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July 11, 1977

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KINETIC ENERGY CONSIDERATIONS AND THE VACUUM

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ABSTRACT

The suppression of infinite kinetic-energy trajectories is related to the topological classification in non abelian gauge theories.

The quantum-mechanical states of an $SU(2)$ gauge theory in the $A_0 = 0$ gauge have been recently considered.^{1,2,3} The potential energy of a configuration $A_1(\vec{x})$ (at a given time) is given by $V[A] = \int d^3x \text{Tr} F_{ij} F^{ij}$. The configurations that correspond to the classical vacuum satisfy $V[A] = 0$, and are given by $A_1(\vec{x}) = g^{-1}(\vec{x}) \partial_1 g(\vec{x})$, where $g(\vec{x})$ is any continuous mapping from 3-space to the group. For simplicity we consider only such mappings for which the following limit exists: $\lim_{r \rightarrow \infty} g(r, \Omega) = g(\Omega)$. The manifold of these vacuum configurations is simply connected. This means that $V[A]$ has one continuous minima valley (unlike the double-well potential in quantum mechanics with one degree of freedom, in which there are two discrete minima which communicate only through the tunneling effect.)

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Jackiw and Rebbi^{1,4} postulated the condition $g(\Omega) = I$. As a result, the vacuum manifold is decomposed into disconnected sections, which they called the $|\eta\rangle$ vacua. The purpose of this comment is to show that this condition is a consequence of kinetic energy considerations.

We first recall that among all the functions $q(t)$ which satisfy $q(t_0) = q_0$ and $q(t_1) = q_1$, the minimal value of the "kinetic energy" $T = \int_{t_0}^{t_1} dt \left(\frac{dq}{dt} \right)^2$, is achieved by the linear interpolating function (This is equivalent to minimizing the action of a free particle). The minimal value of T is $T_{\min} = (q_1 - q_0)^2 / (t_1 - t_0)$. In the $A_0 = 0$ gauge, $T = \int_{t_0}^{t_1} dt \int d^3x \sum_{i,a} \left[\frac{\partial}{\partial t} A_i^a(\vec{x}, t) \right]^2$.

Since T does not couple different points in ordinary ($\vec{x}$) space, its minimal value under the constraints $A_i^a(\vec{x}, t_0) = A_{i,0}^a(\vec{x})$ and $A_i^a(\vec{x}, t_1) = A_{i,1}^a(\vec{x})$ is

$$T_{\min} = 1/(t_1 - t_0) \int d^3x \sum_{i,a} \left[A_{i,1}^a(\vec{x}) - A_{i,0}^a(\vec{x}) \right]^2 \quad (1)$$

This expression is finite if $A_{i,1}^a(\vec{x}) - A_{i,0}^a(\vec{x}) \sim 1/r^{3/2 + \epsilon}$. Suppose $A_{i,0}^a$ and $A_{i,1}^a$ are vacuum configurations, which are derived from the mappings $g(\vec{x})$ and $g'(\vec{x})$ respectively (with the corresponding limits $g(\Omega)$ and $g'(\Omega)$). Unless $g'(\Omega) = \bar{g}g(\Omega)$ where $\bar{g}$ is a constant element of the group, the two configurations are different, and their difference goes like $1/r$. Therefore, $T_{\min} = \infty$, and there will be no communication between these two vacuum configurations in the functional integral formalism.^{4,5} Consequently, if we start at $t = t_0$ with some vacuum configuration with a given $g(\Omega)$, we can ignore at any future time all vacuum configurations which correspond to different $g(\Omega)$. In particular, if we start with the Jackiw-Rebbi

condition, $g(\Omega) = \text{const.}$ we can ignore all other vacuum configurations.

At that point one can use the result that the mappings $g(\vec{x})$ with $g(\Omega) = \text{const.}$ fall into homotopy classes, labelled by the integers.

We thus see that the classification of vacuum configurations into the n -classes, advocated by Jackiw and Rebbi, rises in a natural way.

Let us now consider the mappings $g(\vec{x}) = \exp[-i\alpha(r) \hat{r} \cdot \vec{\tau}/2]$ where $\hat{r} = \frac{\vec{x}}{r}$ and $\tau^i/2$ are the (2×2) generators of the $SU(2)$ group. To get a continuous mapping at $x = 0$ we set $\alpha(0) = 0$. If $\alpha(\infty) = 2\pi n$, the corresponding mapping, $g_n(\vec{x})$, is in the n class (with $g_n(\Omega) = (-1)^n I$). As an illustration to our arguments we construct a trajectory $A_1^a(\vec{x}, t)$, which interpolates the $n = 0$ configuration (namely, $g_0^{-1} \partial_i g_0$) at $t = t_0$ with the $n = 1$ configuration at $t = t_1$ through the valley (namely, $V[A_1^a(\vec{x}, t)] = 0$ at each t . Note that any such trajectory cannot satisfy the classical equations of motion). For example, at each t we take $\alpha(r; t) = y(t)\alpha(r)$ with $y(t_0) = 0$, $y(t_1) = 1$ and $\alpha(0) = 0$, $\alpha(\infty) = 2\pi$, and then set $A_1(\vec{x}; t) = g^{-1}(\vec{x}; t) \partial_i g(\vec{x}; t)$. However, since we change $g(\Omega)$ with time, the kinetic energy T must be infinite, and a direct calculation gives

$$T = 2\pi \int_{t_0}^{t_1} dt \left(\frac{dy}{dt} \right)^2 \int_0^\infty dr \left[\left(\frac{d\alpha}{dr} \right)^2 r^2 + 2\alpha^2 \right] = \infty$$

We see that any trajectory which interpolates the $n = 0$ and the $n = 1$ configurations with a finite kinetic energy must cross the potential barrier. The instanton⁶ is an example of such a trajectory, which has a finite action.

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