Imagery as Process Representation in Problem Solving*

Yulin Qin **Herbert A. Simon**

Department of Psychology
Carnegie Mellon University
Pittsburgh, PA 15213
qin@psy.cmu.edu has@a.gp.cs.cmu.edu

Abstract

In this paper, we describe the characteristics of imagery phenomena in problem solving, develop a model for the process of forming and observing mental images in problem solving, and check the model against data obtained from subjects. Then, we describe the interaction between imaging and problem solving observed in our experiments, and discuss the use of our model to simulate it. We also discuss the relation between mental models and mental image briefly.

We are interested in exploring the role and characteristics of mental imagery in problem solving and understanding. Our method of inquiry is to observe how subjects understand , with the help of their mental images, the first, kinematic, part of Einstein's 1905 paper: "On the Electrodynamics of Moving Bodies" (Einstein,1905).

The generation, maintenance, inspection, and transformation of images have been well researched in general and at a fine grain size (cf. Kosslyn 1988 for summary). For example, Shepard and Metzler (1971) found that the more one rotates an imaged pattern, the more time is required. Our discussion will focus on more special situations and a rather larger grain size: on complex information processes used in solving a non-trivial problem. In our experiments, subjects used their images to simulate physical processes, and by finding the relation among the related physical quantities, they acquired the information necessary for problem solving -- deriving the appropriate equations.

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In Qin and Simon (1992) we reported briefly on the major findings of our project. In Qin and Simon (1990), we discussed in a preliminary way what kinds of information subjects could obtain from their mental images and how they used it in their problem solving. In this paper we will discuss how subjects obtain this information. i.e., the processes subjects use to form, watch (make inferences from) and change their images during their problem solving.

1. Experiment

We will be concerned with the imagery subjects used while deriving the first pair of key equations needed to obtain the Lorentz transformation equations. The reading material states:

> "Let there be given a stationary rigid rod... We now imagine the axis of the rod lying along the axis of x of the stationary system of co-ordinates, and that a uniform motion of parallel translation with velocity v along the axis of x in the direction of increasing x is then imparted to the rod...We imagine further that at the two ends A and B of the rod, clocks are placed which synchronize with the clocks of the stationary system...We imagine further ... Let a ray of light depart from A at the time t_A, let it be reflected at B at the time t_B, and reach A again at the time t'_A. Taking into consideration the principle of the constancy of the velocity of light we find that
>
> $$t_B - t_A = r_{AB} / (c-v) \quad (1)$$
>
> and
>
> $$t'_A - t_B = r_{AB} / (c+v) \quad (2)$$
>
> where r_{AB} denotes the length of the moving rod -- measured in the stationary system."

Figure 1 shows the process of light traveling while rod AB is moving relative to the stationary system. Equation (1) can be derived as follows: When light arrives at B at the time t_B, B has moved a distance, $v(t_B - t_A)$, measured in the stationary

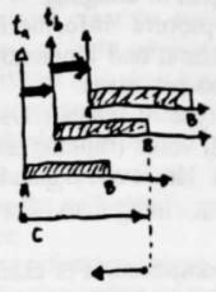


Figure 1

system. So the total distance the light traveled from A to B is $r_{AB} + v(t_B - t_A)$. We assume the constancy of the velocity of light, c, whether it is measured in the stationary system or in the system of the moving rod. Therefore, the total distance the light traveled from A to B is $c(t_B - t_A)$. So we have:

$$c(t_B - t_A) = r_{AB} + v(t_B - t_A).$$

Collecting the terms in $(t_B - t_A)$, we get equation (1). We call this method of derivation the *length method.*

Another method, the *velocity method*, for deriving the equation is to pay attention to velocity instead of length. When light travels from A to B, it travels in the same direction as the moving rod. So, *relative to the moving rod*, still measured in the stationary system, the velocity of the light is c-v, and the distance the light travels is r_{AB}. From the relation: time equals distance divided by velocity, we also get equation (1).

Reference system and *measured system* are key concepts here. Two different reference systems were employed in the two different methods. An interesting phenomenon is that in using the velocity method, but not in using the length method, subjects usually reported the reference system. Equations (1) and (2) contrast with the equations based on Galilean transformations, which assume that the *time* for light to travel, measured in both moving and stationary systems, is the same, and the velocity of light is not a constant.

We obtain information from which we infer what mental images subjects have formed from their protocols, the diagrams they drew, and their gestures.

We have analyzed six subjects' protocols. One subject, with an MS degree in computer science, is a research assistant in psychology at Carnegie Mellon University (CMU). The others are undergraduate or graduate students in electrical engineering or computer science at CMU. None of them were familiar with derivations of the Lorentz equations or could derive them. None were aware that the reading material was from Einstein (1905), until, at the end of the experiment, they were told by the experimenter.

Among these six subjects, S_r and S_m were asked to describe their mental images, draw diagrams, use diagrams; S_b and S_j to describe images, but not to draw diagrams; and S_g and S_s to describe images and draw diagrams, but not to retain the diagrams (The experimenter took each diagram away as soon as it was drawn, and they could not draw diagrams while deriving the equations.) In this paper we will not emphasize differences between the groups, but will try to find some common patterns among them. What the experimental design tells us is that four of six of our subjects could not use diagrams in their problem solving. Allowing some subjects to draw diagrams helps us to clarify their protocol statements about their images. Conversely, protocols can help us to identify the mental images underlying the diagrams they drew, especially in the case of the subjects who were allowed to draw and use diagrams while deriving the equations.

2. The Process of Forming and Watching Mental Images

2.1 Similarities and Individual Differences of Subjects' Mental Images

In his 1905 paper, Einstein invited his readers several times to "imagine" the situations he described. Curiously enough, the published paper contains no diagrams to guide or assist this process. All of our six subjects could form and report their images by means of diagrams and/or protocols. From these we can find the characteristics of their imagery.

Figure 1 is based on S_g's diagram. Figure 2 gives the diagrams drawn by all the other subjects except S_b, who did not draw any diagrams in this experiment. His mental image is inferred by us from his protocol. To make clear the meaning of "triangle image" in S_j's protocol, the experimenter asked S_j to draw the diagram .

To form mental images, subjects needed to add to and clarify the information in the reading material. Without any diagram in the reading material to guide them, there are large individual differences among their images. For example: S_j's "triangle image" has a time dimension (in the horizontal direction), but others do not; Two subjects', S_g and S_s, drew images that had explicit coordinate systems, others did not; S_s' rod and light traveled in a vertical direction, but the others' in a horizontal direction. While traveling from A to B, in most subjects' diagrams, light moved in the same direction as the moving rod, but in S_s' in the opposite direction.

However, underneath the differences, there are some similarities. For example, all the images

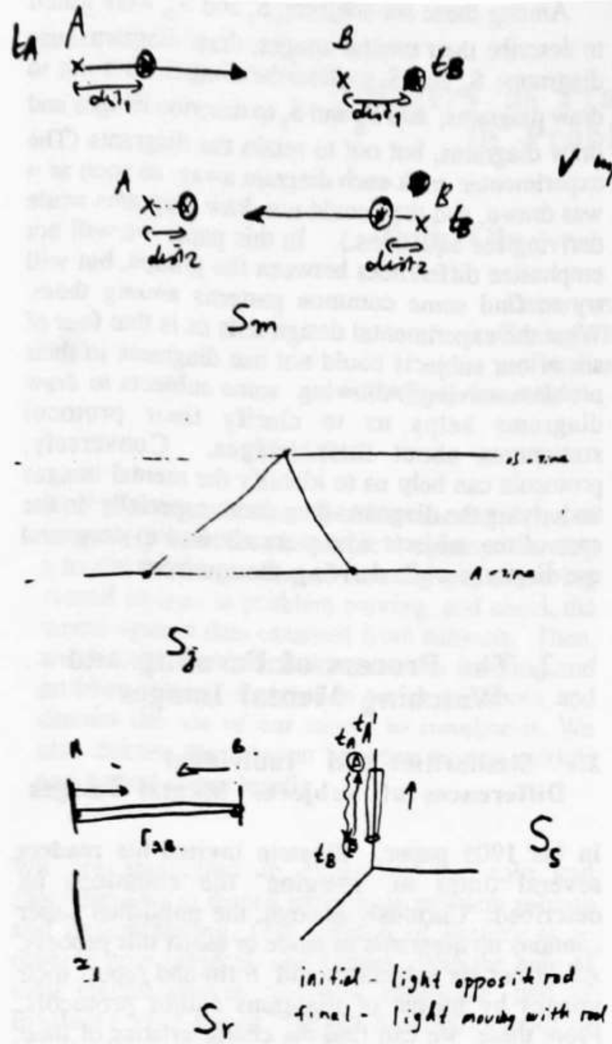


Figure 2

show the process of the light traveling. They represent a process of change. All of them are very simple and neat, containing only information necessary to the subjects' understanding.

2.2 A Model of the Processes of Forming and Watching Mental Images

Based on the data collected in our experiments, we have inferred a model of the processes of forming and watching images, shown in Figure 3.

The boxes in Figure 3 are actions, the circles are data or representations, the solid arrows show the procedure flow. We will discuss them in this section. The dotted arrows in Figure 3 show the information flow, i.e., the influence of problem solving and mental model on forming and watching mental images. We will discuss them in the following two sections.

As shown in Figure 3, the basic process of forming and watching images is as follows:

1. Setting the goal of imaging;

2. Collecting picture information, such as components, structure, and kinematic relations. It consists of these sub-processes:

   a). Choosing type of image: Usually subjects use the default value (running an image of the full situation). However, S_m and S_g also ran a simple partial image to get additional information;

   b). Choosing component: For example, S_j chose the time line as a component in his image;

   c). Assigning attributes and relations: Subjects assign a spatial and a kinematic relation between components (e.g., light traveling from end A to end B of the rod); and assign a given value to an object (e.g.," the length of the *moving rod measured in the stationary system* is r_{AB}".);

   d). Forming stable image: Integrating all the components (This seems necessary, but no subject mentioned it explicitly);

3. Choosing process and frame, such as moving components, moving process, and reference system, etc. It consists of these sub-processes:

   a). Choosing moving components: The default value is all of the movable components. However, in S_m's simple image, only light is represented as a moving component;

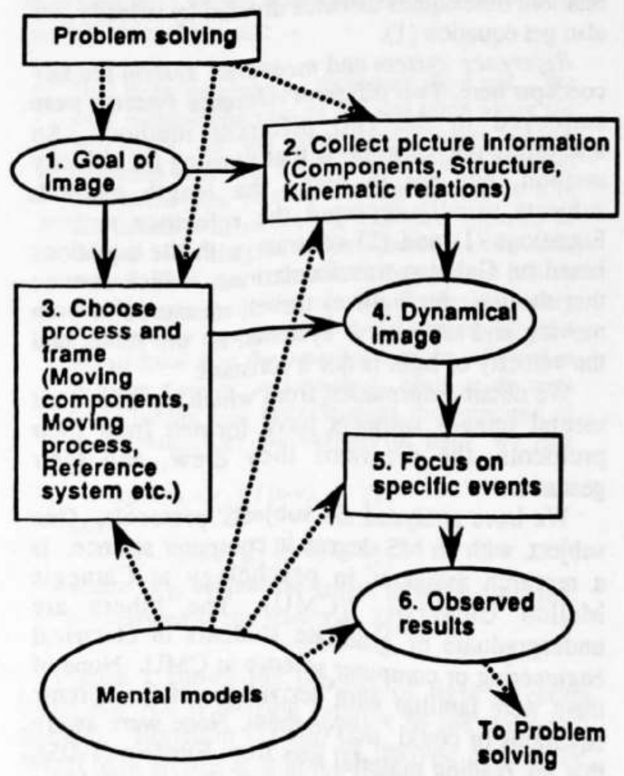


Figure 3

b). Choosing operating method: The choice is based on how clear the dynamic process (e.g., the process of light traveling) is in subjects' minds. When it is clear how to move the moving components, subjects do not report their methods. But when S_m, for example, did not know the path of the light, she tried to determine the path step by step, and then formed her image;

c). Choosing reference system ;

d). Choosing measuring system ;

e). Choosing process, i.e., deciding what process he/she wants to see. For example, S_s used the velocity method to derive the equation. He did not form an image of the whole process of light traveling from A to B and then back to A. Instead, he only imaged light leaving the A end of the rod and the rod moving at the same time;

f). Directing attention: Fixing attention on velocity, or distance, or holding no special focus of attention. For example, a subject using the length method to derive the equation might not pay attention to the velocity of light relative to the moving rod, but instead, focus on the difference between the length of the rod and the distance the light traveled;

4. Forming original dynamical image;
5. Focusing on specific events, forming a snapshot. S_m, for example, reported that when she saw the light arriving at B, she could not see A, but she knew A was moving. It seems possible that, at first, subjects form an image that shows the whole situation, and when trying to measure a particular quantity, they delete some unrelated elements;
6. Getting the result of observing, and giving the explanation of the observed result.

Subjects do not always report all of their steps. A rather complete report, shown below, is S_j's protocol given while he was forming a new image to replace the "triangle image." The number in front of a sentence denotes the (sub-)process to which the sentence corresponds:

(1) I try to figure...(2.b, 2.c) it's the ends of the rod, A is one end of the rod and B is the other end of the rod. (2.b, 2.c) So I guess it [light] is going from one end of the rod to the other and bouncing back to the first end of the rod. (3.c) And I'm trying to figure out if this is all happening in the moving frame of reference as it says RAB goes the length of the moving rod and measured in the stationary system. (3.d) Okay. All right, so this is all measured in the stationary system. (3.e, 4) So it goes from TA to TB it's got, it's going from the beginning of the rod to the end of the rod. (3.f, 5) And it's got a velocity, and the rod is moving at V and the velocity is going and the frame of reference is moving at V. (6) So light impulse then appears to be moving at C minus V, (6) because it's already moving, point A is already moving at V.

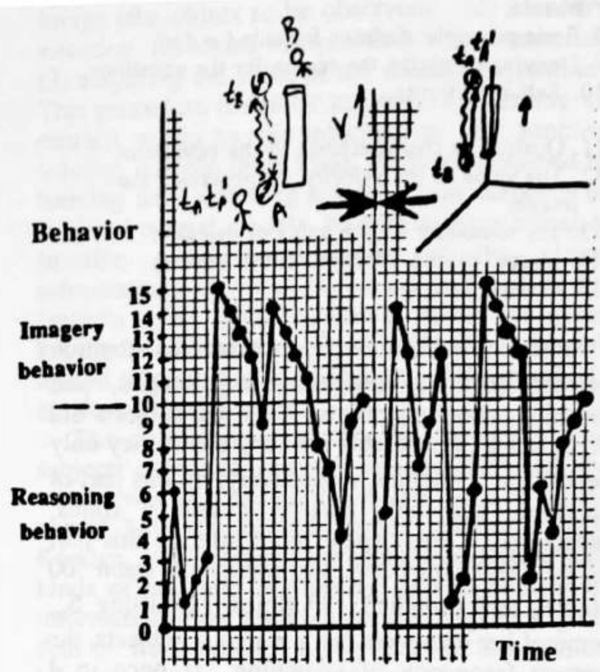


Figure 4

Of the sub-processes that did not appear in this protocol, (2.a) and (3.a) seem to assume their default values; and (2.d) and (3.b) are not usually reported in this kind of situation.

3. The Interaction between Mental Image and Problem Solving

3.1 Subjects' Switches in Attention between Reasoning and Imaging

Figure 4 shows a part of S_s' behavior in the first day of his deriving the equations and the images he formed in this period. The horizontal axis is the time axis, but it only reflects the time order of the behaviors, not time in seconds or minutes. The vertical axis shows his behavior. The meaning of the numbers is as follows (0 to 9 are reasoning behaviors, 11 to 15 are imagery behaviors) :

0. Reading equations;
1. Setting the goal of problem solving;
2. Describing the meaning of tB - tA;
3. Describing the meaning of the equations;
4. The meaning of c - v;
5. The reason for c - v;

6. The meaning of rAB: the length of the rod;
7. The meaning of rAB: the distance the light travels;
8. Basic principle: distance formula t = d/v;
9. Deriving or giving the reason for the equations;
10. Self-monitoring;

11. Qualitative characteristics of the equations;
12. The value of the velocity, by observing the image;
13. The reasoning for the value of velocity;
14. Image, paying attention to the velocity.
15. General image;

As shown in Figure 4, S_s switched his attention between forming and watching images, and reasoning about equations. All of the subjects did so. S_r and S_b's protocols are very short; they only spent 6 and 9 minutes, respectively, in this part of the task, and only switch 2 and 5 times, respectively. Others spent more time on this part and switched more often. For example S_s spent 500 minutes and switched 133 times. Omitting S_m (some of her data were lost), among 5 subjects, the average frequency of switching is once in 4 minutes; the range of frequencies among the subjects is from 0.15 to 0.55 times per minute.

Heavy STM load could be one of the reasons for the attention switching. However, as shown in Figure 4, in the course of this switch process a subject may change his image. So another reason could be the interaction between imaging and problem solving.

3.2 The influence of problem solving on forming and watching images

Images can change the method of problem solving (Qin and Simon, 1990). On the other hand, images are built for the purposes of problem solving, and the problem requirements can change the images.

As shown in Figure 3, our model proposes that problem solving will determine the goal of an image, and influence the processes of collecting picture information and choosing process and frame. We will describe some types of changes in imaging observed in our experiment, and show how we simulate them in our model.

1. Change the image wholly, as S_j does, for example, when the image appears inappropriate to the problem. This can be simulated by changing the choice of components, attributes and relations.

2. Form a new rather simple image to get additional information, as seen in S_m's and S_g's protocols. This can be simulated by choosing a different type of image;

3. Change a local part of an image to clarify a relation. Only after she made this kind of change to show the qualitative relations among unknown quantities, could S_m derive the equations. Such changes can be accomplished by observing the old image to get qualitative results and then matching these results to related quantities;

4. Change the relation among the components of an image and the values of components in the image when a wrong answer is obtained. Figure 4 shows an example from S_s. Such a change can be simulated by altering the mental model and the assignments of attributes and relations;

5. Shift attention from one method to another when difficulties are encountered, e.g. from the velocity to the length method. This is the easiest kind of change, and the one most often used by the subjects. Four of the six subjects changed methods, S_j changing 7 times and S_s 9 times. The method can be changed by altering the focus of attention.

4. Mental Models and Mental Image

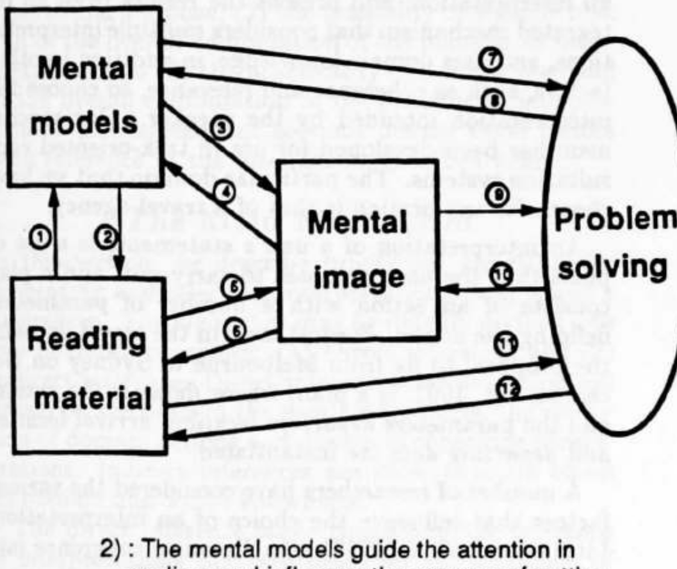
Subjects usually gave reasons for their images. For example, S_m reported while forming her image: "Light traveling needs time. So, B is moved..." In Figure 2, section 2, we have seen the influence of subjects' different prior available knowledge on their images.

As seen in S_j's protocol in section 3, when subjects reported an "observed value", they also usually gave their reasons. It seems that the "observed value" is obtained by combining the visual information (based on "seeing" in the "mind's eye"), and reasoning based on prior available knowledge. We call this prior knowledge, which may be evoked by the language of the problem description, the *mental model*. From another viewpoint, imagery in problem solving seems to integrate information already available in the mental model with new information, and thereby reach conclusions that are impossible or difficult to infer without images.

If there is a bug in a mental model, a subject may form a wrong image or "observe" a wrong result from the image. For example, by watching a correct image, the first one shown in Figure 4, S_s derived a wrong value of light velocity, which was opposite to the equations (1) and (2). He reported in his protocol about the situation of light traveling from A to B: "if you look at the bottom of the rod, the speed of light relative to stationary system is in the same direction as the rod so it would simply be the distance divided by C plus V which is the speed relative to the stationary coordinate system." and about from B back to A:"So you have the thing moving at V and the light is being shined back so you have to subtract the two because they are going in different directions." It appears that S_S took Galilean transformations as his mental model of the situation. (For example, in computing the velocity of a boat traveling in the direction of the current as measured from the bank, the velocity is c+v).

5. Discussion and Conclusion

In this paper, we have described the characteristics of imagery in problem solving, have developed a model for the process of forming, watching and changing mental images in problem solving, and have checked the model in a preliminary way. We have also described the interactions between imaging and problem solving that we observed in our experiment and have discussed how our model's behavior can be simulated. We have also discussed the relation between mental models and mental image briefly.



2) : The mental models guide the attention in reading, and influence the process of getting information from the reading material;
3), 5), 10): Mental models, reading material. and the feedback from problem solving are the information sources for image;
7), 9), 11): Mental models, reading material, and image offer information in problem solving;
6), 12) Image and problem solving also influence reading,

Figure 5

On the one hand, the process of forming and watching mental images involves: (1) forming the image (the object to be observed); (2) focussing attention (choosing the quantities to be measured); (3) acquiring the value of the measured quantities. This procedure is similar to people's observing the outside world to get information for problem solving. On the other hand, the result obtained by forming and watching mental images depends on subjects' mental models. Mental imagery seems to involve some combination of the visual information, which is based on "seeing" in the "mind's eye", and reasoning based on given knowledge. These relations among reading material, mental models, mental images and problem solving are depicted in Figure 5.

The consistency in this experiment of our subjects' drawings, gestures, and protocols, and the consistency of these data with the research on the neural basis of mental imagery (Farah, 1985; Kosslyn, 1987) support the usefulness of these four kinds of mutually reinforcing data as sources of information about invisible mental images. A great deal of work remains to be done on the relation between internal and external representations before we can have complete confidence in the generalizations we have drawn from the evidence, and the conditions under which these generalizations hold.

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