,'-',date_str,'-',st_dir,'-
5.dat'],'r');
    if (fopen(fid 1) == -1)
       disp(message);
    end
  end
  [A4,Count]=fscanf(fid_1,'%f %f %f %f %f',[N_state,N_dat]);
  A4=A4';
  fclose(fid_1);
%%% Convert the time index from float type to integer type. %%
A0(:,1) = round(A0(:,1));
```

A1(:,1)= round(A1(:,1)); A2(:,1)= round(A2(:,1)); A3(:,1)= round(A3(:,1)); A4(:,1)= round(A4(:,1));

```
%% value for each individual
A_Mean = zeros(Total_SampleTime,4);
second
Time_Index=zeros(No_Interval, 1);
                                       %% aggregated value for each
Speed = zeros(No Interval, 1);
sampling time interval
Occupancy = zeros(No Interval, 1);
Flow = zeros(No_Interval, 1);
Density = zeros(No_Interval, 1);
Probe_Density = zeros(No_Interval, 1); %% estimated density from VII
probe vehicles
Speed_Lane = zeros(No_Interval, 5);
                                        %% aggregated value for each
individual lane
Flow_Lane = zeros(No_Interval, 5);
                                        %% aggregated value for each
individual lane
Density Lane = zeros(No Interval, 5); %% aggregated value for each
individual lane
%% Initialize the matrix
% Data_Lane_i stored the sorted data for the next step to calculate the
mean
% for lane group level at the specified time index
Data_Lane_1 = zeros(Total_SampleTime,4); % second_index, occupancy,
speed, flow
Data_Lane_2 = zeros(Total_SampleTime,4);
Data_Lane_3 = zeros(Total_SampleTime,4);
Data_Lane_4 = zeros(Total_SampleTime,4);
Data_Lane_5 = zeros(Total_SampleTime,4);
for i=1:Total SampleTime
   Data_Lane_1(i,1)=i;
   Data\_Lane\_1(i,2)=-1;
    Data\_Lane\_1(i,3)=-1;
    Data Lane 2(i,1)=i;
    Data\_Lane_2(i,2) = -1;
    Data\_Lane\_2(i,3)=-1;
    Data_Lane_3(i,1)=i;
    Data\_Lane\_3(i,2)=-1;
    Data_Lane_3(i,3)=-1;
    Data_Lane_4(i,1)=i;
    Data\_Lane\_4(i,2)=-1;
    Data\_Lane\_4(i,3)=-1;
    Data Lane 5(i,1)=i;
    Data Lane 5(i,2)=-1;
    Data_Lane_5(i,3) = -1;
    A_Mean(i,1)=i;
                        % time index
    A_{Mean(i,2)=-1};
                        % occupancy
   A_Mean(i,3) = -1;
                        % speed
                        % vehicle count for one interval
    A_Mean(i,4)=0;
    A Mean(i,5)=0;
                       % accumulative vehicle count
```

```
% accumulative vehicle count for lane 1
    A_Mean(i,6)=0;
                      % accumulative vehicle count for lane 2
    A_Mean(i,7)=0;
                      % accumulative vehicle count for lane 3
    A_Mean(i,8)=0;
   A_Mean(i,10)=0;
A_Mean(i,10)=0;
                        % accumulative vehicle count for lane 4
                       % accumulative vehicle count for lane 5
end
for i=1:size(A0)
    Data_Lane_1(A0(i, 1)+1,2)=A0(i, 2);
    Data_Lane_1(A0(i, 1)+1,3)=A0(i, 3);
    Data_Lane_1(A0(i,1)+1,4)=1;
end
for i=1:size(A1)
    Data_Lane_2(A1(i,1)+1,2)=A1(i,2);
    Data_Lane_2(A1(i,1)+1,3)=A1(i,3);
    Data Lane 2(A1(i,1)+1,4)=1;
end
for i=1:size(A2)
    Data_Lane_3(A2(i,1)+1,2)=A2(i,2);
    Data_Lane_3(A2(i,1)+1,3)=A2(i,3);
    Data_Lane_3(A2(i,1)+1,4)=1;
end
for i=1:size(A3)
    Data_Lane_4(A3(i,1)+1,2)=A3(i,2);
    Data_Lane_4(A3(i,1)+1,3)=A3(i,3);
    Data_Lane_4(A3(i,1)+1,4)=1;
end
for i=1:size(A4)
    Data_Lane_5(A4(i,1)+1,2)=A4(i,2);
   Data_Lane_5(A4(i,1)+1,3)=A4(i,3);
   Data_Lane_5(A4(i,1)+1,4)=1;
end
%%%%%%% to calculate the average vehicle count %%%%%%%%
%%%% No_Interval = Total_SampleTime/DeltaT_SampleTime; % 5400 / 60 =
90, one hour and half
Sum_Speed = 0;
Sum_Occupancy = 0;
Vehicle Count=0;
Vehicle_Count_1=0;
Vehicle_Count_2=0;
Vehicle_Count_3=0;
Vehicle_Count_4=0;
Vehicle_Count_5=0;
Interval_Vehicle_Count = 0;
                                   % related to volume & density
Interval_Vehicle_Count_1=0;
Interval_Vehicle_Count_2=0;
Interval_Vehicle_Count_3=0;
```

Interval_Vehicle_Count_4=0; Interval_Vehicle_Count_5=0; Accumu_Vehicle_Count = 0; % related to accumulative vechile count for j=1:No_Interval for i = 1:DeltaT_SampleTime if Data_Lane_1((j-1)*DeltaT_SampleTime+i,2)~=-1 Flow_Lane(j,1)=Flow_Lane(j,1)+1; % accutally it is vehicle count now; Speed_Lane(j,1)=Speed_Lane(j,1)+ Data_Lane_1((j-1) * DeltaT_SampleTime+i, 3); % actually it is speed sum now Vehicle_Count= Vehicle_Count+1; Vehicle_Count_1=Vehicle_Count_1+1; Interval_Vehicle_Count= Interval_Vehicle_Count+1; Sum Occupancy = Sum Occupancy + Data Lane 1((j-1)*DeltaT SampleTime+i,2); Sum_Speed = Sum_Speed + Data_Lane_1((j-1)*DeltaT SampleTime+i,3); end if Data_Lane_2((j-1)*DeltaT_SampleTime+i,2)~=-1 Flow_Lane(j,2)=Flow_Lane(j,2)+1; % accutally it is vehicle count now; Speed Lane(j,2)=Speed Lane(j,2)+ Data Lane 2((j-1)*DeltaT_SampleTime+i,3); % actually it is speed sum now Vehicle_Count= Vehicle_Count+1; Vehicle_Count_2=Vehicle_Count_2+1; Interval_Vehicle_Count= Interval_Vehicle_Count+1; Sum_Occupancy = Sum_Occupancy + Data_Lane_2((j-1) * DeltaT_SampleTime+i, 2); Sum_Speed = Sum_Speed + Data_Lane_2((j-1) * DeltaT_SampleTime+i, 3); end if Data Lane 3((j-1)*DeltaT SampleTime+i,2)~=-1 Flow_Lane(j,3)=Flow_Lane(j,3)+1; % accutally it is vehicle count now; Speed_Lane(j,3)=Speed_Lane(j,3)+ Data_Lane_3((j-1)*DeltaT SampleTime+i,3); % actually it is speed sum now Vehicle_Count= Vehicle_Count+1; Vehicle_Count_3=Vehicle_Count_3+1; Interval_Vehicle_Count= Interval_Vehicle_Count+1; Sum_Occupancy = Sum_Occupancy + Data_Lane_3((j-1)*DeltaT_SampleTime+i,2); Sum_Speed = Sum_Speed + Data_Lane_3((j-1)*DeltaT_SampleTime+i,3); end if Data Lane 4((j-1)*DeltaT SampleTime+i,2)~=-1 Flow Lane(j,4)=Flow Lane(j,4)+1; % accutally it is vehicle count now; Speed_Lane(j,4)=Speed_Lane(j,4)+ Data_Lane_4((j-1)*DeltaT_SampleTime+i,3); % actually it is speed sum now Vehicle_Count= Vehicle_Count+1; Vehicle_Count_4=Vehicle_Count_4+1; Interval_Vehicle_Count= Interval_Vehicle_Count+1; Sum_Occupancy = Sum_Occupancy + Data_Lane_4((j-1) * DeltaT_SampleTime+i, 2);

```
Sum_Speed = Sum_Speed + Data_Lane_4((j-
1) * DeltaT_SampleTime+i, 3);
       end
       if Data_Lane_5((j-1)*DeltaT_SampleTime+i,2)~=-1
           Flow_Lane(j,5)=Flow_Lane(j,5)+1;
                                            % accutally it is
vehicle count now;
           Speed_Lane(j,5)=Speed_Lane(j,5)+ Data_Lane_5((j-
1)*DeltaT_SampleTime+i,3); % actually it is speed sum now
           Vehicle_Count= Vehicle_Count+1;
           Vehicle_Count_5=Vehicle_Count_5+1;
           Interval_Vehicle_Count= Interval_Vehicle_Count+1;
           Sum_Occupancy = Sum_Occupancy + Data_Lane_5((j-
1) * DeltaT_SampleTime+i, 2);
           Sum_Speed = Sum_Speed + Data_Lane_5((j-
1)*DeltaT_SampleTime+i,3);
       end
       Accumu Vehicle Count=Accumu Vehicle Count+Vehicle Count;
       A_Mean((j-1)*DeltaT_SampleTime+i,5)=Accumu_Vehicle_Count;
       A_Mean((j-1)*DeltaT_SampleTime+i,6)=Vehicle_Count_1;
       A_Mean((j-1)*DeltaT_SampleTime+i,7)=Vehicle_Count_2;
       A_Mean((j-1)*DeltaT_SampleTime+i,8)=Vehicle_Count_3;
       A_Mean((j-1)*DeltaT_SampleTime+i,9)=Vehicle_Count_4;
       A_Mean((j-1)*DeltaT_SampleTime+i,10)=Vehicle_Count_5;
       Vehicle_Count = 0;
   end
       for m=1:5
           Speed_Lane(j,m)=Speed_Lane(j,m)/Flow_Lane(j,m);
           Flow_Lane(j,m)=Flow_Lane(j,m)*3600/DeltaT_SampleTime;
       end
   Time Index(j,1)=j;
   A Mean(j,4)=Interval Vehicle Count;
   Speed(j,1)=Sum Speed/(Interval Vehicle Count+0.000000000000);
   Occupancy(j,1)=Sum_Occupancy/Total_SampleTime;
   Flow(j,1)=Interval_Vehicle_Count*3600/(5*DeltaT_SampleTime);
   Density(j,1)=Flow(j,1)/Speed(j,1);
   Sum Speed = 0;
   Interval_Vehicle_Count = 0;
end
%%%%%%% to calculate the average vechile count %%%%%%%%
for j=2:No Interval
   if Speed(j,1)==0
       Speed(j,1)=Speed(j-1,1);
   end
   Density(j,1)=Flow(j,1)/Speed(j,1);
   for m=1:5
       if Speed_Lane(j,m)==0
       Speed_Lane(j,m)=Speed_Lane(j-1,m);
```

```
end
Density_Lane(j,m)=Flow_Lane(j,m)/(Speed_Lane(j,m)+0.000000001);
   end
end
for i=2:Total SampleTime
   if Data_Lane_1(i,2)==-1
       Data\_Lane\_1(i,2)=Data\_Lane\_1(i-1,2);
   end
   if Data_Lane_1(i,3)==-1
       Data_Lane_1(i,3)=Data_Lane_1(i-1,3);
   end
   if Data_Lane_2(i,2)==-1
       Data\_Lane\_2(i,2)=Data\_Lane\_2(i-1,2);
   end
    if Data_Lane_2(i,3)==-1
       Data\_Lane\_2(i,3)=Data\_Lane\_2(i-1,3);
   end
   if Data_Lane_3(i,2)==-1
       Data_Lane_3(i,2)=Data_Lane_3(i-1,2);
   end
   if Data Lane 3(i,3)==-1
       Data\_Lane\_3(i,3)=Data\_Lane\_3(i-1,3);
   end
   if Data Lane 4(i,2) = -1
       Data_Lane_3(i,2)=Data_Lane_4(i-1,2);
   end
   if Data_Lane_4(i,3)==-1
       Data\_Lane\_3(i,3)=Data\_Lane\_4(i-1,3);
   end
   if Data Lane 5(i,2) = -1
       Data Lane 5(i,2)=Data Lane 5(i-1,2);
   end
   if Data_Lane_5(i,3)==-1
       Data\_Lane\_5(i,3)=Data\_Lane\_5(i-1,3);
   end
end
for i=1:Total_SampleTime
   sum_Occupancy=0;
   sample_Occupancy = 0;
   sum_Speed=0;
   sample_Speed = 0;
   %%%%%%% to calculate the average occupancy %%%%%%%%
```

```
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```

```
if Data_Lane_1(i,2)~=-1
       sum_Occupancy = sum_Occupancy + Data_Lane_1(i,2);
       sample_Occupancy=sample_Occupancy+1;
   end
   if Data_Lane_2(i,2)~=-1
       sum Occupancy = sum Occupancy + Data Lane 2(i,2);
       sample Occupancy=sample Occupancy+1;
   end
   if Data Lane 3(i,2)~=-1
       sum_Occupancy = sum_Occupancy + Data_Lane_3(i,2);
       sample_Occupancy=sample_Occupancy+1;
   end
   if Data_Lane_4(i,2)~=-1
       sum_Occupancy = sum_Occupancy + Data_Lane_4(i,2);
       sample_Occupancy=sample_Occupancy+1;
   end
   if Data_Lane_5(i,2)~=-1
       sum_Occupancy = sum_Occupancy + Data_Lane_5(i,2);
       sample_Occupancy=sample_Occupancy+1;
   end
   A_Mean(i,2)=sum_Occupancy/(sample_Occupancy+0.00000000001);
   %%%%%%%% to calculate the average speed %%%%%%%%
   if Data_Lane_1(i,3)~=-1
       sum_Speed = sum_Speed + Data_Lane_1(i,3);
       sample_Speed=sample_Speed+1;
   end
   if Data_Lane_2(i,3)~=-1
       sum_Speed = sum_Speed + Data_Lane_2(i,3);
       sample_Speed=sample_Speed+1;
   end
   if Data Lane 3(i,3)~=-1
       sum Speed = sum Speed + Data Lane 3(i,3);
       sample Speed=sample Speed+1;
   end
   if Data_Lane_4(i,3)~=-1
       sum_Speed = sum_Speed + Data_Lane_4(i,3);
       sample_Speed=sample_Speed+1;
   end
   if Data_Lane_5(i,3)~=-1
       sum_Speed = sum_Speed + Data_Lane_5(i,3);
       sample_Speed=sample_Speed+1;
   end
   A Mean(i,3)=sum Speed/(sample Speed++0.00000000001);
%
    A_Mean(i,4)=sample_Speed;
end
   A_Mean(:,2)=butt_flt(A_Mean(:,2),15);
   A Mean(:,3)=butt flt(A Mean(:,3),15);
```

```
fid_mean=0;
    while fid_mean<1</pre>
                                         %% Open file for writing
       [fid_mean, message]=fopen(['st',st_str,'-',st_dir,'-
mean.dat'],'w+');
      if (fopen(fid mean) == -1)
        disp(message);
      end
    end
    for i=1:size(A_Mean)
        fprintf(fid_mean, '%d\t %f\t %f\t %d\t %d\t %d\t %d\t %d\t %d\t
%d\n',A_Mean(i,1),A_Mean(i,2),A_Mean(i,3),A_Mean(i,4),A_Mean(i,5),A_Mea
n(i,6),A_Mean(i,7),A_Mean(i,8),A_Mean(i,9),A_Mean(i,10));
    end
    fclose(fid_mean);
                         %% Close written file
figure(1);
    subplot(3,1,1);
   axis([1,Total_SampleTime,0,1]);
   hold on;
   plot(A_Mean(:,1), A_Mean(:,2));
   set(qca, 'YTick', 0:0.2:1);
    set(gca,'XTick',0:900:Total_SampleTime);
set(gca,'XTickLabel',{'9:00','9:15','9:30','9:45','10:00','10:15','10:3
0'});
   grid;
   hold off;
   ylabel(['Mean-Occupancy @ Station -',st_str]);
    subplot(3,1,2);
    axis([1,Total_SampleTime,0,100]);
   hold on;
   plot(A_Mean(:,1), A_Mean(:,3));
    set(gca,'YTick',0:20:100);
    set(gca,'XTick',0:900:Total_SampleTime);
set(gca,'XTickLabel',{'9:00','9:15','9:30','9:45','10:00','10:15','10:3
0'});
   grid;
   hold off;
   ylabel(['Mean-Speed @ Station-',st_str]);
    subplot(3,1,3);
   axis([1,Total_SampleTime,0,2400]);
   hold on;
   plot(A_Mean(:,1)*DeltaT_SampleTime, A_Mean(:,4)*12);
    set(gca,'YTick',0:300:2400);
    set(gca,'XTick',0:900:Total_SampleTime);
```

```
set(gca,'XTickLabel',{'9:00','9:15','9:30','9:45','10:00','10:15','10:3
0'});
   grid;
   hold off;
   ylabel(['Mean-Flow @ Station-',st str]);
figure(2);
   subplot(3,1,1);
   axis([1,No_Interval,0,100]);
   hold on;
   plot(Density(:,1));
   set(gca,'YTick',0:10:100);
   set(gca,'XTick',0:(No_Interval/6):No_Interval);
set(gca,'XTickLabel',{'9:00','9:15','9:30','9:45','10:00','10:15','10:3
0'});
   grid;
   hold off;
   ylabel(['Density @ Station -',st_str]);
%
   ylabel('Density (veh/mile)');
   xlabel('Density between Station 6 and Station 7');
   subplot(3,1,2);
   axis([1,No_Interval,0,100]);
   hold on;
   plot(Speed(:,1));
   set(gca, 'YTick', 0:20:100);
   set(gca,'XTick',0:(No_Interval/6):No_Interval);
set(gca,'XTickLabel',{'9:00','9:15','9:30','9:45','10:00','10:15','10:3
0'});
   grid;
   hold off;
   ylabel(['Speed (mph)']);
%
   ylabel(['Speed @ Station-',st_str]);
   xlabel('Speed between Station 6 and Station 7');
   subplot(3,1,3);
   axis([1,No_Interval,0,2400]);
   hold on;
   plot(Flow(:,1));
   set(gca,'YTick',0:300:2400);
   set(gca,'XTick',0:(No_Interval/6):No_Interval);
set(gca,'XTickLabel',{'9:00','9:15','9:30','9:45','10:00','10:15','10:3
0'});
   grid;
   hold off;
   ylabel('Flow (veh/h/l)');
   xlabel('Flow at Station 6');
%%%%% Plot the figures for density comparison %%%%%%%%%%%
```

```
fid_2=0;
   while fid_2<1</pre>
                                      %% Open file for reading
     [fid_2, message]=fopen('ComDensity_1.txt','r');
     if (fopen(fid_2) == -1)
        disp(message);
     end
   end
   [Com_Density_1,Count]=fscanf(fid_2,'%d %d',[2,5]);
   Com_Density_1=Com_Density_1';
   fclose(fid_2);
fid_2=0;
   while fid 2<1
                                      %% Open file for reading
     [fid_2, message]=fopen('ComDensity_2.txt','r');
     if (fopen(fid_2) == -1)
        disp(message);
     end
   end
   [Com_Density_2,Count]=fscanf(fid_2,'%d %d',[2,5]);
   Com_Density_2=Com_Density_2';
   fclose(fid_2);
   Probe_Lane_Density = zeros(2,2,5); %% (density,time index, lane
number)
   Probe_Lane_Density(1,1,5)=Com_Density_1(1,1);
   Probe_Lane_Density(1,2,5)=Com_Density_1(1,2);
   Probe_Lane_Density(2,1,5)=Com_Density_2(1,1);
   Probe_Lane_Density(2,2,5)=Com_Density_2(1,2);
   Probe_Lane_Density(1,1,4)=Com_Density_1(2,1);
   Probe_Lane_Density(1,2,4)=Com_Density_1(2,2);
   Probe_Lane_Density(2,1,4)=Com_Density_2(2,1);
   Probe_Lane_Density(2,2,4)=Com_Density_2(2,2);
   Probe_Lane_Density(1,1,3)=Com_Density_1(3,1);
   Probe_Lane_Density(1,2,3)=Com_Density_1(3,2);
   Probe_Lane_Density(2,1,3)=Com_Density_2(3,1);
   Probe_Lane_Density(2,2,3)=Com_Density_2(3,2);
   Probe_Lane_Density(1,1,2)=Com_Density_1(4,1);
   Probe_Lane_Density(1,2,2)=Com_Density_1(4,2);
   Probe_Lane_Density(2,1,2)=Com_Density_2(4,1);
   Probe_Lane_Density(2,2,2)=Com_Density_2(4,2);
   Probe_Lane_Density(1,1,1)=Com_Density_1(5,1);
   Probe_Lane_Density(1,2,1)=Com_Density_1(5,2);
   Probe_Lane_Density(2,1,1)=Com_Density_2(5,1);
   Probe_Lane_Density(2,2,1)=Com_Density_2(5,2);
   figure(3);
   axis([1,5400,0,100]);
   hold on;
```

```
plot(Com_Density_1(:,2),Com_Density_1(:,1),'b*',Time_Index(:,1)*DeltaT_
SampleTime,Density(:,1),'g-');
    set(gca,'YTick',0:10:100);
    set(gca,'XTick',0:900:5400);
set(qca,'XTickLabel',{'9:00','9:15','9:30','9:45','10:00','10:15','10:3
0'});
    legend('Density from Loop and Probe Vehicle', 'Density from Loop
Data');
    grid;
   hold off;
    xlabel('Estimated Density between Station 6 and 7');
    ylabel(['Density (veh/mile)']);
    fid 2=0;
   while fid 2<1
                                      %% Open file for reading
     [fid_2, message]=fopen('Probe_Vehicle.txt','r');
     if (fopen(fid_2) == -1)
        disp(message);
     end
   end
  N_Trips=10;
   Length = 334;
  Lane_Number = 5;
   Probe_Station_Density=zeros(1, N_Trips);
   [T_Probe,Count]=fscanf(fid_2,'%d %d',[2,N_Trips]);
   T_Probe=T_Probe'; % T_Probe(:,1) arrival time for upstream station;
T_Probe(:,2) arrival time for downstream station
   fclose(fid_2);
    for m=1:N Trips
        % accumulative vehicle count change
        Probe_Station_Density(m)=round((A_Mean(T_Probe(m,2),5)-
A Mean(T Probe(m,1),5))*1600/(334*Lane Number));
    end
    for j=1:No_Interval
        Probe_Density(j)=-1;
        Smpling_Probe_No = 0;
        Sum_Probe_Density = 0;
        for m=1:N_Trips
            if (T_Probe(m,2)>(j-
1) *DeltaT_SampleTime) & (T_Probe(m,2) <= j*DeltaT_SampleTime)
                Smpling_Probe_No=Smpling_Probe_No+1;
Sum Probe Density=Sum Probe Density+Probe Station Density(m);
            end
        end
        if Smpling_Probe_No>0
           Probe_Density(j)=Sum_Probe_Density/Smpling_Probe_No;
        \operatorname{end}
    end
    figure(4);
    axis([1,5400,0,100]);
    hold on;
```

```
plot(Time_Index(:,1)*DeltaT_SampleTime,Probe_Density(:,1),'b*',Time_Ind
ex(:,1)*DeltaT_SampleTime,Density(:,1),'g-');
    set(gca,'YTick',0:10:100);
    set(gca,'XTick',0:900:5400);
set(gca,'XTickLabel',{'9:00','9:15','9:30','9:45','10:00','10:15','10:3
0'});
    legend('Density from Loop and Probe Vehicle', 'Density from Loop
Data');
    grid;
    hold off;
    xlabel('Estimated Density between Station 6 and 7');
    ylabel(['Density (veh/mile)']);
    figure(5);
    for m=1:5
    %Density_Lane(:,m)=butt_flt(Density_Lane(:,m),800);
    subplot(5,1,m);
    %figure(3+m);
    axis([1,5400,0,100]);
    hold on;
plot(Probe_Lane_Density(:,2,m),Probe_Lane_Density(:,1,m),'r*',Time_Inde
x(:,1)*DeltaT_SampleTime,Density_Lane(:,m),'b-');
    set(gca,'YTick',0:20:100);
    set(gca,'XTick',0:900:5400);
set(gca,'XTickLabel',{'9:00','9:15','9:30','9:45','10:00','10:15','10:3
0'});
%
    legend('Density from Loop and Probe Vehicle', 'Density from Loop
Data');
    grid;
    hold off;
    xlabel(['Estimated Density for Lane ',int2str(m), ' between Station
6 and 7']);
    ylabel(['Density (veh/mile)']);
    end
```