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Quantum feedback control of a solid-state two-level system

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We have studied theoretically the basic operation of a quantum feedback loop designed to maintain the desired phase of quantum coherent oscillations in a two-level system. Such feedback can suppress the dephasing of oscillations due to interaction with environment. Prospective experiments can be realized using metallic single-electron devices or GaAs technology.

The principle of feedback control is used in a wide variety of physical and engineering problems. In particular, it can be applied in a straightforward way to tune the oscillation phase of a harmonic oscillator in order to achieve a desired synchronization with some reference oscillator. An intriguing and fundamental question is whether continuous feedback can be used to control quantum systems; for instance, if it is possible or not to tune the phase of quantum coherent (Rabi) oscillations in a two-level system (TLS).

At first sight the quantum feedback seems to be impossible because according to the “orthodox” collapse postulate¹ the quantum state is abruptly destroyed by the act of measurement. However, as was shown two decades ago, in particular by Leggett,² in a typical solid-state setup the collapse of a TLS state should be considered as a continuous process rather than as instantaneous event. The reason is typically weak coupling between the quantum system and the detector and also the finite noise of the detector, so that it takes some time until acceptable signal-to-noise ratio is reached and the measurement can be regarded as completed.

While the Leggett’s theory as well as the majority of similar approaches can describe only *ensembles* of quantum systems, the theory describing the gradual collapse of a *single* solid-state TLS was developed only recently.^{3–5} (A similar problem in optics was solved much earlier – see, e.g. Refs.^{6,7} and references in⁴.) Basically, the theory says that the evolution of a single quantum system due to continuous measurement is governed by the information continuously acquired from the detector. Similarly to classical probability, the Bayes formula⁸ which naturally takes into account incomplete information from the detector, can still be applied to the density matrix of the measured quantum system; thus the formalism is called Bayesian.³

In case of a poor detector the extra noise acting on the input disturbs the measured system stronger than the limit determined by the uncertainty principle; this leads to gradual decoherence of the measured system. In contrast, when measured with a good (quantum-limited)

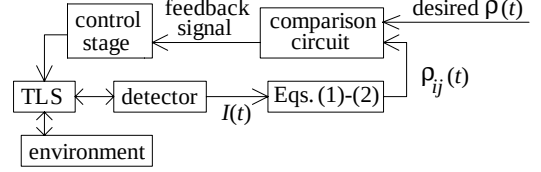


FIG. 1. Schematic of the quantum feedback loop maintaining the quantum oscillations in a two-level system (TLS).

detector, the quantum system does not lose the coherence (even though the quantum state evolves randomly); moreover, its density matrix can be gradually purified³ that basically means acquiring as much information about the system as permitted by quantum mechanics.

Since the Bayesian formalism allows us to monitor the continuous evolution of a quantum system in a process of measurement, this naturally gives rise to a possibility of continuous feedback control of a quantum system. In this paper we will study the operation of a feedback loop proposed in Ref.⁴ and designed to maintain a desired phase of quantum coherent oscillations in a solid-state TLS. (Quantum feedback in optics has been proposed and studied earlier – see, e.g., Refs.^{7,9,10}.) In particular, we will study the dependence of the loop operation on the feedback factor F , the available bandwidth τ_a^{-1} , and the dephasing rate γ_e due to environment.

As an example of the measurement setup (Fig. 1) we consider a TLS represented by a single electron in a double quantum dot (DQD), the location of which is measured by a quantum point contact (QPC) nearby in a way used in Ref.¹¹. If the electron is in the dot 2 (state $|2\rangle$) which is closer to QPC than dot 1, then the QPC tunnel barrier is higher and so the average current I_2 through QPC is smaller than the average current I_1 corresponding to electron in the dot 1 (state $|1\rangle$). Consequently, from the QPC current one gets information about the electron location. We consider a realistic case of weak response, $\Delta I \equiv I_1 - I_2 \ll I_0 \equiv (I_1 + I_2)/2$. In this case the measurement time $S_I/2(\Delta I)^2$, which is necessary to achieve signal-to-noise ratio equal to 1 (here S_I is the shot noise of the QPC current), is much larger than e/I_0 , so the QPC current $I(t)$ is continuous on the measurement timescale and we do not need to consider individual tunneling events in QPC.

The evolution of the TLS density matrix ρ during the measurement process is described within the Bayesian formalism by equations^{3,4}

$$\dot{\rho}_{11} = -\dot{\rho}_{22} = -2\frac{H}{\hbar}\text{Im}\rho_{12} + \rho_{11}\rho_{22}\frac{2\Delta I}{S_I}[I(t) - I_0], \quad (1)$$

$$\begin{aligned} \dot{\rho}_{12} = & i\frac{\varepsilon}{\hbar}\rho_{12} + i\frac{H}{\hbar}(\rho_{11} - \rho_{22}) \\ & - (\rho_{11} - \rho_{22})\frac{\Delta I}{S_I}[I(t) - I_0]\rho_{12} - \gamma\rho_{12}, \end{aligned} \quad (2)$$

where ε and H are, respectively, the energy asymmetry and tunneling strength of the TLS [the TLS Hamiltonian is $\mathcal{H}_{TLS} = (\varepsilon/2)(c_1^\dagger c_1 - c_2^\dagger c_2) + H(c_1^\dagger c_2 + c_2^\dagger c_1)$], and $\gamma = \gamma_d + \gamma_e$ is the dephasing rate due to detector nonideality and coupling with environment. Theoretically, $\gamma_d = 0$ when TLS is measured by a QPC; however, if instead of QPC we use a single-electron transistor, then dephasing γ_d is always significant.⁴

Notice that the ensemble dephasing rate $\Gamma = \gamma + (\Delta I)^2/4S_I$ is larger than γ because of different evolution of the ensemble members due to random $I(t)$. Individual realizations can be simulated using the formula

$$I(t) - I_0 = (\rho_{11} - \rho_{22})\Delta I/2 + \xi(t), \quad (3)$$

where $\xi(t)$ is the pure white noise with spectral density $S_\xi = S_I$. If Eqs. (1)–(2) are averaged over $\xi(t)$ (we use Stratonovich definition for stochastic differential equations), then we get usual ensemble-averaged equations for TLS evolution (terms proportional to ΔI will disappear and γ will be replaced by Γ).

It is natural to characterize the effect of extra dephasing γ_d by the detector ideality (efficiency) $\eta \equiv 1/[1 + \gamma_d 4S_I/(\Delta I)^2]$. One can show^{4,12} that $\eta = (\hbar/2\epsilon_d)^2$ where ϵ_d is the energy sensitivity of the detector including the effect of backaction but neglecting the correlation between output and backaction noises. So, an ideal case $\eta = 1$ corresponds to a detector with quantum-limited sensitivity.

To realize a feedback loop (Fig. 1), we can monitor the TLS evolution using the detector current $I(t)$ plugged into Eqs. (1)–(2). Then the TLS state is compared with the desired state, and the difference signal is used to control the TLS parameters H and/or ε . In the example studied in this paper the feedback loop is designed to stabilize the quantum oscillations of the state of a symmetric TLS ($\varepsilon = 0$), so the desired evolution is $\rho_{11}(t) = 1 - \rho_{22}(t) = [1 + \cos(\Omega t)]/2$, $\rho_{12}(t) = \rho_{21}^*(t) = i \sin(\Omega t)/2$, where the frequency is $\Omega = (4H^2 + \varepsilon^2)^{1/2}/\hbar = 2H/\hbar$. As a difference (“error”) signal we use the phase difference $\Delta\phi$ between the desired value $\phi_0(t) = \Omega t$ and the monitored value $\phi(t) \equiv \arctan\{2\text{Im}\rho_{12}(t)/[\rho_{11}(t) - \rho_{22}(t)]\}$. This difference is used to control the TLS parameter H (controlling the barrier height of DQD); we assume a linear dependence: $H_{fb} = H[1 - F \times \Delta\phi]$, where F is the dimensionless feedback factor.

In this paper we neglect additional time delay⁴ in the feedback network, however, we take into account the finite bandwidth of a line carrying detector current (that is the critical parameter for a possible experiment). More specifically, we average the current $I(t)$ with a rectangular window of duration τ_a , $I_a(t) \equiv \tau_a^{-1} \int_{t-\tau_a}^t I(t') dt'$,

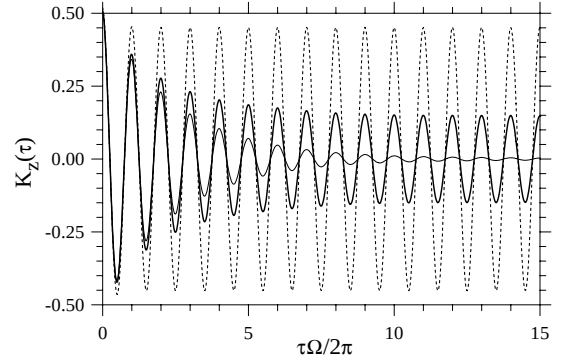


FIG. 2. Correlation function $K_z(\tau)$ of the TLS quantum oscillations for $\mathcal{C} = 1$ and feedback factors $F = 0$ (thin solid line), 0.05 (thick solid line), and 0.5 (dashed line). Nondecaying oscillations are due to synchronization by the feedback.

before plugging it into Eqs. (1)–(2), so that the “available” density matrix $\rho_a(t)$ differs from the “true” density matrix $\rho(t)$. Also, to compensate for the corresponding implicit time delay, we use $\Delta\phi = \phi_a - \Omega(t - \kappa\tau_a)$ with $\kappa = 1/2$ (we tried various κ and found that $\kappa = 1/2$ provides the best operation of the feedback loop).

Let us start with the case of ideal detector, $\eta = 1$, absence of extra environment, $\gamma_e = 0$, and infinite bandwidth, $\tau_a = 0$. Figure 2 shows numerically calculated correlation function $K_z(\tau) \equiv \langle z(t+\tau)z(t) \rangle$ where $z \equiv \rho_{11} - \rho_{22}$, for several feedback factors: $F = 0$, 0.05, and 0.5. The curves are obtained using Monte Carlo simulation^{3,4} of the measurement process for moderately weak coupling between TLS and the detector: $\mathcal{C} \equiv \hbar(\Delta I)^2/S_I H = 1$ (notice that the Q -factor of oscillations¹³ is equal to $8/\mathcal{C}$, so $\mathcal{C} = 1$ is still a weak coupling). In absence of feedback ($F = 0$) the correlation function decays to zero (Fig. 2) while for finite feedback factor the correlations remain for indefinitely long time (of course, assuming perfect reference oscillator which determines desired evolution). The nondecaying correlations show that the quantum feedback loop really provides the synchronization of quantum oscillations.

The degree of synchronization depends on the feedback factor F . One can see that for a moderate value of $F = 0.5$ the synchronization is already very good [the ideal case would be $K_z(\tau) = \cos(\Omega\tau)/2$]. For the case of moderate or good synchronization ($\mathcal{C}/16F \lesssim 1$, $\mathcal{C}/8 \ll 1$) we have derived the following analytical expression:

$$K_z(\tau) = \frac{\cos \Omega\tau}{2} \exp \left[\frac{\mathcal{C}}{16F} \left(e^{-2FH\tau/\hbar} - 1 \right) \right]. \quad (4)$$

The correlation function $K_I(\tau) \equiv \langle I(t+\tau)I(t) \rangle$ of the detector current $I(t)$ have somewhat similar dependence, however, it also has the decaying contribution¹³ due to correlation $K_{z\xi}$ and δ -function contribution due to detector noise. The analytical result,

$$K_I(\tau) = \frac{S_I}{2} \delta(\tau) + \frac{(\Delta I)^2}{4} \frac{\cos(\Omega\tau)}{2} \left(1 + e^{-2FH\tau/\hbar} \right)$$

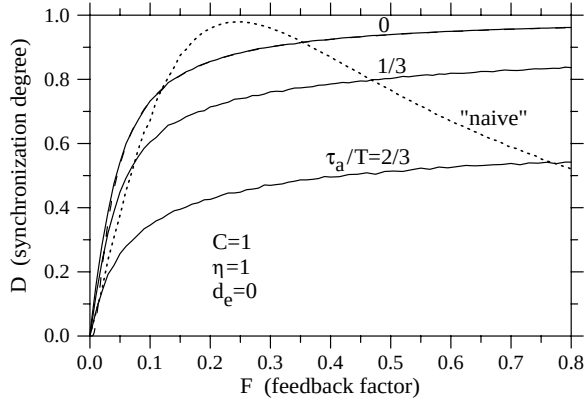


FIG. 3. Synchronization degree D as a function of feedback factor F for several values τ_a of detector signal averaging: $\tau_a/T = 0, 1/3$, and $2/3$, where $T = 2\pi/\Omega$. Dashed line $D = \exp(-C/32F)$ almost coincides with the upper curve. Dotted line corresponds to “naive” feedback with $\tau_a = T/10$.

$$\times \exp\left[\left(\mathcal{C}/16F\right)\left(e^{-2FH\tau/\hbar} - 1\right)\right], \quad (5)$$

agrees well with the Monte Carlo results.

The spectral density $S_I(\omega)$ of the detector current can be obtained as a Fourier transform of $K_I(\tau)$. While in absence of feedback the quantum oscillations in TLS can provide only a moderate peak of $S_I(\omega)$ around frequency Ω (the peak height cannot be larger than 4 times the noise pedestal¹³), the feedback synchronization leads to the appearance of a δ -function at the frequency of desired oscillations. (In principle the desired frequency can differ a little from Ω ; however, in this case the performance of the feedback loop worsens.)

Besides the correlation function and spectral density, we have studied one more characteristic, D , of the synchronization degree. We define D as the average scalar product of the unity-length vector on the Bloch sphere corresponding to the desired state and the vector corresponding to the actual state of TLS. The equivalent definition is $D \equiv 2\langle\text{Tr}\rho\rho_d\rangle - 1$, where ρ_d is the density matrix of the desired pure state. Perfect synchronization corresponds to $D = 1$. It is simple to show that in the case of weak coupling and unshifted desired frequency, $D^2/2$ is equal to the nondecaying amplitude of $K_z(\tau)$ dependence.

Upper solid line in Fig. 3 shows the dependence of D on the feedback factor F for $\mathcal{C} = 1$ and $\tau_a = 0$. One can see that D is proportional to F for small F (“soft” onset of synchronization) and D is asymptotically approaching 1 at large F . Using Eq. (4) it is simple to obtain expression $D = \exp(-\mathcal{C}/32F)$ which is very close to numerical results for moderate and good synchronization (dashed line in Fig. 3).

Finite available bandwidth of the detector current $I(t)$ (finite averaging time τ_a in our formalism) worsens the performance of the quantum feedback loop. The solid lines in Fig. 3 show the dependence of the synchroniza-

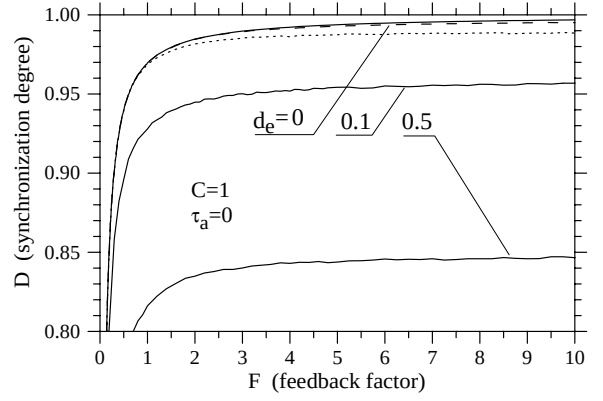


FIG. 4. Dependence $D(F)$ for $\mathcal{C} = 1$, $\tau_a = 0$, and several magnitudes of dephasing due to environment: $d_e = 0, 0.1$, and 0.5 . Dashed and dotted lines correspond to $d_e = 0$ and limitation of H_{fb} by 0 and $H/2$, respectively.

tion degree $D(F)$ for $\tau_a/T = 0, 1/3$, and $2/3$, where $T = 2\pi/\Omega$ is the oscillation period. Obviously, a significant information loss occurs when τ_a becomes comparable to T , leading to a decrease of D . The curves $D(F)$ saturate at large F allowing us to introduce the dependence $D_{max}(\tau)$. Calculations for parameters of Fig. 3 show pretty good synchronization, $D_{max} = 0.993$, for $\tau_a = T/30$, while $D_{max} = 0.98$ for $\tau_a = T/10$, $D_{max} = 0.92$ for $\tau_a = T/3$, and $D_{max} = 0.57$ for $\tau_a = 2T/3$.

The main potential practical importance of the quantum feedback is the ability to suppress the effect of TLS dephasing caused by interaction with environment (see Fig. 1). Solid lines in Fig. 4 show the dependence $D(F)$ for several magnitudes of the dephasing due to environment, $d_e = 0, 0.1$, and 0.5 , where $d_e \equiv \gamma_e/[(\Delta I)^2/4S_I]$ is the ratio between TLS coupling to the environment and to the detector (we still assume an ideal detector). First of all, we see that feedback still maintains the phase coherence of TLS for infinitely long time. However, for finite d_e the degree of synchronization D saturates at the level less than unity. The dependence $D_{max}(d_e)$ is linear at small d_e , numerically we obtained $D_{max} \simeq 1 - 0.5d_e$ for parameters of Fig. 4.

Notice that the solid lines shown in Figs. 3 and 4 are calculated assuming the TLS feedback control $H_{fb} = H[1 - F \times \Delta\phi]$ even when H_{fb} becomes negative (this is also an assumption for analytical results). To eliminate this unphysical assumption we have also performed numerical calculations with restriction $H_{fb} > 0$ and with restriction $H_{fb} > H/2$. This leads to rather minor modifications of the presented curves (dashed and dotted lines in Fig. 4 show the results for $d_e = 0$ and $\tau_a = 0$). However, important difference is that $D(F)$ goes down at large F , so the optimum D_{max} is achieved at some finite value of F . More detailed study of this problem will be presented elsewhere.

Besides the discussed feedback based on $\Delta\phi$ calcula-

tion, we have also studied much simpler feedback loop in which $H_{fb}(t)/H - 1 = F\{2[I_a(t) - I_0]/\Delta I - \cos[\Omega(t - \tau_a/2)]\} \sin[\Omega(t - \tau_a/2)]$ (in an experiment it would eliminate the necessity to solve fast the Bayesian equations). Quite surprisingly, such “naive” feedback can also provide a good phase synchronization of quantum oscillations if F/C is close to $1/4$ (see dotted line in Fig. 3). However, it requires more careful choice of F and τ_a than for the Bayesian feedback, and also suffers more significantly from the restriction on H_{fb} variation. The results in more detail will be discussed elsewhere.

Experimentally, besides the realization of quantum feedback control of a DQD continuously measured by a QPC, one can also think about the TLS based on a single-Cooper-pair box measured by a single-electron transistor (see discussion in⁴). This realization can be preferable because of a rapid progress of metallic single-electronics technology. However, the problems are high output impedance of the single-electron transistor and its significant nonideality as a quantum detector. The third potential realization can be based on SQUIDS. For any realization the major problem is bandwidth: the feedback should be faster than the TLS dephasing due to environment. Because of that, the quantum feedback of a TLS should probably be attempted only after the realization of recently proposed Bell-type two-detector correlation experiment.¹⁴

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- ¹ J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton Univ. Press, Princeton, 1955).
 - ² A. O. Caldeira and A. J. Leggett, *Ann. Phys. (N.Y.)* **149**, 374 (1983).
 - ³ A. N. Korotkov, *Phys. Rev. B* **60**, 5737 (1999).
 - ⁴ A. N. Korotkov, *Phys. Rev. B* **63**, 115403 (2001).
 - ⁵ H.-S. Goan, G. J. Milburn, H. M. Wiseman, and H. B. Sun, *cond-mat/0006333*.
 - ⁶ H. J. Carmichael, *An open system approach to quantum optics*, Lecture notes in physics (Springer, Berlin, 1993).
 - ⁷ H. M. Wiseman and G. J. Milburn, *Phys. Rev. Lett.* **70**, 548 (1993).
 - ⁸ W. Feller, *An Introduction to Probability Theory and Its Applications*, v. 1 (Wiley, NY, 1968).
 - ⁹ P. Tombesi and D. Vitali, *Phys. Rev. A* **51**, 4913 (1995).
 - ¹⁰ A. C. Doherty and K. Jacobs, *Phys. Rev. A* **60**, 2700 (1999).
 - ¹¹ E. Buks, R. Schuster, M. Heiblum, D. Mahalu, and V. Umansky, *Nature* **391**, 871 (1998).
 - ¹² D. V. Averin, *cond-mat/0004364*.
 - ¹³ A. N. Korotkov, *Phys. Rev. B* **63**, 085312 (2001).
 - ¹⁴ A. N. Korotkov, *cond-mat/0008003*.