

UCLA

UCLA Previously Published Works

Title

A New Scaling Law for Nondipolar Magnetic Fields in Rapidly Rotating Stars and Planets

Permalink

<https://escholarship.org/uc/item/1r83d7hv>

Journal

The Astrophysical Journal, 1006(2)

ISSN

0004-637X

Authors

Wulff, Paula N

Cao, Hao

Aurnou, Jonathan M

Publication Date

2026-08-01

DOI

10.3847/1538-4357/ae81a0

Copyright Information

This work is made available under the terms of a Creative Commons Attribution License, available at <https://creativecommons.org/licenses/by/4.0/>

Peer reviewed



A New Scaling Law for Nondipolar Magnetic Fields in Rapidly Rotating Stars and Planets

Paula N. Wulff¹, Hao Cao¹, and Jonathan M. Aurnou¹

Department of Earth, Planetary, and Space Sciences, University of California, Los Angeles (UCLA), 595 Charles E Young Dr East, Los Angeles, CA 90095, USA;

paulawulff@epss.ucla.edu

Received 2026 April 20; revised 2026 June 5; accepted 2026 June 22; published 2026 July 29

Abstract

Magnetic field generation in giant planets and rapidly rotating stars produces a diverse range of field geometries, from large-scale dipole-dominated configurations to complex, small-scale multipolar structures. Earlier dynamo studies have suggested that multipolar solutions tend to arise when rotational effects become less dominant. We investigate the strength of nondipolar magnetic fields generated in systems dominated by rotation. 40 three-dimensional, spherical-shell dynamo simulations were carried out using the MagIC code, primarily made up of bistable pairs—simulations with the same control parameters that can settle in both a dipolar and nondipolar steady state regime. We use this suite of models to test how their magnetic field strength scales with heat flux and velocity. Our dynamo simulations produce magnetic fields with morphologies that fall on the two distinct branches, dipolar and nondipolar, yet have very similar convective velocities. The strength of the dipole component differs by an order of magnitude between the two regimes, when scaled as a function of driving power. However, their nondipolar magnetic field strengths are very similar. We conclude that when attempting to predict the magnetic field strength of rapidly rotating planets and stars, one cannot assume that it will have a dipole-dominated geometry. In particular, the amplitude of the dipole component is expected to be an order of magnitude smaller in the nondipolar regime.

Unified Astronomy Thesaurus concepts: [Magnetic fields \(994\)](#); [Solar system gas giant planets \(1191\)](#); [Uranus \(1751\)](#); [Neptune \(1096\)](#); [Exoplanets \(498\)](#); [M dwarf stars \(982\)](#)

1. Introduction

Planetary and stellar magnetic fields are generated by dynamo action in which convective motions of electrically conducting fluids sustain magnetic fields against ohmic dissipation (H. K. Moffatt 1978; P. H. Roberts & G. A. Glatzmaier 2000; H. K. Moffatt & E. Dormy 2019). Identifying robust scaling laws that relate the amplitude and morphology of these fields to underlying control parameters, such as rotation rate and heat flux, has been a central goal of dynamo theory. In rapidly rotating systems—relevant to Earth, the gas giants, and many low-mass stars—the Coriolis force strongly constrains fluid motions, often leading to large-scale magnetic fields with a dominant axial dipole component (e.g., U. R. Christensen & J. Aubert 2006; J. Aubert et al. 2009; M. A. Calkins et al. 2021). Scaling arguments, based on a balance between the ohmic dissipation associated with the magnetic field and the energy flux available to drive dynamo action, have successfully described aspects of dipole-dominated dynamos, providing benchmarks that connect numerical models with planetary and stellar observations (e.g., U. R. Christensen & J. Aubert 2006; U. R. Christensen et al. 2009; U. R. Christensen 2010; P. A. Davidson 2013).

However, many natural dynamos operate outside this dipolar regime. Nondipolar, or multipolar, fields are common, ranging from the complex morphologies of Uranus and Neptune (N. F. Ness et al. 1986; J. E. P. Connerney et al. 1991; R. Holme & J. Bloxham 1996) to the variable large-

scale fields observed in rapidly rotating M dwarfs (e.g., J. Morin et al. 2010; D. Shulyak et al. 2017; S. Bellotti et al. 2024). Numerical studies have revealed that the transition from dipolar to multipolar regimes is controlled by a delicate balance between buoyancy forcing, rotation, and magnetic feedbacks (M. D. Menu et al. 2020; T. Tassin et al. 2021; B. Zaire et al. 2022; K. M. Soderlund et al. 2025). Yet the scaling behavior of nondipolar dynamos remains less well constrained. Very few previous studies consider whether the laws derived in dipolar contexts extend to multipolar fields, or whether distinct force balances and scaling regimes emerge (R. K. Yadav et al. 2013a, 2013b).

In this study, we investigate dynamo scaling laws in the rapidly rotating regime with a particular focus on nondipolar field configurations. Using numerical dynamo simulations across a broad range of Rayleigh and Ekman numbers, we quantify how magnetic energy, morphology, and characteristic length scales depend on buoyancy flux and rotation rate. We focus on the energy flux scaling laws of U. R. Christensen & J. Aubert (2006) and U. R. Christensen et al. (2009) (we will later refer to the nondimensional form of this law as [CA06], and the dimensional form as applied to stars as [C09]). By systematically comparing dipolar and nondipolar cases, we evaluate the universality of the existing scaling law and identify necessary extensions to account for multipolar magnetic fields.

Unlike previous studies, we perform separate analyses of the dipole component in both regimes. First, the dipole is the most distinguishing difference between the magnetic fields in the two regimes. It is further motivated by the fact that it is the component that can be most reliably determined by observations. For all solar system planets, and for most exoplanets, the dynamo region does not extend to the surface. As a result, even



Original content from this work may be used under the terms of the [Creative Commons Attribution 4.0 licence](#). Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.

for planetary magnetic fields measured in situ, we can accurately determine only their large-scale components. This is particularly true for Uranus and Neptune, given the single flyby Voyager 2 measurements. The small-scale components lose power rapidly before reaching the planetary surface. Consequently, it becomes difficult to test any scaling relation for the total magnetic field strength without making assumptions about how it relates to the observed large-scale field. Moreover, many key phenomena linked to a star's or planet's magnetic field outside the body—such as the stellar wind or a planetary magnetosphere—depend primarily on the dipole component. This further underscores the importance of developing reliable methods for estimating it.

Our results aim to provide a more comprehensive framework for connecting dynamo theory with the observed diversity of planetary and stellar magnetic fields.

2. Methods

Systems where rotation plays a significant role often feature dipolar magnetic fields. Analysing multipolar magnetic field generation in the same regimes requires exploring a parameter space where there is a bistability of the two states. Thus, we base our modeling approach on one of the previous studies where multiple bistable cases were identified (R. K. Yadav et al. 2013a) and expand the dual (dipolar–multipolar) parameter space from there. Numerous bistable dynamo states have also been observed in studies using the anelastic approximation (e.g., T. Gastine et al. 2012), and including an anelastic radial density contrast is no doubt closer to reality for most planets and stars. However, as this is the first study to explicitly focus on multipolar magnetic fields in rotation-dominated dynamo models, we choose a Boussinesq approach to lay down the groundwork before introducing more complexity.

2.1. Governing Equations

Our numerical setup consists of a spherical shell that rotates along the z -axis and is bounded by inner radius r_i and outer radius r_o . The aspect ratio r_i/r_o is 0.35. A linear variation of gravity with radius is assumed. We nondimensionalize the magnetohydrodynamic (MHD) equations by using the shell thickness $r_o - r_i = D$ as the reference length scale and viscous diffusion time $\tau_\nu = D^2/\nu$, where ν is the fluid viscosity, as the time unit. The magnetic field $\mathbf{B}$ is scaled by $\sqrt{\rho\mu\lambda\Omega}$, where ρ is the constant fluid density, μ is the magnetic permeability, λ is the constant fluid magnetic diffusivity, and Ω is the rotation rate. The velocity is scaled by the Reynolds number $Re = uD/\nu$.

The temporal evolution of velocity $\mathbf{u}$, temperature T , and magnetic field $\mathbf{B}$ is described by the MHD equations under the Boussinesq approximation:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \frac{2}{E} \hat{\mathbf{e}}_z \times \mathbf{u} + \nabla P = \frac{Ra}{Pr} g(r) T \hat{\mathbf{e}}_r + \frac{1}{E Pm} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nabla^2 \mathbf{u}, \quad (1)$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{Pr} \nabla^2 T, \quad (2)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{Pm} \nabla^2 \mathbf{B}, \quad (3)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (4)$$

$$\nabla \cdot \mathbf{B} = 0. \quad (5)$$

This system of equations is governed by several nondimensional control parameters: Ekman number $E = \nu/\Omega D^2$; the Rayleigh number $Ra = \alpha g_o D^3 \Delta T / \nu \kappa$, where g_o is gravity at the outer boundary, α is the thermal expansivity, and κ is the thermal conductivity; magnetic Prandtl number $Pm = \nu/\lambda$; and Prandtl number $Pr = \nu/\kappa$. The values of control parameters used in our suite of simulations are tabulated in Appendix E.

We also consider the convective Rossby number $Ro_c = \sqrt{Ra E^2 / Pr}$, which estimates the ratio of buoyancy and Coriolis forces (e.g., J. M. Aurnou et al. 2020). When $Ro_c \ll 1$, we consider a system to be in the rapidly rotating regime. The $Ro_c > 1$ regime is not explored in this study.

We assume free-slip mechanical boundaries at both the inner and outer radii. The magnetic field matches a potential field at both boundaries. A fixed temperature contrast ΔT is maintained between the top and the bottom.

2.2. Numerical Methods

We use the open-source three-dimensional MHD code MagIC (<https://magic-sph.github.io/>, U. R. Christensen et al. 2001; J. Wicht 2002; T. Gastine & J. Wicht 2012) where Chebyshev polynomials are used in the radial direction and spherical harmonic decomposition in the horizontal directions.

The first models at $E = 10^{-4}$ and $Ra = 6 \times 10^6$, $Ra = 8 \times 10^6$, and $Ra = 1 \times 10^7$ are started from random temperature fluctuations and either (i) a strong dipolar magnetic field (as is common practice for geodynamo studies, e.g., T. Frasson et al. 2026) or (ii) a small-scale magnetic field, which will lead the solution to land in the dipolar or multipolar regime respectively (see Appendix A for more details). Simulations with higher Rayleigh and lower Ekman numbers are initiated from this first set of models, to reduce convergence time and save computing resources.

The time-averaging for the diagnostic parameters described in Section 2.3 is started after the initial transient has passed and a steady state is reached. It is performed for a minimum of $0.1 \tau_\nu$.

2.3. Diagnostic Parameters

To assess whether a model is in the rapidly rotating regime, we use the previously defined convective Rossby number Ro_c as well as the a posteriori local Rossby number Ro_ℓ , which is a diagnostic to estimate the importance of inertia relative to rotation in a dynamical system.

The kinetic energy $\mathcal{EK}$ in each system is used to obtain flow amplitudes as a function of time. Here we consider the convective components, from both the poloidal and toroidal nonaxisymmetric parts of $\mathcal{EK}$, to obtain the system-scale Rossby number $Ro (= u'/\Omega D)$:

$$\mathcal{EK}_{\text{conv}} = \frac{1}{2} \int \mathbf{u}' \cdot \mathbf{u}' dV, \quad (6)$$

$$Ro = E \left(2 \frac{\mathcal{EK}_{\text{conv}}}{V_s} \right)^{1/2}, \quad (7)$$

where V_s is the volume of the shell and primes denote the nonaxisymmetric components of the velocity. We also evaluate the characteristic nondimensional convective flow

Table 1

Definitions of the Various Forms of the Lorentz Number Used in This Work

Parameter	Definition
Lo	$B/(\sqrt{\mu_0 \rho} \Omega D)$
Lo_o	$E(2\langle \mathcal{EM}(r_o) \rangle_t / S_s)^{1/2}$
$Lo_o(\ell = 1)$	$E(2\langle f_{\text{dip},r_o} \mathcal{EM}(r_o) \rangle_t / S_s)^{1/2}$
$Lo_o(\ell > 1)$	$E(2\langle (1 - f_{\text{dip},r_o}) \mathcal{EM}(r_o) \rangle_t / S_s)^{1/2}$
Lo_v	$E(2\langle \mathcal{EM} \rangle_t / V_s)^{1/2}$
$Lo_v(\ell = 1)$	$E(2\langle f_{\text{dip},v} \mathcal{EM} \rangle_t / V_s)^{1/2}$
$Lo_v(\ell > 1)$	$E(2\langle (1 - f_{\text{dip},v}) \mathcal{EM} \rangle_t / V_s)^{1/2}$
$f_{\text{dip},v}$	$\mathcal{P}_B(\ell = 1) / \sum_{\ell=1}^{\ell_{\text{max}}} \mathcal{P}_B(\ell)$
f_{dip,r_o}	$\mathcal{P}_B(\ell = 1, r_o) / \sum_{\ell=1}^{\ell_{\text{max}}} \mathcal{P}_B(\ell, r_o)$

Note. S_s is the spherical shell surface and $\mathcal{EM}(r_o)$ is the surface-integrated total magnetic field strength (at the outer boundary). We also include the definition of the volume- and surface-averaged dipolarities, $f_{\text{dip},v}$ and f_{dip,r_o} , respectively. Here, $\mathcal{P}_B(\ell = 1)$ is the power in the magnetic dipole.

length scale:

$$\frac{\mathcal{L}_U}{D} = \pi \left(\sum_{\ell} \ell \frac{\mathbf{u}'_{\ell} \cdot \mathbf{u}'_{\ell}}{\mathbf{u}' \cdot \mathbf{u}'} \right)^{-1}, \quad (8)$$

where ℓ is the spherical harmonic degree and $\mathbf{u}'_{\ell}$ is the nonaxisymmetric velocity field at the corresponding degree. We can then obtain the time-averaged local Rossby number of each model:

$$Ro_{\ell} = \langle Ro D / \mathcal{L}_U \rangle_t, \quad (9)$$

where $\langle \dots \rangle_t$ denotes averaging over time. The magnetic energy (including both the poloidal and toroidal components) is used to calculate the total magnetic field strength in each system:

$$\mathcal{EM} = \frac{1}{2} \int \mathbf{B} \cdot \mathbf{B} dV. \quad (10)$$

Following U. R. Christensen & J. Aubert (2006) and R. K. Yadav et al. (2013a), we express the nondimensional magnetic field strength in terms of the Lorentz number

$$Lo = B/(\sqrt{\mu_0 \rho} \Omega D), \quad (11)$$

which is evaluated for our models using the expressions given in Table 1. This can be related to the often-used Elsässer number, expressing the ratio of Lorentz to Coriolis forces, through $\Lambda = Lo^2 Pm / E$. While the volumetric Lorentz number Lo_v is the classical parameter used in previous works, to make our analysis more comparable with measurements we also evaluate the surface Lorentz number Lo_o , also defined in Table 1.

The table also gives the definitions for dipolarity, where we again define the surface, f_{dip,r_o} , and the volumetric, $f_{\text{dip},v}$, values. In general, f_{dip,r_o} is the quantity we refer to when classifying a model as dipolar or nondipolar. Similarly to previous studies, we typically find that this value is fairly constant for dipolar models while it fluctuates much more for nondipolar systems. This is information we lose when performing a time-average, yet for the purposes of this study it allows for a simple, classical, characterization.

The [CA06] scaling law, relating magnetic field strength to available driving power, is expressed in terms of the nondimensional parameters Lo and the flux-based Rayleigh number Ra_Q^* (U. R. Christensen & J. Aubert 2006; R. K. Yadav et al. 2013a):

$$Lo / \sqrt{f_{\text{ohm}}} \propto Ra_Q^{*1/3}, \quad (12)$$

where f_{ohm} is the ratio of ohmic dissipation to total dissipation. The flux-based Rayleigh number,

$$Ra_Q^* = \frac{1}{4\pi r_o r_i} \frac{\alpha g_o Q_{\text{adv}}}{\rho c_P \Omega^3 D^2}, \quad (13)$$

(where c_P is the heat capacity) is used rather than the ΔT -based Rayleigh number, as this can be estimated more easily for stars and planets by using approximations of their advected heat flux Q_{adv} .

In terms of our model diagnostics this is equivalent to

$$Ra_Q^* = \frac{(Nu - 1) Ra E^3}{Pr^2}, \quad (14)$$

where Nu is the Nusselt number.

However, we can also frame the scalings in terms of Rossby numbers, using the asymptotic scaling relations for rapidly (or slowly) rotating systems (K. Julien et al. 2012; J. M. Aurnou et al. 2020). In terms of the convective Rossby number we have

$$\frac{Nu - 1}{(Ra Pr)^{1/2}} \sim \begin{cases} Ro_c^2, & \text{for } Ro_c \ll 1 \\ \text{const.}, & \text{for } Ro_c \gg 1. \end{cases} \quad (15)$$

Further, asymptotic estimates of the convective flow velocities (see Equation (7)) yield

$$Ro \sim \begin{cases} Ro_c^2, & \text{for } Ro_c \ll 1 \\ Ro_c, & \text{for } Ro_c \gg 1, \end{cases} \quad (16)$$

for the system-scale Rossby number, and the local Rossby number (see Equation (9))

$$Ro_{\ell} \sim Ro_c. \quad (17)$$

These scalings lead to

$$Ra_Q^* \sim \begin{cases} Ro_c^5 \sim Ro^{5/2} \sim Ro_{\ell}^5, & \text{for } Ro_c \ll 1 \\ Ro_c^3 \sim Ro^3 \sim Ro_{\ell}^3, & \text{for } Ro_c \gg 1. \end{cases} \quad (18)$$

Using the convective Rossby number for our numerical framework has the advantage of Ro_c being an input parameter for simulations, as well as it being easy to judge whether one is in an inertia-dominated ($Ro_c \gg 1$) or rotation-dominated ($Ro_c \ll 1$) regime. However, Ro_c is difficult to estimate for planets and stars as it requires approximating the convective velocities, or convective turnover times. Thus, we present the scaling laws in terms of Ra_Q^* in the results section, but include scalings with various Rossby numbers in Appendix B.

3. Results

Figure 1 shows the surface dipolarity as a function of local Rossby number (Equation (9)) for all simulations in this study. The bistable regime is seen to extend from the lowest local Rossby numbers we reach to around $Ro_{\ell} = 0.1$. The lines connecting bistable pairs in Figure 1 highlight that these

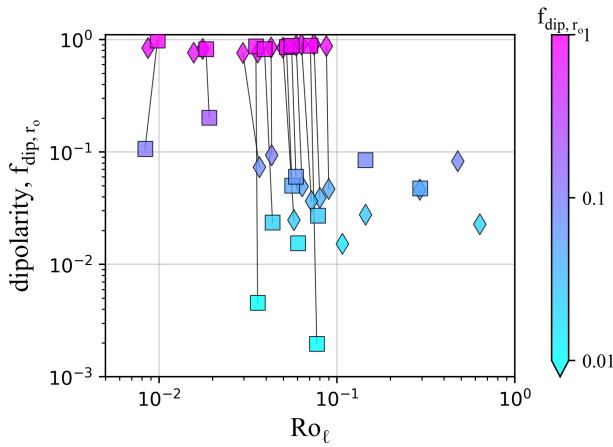


Figure 1. Surface dipolarity as a function of local Rossby number. Bistable pairs (dipolar and nondipolar systems that have identical control parameters) are connected by black lines. Diamonds (squares) represent models with $Pm = 1$ ($Pm = 0.5$).

models not only have the same input parameters and therefore the same value of Ro_c , but the *diagnostic* local Rossby number is also very similar.

A three-dimensional visualization of bistable models ($E = 10^{-5}$, $Ra = 2 \times 10^8$, $Pm = 0.5$, $Ro_c = 0.14$) is shown in Figures 2(a) and (b). This highlights the similarity between the convective flow structures, illustrated by the red/blue isocontours, which are strongly aligned with the axis of rotation ($Ro_\ell \approx 0.04$ for this pair). In contrast, the magnetic field structures differ dramatically, as the dipole component dominates over the smaller scales for the dipolar model. This can be seen more clearly by comparing Figures 3(a) and (d), which show snapshots of the fully resolved radial magnetic fields at the surface of the two models. The two differ dramatically. Figures 3(c) and (f) show the same field snapshots but excluding the $\ell = 1$ component. They highlight how insignificant the dipole contribution is for the nondipolar system.

The spatial distribution of the kinetic and magnetic energy of one pair of models is shown in terms of their spectra in Figure 4. We plot (a) the total nonaxisymmetric kinetic energy (representative of the convection) and both (b) the volumetric and (c) the surface spectra of the poloidal component of the magnetic energies, as a function of spherical harmonic degree ℓ . Figure 4(a) shows that the convection is slightly stronger in the multipolar case for this particular bistable pair of models, indicating that the stronger magnetic field of the dipolar model is suppressing the convection more than in the multipolar system. Apart from this small difference in amplitude (up to $\ell \sim 70$) the shapes of the spectra are very similar. We observe that the shapes and amplitudes of the magnetic energy spectra are very similar for both models—with the stark exception of the dipole, $\ell = 1$, component. The only other noticeable differences are in the volumetric poloidal magnetic energy in the range $7 \lesssim \ell \lesssim 40$ and in the surface energy for $\ell \gtrsim 80$, where there is more energy at those scales in the dipolar model.

3.1. Rossby Number Scaling

As described in Section 2.3 and summarized in Equation (18), we have predictions of how the three forms of the Rossby number introduced here (Ro_c , Ro , and Ro_ℓ)

should scale with respect to each other and to the flux-based Rayleigh number Ra_Q^* . We show how well these scalings apply to our set of models in Figure 5.

While we do not plot $Ro(Ra_Q^*)$ directly, Figure 5 shows that both Ra_Q^* and Ro follow the expected scaling with Ro_c very well. Only the $Ro_\ell \sim Ro_c$ scaling shows a little more shingling of the data points, with more supercritical models better hitting the trend line. This could perhaps be improved if a different definition of the length scale $\mathcal{L}_U$ were used, but we did not explore this further here.

U. R. Christensen & J. Aubert (2006) make a fit of $Ro \propto Ra_Q^{*0.41}$ (although this is calculated using the total kinetic energy, i.e., including zonal and meridional flows). Meanwhile, R. K. Yadav et al. (2013a, 2013b) make separate fittings for the nonaxisymmetric Ro , obtaining exponents of 0.44 and 0.42, respectively. Thus, all three studies agree well with the asymptotically predicted value of $2/5$ (see Equation (18)).

3.2. Magnetic Field Strength Scaling

We fit our data to Equation (12), and the value of the coefficient thus obtained is given for each of the variations of Lo described in Section 2.3 in Table 2. We give the fitting when assuming the exponent is $1/3$, the exponent that emerges from theory (e.g., P. A. Davidson 2013), and also that from dimensional analysis, as well as the scaling when the exponent is left as a fitting parameter. In general, we find that the fitted exponent is a little greater than $1/3$, particularly when evaluated from the values of surface Lorentz number. The multipolar fitting for $Lo_V(\ell = 1)$ marks the exception to this, and has an exponent of only 0.30 ± 0.08 . The steeper slope for the surface Lo fitting could be attributed to the weak magnetic fields of our lowest Ra_Q^* cases, which sit slightly below the rest (see Figures 6(a) and (b)). This may be due to these systems being fairly close to the onset of convection.

Comparing the panels of Figure 6, we find that for the total, volume-averaged, magnetic field (panel (d)) the exponential prefactors are fairly similar (0.646 ± 0.072 and 0.471 ± 0.061 for dipolar/nondipolar models, respectively) yet the two sets are distinct. This is similar to what was found by R. K. Yadav et al. (2013a, 2013b), although their prefactors of 0.9 and 0.7, respectively, differ a little from ours. Figure 6(a) shows that this difference between dipolar and nondipolar models is enhanced at the surface. When focusing exclusively on the dipole component of the models (panels (b) and (e)), we find an even more significant difference between the regimes (which is to be expected) and the prefactors differ by an order of magnitude, both in the volume and at the surface. We also observe slightly more scatter for the nondipolar models around the line of best fit, which reflects the fact that the dipole component does not play a significant role in the dynamics of these systems and can therefore vary more (also with time, as mentioned in Section 2.3).

When comparing the strength of the nondipole components of the two regimes, we find that they follow the same scaling (shown in Figures 6(c) and (f)), in particular at the surface, $Lo_\ell(\ell > 1)$. This is consistent with what the spectra in Figure 4 indicated. This finding suggests that the energy in all $\ell > 1$ magnetic field components is directly set by the available driving power, irrespective of whether the regime is dipolar or nondipolar.

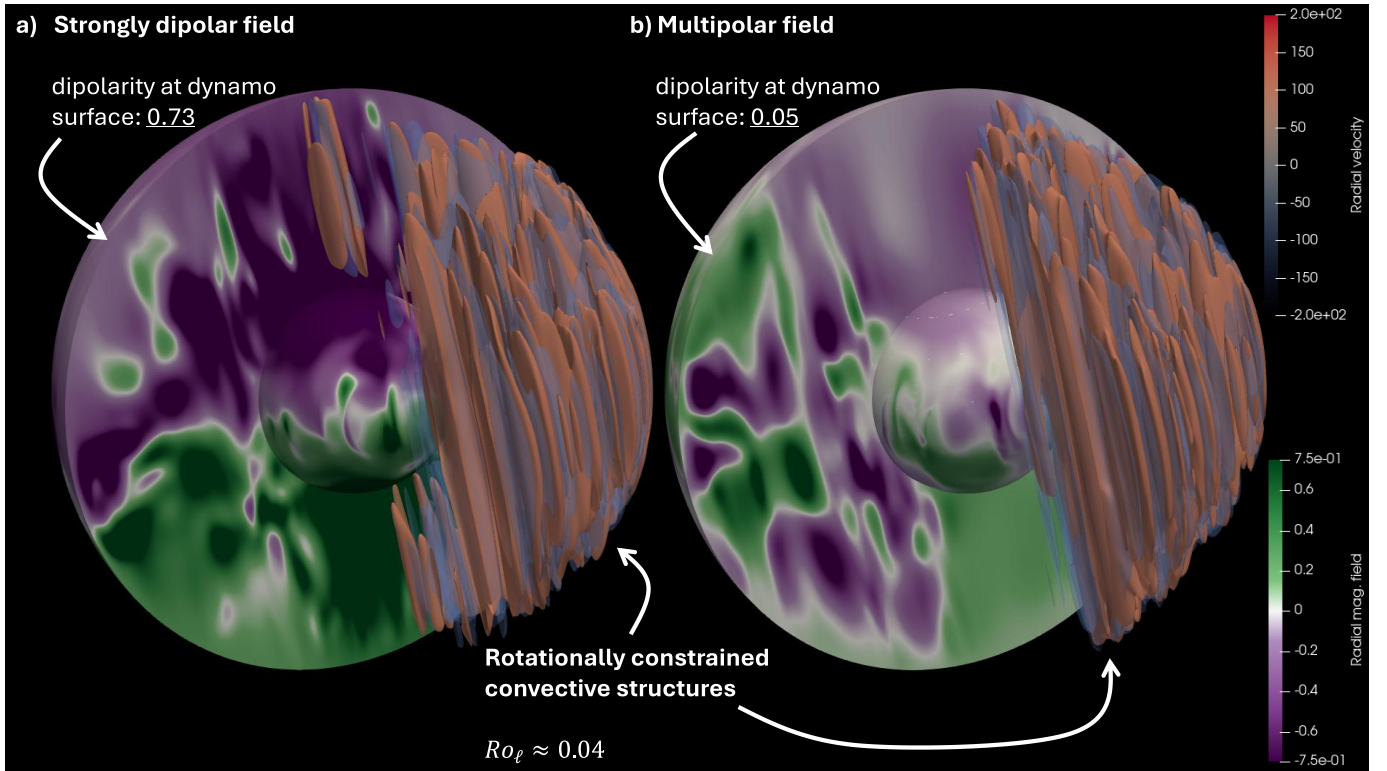


Figure 2. Three-dimensional visualizations of (a) dipolar and (b) nondipolar models, both with $E = 1 \times 10^{-5}$, $Ra = 2 \times 10^8$, and $Pm = 0.5$. Isocontours of outward/inward radial flows are shown in red/blue. Radial magnetic field is shown in green/purple.

4. Magnetic Field Estimates

We now explore how our newly obtained scaling laws may be applied to objects inside and outside our solar system.

4.1. *M* Dwarfs

We harvest magnetic field data for *M* dwarfs from several studies, obtained using different observational methods.

For the large-scale magnetic field components we consult the composite study of V. See et al. (2025). We focus on fully convective stars with $M < 0.35 M_{\odot}$ as we can expect stars with a significant radiative region to deviate from our scaling laws that are based on fully convective spherical shell models. This leads us to restrict our data harvesting to J. Morin et al. (2008a, 2008b, 2010), S. Bellotti et al. (2024), and L. T. Lehmann et al. (2024),¹ where magnetic field strengths are obtained via Zeeman–Doppler Imaging, ZDI (as well as principal component analysis by S. Bellotti et al. 2024 and L. T. Lehmann et al. 2024), relying on circularly polarized light, i.e., Stokes *V* signatures. This is necessary because only the Stokes *V* signatures are sensitive to magnetic field topology.

From these studies we take the average (poloidal) dipole field strength from ZDI, $\langle B_{\text{dip}} \rangle$, which we name $B(\ell = 1)$ in our framework.

However, small-scale magnetic field features with opposite polarities are canceled out when using the Stokes *V*, so it does not capture a large fraction ($\sim 90\%$, A. Reiners & G. Basri 2009) of the total magnetic energy. Therefore, for the total magnetic field strength we consider D. Shulyak et al.

(2017) and A. Reiners et al. (2022), who analyze unpolarized light, i.e., Stokes *I* signatures. These give us the global average magnetic field strength B .

These two measurements are shown in Figure 7(a) (and tabulated, including the respective star names, in Appendix C). To quantify dipolarity as consistently as possible with our numerical study, we estimate

$$f_{\text{dip}} \approx B(\ell = 1)^2 / B^2. \quad (19)$$

This is indicated by the marker color in Figure 7. However, we use different limits for our color scale compared to our numerical analysis. This is due to stars having a much larger fraction of their magnetic energy at smaller scales compared to our simulations. We attribute this to not being able to reach high enough magnetic Reynolds numbers, Rm , with the limitations we have on computing power— $Rm \sim \mathcal{O}(10^8)$ for stars while $Rm \sim \mathcal{O}(10^4 - 10^5)$ for gas planets. Thus, for these objects we consider $f_{\text{dip}} = 3 \times 10^{-3}$ as our classification boundary.

This is also why, when considering the total average magnetic field strength, we only see a vague indication that for nondipolar magnetic fields B lies slightly under the predicted value (as was noticed by D. Shulyak et al. 2017)—see Figure 7(b). The dipole component makes up much less of the total field, even in the dipolar systems, than in our models and therefore barely impacts the total average magnetic field strength. This is therefore very similar for both regimes (see Figures 6(c) and (f)).

To test how these measurements fit with respect to our power-based scaling laws,

$$B^2 \propto P^{1/3}, \quad (20)$$

¹ We exclude the 2019 $B(\ell = 1)$ estimate for EV Lac from S. Bellotti et al. (2024) as they comment that the magnetic field strength for this epoch is likely underestimated.

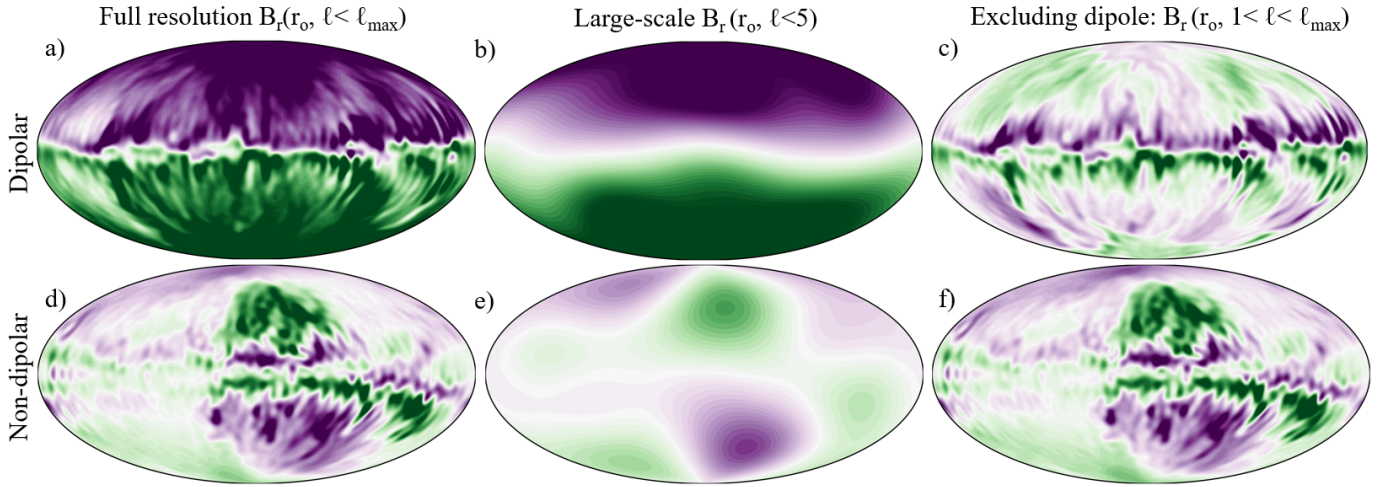


Figure 3. Surface radial magnetic fields for (a)–(c) a dipolar model and (d)–(f) a nondipolar model, both with $E = 1 \times 10^{-5}$, $Ra = 2 \times 10^8$, and $Pm = 0.5$. (a) and (d) use the full resolution, (b) and (e) include only degrees $\ell < 5$ (mirroring our current resolution of the ice giants’ magnetic fields), and (c) and (f) exclude the dipole component. The same color scale of ± 0.25 (in Elsasser units) is used in all panels.

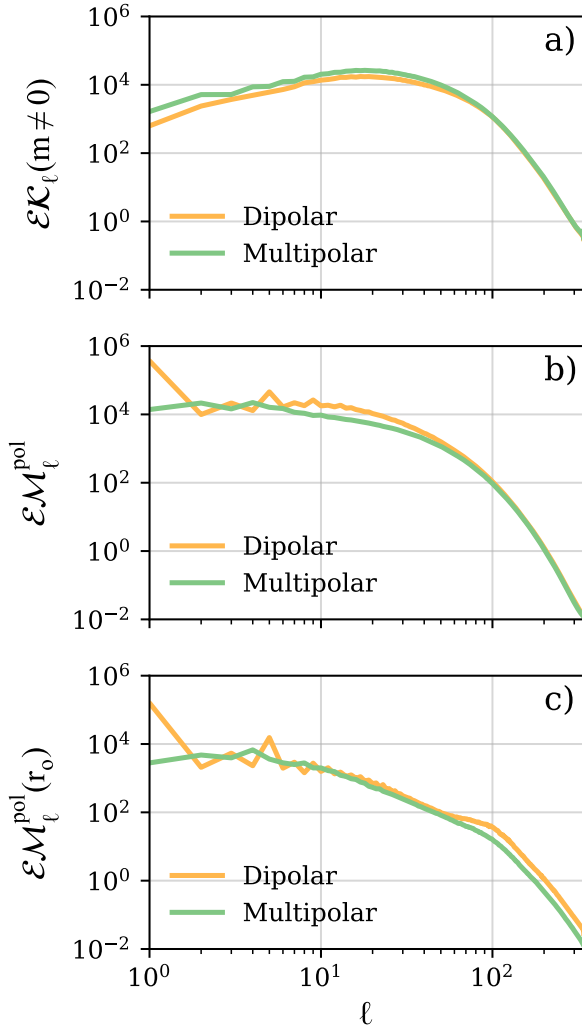


Figure 4. Energy spectra of a bistable pair of models ($E = 10^{-5}$, $Ra = 2 \times 10^8$, $Pm = 0.5$, $Ra/Ra_c = 19.8$, $Ro_c = 0.14$). $\mathcal{EK}(m \neq 0)$ refers to the poloidal and toroidal nonaxisymmetric components of the time-averaged kinetic energy spectra. $\mathcal{EM}^{\text{pol}}$ ($\mathcal{EM}^{\text{pol}}(r_o)$) refers to the time-averaged, volumetric (surface) poloidal magnetic energy spectra. Both models have $\ell_{\text{max}} = 341$.

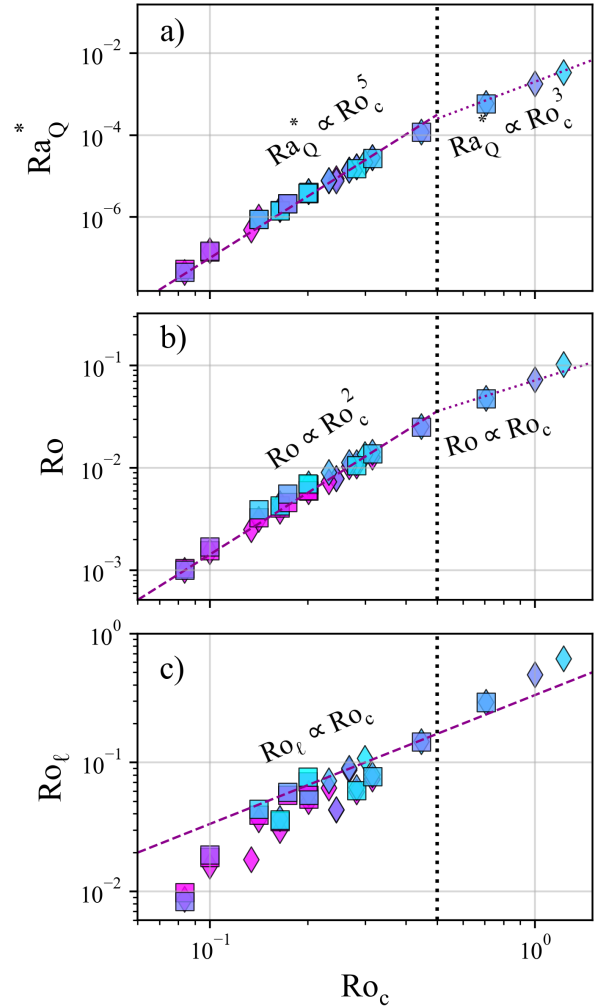


Figure 5. Ra_Q^* , Ro , and Ro_ℓ as a function of convective Rossby number, with the expected rapidly/slowly rotating scalings indicated by dashed/dotted purple lines (see Equation (18)). Symbol colors represent dipolarity, as in Figure 1. The vertical dotted black line denotes the limit of the data we use to make our fittings, $Ro_c < 0.5$, as we observe a clear deviation from the rapidly rotating limit beyond this point.

Table 2

Coefficients, c , and Their Associated 1σ Values when Fitting Equation (12) for Various Forms of Lo for the Dipolar (D), Multipolar (M), and Universal (U) Regimes

Lo , Regime	$Ra_Q^{*1/3}$ Coeff.	Free Power Fitting
Lo_o , D	0.168 ± 0.022	$(0.60 \pm 0.12) Ra_Q^{*0.44 \pm 0.01}$
Lo_o , M	0.075 ± 0.008	$(0.32 \pm 0.11) Ra_Q^{*0.47 \pm 0.03}$
$Lo_o(\ell = 1)$, D	0.157 ± 0.022	$(0.58 \pm 0.11) Ra_Q^{*0.45 \pm 0.01}$
$Lo_o(\ell = 1)$, M	0.015 ± 0.003	$(0.03 \pm 0.03) Ra_Q^{*0.41 \pm 0.07}$
$Lo_o(\ell > 1)$, U	0.069 ± 0.006	$(0.30 \pm 0.08) Ra_Q^{*0.47 \pm 0.02}$
Lo_v , D	0.646 ± 0.072	$(1.26 \pm 0.41) Ra_Q^{*0.39 \pm 0.02}$
Lo_v , M	0.471 ± 0.061	$(0.94 \pm 0.26) Ra_Q^{*0.40 \pm 0.02}$
$Lo_v(\ell = 1)$, D	0.559 ± 0.067	$(0.96 \pm 0.28) Ra_Q^{*0.38 \pm 0.02}$
$Lo_v(\ell = 1)$, M	0.069 ± 0.019	$(0.04 \pm 0.04) Ra_Q^{*0.30 \pm 0.08}$
$Lo_v(\ell > 1)$, U	0.424 ± 0.052	$(1.64 \pm 0.45) Ra_Q^{*0.46 \pm 0.02}$

Note. The third column gives the coefficient and Ra_Q^* exponent when making no assumption of the power law. We exclude simulations with $Ro_c > 0.5$ from the fitting.

we estimate the driving power available as derived by U. R. Christensen et al. (2009) and A. Reiners et al. (2009):

$$P = \left(\frac{M}{M_\odot} \right) \left(\frac{L}{L_\odot} \right)^2 \left(\frac{R}{R_\odot} \right)^{-7}, \quad (21)$$

where we omit a prefactor for now. Here, M is the mass of the star, L is its bolometric luminosity, and R is its radius. In Figures 7(b) and (c) we plot B and $B(\ell = 1)$, respectively, divided by $P^{1/6}$ as a function of rotation period P_{rot} .

Figure 7(b) shows that the total magnetic field strength of the rapidly rotating stars in our sample is estimated very well by the [C09] scaling law with

$$B_{\text{pred}} = 4800 \text{ G} \times P^{1/6}. \quad (22)$$

For stars with $P_{\text{rot}} < 10$ days there is no dependence on rotation rate, as the scaling law predicts.

In Figure 7(c) we show the same stars but now study their average poloidal dipole magnetic field strength. We see that rapidly rotating stars with dipolar ($f_{\text{dip}} > 3 \times 10^{-3}$) and nondipolar ($f_{\text{dip}} < 3 \times 10^{-3}$) magnetic field topologies cluster around two fairly distinct regions in $B(\ell = 1)/P^{1/6}$ space. Taking an average for the samples in each regime we obtain the predictive scalings:

$$B_{\text{pred,D}}(\ell = 1) \approx 710 \text{ G} \times P^{1/6}, \quad (23)$$

$$B_{\text{pred,ND}}(\ell = 1) \approx 67 \text{ G} \times P^{1/6}. \quad (24)$$

It is difficult to say whether the spectropolarimetry measurements of these stars are more analogous to the surface values from our models or deeper. They do recover toroidal magnetic field strengths (from the limbs) that are zero at the surface of our models, due to the potential field boundary condition. In numerical models magnetic field amplitudes—in all components—decrease steeply as the outer boundary is approached (compare our scaling coefficients for Lo_o and Lo_v , and see, for example, Figure 4 from R. K. Yadav et al. 2015). Also, dipolarity tends to decrease toward the outer boundary—noting that in the bulk dipolarity also includes the

toroidal contribution. However, the scalings for the dipole magnetic field strength that we obtain from our models have an offset that differs by an order of magnitude, both for $Lo_o(\ell = 1)$ and $Lo_v(\ell = 1)$, between dipolar and nondipolar models. This makes both of them consistent with the coefficients of $C_{\ell=1,D} = 710 \text{ G}$ and $C_{\ell=1,ND} = 67 \text{ G}$ we find for these stars.

Interestingly, as with our numerical models, we do not find any star in our sample that indicates a bistability within a single system, i.e., even $B(\ell = 1)$ measurements at different times, while showing some variability, do not jump between dipolar and nondipolar regimes. However, both for our simulations and for these stars, this may occur with longer run time/observation time.

We have excluded the very slow rotators (represented by hexagons in Figure 7) in the analysis outlined above. In the context of the study by L. T. Lehmann et al. (2024) from which these data are taken, these stars have stronger magnetic fields than expected. This is because, when considered purely as a function of rotation rate, magnetic field strength is found to decrease with increasing period, for $P_{\text{rot}} \gtrsim 10$ days (see, e.g., A. Reiners et al. 2022). However, as shown by Figures 7(b) and (c), their magnetic field strengths are actually weaker with respect to their available driving power than those of the fast rotators. The derivation of the power-based scaling law explicitly requires the system to be rapidly rotating. Therefore, we do not expect it to correctly predict the magnetic field strengths of these slow rotators.

4.2. Solar System Planets

One major snag when attempting to apply these scaling laws to planetary dynamos is the fact that planetary interiors could feature multiple stably stratified regions. This has become more apparent in the last decade with the measurements from Juno and Cassini at Jupiter and Saturn, respectively. It is now thought that both gas giants feature fuzzy cores (C. R. Mankovich & J. Fuller 2021; R. Helled & D. J. Stevenson 2024; B. Militzer & W. B. Hubbard 2024) as well as helium rain layers in shallower regions (e.g., D. J. Stevenson 1982; S. Markham & T. Guillot 2024; P. N. Wulff et al. 2025). The ice giants are much less constrained due to lack of in situ data, apart from Voyager 2's flyby, but are also likely to host regions of compositional or thermal stratification, or phase changes (M. Cano Amoros et al. 2024; B. Militzer 2024; A. Gupta et al. 2025; L. Morf & R. Helled 2025).

The presence of these regions/boundaries complicates the application of the scaling laws. First, as even for Jupiter and Saturn their exact location is still poorly constrained, it is difficult to estimate the lower and upper bounds of the dynamo region. Second, the presence of stable stratification *above* a dynamo region, but at a depth where electrical conductivity is still significant, has drastic effects on the magnetic field morphology. In particular, if differential rotation is strong in this layer, nonaxisymmetric magnetic field components are filtered out effectively (e.g., D. J. Stevenson 1982; U. R. Christensen 2006). Thus, the field that is measured at the planetary surface can be very different to the field at the top of the dynamo region and cannot simply be extrapolated down as a potential field (P. N. Wulff et al. 2025). Third, the case of a planet that has a truly nested dynamo system—i.e., multiple convective regions, separated by stable layers or phase changes, that all host dynamo

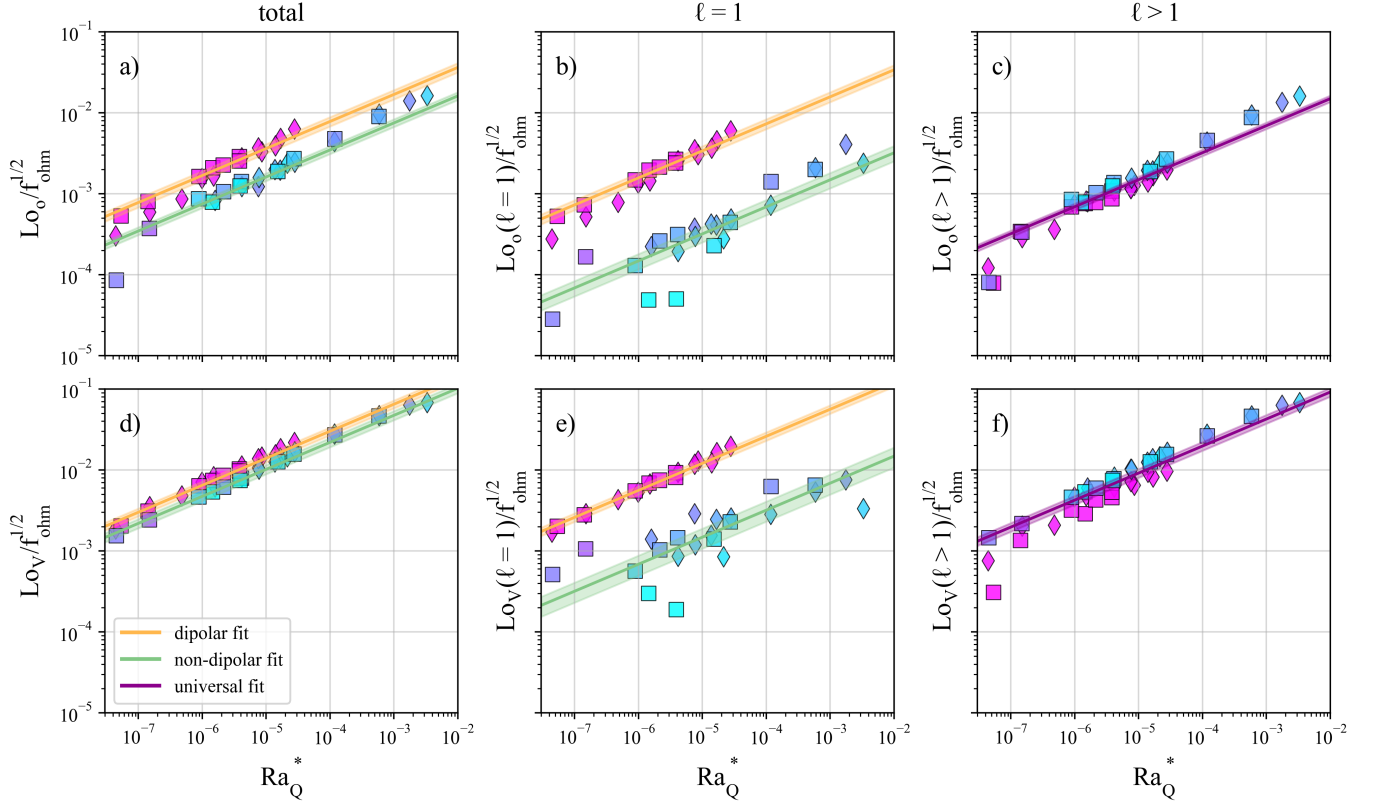


Figure 6. Lorentz number, adjusted by the ohmic dissipation fraction, as a function of the modified Rayleigh number based on the advected heat flux. Panels (a)–(c) show surface values of Lo while panels (d)–(f) show volumetric values. (a) and (d) show total Lo , (b) and (e) $Lo(\ell = 1)$, and (c) and (f) $Lo(\ell > 1)$ (see Table 1). Diamonds (squares) represent models with $Pm = 1$ ($Pm = 0.5$), while color indicates surface dipolarity: dipolar/nondipolar in magenta/cyan (see color bar of Figure 1). Orange/green lines in (a), (b), (d), and (e) show dipolar/nondipolar fits to $Lo/f_{\text{ohm}}^{1/2} \sim Ra_Q^{1/3}$ while purple lines in (d) and (f) show the universal fits (see Table 2).

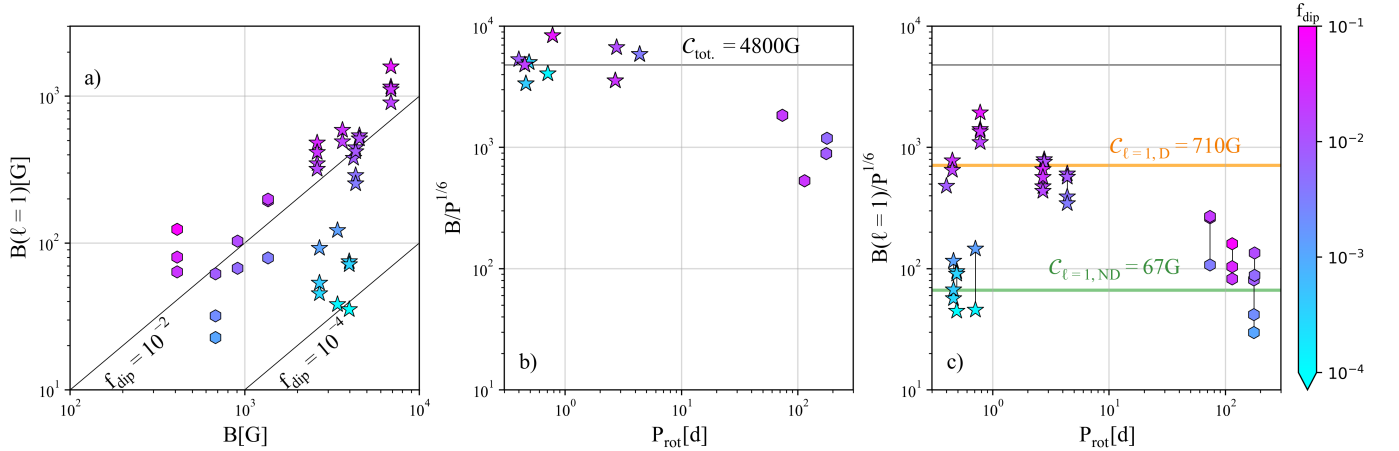


Figure 7. (a) Average (poloidal) dipole field strength $B(\ell = 1)$ as a function of average total field strength B . (b) B , and (c) $B(\ell = 1)$, relative to the power $P \sim ML^2/R^7$ according to the [C09] scaling law (Equation (20)), as a function of rotation period. All stars in our sampled are fully convective ($M_* < 0.35 M_\odot$). See Appendix C, Table 5, for data sources. As the same stars have multiple measurements for $B(\ell = 1)$ we join all markers for the same star with a black line. M , L , and R to calculate P , as well as P_{rot} , are taken from V. See et al. (2025)—see that paper for original sources. Symbol color indicates dipolarity; star markers represent rapidly rotating bodies while hexagons represent those with $P_{\text{rot}} > 10$ days. On (b) we indicate the coefficient of $C_{\text{tot}} = 4800$ G while on (c) we indicate the average ratio $C_{\ell=1} = B(\ell = 1)/P^{1/6}$, for dipolar and nondipolar separately, excluding the slow rotators.

action, not just magnetoconvection—has not been systematically explored.

Thus, with these caveats in mind, we test the accuracy of our scaling laws for the dipole magnetic field strength of the four giant planets in our solar system. Given that $\mathcal{P}_B(\ell, r) = \mathcal{P}_B(\ell, a) (a/r)^{2\ell+4}$, where a is the planetary radius,

the expression for $Lo(\ell = 1)$ (see Equation (11)) at a planet’s dynamo radius r_{dyn} is

$$Lo_{\text{dyn}}(\ell = 1) = \left(\frac{\mathcal{P}_B(\ell = 1, a) (a/r_{\text{dyn}})^6}{\rho \mu_0 \Omega^2 D_{\text{dyn}}^2} \right)^{1/2}, \quad (25)$$

where $\mathcal{P}_B(\ell = 1, a)$ is the energy in its magnetic dipole at the planetary surface, and D_{dyn} is the extent of the dynamo region (equal to $r_o - r_i$ in our numerical models).

The flux-based expression for Ra_Q^* is

$$Ra_Q^* = \frac{1}{4\pi r_{\text{dyn}}(r_{\text{dyn}} - D_{\text{dyn}})} \frac{\alpha g Q_{\text{adv}}}{\rho c_P \Omega^3 D_{\text{dyn}}^2}. \quad (26)$$

We then substitute these definitions into the expression for the dipole Lorentz number $Lo_o(\ell = 1) = c Ra_Q^{*1/3}$, where we have followed the typically made assumption that $f_{\text{ohm}} = 1$ in the planets, as we have no way of measuring this directly. See Appendix B for comments on the impact of this assumption. In our expression c is the prefactor from Table 2, where we test the dipolar prefactor for Jupiter and Saturn and the nondipolar one for Uranus and Neptune. This yields

$$= c \left(\frac{\mathcal{P}_B(\ell = 1, a) a^6}{\rho \mu_0 \Omega^2 D_{\text{dyn}}^2 r_{\text{dyn}}^6} \right)^{1/2} = c \left(\frac{1}{4\pi r_{\text{dyn}}(r_{\text{dyn}} - D_{\text{dyn}})} \frac{\alpha g Q_{\text{adv}}}{\rho c_P \Omega^3 D_{\text{dyn}}^2} \right)^{1/3}, \quad (27)$$

and using the expression $Q_{\text{adv}} = 4\pi r_{\text{dyn}}^2 q_0$ we obtain

$$\frac{\mathcal{P}_B(\ell = 1, a)}{\mu_0} = c^2 \left(\frac{r_{\text{dyn}}^{10} D_{\text{dyn}}}{a^9 (r_{\text{dyn}} - D_{\text{dyn}})} \frac{\alpha g q_0}{c_P} \right)^{2/3} \rho^{1/3}. \quad (28)$$

We simplify the right-hand side of this equation and make it analogous to U. R. Christensen et al. (2009), by defining

$$F = \frac{r_{\text{dyn}}^{10} D_{\text{dyn}}}{a^9 (r_{\text{dyn}} - D_{\text{dyn}})} \frac{\alpha g}{c_P} \quad (29)$$

The values for α , g , c_P , and q_0 used are given in Appendix D. Note that the highest-order dependence is on the dynamo radius. Thus, we mainly explore this dependence, though uncertainties in α , g , c_P , and q_0 will also influence our results.

Figure 8(a) shows how well the scaling laws given in Table 3 agree with the measured dipole magnetic field energy for the solar system gas planets, with the two regimes indicated by the black lines. We make an estimate of the dynamo radius for each planet based on interior models and magnetic field measurements (M. French et al. 2012; M. Preising et al. 2023; P. N. Wulff et al. 2025). The extent of each line on the x-axis is based on a range of values for D_{dyn} : 0.2–0.65 a for Jupiter, 0.2–0.4 a for Saturn, and 0.2–0.6 a for Uranus and Neptune. Jupiter, Neptune, and Uranus fall nicely within the range that we would expect from our scaling laws. However, Saturn is further from our predictions, based on our assumed dynamo radius and D_{dyn} values.

If one makes no assumption of r_{dyn} or D_{dyn} , one can map out where the scaling law holds in the r_{dyn} , D_{dyn} parameter space, as illustrated in Figure 8(b). If our scaling law holds, this indicates that Jupiter and Neptune both have dynamo regions with similar trade-offs between dynamo region depth and extent. For Jupiter we have a better constraint on the dynamo radius, and if $r_{\text{dyn}} = 0.83 R_J$, then this indicates that the dynamo region spans $\sim 30\%$ of the planet, i.e., 0.55–0.83 R_J . Meanwhile, due to Uranus' extremely low heat flux, the scaling law implies that the dynamo region extends almost to the center of the planet and starts near to the surface. For

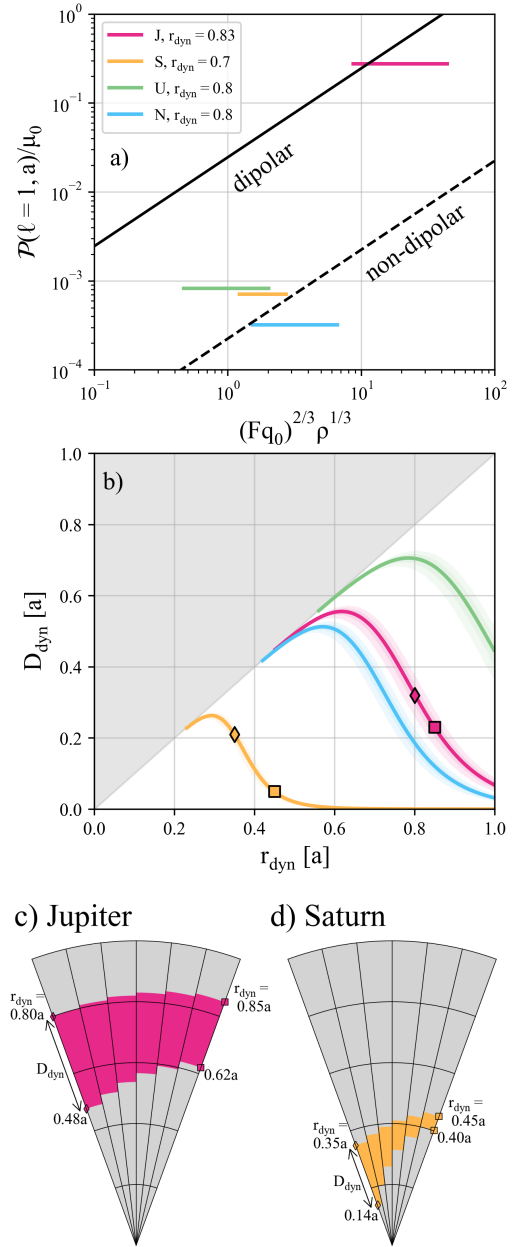


Figure 8. (a) Dipole magnetic field strength at the planetary surface, as a function of driving power, for the four giant planets in our solar system (analogous to figures in e.g., U. R. Christensen et al. 2009). The value we take for the dynamo radius for each planet is given in the legend, and the horizontal extent of the power estimate comes from allowing for a range of D_{dyn} . The black solid/dashed line indicates the prediction for dipolar/nondipolar magnetic fields. (b) The r_{dyn} , D_{dyn} parameter space in which the power in the planet's dipole matches the prediction within 20%. We use the dipolar scaling prefactor for Jupiter and Saturn and the nondipolar scaling prefactor for Uranus and Neptune. Dynamo region geometry of (c) Jupiter and (d) Saturn, from the deepest realistic estimate (indicated by diamond markers) to shallowest estimate (square markers) indicated on panel (b).

Table 3
Predictive Scalings for the Dipole Magnetic Field Strength at the Surface of a Planet with a Dynamo Region Spanning $r_{\text{dyn}} - D_{\text{dyn}}$ to r_{dyn}

Regime	Scaling Law
Dipolar	$\frac{\mathcal{P}_B(\ell=1, a)}{\mu_0} = (0.157 \pm 0.022)^2 (Fq_0)^{2/3} \rho^{1/3}$
Nondipolar	$\frac{\mathcal{P}_B(\ell=1, a)}{\mu_0} = (0.015 \pm 0.003)^2 (Fq_0)^{2/3} \rho^{1/3}$

Saturn, the scaling law suggests a deeper dynamo radius than what we assumed, based on its electrical conductivity profile.

We illustrate our favored solutions for Jupiter and Saturn in Figures 8(c) and (d), respectively. Here, the solutions for Jupiter with $0.8 R_J \leq r_{\text{dyn}} \leq 0.85 R_J$ and for Saturn with $0.35 R_S \leq r_{\text{dyn}} \leq 0.45 R_S$ are shown by highlighting the extent of the corresponding dynamo region, where the range is also indicated by the square and diamond markers on Figure 8(b).

5. Conclusions

We confirm the bistability of dynamo action over a wide range in Ro_ℓ (inertia versus Coriolis) space, where nondipolar dynamos at even lower Ro_ℓ than presented here seem limited only by computational restrictions (i.e., they require lower Ekman numbers, which are more costly to simulate).

We find that the main difference in magnetic field amplitude lies in the power of the $\ell = 1$ component, and that across the remaining length scales, power is distributed in a very similar way when comparing bistable systems.

We present scaling laws for both the volumetric and the surface total magnetic field amplitudes, as well as splitting them into their dipolar ($\ell = 1$) and nondipolar ($\ell > 1$) parts. This allows for a better comparison with measurements of large-scale magnetic fields, as well as improved estimates for the dipole field strength, which is an important controlling factor for stellar and planetary magnetospheres.

We show that these driving power-based scaling relations can also be formed in the framework of the system’s characteristic Rossby number, according to the Coriolis–inertial–Archimedean (CIA) balance.

Naturally, the next step is to further investigate the dynamo mechanisms leading to the two regimes, and how they are linked to differences in zonal flow structures.

5.1. M Dwarfs

Bistability has previously been referred to when explaining the differences in the large-scale magnetic field structures inferred by ZDI for M dwarfs (e.g., J. Morin et al. 2011; T. Gastine et al. 2013). Here, we intentionally use the nomenclature “dipolar” and “nondipolar,” as opposed to “strong-field” and “weak-field” because our analysis showed that in this dual regime, the primary difference lies in the $\ell = 1$ component of the magnetic field, while the strong- and weak-field nomenclature sometimes also refers to strong and weak dipolar bistable systems (e.g., R. J. Teed & E. Dormy 2025).

We have built upon the scaling law that predicts the total magnetic field amplitude (Equation (22); U. R. Christensen et al. 2009), by expanding to predictions of the magnetic dipole, Equations (23) and (24), for dipolar and nondipolar systems, respectively. We show that these differ by an order of magnitude between the two regimes. If one has an estimate of the differential rotation occurring in the star, this can help to distinguish which regime it is likely to be in, and which scaling law one should apply (T. Gastine et al. 2014; B. Zaire et al. 2022). If it is a rapid rotator, and the differential rotation is strong and solar-like, then it is more likely to be in the nondipolar regime. If the differential rotation is weak, then there is likely a strongly dipolar magnetic field braking the zonal flow and one should apply the dipolar scaling law.

J.-F. Donati et al. (2025) investigate the effect of adding more information into the ZDI analysis, using not just Stokes V , but

also Stokes IV and Stokes $IVQU$ profiles. They find that the inferred dipole is stronger than the results when only using Stokes V signatures. However, they express this as the polar field strength, and only give the axisymmetric dipolarity. This can differ quite significantly from the total dipolarity for nondipolar magnetic field morphologies where the dipole has less preference for being aligned with the rotation axis. Nonetheless, their analysis provides a note of caution that our predictions for M dwarf dipole magnetic field strength are based on retrievals from Stokes V and may need to be adapted as measured estimates of the large-scale magnetic field components of low-mass stars are improved.

Our models miss the very small-scale magnetic field structures. We do not analyze the scale-filtering and comparison between models and observational methods in more detail in this work (see R. K. Yadav et al. 2015; L. T. Lehmann et al. 2018). However, P. Lang et al. (2014) find that while adding small-scale field increases the surface flux, the large-scale open flux that governs the stellar magnetosphere remains unaffected.

Thus, our predictions are useful when considering the magnetic environment of exoplanets, which are primarily shaped by the host star’s large-scale field (A. A. Vidotto et al. 2013; O. Cohen et al. 2014). In particular, M dwarfs with nondipolar magnetic fields may make the best host stars for habitable exoplanets, as A. A. Vidotto et al. (2014) find that less axisymmetric stellar magnetic fields help to shield the planets from galactic cosmic rays more efficiently.

5.2. Solar System Planets

Previous suggestions for the reason behind the nondipolar nature of the ice giants’ magnetic fields have included the presence of a stably stratified layer below the dynamo region (S. Stanley & J. Bloxham 2004, 2006), reducing the dynamo region to a thinner layer, or the dominance of inertia over rotational effects in their internal dynamics (K. M. Soderlund et al. 2012, 2013). However, we show that the nondipolar regime has a wide range of overlap with the dipolar regime in Ro_ℓ space and that neither of these ideas needs to be invoked to explain their weak $\ell = 1$ magnetic field components. Our scaling laws do indicate a deeper dynamo region on Uranus, compared to Neptune, due to its extremely low heat flux. However, we observed more variation of the energy in the dipole in our nondipolar models, due to its subservience in these systems. Thus, the energy in the magnetic dipole of Uranus/Neptune may currently be in a stronger/weaker phase, respectively. In general, these systems are better characterized by the energy in the nondipolar large-scale magnetic field, which we hope may be measured by future missions.

Our dipolar scaling law applied to Saturn indicates that its dynamo operates much deeper than one would infer from electrical conductivity profiles alone (as also suggested by U. R. Christensen et al. 2018). This is also the case in Jupiter. P. N. Wulff et al. (2025) show that estimating the magnetic Reynolds number (an indicator of dynamo action) as a function of radius predicts a dynamo radius of around $0.91 R_J$ (electrical conductivity $\sigma \approx 10^5 \text{ S m}^{-1}$ at this depth; M. French et al. 2012). A similar electrical conductivity is reached at $0.7\text{--}0.75 R_S$ in Saturn (M. Preising et al. 2023).

However, the magnetic field spectrum at the Jovian surface indicates $r_{\text{dyn}} \approx 0.83 R_J$ (obtained by carrying out a Lowes analysis, J. E. P. Connerney et al. 2022; P. N. Wulff et al.

2025). This mismatch is due to the presence of a stably stratified helium rain layer from $\sim 0.83 R_J$ to around $0.87\text{--}0.9 R_J$. Such a layer is also present in Saturn and is widely held responsible for the extremely axisymmetric nature of its magnetic field at the surface (D. J. Stevenson 1982; H. Cao et al. 2020; R. K. Yadav et al. 2022). As this is a much more pronounced effect than at Jupiter, the helium rain layer is thought to be much thicker inside Saturn, which is also endorsed by stability analysis in the two planets (S. Markham & T. Guillot 2024).

A Lowes radius analysis cannot be easily performed at Saturn due to the irregular zigzags in its surface spectrum—markers of the magnetic field filtering process occurring in the stable layer. However, the distribution of magnetic energy in its odd spectral components (excluding $\ell = 5$) closely mirrors that of the Earth, which has a dynamo radius of $\sim 0.55 R_E$. This indicates that the dynamo radius is deeper than the $0.7\text{--}0.75 R_S$ implied by conductivity alone.

Thus, both the axisymmetric nature of the Saturnian magnetic field and the gradient of the magnetic power spectrum are consistent with the deeper dynamo radius ($r_{\text{dyn}} \approx 0.45 R_S$) suggested by our scaling law analysis. This is at odds with ring seismology analysis, which suggests the inner $\sim 60\%$ of Saturn is stably stratified (C. R. Mankovich & J. Fuller 2021). If we take $r_{\text{dyn}} = 0.7 R_S$ and $D_{\text{dyn}} = 0.1 R_S$, as this would imply, then this would require $\alpha g q_0 / c_P \approx 2.18 \times 10^{-17}$ for the scaling law to hold, which is 0.00052% of our estimate, based on Saturnian interior models. It is unlikely that our estimates of α , g , q_0 , and c_P are so far off. The $\mathcal{P}_B(\ell = 1, a) \sim r_{\text{dyn}}^6$ dependence in Equation (28) means that small adjustments in the estimate of r_{dyn} are much more effective in finding feasible solutions that obey our scaling laws.

As alluded to in Section 4.2, the filtering process occurring in the stable helium rain layer in Saturn also means that we must take caution when testing the scaling law on the dipole field strength measured at the surface. The assumption that the field behaves as a potential in this layer, and thus that $\mathcal{P}_B(\ell, r) = \mathcal{P}_B(\ell, a) (a/r)^{2\ell+4}$ holds, does not reflect the more complex effect of the stable stratification, in the presence of strong differential rotation.

Furthermore, the mechanical, thermal, and magnetic boundary conditions used in our suite of fully conducting, fully convective dynamo simulations do not reflect the complexity of the interfaces with the regions above and below the dynamo regions in the giant planets. However, R. K. Yadav et al. (2013b) showed that the scaling of Lo_V held for a variety of numerical models, including those with different boundary conditions and varying background density and conductivity profiles. This gives us confidence that our simpler models can be extended to planetary interiors, within reasonable uncertainties.

5.3. Exoplanets

When it comes to predicting the morphology of the magnetic field strength of exoplanets the main motivations, currently, are either to investigate the induction that may take place in the thermally ionized atmospheres of hot Jupiters or to identify likely candidates for magnetic field detection. In both cases it is always assumed that the planet has a dipolar magnetic field morphology (e.g., K. Batygin & D. J. Stevenson 2010; W. Dietrich et al. 2022; K. Kilmetis et al. 2024; A. Elias-López et al. 2025).

U. R. Christensen & J. Aubert (2006) and U. R. Christensen (2010) suggest a factor of around 3.5 for scaling up the magnetic field strength at the top of a dynamo region to the interior field strength. We obtain a similar ratio for the scalings of total magnetic field in dipolar dynamo cases because comparing the prefactors given in Table 2 yields $0.646/0.168 \approx 3.85$. Thus, our results indicate that using the established [CA06]/[C09] scaling laws to first infer internal magnetic field strength and then scaling down using this factor to estimate total surface magnetic field strength should work well for *dipolar magnetic field morphologies*. However, if one has no measurement of the total magnetic field at the surface or if one wants to estimate the dipole field strength specifically—as it is the most relevant for many space physics processes—one must consider the scalings for $B(\ell = 1)$ specifically. This study highlights that, in order to do this, one must take into account that the magnetic field morphology may be non-dipolar, and that this makes up an order-of-magnitude difference. Further, it is then much more likely that the dipole component is not aligned with the rotation axis and is fluctuating more in time.

Acknowledgments

We thank Denis Shulyak for acting as consultant on magnetic field measurements of M dwarfs. P.N.W. and H.C. thank the NASA Juno project and the NASA CDAP grant No. 80NSSC23K0511 for support. H.C. also acknowledges the support from the Sloan Research Fellowship.

Appendix A Initialization of Simulations

We initialize the first dipole-dominated models with an axial dipole with amplitude $\Lambda \sim 5$ at $r = r_i$, and the nondipolar models with a weak $\Lambda \sim 0.1$ field across all modes (ℓ and m) randomly. The evolution of the total magnetic energy and the energy in the dipole (axial and equatorial) for one pair of models is shown in Figure 9. These both have control parameters $Pm = 1$, $E = 10^{-4}$, and $Ra = 10^7$.

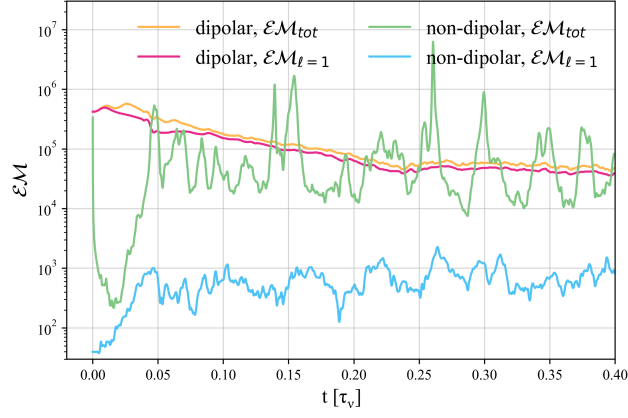


Figure 9. Time series for two simulations with the same control parameters. The orange (pink) curve represents the evolution of the total (dipole) energy for the model that has a steady state dominated by the dipole. Green (blue) is the equivalent for the model in the nondipolar regime.

Appendix B

Magnetic Field Scaling with Rossby number

In Table 4 we present the coefficients for the fitting of Lorentz numbers with $Ra_Q^{*1/3}$ (i.e., omitting the factor of $f_{\text{ohm}}^{1/2}$). Comparing these values with those given in Table 2 highlights the implications of assuming $f_{\text{ohm}} = 1$ for planets: if we take values of the ohmic dissipation fraction that are more similar to our models, we would infer significantly thicker dynamo regions

for Jupiter and Saturn. As an example, for a dynamo radius of $0.8 R_J$ for Jupiter, we would estimate a lower dynamo boundary of $\sim 0.29 R_J$, compared with $0.48 R_J$ obtained with the scaling that assumes that in Jupiter $f_{\text{ohm}} = 1$ (see Figure 8(c)).

Table 4 also gives the coefficients for the fitting Lo with the various Rossby numbers, with their respective exponents. The equivalent of Figure 6, but with the input parameter Ro_c on the x -axis is shown in Figure 10.

Table 4
Coefficients Obtained when Fitting $Lo \sim Ra_Q^{*1/3}$, $Lo \sim Ro_c^{5/3}$, $Lo \sim Ro^{5/6}$, and $Lo \sim Ro_\ell^{5/3}$

Scaling	Prefactor			
	$Ra_Q^{*1/3}$	$Ro_c^{5/3}$	$Ro^{5/6}$	$Ro_\ell^{5/3}$
Lo_o dipolar	0.121 ± 0.016	0.027 ± 0.003	0.138 ± 0.019	0.236 ± 0.067
Lo_o nondipolar	0.047 ± 0.006	0.009 ± 0.001	0.049 ± 0.006	0.068 ± 0.010
$Lo_o(\ell = 1)$ dipolar	0.113 ± 0.015	0.025 ± 0.003	0.129 ± 0.019	0.221 ± 0.063
$Lo_o(\ell = 1)$ nondipolar	0.009 ± 0.002	0.002 ± 0.000	0.010 ± 0.002	0.013 ± 0.003
$Lo_o(\ell > 1)$ universal	0.045 ± 0.004	0.009 ± 0.001	0.048 ± 0.004	0.069 ± 0.010
Lo_v dipolar	0.465 ± 0.052	0.102 ± 0.011	0.531 ± 0.066	0.898 ± 0.271
Lo_v nondipolar	0.294 ± 0.041	0.059 ± 0.008	0.309 ± 0.041	0.419 ± 0.084
$Lo_v(\ell = 1)$ dipolar	0.402 ± 0.044	0.088 ± 0.009	0.459 ± 0.057	0.769 ± 0.245
$Lo_v(\ell = 1)$ nondipolar	0.042 ± 0.011	0.009 ± 0.002	0.044 ± 0.012	0.058 ± 0.021
$Lo_v(\ell > 1)$ universal	0.274 ± 0.032	0.056 ± 0.006	0.295 ± 0.031	0.421 ± 0.065

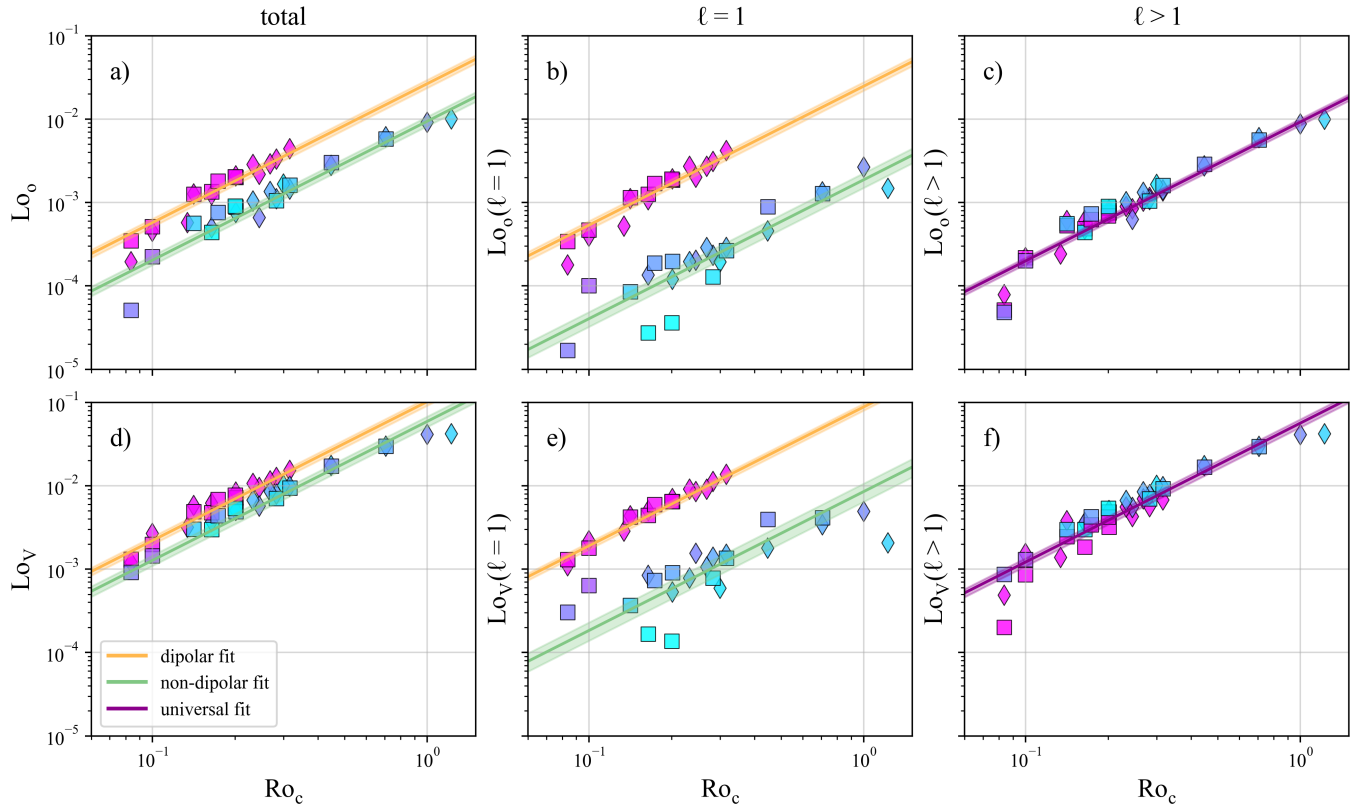


Figure 10. The equivalent of Figure 6, but with convective Rossby number Ro_c on the x -axis and with Lo rather than $Lo/f_{\text{ohm}}^{1/2}$ on the y -axis. See Table 4 for the parameters of the lines of best fit.

Appendix C

M Dwarf Data

Here we provide the data used for our investigation of M dwarf magnetic field amplitudes. Table 5 gives both the data and the sources (see caption for abbreviations).

Table 5
Stellar Properties

Star	M ($M_{\odot}$)	L ($L_{\odot}$)	R ($R_{\odot}$)	P_{rot} (days)	$B(\ell = 1)$ (G)	B (G)	References (Stokes V)	References (Stokes I)
GJ 1111 (2007)	0.1	7.09×10^{-4}	0.11	0.46	92.3	2680	M10	R22
GJ 1111 (2008)	0.1	7.09×10^{-4}	0.11	0.46	45.3	2680	M10	R22
GJ 1111 (2009)	0.1	7.09×10^{-4}	0.11	0.46	53.5	2680	M10	R22
Gl 412 B (2006)	0.1	1.05×10^{-3}	0.12	0.78	904	6880	M10	R22
Gl 412 B (2007)	0.1	1.05×10^{-3}	0.12	0.78	1160	6880	M10	R22
Gl 412 B (2008)	0.1	1.05×10^{-3}	0.12	0.78	1110	6880	M10	R22
Gl 412 B (2009)	0.1	1.05×10^{-3}	0.12	0.78	1590	6880	M10	R22
Gl 406 (2019a)	0.1	7.41×10^{-4}	0.12	2.7	348	2600	B24	S17
Gl 406 (2019b2020a)	0.1	7.41×10^{-4}	0.12	2.7	319	2600	B24	S17
Gl 406 (2020b2021a)	0.1	7.41×10^{-4}	0.12	2.7	482	2600	B24	S17
Gl 406 (2021b2022a)	0.1	7.41×10^{-4}	0.12	2.7	415	2600	B24	S17
GJ 1245 B (2006)	0.12	1.74×10^{-3}	0.14	0.71	122	3400	M10	R22
GJ 1245 B (2008)	0.12	1.74×10^{-3}	0.14	0.71	38.2	3400	M10	R22
GJ 1286 (2020)	0.12	1.40×10^{-3}	0.142	178	103	910	L24	R22
GJ 1286 (2021)	0.12	1.40×10^{-3}	0.142	178	67.3	910	L24	R22
GJ 1156 (2007)	0.14	2.16×10^{-3}	0.16	0.49	35.3	3970	M10	R22
GJ 1156 (2008)	0.14	2.16×10^{-3}	0.16	0.49	74.9	3970	M10	R22
GJ 1156 (2009)	0.14	2.16×10^{-3}	0.16	0.49	71.4	3970	M10	R22
Gl 905 (2019)	0.15	2.18×10^{-3}	0.165	114.3	124	410	L24	R22
Gl 905 (2020)	0.15	2.18×10^{-3}	0.165	114.3	80.2	410	L24	R22
Gl 905 (2021)	0.15	2.18×10^{-3}	0.165	114.3	63.6	410	L24	R22
GJ 1151 (2019)	0.17	3.42×10^{-3}	0.193	175.6	22.7	680	L24	R22
GJ 1151 (2020)	0.17	3.42×10^{-3}	0.193	175.6	31.9	680	L24	R22
GJ 1151 (2021)	0.17	3.42×10^{-3}	0.193	175.6	61.7	680	L24	R22
GJ 1289 (2019)	0.21	5.38×10^{-3}	0.233	73.66	79.2	1360	L24	R22
GJ 1289 (2020)	0.21	5.38×10^{-3}	0.233	73.66	194	1360	L24	R22
GJ 1289 (2021)	0.21	5.38×10^{-3}	0.233	73.66	199	1360	L24	R22
EQ Peg B (2006)	0.25	7.70×10^{-3}	0.25	0.4	379	4200	M08b	R22
V374 Peg (2005)	0.28	9.47×10^{-3}	0.28	0.45	588	3630	M08a	R22
V374 Peg (2006)	0.28	9.47×10^{-3}	0.28	0.45	491	3630	M08a	R22
EV Lac (2006)	0.32	1.05×10^{-2}	0.3	4.37	447	4320	M08b	R22
EV Lac (2007)	0.32	1.05×10^{-2}	0.3	4.37	420	4320	M08b	R22
EV Lac (2019b)	0.32	1.05×10^{-2}	0.3	4.37	63.1	4320	B24	R22
EV Lac (2020b2021a)	0.32	1.05×10^{-2}	0.3	4.37	289	4320	B24	R22
EV Lac (2021b)	0.32	1.05×10^{-2}	0.3	4.37	254	4320	B24	R22
Gl 285 (2007)	0.32	7.27×10^{-3}	0.29	2.77	540	4540	M08b	R22
Gl 285 (2008)	0.32	7.27×10^{-3}	0.29	2.77	514	4540	M08b	R22

Note. In the references M10 refers to J. Morin et al. (2010), B24 to S. Bellotti et al. (2024), L24 to L. T. Lehmann et al. (2024), M08b to J. Morin et al. (2008b), M08a to J. Morin et al. (2008a), R22 to A. Reiners et al. (2022), and S17 to D. Shulyak et al. (2017).

Appendix D

Data on the Solar System Gas Planets

Data used in Section 4.2 are provided here in Table 6.

Table 6
Parameters Used for the Approximation of $Lo(\ell = 1)$ at the Dynamo Surface of the Solar System Planets

Planet	$\mathcal{P}_B(\ell = 1, a)$ (nT ²)	a (m)	α (K ⁻¹)	g (m s ⁻²)	q_0 (W m ⁻²)	C_P (J K ⁻¹ kg ⁻¹)	ρ (kg m ⁻³)
Jupiter	3.49×10^{11}	6.9911×10^7	1.2×10^{-5}	25.9	7.5	1.4×10^4	2000
Saturn	8.94×10^8	5.8232×10^7	2.5×10^{-5}	11	2	1.7×10^4	1500
Uranus	1.04×10^9	2.5362×10^7	4.4×10^{-5}	9	0.089	4.2×10^3	1750
Neptune	4.06×10^8	2.4622×10^7	4.4×10^{-5}	11.3	0.433	4.2×10^3	1750

Appendix E

Model Parameters and Diagnostics

Table 7 gives the model control parameters (Ekman, Rayleigh and magnetic Prandtl numbers) and relevant diagnostics as defined in the main text.

Table 7
Models and Their Relevant Control and Diagnostic Parameters

E	Ra	Pm	Ro_c	Nu	Ro'	$\mathcal{L}_U$	$f_{\text{dip},ro}$	$f_{\text{dip},V}$	Λ_o	Λ_V	f_{ohm}
10^{-4}	6×10^6	1	0.24	2.43	0.008	0.185	0.847	0.798	0.047	0.906	0.438
10^{-4}	6×10^6	1	0.24	2.26	0.008	0.182	0.093	0.075	0.004	0.325	0.29
10^{-4}	8×10^6	1	0.28	3.11	0.01	0.177	0.879	0.8	0.11	1.578	0.473
10^{-4}	8×10^6	1	0.28	3.08	0.011	0.176	0.049	0.033	0.012	0.627	0.322
10^{-4}	8×10^6	0.5	0.28	2.9	0.011	0.174	0.015	0.013	0.006	0.247	0.308
10^{-4}	10^7	1	0.32	3.79	0.013	0.171	0.903	0.804	0.195	2.33	0.492
10^{-4}	10^7	1	0.32	3.82	0.014	0.177	0.039	0.023	0.021	0.973	0.35
10^{-4}	10^7	0.5	0.32	3.72	0.014	0.176	0.027	0.021	0.013	0.438	0.352
10^{-4}	2×10^7	1	0.45	6.91	0.025	0.174	0.028	0.011	0.078	3.62	0.4
10^{-4}	2×10^7	0.5	0.45	6.89	0.025	0.173	0.085	0.052	0.046	1.473	0.401
10^{-4}	5×10^7	1	0.71	12.81	0.048	0.163	0.046	0.012	0.377	8.9	0.409
10^{-4}	5×10^7	0.5	0.71	12.63	0.047	0.161	0.048	0.019	0.168	4.443	0.412
10^{-4}	10^8	1	1.0	18.68	0.072	0.151	0.083	0.014	0.836	17.013	0.427
10^{-4}	1.5×10^8	1	1.22	23.22	0.102	0.16	0.023	0.002	1.011	17.627	0.385
3×10^{-5}	2×10^7	1	0.13	1.89	0.002	0.142	0.823	0.809	0.011	0.335	0.443
3×10^{-5}	3×10^7	1	0.16	2.87	0.004	0.128	0.758	0.662	0.051	1.273	0.563
3×10^{-5}	3×10^7	1	0.16	2.97	0.004	0.122	0.073	0.053	0.008	0.458	0.365
3×10^{-5}	3×10^7	0.5	0.16	2.79	0.004	0.116	0.87	0.85	0.03	0.381	0.408
3×10^{-5}	3×10^7	0.5	0.16	2.77	0.004	0.119	0.005	0.004	0.003	0.149	0.311
3×10^{-5}	4.5×10^7	1	0.2	4.43	0.006	0.118	0.843	0.696	0.145	2.347	0.565
3×10^{-5}	4.5×10^7	1	0.2	4.44	0.007	0.122	0.025	0.011	0.019	0.856	0.384
3×10^{-5}	4.5×10^7	0.5	0.2	4.13	0.006	0.115	0.857	0.803	0.067	0.866	0.484
3×10^{-5}	4.5×10^7	0.5	0.2	4.36	0.007	0.123	0.05	0.035	0.013	0.404	0.385
3×10^{-5}	6×10^7	1	0.23	5.69	0.007	0.116	0.901	0.729	0.272	3.785	0.602
3×10^{-5}	6×10^7	1	0.23	5.79	0.009	0.125	0.036	0.014	0.036	1.499	0.432
3×10^{-5}	8×10^7	1	0.27	7.5	0.01	0.117	0.873	0.631	0.282	4.359	0.555
3×10^{-5}	8×10^7	1	0.27	7.37	0.011	0.125	0.047	0.015	0.061	2.426	0.465
3×10^{-5}	10^8	1	0.3	8.99	0.013	0.125	0.015	0.003	0.092	3.411	0.485
10^{-5}	7×10^7	1	0.08	1.63	0.001	0.115	0.838	0.835	0.004	0.144	0.416
10^{-5}	7×10^7	0.5	0.08	1.64	0.001	0.12	0.106	0.103	0.0	0.042	0.353
10^{-5}	10^8	1	0.1	2.51	0.002	0.097	0.762	0.669	0.02	0.717	0.58
10^{-5}	10^8	0.5	0.1	2.4	0.002	0.085	0.821	0.812	0.013	0.196	0.406

Table 7
(Continued)

E	Ra	Pm	Ro_c	Nu	Ro'	$\mathcal{L}_U$	f_{dip,r_0}	$f_{\text{dip},V}$	Λ_o	Λ_V	f_{ohm}
10^{-5}	10^8	0.5	0.1	2.48	0.002	0.089	0.202	0.192	0.003	0.105	0.36
10^{-5}	2×10^8	1	0.14	5.96	0.003	0.088	0.765	0.562	0.16	3.345	0.676
10^{-5}	2×10^8	0.5	0.14	5.43	0.003	0.083	0.824	0.747	0.079	1.193	0.585
10^{-5}	2×10^8	0.5	0.14	5.39	0.004	0.091	0.024	0.015	0.016	0.457	0.423
10^{-5}	3×10^8	0.5	0.17	8.08	0.005	0.083	0.881	0.754	0.164	2.351	0.633
10^{-5}	3×10^8	0.5	0.17	8.15	0.006	0.094	0.06	0.028	0.029	0.957	0.507
10^{-5}	4×10^8	0.5	0.2	10.52	0.006	0.085	0.884	0.699	0.207	2.981	0.634
10^{-5}	4×10^8	0.5	0.2	10.68	0.007	0.091	0.002	0.001	0.041	1.436	0.522

Note. Critical Rayleigh numbers are $Ra_c = 6.51 \times 10^5$ for $E = 10^{-4}$, $Ra_c = 2.68 \times 10^6$ for $E = 3 \times 10^{-5}$, and $Ra_c = 1.01 \times 10^7$ for $E = 10^{-5}$. All simulations are Boussinesq, have aspect ratio $r_i/r_o = 0.35$, $Pr = 1$, and the same boundary conditions.

ORCID iDs

Paula N. Wulff  <https://orcid.org/0000-0002-6788-8898>
Hao Cao  <https://orcid.org/0000-0002-6917-8363>
Jonathan M. Aurnou  <https://orcid.org/0000-0002-8642-2962>

References

- Aubert, J., Labrosse, S., & Poitou, C. 2009, *GeoJI*, **179**, 1414
Aurnou, J. M., Horn, S., & Julien, K. 2020, *PhRvR*, **2**, 043115
Batygin, K., & Stevenson, D. J. 2010, *ApJL*, **714**, L238
Bellotti, S., Morin, J., Lehmann, L. T., et al. 2024, *A&A*, **686**, A66
Calkins, M. A., Orvedahl, R. J., & Featherstone, N. A. 2021, *GeoJI*, **227**, 1228
Cano Amoros, M., Nettelmann, N., Tosi, N., Baumeister, P., & Rauer, H. 2024, *A&A*, **692**, A152
Cao, H., Dougherty, M. K., Hunt, G. J., et al. 2020, *Icar*, **344**, 113541
Christensen, U. R. 2006, *Natur*, **444**, 1056
Christensen, U. R. 2010, *SSRv*, **152**, 565
Christensen, U. R., & Aubert, J. 2006, *GeoJI*, **166**, 97
Christensen, U. R., Aubert, J., Cardin, P., et al. 2001, *PEPI*, **128**, 25
Christensen, U. R., Cao, H., Dougherty, M. K., & Khurana, K. 2018, in *Saturn's Magnetic Field and Dynamo*, ed. K. H. Baines et al. (Cambridge Univ. Press), 69
Christensen, U. R., Holzwarth, V., & Reiners, A. 2009, *Natur*, **457**, 167
Cohen, O., Drake, J. J., Gloer, A., et al. 2014, *ApJ*, **790**, 57
Connerney, J. E. P., Acuna, M. H., & Ness, N. F. 1991, *JGR*, **96**, 19023
Connerney, J. E. P., Timmins, S., Oliverson, R. J., et al. 2022, *JGRE*, **127**, e07055
Davidson, P. A. 2013, *GeoJI*, **195**, 67
Dietrich, W., Kumar, S., Poser, A. J., et al. 2022, *MNRAS*, **517**, 3113
Donati, J.-F., Cristofari, P. I., Klein, B., Finocietty, B., & Moutou, C. 2025, *A&A*, **700**, A122
Elias-López, A., Cantiello, M., Viganò, D., et al. 2025, *ApJ*, **990**, 38
Frasson, T., Schaeffer, N., Nataf, H.-C., & Labrosse, S. 2026, *E&PSL*, **687**, 120089
French, M., Becker, A., Lorenzen, W., et al. 2012, *ApJS*, **202**, 5
Gastine, T., Duarte, L., & Wicht, J. 2012, *A&A*, **546**, A19
Gastine, T., Morin, J., Duarte, L., et al. 2013, *A&A*, **549**, L5
Gastine, T., & Wicht, J. 2012, *Icar*, **219**, 428
Gastine, T., Yadav, R. K., Morin, J., Reiners, A., & Wicht, J. 2014, *MNRAS*, **438**, L76
Gupta, A., Stixrude, L., & Schlichting, H. E. 2025, *ApJL*, **982**, L35
Helled, R., & Stevenson, D. J. 2024, *AGUA*, **5**, e2024AV001171
Holme, R., & Bloxham, J. 1996, *JGR*, **101**, 2177
Julien, K., Knobloch, E., Rubio, A. M., & Vasil, G. M. 2012, *PhRvL*, **109**, 254503
Kilmetis, K., Vidotto, A. A., Allan, A., & Kubyskhina, D. 2024, *MNRAS*, **535**, 3646
Lang, P., Jardine, M., Morin, J., et al. 2014, *MNRAS*, **439**, 2122
Lehmann, L. T., Donati, J.-F., Fouqué, P., et al. 2024, *MNRAS*, **527**, 4330
Lehmann, L. T., Jardine, M. M., Mackay, D. H., & Vidotto, A. A. 2018, *MNRAS*, **478**, 4390
Mankovich, C. R., & Fuller, J. 2021, *NatAs*, **5**, 1103
Markham, S., & Guillot, T. 2024, *PSJ*, **5**, 269
Menu, M. D., Pettdemange, L., & Galtier, S. 2020, *PEPI*, **307**, 106542
Militzer, B. 2024, *PNAS*, **121**, e2403981121
Militzer, B., & Hubbard, W. B. 2024, *Icar*, **411**, 115955
Moffatt, H. K. 1978, *Magnetic Field Generation in Electrically Conducting Fluids* (Cambridge Univ. Press)
Moffatt, H. K., & Dormy, E. 2019, *Self-Exciting Fluid Dynamos* (Cambridge Univ. Press)
Morf, L., & Helled, R. 2025, *A&A*, **704**, A183
Morin, J., Donati, J.-F., Forveille, T., et al. 2008a, *MNRAS*, **384**, 77
Morin, J., Donati, J.-F., Petit, P., et al. 2008b, *MNRAS*, **390**, 567
Morin, J., Donati, J. F., Petit, P., et al. 2010, *MNRAS*, **407**, 2269
Morin, J., Dormy, E., Schrunner, M., & Donati, J.-F. 2011, *MNRAS*, **418**, L133
Ness, N. F., Acuna, M. H., Behannon, K. W., et al. 1986, *Sci*, **233**, 85
Preisung, M., French, M., Mankovich, C., Soubiran, F., & Redmer, R. 2023, *ApJS*, **269**, 47
Reiners, A., & Basri, G. 2009, *A&A*, **496**, 787
Reiners, A., Basri, G., & Christensen, U. R. 2009, *ApJ*, **697**, 373
Reiners, A., Shulyak, D., Käpylä, P. J., et al. 2022, *A&A*, **662**, A41
Roberts, P. H., & Glatzmaier, G. A. 2000, *RvMP*, **72**, 1081
See, V., Amard, L., Bellotti, S., et al. 2025, *MNRAS*, **542**, 1318
Shulyak, D., Reiners, A., Engeln, A., et al. 2017, *NatAs*, **1**, 0184
Soderlund, K. M., Heimpel, M. H., King, E. M., & Aurnou, J. M. 2013, *Icar*, **224**, 97
Soderlund, K. M., King, E. M., & Aurnou, J. M. 2012, *E&PSL*, **333-334**, 9
Soderlund, K. M., Wulff, P., Käpylä, P. J., & Aurnou, J. M. 2025, *MNRAS*, **541**, L186
Stanley, S., & Bloxham, J. 2004, *Natur*, **428**, 151
Stanley, S., & Bloxham, J. 2006, *Icar*, **184**, 556
Stevenson, D. J. 1982, *GApFD*, **21**, 113
Tassin, T., Gastine, T., & Fournier, A. 2021, *GeoJI*, **226**, 1897
Teed, R. J., & Dormy, E. 2025, *GeoRL*, **52**, e2025GL118078
Vidotto, A. A., Jardine, M., Morin, J., et al. 2013, *A&A*, **557**, A67
Vidotto, A. A., Jardine, M., Morin, J., et al. 2014, *MNRAS*, **438**, 1162
Wicht, J. 2002, *PEPI*, **132**, 281
Wulff, P. N., Cao, H., & Aurnou, J. M. 2025, *ApJ*, **992**, 50
Yadav, R. K., Cao, H., & Bloxham, J. 2022, *ApJ*, **940**, 185
Yadav, R. K., Christensen, U. R., Morin, J., et al. 2015, *ApJL*, **813**, L31
Yadav, R. K., Gastine, T., & Christensen, U. R. 2013a, *Icar*, **225**, 185
Yadav, R. K., Gastine, T., Christensen, U. R., & Duarte, L. D. V. 2013b, *ApJ*, **774**, 6
Zaire, B., Jouve, L., Gastine, T., et al. 2022, *MNRAS*, **517**, 3392