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Modified Fragmentation Function from Quark Recombination

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Within the framework of the constituent quark model, it is shown that the single hadron fragmentation function of a parton can be expressed as a convolution of shower diquark or triquark distribution function and quark recombination probability, if the interference between amplitudes of quark recombination with different momenta is neglected. The recombination probability is determined by the hadron's wavefunction in the constituent quark model. The shower diquark or triquark distribution functions of a fragmenting jet are defined in terms of overlapping matrices of constituent quarks and parton field operators. They are similar in form to dihadron or trihadron fragmentation functions in terms of parton operator and hadron states. Extending the formalism to the field theory at finite temperature, we automatically derive contributions to the effective single hadron fragmentation function from the recombination of shower and thermal constituent quarks. Such contributions involve single or diquark distribution functions which in turn can be related to diquark or triquark distribution functions via sum rules. We also derive QCD evolution equations for quark distribution functions that in turn determine the evolution of the effective jet fragmentation functions in a thermal medium.

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I. INTRODUCTION

In the study of the properties of dense matter and search for quark gluon plasma in high-energy heavy-ion collisions, jet quenching [1] or the suppression of leading high p_T hadrons [2] has become a powerful diagnostic tool. The theoretical concept relies on the study of parton propagation in medium and induced radiative energy loss [3–6]. Using perturbative QCD (pQCD) and taking into account the intricate Landau-Pomeranchuk-Migdal (LPM) [7] interference effect, one can calculate the radiative energy loss. It has a unique dependence on the path length traversed by the parton and the local gluon density of the medium. Such energy loss is manifested in the modification of the effective parton fragmentation functions [8] which in turn leads to the suppression of the single inclusive hadron spectra. In non-central heavy-ion collisions, the path length dependence of the energy loss gives rise to the azimuthal angle dependence of the high p_T single hadron spectra [9]. Therefore, experimental measurements of the suppression of single hadron spectra or the modification of the fragmentation functions and their centrality dependence can provide important information about the initial gluon density and geometry of the produced dense matter. Moreover, one can go beyond single hadron spectra and study the modification of multiple hadron correlations inside jets due to multiple scattering of the parton in the medium [10].

Data from experiments at the Relativistic Heavy-ion Collider (RHIC) have indeed confirmed the predicted features of jet quenching [11]. One has seen not only a significant suppression of single inclusive high p_T hadron spectra [12, 13] and strong azimuthal angle dependence [14], but also the suppression of high p_T hadrons on the opposite side of a triggered high p_T hadron in the central

$Au + Au$ collisions [15]. These are all consistent with the qualitative features of jet quenching due to parton energy loss [16]. The extracted initial gluon density in the most central $Au + Au$ collisions at $\sqrt{s} = 200$ GeV is found to be about 30 times higher than that in a cold Au nuclei [16–18]. Combined with the enormous collective expansion as measured by the elliptic flow of the final bulk hadrons [19], current experimental data points to the formation of a strongly interactive quark-gluon plasma in central $Au + Au$ collisions at RHIC.

During the propagation and interaction inside a deconfined hot partonic medium, a fast parton can have not only induced gluon radiation but also induced absorption of the surrounding thermal gluons. The detailed balance leads to an energy dependence of the net energy loss that is stronger than without for an intermediate energy parton [20]. In principle, one can consider gluon absorption as a parton recombination process and it continues until the hadronization of the bulk partonic matter. Eventually, during the hadronization, partons from the jet can combine with those from the medium to form the final hadrons. Indeed, there already exists some evidence for such parton recombination in the measured hadron spectra in heavy-ion collisions at RHIC. At intermediate $p_T = 2 - 4$ GeV/ c , the suppression of baryons is significantly smaller than mesons, leading to a baryon to meson ratio larger than 1 [21]. This is about a factor of 5 increase over the value in $p + p$ collisions. On the other hand, the azimuthal anisotropy of the baryon spectra is larger than that of meson spectra. Such a flavor dependence of the nuclear modification of the hadron spectra and their azimuthal anisotropy is not consistent with a picture of pure parton energy loss followed by vacuum fragmentation. The most striking empirical observation of the flavor dependence, that could reveal the under-

lying hadronization mechanism, is the scaling behavior between the azimuthal anisotropy of baryon and meson spectra [22], $v_2^M(p_T/2)/2 = v_2^B(p_T/3)/3$. Such an observation is inspired by a schematic model of hadron production by constituent quark recombination [23]. Here, $v_2^M(v_2^B)$ is the second coefficient of the Fourier transformation of the azimuthal angle distribution of mesons (baryons).

Many quark recombination models [24–26] are successful in describing the observed flavor dependence of the nuclear modification of hadron spectra at intermediate p_T . These models in general involve thermal quarks in the medium and employ the constituent quark model for hadron wavefunctions which determine the recombination probabilities. They, however, differ in the handling of specific recombination processes. Some considered only recombination of thermal quarks, with inherent correlations caused by jet quenching. Others also include recombination of thermal and shower quarks from the fragmenting jet, which dominate the hadron spectra at intermediate p_T . They also differ in the determination of the constituent quark distributions from high p_T jets and there exist ambiguities in the connection between partons from pQCD hard processes and constituent quarks that form the final hadrons.

One of the models that we will follow closely in this paper is by Hwa and Yang [24]. In this model, quark recombination processes are traced back to parton fragmentation processes in vacuum. They assume that the initial produced hard partons will evolve into a shower of constituent quarks which then recombine to form the final hadrons in the hadronization process. The formulation of the recombination of the shower quarks of the partonic jets and the medium quarks in heavy-ion collisions is straightforward, given both the shower and medium quark distributions. Since the Hwa-Yang model is a phenomenological one, the nuclear modification of the shower quark distributions and their QCD evolution cannot be calculated. The model has to rely on fitting to the experimentally measured hadron spectra to obtain the corresponding nuclear modified shower quark distributions for each centrality of heavy-ion collisions and correlations between shower quarks are completely neglected.

In this paper, we make a first attempt to derive the recombination model of jet fragmentation functions from a field theoretical formulation and the constituent quark model of hadron structure. Within the constituent quark model, we consider the parton fragmentation as a two stage process. The initial parton first evolves into a shower of constituent quarks that subsequently will combine with each other to form the final hadrons. Since constituent quarks are non-perturbative objects in QCD just like hadrons, the conversion of hard partons into showers of constituent quarks is not calculable in pQCD. However, we can define constituent quark distributions in a jet as overlapping matrices of the parton field operator and the constituent quark states, similarly as one

has defined hadron fragmentation functions. Ignoring interferences in the process of quark recombination, we demonstrate that the single inclusive meson (baryon) fragmentation functions can be cast as a convolution of the diquark (triquark) distribution functions and the recombination probabilities, which are determined by the hadrons' wavefunctions. This is similar in spirit to the Hwa-Yang recombination model. Given a form of the hadrons' wavefunctions in the constituent quark model, one can in principle extract constituent quark distribution functions from measured jet fragmentation functions. We are also able to derive the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) [27–29] evolution equations for the diquark and triquark distribution functions, which in turn give rise to the DGLAP evolution equations for the hadron fragmentation functions within the quark recombination formalism. Such a reformulation of the jet fragmentation functions does nothing to simplify the description of jet hadronization. However, extending the formalism to the case of jet fragmentation in a thermal medium and working within field theory at finite temperature, we can automatically derive contributions from recombination between shower and thermal quarks in addition to soft hadron production from recombination of thermal quarks and leading hadrons from recombination of shower quarks. The shower and thermal quark recombination involves single (diquark) quark distribution functions which can be obtained from diquark (triquark) distributions through sum rules. Therefore, one can consistently describe three different processes within this formalism. One can also consistently take into account parton energy loss and detailed balance effect for jet fragmentation inside a thermal medium.

For convenience in this paper, we will simply refer to the constituent quarks as quarks and the initial current quarks and gluons as partons. The quarks from jet fragmentation will be denoted as shower quarks in contrast to quarks in the thermal medium in heavy-ion collisions. The remainder of the paper is organized as follows: In Section II, we review single inclusive hadron fragmentation functions in terms of overlapping matrix elements between parton field operators and hadronic states. In Section III and IV, we introduce the hadronic wavefunctions in the constituent quark model and reformulate the parton fragmentation functions in terms of quark recombination probabilities and shower quark distribution functions of the fragmenting partons, which are defined as the overlapping matrices of the parton field operators and constituent quark states. We also derive the QCD evolution equations for the shower quark distribution functions and discuss possible sum rules relating triquark, diquark and single quark distribution functions. In Section V, we extend the formalism to parton fragmentation in a thermal medium within the framework of field theory at finite temperature and derive various contributions to the effective fragmentation functions from shower-thermal and thermal-thermal quark recombination. We summarize the results and discuss future work

in Section VI.

II. SINGLE HADRON FRAGMENTATION FUNCTION

For hadron production processes, in e^+e^- annihilation for example, that involve a large momentum scale, the inclusive cross section at leading twist can be factorized into a hard part for parton scattering at short distances and a soft part for hadronization at long distances as the parton fragmentation function. Though one can systematically calculate the hard part in pQCD due to asymptotic freedom, the parton fragmentation function is nonperturbative and currently can only be measured in experiments. However, one can derive the DGLAP evolution equations with the momentum scale, which have been tested successfully against the experimental data [30]. In this section, we review the parton fragmentation functions as defined in the form of matrix elements of parton operators and the corresponding DGLAP evolution equations.

As in a previous study of medium modification of fragmentation functions with detailed balance [31], we consider e^+e^- annihilation to illustrate the parton operator definition of the quark fragmentation functions. Within the collinear factorization approximation, the inclusive differential cross section can be expressed as

$$\frac{d\sigma_{e^+e^- \rightarrow h}}{dz_h} = \frac{1}{2s} \frac{e^4}{q^4} L_{\mu\nu}(p_a, p_b) \frac{dW^{\mu\nu}}{dz_h} = \sum_q \sigma_0^{q\bar{q}} [D_q^h(z_h) + D_{\bar{q}}^h(z_h)] , \quad (1)$$

where $q = p_a + p_b$ is the four-momentum of virtual photon and $s = q^2 \equiv Q^2$ is the invariant mass of e^+e^- pair. The leptonic and hadronic tensors are given by $L_{\mu\nu}(p_a, p_b) = (1/4)\text{Tr}(\gamma_\mu \not{p}_a \gamma_\nu \not{p}_b)$ and

$$W^{\mu\nu}(q) = \sum_X \langle 0 | J^\mu(0) | X \rangle \langle X | J^\nu(0) | 0 \rangle (2\pi)^4 \delta^4(p_X - q) = \int d^4x e^{-iq \cdot x} \langle 0 | J^\mu(0) J^\nu(x) | 0 \rangle , \quad (2)$$

where $\sum_X$ runs over all possible intermediate states and the quark electromagnetic current is $J_\mu = \sum_q e_q \bar{\psi}_q \gamma_\mu \psi_q$. Here, e_q is the fractional charge of the quark in units of an electron charge. The total cross-section at the lowest order in pQCD is $\sigma_0^{q\bar{q}} \equiv N_c 4\pi\alpha^2 e_q^2 / 3s$, where $N_c = 3$ is the number of colors in the fundamental representation of SU(3) and α is the electromagnetic coupling constant. The single inclusive fragmentation functions of a current quark and antiquark are defined as [32–35],

$$D_q^h(z_h) = \frac{z_h^3}{2} \int \frac{d^4p}{(2\pi)^4} \delta\left(z_h - \frac{p_h \cdot n}{p \cdot n}\right) \int d^4x e^{-ip \cdot x} \text{Tr} \left[\frac{\gamma \cdot n}{2p_h \cdot n} \sum_S \langle 0 | \psi(0) | S, p_h \rangle \langle p_h, S | \bar{\psi}(x) | 0 \rangle \right] , \quad (3)$$

$$D_{\bar{q}}^h(z_h) = \frac{z_h^3}{2} \int \frac{d^4p}{(2\pi)^4} \delta\left(z_h - \frac{p_h \cdot n}{p \cdot n}\right) \int d^4x e^{-ip \cdot x} \text{Tr} \left[\sum_S \langle 0 | \bar{\psi}(0) | S, p_h \rangle \frac{\gamma \cdot n}{2p_h \cdot n} \langle p_h, S | \psi(x) | 0 \rangle \right] , \quad (4)$$

respectively, where p_h is the four-momentum of the identified hadron and z_h is its momentum fraction with respect to the initial parton momentum. Sum over the color index of the quark field is implicitly implied. The light-like vector is defined as $n^\mu = [n^+, n^-, n_\perp] = [0, 1, 0_\perp]$. These fragmentation functions $D_{q(\bar{q})}^h(z_h)$ can be interpreted as the multiplicity distribution of hadrons with fractional momenta between z_h and $z_h + dz_h$ produced in the fragmentation of the initial parton quark q or antiquark $\bar{q}$.

In deriving Eqs. (1), (3) and (4) from Eq. (2), the intermediate states $\sum_X |X\rangle \langle X|$ have been replaced by two complete subsets $\sum_{S, S'} |S, p_h; S'\rangle \langle S'; S, p_h|$, corresponding to independent fragmentation of the quark and anti-quark created from the e^+e^- annihilation. One can further replace $|S'\rangle$ by a complete set consisting of quarks and gluons. To the lowest order of pQCD, only a single quark or anti-quark state $|p'\rangle$ of $|S'\rangle$ contributes to the hadronic tensor in Eq. (2). Thus, one can effectively express,

$$\sum_X |X\rangle \langle X| = \sum_S \int \frac{d^3p_h}{(2\pi)^3 2E_h} \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} |S, p_h; p'\rangle \langle p'; p_h, S| , \quad (5)$$

and the factorized form of the differential cross section in Eq. (1) can be represented by a cut diagram as shown in Fig. 1, in which the amplitudes of the fragmentation processes are represented by the blob connecting initial quark and final hadron states. This is normally presented as cut-vertices [31]. Since the hadronic states $|X\rangle$ are all color singlet, the above approximation implies that $|S\rangle$ also carries color which can only be neutralized via exchange of soft partons. These soft processes are higher twist and are suppressed by powers of $1/Q^2$. Soft gluon exchanges also lead

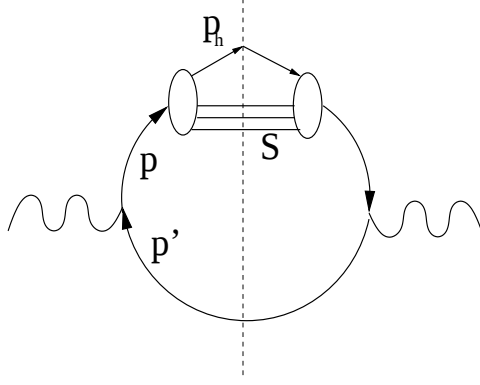


FIG. 1: diagram for single hadron fragmentation in e^+e^- annihilation.

to eikonal contributions or gauge links that ensure the gauge invariance of the defined quark fragmentation functions. They, however, do not appear in the light-cone gauge, $n \cdot A = 0$.

Similarly, the hadron fragmentation function $D_g^h(z_h)$ from a gluon parton is defined as

$$D_g^h(z_h) = \frac{z_h^2}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_h - \frac{p_h \cdot n}{p \cdot n}\right) \int d^4 x e^{-ip \cdot x} d_{\mu\nu}(p) \sum_S \langle 0 | A^\mu(0) | S, p_h \rangle \langle p_h, S | A^\nu(x) | 0 \rangle, \quad (6)$$

where gluonic color indices are also implicitly summed over. The gluon polarization tensor is defined in terms of the polarization vectors $\varepsilon^\mu(k)$ and has the form

$$\begin{aligned} d_{\mu\nu}(p) &= \sum_{\lambda=1,2} \varepsilon_\mu(p, \lambda) \varepsilon_\nu(p, \lambda) \\ &= -g_{\mu\nu} + \frac{p_\mu n_\nu + p_\nu n_\mu}{n \cdot p}, \end{aligned} \quad (7)$$

in the light-cone gauge.

The scale dependence of the single inclusive hadron fragmentation functions can be calculated by considering the radiative corrections to the fragmentation functions at the next-to-leading order in perturbation QCD [33, 34]. It is governed by the DGLAP evolution equations [27–29],

$$Q^2 \frac{d}{dQ^2} D_q^h(z_h, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_h}^1 \frac{dz}{z} \left[\gamma_{qq}(z) D_q^h\left(\frac{z_h}{z}, Q^2\right) + \gamma_{qg}(z) D_g^h\left(\frac{z_h}{z}, Q^2\right) \right], \quad (8)$$

$$Q^2 \frac{d}{dQ^2} D_g^h(z_h, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_h}^1 \frac{dz}{z} \left[\gamma_{gq}(z) D_q^h\left(\frac{z_h}{z}, Q^2\right) + \gamma_{gg}(z) D_g^h\left(\frac{z_h}{z}, Q^2\right) \right], \quad (9)$$

where $D_s^h(z_h/z, Q^2)$ is the singlet quark fragmentation function

$$D_s^h(z, Q^2) \equiv \sum_q [D_q^h(z, Q^2) + D_{\bar{q}}^h(z, Q^2)], \quad (10)$$

and the splitting functions are [36]

$$\gamma_{qq}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(1-z) \right], \quad (11)$$

$$\gamma_{qg}(z) = C_F \frac{1+(1-z)^2}{z}, \quad (12)$$

$$\gamma_{gq}(z) = T_F [z^2 + (1-z)^2], \quad (13)$$

$$\gamma_{gg}(z) = 2C_A \left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) \right] + \delta(1-z) \left[\frac{11}{6} C_A - \frac{2}{3} n_f T_F \right]. \quad (14)$$

Here, n_f is the number of quark flavors, the $SU(N_c)$ Casimirs are given by $C_F = (N_c^2 - 1)/2N_c$, $C_A = N_c$ and $T_F = 1/2$. The ‘+’-function is defined such that the replacement

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z) - f(1)}{1-z} \quad (15)$$

is valid for any function $f(z)$ that is continuous at $z = 1$.

The single hadron fragmentation functions as defined in Eqs. (3), (4) and (6) satisfy the following momentum sum rules,

$$\sum_h \int dz z D_a^h(z, Q^2) = 1, (a = q, \bar{q}, g). \quad (16)$$

One can also naively define the zeroth moments of the fragmentation functions,

$$\sum_h \int dz D_a^h(z, Q^2) = \bar{N}_q^h, \quad (17)$$

$$(18)$$

which are simply the average hadron multiplicities of the parton jets. In principle, the average hadron multiplicities are not infrared safe and therefore not well defined in the simple leading log and leading twist approximation. One has to go beyond these approximations and take into account the coherence of parton cascade. In practice, one can introduce a cut-off to regularize the infrared behavior.

III. QUARK RECOMBINATION AND PARTON FRAGMENTATION

A. The Constituent Quark Model

In hard processes involving a large momentum scale, the produced partons are normally off-shell and subject to radiative processes following the hard scattering. The DGLAP evolution equations describe these radiative processes for a highly virtual parton in pQCD. Eventually, however, when the parton’s virtuality becomes smaller than allowed for the applicability of pQCD, further interaction among produced or shower partons and the ensuing hadronization can never be described by pQCD. Therefore, one cannot calculate perturbatively the parton fragmentation functions with momentum scale $Q < Q_0$. Conventionally, one uses experimental data to parameterize the fragmentation functions with $Q \leq Q_0$ (called initial condition) and the DGLAP equations [Eqs. (8) and (9)] predict the evolution of the fragmentation functions with the momentum scale.

In this paper, we will employ a constituent quark model to describe the soft interaction and hadronization of shower partons. In this approach, soft interaction between shower partons below the scale Q_0 will be effectively represented by constituent quarks and their interaction. The non-perturbative conversion between shower partons and constituent quarks will be described by the constituent quark distribution functions in the fragmenting jet. Further interaction among constituent quarks during the hadronization will be given by the hadrons’ wave functions in the constituent quark model.

In this constituent quark model, a hadronic state with momentum $p_h = [p_h^+, 0, 0_\perp]$ can be expressed as

$$|p_h\rangle = \int [d^2 k_\perp] [dx] \varphi_h(k_{1\perp}, x_1; k_{2\perp}, x_2) |k_{1\perp}, x_1; k_{2\perp}, x_2\rangle, \quad (19)$$

for a meson and

$$|p_h\rangle = \int [d^2 k_\perp] [dx] \varphi_h(k_{1\perp}, x_1; k_{2\perp}, x_2; k_{3\perp}, x_3) |k_{1\perp}, x_1; k_{2\perp}, x_2; k_{3\perp}, x_3\rangle, \quad (20)$$

for a baryon. Here,

$$[d^2 k_\perp] \equiv 2(2\pi)^3 \delta^{(2)} \left(\sum_{i=1}^n k_{i\perp} \right) \prod_{i=1}^n \frac{d^2 k_{i\perp}}{2(2\pi)^3}, \quad (n = 2 \text{ for a meson and } 3 \text{ for a baryon}), \quad (21)$$

$$[dx] \equiv \delta \left(1 - \sum_{i=1}^n x_i \right) \prod_{i=1}^n \frac{dx_i}{\sqrt{x_i}}, \quad x_i = \frac{k_i^+}{p_h^+}, \quad (22)$$

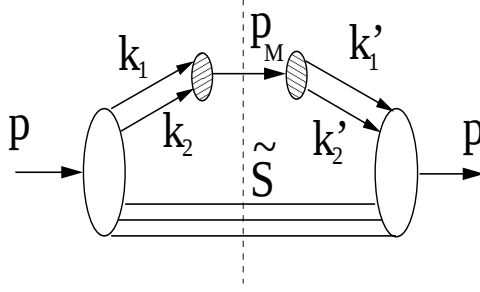


FIG. 2: The cut diagram for single meson fragmentation functions in terms of quark recombination. The blob connecting initial quark and final constituent quarks before recombination represents the amplitudes of shower-to-constituent quark conversion of the fragmenting quark. Hadronic wavefunction is presented by the shaded blob.

and φ_h is the hadronic wavefunction. In the infinite momentum frame, the constituent quarks can be considered approximately on shell. The normalization of the hadronic wavefunction is

$$\int [d^2 k_\perp] [dx] |\varphi_h(k_{i\perp}, x_i)|^2 = 1, \quad (23)$$

given the normalization of single constituent quark states,

$$\langle k_{i\perp}, x_i | k_{j\perp}, x_j \rangle = (2\pi)^3 2x_i \delta^{(2)}(k_{i\perp} - k_{j\perp}) \delta(x_i - x_j). \quad (24)$$

B. Double Constituent Quark Distribution Functions

Substituting the hadronic state in Eq.(19) in the constituent quark model into Eq. (3), we can express the meson fragmentation as

$$\begin{aligned} D_q^M(z_M) &= \frac{z_M^3}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_M - \frac{p_M^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} [d^2 k_\perp] [dx] [d^2 k'_\perp] [dx'] \\ &\quad \varphi_M(k_{1\perp}, x_1; -k_{1\perp}, 1 - x_1) \varphi_M^*(k'_{1\perp}, x'_1; -k'_{1\perp}, 1 - x'_1) \\ &\quad \text{Tr} \left[\frac{\gamma^+}{2p_M^+} \sum_S \langle 0 | \psi(0) | S; k_{1\perp}, x_1; -k_{1\perp}, 1 - x_1 \rangle \langle -k'_{1\perp}, 1 - x'_1; k'_{1\perp}, x'_1; S | \bar{\psi}(x) | 0 \rangle \right]. \end{aligned} \quad (25)$$

Since the constituent quarks can be regarded as a complete set of intermediate states $|\tilde{S}\rangle$, they are equivalent to and can replace the complete set of hadronic states $|S\rangle$ in the above expression, $\sum_S |S\rangle \langle S| = \sum_{\tilde{S}} |\tilde{S}\rangle \langle \tilde{S}|$. One can then interpret the above expression in terms of quark recombination as shown in Fig. 2: a quark and anti-quark with momentum $[x_1 p_M^+, k_{1\perp}]$ and $[(1 - x_1) p_M^+, -k_{1\perp}]$ from the fragmentation of the parton jet will combine to form a meson with momentum $p_M = [p_M^+, 0, 0_\perp]$. Since the fragmentation function $D_q^M(z_M)$ is proportional to the single inclusive cross section for hadron production, it should naturally include the interference between recombination of quark and anti-quark pairs with different momenta that sum to the meson's momentum. With the presence of the interference contribution, one can never arrive at a probabilistic interpretation as postulated in all the current recombination models [24–26]. However, if the hadronic wavefunction $\varphi_M(k_{1\perp}, x_1; k_{2\perp}, x_2)$ in the constituent quark model is sharply peaked, one may neglect the interference contributions. Effectively, one can complete the integration $\int [d^2 k'_\perp] [dx']$ in Eq. (25) and assume that the final result is proportional to the diagonal term with a coefficient C_M . One has then approximately,

$$\begin{aligned} D_q^M(z_M) &\approx C_M \frac{z_M^3}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_M - \frac{p_M^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \int \frac{d^2 k_{1\perp}}{(2\pi)^3} \frac{dx_1}{4x_1(1 - x_1)} |\varphi_M(k_{1\perp}, x_1; -k_{1\perp}, 1 - x_1)|^2 \\ &\quad \text{Tr} \left[\frac{\gamma^+}{2p_M^+} \sum_{\tilde{S}} \langle 0 | \psi(0) | \tilde{S}; k_{1\perp}, x_1; -k_{1\perp}, 1 - x_1 \rangle \langle -k_{1\perp}, 1 - x_1; k_{1\perp}, x_1; \tilde{S} | \bar{\psi}(x) | 0 \rangle \right], \end{aligned} \quad (26)$$

where C_M is a constant with dimensions of momentum that accounts for the non-diagonal (or interference) contributions. Note that

$$x_i = \frac{k_i^+}{p_M^+} = \frac{k_i^+}{p^+} \frac{p^+}{p_M^+} = \frac{z_i}{z_M}, \quad (27)$$

and z_i is the fractional momentum carried by the constituent quark. Defining the diquark recombination probability R_M as

$$R_M(k_{1\perp}, \frac{z_1}{z_M}) \equiv \left| \varphi_M(k_{1\perp}, \frac{z_1}{z_M}; -k_{1\perp}, 1 - \frac{z_1}{z_M}) \right|^2, \quad (28)$$

we can approximate the meson fragmentation function [Eq. (26)] as

$$D_q^M(z_M) \approx C_M \int_0^{z_M} \frac{dz_1}{2} R_M(0_\perp, \frac{z_1}{z_M}) F_q^{q_1 \bar{q}_2}(z_1, z_M - z_1), \quad (29)$$

where the double constituent quark (or diquark) distribution function in a quark jet is defined as

$$F_q^{q_1 \bar{q}_2}(z_1, z_2) = \frac{z_M^4}{2z_1 z_2} \int^\Lambda \frac{d^2 k_{1\perp}}{2(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_M - \frac{p_M^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \text{Tr} \left[\frac{\gamma^+}{2p_M^+} \sum_{\tilde{S}} \langle 0 | \psi(0) | \tilde{S}, k_1, k_2 \rangle \langle k_2, k_1, \tilde{S} | \bar{\psi}(x) | 0 \rangle \right]. \quad (30)$$

Here, $p_M = k_1 + k_2$, $z_M = z_1 + z_2$, and Λ is the cutoff for the intrinsic transverse momentum of the constituent quark inside a hadron, as provided by the hadron wavefunction. One can consider Λ as the scale for hadronization. One can also similarly express single inclusive hadron fragmentation functions for antiquarks and gluons in terms of the same diquark recombination probability $R_M(x_1, x_2)$ and double constituent quark distribution functions from antiquark and gluon jets, $F_{\bar{q}}^{q_1 \bar{q}_2}(z_1, z_2)$ and $F_g^{q_1 \bar{q}_2}(z_1, z_2)$, respectively, which are defined as

$$F_{\bar{q}}^{q_1 \bar{q}_2}(z_1, z_2) = \frac{z_M^4}{2z_1 z_2} \int^\Lambda \frac{d^2 k_{1\perp}}{2(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_M - \frac{p_M^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \text{Tr} \left[\sum_{\tilde{S}} \langle 0 | \bar{\psi}(0) | \tilde{S}, k_1, k_2 \rangle \frac{\gamma^+}{2p_M^+} \langle k_2, k_1, \tilde{S} | \psi(x) | 0 \rangle \right], \quad (31)$$

$$F_g^{q_1 \bar{q}_2}(z_1, z_2) = \frac{z_M^3}{2z_1 z_2} \int^\Lambda \frac{d^2 k_{1\perp}}{2(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_M - \frac{p_M^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} d_{\mu\nu}(p) \sum_{\tilde{S}} \langle 0 | A^\mu(0) | \tilde{S}, k_1, k_2 \rangle \langle k_2, k_1, \tilde{S} | A^\nu(x) | 0 \rangle. \quad (32)$$

As we will show in the next subsection, the above definitions of diquark distribution functions have almost the same form as dihadron quark fragmentation functions [35], except the cutoff Λ for the intrinsic transverse momentum which is similar to the cut-off $\mu_\perp$ for the intra-jet transverse momentum in dihadron fragmentation functions.

The quark recombination process in our constituent quark model happens at a scale $\mu \sim \Lambda$ during the evolution of the jet. Therefore, the expression for hadron fragmentation function, Eq. (29), in this model is valid for jets produced at any initial scale Q . Since the recombination happens at a fixed momentum scale μ , the recombination probability should be independent of the initial scale Q . The scale dependence of the hadron fragmentation functions $D_q^M(z_M, Q^2)$ will be completely determined by the scale dependence of the diquark distribution functions $F_q^{q_1 \bar{q}_2}(z_1, z_2, Q^2)$. Similar to the derivation of the evolution equations of the dihadron fragmentation function [35], one can also obtain the

radiative corrections to the double constituent quark distributions from parton fragmentation,

$$\begin{aligned}
F_q^{q_1 \bar{q}_2}(z_1, z_2, Q^2) &= F_q^{q_1 \bar{q}_2}(z_1, z_2, \mu^2) + \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_{z_1+z_2}^1 \frac{dz}{z^2} \gamma_{qq}(z) F_q^{q_1 \bar{q}_2}\left(\frac{z_1}{z}, \frac{z_2}{z}, \mu^2\right) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_{z_1+z_2}^1 \frac{dz}{z^2} \gamma_{qg}(z) F_g^{q_1 \bar{q}_2}\left(\frac{z_1}{z}, \frac{z_2}{z}, \mu^2\right) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{\Lambda^2} \frac{dl_{\perp}^2}{l_{\perp}^2} \sum_{i=1}^2 \int_{z_i}^{1-z_i} \frac{dz}{z(1-z)} \hat{\gamma}_{qq}(z) F_q^{q_i}\left(\frac{z_i}{z}, \mu^2\right) F_g^{\bar{q}_i}\left(\frac{z_i}{1-z}, \mu^2\right), \quad (33)
\end{aligned}$$

$$\begin{aligned}
F_g^{q_1 \bar{q}_2}(z_1, z_2, Q^2) &= F_g^{q_1 \bar{q}_2}(z_1, z_2, \mu^2) + \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_{z_1+z_2}^1 \frac{dz}{z^2} \gamma_{gq}(z) F_s^{q_1 \bar{q}_2}\left(\frac{z_1}{z}, \frac{z_2}{z}, \mu^2\right) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_{z_1+z_2}^1 \frac{dz}{z^2} \gamma_{gg}(z) F_g^{q_1 \bar{q}_2}\left(\frac{z_1}{z}, \frac{z_2}{z}, \mu^2\right) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{\Lambda^2} \frac{dl_{\perp}^2}{l_{\perp}^2} \sum_{i=1}^2 \sum_q \int_{z_i}^{1-z_i} \frac{dz}{z(1-z)} \gamma_{gq}(z) F_q^{q_i}\left(\frac{z_i}{z}, \mu^2\right) F_g^{\bar{q}_i}\left(\frac{z_i}{1-z}, \mu^2\right) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{\Lambda^2} \frac{dl_{\perp}^2}{l_{\perp}^2} \int_{z_1}^{1-z_2} \frac{dz}{z(1-z)} \hat{\gamma}_{gg}(z) F_g^{q_1}\left(\frac{z_1}{z}, \mu^2\right) F_g^{\bar{q}_2}\left(\frac{z_2}{1-z}, \mu^2\right). \quad (34)
\end{aligned}$$

The singlet constituent diquark distribution function is defined as

$$F_s^{q_1 \bar{q}_2}(z_1, z_2, \mu^2) \equiv \sum_q [F_q^{q_1 \bar{q}_2}(z_1, z_2, \mu^2) + F_{\bar{q}}^{q_1 \bar{q}_2}(z_1, z_2, \mu^2)]. \quad (35)$$

The splitting γ -functions are the same as in the evolution equation of single inclusive hadron fragmentation functions given in Eqs.(11)-(14). The first two terms in Eqs. (33) and (34) represent diquark production from a single parton after the radiative splitting. The third term in Eq. (33) and the third and fourth terms in Eq. (34) are from independent single quark production from each of the partons after the radiative splitting. The indices $\bar{i} = 2, 1$ and $\bar{q}_i = \bar{q}_2, q_1$ for $i = 1, 2$ and $q_i = q_1, \bar{q}_2$, representing the exchange between q_1 and $\bar{q}_2$ in the independent single quark production process. These terms from independent single quark production are proportional to the products of two single constituent quark distribution functions,

$$F_q^q(z_q) = \frac{z_q^3}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_q - \frac{p_q^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \text{Tr} \left[\frac{\gamma \cdot n}{2p_q^+} \sum_{\tilde{S}} \langle 0 | \psi(0) | \tilde{S}, p_q \rangle \langle p_q, \tilde{S} | \bar{\psi}(x) | 0 \rangle \right], \quad (36)$$

$$F_{\bar{q}}^q(z_q) = \frac{z_q^3}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_q - \frac{p_q^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \text{Tr} \left[\sum_{\tilde{S}} \langle 0 | \bar{\psi}(0) | \tilde{S}, p_q \rangle \frac{\gamma \cdot n}{2p_q^+} \langle p_q, \tilde{S} | \psi(x) | 0 \rangle \right], \quad (37)$$

$$F_g^q(z_q) = \frac{z_q^2}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_q - \frac{p_q^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} d_{\mu\nu}(p) \sum_{\tilde{S}} \langle 0 | A^\mu(0) | \tilde{S}, p_q \rangle \langle p_q, \tilde{S} | A^\nu(x) | 0 \rangle, \quad (38)$$

whose definitions have the same form as single hadron fragmentation functions in Eqs. (3),(4) and (6), replacing the single hadron state h with a single constituent quark q .

In the independent single quark production, there are no virtual corrections. Therefore, the corresponding splitting $\hat{\gamma}$ -functions

$$\hat{\gamma}_{qq}(z) = C_F \frac{1+z^2}{1-z}, \quad (39)$$

$$\hat{\gamma}_{gg}(z) = 2C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right], \quad (40)$$

have no '+'-function and delta functions, unlike γ_{qq} and γ_{gg} in Eqs.(11)-(14).

Since the hadrons' wavefunctions in the constituent quark model restrict the relative transverse momentum of the constituent quarks within a hadron to a finite value, we have an intrinsic transverse momentum cutoff Λ in the

definition of the double constituent quark distributions that are relevant for quark recombination in Eqs. (30)–(32). Similarly, such a restriction should also be applied to the relative transverse momentum between the two quarks from the independent single quark production processes in the radiative corrections. This is why there is a cutoff Λ in the relative transverse momentum in the third term in Eq. (33) and the third and four terms in Eq. (34) for the independent production of two constituent quarks that will form a final hadron. This cutoff is determined by the hadronic structure in the constituent quark model and is independent of the momentum scale of the initial hard parton scattering. Therefore, these radiative corrections do not contribute to the DGLAP evolution equations for the diquark distribution functions. Differentiating Eqs. (33) and (34) with respect to Q^2 , one obtains,

$$Q^2 \frac{d}{dQ^2} F_q^{q_1 \bar{q}_2}(z_1, z_2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_1+z_2}^1 \frac{dz}{z^2} \left[\gamma_{qq}(z) F_q^{q_1 \bar{q}_2}\left(\frac{z_1}{z}, \frac{z_2}{z}, Q^2\right) + \gamma_{qg}(z) F_g^{q_1 \bar{q}_2}\left(\frac{z_1}{z}, \frac{z_2}{z}, Q^2\right) \right], \quad (41)$$

$$Q^2 \frac{d}{dQ^2} F_g^{q_1 \bar{q}_2}(z_1, z_2, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_1+z_2}^1 \frac{dz}{z^2} \left[\gamma_{gq}(z) F_s^{q_1 \bar{q}_2}\left(\frac{z_1}{z}, \frac{z_2}{z}, Q^2\right) + \gamma_{gg}(z) F_g^{q_1 \bar{q}_2}\left(\frac{z_1}{z}, \frac{z_2}{z}, Q^2\right) \right]. \quad (42)$$

One can show that combining the above evolution equations for the double constituent quark distribution functions with the expression of jet fragmentation function in Eq. (29), the DGLAP evolution equations for single inclusive hadron fragmentation functions in Eqs. (8) and (9) can be recovered.

Similar to the single and dihadron fragmentation functions, one can also derive sum rules for the single and diquark distribution functions in a parton jet. One can define the mean constituent quark multiplicities from the single quark distributions as

$$\int_0^1 dz F_a^q(z) \equiv \bar{N}_a^q, (a = q, \bar{q}, g). \quad (43)$$

Similarly, single quark distribution functions also obey the momentum sum rule,

$$\int_0^1 dz \sum_q z F_a^q(z) = 1. \quad (44)$$

For diquark distribution functions $F_a^{q_1, q_2}(z_1, z_2, Q^2)$, the multiplicity sum rule leads to the second cumulant moments of the multiplicity distribution,

$$\int dz_1 dz_2 F_a^{q_1 q_2}(z_1, z_2, Q^2) \equiv \overline{N_a^{q_1} (N_a^{q_2} - \delta_{q_1 q_2})}, \quad (45)$$

where $\delta_{q_1 q_2} = 1$ if q_1 and q_2 are identical quarks and $\delta_{q_1 q_2} = 0$ if q_1 and q_2 are different. It illustrates the importance of quark correlations in the diquark distribution functions. Any initial conditions for the diquark distributions should contain the correlation between two quarks within a jet. For two identical quark distributions, the above equation also illustrates the information on the multiplicity fluctuation contained in the diquark distributions. Because of the correlation and fluctuation contained in the diquark distributions, one cannot find any rigorous sum rules relating single and diquark distributions. Following the work on dihadron fragmentation functions [35], an approximate ansatz that contains the minimum correlation and fluctuation is

$$\sum_{q_2} \int dz_2 F_a^{q_1 q_2}(z_1, z_2, Q^2) \approx \frac{\overline{N_a^{q_1} (N_a^{q_2} - \delta_{q_1, q_2})}}{\bar{N}_a^{q_1}} F_a^{q_1}(z_1, Q^2). \quad (46)$$

Using the momentum sum rule Eq. (44) for single quark distributions, the above ansatz will lead to the momentum sum rule for double constituent quark distributions,

$$\sum_{q_1 q_2} \int dz_1 dz_2 \frac{1}{2} (z_1 + z_2) F_a^{q_1 q_2}(z_1, z_2, Q^2) = \frac{\overline{N_a^{q_1} (N_a^{q_2} - \delta_{q_1, q_2})}}{2 \bar{N}_a^{q_1} \bar{N}_a^{q_2}} (\bar{N}_a^{q_1} + \bar{N}_a^{q_2}). \quad (47)$$

Whether such ansatz is a good approximation remains to be explored.

C. Dihadron Fragmentation Functions

The definitions of diquark distribution functions of a jet are very similar to that of dihadron fragmentation functions, except that the constituent quarks in the distributions are restricted by the hadron wavefunction in the phase space

for quark recombination during hadronization. According to Ref. [35], the dihadron fragmentation functions of a quark or gluon parton are defined as,

$$D_q^{h_1 h_2}(z_{h_1}, z_{h_2}) = \frac{z_h^4}{2z_{h_1} z_{h_2}} \int \frac{d^2 p_{h_1 \perp}}{2(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_h - \frac{p_h^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \text{Tr} \left[\frac{\gamma^+}{2p_h^+} \sum_S \langle 0 | \psi(0) | S, p_{h_1}, p_{h_2} \rangle \langle p_{h_2}, p_{h_1}, S | \bar{\psi}(x) | 0 \rangle \right], \quad (48)$$

$$D_{\bar{q}}^{h_1 h_2}(z_{h_1}, z_{h_2}) = \frac{z_h^4}{2z_{h_1} z_{h_2}} \int \frac{d^2 p_{h_1 \perp}}{2(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_h - \frac{p_h^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \text{Tr} \left[\sum_S \langle 0 | \bar{\psi}(0) | S, p_{h_1}, p_{h_2} \rangle \frac{\gamma^+}{2p_h^+} \langle p_{h_2}, p_{h_1}, S | \psi(x) | 0 \rangle \right], \quad (49)$$

$$D_g^{h_1 h_2}(z_{h_1}, z_{h_2}) = \frac{z_h^3}{2z_{h_1} z_{h_2}} \int \frac{d^2 p_{h_1 \perp}}{2(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_h - \frac{p_h^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} d_{\mu\nu}(p) \sum_S \langle 0 | A^\mu(0) | S, p_{h_1}, p_{h_2} \rangle \langle p_{h_2}, p_{h_1}, S | A^\nu(x) | 0 \rangle, \quad (50)$$

where $p_h = p_{h_1} + p_{h_2}$ and $z_h = z_{h_1} + z_{h_2}$. Note that the transverse momentum, $p_{h_1 \perp}$ and $p_{h_2 \perp}$, are defined as perpendicular to the summed total momentum p_h . The relative transverse momentum q_T used in Ref. [35] is then $\vec{q}_\perp = 2\vec{p}_{h_1 \perp} = -2\vec{p}_{h_2 \perp}$.

The radiative corrections to the dihadron fragmentation functions are similar to that of diquark distribution functions, except that the transverse momentum between two individual hadrons is not limited as in the case for diquark distributions due to quark recombination during hadronization. Therefore, the cutoff Λ in Eq. (33) and (34) should be replaced by Q^2 . The DGLAP evolution equations for dihadron fragmentation functions then become [35],

$$Q^2 \frac{d}{dQ^2} D_q^{h_1 h_2}(z_{h_1}, z_{h_2}, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \left[\int_{z_{h_1} + z_{h_2}}^1 \frac{dz}{z^2} \gamma_{qq}(z) D_q^{h_1 h_2}\left(\frac{z_{h_1}}{z}, \frac{z_{h_2}}{z}, Q^2\right) + \sum_{i=1}^2 \int_{z_{h_i}}^{1-z_{h_i}} \frac{dz}{z(1-z)} \hat{\gamma}_{qq}(z) D_q^{h_i}\left(\frac{z_{h_i}}{z}, Q^2\right) D_g^{h_i}\left(\frac{z_{h_i}}{1-z}, Q^2\right) + \int_{z_{h_1} + z_{h_2}}^1 \frac{dz}{z^2} \gamma_{qg}(z) D_g^{h_1 h_2}\left(\frac{z_{h_1}}{z}, \frac{z_{h_2}}{z}, Q^2\right) \right], \quad (51)$$

$$Q^2 \frac{d}{dQ^2} D_g^{h_1 h_2}(z_{h_1}, z_{h_2}, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \left[\int_{z_{h_1} + z_{h_2}}^1 \frac{dz}{z^2} \gamma_{gq}(z) D_s^{h_1 h_2}\left(\frac{z_{h_1}}{z}, \frac{z_{h_2}}{z}, Q^2\right) + \sum_{i=1}^2 \sum_q \int_{z_{h_i}}^{1-z_{h_i}} \frac{dz}{z(1-z)} \gamma_{gq}(z) D_q^{h_i}\left(\frac{z_{h_i}}{z}, Q^2\right) D_{\bar{q}}^{h_i}\left(\frac{z_{h_i}}{1-z}, Q^2\right) + \int_{z_{h_1} + z_{h_2}}^1 \frac{dz}{z^2} \gamma_{gg}(z) D_g^{h_1 h_2}\left(\frac{z_{h_1}}{z}, \frac{z_{h_2}}{z}, Q^2\right) + \int_{z_{h_1}}^{1-z_{h_2}} \frac{dz}{z(1-z)} \hat{\gamma}_{gg}(z) D_g^{h_1}\left(\frac{z_{h_1}}{z}, Q^2\right) D_g^{h_2}\left(\frac{z_{h_2}}{1-z}, Q^2\right) \right]. \quad (52)$$

The additional terms in comparison to the DGLAP evolution equations for single hadron fragmentation functions that are proportional to the convolution of two single hadron fragmentation functions arise from independent fragmentation of both the leading parton and radiated gluon. They can be attributed to the dependence of the relative transverse momentum between two hadrons from the independent fragmentation on the momentum scale. This is the main difference between the evolution of the above dihadron fragmentation functions and the diquark distributions in the quark recombination model.

IV. BARYON FRAGMENTATION FUNCTIONS IN QUARK RECOMBINATION MODEL

A. Triple Constituent Quark Distribution Function

Since a baryon has three constituent quarks, the fragmentation functions for a parton into baryons naturally involve triple quark distribution functions in the quark recombination model. Similar to the meson fragmentation functions, we can also express the baryon fragmentation functions in terms of baryon wave functions and overlapping matrix elements of parton fields and constituent quark states, as illustrated in Fig. 3,

$$\begin{aligned}
D_q^B(z_B) &= \frac{z_B^3}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_B - \frac{p^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} [d^2 k_\perp] [dx] [d^2 k'_\perp] [dx'] \\
&\quad \varphi_B(k_{1\perp}, x_1; k_{2\perp}, x_2; k_{3\perp}, x_3) \varphi_B^*(k'_{1\perp}, x'_1; k'_{2\perp}, x'_2; k'_{3\perp}, x'_3) \\
&\quad \text{Tr} \left[\frac{\gamma^+}{2p_B^+} \sum_{\tilde{S}} \langle 0 | \psi(0) | \tilde{S}; k_{1\perp}, x_1; k_{2\perp}, x_2; k_{3\perp}, x_3 \rangle \langle k'_{3\perp}, x'_3; k'_{2\perp}, x'_2; k'_{1\perp}, x'_1; \tilde{S} | \bar{\psi}(x) | 0 \rangle \right] \\
&\approx C_B \frac{z_B^3}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_B - \frac{p^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \int \frac{d^2 k_{1\perp}}{(2\pi)^3} \frac{dx_1}{4x_1} \int \frac{d^2 k_{2\perp}}{(2\pi)^3} \frac{dx_2}{4x_2} \frac{1}{x_3} |\varphi_b(k_{1\perp}, x_1; k_{2\perp}, x_2; k_{3\perp}, x_3)|^2 \\
&\quad \text{Tr} \left[\frac{\gamma^+}{2p_B^+} \sum_{\tilde{S}} \langle 0 | \psi(0) | \tilde{S}; k_{1\perp}, x_1; k_{2\perp}, x_2; k_{3\perp}, x_3 \rangle \langle k_{3\perp}, x_3; k_{2\perp}, x_2; k_{1\perp}, x_1; \tilde{S} | \bar{\psi}(x) | 0 \rangle \right], \tag{53}
\end{aligned}$$

where $[d^2 k_\perp]$ and $[dx]$ are given by Eqs. (21) and (22) for baryons, $x_3 = 1 - x_1 - x_2$, $k_{3\perp} = -k_{1\perp} - k_{2\perp}$, and C_B is a constant with the dimension of momentum. Notice again that

$$x_1 = \frac{k_1^+}{p_B^+} = \frac{k_1^+ p^+}{p^+ p_B^+} = \frac{z_1}{z_B}; \quad x_2 = \frac{k_2^+}{p_B^+} = \frac{k_2^+ p^+}{p^+ p_B^+} = \frac{z_2}{z_B}. \tag{54}$$

As in the quark recombination model for meson fragmentation functions, we also define the recombination probability for a baryon as

$$R_B(k_{1\perp}, \frac{z_1}{z_B}; k_{2\perp}, \frac{z_2}{z_B}) \equiv |\varphi_B(k_{1\perp}, \frac{z_1}{z_B}; k_{2\perp}, \frac{z_2}{z_B}; -k_{1\perp} - k_{2\perp}, 1 - \frac{z_1 + z_2}{z_B})|^2. \tag{55}$$

The baryon fragmentation function from Eq. (53) can then be cast in the form

$$D_q^B(z_B) \approx C_B \int_0^{z_B} \frac{dz_1}{2} \int_0^{z_B - z_1} \frac{dz_2}{2} R_B(0_\perp, \frac{z_1}{z_B}; 0_\perp, \frac{z_2}{z_B}) F_q^{q_1 q_2 q_3}(z_1, z_2, 1 - z_1 - z_2). \tag{56}$$

This relation is very similar to that for meson fragmentation functions in Eq. (29). The triple constituent quark distribution function is defined as

$$\begin{aligned}
F_q^{q_1 q_2 q_3}(z_1, z_2, z_3) &= \frac{z_B^4}{2z_1 z_2 z_3} \int^\Lambda \frac{d^2 k_{1\perp}}{2(2\pi)^3} \int^\Lambda \frac{d^2 k_{2\perp}}{2(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_B - \frac{p^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \\
&\quad \text{Tr} \left[\frac{\gamma^+}{2p_B^+} \sum_{\tilde{S}} \langle 0 | \psi(0) | \tilde{S}, k_1, k_2, k_3 \rangle \langle k_3, k_2, k_1, \tilde{S} | \bar{\psi}(x) | 0 \rangle \right], \tag{57}
\end{aligned}$$

where, $p_B = k_1 + k_2 + k_3$, $z_B = z_1 + z_2 + z_3$ and Λ is the cutoff for the intrinsic transverse momentum of the constituent quarks in a baryon. Similarly, the triple constituent quark distribution function from the antiquark and gluon parton are defined as

$$\begin{aligned}
D_{\bar{q}}^{q_1 q_2 q_3}(z_1, z_2, z_3) &= \frac{z_B^4}{2z_1 z_2 z_3} \int^\Lambda \frac{d^2 k_{1\perp}}{2(2\pi)^3} \int^\Lambda \frac{d^2 k_{2\perp}}{2(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_B - \frac{p^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \\
&\quad \text{Tr} \left[\sum_{\tilde{S}} \langle 0 | \bar{\psi}(0) | \tilde{S}, k_1, k_2, k_3 \rangle \frac{\gamma^+}{2p_B^+} \langle k_3, k_2, k_1, \tilde{S} | \psi(x) | 0 \rangle \right], \tag{58}
\end{aligned}$$

$$\begin{aligned}
D_g^{q_1 q_2 q_3}(z_1, z_2, z_3) &= \frac{z_B^3}{2z_1 z_2 z_3} \int^\Lambda \frac{d^2 k_{1\perp}}{2(2\pi)^3} \int^\Lambda \frac{d^2 k_{2\perp}}{2(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_B - \frac{p^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \\
&\quad d_{\mu\nu}(p) \sum_{\tilde{S}} \langle 0 | A^\mu(0) | \tilde{S}, k_1, k_2, k_3 \rangle \langle k_3, k_2, k_1, \tilde{S} | A^\nu(x) | 0 \rangle. \tag{59}
\end{aligned}$$

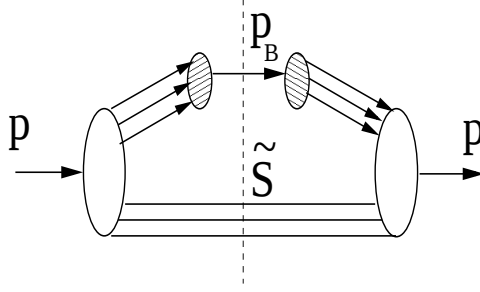


FIG. 3: The cut diagram for single baryon fragmentation function in terms of quark recombination.

Similarly as for the diquark distribution functions, one can also obtain the radiative corrections to the triple constituent quark distribution functions,

$$\begin{aligned}
F_q^{q_1 q_2 q_3}(z_1, z_2, z_3, Q^2) &= F_q^{q_1 q_2 q_3}(z_1, z_2, z_3, \mu^2) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_{z_1+z_2+z_3}^1 \frac{dz}{z^3} \gamma_{qq}(z) F_q^{q_1 q_2 q_3}\left(\frac{z_1}{z}, \frac{z_2}{z}, \frac{z_3}{z}, \mu^2\right) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_{z_1+z_2+z_3}^1 \frac{dz}{z^3} \gamma_{qg}(z) F_g^{q_1 q_2 q_3}\left(\frac{z_1}{z}, \frac{z_2}{z}, \frac{z_3}{z}, \mu^2\right) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{\Lambda^2} \frac{dl_{\perp}^2}{l_{\perp}^2} \sum_{i=1}^3 \int_{z_i}^{1-z_{i'}-z_{i''}} \frac{dz}{z(1-z)^2} \hat{\gamma}_{qq}(z) F_q^{q_i}\left(\frac{z_i}{z}, \mu^2\right) F_g^{q_{i'} q_{i''}}\left(\frac{z_{i'}}{1-z}, \frac{z_{i''}}{1-z}, \mu^2\right) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{\Lambda^2} \frac{dl_{\perp}^2}{l_{\perp}^2} \sum_{i=1}^3 \int_{z_{i'}+z_{i''}}^{1-z_i} \frac{dz}{z^2(1-z)} \hat{\gamma}_{qq}(z) F_q^{q_{i'} q_{i''}}\left(\frac{z_{i'}}{z}, \frac{z_{i''}}{z}, \mu^2\right) F_g^{q_i}\left(\frac{z_i}{1-z}, \mu^2\right), \tag{60}
\end{aligned}$$

$$\begin{aligned}
F_g^{q_1 q_2 q_3}(z_1, z_2, z_3, Q^2) &= F_g^{q_1 q_2 q_3}(z_1, z_2, z_3, \mu^2) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_{z_1+z_2+z_3}^1 \frac{dz}{z^3} \gamma_{gq}(z) F_s^{q_1 q_2 q_3}\left(\frac{z_1}{z}, \frac{z_2}{z}, \frac{z_3}{z}, \mu^2\right) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{Q^2} \frac{dk_{\perp}^2}{k_{\perp}^2} \int_{z_1+z_2+z_3}^1 \frac{dz}{z^2} \gamma_{gg}(z) F_g^{q_1 q_2 q_3}\left(\frac{z_1}{z}, \frac{z_2}{z}, \frac{z_3}{z}, \mu^2\right) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{\Lambda^2} \frac{dl_{\perp}^2}{l_{\perp}^2} \sum_{i=1}^3 \sum_q \int_{z_i}^{1-z_{i'}-z_{i''}} \frac{dz}{z(1-z)^2} \gamma_{gq}(z) F_q^{q_i}\left(\frac{z_i}{z}, \mu^2\right) F_q^{q_{i'} q_{i''}}\left(\frac{z_{i'}}{1-z}, \frac{z_{i''}}{1-z}, \mu^2\right) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{\Lambda^2} \frac{dl_{\perp}^2}{l_{\perp}^2} \sum_{i=1}^3 \sum_q \int_{z_{i'}+z_{i''}}^{1-z_i} \frac{dz}{z^2(1-z)} \gamma_{gq}(z) F_q^{q_{i'} q_{i''}}\left(\frac{z_{i'}}{z}, \frac{z_{i''}}{z}, \mu^2\right) F_q^{q_i}\left(\frac{z_i}{1-z}, \mu^2\right) \\
&+ \frac{\alpha_s(\mu^2)}{2\pi} \int_{\mu^2}^{\Lambda^2} \frac{dl_{\perp}^2}{l_{\perp}^2} \sum_{i=1}^3 \int_{z_i}^{1-z_{i'}-z_{i''}} \frac{dz}{z(1-z)^2} \hat{\gamma}_{gg}(z) F_g^{q_i}\left(\frac{z_i}{z}, \mu^2\right) F_g^{q_{i'} q_{i''}}\left(\frac{z_{i'}}{1-z}, \frac{z_{i''}}{1-z}, \mu^2\right), \tag{61}
\end{aligned}$$

where $(i', i'') = (2, 3), (1, 3), (1, 2)$ as $i = 1, 2, 3$ and the singlet triple constituent quark distribution is given by

$$F_s^{q_1 q_2 q_3}(z_1, z_2, z_3, \mu^2) \equiv \sum_q [F_q^{q_1 q_2 q_3}(z_1, z_2, z_3, \mu^2) + F_{\bar{q}}^{q_1 q_2 q_3}(z_1, z_2, z_3, \mu^2)]. \tag{62}$$

The DGLAP evolution equations for triple constituent quark distribution functions can be obtained from the above,

$$Q^2 \frac{d}{dQ^2} D_q^{q_1 q_2 q_3}(z_1, z_2, z_3, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_1+z_2+z_3}^1 \frac{dz}{z^3} \left[\gamma_{qq}(z) D_q^{q_1 q_2 q_3}\left(\frac{z_1}{z}, \frac{z_2}{z}, \frac{z_3}{z}, Q^2\right) + \gamma_{qg}(z) D_g^{q_1 q_2 q_3}\left(\frac{z_1}{z}, \frac{z_2}{z}, \frac{z_3}{z}, Q^2\right) \right], \quad (63)$$

$$Q^2 \frac{d}{dQ^2} D_g^{q_1 q_2 q_3}(z_1, z_2, z_3, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{z_1+z_2+z_3}^1 \frac{dz}{z^3} \left[\gamma_{gq}(z) D_s^{q_1 q_2 q_3}\left(\frac{z_1}{z}, \frac{z_2}{z}, \frac{z_3}{z}, Q^2\right) + \gamma_{gg}(z) D_g^{q_1 q_2 q_3}\left(\frac{z_1}{z}, \frac{z_2}{z}, \frac{z_3}{z}, Q^2\right) \right]. \quad (64)$$

Again, the radiative corrections from the independent fragmentation of the leading and radiated partons do not contribute to the DGLAP evolution because of the restriction on the relative transverse momentum between constituent quarks by the baryon's wavefunction during quark recombination. As in the case for diquark distribution functions and meson fragmentation functions, the above DGLAP evolution equations for triple quark distributions will give rise to the DGLAP equations for baryon fragmentation functions within the quark recombination model as given by Eq. (56).

One can also obtain the multiplicity sum rule for the triple quark distribution functions of a parton jet,

$$\int dz_1 dz_2 dz_3 F_a^{q_1 q_2 q_3}(z_1, z_2, z_3, Q^2) = \overline{N_a^{q_1} (N_a^{q_2} - \delta_{q_1 q_2}) (N_a^{q_3} - \delta_{q_2 q_3} - \delta_{q_1 q_3})}. \quad (65)$$

Similar to the ansatz of the sum rule relating single and diquark distributions, one can also have the following ansatz for a sum rule relating diquark and triple quark distributions,

$$\sum_{q_3} \int dz_3 F_a^{q_1 q_2 q_3}(z_1, z_2, z_3, Q^2) \approx \frac{\overline{N_a^{q_1} (N_a^{q_2} - \delta_{q_1 q_2}) (N_a^{q_3} - \delta_{q_2 q_3} - \delta_{q_1 q_3})}}{\overline{N_a^{q_1} (N_a^{q_2} - \delta_{q_1 q_2})}} F_a^{q_1 q_2}(z_1, z_2, Q^2). \quad (66)$$

B. Trihadron Fragmentation Function

It is straightforward to generalize the result for the triple quark distribution functions to the trihadron fragmentation functions. The operator expressions of the trihadron fragmentation functions from quark, antiquark or a gluon jet are defined as

$$D_q^{h_1 h_2 h_3}(z_{h_1}, z_{h_2}, z_{h_3}) = \frac{z_h^4}{2z_{h_1} z_{h_2} z_{h_3}} \int \frac{d^2 p_{h_1 \perp}}{2(2\pi)^3} \int \frac{d^2 p_{h_2 \perp}}{2(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_h - \frac{p_h^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \text{Tr} \left[\frac{\gamma^+}{2p_h^+} \sum_S \langle 0 | \psi(0) | S, p_{h_1}, p_{h_2}, p_{h_3} \rangle \langle p_{h_3}, p_{h_2}, p_{h_1}, S | \bar{\psi}(x) | 0 \rangle \right], \quad (67)$$

$$D_{\bar{q}}^{h_1 h_2 h_3}(z_{h_1}, z_{h_2}, z_{h_3}) = \frac{z_h^4}{2z_{h_1} z_{h_2} z_{h_3}} \int \frac{d^2 p_{h_1 \perp}}{2(2\pi)^3} \int \frac{d^2 p_{h_2 \perp}}{2(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_h - \frac{p_h^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \text{Tr} \left[\sum_S \langle 0 | \bar{\psi}(0) | S, p_{h_1}, p_{h_2}, p_{h_3} \rangle \frac{\gamma^+}{2p_h^+} \langle p_{h_3}, p_{h_2}, p_{h_1}, S | \psi(x) | 0 \rangle \right], \quad (68)$$

$$D_g^{h_1 h_2 h_3}(z_{h_1}, z_{h_2}, z_{h_3}) = \frac{z_h^3}{2z_{h_1} z_{h_2} z_{h_3}} \int \frac{d^2 p_{h_1 \perp}}{2(2\pi)^3} \int \frac{d^2 p_{h_2 \perp}}{2(2\pi)^3} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_h - \frac{p_h^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} d_{\mu\nu}(p) \sum_S \langle 0 | A^\mu(0) | S, p_{h_1}, p_{h_2}, p_{h_3} \rangle \langle p_{h_3}, p_{h_2}, p_{h_1}, S | A^\nu(x) | 0 \rangle, \quad (69)$$

where $p_h = p_{h_1} + p_{h_2} + p_{h_3}$ and $z_h = z_{h_1} + z_{h_2} + z_{h_3}$.

One can show that in the leading twist and collinear factorization approximations, the semi-inclusive cross section $\sigma_{e^+ e^- \rightarrow h_1 h_2 h_3}$ can be expressed in terms of the trihadron fragmentation functions as,

$$\frac{d\sigma_{e^+ e^- \rightarrow h_1 h_2 h_3}}{dz_{h_1} dz_{h_2} dz_{h_3}} = \sum_q \sigma_0^{q\bar{q}} \left[D_q^{h_1 h_2 h_3}(z_{h_1}, z_{h_2}, z_{h_3}) + D_{\bar{q}}^{h_1 h_2 h_3}(z_{h_1}, z_{h_2}, z_{h_3}) \right]. \quad (70)$$

Again, the radiative corrections for the trihadron fragmentation functions are similar to that of triple quark distribution functions in Eqs.(60) and (61), except that the cutoff in the intrinsic transverse momentum Λ should be replaced by Q which is the only limit of the relative transverse momenta between hadrons. The DGLAP equations for the scale evolution for the trihadron fragmentation functions are then,

$$\begin{aligned}
Q^2 \frac{\partial}{\partial Q^2} D_q^{h_1 h_2 h_3}(z_{h_1}, z_{h_2}, z_{h_3}, Q^2) &= \frac{\alpha_s(Q^2)}{2\pi} \left[\int_{z_{h_1}+z_{h_2}+z_{h_3}}^1 \frac{dz}{z^3} \gamma_{qq}(z) D_q^{h_1 h_2 h_3} \left(\frac{z_{h_1}}{z}, \frac{z_{h_2}}{z}, \frac{z_{h_3}}{z}, Q^2 \right) \right. \\
&+ \sum_{i=1}^3 \int_{z_{h_i}}^{1-z_{h_{i'}}-z_{h_{i''}}} \frac{dz}{z(1-z)^2} \hat{\gamma}_{qq}(z) D_q^{h_i} \left(\frac{z_{h_i}}{z}, Q^2 \right) D_g^{h_{i'} h_{i''}} \left(\frac{z_{h_{i'}}}{1-z}, \frac{z_{h_{i''}}}{1-z}, Q^2 \right) \\
&+ \sum_{i=1}^3 \int_{z_{h_{i'}}+z_{h_{i''}}}^{1-z_{h_i}} \frac{dz}{z^2(1-z)} \hat{\gamma}_{qq}(z) D_q^{h_{i'} h_{i''}} \left(\frac{z_{h_{i'}}}{z}, \frac{z_{h_{i''}}}{z}, Q^2 \right) D_g^{h_i} \left(\frac{z_{h_i}}{1-z}, Q^2 \right) \\
&\left. + \int_{z_{h_1}+z_{h_2}+z_{h_3}}^1 \frac{dz}{z^3} \gamma_{qq}(z) D_g^{h_1 h_2 h_3} \left(\frac{z_{h_1}}{z}, \frac{z_{h_2}}{z}, \frac{z_{h_3}}{z}, Q^2 \right) \right], \tag{71}
\end{aligned}$$

$$\begin{aligned}
Q^2 \frac{\partial}{\partial Q^2} D_g^{h_1 h_2 h_3}(z_{h_1}, z_{h_2}, z_{h_3}, Q^2) &= \frac{\alpha_s(Q^2)}{2\pi} \left[\int_{z_{h_1}+z_{h_2}+z_{h_3}}^1 \frac{dz}{z^3} \gamma_{gq}(z) D_s^{h_1 h_2 h_3} \left(\frac{z_{h_1}}{z}, \frac{z_{h_2}}{z}, \frac{z_{h_3}}{z}, Q^2 \right) \right. \\
&+ \sum_{i=1}^3 \sum_q \int_{z_{h_i}}^{1-z_{h_{i'}}-z_{h_{i''}}} \frac{dz}{z(1-z)^2} \gamma_{gq}(z) D_q^{h_i} \left(\frac{z_{h_i}}{z}, Q^2 \right) D_{\bar{q}}^{h_{i'} h_{i''}} \left(\frac{z_{h_{i'}}}{1-z}, \frac{z_{h_{i''}}}{1-z}, Q^2 \right) \\
&+ \sum_{i=1}^3 \sum_q \int_{z_{h_{i'}}+z_{h_{i''}}}^{1-z_{h_i}} \frac{dz}{z^2(1-z)} \gamma_{gq}(z) D_q^{h_{i'} h_{i''}} \left(\frac{z_{h_{i'}}}{z}, \frac{z_{h_{i''}}}{z}, Q^2 \right) D_{\bar{q}}^{h_i} \left(\frac{z_{h_i}}{1-z}, Q^2 \right) \\
&+ \int_{z_{h_1}+z_{h_2}+z_{h_3}}^1 \frac{dz}{z^3} \gamma_{gg}(z) D_g^{h_1 h_2 h_3} \left(\frac{z_{h_1}}{z}, \frac{z_{h_2}}{z}, \frac{z_{h_3}}{z}, Q^2 \right) \\
&\left. + \sum_{i=1}^3 \int_{z_{h_i}}^{1-z_{h_{i'}}-z_{h_{i''}}} \frac{dz}{z(1-z)^2} \hat{\gamma}_{gg}(z) D_g^{h_i} \left(\frac{z_{h_i}}{z}, Q^2 \right) D_g^{h_{i'} h_{i''}} \left(\frac{z_{h_{i'}}}{1-z}, \frac{z_{h_{i''}}}{1-z}, Q^2 \right) \right]. \tag{72}
\end{aligned}$$

V. QUARK RECOMBINATION AND JET FRAGMENTATION IN A THERMAL MEDIUM

So far we have reformulated jet fragmentation in vacuum in the quark recombination model in which we related the parton fragmentation functions to the quark recombination probability as determined by the hadron wavefunction in a constituent quark model and multi-quark distribution functions of a parton jet before hadronization. In the following sections we will extend the formula to the case of jet fragmentation in a thermal medium which is relevant to jet fragmentation in the environment of high-energy heavy-ion collisions. Such problems have been considered before [31]. But the attention has been focused on parton emission and absorption by the propagating parton jet in a thermal medium before hadronization. It has been assumed that the thermal medium is in a deconfined phase so that partons from the jet will eventually hadronize together with the medium.

In this paper, however, we focus on the physical process during the hadronization of the parton jet in a thermal medium and the modification of the parton fragmentation functions with respect to that in the vacuum. We assume that the effective degrees of freedom can be described by constituent quarks just before and during hadronization. Therefore, we consider constituent quarks not only as the effective states in the process of jet fragmentation but also the effective constituent of the medium just before hadronization. In this framework, one naturally encounters recombination between thermal constituent quarks and shower quarks from parton fragmentation, in addition to recombination of shower quarks as in the vacuum. They both contribute to hadron production associated with an energetic parton jets in a thermal medium.

As in the previous study [31], one can describe the fragmentation of a parton jet in medium simply by replacing the vacuum expectation in the S matrix of the processes or the operator definition of the parton fragmentation functions by their thermal expectation values, $\langle 0 | \mathcal{O} | 0 \rangle \rightarrow \langle \langle \mathcal{O} \rangle \rangle$,

$$\langle \langle \mathcal{O} \rangle \rangle = \frac{\text{Tr}[e^{-\hat{H}\beta} \mathcal{O}]}{\text{Tr} e^{-\hat{H}\beta}}, \tag{73}$$

where, $\hat{H}$ is the Hamiltonian operator of the system and $1/\beta = T$ is the temperature. Therefore, the single hadron fragmentation functions at finite temperature for a quark, antiquark and gluon jet can be defined as

$$\tilde{D}_q^h(z_h, p^+) = \frac{z_h^3}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_h - \frac{p_h^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \text{Tr} \left[\frac{\gamma^+}{2p_h^+} \sum_S \langle \langle \psi(0) | S, p_h \rangle \langle p_h, S | \bar{\psi}(x) \rangle \rangle \right], \quad (74)$$

$$\tilde{D}_{\bar{q}}^h(z_h, p^+) = \frac{z_h^3}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_h - \frac{p_h^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \text{Tr} \left[\sum_S \langle \langle \bar{\psi}(0) | S, p_h \rangle \frac{\gamma^+}{2p_h^+} \langle p_h, S | \psi(x) \rangle \rangle \right], \quad (75)$$

$$\tilde{D}_g^h(z_h, p^+) = \frac{z_h^2}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_h - \frac{p_h^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} d_{\mu\nu}(p) \sum_S \langle \langle A^\mu(0) | S, p_h \rangle \langle p_h, S | A^\nu(x) \rangle \rangle, \quad (76)$$

where p_h and p are the four-momentum of the hadron and the initial parton, respectively. In the above definition of the parton fragmentation function in medium, we have explicitly kept the dependence on the initial parton energy p^+ . Such dependence arises at finite temperature from the thermal average which also introduces dependence on the temperature T . An alternative definition of the fragmentation functions [31],

$$\tilde{D}_q^h(p_h^+, p^+) = \frac{p_h^+}{4p^+} \int \frac{d^4 k}{(2\pi)^4} \delta(k^+ - p^+) \int d^4 x e^{-ik \cdot x} \text{Tr} \left[\gamma^+ \sum_S \langle \langle \psi(0) | S, p_h \rangle \langle p_h, S | \bar{\psi}(x) \rangle \rangle \right], \quad (77)$$

$$\tilde{D}_{\bar{q}}^h(p_h^+, p^+) = \frac{p_h^+}{4p^+} \int \frac{d^4 k}{(2\pi)^4} \delta(k^+ - p^+) \int d^4 x e^{-ik \cdot x} \text{Tr} \left[\sum_S \langle \langle \bar{\psi}(0) | S, p_h \rangle \gamma^+ \langle p_h, S | \psi(x) \rangle \rangle \right], \quad (78)$$

$$\tilde{D}_g^h(p_h^+, p^+) = -\frac{p_h^+}{2p^{+2}} \int \frac{d^4 k}{(2\pi)^4} \delta(k^+ - p^+) \int d^4 x e^{-ik \cdot x} \sum_S \langle \langle F^{+\mu}(0) | S, p_h \rangle \langle p_h, S | F_\mu^+(x) \rangle \rangle, \quad (79)$$

explicitly takes into account the dependence on the absolute initial parton energy and the medium temperature. The two expressions can be related via the identity,

$$\frac{p_h^+}{p^+} \delta(k^+ - p^+) = \frac{z_h^3}{p_h^+} \delta(z_h - \frac{p_h^+}{k^+}). \quad (80)$$

At zero temperature $T = 0$, the above fragmentation functions will be reduced to the parton fragmentation functions in vacuum as defined in Eqs.(3), (4) and (6). At leading twist, they depend only on the scaling variable, $z_h = p_h^+/p^+$, *i.e.*, the ratio of the hadron and parton energies.

A. Meson Production from Thermal Quark Recombination

From Eq. (74), the meson fragmentation function at finite temperature from the quark parton can be expressed as

$$\begin{aligned} \tilde{D}_q^M(z_M, p^+) &= \frac{z_M^3}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_M - \frac{p_M^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \\ &\int [d^2 k_\perp] [dx] \int [d^2 k'_\perp] [dx'] \varphi_M(k_{1\perp}, x_1; k_{2\perp}, x_2) \varphi_M^*(k'_{1\perp}, x'_1; k'_{2\perp}, x'_2) \\ &\text{Tr} \left[\frac{\gamma^+}{2p_M^+} \sum_{\tilde{S}} \langle \langle \psi(0) | \tilde{S}; k_{1\perp}, x_1; k_{2\perp}, x_2 \rangle \langle k'_{2\perp}, x'_2; k'_{1\perp}, x'_1; \tilde{S} | \bar{\psi}(x) \rangle \rangle \right]. \end{aligned} \quad (81)$$

To evaluate the thermal average in Eq. (81), it is convenient to use the finite volume quantization (FVQ) in a cubic box with length L . Employing FVQ, the momentum integral, δ -function and continuous states can be replaced by following summation, Kronecker functions and discrete states, respectively.

$$\int_{-\infty}^{\infty} dq_x \int_{-\infty}^{\infty} dq_y \int_{-\infty}^{\infty} dq_z f(q_x, q_y, q_z) \longleftrightarrow \left(\frac{2\pi}{L}\right)^3 \sum_{n_x=-\infty}^{\infty} \sum_{n_y=-\infty}^{\infty} \sum_{n_z=-\infty}^{\infty} f\left(\frac{2\pi n_x}{L}, \frac{2\pi n_y}{L}, \frac{2\pi n_z}{L}\right), \quad (82)$$

$$(2\pi)^3 \delta(q_x - q'_x) \delta(q_y - q'_y) \delta(q_z - q'_z) \longleftrightarrow L^3 \delta_{n_x, n'_x} \delta_{n_y, n'_y} \delta_{n_z, n'_z}, \quad (83)$$

$$\frac{1}{\sqrt{2E_q L^3}} |q\rangle \longleftrightarrow |q_i\rangle. \quad (84)$$

Using Eq.(73), we can express the thermal average of the matrix element in Eq. (81) as

$$\begin{aligned} & \langle \langle \psi(0) | \tilde{S}; k_{1\perp}, x_1; k_{2\perp}, x_2 \rangle \langle k'_{2\perp}, x'_2; k'_{1\perp}, x'_1; \tilde{S} | \bar{\psi}(x) \rangle \rangle \\ &= \frac{\sum_{\{n_i\}} \langle \{ (q_i, n_i) \} | e^{-\beta \hat{H}} \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | \{ (q_i, n_i) \} \rangle}{\sum_{\{n_i\}} \langle \{ (q_i, n_i) \} | e^{-\beta \hat{H}} | \{ (q_i, n_i) \} \rangle}, \end{aligned} \quad (85)$$

where $n_i = 0, 1$ (for $i = 1, 2, \dots, \infty$) since we have assumed the thermal medium to consist of constituent quarks that obey Fermi statistics. The denominator in Eq.(85) can be reduced to

$$\sum_{\{n_i\}} \langle \{ (q_i, n_i) \} | e^{-\beta \hat{H}} | \{ (q_i, n_i) \} \rangle = \sum_{\{n_i\}} e^{-\beta \sum_{i=1}^{\infty} n_i E_i} = \prod_{i=1}^{\infty} (1 + e^{-\beta E_i}). \quad (86)$$

Taking into account that the intermediate states of constituent quarks $|k_1\rangle, |k_2\rangle, |k'_1\rangle, |k'_2\rangle$ can contract both with the field operators ψ and $\bar{\psi}$ and the constituent quark states $|\{ (q_i, n_i) \}\rangle$ in the thermal medium, we get

$$\begin{aligned} & \langle \langle \psi(0) | \tilde{S}; k_{1\perp}, x_1; k_{2\perp}, x_2 \rangle \langle k'_{2\perp}, x'_2; k'_{1\perp}, x'_1; \tilde{S} | \bar{\psi}(x) \rangle \rangle \\ &= \left(\prod_{i=1}^{\infty} (1 + e^{-\beta E_i}) \right)^{-1} \sum_{\{n_i\}} e^{-\beta \sum_{i=1}^{\infty} n_i E_i} \\ & \quad \left[\langle (q_1, n_1); \dots; (q_{\infty}, n_{\infty}) | \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | (q_{\infty}, n_{\infty}); \dots; (q_1, n_1) \rangle \right. \\ & \quad + \sum_{i,j} \langle (q_1, n_1); \dots; (q_i, n_i); \dots | \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | \dots; (q_j, n_j); \dots; (q_1, n_1) \rangle \\ & \quad + \sum_{i,j} \langle (q_1, n_1); \dots; (q_i, n_i); \dots | \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | \dots; (q_j, n_j); \dots; (q_1, n_1) \rangle \\ & \quad \left. + \sum_{i,j,l,m} \langle \dots; (q_i, n_i); \dots; (q_l, n_l); \dots | \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | \dots; (q_m, n_m); \dots; (q_j, n_j); \dots \rangle \right]. \end{aligned} \quad (87)$$

For simplicity in this paper, the states $|k_1\rangle, |k_2\rangle, |k'_1\rangle, |k'_2\rangle$ without the connecting lines are meant to be only contracted with the field operators ψ and $\bar{\psi}$. The states $|\{ (q_i, n_i) \}\rangle$ with the connecting lines are meant to contract only with the final states $|k_{1,2}\rangle$ and $|k'_{1,2}\rangle$ but without contraction with the field operators ψ and $\bar{\psi}$. The states $|\{ (q_i, n_i) \}\rangle$ without the connecting lines can still contract with the field operators ψ and $\bar{\psi}$ but will not contract with the final states $|k_{1,2}\rangle$ and $|k'_{1,2}\rangle$.

Let us for a moment neglect the contraction between the thermal states $|\{ (q_i, n_i) \}\rangle$ and the field operators ψ and $\bar{\psi}$. Using

$$\langle (q_1, n_1); \dots; (q_{i-1}, n_{i-1}); (q_{i+1}, n_{i+1}); \dots | \dots; (q_{j+1}, n_{j+1}); (q_{j-1}, n_{j-1}); \dots; (q_1, n_1) \rangle = \delta_{ij}, \quad (88)$$

one can obtain from Eq. (87)

$$\begin{aligned} & \langle \langle \psi(0) | \tilde{S}; k_{1\perp}, x_1; k_{2\perp}, x_2 \rangle \langle k'_{2\perp}, x'_2; k'_{1\perp}, x'_1; \tilde{S} | \bar{\psi}(x) \rangle \rangle \\ &= \left(\prod_{i=1}^{\infty} (1 + e^{-\beta E_i}) \right)^{-1} \left[\sum_{\{n_i\}} e^{-\beta \sum_{i=1}^{\infty} n_i E_i} \langle 0 | \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | 0 \rangle \right. \\ & \quad + \sum_j e^{-\beta E_j} \sum'_{\{n_i\}} e^{-\beta \sum_{i=1}^{\infty} n_i E_i} \langle (q_j, n_j) | \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | (q_j, n_j) \rangle \\ & \quad + \sum_j e^{-\beta E_j} \sum'_{\{n_i\}} e^{-\beta \sum_{i=1}^{\infty} n_i E_i} \langle (q_j, n_j) | \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | (q_j, n_j) \rangle \\ & \quad \left. + \sum_{j,m} e^{-\beta(E_j+E_m)} \sum''_{\{n_i\}} e^{-\beta \sum_{i=1}^{\infty} n_i E_i} \langle (q_j, n_j); (q_m, n_m) | \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | (q_m, n_m); (q_j, n_j) \rangle \right], \end{aligned} \quad (89)$$

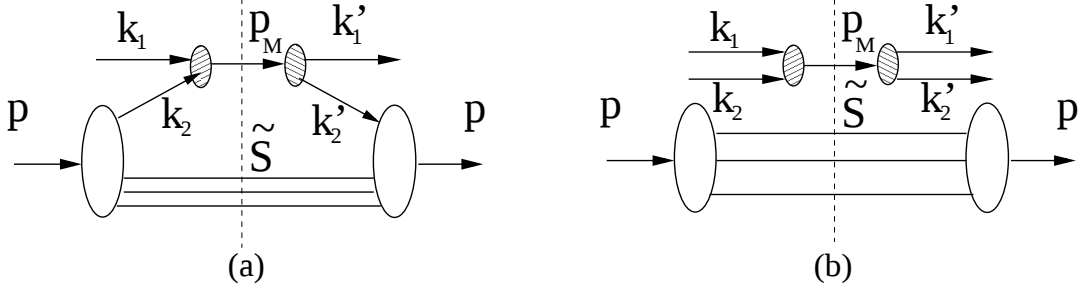


FIG. 4: The cut diagram for contributions from (a) shower-thermal and (b) thermal-thermal quark recombination to the single meson fragmentation function.

where Σ', Σ'' indicates that $i \neq j$ and $i \neq j, m$ in their summation, respectively. One can generalize the above results to include the contraction between thermal states and the quark field operators. In this general case, the vacuum expectation will be replaced by a thermal average without contraction between thermal states and the final constituent quark states. Using Eq.(86) and replacements (82),(84), we obtain the thermal average value of the matrix elements in the meson fragmentation function as

$$\begin{aligned} \langle \langle \psi(0) | \tilde{S}; k_{1\perp}, x_1; k_{2\perp}, x_2 \rangle \langle k'_{2\perp}, x'_2; k'_{1\perp}, x'_1; \tilde{S} | \bar{\psi}(x) \rangle \rangle &= {}_k \langle \langle \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) \rangle \rangle_k \\ &+ \int \frac{d^3 q_1}{(2\pi)^3 2E_{q_1}} \frac{1}{e^{\beta E_{q_1}} + 1} \left[{}_k \langle \langle q_1 | \psi(0) | S; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | q_1 \rangle \rangle_k + {}_k \langle \langle q_1 | \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | q_1 \rangle \rangle_k \right] \\ &+ \int \frac{d^3 q_1}{(2\pi)^3 2E_{q_1}} \frac{d^3 q_2}{(2\pi)^3 2E_{q_2}} \frac{1}{e^{\beta E_{q_1}} + 1} \frac{1}{e^{\beta E_{q_2}} + 1} {}_k \langle \langle q_2; q_1 | \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | q_1; q_2 \rangle \rangle_k, \end{aligned} \quad (90)$$

where the notation ${}_k \langle \langle \dots \rangle \rangle_k$ represents thermal averaging without the contraction between thermal states and the final constituent quark states. The form $(e^{\beta E} + 1)^{-1}$ is the Fermi-Dirac thermal distribution function in the co-moving frame of a fluid that maintains only local equilibrium. In the reaction frame in which a parton jet propagates, it should be replaced by

$$f(q) = \frac{1}{e^{\beta q \cdot u} + 1}, \quad (91)$$

for the thermal quark distribution in a fluid element with flow four-velocity u in the thermal medium. The momentum of a thermal constituent quark is decomposed to $q = (q^+, 0, q_\perp)$ with respect to the direction of the propagating parton jet. In the following, we also sometimes express the longitudinal component of the thermal momentum as a fraction of the initial parton momentum, $q^+ = z_q p^+$. In this case, the value of z_q is not bound. With this, we can finally express the thermal averaged matrix element in the fragmentation function as

$$\begin{aligned} \langle \langle \psi(0) | \tilde{S}; k_{1\perp}, x_1; k_{2\perp}, x_2 \rangle \langle k'_{2\perp}, x'_2; k'_{1\perp}, x'_1; \tilde{S} | \bar{\psi}(x) \rangle \rangle &= {}_k \langle \langle \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) \rangle \rangle_k \\ &+ \int \frac{d^2 q_{1\perp}}{2(2\pi)^3} \frac{dq_1^+}{q_1^+} f(q_{1\perp}, q_1^+) \left[{}_k \langle \langle q_1 | \psi(0) | S; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | q_1 \rangle \rangle_k + {}_k \langle \langle q_1 | \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | q_1 \rangle \rangle_k \right] \\ &+ \int \frac{d^2 q_{1\perp}}{2(2\pi)^3} \frac{dq_1^+}{q_1^+} \int \frac{d^2 q_{2\perp}}{2(2\pi)^3} \frac{dq_2^+}{q_2^+} f(q_{1\perp}, q_1^+) f(q_{2\perp}, q_2^+) {}_k \langle \langle q_2; q_1 | \psi(0) | \tilde{S}; k_1; k_2 \rangle \langle k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | q_1; q_2 \rangle \rangle_k. \end{aligned} \quad (92)$$

The above equation has three distinct contributions. One can immediately identify the first term as the contribution from recombination of two shower quarks from parton fragmentation,

$$\tilde{D}_q^{M(SS)}(z_M) \approx C_M \int_0^{z_M} \frac{dz_1}{2} R_M(0_\perp, \frac{z_1}{z_M}) \tilde{F}_q^{q_1 \bar{q}_2}(z_1, z_M - z_1), \quad (93)$$

This contribution is normally referred [24] to as “shower-shower” quark recombination. It has exactly the same form as the parton fragmentation functions in vacuum [Eq.(29)] in the framework of quark recombination, except that the diquark distribution functions $\tilde{F}_q^{q_1 \bar{q}_2}(z_1, z_2)$ are now medium modified. Their definitions are similar to that in vacuum in Eq. (30) but the vacuum expectation is replaced by a thermal average. These modified diquark distribution

functions should in principle contain effects of multiple scattering, induced gluon radiation and absorption, in the same way as the modification of hadron fragmentation functions in a thermal medium [31]. We have assumed that the meson wavefunction and therefore the recombination probability during the hadronization of the parton jets in a quark-gluon plasma are the same as in vacuum.

In the second term, one of the constituent quarks is contracted with the thermal quark states while the other with the quark field operators. This corresponds to processes in which a quark from the thermal medium combines with another quark from the parton fragmentation (or shower) to form the final meson, as illustrated in Fig. 4(a). Using the convention of previous studies [24], these processes are called “shower-thermal” quark recombination. Finally, the third term where both of the final constituent quark states contract with the thermal quark states corresponds to the formation of the final meson from two thermal quarks in the quark recombination model, as shown in Fig. 4(b). This is referred to as “thermal-thermal” quark recombination.

Using Eqs. (28) and (81) and the contraction between two single-particle states,

$$\langle \underline{q_i | k_j} \rangle \equiv \langle q_{i\perp}, x_{qi} | k_{j\perp}, x_j \rangle = (2\pi)^3 2x_i \delta^{(2)}(q_{i\perp} - k_{j\perp}) \delta(x_{qi} - x_j), \quad (94)$$

we can express the contribution from “shower-thermal” quark recombination to the meson fragmentation function as

$$\begin{aligned} \tilde{D}_q^{M(ST)}(z_M, p^+) &= \int_0^{z_M} \frac{dz_q}{z_M} \int \frac{d^2 q_\perp}{2(2\pi)^3} R_M(q_\perp, \frac{z_q}{z_M}) \frac{1}{(1 - z_q/z_M)^2} \\ &\quad \left[f_{q_1}(q_\perp, z_q p^+) \tilde{F}_q^{\bar{q}2}(z_M - z_q) + f_{\bar{q}_2}(q_\perp, z_q p^+) \tilde{F}_q^{q1}(z_M - z_q) \right], \end{aligned} \quad (95)$$

where $x_{qi} = z_{qi}/z_M$ is the thermal quark’s momentum as a fraction of the produced meson’s momentum, $\tilde{F}_q^{q_i}(z)$ and $\tilde{F}_q^{\bar{q}_i}(z)$ are single constituent quark or anti-quark distributions of the fragmenting parton jet in a thermal medium defined similarly as in Eqs. (36) and (37), except that the vacuum expectation values are replaced again by thermal averaged expectation. They should be different from the corresponding quark distributions in vacuum because of multiple scattering, induced gluon bremsstrahlung and parton absorption.

The contribution due to “thermal-thermal” quark recombination to the meson fragmentation function in Eq. (92) comes from recombination of two thermal constituent quarks. This contribution would never be associated with the parton jet had we not used the parton’s momentum as a reference to calculate the distribution of produced hadrons from such thermal quark recombination. Again, using Eq. (94), we can express this “thermal-thermal” contribution as

$$\begin{aligned} \tilde{D}_q^{M(TT)}(z_M, p^+) &= \int \frac{d^2 q_{1\perp} dz_{q_1}}{2(2\pi)^3} \int \frac{d^2 q_{2\perp} dz_{q_2}}{2(2\pi)^3} f_{q_1}(q_{1\perp}, z_{q_1} p^+) f_{\bar{q}_2}(q_{2\perp}, z_{q_2} p^+) S_q^0(z_M) \\ &\quad \times 4 \left| \varphi_M(q_{1\perp}, \frac{z_{q_1}}{z_M}; q_{2\perp}, \frac{z_{q_2}}{z_M}) \right|^2 \left[(2\pi)^3 \delta^{(2)}(p_{M\perp} - q_{1\perp} - q_{2\perp}) \delta(z_M - z_{q_1} - z_{q_2}) \right]^2, \end{aligned} \quad (96)$$

where

$$\begin{aligned} S_q^0(z_M) &\equiv \frac{z_M^3}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_M - \frac{p^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \text{Tr} \left[\frac{\gamma^+}{2p_M^+} \sum_{\tilde{S}} \langle \langle \psi(0) | \tilde{S} \rangle \langle \tilde{S} | \bar{\psi}(x) | 0 \rangle \rangle_k \right] \\ &= \frac{z_M}{2} \frac{1}{(2\pi)^3} \int d^2 p_\perp = \frac{1}{2(2\pi)^3} \frac{1}{z_M} \int d^2 p_{M\perp}, \end{aligned} \quad (97)$$

is just a factor associated with the phase-space integration. Since the fragmentation function represents the total yield of particle production, the thermal contribution naturally contains a volume factor, which leads to the δ -function squared in Eq. (96). To factor out this volume factor, we again employ the finite volume quantization with $V = L^3$ in a cubic box. The momentum integral and δ -functions can be expressed by using replacements Eqs. (82) and (83). Then we can rewrite Eq. (96) as

$$\begin{aligned} \tilde{D}_q^{M(TT)}(z_M, p^+) &= V p^+ \int \frac{d^2 p_{M\perp}}{(2\pi)^3} \int_0^{z_M} \frac{dz_q}{2z_M} \int \frac{d^2 q_\perp}{(2\pi)^3} \\ &\quad f_{q_1}(q_\perp, z_q p^+) f_{\bar{q}_2}(p_{M\perp} - q_\perp, (z_M - z_q) p^+) R_M(q_\perp, \frac{z_q}{z_M}). \end{aligned} \quad (98)$$

Since hadron production from the thermal quark recombination is not correlated with the parton jet and its fragmentation, the above expression is a little unnatural. One should be able to rewrite it in a form that has no

dependence on the parton jet. Considering,

$$\tilde{D}_q^{M(TT)}(z_M, p^+) = \frac{dN^{M(TT)}}{dz_M} = \frac{dN^{M(TT)}}{dp_M^+} p^+, \quad (99)$$

one can obtain the invariant hadron spectrum from thermal quark recombination,

$$(2\pi)^3 \frac{dN^{M(TT)}}{dp_M^+ d^2 p_{M\perp}} = V \int_0^1 dx_q \int \frac{d^2 q_\perp}{2(2\pi)^3} f_{q_1}(q_\perp, x_q p_M^+) f_{\bar{q}_2}(p_{M\perp} - q_\perp, (1-x_q)p_M^+) R_M(q_\perp, x_q). \quad (100)$$

which coincides with results from other recombination models [25, 26]. In this expression, the hadron spectra from thermal quark recombination are not correlated and therefore do not depend on the parton jet fragmentation. Even though one can sum over the three contributions and obtain the effective meson fragmentation in a thermal medium,

$$\tilde{D}_q^M(z_M, p^+) = \tilde{D}_q^{M(SS)}(z_M) + \tilde{D}_q^{M(ST)}(z_M, p^+) + \tilde{D}_q^{M(TT)}(z_M, p^+), \quad (101)$$

the last term from thermal quark recombination is not correlated with the initial parton jet and therefore should not be considered as part of the medium modified jet fragmentation function. The contributions that are correlated with the initial parton jets are from “shower-shower” and “shower-thermal” quark recombination. For a thermalized medium, the thermal quark distribution follows a Fermi-Dirac form that is determined by the local temperature and flow velocity. The contribution from “shower-thermal” recombination is most important for hadron spectra in the intermediate transverse momentum region. This contribution, however, will be negligible relative to the “shower-shower” recombination which dominates hadron spectra at large transverse momentum because that the power-law-like spectra of initially produced partons will win over the exponential-like distribution of thermal quarks.

B. Baryon Production from Thermal Quark Recombination

We can similarly generalize the quark recombination model for baryon fragmentation functions to the case in a thermal medium,

$$\begin{aligned} \tilde{D}_q^B(z_B, p^+) &= \frac{z_B^3}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_B - \frac{p^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \\ &\quad \int \frac{[d^2 k_\perp][dx]}{\sqrt{x_1} \sqrt{x_2} \sqrt{x_3}} \int \frac{[d^2 k'_\perp][dx']}{\sqrt{x'_1} \sqrt{x'_2} \sqrt{x'_3}} \varphi_B(k_{1\perp}, x_1; k_{2\perp}, x_2; k_{3\perp}, x_3) \varphi_B^*(k'_{1\perp}, x'_1; k'_{2\perp}, x'_2; k'_{3\perp}, x'_3) \\ &\quad \text{Tr} \left[\frac{\gamma^+}{2p_B^+} \sum_{\tilde{S}} \langle \langle \psi(0) | \tilde{S}; k_{1\perp}, x_1; k_{2\perp}, x_2; k_{3\perp}, x_3 \rangle \langle k'_{3\perp}, x'_3; k'_{2\perp}, x'_2; k'_{1\perp}, x'_1; \tilde{S} | \bar{\psi}(x) \rangle \rangle \right]. \end{aligned} \quad (102)$$

As in the case of meson fragmentation functions, the thermal average of the matrix element in the above baryon fragmentation function can be expressed as

$$\begin{aligned} &\langle \langle \psi(0) | \tilde{S}; k_{1\perp}, x_1; k_{2\perp}, x_2; k_{3\perp}, x_3 \rangle \langle k'_{3\perp}, x'_3; k'_{2\perp}, x'_2; k'_{1\perp}, x'_1; \tilde{S} | \bar{\psi}(x) \rangle \rangle \\ &= {}_k \langle \langle \psi(0) | \tilde{S}; k_1; k_2; k_3 \rangle \langle k'_3; k'_2; k'_1; \tilde{S} | \bar{\psi}(x) \rangle \rangle_k \\ &+ \int \frac{d^2 q_{1\perp}}{2(2\pi)^3} \frac{dq_1^+}{q_1^+} f(q_{1\perp}, q_1^+) \left[{}_k \langle \langle q_1 | \psi(0) | \tilde{S}; k_1; k_2; k_3 \rangle \langle k'_3; k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | q_1 \rangle \rangle_k \right. \\ &+ {}_k \langle \langle q_1 | \psi(0) | \tilde{S}; k_1; k_2; k_3 \rangle \langle k'_3; k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | q_1 \rangle \rangle_k + {}_k \langle \langle q_1 | \psi(0) | \tilde{S}; k_1; k_2; k_3 \rangle \langle k'_3; k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | q_1 \rangle \rangle_k \left. \right] \\ &+ \int \frac{d^2 q_{1\perp}}{2(2\pi)^3} \frac{dq_1^+}{q_1^+} \int \frac{d^2 q_{2\perp}}{2(2\pi)^3} \frac{dq_2^+}{q_2^+} f(q_{1\perp}, q_1^+) f(q_{2\perp}, q_2^+) \left[{}_k \langle \langle q_2; q_1 | \psi(0) | \tilde{S}; k_1; k_2; k_3 \rangle \langle k'_3; k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | q_1; q_2 \rangle \rangle_k \right. \\ &+ {}_k \langle \langle q_2; q_1 | \psi(0) | \tilde{S}; k_1; k_2; k_3 \rangle \langle k'_3; k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | q_1; q_2 \rangle \rangle_k + {}_k \langle \langle q_2; q_1 | \psi(0) | \tilde{S}; k_1; k_2; k_3 \rangle \langle k'_3; k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | q_1; q_2 \rangle \rangle_k \left. \right] \\ &+ \int \frac{d^2 q_{1\perp}}{2(2\pi)^3} \frac{dq_1^+}{q_1^+} \int \frac{d^2 q_{2\perp}}{2(2\pi)^3} \frac{dq_2^+}{q_2^+} \int \frac{d^2 q_{3\perp}}{2(2\pi)^3} \frac{dq_3^+}{q_3^+} \\ &\quad f(q_{1\perp}, q_1^+) f(q_{2\perp}, q_2^+) f(q_{3\perp}, q_3^+) {}_k \langle \langle q_3; q_2; q_1 | \psi(0) | \tilde{S}; k_1; k_2; k_3 \rangle \langle k'_3; k'_2; k'_1; \tilde{S} | \bar{\psi}(x) | q_1; q_2; q_3 \rangle \rangle_k. \end{aligned} \quad (103)$$

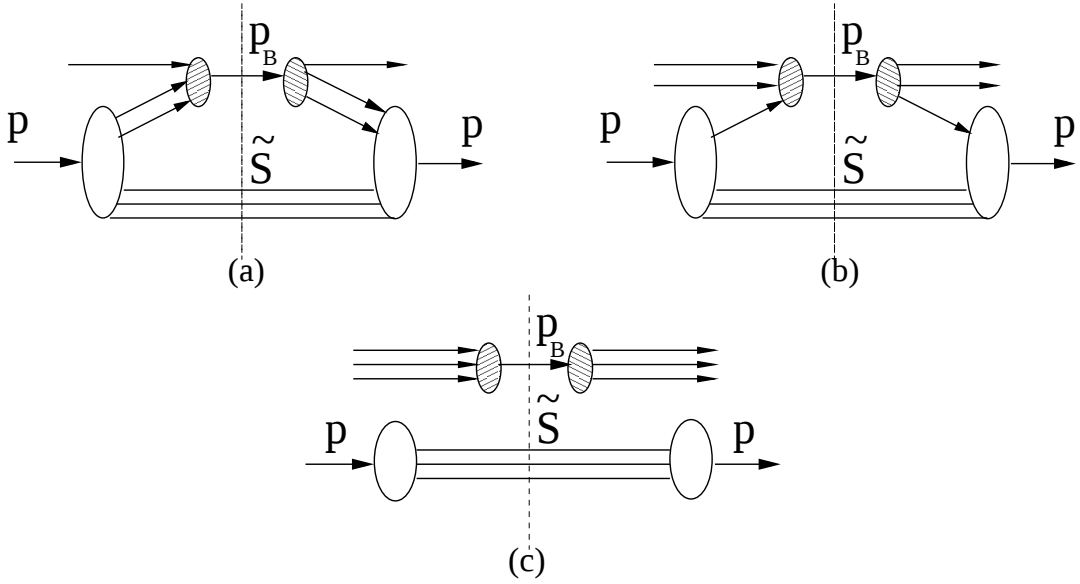


FIG. 5: The cut diagrams for contributions from (a) shower-shower-thermal, (b) shower-thermal-thermal and (c) thermal-thermal-thermal quark recombination to the single baryon fragmentation function.

Here, one can similarly identify the first term and the last term as the contributions from "shower-shower-shower" and "thermal-thermal-thermal" quark recombination, respectively. The others terms are the contributions from the "shower-shower-thermal" and "shower-thermal-thermal" quark recombination. These last three processes involving thermal quarks are illustrated in Fig. 5.

The contributions from "shower-shower-shower" quark recombination, denoted as $\tilde{D}_q^{B(SSS)}$, has the same expression as in vacuum [Eq.(56)] with the replacement of the triple quark distribution functions $F_q^{q_1 q_2 q_3}(z_1, z_2, z_3)$ in vacuum by its counterpart in medium $\tilde{F}_q^{q_1 q_2 q_3}(z_1, z_2, z_3)$. Working along the same approach, one can also find out the contribution from "thermal-thermal-thermal" quark recombination,

$$\begin{aligned} \tilde{D}_q^{B(TTT)}(z_B, p^+) &= \int \frac{d^2 q_{1\perp} dz_{q_1}}{2(2\pi)^3} \int \frac{d^2 q_{2\perp} dz_{q_2}}{2(2\pi)^3} \int \frac{d^2 q_{3\perp} dz_{q_3}}{2(2\pi)^3} f_{q_1}(q_{1\perp}, z_{q_1} p^+) f_{q_2}(q_{2\perp}, z_{q_2} p^+) f_{q_3}(q_{3\perp}, z_{q_3} p^+) S_q^0(z_B) \\ &\times \frac{4}{z_B} \left| \varphi_B(q_{1\perp}, \frac{z_{q_1}}{z_B}; q_{2\perp}, \frac{z_{q_2}}{z_B}; q_{3\perp}, \frac{z_{q_3}}{z_B}) \right|^2 \left[(2\pi)^3 \delta^{(2)}(p_{B\perp} - q_{1\perp} - q_{2\perp} - q_{3\perp}) \delta(z_B - z_{q_1} - z_{q_2} - z_{q_3}) \right]^2, \quad (104) \end{aligned}$$

where $S_q^0(z_B)$ is the same factor associated with phase-space integration as in Eq. (97). After extracting the spatial volume from the δ -function squared, one has

$$\begin{aligned} \tilde{D}_q^{B(TTT)}(z_B, p^+) &= V p^+ \int \frac{d^2 p_{B\perp}}{(2\pi)^3} \int_0^{z_B} \frac{dz_{q_1}}{2z_B} \int_0^{z_B - z_{q_1}} \frac{dz_{q_2}}{2z_B} \int \frac{d^2 q_{1\perp}}{(2\pi)^3} \int \frac{d^2 q_{2\perp}}{(2\pi)^3} \\ &f_{q_1}(q_{1\perp}, z_{q_1} p^+) f_{q_2}(q_{2\perp}, z_{q_2} p^+) f_{q_3}(p_{B\perp} - q_{1\perp} - q_{2\perp}, (z_B - z_{q_1} - z_{q_2}) p^+) \\ &R_B(q_{1\perp}, \frac{z_{q_1}}{z_B}; q_{2\perp}, \frac{z_{q_2}}{z_B}), \quad (105) \end{aligned}$$

which corresponds to an invariant inclusive baryon spectrum from thermal quark recombination,

$$\begin{aligned} (2\pi)^3 \frac{dN^{B(TTT)}}{dp_B^+ d^2 p_{B\perp}} &= V \int_0^1 dx_{q_1} \int_0^{1-x_{q_1}} dx_{q_2} \int \frac{d^2 q_{1\perp}}{2(2\pi)^3} \int \frac{d^2 q_{2\perp}}{2(2\pi)^3} f_{q_1}(q_{1\perp}, x_{q_1} p_b^+) f_{q_2}(q_{2\perp}, x_{q_2} p_b^+) \\ &\times f_{q_3}(p_{B\perp} - q_{1\perp} - q_{2\perp}, (1-x_{q_1}-x_{q_2}) p_B^+) R_B(q_{1\perp}, x_{q_1}; q_{2\perp}, x_{q_2}). \quad (106) \end{aligned}$$

The derivation of the contribution from the "shower-thermal-thermal" quark recombination to the baryon fragmen-

tation function is also straightforward. One has,

$$\begin{aligned} \tilde{D}_q^{B(STT)}(z_B, p^+) = & \sum_{q_3 \in B} \int \frac{d^2 q_{1\perp}}{(2\pi)^3} \int \frac{d^2 q_{2\perp}}{(2\pi)^3} \int_0^{z_B} \frac{dz_{q_1}}{2z_B} \int_0^{z_B - z_{q_1}} \frac{dz_{q_2}}{2z_B} f_{q_1}(q_{1\perp}, z_{q_1} p^+) f_{q_2}(q_{2\perp}, z_{q_2} p^+) \\ & \frac{1}{(1 - z_{q_1}/z_B - z_{q_2}/z_B)^2} R_B(q_{1\perp}, \frac{z_{q_1}}{z_B}; q_{2\perp}, \frac{z_{q_2}}{z_B}) \tilde{F}_q^{q_3}(z_B - z_{q_1} - z_{q_2}), \end{aligned} \quad (107)$$

where the summation is over different quark flavors of $q_3 \in B$ among the constituent quarks of the baryon B .

In deriving the contribution from “shower-shower-thermal” quark recombination, one encounters again the problem of interference between amplitudes of different shower quark recombination. We have to make the same approximation that one can neglect the interference term and the final result is proportional to the diagonal contribution with a constant C_{B2} . Therefore, we have

$$\begin{aligned} \tilde{D}_q^{B(SST)}(z_B, p^+) = & \sum_{(q_1 q_2) \in B} \frac{z_B^3}{2} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_B - \frac{p_B^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \int \frac{d^2 k_{1\perp}}{(2\pi)^3} \frac{dx_{k_1}}{2\sqrt{x_{k_1}}} \int \frac{d^2 k'_{1\perp}}{(2\pi)^3} \frac{dx'_{k_1}}{2\sqrt{x'_{k_1}}} \int \frac{d^2 q_{\perp}}{(2\pi)^3} \frac{dx_q}{2} \\ & \frac{1}{\sqrt{x_{k_2} x'_{k_2}}} f_{q_3}(q_{\perp}, x_q p_B^+) \varphi_B(q_{\perp}, x_q; k_{1\perp}, x_{k_1}; k_{2\perp}, x_{k_2}) \varphi_B^*(q_{\perp}, x_q; k'_{1\perp}, x'_{k_1}; k'_{2\perp}, x'_{k_2}) \\ & \text{Tr} \left[\frac{\gamma^+}{2p_B^+} \sum_S k \langle \langle \psi(0) | S; k_{1\perp}, x_{k_1}; k_{2\perp}, x_{k_2} \rangle \langle k'_{2\perp}, x'_{k_2}; k'_{1\perp}, x'_{k_1}; S | \bar{\psi}(x) \rangle \rangle_k \right] \\ \approx & \tilde{C}_{B2} \frac{z_B^3}{2} \sum_{(q_1 q_2) \in B} \int \frac{d^4 p}{(2\pi)^4} \delta\left(z_B - \frac{p_B^+}{p^+}\right) \int d^4 x e^{-ip \cdot x} \int \frac{d^2 k_{1\perp}}{(2\pi)^3} \frac{dx_{k_1}}{4x_{k_1}} \int \frac{d^2 q_{\perp}}{(2\pi)^3} \frac{dx_q}{2} \\ & \frac{1}{x_{k_2}} f_{q_3}(q_{\perp}, x_q p_B^+) |\varphi_B(q_{\perp}, x_q; k_{1\perp}, x_{k_1}; k_{2\perp}, x_{k_2})|^2 \\ & \text{Tr} \left[\frac{\gamma^+}{2p_B^+} \sum_S k \langle \langle \psi(0) | S; k_{1\perp}, x_{k_1}; k_{2\perp}, x_{k_2} \rangle \langle k_{2\perp}, x_{k_2}; k_{1\perp}, x_{k_1}; S | \bar{\psi}(x) \rangle \rangle_k \right], \end{aligned} \quad (108)$$

where $k_{2\perp} = p_{B\perp} - q_{1\perp} - k_{1\perp}$, $k'_{2\perp} = p_{B\perp} - q_{1\perp} - k'_{1\perp}$, $x_{k_2} = 1 - x_{q_1} - x_{k_1}$, $x'_{k_2} = 1 - x_{q_1} - x'_{k_1}$, $\tilde{C}_{B2}$ is a constant with the dimension of momentum and the summation is over all possible $(q_1 q_2) \in B$ quark pairs among the three constituent quarks of the baryon B . Changing the variables $x_{k_1}, x_{k_2}, x_{q_1}$ to $z_{k_1}, z_{k_2}, z_{q_1}$ and neglecting the intrinsic momentum in the baryon wavefunction, $|\varphi_B(q_{\perp}, x_q; k_{1\perp}, x_{k_1}; k_{2\perp}, x_{k_2})|^2 \approx R_B(q_{\perp}, x_q; 0_{\perp}, x_{k_1})$, we have

$$\begin{aligned} \tilde{D}_q^{B(SST)}(z_B, p^+) \approx & \sum_{(q_1 q_2) \in B} \tilde{C}_{B2} \int \frac{d^2 q_{\perp}}{(2\pi)^3} \int_0^{z_B} \frac{dz_q}{2z_B} \int_0^{z_B - z_q} \frac{dz}{2z_B} f_{q_3}(q_{\perp}, z_q p^+) \\ & \times \frac{1}{(1 - z_q/z_B)^2} R_B(q_{\perp}, \frac{z_q}{z_B}; 0_{\perp}, \frac{z}{z_B}) \tilde{F}_q^{q_1 q_2}(z, z_B - z_q - z). \end{aligned} \quad (109)$$

The diquark distribution $\tilde{F}_q^{q_1 q_2}(z_1, z_2)$ is the same as in the “shower-shower” contribution to the meson fragmentation in Eq. (29), which is similarly defined as in the vacuum except that the thermal average in principle should include medium effects such as induced radiation and absorption.

Finally, the effective baryon fragmentation function is the sum over the contributions resulting from “shower-shower-shower”, “shower-shower-thermal”, “shower-thermal-thermal” and “thermal-thermal-thermal” constituent quark recombination,

$$\tilde{D}_q^B(z_B, p^+) = \tilde{D}_q^{B(SSS)}(z_B) + \tilde{D}_q^{B(SST)}(z_B, p^+) + \tilde{D}_q^{B(STT)}(z_B, p^+) + \tilde{D}_q^{B(TTT)}(z_B, p^+), \quad (110)$$

though the “thermal-thermal-thermal” contribution is not correlated with the initial parton jet and thus is only the background from the hadronization of the thermal medium.

VI. CONCLUSION

In this paper, we have studied medium modification to the effective parton fragmentation functions due to quark

recombination during the hadronization of the parton jet together with a quark-gluon plasma.

We started with a formulation of the vacuum par-

ton fragmentation functions in the parton operator form within a constituent quark model. In this model, constituent quarks are the effective degrees of freedom during the hadronization of both the parton jet and the quark-gluon plasma, which is essentially a thermalized gas of constituent quarks. Final hadrons are composed of constituent quarks with given wavefunctions. We showed that for sharply peaked hadron wavefunctions, one can neglect the interference between amplitudes of hadron formation from recombination of constituent quarks with different momenta. Consequently, we were able to cast the meson (baryon) fragmentation functions into a convolution of the recombination probability and constituent diquark (triquark) distribution functions of the fragmenting parton. The recombination probability is determined by the hadron's wavefunction in the constituent quark model. The diquark or triquark quark distribution functions of the fragmenting parton are defined as the overlapping matrices between parton field operator and the final constituent quark states, similarly as the dihadron or trihadron fragmentation functions defined as the overlapping matrices between parton operator and final hadron states. We derived the QCD DGLAP evolution equations for the diquark and triquark distributions functions which are a little different from that of dihadron and trihadron fragmentation functions because of the kinematic constraints imposed by the hadrons' wavefunctions in the constituent quark model. We further discussed possible connections between triquark, diquark and single quark distribution functions through sum rules.

Working within the framework of field theory at finite temperature, we extended the formulation of parton fragmentation functions in terms of quark recombination to the case of jet fragmentation in medium, assuming that the medium is hadronizing together with the parton jets. Replacing the vacuum expectation in the overlapping matrices by the thermal average, we were able to derive the medium modification to the fragmentation functions via quark recombination. The medium modification comes not only as the medium modification of the diquark or triquark distribution functions in the recombination of constituent quarks from the fragmenting parton, called shower quark recombination, but also as additional con-

tributions from recombination between shower and thermal constituent quarks. We also obtained contributions from recombination of thermal quarks within the same formalism. However, such contributions of thermal quark recombination are disconnected with parton jet fragmentation and therefore are just a thermal background to jet fragmentation processes in the medium.

The formulation and derivation of jet fragmentation functions in terms of quark recombination in the vacuum do not simplify the theoretical description of the jet fragmentation processes. It is only a model of the non-perturbative process of hadronization. However, once extended to jet fragmentation in medium, it provides a natural framework for the description of jet hadronization in medium and the study of effective medium modification to the fragmentation functions. Within this formalism, one can not only include contributions to jet fragmentation from the recombination of constituent quarks from the parton jet and the thermal medium but also the medium modification of the quark distributions of the jet parton due to induced gluon bremsstrahlung and absorption. This framework will effectively unify parton energy loss via induced gluon radiation and quark recombination processes during hadronization.

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