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# Nowhere to Hide:

# Time-Varying Inflation Risk and Bond-Stock Correlation

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#### Abstract

Long-term nominal Treasury risks show substantial time variation over the last several decades. In this paper, I explore a New Keynesian asset pricing model to study the interaction of monetary policy and nominal bond risk when a central bank's inflation target is time-varying. I first document that there was a structural break in bond-stock relationship around the time when the chairman Greenspan took the office. The identified break date indicates a sign change in bond-stock beta. I propose the inflation risk channel, through which persistent increase in inflation has contractionary effects and a positive bond-stock correlation. The channel predicts nominal bond risks heavily depend on inflation risks driven by time-varying inflation target. The estimated New Keynesian model confirms the economic significance of the channel. Estimated long-run inflation target decreases significantly over time, which accounts for the time-varying bond-stock correlation. Counterfactual analysis shows that long-run inflation target and the inflation coefficient in Taylor rule play important roles to explain the sign-switching pattern in bond-stock correlation. I also estimate a SVAR model with sign restriction and confirm that persistent inflationary shocks have state-dependent effect on macro aggregates and asset prices, consistent with the theoretical finding.

JEL classification: E31, E43, E44, E52, G12

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When people begin anticipating inflation, it doesn't do you any good anymore, because any benefit of inflation comes from the fact that you do better than you thought you were going to do.

- Paul Volcker, *The First Measured Century* (2000)

### 1 Introduction

In financial markets, US Treasuries are perceived as one of hedge assets, which refer to a class of assets that can be used to offset the risks in other risky assets, such as stocks. Because hedges are expected to perform relatively well when other risky assets lose its value, the hedging property of Treasuries implies that the correlation between return on nominal Treasuries and risky assets tends to be negative, which has been observed in US financial markets for the last two decades. For instance, during the last three recessions in 2001, 2008, and 2020, yields on long-term nominal Treasuries experienced significant drop due to the demand shift from risky assets to safe and liquid assets.<sup>1</sup> But the perception of the hedging property of the Treasuries has been established only since the mid 1990s, or later (Campbell et al., 2009). Before that, long-term nominal Treasuries are considered as assets that tend to move together with other risky, long-term assets. According to Campbell and Ammer (1993), it was general perception in financial markets that "conventional wisdom is that long-term asset prices move (or should move) together, and indeed it is true that stock and bond returns are always positively correlated. (Campbell and Ammer 1993, p.17)". In fact, not only the perception about the cyclical property of Treasuries was different, but the observed correlation between the two assets was positive and well above zero. This shows there has been a shift in the dynamics governing the prices of long-term Treasuries and risky assets.

In this paper, I explore a New Keynesian asset pricing model with time-varying inflation target, and propose *the inflation risk channel* through which a monetary policy affects macroeconomic outcome and asset prices. A novel prediction of the model is that when inflation target set by a central bank is higher than a certain threshold and firms subject to Calvo pricing friction, a persistent increase in inflation can be contractionary, raising inflation but causing output to fall. This, in turn, affects asset pricing dynamics and predicts a positive bond-stock correlation. This mechanism, *the inflation risk channel*, reflects the cost of inflation, or the inefficiency driven by nominal rigidity. When inflation target is already high, additional increase in inflation to boost an economy is very costly because of its distortionary effects on price. When firms expect persistently high inflation in the future, their pricing decision becomes more responsive to inflation, which

<sup>&</sup>lt;sup>1</sup>One of few exceptional periods is studied in Vissing-Jorgensen (2021). From March 9th to 18th in 2020, when the news about the COVID-driven pandemic materialized. Before the Federal Reserve decided to intervene, the yield on 10-year Treasuries spiked sharply by 64 basis points, in the mean time the stock market crashed. Vissing-Jorgensen (2021) concludes that the yield spike was largely caused by the liquidity needs from financial intermediaries prone to run, such as mutual funds and hedge funds.

brings about inefficient demand allocation and output loss. According to the Bayesian estimation of the model, the US economy suffered from substantial inflation risks, which can explain countercyclical inflation and a positive Treasury-stock correlation that the US economy has been through between 1960s to 1980s.

The main contribution of this paper is to shed light on the role of monetary policy on the cyclicality of nominal bond risks, with emphasis on central bank's inflation target. There has been an increasing body of literature that deals with the sign-changing bond-stock return correlation, but the role of monetary policy, in particular inflation target, has not attracted much of the attention.<sup>2</sup> In macro literature, how policy makers set the target rate of inflation and how it interacts with macro and financial outcomes have gained some attention,<sup>3</sup> but my finding indicates that it can play a critical role in shaping macroeconomic and financial dynamics.<sup>4</sup> According to my estimates, the cost of inflation is not negligible and the benefit of anchoring inflation target is substantial. This is because with well-anchored inflation target, monetary authority is able to not only stabilize macroeconomy but also financial markets by providing hedge assets.

I begin by testing a structural break in nominal bond-stock relationship. To identify a break, I conduct a Quandt likelihood ratio (QLR) test following Andrews (1993, 2003) for an unknown break date, using weekly data on return on 5-year nominal Treasuries and S&P500 excess of 3-month T-bills. The test identifies a structural break at the first week of October, 1987, two months after the chairman Greenspan started his tenure. Before and after the identified break date, the bond-stock beta turns from positive to negative, from 0.1 to -0.04. The timing of the structural change is informative. The Volcker disinflation that started in the late 1970s, seemed to successfully contain the volatile inflation, and finally around 1987, various inflation measures marked two decades low. In the meantime, inflation expectation and the forecasting uncertainty had also significantly decreased during the first half of 1980s. The conditional response of stock returns to a positive inflation shock is negative before the structural break, while the response of stock returns to a positive after the break. This set of evidence speaks to hypothesis that the regime change in monetary policy that alters inflation dynamics could be a cause behind the shift in the bond-stock beta.

<sup>&</sup>lt;sup>2</sup>In asset pricing literature, the sign-change in bond-stock correlation is often related to the change in inflation cyclicality, measured by output (or consumption) growth-inflation correlation. For example, Campbell et al. (2020) estimate that the output gap-inflation correlation switched from negative to positive, and it explains why the bond-stock return correlation changed its sign. Similarly, Song (2017) estimates a consumption-based asset pricing model and finds that bond-stock return correlation is positive when inflation is counter-cyclical.

<sup>&</sup>lt;sup>3</sup>Some examples include Cogley et al. (2010), where the authors estimate time-varying inflation target process to explain inflation persistence but in the context of full indexation. When firms who do not optimally choose prices are allowed to index their prices following the previous (or steady state) inflation, the cost of higher long-run inflation is negligible. Another example is Coibion et al. (2012) where the authors explore welfare implications of inflation target.

<sup>&</sup>lt;sup>4</sup>In existing New Keynesian models, long-run inflation target or steady state inflation has played very limited (or even no) role for the model dynamics. This is because either those models assume zero steady state inflation for simplicity, or assume full indexation of price (Christiano et al., 2005; Smets and Wouters, 2007)

Next, I explore a New Keynesian (NK) asset pricing model with Calvo nominal frictions to investigate the implication of monetary policy and inflation dynamics on the nominal bond risks. The model has two main ingredients, which are also crucial for the model prediction. First, a central bank sets not only short-term interest rate as in a standard NK model, but also determines short-run inflation target, following Cogley et al. (2010), given their choice of long-run inflation target as in a generalized New Keynesian model (Ascari and Sbordone, 2014). Second, the pricing behavior of firms that do not optimally choose their prices does not feature the full indexation to previous or steady state inflation<sup>5</sup>. In contrast to a standard NK model in the literature (Smets and Wouters, 2007) in which either steady state inflation, which I refer to as long-run inflation target. In this simple modification of a NK model, *the inflation risk channel* arises and makes inflation more costly because of inefficient distribution of demand caused by price distortion.<sup>6</sup>

The model provides novel predictions about the effects of a persistent change in inflation target. An increase in short-run inflation target has an expansionary effect and the conditional bond-stock correlation is negative when long-run inflation target is sufficiently low. But once the long-run target passes a certain threshold, the effect of a persistent increase in short-run inflation target becomes contractionary and drives a positive conditional bond-stock correlation. This state-dependent effect of inflation target shocks is determined by the two effects that turn up due to firms' pricing behavior. In Calvo model, only a fraction of firms are allowed to change their prices optimally in each period. On the one hand, firms that adjust prices take into account high inflation in the future as well as the possibility of future non-adjusting, being more responsive to inflation change and charging prices inefficiently high. On the other hand, remaining firms that do not adjust are stuck with the old prices. Prices of those firms are inefficiently low in high inflation environment, making cross-sectional price *effect*, create output loss due to inefficient price dispersion.<sup>7</sup> When the inefficiency generated by inflation dominates the expansionary effect, output response ends

<sup>&</sup>lt;sup>5</sup>This setting is often called the Generalized New Keynesian model(or the GNK model). Examples of GNK model include Ascari and Ropele (2007), Amano et al. (2007), and Ascari et al. (2018). Most of previous study using NK model is based on either zero steady state inflation (canonical three equation NK model in Galí 2015 or Woodford 2003) or full indexation (firms automatically adjust their price according to the previous inflation or steady state inflation as in Christiano et al. 2005 or Smets and Wouters 2007). In either case, inflation target determines only the average level of inflation, and does not affect or is affected by any other endogenous part of the model.

<sup>&</sup>lt;sup>6</sup>Recently, an increasing number of papers have focused on micro-level evidence on firms' pricing behavior. These papers include Klenow and Kryvtsov (2008), Nakamura and Steinsson (2008) and Nakamura et al. (2018). Those papers argue that micro evidence is more favorable to state-dependent model such as the ones developed in Golosov and Lucas (2007), Gertler and Leahy (2008) and Nakamura and Steinsson (2010), rather than time-dependent model as Calvo (1983). But recent paper by Auclert et al. (2022) shows that there exists an equivalence between time- and state-dependent pricing models when parameters are appropriately chosen.

<sup>&</sup>lt;sup>7</sup>Note that this channel does not exist in New Keynesian models with Rotemberg pricing friction. The main difference is that under Calvo assumption, firms face possibility of sticking at past prices, so they make more forward looking pricing decision, making cost-push shock-like effects endogenously. But under Rotemberg setting, firms always adjust price and as a result, there is no practical difference between zero and non-zero long-run inflation.

up falling. Then, the linkage between output-stock prices and inflation-nominal bond prices<sup>8</sup> implies that countercyclical inflation translates into a positive bond-stock correlation.

In the next step, I estimte the structural parameters using the Bayesian technique (Smets and Wouters, 2007) with US time series data. The estimated model confirms the economic significance of the inflation risk channel. First, the estimated long-run inflation dramatically changes across periods. The long-run target is 4.1% in the earlier sample but it drops to 1.75% in the recent period. Because the magnitude of the inflation risk channel increases in the long-run target, the estimates imply much larger effects from the inflation risks in the earlier period. Second, the model-implied bond-stock correlation captures the sign-switching pattern, even without feeding financial data to the model. Recall the linkage between output-stock return, and inflation-nominal bond return, where the former is positive and the latter is negative. Sizable inflation risks in the earlier period imply countercyclical inflation, or lower output with higher inflation, which predicts a positive bond-stock beta. In contrast, with low long-run target, in the latter period, the size of inflation risks is limited, implying procyclical inflation and a negative bond-stock beta.

My finding sheds light on an important asset pricing implication of monetary policy. When policy makers allow for higher inflation, the cost of inflation is charged not only in real side, but also financial side due to its threat to the hedging property of nominal long-term bonds. Note that government-issued nominal long-term bonds play a key role in financial markets by providing hedging opportunity to investors. When nominal bonds move similar to risky assets, the volatility in financial markets would worsen, possibly triggering a market crash. In particular, this aspect of monetary policy is important for financial stability in small open economy, where financial markets are exposed to foreign markets and financial stability is one of the major concern of policy makers. My counterfactual analysis shows that when long-run target is high, the role of inflation coefficient in Taylor rule is even more important because by being responsive to inflation, monetary authority can attenuate the inflation risks.

**Related literature.** This paper is related to several branches of literature. First, a growing body of literature studies determinants of the bond-stock return correlation, including Campbell et al. (2020), Song (2017), Chernov et al. (2021b), Li et al. (2022), David and Veronesi (2013), Pflueger (2022), and Campbell and Ammer (1993). Among related papers, Campbell et al. (2020) construct and estimate a consumption-based asset pricing model. Using their model, they find that the sign change in the bond-stock return correlation can be attributed to the sign change in the output gap-inflation relationship. Song (2017) and David and Veronesi (2013) find similar prediction by estimating a regime switching model. All of three papers find that countercyclical inflation is a key to predict positive bond-stock return correlation, while depending on dif-

<sup>&</sup>lt;sup>8</sup>Stock prices are positively associated with output while nominal bond prices are negatively related to inflation.

ferent mechanisms to explain inflation cyclicality. These papers focus on regime change in macroeconomic shocks, which is purely exogenous. My analysis takes a step forward from their findings and provide a theoretical foundation for the shift in the cyclical property of macroeconomic shocks, which can be accounted for by the change in monetary policy regime from high inflation target with less inflation-responsive rule to low inflation target with more responsive rule. The long-run inflation risk in the former regime makes nominal bonds non hedge assets, while making inflation countercyclical.<sup>9</sup>. My results give an interpretation that regime shifts in macroeconomic shocks and resulting change in bond-stock return correlation can be the consequence of the shift in monetary policy framework and how a central sets long-run inflation target.

Second, my paper fits into the large literature on monetary policy and asset prices. Hanson and Stein (2015) and Gertler and Karadi (2015) use high frequency identification for monetary policy shocks and show that monetary policy surprise affects long-term asset prices, such as long-term Treasuries. Bianchi et al. (2016) build and estimate a regime switching model and show that regime change in monetary policy has large and long-lasting effects on asset prices. Cieslak and Povala (2015), Davis et al. (2019), and Bauer and Rudebusch (2020) study the implications of persistent components of inflation (or trend inflation) on asset prices. These papers find that incorporating slow moving parts of inflation significantly improves the explanatory power of asset pricing models for explaining long-term bonds. My analysis extends this finding and shows that how the choice of target inflation drives low frequency movements in inflation, and how shocks to inflation targets can propagate into macro and financial variables. By doing so, I reveal a novel channel through which monetary policy affects return on long-term bonds and its cyclical property.

Third, this paper is also related to the work of Clarida et al. (2000), Smets and Wouters (2007), Coibion and Gorodnichenko (2011), Cogley and Sbordone (2008), Cogley et al. (2010), and Ascari and Ropele (2009) that study the interaction of monetary policy, trend inflation, and macroeconomic dynamics. Clarida et al. (2000) and Coibion and Gorodnichenko (2011) estimate monetary policy rule before and after the appointment of Paul Volcker and find that there has been a structural break, and monetary policy became more aggressive to inflation. Cogley and Sbordone (2008) and Cogley et al. (2010) estimate a version of New Keynesian model with trend inflation. They find that trend inflation was higher and more volatile during the 1970s, and this contributed to the decline in the persistence of inflation. A common finding in this literature is that policy stance of the Federal Reserve during the Great Inflation was much accommodative in the sense that it allowed for higher and volatile inflation, which sharply contrasts with the policy stance in more

<sup>&</sup>lt;sup>9</sup>Relatedly, Campbell et al. (2009) document nominal and real bond returns and their correlation with stock returns. They argue that the risk premia of nominal bonds should have been time-varying because the cyclicality of inflation has changed. Baele et al. (2010)) use a dynamic factor model to study the sources of bond-stock return correlations and find that bond and stock market liquidity factors play an important role. Chernov et al. (2021b) argue that regime change, in which either permanent or transitory consumption shocks are dominant, makes the bond-stock return correlation switch the sign.

recent decades.<sup>10</sup> My results contribute to this strand of literature by showing that the structural breaks in monetary policy and long-run inflation have important implications for macroeconomic dynamics. Higher target inflation with less aggressive interest rate rule generates negative comovement between output gap and inflation, while generating large volatility in output. Therefore, through the lens of my estimated model, there is a possibility that unanchored inflation target contributed to the macroeconomic instability the US economy experienced in 1970s.

Lastly, this paper has a contribution to New Keynesian asset pricing literature. Rudebusch and Swanson (2012) construct an asset pricing model based on a standard NK framework with recursive preferences following Epstein and Zin (1989). Caramp and Silva (2021) investigates the role of wealth effects in the monetary transmission mechanism. Kung (2015) also develops an asset pricing model with endogenous growth in a NK framework to explain the term structure of interest rates. Diercks (2015) constructs a New Keynesian model to study long-run risks, equity premia, and optimal monetary policy. Gourio and Ngo (2020) study the correlation between inflation and stock returns and show that when the Zero Lower Bound (ZLB) constraint on nominal interest rates is binding, inflation and stock returns become positively related. I contribute to this literature by analyzing the implications of positive and volatile trend inflation for asset prices. I show that when a model is approximated around positive steady state inflation, nominal rigidity has a new channel to affect asset pricing dynamics, which has not been revealed in previous models.

### 2 Empirical evidence

### 2.1 Bond-stock correlation and the cyclicality of inflation

In this subsection, I document historical bond-stock correlation and the cyclicality of inflation. I conduct a formal structural break test with an unknown break date according to Andrews (1993, 2003), which identifies a structural break in bond-stock beta in 1987.

In the last couple of decades, US Treasuries have been perceived as hedge assets<sup>11</sup> in financial markets. For instance, during the recessions in 2001 and 2008-2010, the two most recent recessions before the Covid crisis, excess return on 5-year nominal Treasuries were 3.9 and 5.7 percentage points, whereas return on US stocks were -14.0 and -30.8 percentage point, far beneath from the Treasury returns. However, this

<sup>&</sup>lt;sup>10</sup>There exists another strand of literature that studies New Keynesian model under positive steady state inflation. Ascari and Ropele (2009) investigate determinacy of the rational expectations equilibrium of New Keynesian model when steady state inflation is positive. Ascari and Ropele (2007) study optimal monetary policy under positive but low trend inflation. Coibion et al. (2012) focus on the optimal inflation rate when there exists a possibility of nominal interest rate hitting the Zero lower bound.

<sup>&</sup>lt;sup>11</sup>Hedge assets can be defined as assets that perform relatively better when other assets are losing its value or an economy is doing bad.



Figure 1: Bond-stock correlation and bond-stock beta

perception of Treasuries that considers the bonds as hedges was not prevalent before 1990s. In fact, up to 1990s, US Treasuries had performed very differently, and it was even a 'conventional wisdom' to expect long-term Treasuries and stock prices move together.

Figure 1 shows the historical bond-stock correlation and bond-stock beta, measured by using 5-year nominal Treasuries and US stock market returns excess of 3 month Treasury yields.<sup>12</sup> The figure exhibits there has been a substantial time variation in bond-stock relationship, and even the relation even changed its sign. From 1960s to 1990s, bond-stock correlations and betas were mostly positive, as high as 0.55 and 0.3, respectively. But there was a sharp decline in both correlation and beta in 1987, which is followed by a sharp increase in both measures in 1989. Two measures reversed its course again in the late 1990s, changed its sign again around 2000, then have remained negative. The observed sign change implies there could have been low-frequency change in the relationship between bond and stock returns.

To formally test whether there has been a structural break and if so, when it was, I conduct a test for a structural break with an unknown break date. Following Andrews (1993, 2003), conduct a Quandt Likelihood Ratio (QLR) test for an unknown break date of bond-stock beta. For each time period  $\tau$ , estimate the following dummy regression using full sample.

$$xr_t^b = \alpha_\tau + \beta_\tau^1 I_{t \ge \tau} + \beta_\tau^2 xr_t^s + \beta_\tau^3 xr_t^s I_{t \ge \tau} + \varepsilon_t,$$
(1)

Notes: 3-year rolling correlations and betas of bond and stock excess returns. Bond return  $(xr^b)$  is measured by onequarter holding return of 5-year Treasury bonds excess of 3-month T-bills. Stock return  $(xr^s)$  is measured by one-quarter return from the value-weighted S&P 500 including dividends excess of 3-month T-bills. The shaded area indicates the recession date identified by NBER.

<sup>&</sup>lt;sup>12</sup>Correlations for other maturities are shown in Appendix B.



Figure 2: Structural break test for bond-stock beta and inflation expectation

Notes: Structural break test is based on Andrews (1993, 2003). Bond return  $(xr^b)$  is measured by one-quarter holding return of 5-year Treasury bonds excess of 3-month T-bills. Stock return  $(xr^s)$  is measured by one-quarter return from the value-weighted S&P 500 including dividends excess of 3-month T-bills. Inflation is measured by the annualized log difference in the GDP deflator. Inflation expectation is measured by one-year ahead forecast on GDP deflator from the Survey of the Professional Forecasters (SPF). Dispersion is the inter-quantile range in inflation expectation from the SPF. The shaded area indicates the recession date identified by NBER.

where  $I_{t \ge \tau}$  is a dummy variable that takes the value of one when  $t \ge \tau$  and zero otherwise. For each candidate break date  $\tau$ , compute the F-statistic corresponding to  $\beta_{\tau}^3$ . The QLR test statistic is the maximum F-statistic, and the estimated break date is the period  $\tau$  with the highest F-statistic.<sup>13</sup> I conduct the QLR test using

<sup>&</sup>lt;sup>13</sup>The critical values for the statistical test can be found in Andrews (2003).

weekly excess bond return and stock return, trimming 15% of sample from the beginning and the end date.

Panel (a) of figure 2 depicts the estimated F-statistic over the sample periods. F-statistic gradually increases from the mid 1970s, and start to increase rapidly from 1980. It peaks in the week of October 5th 1987, which is identified as the structural break date. After the break, the statistic drops sharply and gradually increases again until 2000, although the maximum occurs in the week of October 5th, 1987. Before and after the structural break date, the bond-stock beta changes its sign, from 0.10 to -0.04, respectively. The timing of the identified structural break is noteworthy because October 1987 is two months after the chairman Greenspan took the office and the US inflation had been stabilized following the Volcker disinflation.

In panel (b) of the figure, I plot historical paths of inflation (annualized growth rate of the GDP deflator), one-year inflation expectation and forecasting dispersion in one-year inflation measured by inter-quantile range. During 1970s, inflation is mostly higher than 4 percent, and even reached nearly 12 percent. From the mid 1960s to the early 1980s, the so-called "Great Inflation" (Clarida et al., 2000), the macroeconomic dynamics in the US can be characterized by high and volatile inflation, and the acceleration of the deanchoring of long-run inflation expectations (Levin and Taylor, 2013) as well as the negative comovement between inflation and output (see table 1).

Since the chairman Volcker took over the office, his famous disinflationary policy began and inflation started to subside during the 1980s. During the same period, inflation expectation had also rapidly increased, from 3 percent in 1970 to 9 percent in 1980. High expected inflation came along with higher inflation uncertainty. Inflation uncertainty, measured by dispersion in inflation forecast, also peaked in 1980, and slowly decreased through the 1980s. The estimated structural break of the third quarter of 1987 is when inflation hit two decades low, and both expectation of and uncertainty around inflation came back to its pre-1970 level. From figure 1, the observed bond-stock correlation and beta changed abruptly towards negative in 1987, when actual and expected inflation start to settle down.

However, with the stock market crash and recession, the Federal Reserve stopped fighting against inflation and conducted accommodative policy to support US economy (Bordo and Schwartz, 1999; Benati and Goodhart, 2010). Because in 1980s, a period of low and stable inflation was relatively shorter than a period of wild inflation, this accommodative policy induced inflation fear to work again. This can be seen in figure 1 that bond-stock correlation popped up again in 1989 when inflation bounced back. Bond-stock correlation turned negative once inflation was stabilized again since the mid-1990s and the inflation expectation started to anchor around 2%. The timing of the structural break, therefore, points to the time when inflation started to be contained, and expectation about future inflation became stabilized, although it didn't last long.

Table 1 summarizes bond-stock correlation, output gap-inflation correlation, and the average and volatility of inflation for the two periods. In the earlier period, inflation is more than two times more volatile and

	$\sigma(\pi)$	$E(\pi)$	$Corr(xr^b, xr^s)$	$Corr(\tilde{y},\pi)$
1965-1987	2.47%	5.21%	0.40	-0.21
1987-2019	0.98%	2.12%	-0.37	0.54

Table 1: Inflation dynamics and bond-stock correlation in the US

Notes: Inflation data is annualized CPI inflation. Short-term interest rate is the federal funds rate. Bond-stock correlation is a rolling 3-year window of quarterly excess return on 5-year zero coupon Treasuries bonds and S&P 500 excess returns. Bond return  $(xr^b)$  is measured by one-quarter holding return of 5-year Treasury bonds excess of 3-month Tbills. Stock return  $(xr^s)$  is measured by one-quarter return from the value-weighted S&P 500 including dividends excess of 3-month T-bills. Output gap (x) data is collected from the Congressional Budget Office.

higher than the recent period, which constitute decisive features of the Great Inflation. As can be seen in panel (b) figure 2, inflation expectation was deanchoring and raising rapidly. During the same period, Treasuries were not hedges in the sense that its return is moving together with stock returns. This coincides with countercyclical inflation, i.e. negative correlation between output gap and inflation.

After the structural break, the empirical pattern of inflation and asset prices changed remarkably. The average level and volatility of inflation decreases more than half, which characterizes the so-called Great Moderation. When it comes to the 'conquest' of the US inflation (Cogley and Sargent, 2005), it is widely believed that strict policy framework of Volcker was crucial to stabilize price and long-term inflation expectation.<sup>14</sup> After the former chairman Paul Volcker took over the office, he put strong emphasis on price stabilization, which is well-known as the Volcker disinflation. Households and firms adjust their long-run inflation expectation as the Federal Reserve aggressively reacts to inflation by raising policy rates, which gives rise to the stabilized inflation. This, the Great Moderation, period features (re)anchoring of long-term inflation expectation and broad decline in volatility of the overall economy. Bond-stock correlation becomes negative, which means long-term nominal Treasuries counterbalance stock market movements. As for the cyclical property of inflation, it moves together with output and shows a positive correlation with output gap, hence procyclical inflation.

Note that the cyclical property of inflation is closely related to bond-stock correlation and inflation risks. On the one hand, there is a negative association between inflation and nominal bond prices because inflation erodes real vale of nominal long-term bonds. When an unexpected inflation arises, bond holders are exposed to fall in asset value in real term. On the other hand, when an economy is in expansion and output grows, stocks also perform well, and exhibits positive return. Therefore, the change in bond and stock returns reflect the fluctuations in macroeconomic fundamental, summarized by the cyclicality of inflation.

<sup>&</sup>lt;sup>14</sup>Relatedly, recent papers have found the importance of fiscal backing for the success of active monetary policy. According to this line of research, passive fiscal policy that accommodates active monetary policy is critical to obtain stable equilibrium in monetary models. For more discussion, see Leeper (1991), Bianchi and Melosi (2019) and Caramp (2021).

Let's consider a case where inflation is procyclical. Inflation is high when output and consumption are also high, hence low marginal utility. Then, the periods of high marginal utility coincide with high values of nominal bonds and low values of stocks. When inflation is countercyclical, however, the opposite mechanism holds and nominal bond prices are low when marginal utility is high. Because long maturity bonds are even more exposed to inflation, nominal bond risks are high and bond returns are low. In this regards, nominal bonds are exposed to inflation risks even more when inflation is countercyclical. Therefore, households are more willing to save via nominal bonds when inflation is procyclical but the opposite motivation would arise when inflation is countercyclical.

#### 2.2 SVAR evidence with identification by sign restriction

In this section, I estimate a structural vector autoregression (SVAR) model with sign restriction, and show that persistent monetary shock has state-dependent effects. Identification is achieved by the sign restriction on the impact effect of inflation target shock, which has the same sign for both inflation and nominal interest rate. A positive inflation target shock generates expansionary effects after the structural break, but causes contractionary effects before the structural break. This is an evidence that implies the effects of inflation target shock is state-dependent, and it explains the countercyclical inflation and positive bond-stock correlation before the structural break.

The roots of the Great Inflation have been studied broadly in the literature<sup>15</sup>. Among many views, one of the convincing argument is that monetary policy played an important role, either directly or indirectly. Before 1980s, monetary policy may have failed to satisfy the Taylor principle, hence any sunspot hitting the US economy could have brought about the Great Inflation (Clarida et al., 2000). When the Taylor principle is not satisfied, a sunspot shock can cause dire spiral in inflation, which leads to a burst of inflationary periods. On the other side, more direct role of monetary policy is supported by the persistence of inflation during this period. US trend inflation is estimated to be well above 2 percent, and even hits 8 percent, which implies that there was a low frequency movement in inflation that brought up inflation. That is, monetary policy 'allowed' higher inflation.<sup>16</sup>

Taking the side of monetary policy explanation for the Great Inflation, this paper tries to explain the change in bond-stock relationship based on persistent movement in inflation, initiated by a central bank.

<sup>&</sup>lt;sup>15</sup>For instance, Clarida et al. (2000) estimates different Taylor rules before and after the Great Inflation, Cogley and Sargent (2005), and Primiceri (2006) argue that there was a policy mistake by the Federal Reserve, Sims and Zha (2006) estimate volatilities of exogenous shocks have changed over time, and Bianchi and Ilut (2017) emphasizes fiscal dominant regimes to explain high and persistent inflation.

<sup>&</sup>lt;sup>16</sup>Cogley and Sargent (2005) argue that the Federal Reserve believed that they can shift Phillips curve so that they lower unemployment rate at the cost of 'slightly more' inflation. Bianchi and Ilut (2017) argue that the Great Inflation period is when fiscal policy dominates monetary policy. Therefore, large unfunded fiscal expansion in 1960s causes higher inflation in the subsequent period.

If inflation diverges from its historical average for an extended period of time, one can interpret this is the result of the decision that monetary authority made, either directly increasing the target inflation level or indirectly response to inflation less than one-for-one. To focus on monetary policy and abstract from fiscal decision, I assume that persistent movement in inflation is caused by the change in inflation target by a central bank. This view of low frequency movement in inflation is consistent with the literature (Cogley et al., 2010).

In the following, I estimate empirical impulse responses of macro aggregates (output, inflation and shortterm interest rate) and asset prices (bond and stock returns) to investigate whether persistent change in inflation gave rise to the time-varying patterns in inflation cyclicality and bond-stock correlation. Specifically, I estimate a SVAR model with sign restrictions, which come from a workhorse New Keynesian model. In a workhorse New Keynesian model such as the one in Galí (2015), conventional monetary policy shocks move short-term interest rates, and move inflation and short-term interest rates in the opposite direction. Furthermore, due to nominal rigidity, monetary shocks move output via various channels. The non-neutrality of monetary shocks is also widely studied in the literature. For instance, Gertler and Karadi (2015) and Hanson and Stein (2015) show that identified monetary policy shocks have significant effects on macro aggregates and asset prices.

But, the effect of persistent monetary shocks, or inflation target shocks in this paper, has attracted attention only recently.<sup>17</sup> Recent monetary economics literature has studied that there exist persistent components in monetary shocks that affect nominal interest rates and inflation (Cogley and Sargent, 2005; Coibion et al., 2017). In this strand of literature, persistent monetary shocks have driven the low frequency movements in inflation, and more importantly, it is estimated that the persistent components account for significant portion of the inflation the US economy has been through, including the Great Inflation era (Cogley et al., 2010; Ireland, 2007). The persistent monetary shock, which is interpreted as the change in inflation target in this paper, has long-lasting effect on inflation and its expectation, such as the prolonged inflation in 1970s. However, it is tricky to identify inflation target shocks from other shocks, especially conventional transitory monetary policy shocks.

To identify the persistent shocks to inflation, I exploit the sign restriction framework following Uhlig (2005). Identification of inflation target shock can be achieved by imposing the assumption that the sign of the impact effect on inflation and short-term interest rate is the same after persistent inflation shocks, but

<sup>&</sup>lt;sup>17</sup>One recent example is Uribe (2022). The author estimates a SVAR model exploiting both long-run identification and signrestrictions. Identification is obtained by three assumptions. First, assume that the nominal interest rate and inflation is co-integrated with the permanent component of monetary shocks with co-integrating vector of (1, -1). Second, the inflation target shock is the only shock that has long-run effect on the nominal interest rate. Another assumption is that monetary policy shock has non-positive impact effects on output and inflation. The author finds that permanent monetary shocks account for more than half of the US inflation.

#### Table 2: Sign restriction on structural shocks

	Inflation target	MP	Supply
Output	*	-	—
Inflation	+	_	+
Interest rate	+	+	*
Bond return	_	_	—
Stock return	*	_	_

Notes: Sign restrictions on each shock. The restrictions are only imposed on impact.

different after transitory monetary shocks.<sup>18</sup> To further deal with the concern that large unexpected supply shocks (eg. oil shocks in 1970s) could cause the stagflationary outcome, I explicitly identify supply shocks by imposing sign restrictions that are predicted by theory.

In summary, my empirical specification is based on a structural VAR that identifies structural shocks including inflation target shocks (persistent monetary shocks), traditional monetary policy shocks (transitory monetary shocks) and supply shocks (such as oil price shocks) using sign restrictions on the impact effect of variables following each shock.<sup>19</sup> I estimate the model using quarterly US date spanning two subperiods based on the identified structural break date, from 1965Q1 to 1987Q3 and 1987Q3 to 2019Q4 with one year lag. I collected macro data (GDP, GDP deflator and the federal funds rate) from the St. Louis Fed website. Bond return  $(xr^b)$  is measured by one-quarter holding return of 5-year Treasury bonds excess of 3-month T-bill rate, where Treasury yields are based on the estimates of Gürkaynak et al. (2007). Stock return  $(xr^s)$  is measured by one-quarter return from the value-weighted S&P 500 including dividends excess of 3-month T-bills, where S&P 500 date is from CRSP. I define the vector of five variables  $Y_t \equiv (\log GDP_t, \log P_t, R_t, xr_t^b, xr_t^s)'$  where  $\log GDP_t$  is log real GDP per capita,  $\log P_t$  is  $\log$  GDP deflator,  $R_t$  is the effective federal funds rate,  $xr_t^b$  and  $xr_t^s$  are excess return on bonds and stocks, respectively. For drawing random rotation matrix, I use uninformative uniform distributions following Uhlig (2005).

Figure 3 displays the estimated median impulse responses of output, inflation, nominal short-term interest rate, bond and stock returns following a one standard deviation positive inflation target shock (a persistent monetary shock) for each sample period. In both periods, the responses of inflation and short-term nominal interest rate are statistically significant for more than 3 years horizon, but quantitatively much

<sup>&</sup>lt;sup>18</sup>Theoretical prediction of a workhorse New Keynesian model when long-run inflation target (i.e. steady state inflation) is not zero will be discussed in the next section. If the persistence of inflation target shock is sufficiently high, then it increases both inflation and nominal short-term interest rates.

<sup>&</sup>lt;sup>19</sup>In this exercise, I only impose the sign restriction on the impact period (static sign restrictions). In contrast, dynamic sign restrictions can be considered where sign restrictions are imposed up to h periods after the shock hitting Uhlig (2005). Also, following Uhlig (2005), I do not include a constant or a time trend. This specification makes more robust results. For detail, see Uhlig (2005).



(a) Impulse responses to inflation target shock (persistent monetary shock): 1965-1987

(b) Impulse responses to inflation target shock (persistent monetary shock): 1987-2019



Figure 3: Impulse responses of nominal interest rate, real interest rate, stock returns and bond returns

Notes: Impulse responses to a one standard deviation permanent monetary shock to nominal interest rates. Identification scheme is described in the main test. The SVAR model is estimated by OLS. The shaded area represents 68% confidence interval. GDP is log real GDP per capita. Inflation is measured using the annualized log difference in GDP deflator. Short-term interest rate is the effective federal funds rate. Bond return  $(xr^b)$  is measured by one-quarter holding return of 5-year Treasury bonds excess of 3-month T-bills. Stock return  $(xr^s)$  is measured by one-quarter return from the value-weighted S&P 500 including dividends excess of 3-month T-bills. smaller in the earlier period. Following one standard deviation inflation target shock, inflation increases by 0.8 percent and 0.25 percent, which are around one-third and one-fourth of inflation in each period, respectively. The responses of inflation are also persistent. Before the structural break, inflation response remains around the peak level, and even after the break, inflation response is still positive. Federal funds responses are also economically large and significant, which exhibit peak responses of 0.8 percent and 0.4 percent. Positive comovement of inflation and short-term interest responses<sup>20</sup> account for negative bond return responses in both periods. This is because higher than expected inflation erodes the real value of nominal long-term bonds, even more than that of shorter maturity nominal bonds. Unlike macro variables, the effects on financial variables disappear very quickly.

Turning to my variables of interest, following a positive inflation target shock, the responses of GDP and stock returns show sharp state dependent effects of persistent inflationary shock. Before the estimated break date, following an increase in inflation target, real GDP per capita and stock returns fall and the response is economically large and statistically significant. GDP decreases nearly 1 percent over the 5 year after the initial shock, which coincides with drop in stock returns. As my previous intuition expects, fall in GDP is followed by poor stock return, which makes bonds and stocks comove together. On the other hand, after the structural break, GDP responses are much smaller, and the median response is positive for the 10 quarters from the initial shock. This also associated with positive median stock return response in the period. Again, GDP and stock returns comove, but in this period, positive responses of stock returns are inversely related to bond returns, generating a negative bond-stock correlation.

The key results in this section that the conditional responses of real GDP and stock returns are statedependent is the crucial empirical support for the idea of the inflation risk channel, which will be discussed in more detail in the following sections. When an economy is in a state where inflation is unstable, a persistent increase in inflation target generates substantial inflation risks, which drive both stock and bond prices to fall. It is also consistent with the observed negative correlation between inflation and output gap, which implies that in such a state, inflation is countercyclical and nominal long-term bonds lose its value when an economy is in downturn, i.e. marginal utility of households is high. In contrast, if an economy is in a stable inflation, the same increase in inflation target does not drive the positive comovement of bond and stock prices. Rather, in this case, positive inflation is associated with higher return from stocks and hence a negative correlation between the two assets. The SVAR evidence in this section, therefore, shows an evidence that the response to persistent inflationary shocks is state-dependent, and in particular, dependent upon how unstable inflation is.

<sup>&</sup>lt;sup>20</sup>This is the so-called 'Neo-Fisherian' effect (Uribe, 2022), which refers to the short-run positive comovement of nominal interest rate and inflation.

In the remaining part of the paper, I will analytically show that a workhorse New Keynesian model with positive and time-varying inflation target can explain this empirical finding, and more importantly, inflation target explains the majority of the sign change in bond-stock correlation and inflation cyclicality.

### 3 The inflation risk channel in New Keynesian model

In this section, I study a simple linearized New Keynesian model with time-varying inflation target. When long-run inflation target is positive ( $\bar{\pi} \ge 0$ ), the supply side of economy differs from a standard New Keynesian Phillips curve (NKPC) and features the inflation risk terms. The channel is strengthened as the target increases and brings about substantial inflation risks. Through this channel, inflation target shocks generate a negative or positive bond-stock correlation, depending on the size of inflation risks.

#### 3.1 A linearized New Keynesian model

The model environment is standard, and mostly follows a standard NK model described in chapter 3 of Galí (2015). The demand side of the model consists of a continuum of identical and infinitely lived households that consume and supply labor. Each household maximizes lifetime utility following CRRA preferences with  $\sigma = 1$ . The supply side consists of a continuum of intermediate goods firms and final goods firms. Final goods producers operate in a competitive market and use intermediate goods to produce final goods using a CES aggregator with the elasticity of substitution  $\epsilon > 1$ . Monopolistically competitive intermediate good firms produce differentiated intermediate goods using labor as the only input and a CRS technology. Each intermediate goods firms are randomly chosen and only those firms are allowed to reset their prices, holding other prices fixed.<sup>21</sup> There is a central bank that sets a short-term nominal interest rate following a Taylor rule. The central bank also sets an inflation target in a persistent way, which will be specified below. The model is linearized around a long-run inflation target ( $\overline{\Pi}$  or  $1 + \overline{\pi}$ ), which is possibly different from zero.

In this simple setting, the demand side can be summarized by the following IS curve using the Euler equation of households:

$$\tilde{y}_t = E_t \tilde{y}_{t+1} - (\hat{i}_t - E_t \hat{\pi}_{t+1})$$
(2)

where  $\beta$  is household's discount factor,  $\tilde{y}_t$  denotes the log-deviation of output gap from its steady state, and

<sup>&</sup>lt;sup>21</sup>For simplicity, indexation to previous inflation is not considered here. But in the quantitative model in the next section, partial indexation will be allowed.

 $\hat{\pi}_t$  is the deviation of inflation rate from its long-run target ( $\bar{\pi}$ ), and  $\hat{i}_t$  denotes the deviation of short-term nominal interest rate. Here, the intertemporal elasticity of substitution ( $\sigma$ ) is assumed to be 1 for simplicity. Monetary policy rule and time-varying inflation target are characterized as following:

$$\hat{i}_t = \phi_\pi (\hat{\pi}_t - \hat{\pi}_t^*) + \phi_y \tilde{y}_t \tag{3}$$

$$\hat{\pi}_t^* = \rho_\pi \hat{\pi}_{t-1}^* + \epsilon_t^\pi, \tag{4}$$

where  $\hat{\pi}^*$  is the deviation of  $\pi^*$  from its long-run target.  $\epsilon^{\pi}$  is an iid innovation to inflation target, following  $N(0, \sigma_{\pi}^2)$ .  $\phi_{\pi}$  and  $\phi_y$  denote a central bank's response to inflation and output gap. The stochastic nature of inflation target can reflect policy mistake or misperception of the current state. For instance, Primiceri (2006), Cogley et al. (2010), and Justiniano et al. (2013) interpret that policymakers learn about the structure of the economy, and the time-varying inflation target reflects the evolution of beliefs policymakers have. As Primiceri (2006) hypothesizes, change in beliefs by policymakers gives rise to low-frequency movements in inflation. Another possible interpretation is miscommunication or uncertainty around the target inflation rate. Uncertainty can arise due to the time lag between the point when a central bank changes its target and the point when it is publicly announced. This is because oftentimes a major tool of monetary policy is not changing the target inflation, but a short-term nominal interest rate.<sup>22</sup>

**Supply side:**  $\bar{\pi} \ge 0$ . In this case, the New Keynesian Phillips curve block involves additional terms which consist of a linear combination of output gap, price dispersion and expected inflation. Specifically, the NKPC block is composed of the following equations:

$$\hat{\pi}_{t} = \beta(\bar{\Pi})E_{t}\hat{\pi}_{t+1} + \kappa(\bar{\Pi})\tilde{y}_{t} + \underbrace{(\bar{\Pi}-1)\Big[\lambda(\bar{\Pi})\hat{s}_{t} + \gamma(\bar{\Pi})E_{t}\hat{\psi}_{t+1}\Big]}_{\text{Inflation risk term}}$$
(5)

$$\hat{s}_t = \xi \bar{\Pi}^\epsilon \hat{s}_{t-1} + \frac{\epsilon \xi \bar{\Pi}^{\epsilon-1}}{1 - \xi \bar{\Pi}^{\epsilon-1}} (\bar{\Pi} - 1) \hat{\pi}_t$$
(6)

$$\hat{\psi}_t = (1 - \xi \beta \bar{\Pi}^{\epsilon}) \Big( \varphi \hat{s}_t + (\varphi + 1) \tilde{y}_t \Big) + \xi \beta \bar{\Pi}^{\epsilon} (E_t \hat{\psi}_{t+1} + \epsilon E_t \hat{\pi}_{t+1})$$
(7)

where  $\bar{\kappa} = \frac{(1-\xi\bar{\Pi}^{\epsilon-1})(1-\xi\beta\bar{\Pi}^{\epsilon})}{\xi\bar{\Pi}^{\epsilon-1}}(\varphi+1), \bar{\beta} = \beta(1+\epsilon(\bar{\Pi}-1)(1-\xi\bar{\Pi}^{\epsilon-1})), \bar{\lambda} = \bar{\kappa}(\varphi+1)^{-1}\varphi/(\bar{\Pi}-1), \bar{\gamma} = \beta(1-\xi\bar{\Pi}^{\epsilon-1}).$  $\varphi$  is the inverse Frisch elasticity of labor supply, and  $\hat{s}_t$  denotes log deviation of price dispersion from its

<sup>&</sup>lt;sup>22</sup>Recent adoption of the average inflation targeting (AIT) framework is an exception (Powell et al., 2020). A distinguishing feature of the AIT is that under the AIT, the Fed is willing to allow for inflation higher than its long-run target of 2% temporarily, until the average of inflation reaches its 2% target. In this regard, the AIT introduces an uncertainty around the short-run inflation target,  $\pi_t$ 

steady state.<sup>23</sup> In the NKPC block above,  $\hat{s}_t$  represents cross-sectional price dispersion in each period. Note that equations (5) - (7) collapse to the simple model with the standard Phillips curve if  $\bar{\Pi} = 1$  (or  $\bar{\pi} = 0$ ). But if once long-run inflation target deviates from zero inflation, or  $\bar{\pi} > 0$ , the NKPC block in this economy is different from the conventional three equation model, such as the one in Galí (2015).

The third term in equation (5) emerges under positive inflation target under Calvo pricing, which I label the *inflation risk* term.<sup>24</sup> The inflation risk term is a linear combination of the weighted sum of the history of price dispersions ( $\hat{s}_t$ ) and the weighted average of future marginal costs ( $\varphi \hat{s}_{t+1} + \varphi \tilde{y}_{t+1}$ ) and inflation. The generalized New Keynesian Phillips curve in (5)-(7) is both more backward-looking and forward-looking.<sup>25</sup> That is, current inflation is affected more by the past inflation, expected inflation and expected future cost conditions. On the one hand, price adjusting firms facing future inflation ( $\bar{\pi} > 0$ ) decide optimal prices given the possibility that they may not have chances to adjust their prices in the future, hence taking into account future inflation and cost conditions with more weights. On the other hand, non-adjusting firms stick with their old prices. This puts price distortion in the economy because on average, price increases at the rate of  $\bar{\pi} > 0$ . Under positive long-run inflation, sticking to old prices implies going farther from optimal prices, and getting inefficiently cheaper over time. The inflation risk channel has qualitatively and quantitatively important effects on inflation, output and asset price dynamics.

#### 3.2 Macroeconomic dynamics

In this subsection, I study the linearized NK model described above when inflation target shocks hit. I explore the economic significance of time-varying inflation target ( $\pi_t^*$ ), and how it interacts with long-run inflation target ( $\bar{\pi}$ ).

For analytical tractability and simplicity, assume  $\varphi = 0$ , i.e. labor choice is indivisible. Note that under this assumption, price dispersion is irrelevant for Phillips curve (see equation 6 and 7), hence the equilibrium consists of four equations (2), (3), (5), and (7). In this equilibrium, all variables are forward-looking and thus free variables. By substituting out the nominal interest rate  $\hat{i}_t$  (3) into IS equation (2), the system of

$$s_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^\epsilon di$$

<sup>&</sup>lt;sup>23</sup>The price dispersion is defined as a cross-sectional dispersion of prices across intermediate goods producers,

The price dispersion arises in Calvo model because not every firm has chance to reoptimize its price in a given period.

<sup>&</sup>lt;sup>24</sup>Note that under alternative pricing frictions such as Rotemberg pricing, the inflation risk terms do not arise in general.

<sup>&</sup>lt;sup>25</sup>Phillips curve featuring backward-looking terms is also studied in Gali and Gertler (1999). This reflects They derive the hybrid New Keynesian Phillips curve which depends on previous inflation ( $\hat{\pi}_{t-1}$ ) and future expected inflation ( $E_t \hat{\pi}_{t+1}$ ). The underlying pricing behavior in their model is that among  $1 - \xi$  fraction of adjusting firms, a fraction  $1 - \omega$  of firms are 'forward looking' and reoptimize prices considering future inflation while the remaining  $\omega$  fraction of firms adjust their prices but non-optimally index their prices to the past inflation. Hence, in each period,  $(1 - \xi)\omega$  fraction of firms adopt the past inflation, which introduces persistence in the Phillips curve. The expression in (5) is even more generalized, and it includes all the past inflation and expected inflation.

equations (2), (5), and (7) can be rewritten in the following matrix form

$$x_t = BE_t x_{t+1} + \varepsilon_t, \tag{8}$$

where  $x_t = [\tilde{y}_t, \hat{\pi}_t, \hat{\psi}_t]'$  and

$$B = \begin{bmatrix} \sigma & 1 - \phi_{\pi}\bar{\beta} & -\phi_{\pi}(\bar{\Pi} - 1)\bar{\gamma} \\ \bar{\kappa} & \bar{\kappa} + \bar{\beta}(\sigma + \phi_{y}) & (\sigma + \phi_{y})(\bar{\Pi} - 1)\bar{\gamma} \\ (1 - \xi\beta\bar{\Pi}^{\epsilon})\sigma & (\sigma + \phi_{\pi}\bar{\kappa} + \phi_{y})\epsilon\xi\beta\bar{\Pi}^{\epsilon} + (1 - \bar{\beta})(1 - \xi\beta\bar{\Pi}^{\epsilon}) & (\sigma + \phi_{\pi}\bar{\kappa} + \phi_{y})\xi\beta\bar{\Pi}^{\epsilon} + (1 - \bar{\beta}\phi_{\pi})(1 - \xi\beta\bar{\Pi}^{\epsilon}) \end{bmatrix}$$

In this case, determinacy of rational expectation equilibrium can be achieved when all eigenvalues of *B* lie inside the unit circle (Blanchard and Kahn, 1980). The condition for determinacy under zero inflation target collapses to the textbook determinacy condition

$$\kappa(\phi_{\pi} - 1) + (1 - \beta)\phi_{y} > 0 \tag{9}$$

when  $\overline{\Pi} = 1$ , hence  $\psi_t$  becomes irrelevant for determinacy (Bullard and Mitra, 2002). But with a generic long-run inflation target ( $\overline{\pi} \ge 0$ ), the condition is more involved because of the expectation term,  $\psi_t$ . The following proposition provides characterization of the sufficient and necessary condition for determinacy.

**Proposition 1** (Determinacy). *The sufficient and necessary condition for a unique rational expectations equilibrium for the system* (51) *is* 

1.  $1 + \frac{1}{2}(tr(B)^2 - tr(B^2)) > |det(B) + tr(B)|$ 

**2.** 
$$1 - det(B)^2 > |det(B)tr(B) - \frac{1}{2}(tr(B)^2 - tr(B^2))|$$

where tr(B) is the trace of B and det(B) is the determinant of B.

Proof. See Appendix.

To get a sense of the determinacy region with  $\bar{\pi} \geq 0$ , figure 4 depicts the determinacy region over  $\phi_{\pi}$  and  $\bar{\pi}$  plane with a textbook calibration of the model. Similar to the case in zero inflation target, it is more likely to achieve determinacy when  $\phi_{\pi}$  is higher. Higher  $\bar{\pi}$ , however, reduces determinacy region and it is increasingly shrinking determinacy area. Consistent with previous results by Ascari and Ropele (2009) and Coibion and Gorodnichenko (2011), when  $\bar{\pi}$  is above zero, the basic Taylor principle breaks down and the minimum inflation response by a central bank becomes larger.

To characterize equilibrium dynamics of the model, I only consider time-varying inflation target as the only source of uncertainty as it is the primary interest of the paper. Further, I use undetermined coefficients



Figure 4: Determinacy region of the GNK model

Notes: This figure is based on the following standard calibration  $-\beta = 0.99$ ,  $\sigma = 1$ ,  $\varphi = 0$ ,  $\xi = 0.75$ ,  $\epsilon = 7$ ,  $\phi_y = 0.15$ .

method to find the functional form of endogenous variables as a function of the exogenous variable,  $\hat{\pi}_t^*$ , which is specified in equation (4). For simplicity, I assume that the economy starts in deterministic steady state. The following lemma characterizes the equilibrium dynamics in the model as a linear function of  $\pi_t^*$  when inflation target is the only source of uncertainty.

**Lemma 1** (Equilibrium dynamics). Suppose the sufficient and necessary condition for a unique rational expectations equilibrium holds. Further assume that the economy is initially in steady state. At the beginning of period t, an unexpected inflation target shock  $\varepsilon_t$  hits the economy. Then, the equilibrium dynamics can be characterized by the following forms

$$[\hat{\pi}_t, \tilde{y}_t, \hat{i}_t, \hat{\psi}_t]' = [\Gamma_\pi, \Gamma_y, \Gamma_i, \Gamma_\psi]' \hat{\pi}_t^*$$
(10)

where  $\Gamma_{\pi}(\bar{\Pi}), \Gamma_{y}(\bar{\Pi}), \Gamma_{i}(\bar{\Pi}), \Gamma_{\psi}(\bar{\Pi})$  have the following form

$$\Gamma_{\pi}(\bar{\Pi}) = \frac{\frac{\bar{\kappa}\phi_{\pi}}{1-\rho_{\pi}} + \frac{\bar{\gamma}\rho_{\pi}\phi_{\pi}(\bar{\Pi}-1)(1-\xi\beta\bar{\Pi}^{\epsilon})}{(1-\rho_{\pi}\xi\beta\bar{\Pi}^{\epsilon})(1-\rho_{\pi})}}{1-\bar{\beta}\rho_{\pi} + \frac{\bar{\kappa}(\phi_{\pi}-\rho_{\pi})}{1-\rho_{\pi}} + \frac{\bar{\gamma}\rho_{\pi}(\bar{\Pi}-1)}{1-\rho_{\pi}\xi\beta\bar{\Pi}^{\epsilon}}\left(\frac{(1-\xi\beta\bar{\Pi}^{\epsilon})(\phi_{\pi}-\rho_{\pi})}{1-\rho_{\pi}} - \epsilon\xi\beta\bar{\Pi}^{\epsilon}\rho_{\pi}\right)}$$
(11)

$$\Gamma_y(\bar{\Pi}) = \frac{\phi_\pi}{1 - \rho_\pi} - \frac{\phi_\pi - \rho_\pi}{1 - \rho_\pi} \Gamma_\pi$$
(12)

$$\Gamma_i(\bar{\Pi}) = \phi_\pi(\Gamma_\pi - 1) + \phi_y \Gamma_y \tag{13}$$

$$\Gamma_{\psi}(\bar{\Pi}) = \frac{(1 - \xi \beta \bar{\Pi}^{\epsilon}) \Gamma_{y} + \epsilon \xi \beta \bar{\Pi}^{\epsilon} \rho_{\pi} \Gamma_{\pi}}{1 - \rho_{\pi} \xi \beta \bar{\Pi}^{\epsilon}}$$
(14)

Proof. See Appendix.

Lemma 1 confirms that all endogenous variables  $(\hat{\pi}_t, \tilde{y}_t, \hat{i}_t, \hat{\psi}_t)$  are free variables and can be expressed a function of the state variable,  $\pi_t^*$ . The following proposition establishes macroeconomic dynamics following an inflation target shock. The results show that (i) the inflation risk effects of time-varying inflation target, (ii) the responses of inflation and short-term nominal interest rate exhibit the Neo-Fisher property, and (iii) the sign of output gap-inflation covariance depends on  $\overline{\Pi}$ .

**Proposition 2.** Suppose the sufficient and necessary condition for a unique rational expectations equilibrium holds. Further assume that the economy is initially in steady state. Then the following results hold.

1. If  $\rho_{\pi} > \rho_{\pi}^*$  for some  $\rho_{\pi}^* \in (0, 1)$ , then inflation response is increasing in  $\overline{\Pi}$ , or  $\Gamma_{\pi}(\overline{\Pi})$  is increasing in  $\overline{\Pi}$ 

$$\frac{d\Gamma_{\pi}(\bar{\Pi})}{d\bar{\Pi}} > 0 \tag{15}$$

- 2. (The Neo-Fisher effect) If  $\rho_{\pi} > \max\{\bar{\rho}_{\pi}, \rho_{\pi}^*\}$  for some  $\bar{\rho}_{\pi} \in (0, 1)$ , then inflation responds more than one-to-one to inflation target change, i.e.  $\Gamma_{\pi}(\bar{\Pi}) > 1$  for  $\bar{\Pi} > 1$ . Furthermore, if  $\phi_{\pi} > -\frac{\rho_{\pi}\phi_{y}}{1-\rho_{\pi}-\phi_{y}}$ , then nominal short-term interest rate increases following a positive inflation target change, i.e.  $\Gamma_{i}(\bar{\Pi}) > 0$
- 3. (The inflation risk effect) If  $\rho_{\pi} > \rho_{\pi}^*$ , then output response is decreasing in  $\overline{\Pi}$ , or  $\Gamma_y(\overline{\Pi})$  is increasing in  $\overline{\Pi}$

$$\frac{d\Gamma_y(\Pi)}{d\bar{\Pi}} < 0 \tag{16}$$

Furthermore, if  $\overline{\Pi} > \overline{\Pi}^*$  such that  $\Gamma_{\pi} > \Gamma_{\pi}^* \equiv \frac{\phi_{\pi}}{\phi_{\pi} - \rho_{\pi}}$ , then output responses to inflation target shocks switch the sign, i.e.  $\Gamma_y(\overline{\Pi}) < 0$ .

4. Output gap-inflation covariance can be characterized by

$$Cov(\hat{y}_t, \hat{\pi}_t) = \Gamma_y(\bar{\Pi})\Gamma_\pi(\bar{\Pi}) \frac{\sigma_\pi^2}{1 - \rho_\pi^2}$$
(17)

Then there exists  $\overline{\Pi}^* > 1$  such that

$$Cov(\hat{y}_t, \hat{\pi}_t) > 0 \qquad if \quad \bar{\Pi} \le \bar{\Pi}^*$$
  
and 
$$Cov(\hat{y}_t, \hat{\pi}_t) < 0 \qquad if \quad \bar{\Pi} > \bar{\Pi}^*$$

5. Active monetary policy reduces inflation response, i.e.  $\Gamma_{\pi}(\bar{\Pi})$  is decreasing in  $\phi_{\pi}$ ,

$$\frac{d\Gamma_{\pi}(\bar{\Pi})}{d\phi_{\pi}} < 0 \tag{18}$$

Proof. See Appendix.

Proposition 2 uncovers the responses of macro aggregates to a persistent change in target inflation rate. First, inflation response to a positive target change is positive and increasing in  $\bar{\pi}$ . This implies that when a central bank sets already high long-run inflation target, the endogenous response of inflation to a change in inflation target is higher than the response to the same magnitude of shock in low inflation steady state. This provides intuition why higher long-run target shrinks determinacy region. Because higher  $\bar{\pi}$  amplifies the endogenous responses of inflation to exogenous shocks, the required inflation response should be higher.

Second result in Proposition 2 shows that for a sufficiently persistent inflation target process, it generates the Neo-Fisher effect, i.e. a short-run positive inflation-short-term nominal interest rate comovement. When an increase in  $\pi_t^*$  is persistent, real interest rate decreases following an increase in  $\pi_t^*$ , lowering saving and raising consumption by households. Compared to temporary shocks, such as conventional expansionary monetary policy shocks, the expected path of inflation is more persistent and high, the effect on consumption and saving is much stronger. This leads to an increase in short-term interest rate, even though inflation target change looks like a negative increase in the constant in Taylor rule (see equation (3)).

Third, the inflation risk effect confirms the detrimental consequences of high inflation regime. For sufficiently persistent inflation target process, a positive inflation shock is expansionary only when long-run inflation target is low. If  $\overline{\Pi} > \overline{\Pi}^*$ , then output response starts to decrease, hence inflationary shock becomes contractionary. When  $\overline{\Pi}$  is high enough, current inflation is less related to the current economic conditions, but more related to the past and future inflation. This is because when adjusting firms decide the optimal price, it takes account the expected future revenue and costs, given the expected path of inflation. Intuitively speaking, higher  $\overline{\Pi}$  means higher expected loss in the future because of the combination of the possibility of non-adjusting and positive inflation on average. These make the adjusting firms more proactive, and charge higher prices than the prices that would have prevailed under zero long-run inflation. The inefficiency gen-

erated by the pricing behavior of firms and nominal rigidity could dominate the initial expansionary effects, which contributes to the contractionary responses of output. This novel effect of inflation target can be explained through the lens of *the inflation risk channel*, which is captured by the inflation risk term in equation (5). I will discuss the inflation risk channel in more detail in the next subsection.

The fourth result provides a characterization of output gap-inflation covariance, and shows that the sign of the covariance depends on the long-run inflation target. Higher long-run target strengthens the inflation risk effect, and if it is sufficiently strong, then the output gap-inflation covariance can switch the sign from positive to negative as inflation target increases. This is the key result that can rationalize the sign switching pattern in bond-stock correlation because of tight relationship between output-stock return, and inflationbond return.

The last result states that the role of monetary policy on the volatility of inflation given long-run inflation target. Consistent with zero inflation case, the inflation responses are attenuated if a central bank more actively responds to inflation. This also helps to understand how increasing inflation coefficient expands determinacy region in figure 4.

### 3.3 The inflation risk channel

The inflation risk effects in Proposition 2 provides new insights about detrimental effects of persistent inflation. The *inflation risk channel* tells us that when a central bank sets long-run inflation target high, additional attempts to boost an economy by adding more inflationary pressure can be contractionary. The contractionary consequence of the inflation risk channel reflects inefficient output loss that arises due to price distortion. Intuitively speaking, this channel emerges because of expected inflation. When the future path of inflation is expected to be high, then price adjusting firms take the future path into account, charging even higher prices, compared to the prices that if would have charged under low inflation expectation. This puts distortionary forces and makes the current price distribution more inefficient. There is another distortion that contributes to aggregate inefficiency. Firms that are not allowed to adjust should keep their previous prices, but on average, aggregate price level increases by  $\bar{\pi}$ . Thus those previous prices are inefficiently low, compared to the optimal prices under  $\bar{\pi} = 0$ . When the inefficiency is too large, additional attempts to boost an economy by raising inflation fails to support the economy, and even make the situation worse.

Let us rewrite the components of the inflation risk term in NKPC (5) in recursive forms.

$$\hat{s}_t = \frac{\epsilon \xi \bar{\Pi}^{\epsilon - 1}}{1 - \xi \bar{\Pi}^{\epsilon - 1}} (\bar{\Pi} - 1) \sum_{k=0}^{\infty} (\xi \bar{\Pi}^{\epsilon})^k \hat{\pi}_{t-k}$$
<sup>(19)</sup>

$$E\hat{\psi}_{t+1} = (1 - \xi\beta\bar{\Pi}^{\epsilon})E_t\widehat{rmc}_{t+1} + \sum_{k=2}^{\infty}(\xi\beta\bar{\Pi}^{\epsilon})^kE_t\Big[(1 - \xi\beta\bar{\Pi}^{\epsilon})\widehat{rmc}_{t+k} + \epsilon\hat{\pi}_{t+k}\Big]$$
(20)

Thus, the inflation risk channel can be decomposed into the backward looking term (19) and the forward looking term (20). The backward looking term, the price dispersion (19), is a weighted average of past inflation. Because in every period, there is a fraction  $\xi$  of firms that are not allowed to adjust prices, current inflation is affected by the full history of past inflation. In the expression, the weight on farther past is increasing in inflation. If average inflation is high, then the 'distance' between past and current inflation is far, so it serves as bigger distortionary factor. The forward looking term in (20) is the weighted average of future real and nominal conditions. If a firm is chosen to change its price, it takes into account possibilities of future non-adjustment, hence profit loss due to future economic conditions. Note that the relative weight on future inflation is an increasing function of long-run inflation. This is because if the average inflation is high, profit loss due to non-adjusting is increasing.

The former effect (*the staggered price effect*) which refers to the effects of falling behind from the optimal price over time, gives an endogenous force that contributes to persistence of inflation. The latter effect (*the precautionary pricing effect*), more proactive pricing by adjusting firms, reflects higher weights on future inflation and marginal cost conditions, more forward-looking pricing decision. This pricing behavior is endogenous and due to the concern on future inflation and the possibility of non-adjusting. Taking into account this future risks of non-adjusting under positive inflation, firms act with precaution such that they charge higher prices, considering future inflation. By the force of the two effect, expansionary inflation shocks can become contractionary to output by causing inflation.

According to Proposition 2, when long-run inflation target is low, a positive innovation to inflation target has an effect on an economy similar to that of demand shocks. It boosts consumption, output and raises inflation as well as short-term interest rate. However, when  $\bar{\pi}$  is sufficiently high so that the output response flips the sign, then the same positive inflationary shock has a cost-push shock-like effect: lower consumption and output with higher inflation. In this regards, inflation risk channel is an endogenous mechanism that turns inflationary pressure from demand shocks into cost-push shocks.

This endogenous cost-push mechanism operates through the inefficiency generated by inflation and nominal rigidity. In Calvo model, the inefficiency due to nominal rigidity can be summarized by one statistic, the price dispersion.<sup>26</sup> Note that there is no heterogeneity among producers, so non-degenerate distribution in prices would allocate demand inefficiently and the cost of the inefficient allocation of demand is lower

<sup>&</sup>lt;sup>26</sup>From the definition of price dispersion, it is a cross-sectional distribution of prices in a give period. But one can show that it is also a weighted average of past inflation. This is because in each period,  $\xi$  fraction of non-adjusting firms are stuck at the previous prices, and  $\xi(1 - \xi)$  fraction of non-adjusting firms are stuck at the prices in two-periods before, ... Therefore, the price dispersion in the current period not only summarizes the current cross-sectional distribution, but also all the previous inflation.

output loss.<sup>27</sup> The output loss due to high inflation target is the main source of inflation risks analyzed in this paper and can explain countercyclical inflation and a positive bond-stock correlation following an inflation target shock. It is worth mentioning that even if the model is linear and certainty equivalence holds, there exists still 'risk' on firm side. The risk each firm is facing is coming from the Calvo setting. In each period, a firm has chance to be chose to adjust with probability  $1 - \xi$ . When allowed, adjusting firms decide optimal price, taking into account the future expected inflation and cost conditions. The risk arises due to the circumstance that inflation is *on average* positive, which implies that adjusting firms face positive cost because of *expected* inflation. Therefore, the risk in the inflation risk channel is cross-sectional risks coming from the future inflation path.

The price dispersion summarizes the staggered price effect and the precautionary pricing effect, hence reflecting the aggregate distortion due to nominal rigidity. Let us consider aggregate output in the model to understand how this price distortion translates into actual output loss. Aggregate output can be expressed as<sup>28</sup>

$$Y_t = s_t^{-1} A_t L_t, \tag{21}$$

where  $Y_t$  is output,  $A_t$  is aggregate productivity, and  $L_t$  is labor. From this expression, price dispersion is similar to an inverse of aggregate productivity and hence negatively related to output. This is so because higher dispersion means higher distortion in prices, hence higher inefficiency. Note that from equation (6), price dispersion is positively associated with inflation. When inflation increases, it also increases the distance between the prices of adjusting and non-adjusting firms, hence price dispersion, which eventually incurs output loss. In this regards, the effects of inflation target shocks are similar to cost-push shocks when  $\bar{\pi}$  is high. This inflation risk arises from nominal rigidity and inflation, and I will see that time-varying inflation target is one of the major forces that cause inflation risks under high long-run inflation target.

### 3.4 Asset price dynamics with inflation risks

**Asset prices** Given my interest on long-term bond and stock return correlation, consider a (unlevered) consumption claim and two period nominal bonds to keep the analysis simple.<sup>29</sup> A consumption claim is defined as a claim on aggregate consumption, hence output in the model, which does not expire. In each

<sup>&</sup>lt;sup>27</sup>Ascari and Sbordone (2014) show that there exists a negative long-run relationship between output and inflation target. That is, steady state output and inflation target has negative relationship. Compare to their long-run relationship, my results show that there exists short-run negative relationship between output and inflation when inflation target is high.

<sup>&</sup>lt;sup>28</sup>This aggregate output can be derived from integrating production functions of individual intermediate goods producers

<sup>&</sup>lt;sup>29</sup>The analysis can be easily extended to n period nominal bonds.

period, a claim pays out dividend  $Y_t$ . Each claim can be traded in financial markets at an ex-dividend price  $P_t^s$ . In equilibrium, the price of the consumption claim is given by  $P_t^s = E_t m_{t+1}(Y_t + P_{t+1}^s)$ , where  $m_{t+1}$  is the households' stochastic discount factor (SDF). When households have log utility (i.e.  $\sigma = 1$ ), log-deviation of stock returns from steady state  $(\hat{r}_{t+1}^s)$  is same as output growth

$$\hat{r}_{t+1}^s = \Delta \hat{y}_{t+1} \tag{22}$$

Excess return on consumption claim is

$$\hat{xr}_{t+1}^s = \hat{r}_{t+1}^s - \hat{r}_t = \Delta \hat{y}_{t+1} + E_t \hat{m}_{t+1} = y_{t+1} - E_t y_{t+1}$$
(23)

Therefore, excess return on consumption claim is the unexpected realization of output. Next, two period nominal bonds are default-free zero coupon bonds that pay one 'dollar' at maturity. Denote the price of this bond  $P_t^{(2)}$  then the equilibrium price of long-term bonds satisfy  $P_t^{(2)} = E_t m_{t+1} P_{t+1}^{(1)} / \pi_{t+1}$ , where  $P_t^{(1)}$  is the price of one period nominal bond. By linearizing the equation around the steady state, I have the following equation for the return on two period nominal bonds  $(r_{t+1}^b)$ 

$$\hat{r}_{t+1}^b = \hat{p}_{t+1}^{(1)} - \hat{p}_t^{(2)} - \hat{\pi}_{t+1}$$
(24)

where  $p_t^{(1)}$  and  $p_t^{(2)}$  are log prices of one and two period bonds, respectively. Then, excess return on two period bonds is given by

$$\hat{xr}_{t+1}^b = \hat{r}_{t+1}^b - \hat{i}_t = E_{\Delta(t+1)}\hat{m}_{t+2} - E_{\Delta(t+1)}\hat{\pi}_{t+2} - \hat{\pi}_{t+1},$$
(25)

where  $E_{\Delta(t+1)}X_{t+k} = E_{t+1}X_{t+k} - E_tX_{t+k}$ . Thus, excess return on two period bonds consists of two parts: (i) change in expected SDF, (ii) change in expected inflation and (iii) unexpected realization of the current inflation. If SDF is expected to be higher in the future, then holding bonds can hedge consumption risks, hence higher excess return. When future inflation is expected to be higher, then it will deteriorate the real value of bonds, hence lower excess return. The same reasoning applies to the current (unexpected) inflation. Accordingly, bond-stock covariance can be decomposed into the covariances between (i) output-SDF, and (ii) output-inflation path for the remaining bond maturity. For a non-permanent shock such as the inflation target shock in the current analysis, output and SDF correlation is positive. It implies that if it were real bonds that provide one unit of 'consumption' on maturity, then inflation target shock would predict a positive correlation between real long-term bonds and consumption claim because it is a persistent, but non-permanent shock that features a negative correlation between output and future output growth.<sup>30</sup> But nominal bond returns are also exposed to inflation risks, which is ambiguous a priori. If output and inflation over the remaining maturity of the bonds are positively correlated, then bond-stock covariance would be negative, which means nominal bonds are hedge. On the other hand, if output and inflation are negatively associated as the US economy experienced before the structural break, the above real hedging incentive can be dominated by inflation risks driven by countercyclical inflation. This is because countercyclical inflation implies that the value of nominal bonds is low when marginal utility of household is high.

The following proposition characterizes the main interest of the analysis, bond-stock covariance.

**Proposition 3** (Bond-stock covariance). Suppose the sufficient and necessary condition for a unique rational expectations equilibrium holds, and the economy is initially in steady state. Further assume that  $\rho_{\pi} > \max\{\bar{\rho}_{\pi}, \rho_{\pi}^*\}$ . Then bond-stock return covariance can be characterized by

$$Cov(xr_{t+1}^s, xr_{t+1}^b) = \Gamma_y(\bar{\Pi})(\Gamma_y(\bar{\Pi})(1-\rho_\pi) - \Gamma_\pi(\bar{\Pi})(1+\rho_\pi))\frac{\sigma_\pi^2}{1-\rho_\pi^2}.$$
 (26)

Also,  $\Gamma_y(\bar{\Pi})(1 - \rho_\pi) - \Gamma_\pi(\bar{\Pi})(1 + \rho_\pi) < 0$ , which implies bond-stock covariance is negatively associated with output gap-inflation covariance. Formally,

$$Cov(xr_{t+1}^s, xr_{t+1}^b) < 0 \quad \text{if and only if} \quad \Gamma_y(\bar{\Pi}) > 0.$$

$$(27)$$

 $\square$ 

Therefore, there exists  $\overline{\Pi}^* > 1$  such that

$$Cov(xr_{t+1}^{s}, xr_{t+1}^{b}) < 0$$
 if  $\Pi \le \Pi^{*}$   
and  $Cov(xr_{t+1}^{s}, xr_{t+1}^{b}) > 0$  if  $\overline{\Pi} > \overline{\Pi}^{*}$ 

Proof. See Appendix.

Proposition 3 uncovers a macro-financial linkage in the model. It shows that the covariance structure of asset prices heavily depends on macro aggregates, and in particular, on output-inflation relationship, which is also governed by long-run inflation target. For a sufficiently persistent inflation target change, the conditional response of nominal bonds and stocks switch its correlation sign exactly when output-inflation correlation changes its sign. This is because bond-stock covariance is dominated by the covariance between output and expected inflation path over the maturity. If future inflation is expected to be accompanied by output growth, then nominal long-term bonds provide hedging property. But if future inflation is accom-

<sup>&</sup>lt;sup>30</sup>Chernov et al. (2021a) studies the asset pricing implications of permanent and transitory shock regimes.



**Figure 5:** IRFs of output gap, inflation, real stock and bond return to a positive  $\pi^*$  shock

Notes: This figure is based on the following standard calibration –  $\beta = 0.99$ ,  $\sigma = 1$ ,  $\varphi = 1$ ,  $\xi = 0.75$ ,  $\epsilon = 5$ ,  $\phi_{\pi}$ =1.5,  $\phi_y = 0.15$ ,  $\rho_{\pi} = 0.995$ .

panied by lower output, then nominal bonds are risky in the sense that bond prices are moving together with other risky assets, consumption claim. In this case, nominal bonds are not hedge assets because of inflation risks. Therefore, when long-run inflation target is high so that the inflation risk channel operates, the response of output to inflation is negative, changing the signs of both output-inflation correlation and bond-stock correlation.

In figure 5, I depict an illustrative example of the inflation risk channel using a standard calibration from the literature. The figure reports impulse response functions of output gap, inflation and excess returns on the two assets following a one percentage point inflation target shock. I compare IRFs for three different long-run inflation target:  $\bar{\pi} = 0\%$  (black solid line), 1% (blue dashed line), and 2% (red dotted line).

In a zero-inflation case ( $\bar{\pi} = 0\%$ ), when a positive inflation target shock hits, output gap and inflation both rise, implying the effect of the shock is expansionary. Either if long-run inflation is zero ( $\bar{\pi} = 0\%$ ) or full price indexation by non-adjusting firms is assumed, the inflation risk channel disappears as there is neither staggered price effect nor precautionary pricing effect. As a result of an expansionary effect, stock return increases with output. In contrast, higher inflation erodes real value of nominal bonds, hence its return decreases. This case is consistent with the previous SVAR results that show after the structural break, a positive inflation shock increases both output and inflation, and generates a negative comovement between nominal bond and stocks. It also matches with the recent observation that shows inflation is procyclical and bond-stock correlation is well below zero.

Next consider positive long-run inflation cases ( $\bar{\pi} = 1\%$  and 2%). Under a mild inflation ( $\bar{\pi} = 1\%$ ), the expansionary effect weakens but it is still present. The weaker expansion leads to a smaller increase in stock return. Still, inflation target shocks generate procyclical inflation and a negative bond-stock correlation, but the correlations are much attenuated. Under a higher inflation ( $\bar{\pi} = 2\%$ ), the inflation risk channel starts to work and the effect of an inflationary shock is overturned. The increase in inflation target still raises inflation, but it ends up causing recession: a fall in output gap. Due to the recession generated by higher inflation target, consumption claim performs poorly and experiences negative returns. Therefore, under a high long-run inflation target, an increase in inflation. Risk-averse households are willing to hold short-term bonds since nominal long-term bonds do not hedge risks in this circumstance. In turn, this leads to a positive bond-stock correlation, through the inflation risk channel.

The impulse responses under higher long-run inflation target match the SVAR moments estimated earlier for the period before the structural break. Before the break, inflation and its expectation were both much higher and volatile because of the lack of anchoring of expectation. According to the inflation risk channel, this can contribute to a significant amount of inflation risks, which translates into countercyclical inflation and a positive bond-stock correlation.

### 4 Quantitative model

In this section, I describe a version of the standard New Keynesian model, which is based on the model outlined in Woodford (2003) or Galí (2015). To explore asset pricing implications of persistent inflation shock, I model nominal short-term and perpetual bonds as well as consumption claim. Households face a portfolio choice problem between short-term bonds, long-term bonds, and equity. This environment is useful to study the dynamics of bonds and equity prices with the presence of the inflation risk channel. Also, the model incorporates stochastic inflation target for monetary policy to capture low frequency movements in inflation.

#### 4.1 Household

Time is discrete and denoted by  $t = 0, 1, \cdots$ . The economy is populated by a continuum of households, which are identical and infinitely lived. A representative household consumes, supplies labor and earns

labor income. The household also decides how much to save in each class of assets: short-term bonds, long-term bonds and consumption claime.

The representative household maximizes expected utility (28), subject to (29)

$$E_0 \sum_{t=0}^{\infty} \beta^t \left( \log(C_t - hC_{t-1}) - \chi \frac{H_t^{1+\nu}}{1+\nu} \right)$$
(28)

$$P_t C_t + \frac{D_t}{\zeta_t} + \frac{P_t^B}{\zeta_t} (B_t - \kappa B_{t-1}) + P_t^E E_{t+1} = W_t H_t + DIV_t E_t + R_t D_{t-1} + B_{t-1}$$
(29)

where  $C_t$  is consumption,  $H_t$  is labor supply,  $W_t$  is nominal wage,  $D_t/P_t$  is the real holdings of short-term bonds,  $P_t^B B_t/P_t$  is the real holdings of long-term bonds,<sup>31</sup> and  $P_t^E E_t/P_t$  is the real holdings of consumption claim.  $\beta \in (0, 1)$  is a time discount factor, h is consumption habit parameter and  $\nu$  are the inverse of the Frisch elasticity of labor supply, and  $\chi > 0$ .  $\zeta_t$  represents a reduced form risk premium shock following a stationary AR(1) process (Smets and Wouters, 2007):

$$\log(\zeta_{t+1}) = \rho_{rp} \log(\zeta_t) + \epsilon_{t+1}^{rp}, \quad \epsilon_{t+1}^{rp} \sim^{iid} N(0, \sigma_{rp}^2).$$
(30)

As discussed in Smets and Wouters (2007), the risk premium shocks serve as a wedge between the return on bonds and the interest rate controlled by a central bank. It also generates a wedge between financial assets, especially between bonds and equity. A positive shock to  $\zeta_t$  increases demand for bonds and reduces consumption, and demand for equity, which generates negative comovement between bonds and equity. One can think of this shock as a "flight-to-quality" shock to safe and liquid assets. This shock exogenously alters demand for assets, and importantly, drives the flight-to-quality effect from risky assets (stocks) to safe assets (bonds).<sup>32</sup>

#### 4.2 Final good producers

In each period *t*, the final consumption good  $Y_t$  is produced by perfectly competitive firms using a continuum of each intermediate goods  $Y_t(i)$ ,  $i \in [0, 1]$  as inputs. Final good producers have access to a CES production

<sup>&</sup>lt;sup>31</sup>In the model, new issuance of long-term bonds is  $B_t - \kappa B_{t-1}$ , and each unit of long-term bonds pays off the cash flows of 1,  $\kappa$ ,  $\kappa^2$ ,  $\cdots$ . The structure of the long-term bonds will be discussed further below.

<sup>&</sup>lt;sup>32</sup>Fisher (2015) shows that the demand for safe and liquid assets as in Krishnamurthy and Vissing-Jorgensen (2012) can rationalize the risk premium shock in Smets and Wouters (2007). The convenience benefit can be rationalized by the notion that Treasury securities are highly liquid and safe, which can reduce transaction costs that could be incurred.  $\zeta_t$  is a shock to the convenience function, which makes the convenience benefits time-varying. In the current specification, risk premium shocks affect short-term and long-term bonds in the same manner. It is a simplifying assumption but also reflects the fact that Treasuries are very liquid even for longer maturity securities. Another benefit of this simplifying assumption is that it gives the same Euler equation as in Smets and Wouters (2007) in which the authors augment household Euler equation with risk premium shock. The importance of the risk premium shock has been discussed widely, especially in the context of New Keynesian models. For example, Christiano et al. (2015) find that the risk premium shock improves model fit for the data after.

technology that aggregates the continuum of intermediate goods into the single final good:

$$Y_{t} = \left[\int_{0}^{1} Y_{t}(i)^{\frac{1}{1+\lambda_{p,t}}} di\right]^{1+\lambda_{p,t}}.$$
(31)

where  $\lambda_{p,t}$  represents the desired markup over marginal costs for intermediate goods firms. I assume that  $\lambda_{p,t}$  follows a stationary AR(1) process

$$\log(1 + \lambda_{p,t}) = (1 - \rho_p)\log(1 + \lambda_p) + \rho_p\log(1 + \lambda_{p,t-1}) + \epsilon_t^p, \quad \epsilon_t^p \sim^{iid} N(0, \sigma_p^2).$$
(32)

I label this innovation as price markup shocks as in Smets and Wouters (2007). Final good producers' profit maximization and the zero profit condition for the competitive market yield the following condition for the final good price  $P_t$ :

$$P_t = \left[\int_0^1 P_t(i)^{-\frac{1}{\lambda_{p,t}}} di\right]^{-\lambda_{p,t}}$$
(33)

and the downward sloping demand function for each intermediate good *i*:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} Y_t$$
(34)

The demand function for each intermediate good will be used to derive price dispersion, which is central to understand the inflation risk channel.

#### 4.3 Intermediate goods producers

A continuum of monopolistically competitive firms produce a different variety i. Each intermediate goods firm i has access to a CRS technology

$$Y_t(i) = A_t \gamma^t H_t(i)^{1-\alpha} \tag{35}$$

where  $H_t(i)$  denotes the quantity of labor hired by firm *i*.  $\gamma$  represents deterministic growth rate of the economy.  $A_t$  is a neutral exogenous technology that is common across firms, and follows a stationary AR(1) process:

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t^A, \quad \epsilon_t^A \sim^{iid} N(0, \sigma_A^2).$$
(36)

Every intermediate goods firm is subject to the Calvo-type friction. In each period t, a fraction  $\xi_p$  of intermediate goods firms are not allowed to change its price optimally and instead, they reset the price according to the indexation rule

$$P_t(i) = P_{t-1}(i)\pi_{t-1}^{\chi_p}$$
(37)

where  $\pi_t \equiv P_t/P_{t-1}$ , and  $\chi_p$  is the degree of price indexation. In general,  $\chi_p \leq 1$  and therefore, there exists price dispersion in steady state, which is crucial for the inflation risk channel to work. The remaining fraction of firms,  $1 - \xi_p$ , can reset their price optimally to maximize the present value of its future expected cash flow. Because households own firms, realized profits will be distributed to households. Specifically, optimizing firms set their price  $P_t(i)$  solve the following expected profit maximizing problem:

$$\max E_t \sum_{s=0}^{\infty} \xi_p^s m_{t,t+s} \Big[ P_t(i) \Big( \prod_{k=1}^s \pi_{t+k-1}^{\chi_p} \Big) Y_{t+s}(i) - W_{t+s} H_{t+s}(i) - R_{t+s}^k K_{t+s}(i) \Big]$$
(38)

where  $m_{t,t+s}$  is the SDF that will be defined below. Solving this optimization problem, the aggregate price level is determined by

$$P_t = \left[ (1 - \xi_p) (P_t^r)^{-\frac{1}{\lambda_{p,t}}} + \xi_p (\pi_{t-1} P_{t-1})^{-\frac{1}{\lambda_{p,t}}} \right]^{-\lambda_{p,t}}$$
(39)

where  $P_t^r$  is the optimal reset price for adjusting firms, and  $P_{t-1}$  is the average price level in the previous period.

### 4.4 Monetary policy

A monetary authority sets the short-term interest rate  $R_t$  according to the following Taylor-type policy rule:

$$R_{t} = \rho_{R}R_{t-1} + (1 - \rho_{R})\left(\bar{R} + \phi_{\pi}(\pi_{t} - \pi_{t}^{*}) + \phi_{y}\tilde{y}_{t}\right) + \epsilon_{t}^{R}$$
(40)

where  $\bar{R}$  is the steady state short-term interest rate,  $\pi_t^*$  is the inflation target set by the monetary authority, and  $\epsilon_t^{MP}$  is an iid monetary policy shock, which follows  $\epsilon_t^{MP} \sim N(0, \sigma_R^2)$ . Paramters  $\phi_{\pi}$ ,  $\phi_y$  and  $\phi_{\Delta y}$ determine its response to inflation, output growth, and output gap, respectively.

The inflation target  $\pi_t^*$  is set by the monetary authority and follows the below stochastic process:

$$\pi_t^* = (1 - \rho_\pi)\bar{\pi} + \rho_\pi \pi_{t-1}^* + \epsilon_t^\pi$$
(41)

where  $\bar{\pi}$  is the steady state inflation, and  $\epsilon_t^{\pi} \sim N(0, \sigma_{\pi}^2)$  is an independently and identically distributed shock to inflation target (Ireland, 2007; Cogley et al., 2010; Justiniano et al., 2013). It has been documented in the literature that inflation contains very low frequency variation (Coibion and Gorodnichenko, 2011), which also has important implications for asset pricing dynamics (Cieslak and Povala, 2015; Davis et al., 2019; Bauer and Rudebusch, 2020). Equation (41) is one way to incorporate low frequency variation in inflation by incorporating stochastic target inflation. Time-varying inflation target can be interpreted as policy maker's misperception as in Cogley et al. (2010) and Justiniano et al. (2013). Alternatively, it can also reflect uncertainty around a central bank's policy goal. This is so because the now-clear 2% inflation target was first mentioned in 1996 in the conversation between the Chairman Greenspan and the Treasury Secretary Yellen. Before this conversation, central bank's inflation target was not clearly communicated.

When economic agents are not certain about a central bank's inflation target, even if it is small disturbance, this introduces non-trivial uncertainty about monetary policy, which is conceptually distinct from traditional monetary policy shocks to short-term interest rate. As for the calibration of the stochastic process (41), I follow the calibration of Cogley et al. (2010), where authors assume that inflation target follows near-unit root process.

In this calibration, traditional monetary policy shock ( $\epsilon_t^R$ ) and inflation target shock ( $\epsilon_t^{\pi}$ ) is not isomorphic due to their difference in persistence. For example, compare a positive innovation to monetary policy shock ( $\epsilon_t^R > 0$ ) and a negative innovation to inflation target shock ( $\epsilon_t^{\pi} < 0$ ). At a first glance, both shocks are contractionary, hence it might be intuitive that both shocks increase short-term interest rate. For traditional monetary policy shock, this is true: nominal short-term interest rate increases after a positive innovation to  $\epsilon_t^R$ . But for inflation target shock, the opposite is true: nominal short-term interest rate *decreases* after a negative innovation to  $\epsilon_t^{\pi}$ . In other words, inflation and nominal short-term interest rate moves in the same direction after inflation target shock.<sup>33</sup>

#### 4.5 Asset prices

In the model, I assume that long-term bonds are Woodford-type perpetual bonds with a declining coupon payment:  $1, \kappa, \kappa^2, \ldots$ . Let  $P_t^B$  denote the price of a newly issued bond at period t. Let  $CI_t$  denote the amount of new perpetuities issued in period t. Then the total liability on past issues is given by  $B_t = CI_t + \kappa CI_{t-1} + \kappa^2 CI_{t-2} + \cdots$ , which can be rewritten as

$$CI_{t+1} = B_{t+1} - \kappa B_t \tag{42}$$

<sup>&</sup>lt;sup>33</sup>Caramp (2021) formally proves this in the context of wealth effect. They show that if the persistence of the innovation is sufficiently high, on impact response of nominal interest rate to a positive innovation to the policy rule is negative.

Because of the decaying coupon structure, bonds issued in period t - j will trade at  $\kappa^j P_t^B$ . Therefore, it is unnecessary to keep track of the prices of perviously issued bonds, because those prices can be a function of the current price. Also, the bond with decaying rate of  $\kappa$  has the duration of  $\frac{1}{1-\beta\kappa}$ , which will be used to calibrate the average duration of the Treasuries.

Euler equation for long-term bonds are given by

$$(1+\zeta_t)P_t^B = E_t m_{t,t+1} (1+\kappa P_{t+1}^B)(1+\pi_{t+1})^{-1},$$
(43)

where  $m_{t,t+1}$  is the SDF.<sup>34</sup> The holding period return on the long-term bond ( $R_t^B$ ) and the excess return on nominal long-term bonds are respectively

$$R_t^B = \frac{1 + \kappa P_t^B}{P_{t-1}^B} \tag{44}$$

$$xr_{t+1}^B = R_{t+1}^B - i_t$$
(45)

Next, the price of a consumption claim,  $p_t^E$  satisfies the following pricing equation:

$$p_t^E = E_t \Big[ m_{t,t+1} (C_{t+1} + p_{t+1}^E) \Big].$$
(46)

The excess return on consumption claim net of real risk-free rate  $(r_t^f)$ 

$$xr_{t+1}^E = \frac{div_{t+1} + p_{t+1}^E}{p_t^E} - r_t^f.$$
(47)

# 5 Bayesian inference

#### 5.1 Data and measurement equations

I use Bayesian technique to estimate the model parameters using three macroeconomic quarterly time series commonly used in the literature: the log difference of real GDP per capita, the log difference of the GDP deflator and the Federal Funds rate. Because the economy grows over time with a deterministic growth

$$m_{t,t+1} = \beta \frac{u_1(C_{t+1}, H_{t+1})}{u_1(C_t, H_t)}$$

and  $m_{t,t+s}$  is the SDF between period t and t + s:

 $m_{t,t+s} = m_{t,t+1} \times m_{t+1,t+2} \cdots \times m_{t+s-1,t+s}$ 

<sup>&</sup>lt;sup>34</sup>The stochastic discount factor  $m_{t,t+1}$  is given by

Parameters	Value	Description
Н	1/3	Steady-state labor supply
$\lambda_p$	0.11	Steady-state price markup
$\chi_p$	0.4	Price indexation
$\kappa$	0.96	Bond duration = $20$ quarters (5 yrs)
$\gamma$	0.47	Quarterly growth rate

Table 3: Calibrated parameters

rate of  $\gamma$ , I need to rewrite the equilibrium conditions in terms of deviations from the deterministic growth paths. Then I log-linearized the equilibrium conditions around deterministic steady state. To match the observables to the model defined series, I specify the measurement equations for three observables used in estimation, which include real GDP per capita, annualized quarterly inflation using GDP deflator, and the Federal Funds rate.

$$\Delta y_t^o = 100(\hat{y}_t - \hat{y}_{t-1}) \tag{48}$$

$$\pi_t^o = 400(\bar{\pi} - 1) + 400\hat{\pi}_t \tag{49}$$

$$R_t^o = 400(\bar{R} - 1) + 400\bar{R}_t \tag{50}$$

where  $\hat{y}$  is detrended output deviation from deterministic steady state. The data are quarterly US time series and the sample period runs from 1965Q1 to 2019Q4. I collected all data series from the St. Louis Fed website.

Several parameters are calibrated using external information. Time allocation to labor supply in steady state is assumed to be 1/3. Deterministic growth rate is calibrated to 0.47 percent per quarter (or 1.9 percent per year), which is calculated from the average growth rate of real GDP per capita during the sample period. Steady state markup is calibrated to 11 percent which is in line with the literature. This parameter is not identified separately from the Calvo price stickiness parameter ( $\xi$ ). Duration of nominal long-term bonds is set to 20 quarters (5 years) to match the average duration of the US Treasuries. Price indexation parameter  $\chi_p$  is calibrated to 0.4, which means 40 percent of firms adjust prices solely depending on the previous inflation. The calibration of 0.4 is the upper bound of empirical estimates of indexation parameter (Cogley and Sbordone, 2008; Ascari et al., 2011), the choice of this value does not change the results of the paper though.

#### 5.2 Prior distributions and posterior estimates

Prior distributions of parameters are taken from the literature (Justiniano et al., 2013). The prior distributions of shock persistence parameters are beta with mean 0.5 and standard deviations of 0.2. The priors on the standard deviations of the innovations to exogenous shocks are inverse gamma distributions with mean 0.1 and standard deviations of 1, which are quite dispersed.

The parameters governing preferences have the following priors: the discount rate is set at 0.4 with standard deviation of 0.1; the habit parameter is assumed to be around 0.5 with standard deviation of 0.2; the inverse Frisch elasticity is assumed to fluctuate around 2 with standard deviation of 0.5. The priors on the technology parameters are the following: the capital share of income is set to be around 0.25 with standard deviation of 0.05; the Calvo price stickiness is assumed to be around 0.5 with standard deviation of 0.2. All this priors are standard in the literature.

The monetary policy parameters, including the coefficients on Taylor rule and long-run inflation target, also have the priors that are in line with the literature: the reaction on inflation and the output gap are assumed to have normal distributions with mean 2.5 and 0.15, respectively and standard deviations of 1 and 0.05, respectively; the persistence of the Taylor rule is described by beta distribution with mean 0.7 and standard deviation of 0.1; the prior on long-run inflation target is set to be around 2.5 with standard deviation of 1. The prior on  $\pi$  is intentionally chosen to be dispersed around the average inflation.

To quantify the quantitative importance of time-varying inflation on macro-finance dynamics, I take twostep procedure to estimate the structural parameters. In the first step, I estimate the full set of parameters using the full sample spanning from 1965Q1 to 2019Q4. By exploiting six decades of macroeconomic time series, I obtain values for structural parameters including monetary policy and shock processes. In the second step, I fix all the parameters except the parameters regarding monetary policy, and re-estimate the model for the two subsample, before and after the structural break, but only allowing the monetary policy parameters to change. By estimating the subsamples restricting other parameters, I can flesh out the effects of change in monetary policy regime on inflation dynamics and bond-stock correlation. Also, I implicitly assume that exogenous shock processes have been stable over time, and focus on the policy regime change.

Table 4 provides the priors distributions and the posterior modes and standard deviations of structural parameters for the first stage estimation in panel (a), and those for the second stage estimation in panel (b). The data are informative about the structural parameters and the estimates are largely in consistent with those of previous studies. There are several points to discuss. The posterior distribution of the Calvo parameter,  $\theta$ , has the mode of 0.76, which is in line with the consensus value of the literature (Nakamura and Steinsson, 2008). The mode of 0.76 implies that firms re-optimize prices roughly every four quarter. As for

#### Table 4: Prior and posterior distributions of structural parameters

(a) Full sample estimation results (1965-201	9)
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		Prior		Post	erior
	Dist.	Mean	Stdev	Mode	Stdev
$100(\beta^{-1}-1)$	Gamma	0.400	0.1000	0.2747	0.0640
h	Beta	0.500	0.2000	0.4692	0.0548
ν	Gamma	2.000	0.5000	0.9428	0.2490
$\alpha$	Normal	0.250	0.0500	0.3292	0.0385
$\theta$	Beta	0.500	0.2000	0.7676	0.0189
$\phi_{\pi}$	Norm	3.000	1.0000	2.8999	0.3677
$\phi_y$	Norm	0.200	0.0500	0.2379	0.0401
$ ho_i$	Beta	0.700	0.1000	0.6952	0.0486
$\bar{\pi}$	Norm	3.000	1.0000	3.4905	0.5587
$ ho_A$	Beta	0.500	0.2000	0.9860	0.0084
$ ho_b$	Beta	0.500	0.2000	0.8163	0.0309
$ ho_{\lambda}$	Beta	0.500	0.2000	0.1732	0.0886
$ ho_{\pi}$	Beta	0.700	0.2000	0.9527	0.0226
$\sigma_A$	IG	0.100	5.0000	1.6619	0.2872
$\sigma_b$	IG	0.100	5.0000	0.6015	0.1234
$\sigma_{\lambda}$	IG	0.100	5.0000	0.0215	0.0039
$\sigma_{MP}$	IG	0.100	5.0000	0.1672	0.0125
$\sigma_{\pi}$	IG	0.100	5.0000	0.0567	0.0184

Notes: IG refers to inverse Gamma distribution.

#### (b) Subsample estimation results

	Prior			Period I: 1	965Q1:1987Q2	Period II: 1987Q3-2019Q4		
	Dist.	Mean	Stdev	Mode	Stdev	Mode	Stdev	
$\rho_R$	Beta	0.700	0.1000	0.5464	0.0484	0.8689	0.0212	
$\phi_{\pi}$	Gamma	2.500	1.0000	3.5999	0.3411	4.3316	0.6591	
$\phi_y$	Gamma	0.150	0.0500	0.2263	0.0317	0.2372	0.0454	
$\bar{\pi}$	Gamma	3.000	1.0000	4.1005	0.2629	1.7500	0.3675	

exogenous shock processes, TFP shock is estimated to be very persistent, which has persistence parameter of 0.986, and that of price markup shock is relatively low. The estimates of long-run inflation target is 3.8 percent per annum, which is approximately the sample average of inflation. Taylor rule parameters are also in line with the previous findings with non-zero long-run inflation target (Ascari et al., 2011).

I demonstrate the estimation results in panel (b) of table 4 for the subsample estimation, given the estimates of shocks and structural parameters. In the second step of the estimation, I keep the prior distributions same across subperiods. The estimated Taylor rule for the period I (before the structural break) is much less persistent than that for the period II (after the structural break). It can explain the volatility of the federal funds rate in the earlier period. Because the Fed was more actively changing the policy rate, shortterm interest rates became more volatile in this period. The reaction coefficient on inflation in period I is smaller than that in period II. This is consistent with previous studies that estimate New Keynesian model for before- and after- Volcker period (Clarida et al., 2000; Smets and Wouters, 2007), my structural break speaks to the appointment of the chairman Greenspan though. The coefficient on output gap is also estimated to be smaller in period I. The long-run inflation target,  $\bar{\pi}$ , shows the most noticeable change across the two periods. The estimates of long-run inflation target in period I is 4.4 percent per annum and that in period II is 1.76 percent per annum, which exhibits almost 3 percentage point difference. Those estimates are slightly lower than the average inflation in each period. The 3 percentage point difference in inflation target is economically very large and implies that there was a significant magnitude of inflation risks in period I.

There has been a long debate in the literature whether the responsiveness of monetary policy to inflation has changed over time. Clarida et al. (2000) estimate the responsiveness parameter  $\phi_{\pi}$  is smaller then 1 for the pre-Volcker (before 1983) but much higher afterwards.<sup>35</sup>In contrast, Sims and Zha (2006) find that the parameter has been stable over time, and instead, volatilities of shocks has decreased, which contributes to low inflation. The estimates in this paper that long-run inflation target adds an explanation on this debate. When the inflation risk channel operates, given the set of monetary policy reaction function, macroeconomic volatilities are increasing functions of long-run inflation target. In other words, the observed price instability during 1970s can be either from indeterminacy or the inflation risk channel, which is a mix of high long-run target and low responsiveness to inflation but still satisfies determinacy condition.

If one looks at just the monetary policy rule, it can be misleading because if long-run inflation target exhibits substantial change over time, the Great Moderation era is the consequence of the anchoring of inflation target around 2 percent. <sup>36</sup> Therefore, without taking into account the inflation risk channel, it is observationally equivalent that an economy is located in indeterminacy region before the structural break and in determinacy region after the break, or the volatilities of exogenous shocks have changed significantly

<sup>&</sup>lt;sup>35</sup>This, "policy mistake" view stresses that US monetary policy was less responsive to inflation in the 1960s and 1970s. For detailed discussion, see Clarida et al. (2000), Lubik and Schorfheide (2004), Coibion and Gorodnichenko (2011) and Boivin and Giannoni (2006). Relatedly, Ascari and Ropele (2007) study the implication of non-zero inflation target on macroeconomic stability. They find that once inflation target is above 4%, determinate equilibrium is very unlikely, based on standard calibration of New Keynesian model. Benati and Goodhart (2010) also point out that high average inflation during 1960s and 1970s could the economy to remain in indeterminate region. In contrast, the "bad luck" view explanation argue that the instability during the Great Inflation can be attributed to the change in the volatility of the exogenous shocks. For further detail, see Sims and Zha (2006), Smets and Wouters (2007), Justiniano and Primiceri (2008).

<sup>&</sup>lt;sup>36</sup>Cogley et al. (2010) find that  $\sigma_{\pi}$  has decreased substantially after the appointment of Volcker, which contributed to smaller volatility of inflation. But the model the authors estimate does not feature the inflation risk channel as it assumes full price indexation. This enforces the model to favor higher volatility of inflation target shock for the period before Volcker, which is captured by higher  $\bar{\pi}$  in my case.

over time.

The estimated shift in  $\bar{\pi}$  has important implications for the dynamics of macroeconomic aggregates and asset prices. As discussed in the earlier section, when long-run inflation target is high, the inflation risk channel predicts countercyclical inflation and a positive bond-stock correlation. I will discuss the implication of high and low inflation target for inflation cyclicality and nominal bond risks, and to what extent time-varying inflation target explains the sign-switching patters of inflation and bond return cyclicality.

### 6 Implications of the inflation target shocks

#### 6.1 The inflation target shocks and macroeconomic dynamics

Table 5 reports the contribution of each shock to the level of observables at business cycle frequency, calculated at the posterior mode for each period. The importance of inflation target shock (denoted by  $\pi^*$ ) is substantial for inflation for both of the periods. The variance of inflation accounted for by inflation target shock is almost 50% in period I and 30 percent in period II.<sup>37</sup> As discussed in proposition 2, the response of inflation to inflation target shock is increasing in long-run target. When long-run target falls significantly, it also drags down the quantitative importance of inflation target shocks on inflation. The contribution of markup shocks ( $\lambda_p$ ) is also significant and similar across the periods. It accounts for one-third of inflation in both periods, which is economically large but much smaller than what Smets and Wouters (2007) estimates. One reason for the difference is that in the model with the inflation risk channel, the way how inflation target shocks work is observationally equivalent to that of markup shocks. Thus, the data speaks more strongly in favor of inflation target shocks, rather than markup shocks. Remaining variance of inflation is mostly explained by risk premium shocks and the contribution of temporary monetary policy shocks is limited.

The decomposition of output variance exhibits more substantial change over time. In period I, TFP shocks account for 30 percent of output variance, while inflation target shocks are estimated to explain around 70 percent of output variance. This result is surprising given my estimation scheme that keeps the volatilities of shocks same across periods. It turns out that even if the volatilities do not change, the difference in monetary policy parameters including long-run inflation target is enough to generates substantial difference in macroeconomic dynamics. In particular, the magnitude of the effect of time-varying inflation target on output reflects the extent to which the inflation risk channel is at work. The data favors the inflation risk channel, which implies that macroeconomic volatility the US economy experienced in the period I can be attributed to inflation target set too high by the central bank. In contrast, in period II, the importance

<sup>&</sup>lt;sup>37</sup>This is comparable to the estimates of Uribe (2022), where the author estimates permanent and transitory change in inflation targets. The author reports that according to his estimates, 45 percent of the change in inflation is explained by inflation target shock.

	Period I: 1965Q1-1987Q2					Period II: 1987Q3-2019Q4				
	TFP	RP	$\lambda_p$	MP	$\pi^*$	TFP	RP	$\lambda_p$	MP	$\pi^*$
Output	34.21	1.82	0.15	0.14	63.69	59.86	34.44	1.99	3.59	0.12
Inflation	0.44	15.27	26.36	0.03	57.90	0.27	24.01	32.80	4.15	38.77
Interest rate	1.24	73.03	0.64	3.57	21.52	1.01	68.51	7.85	17.70	4.93

Table 5: Variance decomposition at business cycle frequency (percent)

Notes: Variance decompositions are computed at the posterior mode. Each column shows the contribution of each shock: TFP shock, Risk premium shock, price markup shock ( $\lambda_p$ ), Monetary policy shock, and inflation target shock. Output is the log real GDP per capita, inflation is the log difference of the GDP deflator, interest rate is the federal funds rate. Business cycle frequency corresponds to a periodic component with cycles of 6-32 quarters.

of inflation target shocks on output dynamics decline dramatically, from 68.7 percent to 0.5 percent. In this period, TFP shocks account for 60 percent of output variance and risk premium shocks explain 33 percent of it. This sharp change is attributed to the change in long-run inflation target, from above 4 percent to below 2 percent. When long-run inflation decreases, the magnitude of the inflation risk channel also decreases substantially, mostly due to the weakening of the precautionary pricing effects. Intuitively speaking, when  $\bar{\pi}$  is low, precautionary pricing motive is small, hence the price distortion induced by the inflation risk channel is attenuated. This will be discussed in more detail in the later section by decomposing the impulse response to inflation target shocks.

Figure 6 presents the evolution of inflation, the model-implied inflation target and the contribution of inflation target shocks to inflation. The estimated inflation target is well above 4% and it hits 5.5% in 1975 when actual inflation peaks. After the peak, the target starts to gradually decrease over time. The contribution of the time-varying inflation target also peaks in 1975, and gradually decreases after the peak. But after 1982, actual inflation starts to fall sharply, the so-called Volcker disinflation. The model attributes the majority of the Volcker disinflation to the time-varying inflation target. During the mid-1980s, disinflation-ary force driven by the central bank is able to change the course of inflation, and it finally reaches below 2 percent in 1986, one year before the structural break. After the break, the quantitative importance of the time-varying inflation subdues and the wild swing of inflation in the US is 'conquered' (Cogley and Sargent, 2005). Inflation target shocks only account for moderate fluctuation in inflation during the recent period. The time-varying inflation target can also explain the change in inflation persistence (Cogley et al., 2010). In period I, inflation is primarily driven by the low frequency force, implying higher inflation persistence. In contrast, inflation is also driven by higher frequency forces in period II, which reduces the persistence of inflation.



Figure 6: Data and model-generated inflation and short-term interest rates

Notes: Inflation data is the annualized log difference in the GDP deflator. Inflation target is the model-estimated  $\pi^*$ .  $\pi^*$  only series refers to the model generated series that only accounted for by  $\pi^*$  shock. The gray shaded area indicates the recession date identified by NBER.

Table 6: Data and model-implied macro-finance dynamics

	Per	riod I	Period II		
	Data	ta Model Data M		Model	
$Corr(xr^b, xr^s)$	0.40	0.22	-0.37	-0.44	
$Corr(\tilde{y},\pi)$	-0.21	-0.91	0.54	0.16	

Notes: Bond-stock correlation in data is calculated using quarterly returns on 5-year zero coupon Treasuries and S&P 500 returns including dividends, excess of 3-month T-bill rates. Output gap (x) data is collected from the Congressional Budget Office. Inflation is measured by the log-difference in the GDP deflator. Period I is from 1965Q1 to 1987Q2 and Period II is from 1987Q3 to 2019Q4. Model-implied bond-stock correlation is the simulated correlation calculated at posterior modes using 50,000 simulated data.

### 6.2 The inflation risk channel and bond-stock correlation

Table 6 summarizes data and model-implied bond-stock correlation and output gap-inflation correlation. The estimated model successfully matches empirical bond-stock correlations, even though any asset price data is used in estimation. This can be interpreted as an evidence that bond-stock correlation is driven by macroeconomic factor, including monetary policy regime. Bond-stock correlations calculated at posterior mode for period I is 0.22 in model versus 0.40 in data. The positive correlation is primarily accounted for



**Figure 7:** Unconditional bond-stock correlations: calculated at posterior mode in period I (left) and period II (right)

Notes: Model-implied bond-stock correlation is the theoretical correlation calculated at posterior modes. Red dots shows the posterior modes at period I and period II.

by the inflation risk channel; inflation target shocks are quantitatively important for output, and drive both asset prices toward the same direction. At the same time, inflation is countercyclical, meaning a negative output gap-inflation correlation. In period I, when a positive inflation target shock hits, inflation arises and households' marginal utility increases as output falls. But this period is when both nominal long-term bonds and stock prices are lower, hence those assets cannot hedge consumption risks.

In contrast, in period II, inflation features procyclical pattern and it comoves with output gap. Thus, nominal bond prices are higher exactly when households' marginal utility is higher. In this case, long-term bonds provide hedge property, and bond-stock correlation becomes negative. In the estimated model, when  $\bar{\pi}$  is low, the inflation risk channel weakens, and asset prices are driven by other shocks, mostly risk premium shocks. In summary, the characteristic of inflation target shock heavily depends on monetary policy regime because it governs the extent which the inflation risk channel affects inflation and output dynamics, hence asset pricing dynamics. When a central bank sets low inflation target with active monetary policy rule, the inflation risk channel is attenuated and inflation is procyclical. This provides hedging property to nominal long-term bonds, which leads to a negative bond-stock correlation. If long-run inflation target is set to a higher value, then the quantitative significance of the inflation risk channel is substantial, driving countercyclical inflation. When inflation moves to the opposite way to output, nominal long-term bonds move together with risky assets, hence a positive bond-stock correlation.

Figure 7 illustrates the model-implied bond-stock correlation for each pair of  $\phi_{\pi}$  and  $\bar{\pi}$ .<sup>38</sup> From period I to period II, the inflation coefficient in Taylor rule increases and long-run inflation target decreases. Both of these two policy changes contribute to lower bond-stock correlation. When  $\phi_{\pi}$  increases, as shown in the previous section, the response of inflation is attenuated and inflation risks abate. Change in  $\bar{\pi}$  has similar but stronger effect. As  $\bar{\pi}$  gets higher, the transmission via the inflation risk channel is reenforced because it amplifies both the precautionary pricing effect and the staggered price effect. The contour map shows that this force is increasing in the level of  $\bar{\pi}$ , which the narrowing gap between two contours verifies. When  $\bar{\pi}$  is sufficiently low, in the figure below 2 percent, neither change in  $\phi_{\pi}$  nor  $\bar{\pi}$  seems to have effect on the correlation. This is because once  $\bar{\pi}$  sets to be sufficiently low, inflation risks are not severe, hence it does not have primary effect on asset pricing dynamics.

#### 6.3 Impulse responses

Figure 8 shows the impulse responses of output gap, inflation, nominal short-term interest rate, bond and stock returns as well as SDF following a one standard deviation shock to inflation target. All IRFs are the responses following one standard deviation shock to  $\pi^*$ . In panel (a), I depict the IRFs in period II and zero inflation. First consider the IRFs in period II, where  $\bar{\pi}$  is low and  $\phi_{\pi}$  is high. The response of output is positive on impact period (t = 1), but it decreases subsequently and persistently. Inflation peaks on impact, and then moves back to steady state very slowly. In contrast, nominal short-term interest rate peaks around 12 quarters after the shock arrives. All of output gap, inflation and interest rate responses are very persistent. As output increases on impact, stock returns also rise, but quickly come back to steady state. Bond returns, however, decreases by 3 percent on impact. Higher inflation target causes persistently higher inflation and nominal interest rate, long-term nominal bonds prices decline. The SDF, or pricing kernel, decreases on impact due to lower marginal utility. The innovation to  $\pi^*$  increases output, hence consumption, which lowers marginal utility of households. A positive inflation target shock is expansionary in period II, but its expansionary effect is limited, and in the medium-run, it can cause lower output.

Note that zero inflation steady state is identical to positive inflation but with full indexation. Therefore, I can measure the size of inflation risks and the resulting inefficiency by comparing the IRFs with  $\bar{\pi} = 0$ and  $\bar{\pi} > 0$ . When long-run inflation is set to zero,<sup>39</sup> the expansionary effect of positive inflation is stronger. Output gap response is much larger and converges to steady state from above. Due to larger impact on output, the responses of stock returns and the SDF are larger. Comparing the IRFs in period II and with zero

<sup>&</sup>lt;sup>38</sup>Contour plots for model-implied output gap-inflation are shown in Appendix B.

<sup>&</sup>lt;sup>39</sup>Full indexation is often used in the literature to capture internal persistence of inflation and analytical tractability (Christiano et al., 2005).

(a) Impulse responses in period II and zero inflation



Notes: This figure is based on the impulse response of each variable computed at the posterior mode for the parameter estimates in period II.

(b) Impulse responses in period I



Figure 8: Impulse responses to one standard deviation inflation target shock.

Notes: This figure is based on the impulse response of each variable computed at the posterior mode for the parameter estimates in period I.



Figure 9: Decomposition of the inflation risk channel (Period I)

Notes: This figure is based on the impulse response of each variable computed at the posterior mode for the parameter estimates in period I.

inflation, the magnitude of the inflation risk channel is not zero but still weak so that it does not dominate the expansionary effect.

Next, in panel (b), the IRFs in period I is present. Output gap decreases on impact and the absolute size of the output response is around 14 times bigger than that in period II. Inflation and short-term nominal interest rate increase very persistently and strongly, either. As a result, both asset prices fall significantly, and so does SDF. These exhibit stark contrast to the IRFs under low inflation. In period I, long-run inflation target is high and it amplifies inefficiency due to price distortion through the inflation risk channel. Under high inflation target, firms expect future possibility of non-adjusting. Therefore, when it is possible, firms charge much higher prices than the prices that it would have charged under zero inflation. This precautionary pricing effect introduces considerable distortion in cross-sectional price distribution, which dominates the expansionary effect of inflation shock.

Note that the response of inflation in period I is different from that in period II, where inflation response peaks on impact. This is because with higher long-run inflation target, the staggered price effect generates delayed responses in inflation. Those firms that do not have chance to adjust need to wait until they are allowed to adjust. If it gets the chance, its price 'jumps' a lot to keep the optimal price level given persistent path of higher inflation. In the course of this adjustment process, when there are still large number of firms who have not had adjustment, their prices are stuck in the previous prices, which puts downward pressure on inflation in earlier period. Once certain amount of time passes since the initial shock, this staggered firms start to adjust and it puts upward pressure on inflation in the later period after the shock. As a result of stagflationary effect of inflation shock, stock and bond returns fall sharply on impact. The magnitude of the change in asset prices is worth mentioning. In period I, one standard deviation change in target generates only 0.7 percent change in stock return, and 3 percent change in bond returns, both from steady state. The same magnitude of the shock induces more than 10 percent drop in stock returns and 9 percent fall in bond returns, both of which are economically large.

Notably, the sharp contrast in output and stock return responses can be attributed to the magnitude of inflation risks. To better understand the mechanics of the inflation risk channel, figure 9 decomposes the impulses responses of macro and financial variables into the responses due to the precautionary pricing effect and the staggered price effect. As discussed above, the precautionary pricing effect is due to the possibility of non-adjusting. When a persistent inflationary shock hits, the precautionary pricing motive even more increases, and adjusting firms charge higher prices, taking into account a persistent high inflation in the future and the possible non-adjustment. In the figure, this puts inflationary pressure and provides substantial inefficiency due to price distortion, contributing to lower output and stock returns.

The staggered price effect, however, gradually increases and it takes a while to have peak effects on macro aggregates. The staggered price effect is due to the delayed reaction by non-adjusting firms. When an unexpected inflation takes place, the prices charged by those firms are inefficiently low, hence generating nominal distortion. However, the staggered prices are inefficiently low so it puts downward pressure on inflation at first. As time goes, more firms adjust prices to keep their price close to the optimal prices hence the staggered price effect puts less downward pressure. In the later periods, this effect starts to overturn and gives more persistence in inflation and price distortion.<sup>40</sup> In each period, a fraction of firms do not adjust, even after sufficient number of firms have adjusted their prices Firms that already adjusted may want to re-optimize their prices downwardly because the initial shock starts to fade away, but due to the staggered price effect, only a fraction of firms are allowed to change. Therefore, in the later period of the impulse responses, the staggered effect contributes to the increase in inflation and short-term interest rate, while it further decreases output

<sup>&</sup>lt;sup>40</sup>This provides one solution to deal with the lack of internal persistence of inflation in NK models. In the literature, price indexation is often used to fit the empirical persistence of inflation (Gali and Gertler, 1999; Christiano et al., 2005). But this non-optimizing pricing behavior is ad-hoc and not well supported by empirical evidence on price setting behavior of firms (Bils and Klenow, 2004; Nakamura and Steinsson, 2008)



#### Figure 10: Data and counterfactual bond-stock correlation

Notes: Bond-stock correlation in data is calculated using quarterly returns on 5-year zero coupon Treasuries and S&P 500 returns including dividends, excess of 3-month T-bill rates. Period I is from 1965Q1 to 1987Q2 and Period II is from 1987Q3 to 2019Q4. Model-implied bond-stock correlation is the simulated correlation calculated at posterior modes using 50,000 simulated data. Each counterfactual correlation is calculated same way but with the parameter value in the other period.

gap. Consequently, inflation persistence is increasingly higher and its peak response appears much later. Anticipating this, asset prices move more abruptly and show sharp decline and a positive comovement.

### 6.4 Counterfactual analysis

I further explore how much of correlation change be attributed to each policy parameter by calculating counterfactual correlations. Figure 10 compares counterfactual correlations to data and baseline correlation calculated at each posterior mode. The left panel shows correlations in period I, and the right panel is for period II. In each panel, the far left bars present data moment, and the second left bars stands for the baseline prediction of the model at each posterior mode. From the third left to the last, I change each parameter to the estimate for the other period. For example, the third left bar in period I is a counterfactual correlation for period I if  $\bar{\pi}$  were 1.76, the posterior mode in period II, instead of 4.39, the posterior mode in period I. As an exception, I change both  $\bar{\pi}$  and  $\phi_{\pi}$  for the far right bars in the figure to investigate the asset pricing implications of those two parameters of interest.

In both periods, if  $\bar{\pi}$  is set counterfactually, model-implied correlations switch the sign. This shows the primary factor that determines the size of inflation risks is long-run inflation target. When it is low, then even if the Taylor rule is less responsive to inflation, nominal bond risks are muted and it keeps its hedging property in financial markets.

The inflation coefficient in Taylor rule has also considerable implications for asset pricing dynamics. In period I, if the coefficient were counterfactually high, then the model-implied bond-stock correlation would be around zero, a significant fall from 0.22 in the baseline correlation. If the coefficient is high, then the volatility of inflation dwindles and it hinders the inflation risk channel. On the other hand, in period II, the counterfactual change in the inflation coefficient has almost nil. Long-run target is already low, hence the extent which asset prices are explained by inflation risks is limited. As far as  $\bar{\pi}$  is low enough, inflation volatility and the size of inflation risk are small, hence the counterfactual change does not have any noticeable asset pricing implication.

Other parameters in Taylor rule also affect asset market dynamics. If output gap coefficient is counterfactually higher in period I, the correlation increases. The focus of monetary policy shifts toward output, then inflation remains less tightly controlled and it contributes to higher inflation risks. The counterfactual correlations react to the change in persistence parameter asymmetrically. In period I, counterfactual persistence is much higher, and the correlation decreases. However in period II, decrease in persistence would increase the correlation.

## 7 Conclusion

Macroeconomic effects of time-varying inflation targets has only gained attention recently. Given the fact that decision on change in target inflation is based on a persistent and a longer-run perspective, it is not surprising that those change has large and persistent effect on macroeconomic and financial variables, such as long and short interest rates and inflation. In this paper, I explore a New Keynesian asset pricing model to study the interaction of monetary policy, time-varying inflation target, and nominal bond risks.

Using a simple NK model, I show that there exists a rich interaction between monetary policy and the cyclicality of inflation, which determines bond market risks. It is important to note that monetary policy not only includes the interest rate rule (eg. the Taylor rule), but also long-run and short-run inflation target. It turns out that with higher inflation target, the inflation risk channel emerges due to the staggered price effect and the precautionary price effect, which are based on forward-looking pricing behavior with positive long-run inflation and Calvo pricing friction. The inflation risks play an important role that amplifies output and inflation responses to exogenous shocks including inflation target shocks.

When long-run inflation target is low, a positive shock to inflation target generates procyclical inflation, which drives a negative comovement between bond and stock returns. In contrast, if long-run inflation target is higher, for instance 4% as the estimates for the Great Inflation, a positive target shock ends up with sizable output fall, countercyclical inflation and a positive bond-stock correlation. The main mechanism

is the inflation risk channel, which strengthens as long-run inflation target increases. The counterfactual analysis shows that if either long-run inflation targets were low or inflation targets were constant over time, bond market risks were substantially lower than what is observed, and bond-stock correlation would have been negative. The results in this paper indicate that inflation targets have powerful effects on the dynamics of macro aggregates and financial variables.

My results suggest an important policy implications for the countries that suffer from financial market instability. According to my analytical results, high and volatile inflation targets can distort the economic decision of households and firms so that inflation can change the safety of long-term bonds. When public starts to expect inflation, their reaction to inflation deviates from the previous one due to inflation risks. If a central bank fails to convince the public that long-run inflation target is stable, then the consequence of expectation deanchoring would be detrimental, and have serious spillover effects to financial markets. If government-issued bonds are considered as risky assets, fiscal space of a government would be tight and the capacity to stabilize the economy is limited. Therefore, to regain fiscal space and the credibility of government bonds, it is important to stabilize long-run inflation.

### References

- Amano, R., Ambler, S., and Rebei, N. (2007). The macroeconomic effects of nonzero trend inflation. *Journal of Money, Credit and Banking*, 39(7):1821–1838.
- Andrews, D. W. (1993). Tests for parameter instability and structural change with unknown change point. *Econometrica: Journal of the Econometric Society*, pages 821–856.
- Andrews, D. W. (2003). Tests for parameter instability and structural change with unknown change point: A corrigendum. *Econometrica*, pages 395–397.
- Ascari, G., Castelnuovo, E., and Rossi, L. (2011). Calvo vs. rotemberg in a trend inflation world: An empirical investigation. *Journal of Economic Dynamics and Control*, 35(11):1852–1867.
- Ascari, G., Phaneuf, L., and Sims, E. R. (2018). On the welfare and cyclical implications of moderate trend inflation. *Journal of Monetary Economics*, 99:56–71.
- Ascari, G. and Ropele, T. (2007). Optimal monetary policy under low trend inflation. *Journal of monetary Economics*, 54(8):2568–2583.
- Ascari, G. and Ropele, T. (2009). Trend inflation, taylor principle, and indeterminacy. *Journal of Money, Credit and Banking*, 41(8):1557–1584.
- Ascari, G. and Sbordone, A. M. (2014). The macroeconomics of trend inflation. *Journal of Economic Literature*, 52(3):679–739.
- Auclert, A., Rigato, R. D., Rognlie, M., and Straub, L. (2022). New pricing models, same old phillips curves? Technical report, National Bureau of Economic Research.
- Baele, L., Bekaert, G., and Inghelbrecht, K. (2010). The determinants of stock and bond return comovements. *The Review of Financial Studies*, 23(6):2374–2428.
- Bauer, M. D. and Rudebusch, G. D. (2020). Interest rates under falling stars. *American Economic Review*, 110(5):1316–54.
- Benati, L. and Goodhart, C. (2010). Monetary policy regimes and economic performance: the historical record, 1979–2008. In *Handbook of monetary economics*, volume 3, pages 1159–1236. Elsevier.
- Bianchi, F. and Ilut, C. (2017). Monetary/fiscal policy mix and agents' beliefs. *Review of economic Dynamics*, 26:113–139.

- Bianchi, F., Lettau, M., and Ludvigson, S. C. (2016). Monetary policy and asset valuation. Technical report, National Bureau of Economic Research.
- Bianchi, F. and Melosi, L. (2019). The dire effects of the lack of monetary and fiscal coordination. *Journal of Monetary Economics*, 104:1–22.
- Bils, M. and Klenow, P. J. (2004). Some evidence on the importance of sticky prices. *Journal of political economy*, 112(5):947–985.
- Blanchard, O. J. and Kahn, C. M. (1980). The solution of linear difference models under rational expectations. *Econometrica*, 48(5):1305–1311.
- Boivin, J. and Giannoni, M. P. (2006). Has monetary policy become more effective? *The Review of Economics and Statistics*, 88(3):445–462.
- Bordo, M. D. and Schwartz, A. J. (1999). Monetary policy regimes and economic performance: the historical record. *Handbook of macroeconomics*, 1:149–234.
- Bullard, J. and Mitra, K. (2002). Learning about monetary policy rules. *Journal of monetary economics*, 49(6):1105–1129.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3):383–398.
- Campbell, J. Y. and Ammer, J. (1993). What moves the stock and bond markets? a variance decomposition for long-term asset returns. *The journal of finance*, 48(1):3–37.
- Campbell, J. Y., Pflueger, C., and Viceira, L. M. (2020). Macroeconomic drivers of bond and equity risks. *Journal of Political Economy*, 128(8):3148–3185.
- Campbell, J. Y., Sunderam, A., and Viceira, L. M. (2009). Inflation bets or deflation hedges? the changing risks of nominal bonds. Technical report, National Bureau of Economic Research.
- Caramp, N. (2021). Fiscal policy and the monetary transmission mechanism.
- Caramp, N. and Silva, D. H. (2021). Monetary policy and wealth effects: The role of risk and heterogeneity. Technical report, Working Paper.
- Chernov, M., Lochstoer, L. A., and Song, D. (2021a). The real channel for nominal bond-stock puzzles. Working Paper 29085, National Bureau of Economic Research.

- Chernov, M., Lochstoer, L. A., and Song, D. (2021b). The real explanation of nominal bond-stock puzzles. Technical report, National Bureau of Economic Research.
- Christiano, L. J., Eichenbaum, M., and Evans, C. L. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113(1):1–45.
- Christiano, L. J., Eichenbaum, M. S., and Trabandt, M. (2015). Understanding the great recession. *American Economic Journal: Macroeconomics*, 7(1):110–67.
- Cieslak, A. and Povala, P. (2015). Expected returns in treasury bonds. *The Review of Financial Studies*, 28(10):2859–2901.
- Clarida, R., Gali, J., and Gertler, M. (2000). Monetary policy rules and macroeconomic stability: evidence and some theory. *The Quarterly journal of economics*, 115(1):147–180.
- Cogley, T., Primiceri, G. E., and Sargent, T. J. (2010). Inflation-gap persistence in the us. *American Economic Journal: Macroeconomics*, 2(1):43–69.
- Cogley, T. and Sargent, T. J. (2005). The conquest of us inflation: Learning and robustness to model uncertainty. *Review of Economic dynamics*, 8(2):528–563.
- Cogley, T. and Sbordone, A. M. (2008). Trend inflation, indexation, and inflation persistence in the new keynesian phillips curve. *American Economic Review*, 98(5):2101–26.
- Coibion, O. and Gorodnichenko, Y. (2011). Monetary policy, trend inflation, and the great moderation: An alternative interpretation. *American Economic Review*, 101(1):341–70.
- Coibion, O., Gorodnichenko, Y., Kueng, L., and Silvia, J. (2017). Innocent bystanders? monetary policy and inequality. *Journal of Monetary Economics*, 88:70–89.
- Coibion, O., Gorodnichenko, Y., and Wieland, J. (2012). The optimal inflation rate in new keynesian models: should central banks raise their inflation targets in light of the zero lower bound? *Review of Economic Studies*, 79(4):1371–1406.
- David, A. and Veronesi, P. (2013). What ties return volatilities to price valuations and fundamentals? *Journal of Political Economy*, 121(4):682–746.
- Davis, J., Fuenzalida, C., and Taylor, A. M. (2019). The natural rate puzzle: Global macro trends and the market-implied r. Technical report, National Bureau of Economic Research.
- Diercks, A. M. (2015). The equity premium, long-run risk, & optimal monetary policy.

- Epstein, L. G. and Zin, S. E. (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. *Econometrica*, 57(4):937–969.
- Fisher, J. D. (2015). On the structural interpretation of the smets–wouters "risk premium" shock. *Journal of Money, Credit and Banking*, 47(2-3):511–516.
- Galí, J. (2015). Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications. Princeton University Press.
- Gali, J. and Gertler, M. (1999). Inflation dynamics: A structural econometric analysis. *Journal of monetary Economics*, 44(2):195–222.
- Gertler, M. and Karadi, P. (2015). Monetary policy surprises, credit costs, and economic activity. *American Economic Journal: Macroeconomics*, 7(1):44–76.
- Gertler, M. and Leahy, J. (2008). A phillips curve with an ss foundation. *Journal of political Economy*, 116(3):533–572.
- Golosov, M. and Lucas, R. E. (2007). Menu costs and phillips curves. *Journal of Political Economy*, 115(2):171–199.
- Gourio, F. and Ngo, P. (2020). Risk premia at the zlb: A macroeconomic interpretation. Technical report, Federal Reserve Bank of Chicago.
- Gürkaynak, R. S., Sack, B., and Wright, J. H. (2007). The us treasury yield curve: 1961 to the present. *Journal of monetary Economics*, 54(8):2291–2304.
- Hanson, S. G. and Stein, J. C. (2015). Monetary policy and long-term real rates. *Journal of Financial Economics*, 115(3):429–448.
- Ireland, P. N. (2007). Changes in the federal reserve's inflation target: Causes and consequences. *Journal of Money, credit and Banking*, 39(8):1851–1882.
- Justiniano, A. and Primiceri, G. E. (2008). The time-varying volatility of macroeconomic fluctuations. *American Economic Review*, 98(3):604–41.
- Justiniano, A., Primiceri, G. E., and Tambalotti, A. (2013). Is there a trade-off between inflation and output stabilization? *American Economic Journal: Macroeconomics*, 5(2):1–31.
- Klenow, P. J. and Kryvtsov, O. (2008). State-dependent or time-dependent pricing: Does it matter for recent us inflation? *The Quarterly Journal of Economics*, 123(3):863–904.

- Krishnamurthy, A. and Vissing-Jorgensen, A. (2012). The aggregate demand for treasury debt. *Journal of Political Economy*, 120(2):233–267.
- Kung, H. (2015). Macroeconomic linkages between monetary policy and the term structure of interest rates. *Journal of Financial Economics*, 115(1):42–57.
- LaSalle, J. (1986). The stability and control of discrete processes. Applied mathematical sciences, 62.
- Leeper, E. M. (1991). Equilibria under 'active' and 'passive' monetary and fiscal policies. *Journal of monetary Economics*, 27(1):129–147.
- Levin, A. and Taylor, J. B. (2013). Falling behind the curve: a positive analysis of stop-start monetary policies and the great inflation. In *The Great Inflation: The Rebirth of Modern Central Banking*, pages 217–244. University of Chicago Press.
- Li, E. X., Zha, T., Zhang, J., and Zhou, H. (2022). Does fiscal policy matter for stock-bond return correlation? *Journal of Monetary Economics*.
- Lubik, T. A. and Schorfheide, F. (2004). Testing for indeterminacy: An application to us monetary policy. *American Economic Review*, 94(1):190–217.
- Nakamura, E. and Steinsson, J. (2008). Five facts about prices: A reevaluation of menu cost models. *The Quarterly Journal of Economics*, 123(4):1415–1464.
- Nakamura, E. and Steinsson, J. (2010). Monetary non-neutrality in a multisector menu cost model. *The Quarterly journal of economics*, 125(3):961–1013.
- Nakamura, E., Steinsson, J., Sun, P., and Villar, D. (2018). The elusive costs of inflation: Price dispersion during the us great inflation. *The Quarterly Journal of Economics*, 133(4):1933–1980.
- Pflueger, C. E. (2022). What do treasury bond risks say about supply and demand shocks? *University of Chicago, Becker Friedman Institute for Economics Working Paper*, (102).
- Powell, J. H. et al. (2020). Opening remarks: New economic challenges and the fed's monetary policy review. In *Proceedings-Economic Policy Symposium-Jackson Hole*, pages 1–18. Federal Reserve Bank of Kansas City.
- Primiceri, G. E. (2006). Why inflation rose and fell: policy-makers' beliefs and us postwar stabilization policy. *The Quarterly Journal of Economics*, 121(3):867–901.

- Rudebusch, G. D. and Swanson, E. T. (2012). The bond premium in a dsge model with long-run real and nominal risks. *American Economic Journal: Macroeconomics*, 4(1):105–43.
- Sims, C. A. and Zha, T. (2006). Were there regime switches in u.s. monetary policy? *American Economic Review*, 96(1):54–81.
- Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian DSGE approach. *American Economic Review*, 97(3):586–606.
- Song, D. (2017). Bond market exposures to macroeconomic and monetary policy risks. *The Review of Financial Studies*, 30(8):2761–2817.
- Uhlig, H. (2005). What are the effects of monetary policy on output? results from an agnostic identification procedure. *Journal of Monetary Economics*, 52(2):381–419.
- Uribe, M. (2022). The neo-fisher effect: Econometric evidence from empirical and optimizing models. *American Economic Journal: Macroeconomics*, 14(3):133–62.
- Vissing-Jorgensen, A. (2021). The treasury market in spring 2020 and the response of the federal reserve. *Journal of Monetary Economics*, 124:19–47.
- Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press.

# A Proof

Proof of Proposition 1.

$$x_t = BE_t x_{t+1} + \varepsilon_t, \tag{51}$$

where  $x_t = [\tilde{y}_t, \hat{\pi}_t, \hat{\psi}_t]'$  and

$$B = \begin{bmatrix} \sigma & 1 - \phi_{\pi}\bar{\beta} & -\phi_{\pi}(\bar{\Pi} - 1)\bar{\gamma} \\ \bar{\kappa} & \bar{\kappa} + \bar{\beta}(\sigma + \phi_{y}) & (\sigma + \phi_{y})(\bar{\Pi} - 1)\bar{\gamma} \\ (1 - \xi\beta\bar{\Pi}^{\epsilon})\sigma & (\sigma + \phi_{\pi}\bar{\kappa} + \phi_{y})\epsilon\xi\beta\bar{\Pi}^{\epsilon} + (1 - \bar{\beta})(1 - \xi\beta\bar{\Pi}^{\epsilon}) & (\sigma + \phi_{\pi}\bar{\kappa} + \phi_{y})\xi\beta\bar{\Pi}^{\epsilon} + (1 - \bar{\beta}\phi_{\pi})(1 - \xi\beta\bar{\Pi}^{\epsilon}) \end{bmatrix}$$

The characteristic polynomial of the system is

$$p(\lambda) = \lambda^3 - tr(B)\lambda^2 + \frac{1}{2}(tr(B)^2 - tr(B^2))\lambda + det(B).$$
(52)

Following LaSalle (1986) (p.28), a sufficient and necessary condition for the polynomial to have all of its roots inside the unit circle is

$$|a_0 + a_2| < 1 + a_1 \tag{53}$$

$$|a_1 - a_0 a_2| < 1 - a_0^2, \tag{54}$$

where  $a_2, a_1, a_0$  are the coefficients on  $\lambda^2, \lambda$  and constant. The result follows.

*Proof of Lemma 1.* First, guess each endogenous variable as a linear function of  $\hat{\pi}_t^*$ 

$$[\hat{\pi}_t, \tilde{y}_t, \hat{i}_t, \hat{\psi}_t]' = [\Gamma_\pi, \Gamma_y, \Gamma_i, \Gamma_\psi]' \hat{\pi}_t^*$$
(55)

Then, from the IS equation (2),

$$y_t = y_{t+1} - \phi_\pi(\pi_t - \pi_t^*) + E_t \pi_{t+1}$$
(56)

$$\Gamma_{y}\pi_{t}^{*} = \Gamma_{y}\rho_{\pi}\pi_{t}^{*} - \phi_{\pi}(\Gamma_{\pi} - 1)\pi_{t}^{*} + \Gamma_{\pi}\rho_{\pi}\pi_{t}^{*}$$
(57)

$$\Gamma_y(1-\rho_\pi) = \Gamma_\pi(\rho_\pi - \phi_\pi) + \phi_\pi \tag{58}$$

$$\therefore \quad \Gamma_y = \frac{\phi_\pi - \Gamma_\pi(\phi_\pi - \rho_\pi)}{1 - \rho_\pi} \tag{59}$$

Next, from (7),

$$\psi_t = (1 - \xi \beta \bar{\Pi}^\epsilon) y_t + \epsilon \beta \xi \bar{\Pi}^\epsilon E_t \pi_{t+1} + \xi \beta \bar{\Pi}^\epsilon E_t \psi_{t+1}$$
(60)

$$= X_t + \xi \beta \bar{\Pi}^{\epsilon} E_t X_{t+1}, \qquad \text{where } X_t = (1 - \xi \beta \bar{\Pi}^{\epsilon}) y_t + \epsilon \beta \xi \bar{\Pi}^{\epsilon} E_t \pi_{t+1}$$
(61)

$$=\frac{1}{1-\rho_{\pi}\xi\beta\bar{\Pi}^{\epsilon}}X_{t}$$
(62)

$$=\frac{1}{1-\rho_{\pi}\xi\beta\bar{\Pi}^{\epsilon}}\Big[(1-\xi\beta\bar{\Pi}^{\epsilon})\Gamma_{y}+\epsilon\beta\xi\bar{\Pi}^{\epsilon}\rho_{\pi}\Gamma_{\pi}\Big]\pi_{t}^{*}$$
(63)

Now, from (5),

$$\pi_t = \bar{\beta}\rho_\pi\Gamma_\pi\pi_t^* + \bar{\kappa}\Gamma_y\pi_t^* + (\bar{\pi}-1)\bar{\gamma}\frac{\rho_\pi}{1-\rho_\pi\xi\beta\bar{\Pi}^\epsilon}\Big[(1-\xi\beta\bar{\Pi}^\epsilon)\Gamma_y + \epsilon\beta\xi\bar{\Pi}^\epsilon\rho_\pi\Gamma_\pi\Big]\pi_t^*$$
(64)

$$\Gamma_{\pi} = \bar{\beta}\rho_{\pi}\Gamma_{\pi} + \bar{\kappa}\Gamma_{y} + \frac{\rho_{\pi}(\bar{\pi}-1)\bar{\gamma}}{1-\rho_{\pi}\xi\beta\bar{\Pi}^{\epsilon}} \Big[ (1-\xi\beta\bar{\Pi}^{\epsilon})\Gamma_{y} + \epsilon\beta\xi\bar{\Pi}^{\epsilon}\rho_{\pi}\Gamma_{\pi} \Big]$$
(65)

Substituting out  $\Gamma_y$  using (63),

$$\Gamma_{\pi}(\bar{\Pi}) = \frac{\frac{\bar{\kappa}\phi_{\pi}}{1-\rho_{\pi}} + \frac{\bar{\gamma}\rho_{\pi}\phi_{\pi}(\bar{\Pi}-1)(1-\xi\beta\bar{\Pi}^{\epsilon})}{(1-\rho_{\pi}\xi\beta\bar{\Pi}^{\epsilon})(1-\rho_{\pi})}}{1-\bar{\beta}\rho_{\pi} + \frac{\bar{\kappa}(\phi_{\pi}-\rho_{\pi})}{1-\rho_{\pi}} + \frac{\bar{\gamma}\rho_{\pi}(\bar{\Pi}-1)}{1-\rho_{\pi}\xi\beta\bar{\Pi}^{\epsilon}}\left(\frac{(1-\xi\beta\bar{\Pi}^{\epsilon})(\phi_{\pi}-\rho_{\pi})}{1-\rho_{\pi}} - \epsilon\xi\beta\bar{\Pi}^{\epsilon}\rho_{\pi}\right)}.$$
(66)

Then, using (63), (65) and (3)

$$\Gamma_y(\bar{\Pi}) = \frac{\phi_\pi}{1 - \rho_\pi} - \frac{\phi_\pi - \rho_\pi}{1 - \rho_\pi} \Gamma_\pi$$
(67)

$$\Gamma_i(\bar{\Pi}) = \phi_\pi(\Gamma_\pi - 1) + \phi_y \Gamma_y \tag{68}$$

$$\Gamma_{\psi}(\bar{\Pi}) = \frac{(1 - \xi\beta\bar{\Pi}^{\epsilon})\Gamma_{y} + \epsilon\xi\beta\bar{\Pi}^{\epsilon}\rho_{\pi}\Gamma_{\pi}}{1 - \rho_{\pi}\xi\beta\bar{\Pi}^{\epsilon}}$$
(69)

*Proof of Proposition 2.* From (66),  $\frac{d\Gamma_{\pi}}{d\bar{\pi}} > 0$  if and only if  $\frac{df}{d\bar{\pi}} > 0$  where

$$f(\bar{\pi}) = \frac{(\bar{\pi} - 1)(1 - \xi\beta\bar{\pi}^{\epsilon})}{1 - \rho_{\pi}\xi\beta\bar{\pi}^{\epsilon}}$$
(70)

Then,

$$\frac{df}{d\bar{\pi}} = \frac{1}{(1 - \rho_{\pi}\xi\beta\bar{\Pi}^{\epsilon})} \tag{71}$$

$$\frac{df}{d\bar{\pi}} > 0 \tag{72}$$

$$\Leftrightarrow (1 - \xi \beta \bar{\Pi}^{\epsilon}) (1 - \rho_{\pi} \xi \beta \bar{\Pi}^{\epsilon}) > (1 - \rho_{\pi}) (\bar{\Pi} - 1) \epsilon \xi \beta \bar{\Pi}^{\epsilon - 1}$$
(73)

$$\Leftrightarrow -(1-\xi\beta\bar{\Pi}^{\epsilon})\xi\beta\bar{\Pi}^{\epsilon}\rho_{\pi} + (1-\xi\beta\bar{\Pi}^{\epsilon}) > (1-\rho_{\pi})(\bar{\Pi}-1)\epsilon\xi\beta\bar{\Pi}^{\epsilon-1}$$
(74)

$$\Leftrightarrow \rho_{\pi} > \frac{\xi \beta \bar{\Pi}^{\epsilon-1} ((\bar{\Pi} - 1)\epsilon + \bar{\Pi}) - 1}{\xi \beta \bar{\Pi}^{\epsilon-1} ((\bar{\Pi} - 1)\epsilon - (1 - \xi \beta \bar{\Pi}^{\epsilon}) \bar{\Pi})} \equiv \rho_{\pi}^{*}$$

$$(75)$$

Therefore, if  $\rho_{\pi} > \rho_{\pi}^*$ , then  $\frac{d\Gamma_{\pi}}{d\pi} > 0$ .

For the result 5,

$$\frac{d\Gamma_{\pi}}{d\phi_{\pi}} < 0$$

$$\Leftrightarrow \frac{1}{\phi_{\pi}} \Big( \frac{\bar{\kappa}\phi_{\pi}}{1 - \rho_{\pi}} + \frac{\bar{\gamma}\rho_{\pi}\phi_{\pi}(\bar{\Pi} - 1)(1 - \xi\beta\bar{\Pi}^{\epsilon})}{(1 - \rho_{\pi}\xi\beta\bar{\Pi}^{\epsilon})(1 - \rho_{\pi})} \Big) \Big( 1 - \bar{\beta}\rho_{\pi} + \frac{\bar{\kappa}(\phi_{\pi} - \rho_{\pi})}{1 - \rho_{\pi}} + \frac{\bar{\gamma}\rho_{\pi}(\bar{\Pi} - 1)}{1 - \rho_{\pi}\xi\beta\bar{\Pi}^{\epsilon}} \Big( \frac{(1 - \xi\beta\bar{\Pi}^{\epsilon})(\phi_{\pi} - \rho_{\pi})}{1 - \rho_{\pi}} - \epsilon\xi\beta\bar{\Pi}^{\epsilon}\rho_{\pi} \Big) \Big)$$

$$(76)$$

$$(76)$$

$$(77)$$

$$-\left(\frac{\bar{\kappa}\phi_{\pi}}{1-\rho_{\pi}}+\frac{\bar{\gamma}\rho_{\pi}\phi_{\pi}(\bar{\Pi}-1)(1-\xi\beta\bar{\Pi}^{\epsilon})}{(1-\rho_{\pi}\xi\beta\bar{\Pi}^{\epsilon})(1-\rho_{\pi})}\right)\left(\frac{\bar{\kappa}}{1-\rho_{\pi}}+\frac{\bar{\gamma}\rho_{\pi}(\bar{\Pi}-1)}{1-\rho_{\pi}\xi\beta\bar{\Pi}^{\epsilon}}\frac{(1-\xi\beta\bar{\Pi}^{\epsilon})}{1-\rho_{\pi}}\right)<0$$
(78)

Therefore,  $\frac{d\Gamma_{\pi}}{d\phi_{\pi}} < 0$ .

Next,

$$\Gamma_{\pi}|_{\bar{\Pi}=1} = \frac{\frac{\bar{\kappa}\phi_{\pi}}{1-\rho_{\pi}}}{1-\bar{\beta}\rho_{\pi} + \frac{\bar{\kappa}(\phi_{\pi}-\rho_{\pi})}{1-\rho_{\pi}}} > 1$$
(79)

$$\Leftrightarrow \frac{\bar{\kappa}\phi_{\pi}}{1-\rho_{\pi}} > 1 - \bar{\beta}\rho_{\pi} + \frac{\bar{\kappa}(\phi_{\pi} - \rho_{\pi})}{(1-\rho_{\pi})}$$
(80)

$$\Leftrightarrow \bar{\kappa}(\phi_{\pi} - \rho_{\pi}) > (1 - \bar{\beta}\rho_{\pi})(1 - \rho_{\pi})$$
(81)

$$0 > \rho_{\pi}^2 - (1 + \bar{\beta} + \bar{\kappa})\rho_{\pi} + 1$$
(82)

$$\rho_{\pi} > \frac{1 + \bar{\beta} + \bar{\kappa} - \sqrt{(1 + \bar{\beta} + \bar{\kappa})^2 - 4\bar{\beta}}}{2\bar{\beta}} \equiv \bar{\rho_{\pi}}$$
(83)

Therefore, if  $\rho_{\pi} > \max\{\bar{\rho}_{\pi}, \rho_{\pi}^*\}$ , then  $\Gamma_{\pi} > 1$ . From (68),  $\Gamma_i > 0$  under the assumption that  $\phi_y \ge 0$ .

From the above result, when  $\rho_{\pi} > \rho_{\pi}^*$ ,

$$\frac{d\Gamma_y}{d\bar{\Pi}} = -\frac{\phi_\pi - \rho_\pi}{1 - \rho_\pi} \frac{d\Gamma_\pi}{d\bar{\Pi}} < 0$$
(84)

Furthermore, if  $\Gamma_{\pi} > \frac{\phi_{\pi}}{\phi_{\pi} - \rho_{\pi}}$ , then  $\Gamma_{y} < 0$ .

Lastly,

$$Cov(\hat{y}_t, \hat{\pi}_t) = \Gamma_y(\bar{\Pi})\Gamma_\pi(\bar{\Pi}) \frac{\sigma_\pi^2}{1 - \rho_\pi^2}$$
(85)

is immediate from the lemma.

Proof of Proposition 3. Excess return on consumption claim is

$$\hat{x}\hat{r}_{t+1}^s = \hat{r}_{t+1}^s - \hat{r}_t = \Delta\hat{y}_{t+1} + E_t\hat{m}_{t+1} = y_{t+1} - E_t y_{t+1}$$
(86)

and excess return on two period nominal bonds is

$$\hat{xr}_{t+1}^b = \hat{r}_{t+1}^b - \hat{i}_t = E_{\Delta(t+1)}\hat{m}_{t+2} - E_{\Delta(t+1)}\hat{\pi}_{t+2} - \hat{\pi}_{t+1},$$
(87)

The SDF is give by

$$\hat{m}_{t+1} = -\Delta y_{t+1} \tag{88}$$

Then, using the lemma,

$$Cov(xr_{t+1}^s, xr_{t+1}^b) = \Gamma_y(\bar{\Pi})(\Gamma_y(\bar{\Pi})(1-\rho_\pi) - \Gamma_\pi(\bar{\Pi})(1+\rho_\pi))\frac{\sigma_\pi^2}{1-\rho_\pi^2}.$$
(89)

## **B** Additional results



Figure 11: Variance decomposition (percent): Period I

Notes: Variance decompositions are computed at the posterior mode. Each column shows the contribution of each shock: TFP shock, Risk premium shock, price markup shock ( $\lambda_p$ ), Monetary policy shock, and inflation target shock. Output is the log real GDP per capita, inflation is the log difference of the GDP deflator, interest rate is the federal funds rate. Business cycle frequency corresponds to a periodic component with cycles of 6-32 quarters.



Figure 12: Variance decomposition (percent): Period II

Notes: Variance decompositions are computed at the posterior mode. Each column shows the contribution of each shock: TFP shock, Risk premium shock, price markup shock ( $\lambda_p$ ), Monetary policy shock, and inflation target shock. Output is the log real GDP per capita, inflation is the log difference of the GDP deflator, interest rate is the federal funds rate. Business cycle frequency corresponds to a periodic component with cycles of 6-32 quarters.



Figure 13: Bond-stock correlation for different maturity

Notes: 3-year rolling correlations and betas of bond and stock excess returns. Bond return  $(xr^b)$  is measured by onequarter holding return of 2/5/10/20/30-year Treasury bonds excess of 3-month T-bills. Stock return  $(xr^s)$  is measured by one-quarter return from the value-weighted S&P 500 including dividends excess of 3-month T-bills. The shaded area indicates the recession date identified by NBER.



**Figure 14:** Unconditional output gap-inflation correlations: calculated at posterior mode in period I (left) and period II (right)

Notes: Model-implied output gap-inflation correlation is the theoretical correlation calculated at posterior modes. Red dots shows the posterior modes at period I and period II.