Title
Market Value and Patent Citations: A First Look

Permalink
https://escholarship.org/uc/item/1rh8k6z2

Authors
Hall, Bronwyn H.
Jaffee, Adam
Trajtenberg, Manuel

Publication Date
2000-05-04
Market Value and Patent Citations: A First Look

Bronwyn H. Hall
UC Berkeley, Nuffield College Oxford, NBER, and IFS

Adam Jaffe
Brandeis University and NBER

Manuel Trajtenberg
Tel Aviv University and NBER

May 2000

Abstract

As patent data become more available in machine-readable form, an increasing number of researchers have begun to use measures based on patents and their citations as indicators of technological output and information flow. This paper explores the economic meaning of these citation-based patent measures using the financial market valuation of the firms that own the patents. Using a new and comprehensive dataset containing over 4800 U.S. Manufacturing firms and their patenting activity for the past 30 years, we explore the contributions of R&D spending, patents, and citation-weighted patents to measures of Tobin’s Q for the firms. We find that citation-weighted patent stocks are more highly correlated with market value than patent stocks themselves and that this fact is due mainly to the high valuation placed on firms that hold very highly cited patents.
Acknowledgements

This is a revision of a paper prepared for the Conference in Commemoration of Zvi Griliches’ 20 Years as Director of the NBER Program on Productivity and Technological Progress, Cambridge, Massachusetts, March 5 and 6, 1999. As should be clear from the discussion in Section 2, this paper represents but one further step in the research agenda sketched in Zvi’s 1979 Bell Journal paper, reported on in the 1984 NBER volume Zvi edited, and continued by Zvi, his students and associates in the ensuing decade. Earlier versions of this paper were presented at the Conference on Intangibles and Capital Markets, New York University, May 15-16, 1998 and the Conference on the Economics of Science and Technology, University of Urbino, Italy, June 5-6, 1998. We have benefited from comments at these conferences and by seminar participants at Keele University, the University of Paris I, the University of Reading, the University of Manchester, and University College London. The data construction effort described in this paper was partially supported by the National Science Foundation, via grants SBR-9413099 and SBR-9320973. We are extremely grateful to Meg Fernando of REI, Case Western Reserve University for excellent assistance in matching the patenting data to Compustat. Responsibility for anything anyone doesn’t like about the paper lies with the authors.

Correspondence: bhhall@econ.berkeley.edu. Department of Economics, UC Berkeley, Berkeley, CA 94720-3880

Keywords: market value, patent citations, bibliometrics, innovation.

JEL Classification: O31, O38
1. Introduction

Micro-level data on patents that include detailed technology field, citations to other patents, number of claims, geographical location, and a variety of other information are increasingly available in machine-readable form. For economists in the field of technical change and innovation, these data have enormous potential: in addition to providing rich technological, geographic and institutional detail, patent data are publicly available for all kinds of research institutions (firms, universities, other non-profits, and government labs) in virtually every country. At a general level, economists have used patents and/or patents weighted by subsequent citations to measure the inventive output of organizations or geographic units; they have used citation intensity or measures related to the nature of citations that an entity’s patents receive to measure the importance or impact of that entity’s inventions; and they have used aggregate flows of citations to proxy for flows of knowledge to investigate knowledge spillovers across organizational, technological and geographic boundaries.

With a few exceptions discussed below, this work has relied on maintained hypotheses that patents are a proxy for inventive output, and patent citations are a proxy for knowledge flows or knowledge impacts. In this kind of work, these maintained hypotheses cannot really be tested, though they may be supported by results that are consistent with strong priors about the nature of the innovation process, and which are internally consistent. In this paper, we seek to strengthen the foundation for the use of patent and patent citation data, by exploring the extent to which firms’ stock market value is correlated with their stocks of patents and patent citations. Our maintained assumption is that stock market investors hold rational expectations about the extent to which the present value of a firm’s future profits varies with its stock of knowledge. Hence evidence that patent-related measures are correlated with market values represents evidence that
they are proxies for the private economic value of the firm’s knowledge stock. Previous work looking at the relationship between patents and market value suggested that the extremely skewed nature of the value distribution of individual patents makes firm patent totals very noisy as an indicator of the value of firms’ knowledge. In this paper, we explore the extent to which this problem can be mitigated by using citation-weighted patent counts, in the context of a larger and more comprehensive dataset than has been used before.

We begin the paper with a discussion of the meaning of patent citations and a brief survey of prior uses of these data for economic analysis. Then we review what is known about the relationship between patent counts and a firm’s value in the financial markets. The next sections of the paper present our data, which is the product of a large-scale matching effort at the NBER and Case Western Reserve University, and the relatively simple “hedonic” model for market valuation that we use. The primary contribution of this paper, estimates of the market value equation that include patent citations, is contained in Section 6; the conclusions contain an extensive discussion of further work and refinements to be implemented in a revision of this paper. Appendices describe the construction of the data, and discuss the important issue of adjusting patent citation data for the truncation inherent in the fact that we cannot observe the entire citation life of patents, with the extent of this truncation increasing for more recent patents.


A patent, as a matter of definition, is a temporary legal monopoly granted to inventors for the commercial use of an invention. In principle, in order to receive this right, the invention must be nontrivial, in the sense that it must not be obvious to a skilled practitioner of the relevant technology, and it must be useful, meaning that it has potential commercial value. If the patent is granted, an extensive public document is created which contains detailed information about the invention, the inventor(s), the organization to which the inventor assigns the patent property right (usually an employer), and the technological antecedents of the invention. These antecedents,

\[1\] For more comprehensive literature reviews, see Griliches (1990) and Lanjouw and Schankerman (1999).

\[2\] See Appendix B for an example of the front page of such a document, in the form in which it appears on the publicly accessible website of the United States Patent Office (http://www.uspto.gov). Note that no public information is currently available for patent applications that are still pending, or for patent applications that are denied by the patent office.
identified as references or citations, include previous patents and other published material that identify or describe aspects of the relevant technology that were previously publicly known. The citations identify “prior art,” the practice of which is necessarily excluded from the property right granted by the patent. Thus, together with the language of the patent claims—which describe exactly what the patented invention does that has never been done before—the citations help to delimit the property right that the patent represents. As will be discussed further below, the cited patents can be identified by the inventor herself, by a search conducted by the inventor’s patent attorney, or by the patent examiner who reviews the application for the patent office.3

The use of patent data in the economic analysis of technological change has a fairly long, if somewhat unsatisfactory history, which stretches back to the pathbreaking analyses of Schmookler (1966) and Scherer (1965). The availability of information from the U.S. patent office in machine-readable form in the late 1970s spurred greater interest in econometric analyses using these data; much of the resulting early work is reported in Griliches (1984).4 In the late 1980s, patent citation information began to be available in computerized form, which led to a second wave of econometric research, utilizing the citation information to increase the information content of the patent data themselves, as well as to investigate an additional set of questions related to the flow of knowledge across time, space and organizational boundaries.5

2.1. Patent Citations

Viewed optimistically, patent citations can be seen as providing direct observations of technological impact and knowledge spillovers, in that one technological innovation explicitly identifies several

3As can be seen in the patent in Appendix B., patents can make citations both to earlier patents and to non-patent publications. The non-patent citations appear in plain text form, and hence are difficult to manipulate electronically. Research that utilizes the non-patent references includes Trajtenberg, Henderson and Jaffe (1997) and Narin et al (1997).

4See also Pakes (1986) and Griliches, Hall and Pakes (1987). Also in the late 1970s Mark Schankerman and Ariel Pakes pioneered the use of renewal data from the European patent office to estimate the value distribution of patents. (Pakes and Schankerman 1984, Schankerman and Pakes 1986). (Renewal of U.S. patents was not required before the mid-1980s.)

5It is perhaps interesting to chart the effect of computerization on research via the authors’ experience with the acquisition of patent data. In the early 1980s, Trajtenberg collected citations information for hundreds of CT-scanner patents by hand from paper patent documents. In 1989, we paid $10,000 to a private data firm for citation information on about 10,000 patents. In the mid-1990s, we began construction of a database with citations to about 2.5 million patents, using an NSF grant of about $100,000. Today, citations to over 3 million U.S. patents are available free from numerous websites, and a CD-ROM is available from the authors with comprehensive information on all patents granted between 1964 and 1996, and all citations made between 1976 and 1996.
others as constituting the technological state-of-the-art on which it builds. Unfortunately, this optimistic view is somewhat clouded by the reality that there is substantial "noise" in the patent citations data. The nature and extent of such noise depends, to some extent, on the purpose to which the patent data are put. Some authors have used these data to explore questions involving spatial spillovers (e.g., Jaffe, Trajtenberg, and Henderson 1993), knowledge flows among firms in a research consortium (e.g., Ham 1998), and spillovers from public research (e.g., Jaffe and Trajtenberg 1996; Jaffe and Lerner 1999). In using citations as evidence of spillovers, or at least knowledge flows, from cited inventors to citing inventors, it is clearly a problem that many of the citations are added by the inventor’s patent attorney or the patent examiner, and may represent inventions that were wholly unknown to the citing inventor. On the other hand, in using citations received by a patent as an indication of that patent’s importance, impact or even economic value, the citations that are identified by parties other than the citing inventor may well convey valuable information about the size of the technological "footprint" of the cited patent. That is, if a patent stakes out a territory in technology space that is later frequently deemed to abut areas that are patented in the future, this suggests that the cited patent is important, whoever it is that decides that the citation is necessary.

A recent survey of inventors sheds some qualitative light on these issues (Jaffe, Trajtenberg and Fogarty, 2000). Approximately 160 patentees answered questions about their inventions, the relationship of their inventions to patents that were cited by their patents, and the relationship to "placebo" patents that were technologically similar to the cited patents but which were not cited. The cited and placebo patents were not distinguished in the survey questionnaire, although it is possible that the surveyed inventors knew or looked up which patents they actually cited. The results confirm that citations are a noisy measure of knowledge flow, but also suggest that they do have substantial information content. Overall, as many as half of all citations did not seem to correspond to any kind of knowledge flow; indeed, in a substantial fraction of cases the inventors judged that the two patents were not even very closely related to each other. At the same time, the answers revealed statistically and quantitatively significant differences between the

---

6The results also confirmed that the addition of citations by parties other than the inventor is a major explanation for citations that do not correspond to knowledge flow. About 40% of respondees indicated that they first learned of the cited invention during the patent application process.
cited patents and the placebos with respect to whether the citing inventor felt that she had learned from the cited patent, when she learned about it, how she learned about, and what she learned from it. Qualitatively, it appears that something like one-half of citations correspond to some kind of impact of the cited invention on the citing inventor, and something like one-quarter correspond to fairly rich knowledge flow, fairly significant impact, or both.

There are also a small number of studies that "validate" the use of citations data to measure economic impact, by showing that citations are correlated with non-patent-based measures of value. Trajtenberg (1990) collected patents related to a class of medical instruments (computerized tomography, or "CAT" scanners), and related the flow of patents over time to the estimated social surplus attributed to scanner inventions. When simple patent counts are used, there is essentially no correlation with estimated surplus, but when citation-weighted patent counts are used, the correlations between welfare improvements and patenting are extremely high, on the order of 0.5 and above. This suggests that citations are a measure of patent "quality" as indicated by the generation of social welfare. Interesting recent work by Lanjouw and Schankerman (1997, 1999) also uses citations, together with other attributes of the patent (number of claims and number of different countries in which an invention is patented) as a proxy for patent quality. They find that a patent "quality" measure based on these multiple indicators has significant power in predicting which patents will be renewed, and which will be litigated. They infer from this that these quality measures are significantly associated with the private value of patents. With respect to university patents, Shane (1999a, 1999b) finds that more highly cited M.I.T. patents are more likely to be successfully licensed, and also more likely to form the basis of starting a new firm. Sampat (1998) and Ziedonis (1998) explore the relationship between citations and licensing revenues from university patents. Harhoff et al (1999) surveyed the German patentholders of 962 U. S. invention patents that were also filed in Germany, asking them to estimate at what price they would have been willing to sell the patent right in 1980, about three years after the date at which the German patent was filed. They find both that more valuable patents are more likely to be renewed to full term and that the estimated value is correlated with subsequent citations to that patent. The most

---

7 We are not aware of any studies that validate (by reference to non-patent data) the use of citations to trace knowledge flows. Since it is hard to find other measures or proxies for knowledge flows, this kind of validation is inherently difficult.
highly cited patents are very valuable, "with a single U.S. citation implying on average more than $1 million of economic value" (Harhoff, et al 1999).

2.2. Market Value and Patents

Until recently, research that uses patents in the market value equation (in addition to or in place of R&D) has been somewhat limited, primarily because of the difficulty of constructing firm datasets that contain patent data. Most of the work shown in Table 1 and described here has been done by Griliches and his coworkers using the database constructed at the NBER that contained data on patents only through 1981. This dataset did not include information on the citations related to the patents. The other papers in the table use a cross section constructed by Connolly et al. for 1997 of Fortune 500 companies, and datasets involving UK data, one of which uses innovation counts rather than patents.

[Table 1 about here]

When patents are included in a market value equation, they typically do not have as much explanatory power as R&D measures, but they do appear to add information above and beyond that obtained from R&D, as one would expect if they measure the "success" of an R&D program. Griliches, Hall, and Pakes (1987) show that one reason patents may not exhibit very much correlation with dollar-denominated measures like R&D or market value is that they are an extremely noisy measure of the underlying economic value of the innovations with which they are associated. This is because the distribution of the value of patented innovations is known to be extremely skew, i.e., a few patents are very valuable, and many are worth almost nothing. Scherer (1965) was one of the first to make this point, and it has recently been explored further by Scherer and his co-authors (Scherer 1998; Harhoff, et al 1999). Therefore the number of patents held by a firm is a poor proxy for the sum of the value of those patents and we should not expect the correlation to be high. If the number of citations received by a patent is indicative of its value, then weighting patent counts by citation intensity should mitigate the skewness problem and increase the information content of the patents. As will be shown below, the distribution of citations is also quite skewed, suggesting perhaps that it can mirror the value distribution.
Shane (1993) regresses Tobin’s Q for 11 semiconductor firms between 1977 and 1990 on measures of R&D stock, patent stock, and patent stock weighted by citations and finds that the weighted measure has more predictive power than the unweighted measure, entering significantly even when R&D stock is included in the regression; that is, there is independent information about the success of R&D in the weighted patent count measure. Weighted patent counts are also more highly correlated with the R&D input measure than simple patent counts; this implies that ex ante more effort was put into the patents that ultimately yielded more citations. An implication of this finding is that the citations may be measuring something that is not just the luck of the draw; the firms may have known what they were shooting at. Austin (1993a, 1993b) finds that citation-weighted counts enter positively but not significantly (due to small sample size) in an event study of patent grants in the biotechnology industry. This means they add a small amount of information beyond the simple fact of a patent having granted. Many of the important Austin patents were applied for after 1987, which makes this study subject to serious truncation bias (discussed below). In addition, it uses the 3-day CAR (cumulative abnormal return) at the time of the patent grant as the indicator of economic value; this is an underestimate of the actual value of the patent because there is substantial informational leakage before. In fact, the work surveyed in Griliches, Pakes, and Hall (1987) typically finds that patent counts by application date are more tightly linked to market value than counts by granting date.

The present research project was inspired partly by the tantalizing results in these earlier small-scale studies that citation-weighted patent counts might prove a better measure of the economic value of innovative output than simple patent counts. Rather than working with a single product or market segment, we have assembled data on the entire manufacturing sector and the patenting and citing behavior of the firms within it, taking advantage of the computerized databases now available from the United States Patent Office. The remainder of the paper discusses our dataset construction and some preliminary results.

3. Data

Because of the way in which data on patents are collected at the United States Patent Office, matching the patents owned by a firm to firm data is not a trivial task. Firms patent under a
variety of names (their own and those of their subsidiaries) and the Patent Office does not keep a unique identifier for the same patenting entity from year to year. To construct our dataset, a large name-matching effort was undertaken that matched the names of patenting organizations to the names of the approximately 6000 manufacturing firms on the Compustat files available to us and to about 30,000 of their subsidiaries (obtained from the Who Owns Whom directory). As described in Appendix A, the majority of the large patenting organizations have been matched to our data, or we have established why they will not match (because they are foreign or non-manufacturing corporations). However, budget constraints mean that our matching is primarily a snapshot exercise conducted using 1989 ownership patterns; we have limited evidence using patents in the semiconductor industry that this leads to some undercounting of patents for some firms.\(^8\) Thus the precise results here should be viewed with some caution: they are unlikely to change drastically, but they may be affected by a slight undercount of the firms’ patents.

The firm data to which we have matched patenting information is drawn from the Compustat file. The full sample consists of over 6000 publicly traded manufacturing firms with data between 1957 and 1995. After restricting the sample to 1965-1995, dropping duplicate observations and partially-owned subsidiaries, and cleaning on our key variables, we have about 4800 firms in an unbalanced panel (approximately 1700 per year). The variables are described in somewhat more detail in the appendix and the construction of R&D capital and Tobin’s q is described fully in Hall (1990). For the purposes of this paper, we constructed patent and citation-weighted patent stocks using the same declining balance formula that we used for R&D, with a depreciation rate of fifteen percent. Our patent data goes back to 1964, and the first year for which we used a patent stock variable in the pooled regressions was 1975, so the effect of the missing initial condition should be small for the patent variable. However, we only know about citations made by patents beginning in 1976.

Figure 1 shows the fraction of firms in our sample in a given year who reported R&D expenditures to the SEC, the fraction that year who applied for a patent that was ultimately granted, and the fraction who have a nonzero stock of patents in that year.\(^9\) The increase in R&D reporting

\(^8\)See Hall and Ham (1999).

\(^9\)The stock of patents is defined using a declining balance formula and a depreciation rate of 15 percent, by analogy to the stock of R&D spending:
between 1969 and 1972 is due to the imposition of FASB rule no. 2, which mandated the public reporting of ”material” R&D expenditures. The fall in the later years in the number of firms with patent applications is due to the fact that we only know about patent applications when they are granted, and our grant data end in 1996. The fact the share of firms with patent applications and with R&D spending is approximately the same is only a coincidence: although there is substantial overlap, these samples are not nested. 19 percent of the firms have R&D stocks and no patents while 13 percent have patent stocks but no R&D.

We want to construct a citation-weighted stock of patents held by the firm, as a proxy for its stock of knowledge. This requires that we have a measure of the citation intensity for each patent that is comparable across patents with different grant years. The difficulty is that, for each patent, we only observe a portion of the period of time over which it can be cited, and the length of this observed interval varies depending on where the patent’s grant date falls within our data. For patents granted towards the end our data period, we only observe the first few years of citations. Hence, a 1993 patent that has gotten 10 citations by 1996 (the end of our data) is likely to be a higher citation-intensity patent than one that was granted in 1985 and received 11 citations within our data period. To make matters worse, although our basic patent information begins in 1964, with respect to citations we only have data on the citations made by patents beginning in 1976. Hence for patents granted before 1976, we have truncation at the other end of the patent’s life–a patent granted in 1964 that has 10 citations between 1976 and 1996 is probably more citation-intensive than one granted in 1976 that has 11 citations over that same period.

Our solution to this problem is to estimate from the data the shape of the citation-lag distribution, i.e. the fraction of lifetime (defined as the 30 years after the grant date) citations that are received in each year after patent grant. We assume that this distribution is stationary and independent of overall citation intensity. Given this distribution, we can estimate the total (20-year lifetime) citations for any patent for which we observe a portion of the citation life, simply by

\[ PS_t = 0.85PS_{t-1} + P_t \]  

(3.1)
dividing the observed citations by the fraction of the population distribution that lies in the interval for which citations were observed. For patents where we observe the "meat" of the distribution (roughly years 3-10 after grant), this should give an accurate estimate of lifetime citations. For other patents, particularly where we observe only the first few years of patent life, this will give a very noisy estimate of lifetime citations. Many patents receive no citations in their first few years, leading to a prediction of zero lifetime citations despite the fact that some patents with no citations in the first few years are eventually cited.\footnote{Another issue is that the number of citations made by each patent has been rising over time, suggesting a kind of "citation inflation" that renders each citation less significant in later years. It is hard to know, however, to what extent this increased intensity is an artificial artifact of patent office practices, and the extent to which it might represent true secular changes in patent impact. In this paper we choose not to make any correction or deflation for the secular changes in citation rates, with the cost that our extrapolation attempts become somewhat inaccurate later in the sample. For further discussion of this point, see Appendix D. For an attempt to econometrically separate such effects, see Caballero and Jaffe, 1993.}

The details of the estimation of the citation lag distribution and the derived adjustment to citation intensity are described in Appendix D. Figure 2 shows the ratio of total citations to total patents for the firms with patents in our data, both uncorrected and corrected for truncation. The raw numbers decline beginning in about 1983, because citations are frequently made more than 10 years after the original patent is issued, and these later citations are unobserved for patents at the end of the data period. The truncation-corrected citation intensity is flat after about 1988 and then begins to rise again. Recall that we date the patents by year of application so that a patent applied for in 1988 was most likely granted between 1989 and 1991 and hence effectively had only 4-6 years to be cited. In addition, the citing patents were also less and less likely to have been observed as we reach 1995 and 1996. Because of the increasing imprecision in measuring cites per patent as we approach the end of our sample period, our pooled regressions focus first on the 1976-1992 period, and then on the subset of years between 1979 and 1988.

[Figure 2 about here]

Figure 3 shows the total citation and patenting rates per real R&D spending for our sample. The patent counts are adjusted for the application-grant lag and the citation counts are shown both corrected and uncorrected. Although the earlier years (1975-1985) show a steady decline in patenting and citation weighted patenting per R&D dollar, one can clearly see the recent increase
in patenting rate beginning in 1986-87 that has been remarked upon by other authors (Kortum and Lerner 1998, Hall and Ham 1999). However, the yield begins to decline in about 1993, two years before the end of our sample, mostly because real R&D increases during that period. The corrected patent citation yield also begins to increase in 1986-87 but does not decline quite as much as the patent yield, reflecting an increase in citations per patent in the early to mid-nineties.

[Figure 3 about here]

Figure 4 provides some evidence on the skewness of the distribution of citations per patent. In this figure we plot a distribution of the number of citations received by each of the approximately one million patents that we have assigned to manufacturing corporations. Fully one quarter of the patents have no citations, 150,000 have only one, 125,000 have two, and 4 patents have more than 200 citations. Fitting a Pareto distribution to this curve yields a parameter of 1.8, which implies that the distribution has a mean but no variance. However, a Kolmogorov-Smirnov or other distributional test would easily reject that the data are actually Pareto.

The most cited patents since 1976 are a patent for Crystalline silicoaluminophosphates held by Union Carbide Corporation (227 citations through 1995) and a patent for a Transfer imaging system held by Mead Corporation (195 citations through 1995). In Appendix B, we show detailed information for the first of these patents obtained from the USPTO website. It is apparent from the list of citations that the patent is important because the compound it describes is used as a catalyst in many processes. This single example suggests already that a high citation rate may be correlated with the value of a patent right, because such a product is useful both directly (via sales to other users) and in licensing and cross-licensing.

[Figure 4 about here]

11 These two patents are the third and fifth most cited overall. The other 3 in the top 5 were taken out before 1976, so they are not contained in the online database provided by the U.S. Patent Office (http://patents.uspto.gov).

12 See Somaya and Teece (1999) for an interesting discussion of the IP value creation choice between production or licensing.
4. Equation Specification and Estimation Results

4.1. The Market Value Equation

We use a specification of the firm-level market-value function that is predominant in the literature: an additively separable linear specification, as was used by Griliches (1981) and his various co-workers. The advantage of this specification is that it assumes that the marginal shadow value of the assets is equalized across firms. The model is given by

\[ V_{it} = q_t (A_{it} + \gamma_t K_{it})^{\sigma_t} \]  

(4.1)

where \( A_{it} \) denotes the ordinary physical assets of firm \( i \) at time \( t \) and \( K_{it} \) denotes the firm’s knowledge assets. Both variables are in nominal terms.

Taking logarithms of both sides of equation 4.1, we obtain

\[ \log V_{it} = \log q_t + \sigma_t \log A_{it} + \log(1 + \gamma_t K_{it}/A_{it}) \]  

(4.2)

In most of the previous work using this equation, the last term is approximated by \( \gamma_t K_{it}/A_{it} \), in spite of the fact that the approximation can be relatively inaccurate for \( K/A \) ratios of the magnitude that are now common (above 15 percent). In this formulation, \( \gamma_t \) measures the shadow value of knowledge assets relative to the tangible assets of the firm and \( \sigma_t \gamma_t \) measures their absolute value.

The coefficient of \( \log A \) is unity under constant returns to scale or linear homogeneity of the value function. If constant returns to scale holds (as it does approximately in the cross section), the log of ordinary assets can be moved to the left hand side of the equation and the model estimated with the conventional Tobin’s q as the dependent variable, as we do here. The intercept of the model can be interpreted as an estimate of the logarithmic average of Tobin’s q for manufacturing corporations during the relevant period. Thus our estimating equation becomes the following:

\[ \log V_{it}/A_{it} = \log Q_{it} = \log q_t + \log(1 + \gamma_t K_{it}/A_{it}) + \delta_t D(K_{it} = 0) \]  

(4.3)

where the last term is included to control for the overall level of \( Q \) when either R&D or patents are missing. Theory does not give much guidance for the specification of intangible capital stocks...
and it is not clear how we should specify an equation for $K$ that incorporates patents and citation-weighted patents as measures of intangible assets in addition to R&D stocks. There are at least two possible approaches. In our first approach, we simply assume that R&D stocks, patent stocks or citation stocks are all just different measures of the same thing, and compare their performance in a $\log Q$ equation like equation 4.3, one at a time. This is the simplest way to validate our measures and compare their performance, but it implies that each stock (R&D, patent, or citation) is just another indicator of the same underlying concept, the knowledge assets of the firm. In our second approach to the problem, we ask what incremental value patents add in the presence of R&D stocks, and similarly for citations in the presence of patent stocks.

4.2. Citation-weighted patent stocks

The central problem we face in estimation is how to model the stock of intangible assets that is associated with the patents owned by a particular firm. We know that firms apply for patents for a variety of reasons: to secure exclusive production marketing rights to an invention/innovation, to obtain a currency that can be used in trading for the technology of other firms, to serve as a benchmark for the productivity of their research staff, and so forth. We also know that firms in different technology areas have substantially different propensities to patent. For the valuation function, we want a measure of the "book" value of the knowledge capital owned by the firm. That is, ideally we would like to know the cost in current prices of reproducing the knowledge this firm has of how to make new products today and how to undertake future innovation.

When we use past R&D expenditure to proxy for the book value of knowledge capital, we are implicitly assuming that a dollar is a dollar, i.e., that each dollar spent on research generates the same amount of knowledge capital. The reason one might want to use patents as a proxy for knowledge capital is because a patent could represent the "success" of an R&D program. That is, some of the R&D undertaken by the firm produces "dry holes" and although the knowledge gained by doing that research may have some value, such R&D should not be weighted equally with successful innovation-producing R&D in our measure of knowledge capital. Our problem is that to the extent that patents are used as engineer productivity measures and as a currency for technology licensing exchanges, some of the patents held by a firm may represent the same kind of "dry hole,"
in the sense that they document technological avenues that turn out not to be productive. More
generally, it is clear from the work cited earlier that the private value distribution of the patent
right is extremely skewed, making simple counts a noisy measure of value.

The idea of using subsequent citations to a patent as a measure of the patent’s value rests on
the argument that valuable technological knowledge within the firm tends to generate patents that
future researchers build on (and therefore cite) when doing their own innovation. The example we
gave earlier, the highly cited patent for Crystalline silicoaluminophosphates applied for by Union
Carbide in 1984 (and subsequently granted), suggested that this could be the case. From the
abstract and the citing patents it is clear that this class of chemicals has widespread use as a
catalyst in chemical reactions, which doubtless creates value for the holder of the patent.

Appendix C presents the details of the construction of our citation-weighted patent stocks.
Because citations can happen at any time after a patent is applied for,13 a natural question is
whether we should use citation weights based on all the citations to patents applied for this year
and earlier, or whether we should use only citations that have already occurred. That is, do the
citations proxy for an innovation value that is known at the time the patent is applied for, or do
they proxy for the future value of the innovation, for which the current market value of the firm is
only an unbiased forecast? We attempt to explore this question by dividing our stocks into past
and future. First, we construct the “total” citation stock for a given firm as of a given date, based
on the number of citations made through 1996 to patents held by the firm as of the given date
(depreciated). Then, we construct the “future” stock, as the difference between the total citation
stock as of the date, and the stock based only on citations that were actually observed by the given
date. The latter variable represents the future citations that will eventually be made to patents
already held by the firm.

4.3. Basic Results

In Table 2, we show the results from running a "horse race" between R&D stocks, simple patent
stocks, and citation-weighted patent stocks on data pooled across two subperiods (1976-1984 and

13There is at least one citation in our sample that is over 50 years old, to a patent that was applied for in 1921
and granted in 1992! Such very long grant lags usually are the result of the "continuation" process allowed by the
patent rules, under which an inventor can file a modified patent application that retains the application date of the
original.
As others (including some the present authors) have found before, R&D stock is more highly correlated with market value than either patents or citations, but it is also clear that citation-weighted patent stocks are more highly correlated than patents themselves (compare the R-squares).

Comparing the coefficients in these equations is somewhat difficult, because the units are not the same. The coefficient of the R&D stock/assets ratio is in units of dollar for dollar, i.e., market value per R&D dollar, whereas that for the patent stock/assets ratio is in units of millions of dollars per patent. One possibility is to normalize the patent coefficients by the average or median patent per million R&D dollars or citation per million R&D dollars in the sample. Because of the presence of many zeros and the skewness of both the patent and citation distributions, neither measure is very robust, so we have used the ratio of the total patent stock or total citation stock to the total stock of R&D for all firms, rather than the average of these ratio across firms. For the firms in the first period these numbers are 0.62 (that is, approximately 1.6 million 1980 dollars per patent) and 4.7 (that is, about 210,000 1980 dollars per citation). Using this method, the marginal shadow value of a patent (measured in R&D dollars) for this period is approximately 0.37 million 1980 dollars and the marginal shadow value of a citation (again, measured in R&D dollars) is about 0.50 million 1980 dollars. These numbers can be directly compared to the R&D coefficient of 1.75. The magnitudes suggest substantial downward bias from measurement error in the patents or citations variable and from the use of an average patent per R&D yield for normalization. It is noteworthy that the citation coefficient is somewhat higher, and that the difference in explanatory power is more marked for the firms that patent.

Although the results in Table 2 are somewhat encouraging, the extremely oversimplified equation we are using here is likely to obscure much that is of interest. In the next few sections of the paper we explore various ways of looking at this relationship in more detail. But first we examine how it has changed over time.

14 These estimates are computed holding $\gamma_t$ and $\delta_t$ constant across the subperiods for simplicity. The R-squared graph shown later is based on estimates that allow the coefficients to vary over time.
Figures 5a and 5b show the R-squared from the same simple Tobin’s q regression on R&D stock, patent stocks, and citation-weighted patent stocks, estimated year-by-year between 1973 and 1993. Figure 5a shows the result for the R&D-performing firms and Figure 5b for the patenting firms. While neither patents nor citation-weighted patents have as great an explanatory power for the market to book ratio as R&D during the earlier years, by 1984-1986 the citation-weighted patents are doing about as well as R&D, especially when we focus on patenting firms, though this is partly because the explanatory power of R&D has declined.\textsuperscript{15}

It is noteworthy that the date at which the explanatory power of citation-weighted patents converges to that of R&D in Figure 5b coincides roughly with a number of events that led to an increase in patenting activity during the mid-eighties, such as the Kodak-Polaroid decision. One interpretation is that patenting and citation behavior changed around this time because of changing litigation conditions. This might be explored further by looking more closely at which firms are making and receiving the citations. That is, does fear of litigation lead firms to cite others’ patents more carefully in order to fence off their own technology?

[Figures 5a and 5b about here]

Because of the inaccuracy of our citation measures post-1990 and because the shadow value of our measures seems to change over time, in the remainder of this section of the paper we focus on one ten-year period in the middle of our sample where the data are the most complete, and where the valuation coefficients do not change dramatically in Figures 5a and 5b, the 1979-1988 period.

4.4. Explorations(1): Do citations add information?

A second, more informative way to look at the valuation problem is to hypothesize that although patents are clearly correlated with R&D activity at the firm level, they measure something that is distinct from R&D, either ”success” in innovative activity, or perhaps success in appropriating the returns to such activity. This suggests that we should include the yield of patents or citations per R&D dollar as a separate variable in the equation. When interpreting the coefficient, it is important to note that R&D is a nominal quantity while patents are ”real,” so part of what we

\textsuperscript{15}See Hall (1993a,b) for year-by-year measures of the market value of R&D investments.
see is the changing real price of R&D over the sample. It is also likely that the expected yield of patents per R&D varies by industry, although we do not allow for this in the present paper.

In Table 3, we explore two variations of this idea. In column 2, we add a patent yield (the ratio of patent stock to R&D stock) to the equation that already has R&D stock and find that it is significant and has a small amount of additional explanatory power. In column 3, we add the firm’s average cites per patent to the equation to see if the citation rate has any impact on market value above and beyond that due to R&D and patenting behavior. This variable is quite significant and its coefficient is fairly large. To interpret the results, we use the following expression for the semi-elasticity of market value or $Q$ with respect to the citation-patent ratio:

$$\frac{\partial \log Q}{\partial C/P} = \frac{\gamma_2}{1 + \gamma_0 K/A + \gamma_1 P/K + \gamma_2 C/P}$$

where $K$ is the R&D stock, $P$ is the patent stock, and $C$ is the citation stock. In the table below we show this quantity together with the corresponding quantity for the patents-R&D stock ratio, evaluated at a range of values of the independent variables:

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Ratio of Totals</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K/A ($/M)</td>
<td>0.31</td>
<td>0.10</td>
<td>0.15</td>
<td>0.57</td>
</tr>
<tr>
<td>$P/K ($/M)</td>
<td>0.64</td>
<td>0.0</td>
<td>0.47</td>
<td>3.87</td>
</tr>
<tr>
<td>$C/P ($/M)</td>
<td>4.90</td>
<td>3.88</td>
<td>7.48</td>
<td>6.87</td>
</tr>
<tr>
<td>Denominator</td>
<td>1.630</td>
<td>1.311</td>
<td>1.562</td>
<td></td>
</tr>
<tr>
<td>$\frac{\partial \log Q}{\partial C/P}$</td>
<td>0.030</td>
<td>0.038</td>
<td>0.032</td>
<td></td>
</tr>
<tr>
<td>$\frac{\partial \log Q}{\partial P/K}$</td>
<td>0.018</td>
<td>0.023</td>
<td>0.019</td>
<td></td>
</tr>
</tbody>
</table>

Thus an increase of one citation per patent is associated with a three percent increase in market value at the firm level. This is a very large number and may be consistent with the "million dollar" citations reported by Harhoff et al. (1999). The value of additional patents per R&D is somewhat lower: an increased yield of one patent per million dollars of R&D is associated with a two percent increase in the market value of the firm. Note that the statistics in the table above make it clear that the ratios are far from normally or even symmetrically distributed, which suggests that some exploration of the functional form of our equation might be useful. We present a simple version of such an exploration later in the paper.

[Table 3 about here]
4.5. Explorations(2): When do citations add information?

Table 4 shows the results of an investigation into whether there is a difference between the market valuation of past and future citations. The answer is a resounding yes (see columns 2 and 5 of Table 4). Whether we include the citation stock alone (column 2) or use the full model with R&D and patents (column 5), the coefficient of a stock based only on future citations is equal to or greater than the coefficient of the stock based on all citations, and the coefficient of the past citation stock is negative and marginally significant or insignificant. The apparent implication is that future citations are more correlated than past citations with the expected profitability of the patent right.

Because the two stocks, past and future, are highly correlated measures of the same underlying quantity, this finding does not necessarily imply that citations are worthless for forecasting the value of the knowledge assets associated with patents or the expected profit stream from those assets. The past citation stock of a firm could be an excellent forecast of the future citations that are expected for its patent portfolio, even though it is not quite as good as knowing the future citations when predicting the firm’s market value. To explore this idea, we decomposed the total citation stock into the part predicted by the past citation stock and the part that is not predicted:\footnote{We also included a full set of time dummies in the conditioning set.}

\[
K^C(t) = E[K^C(t)|K^{PPC}(t)] + K^C(t) - E[K^C(t)|K^{PPC}(t)]
\]

\[
= \tilde{K}^C(t) + \epsilon_{K^C}(t)
\]

where \(K^C(t)\) denotes the total citation weighted patent stock at time \(t\) and \(K^{PPC}(t)\) denotes the patent stock weighted by the citations received as of time \(t\) (see Appendix C for details on construction of these variables). The results of including the citation-assets and citation-patent ratios partitioned in this way are shown in columns 3 and 6 of Table 4. In all cases, the coefficient of the unexpected portion of the total citation stock is approximately the same as the coefficient of the future citation stock in the preceding column. The coefficient of the predictable portion of the total citation stock is approximately 40 percent lower, although still quite significant. Thus, although future citations are a more powerful indicator of the market value of the patent portfolio held by these firms, past citations clearly also help in forecasting future returns.

Figures 6 and 7 show the coefficients that result when both past and future citation-weighted
patent stocks or predictable and unpredictable citation-weighted stocks are included in the same Tobin’s q equation year-by-year for the 1973-1993 period. In Figure 6, we see that the future citation-weighted patent stock is clearly preferred over the past and that the latter has a coefficient that is zero or negative. When we separate the citation stock via the orthogonal decomposition of predictable based on the past versus unpredictable, we find that both enter, but that the unpredictable portion has a higher shadow value in the equation and that the predictable portion behaves more or less like the total stock (Figure 7).\footnote{Interpretation here is a bit dicey. These coefficients ought to be normalized in some way to put them on a common ground.}

4.6. Explorations(3): How do citations add information?

Our working hypothesis is that citations are an indicator of the (private) value of the associated patent right, and are therefore correlated with the market value of the firm because investors value the firm’s stock of knowledge. For this reason, it is of interest to explore the question of the precise shape of the citation valuation distribution: does the fact that a firm’s patents yield fewer citations than average mean that its R&D has been unproductive? How does the valuation change for firms with patents that have very high citation yields of the sort we saw in Figure 4? Table 5 explores these questions. We broke the average citation stock per patent stock variable up into 5 groups: less than 4, 4-6 (the median for firms with patents), 6-10, 10-20, and more than 20 (see Table 5 for details). The groups are unequal partly because we were interested in the tail behavior. We then included dummy variables for four of the five groups in the valuation regression (the left-out category was 0-4 citations per patent).

The results are quite striking. For firms with less than the median number of citations per patent (6), it makes no difference how far below the median they fall; firms with 4-6 citations per patent have no higher value than firms with less than 4 (the left-out category). However, firms
that average more than the median number of citations per patent have a very significant increase in market value, and one that appears to be approximately linear. The most dramatic effect is for the 508 observations (184 firms) with a stock of more than 20 cites per patent: the market value of these firms is 54-58 percent higher than would be expected given their R&D capital and their patent stock.

Further investigation of these 184 firms revealed the following: they are concentrated in computing, office equipment, semiconductor, and electronics (259 observations on 82 firms), pharmaceuticals and medical instruments (149 observations on 52 firms), and to a lesser extent, in textiles and apparel (28 observations on 7 firms), and machinery (32 observations on 13 firms). They include both quite small (so they have a very few highly cited patents) and medium to large firms (such as Intel, Compaq Computer, Tandem Computer, Alza Corp, and Signal Companies). It appears that the larger firms are primarily in the electronics sector, broadly defined, while those in the pharmaceutical sector that average a high citation rate are more likely to be smaller biotechnology firms. It should be kept in mind that we are focusing here on a period that spans the period during which profound changes in patenting strategy took place in some industries after the creation of the Circuit Court of Appeals and the well-known Kodak-Polaroid decision of 1986 - see Hall and Ham 1999 inter alia, for discussion of this point).

5. Conclusion and Suggestions for Further Research

This paper is a "first look" at these data. We find that augmenting firms’ patent counts with citation intensity information produces a proxy for the firms’ knowledge stocks that is considerably more value-relevant than the simple patent count itself. It remains true that, for most of the time period, patent-related measures cannot win a "horserace" with R&D as an explanator of market value. But this should not surprise us. As emphasized by Sam Kortum in his comments on this paper, even if citations are a reasonably informative signal of success, this does not mean they will be more correlated with value than R&D, because optimizing firms will increase their R&D in response to success. The citation stock is also associated with significant incremental market value after controlling for firms’ R&D. Indeed, firms with very highly cited patents (more than 20 cites per patent), the estimates imply almost implausibly large market value differences, on the
order of a 50% increase in value, relative to firm with the same R&D and patent stocks but with the median citation intensity.

The main truly new result is that market value is correlated, to a significant extent, with the portion of eventual citations that cannot be predicted based on past citations. This suggests that the market "already knows" much about the quality of inventions, which will ultimately be confirmed by the arrival of future citations that are "unexpected" in the sense of unpredictable based only on past citation information. This result clearly requires further exploration. First, it would be useful to explore the use of a functional form or normalization that would allow the relative value of past and future citations to be compared more directly, rather than just asking which adds more to the R-squared. In addition, one could ask how many years' worth of citations does one have to see to know most of what citations will eventually reveal. Is 10 years enough? What fraction of what you "know" (in the sense of correlation with market value) by knowing the lifetime citations do you "know" after 5 years? Also, one could explore whether this result is driven by the tail of the distribution, which we know is associated with much of the value. In other words, to what extent is it possible to predict that a patent will ultimately get >20 citations based only on the first few years' patents? Is it this difficulty of predicting the really big winners that makes the unpredictable portion of the citations total so important?

Other variations on the results include more exploration of the shape of the citations-value relationship. Has the importance of highly cited patents changed over time with changes in the patent regime? As noted, the firm-years with a citation intensity above 20 include both small and large firms. It would be useful to sort out whether these are different from each other, and also the extent to which the results relating to an average citation intensity of more than 20 are themselves driven by a few patents in the extreme tail. Here again, it would be useful to explore other functional forms.

In addition to these variations on the themes already struck, there are other aspects of citation behavior that ought to be value-relevant. These include:

- Self citations. Approximately one-quarter of citations received by corporate patents come from other patents assigned to the same company. Such "self citations may represent construction of patent thickets or other behaviors that are less value-relevant than citations from
outsiders. On the other hand, a self-citation may represent evidence that the firm is successfully appropriating downstream impacts that would otherwise represent spillovers stemming from the patent. This would imply that self-citations are more valuable than citations from others. We could have two counts of citations, and allow for different coefficients on self versus other citations in the market value equation. Preliminary investigation suggests that the self-citation effect is small and positive: if the "self" share of citations is higher, the market value is higher, other things equal.

- Generality. Trajtenberg, Jaffe, and Henderson have proposed a measure of "generality," defined as (1 minus) the Herfindahl Index of concentration of citations over patent classes. The idea is that citations that are spread over a larger number of technological fields are more "general," and vice versa. In terms of impacting the market value of firms, though, one could hypothesize the following: for firms that concentrate in narrow fields of activity, more generality is bad, since the firm will not be able to appropriate the spillovers to other fields. For conglomerates, the opposite may be true. Thus, we could compute the average generality of patents for firm \( j \) in year \( t \), and interact this variable with a dummy for whether or not the firm is a conglomerate. This may be tricky; see the strategy literature on diversification that occurs in order to exploit an innovation resource base (Silverman 1997). We may need to normalize generality as well, since the measure depends on the number of citations. This suggests both conceptual difficulty in separating the effects of citation intensity and generality, and also a complex truncation problem in the generality measure itself.

6. Bibliography


Jaffe, Adam, Michael Fogarty and Bruce Banks. 1998. "Evidence from Patents and Patent


Licht, Georg and Konrad Zos (1996), ”Patents and R&D: An Econometric Investigation Using Applications for German, European, and U.S. Patents by German Companies,” ZEW, Mannheim:


Shane, Scott. 1999b. "Technological Opportunities and New Firm Creation," mimeo


### Table 1

**Market Value - Innovation Studies with R&D and Patents**

<table>
<thead>
<tr>
<th>Study</th>
<th>Country (industry)</th>
<th>Years</th>
<th>Functional Form</th>
<th>Other Variables</th>
<th>R&amp;D Stock Coeff</th>
<th>Patent or Innov Coeff</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Griliches 1981</td>
<td>US</td>
<td>1968-74</td>
<td>Linear (Q)</td>
<td>Time &amp; Firm dummies, [log Q(-1)] Ind dummies, Investment, Earnings</td>
<td>1.0-2.0</td>
<td>.08 to .25</td>
<td>units appear to be 100 pats</td>
</tr>
<tr>
<td>Ben-Zion 1984</td>
<td>US</td>
<td>1969-76</td>
<td>Linear (V)</td>
<td></td>
<td>3.4 (0.5)</td>
<td>.065 (.055)</td>
<td>No time dummies?</td>
</tr>
<tr>
<td>Jaffe 1986</td>
<td>US</td>
<td>1973, 79</td>
<td>Linear (Q)</td>
<td>Time &amp; tech dummies, C4, mkt share, Tech pool, interactions</td>
<td>7.9 (3.3)</td>
<td></td>
<td>3SLS even higher</td>
</tr>
<tr>
<td>Connolly, Hirsch, Hirschey 1986</td>
<td>US</td>
<td>1977</td>
<td>Linear (EV/S)</td>
<td>Growth, risk, age, Mkt share, C4, Adv, Union share, Ind dummies</td>
<td>7.0 (0.8)</td>
<td>4.4 (0.6)</td>
<td>Unexpected patents</td>
</tr>
<tr>
<td>Cockburn, Griliches 1988</td>
<td>US</td>
<td>1977</td>
<td>Linear (Q)</td>
<td>Industry appropriability (Yale survey)</td>
<td>.9-1.2</td>
<td>0.1</td>
<td>patent coef. Is insignificant</td>
</tr>
<tr>
<td>Griliches, Pakes, Hall 1987</td>
<td>US</td>
<td>1977</td>
<td>Linear (EV/S)</td>
<td>Growth, risk, C4, Adv</td>
<td>5.6 (0.6)</td>
<td>5.7 (0.5)</td>
<td>Bayesian estimation</td>
</tr>
<tr>
<td>Connolly, Hirschey 1988</td>
<td>US</td>
<td>1973-91</td>
<td>Linear (V)</td>
<td>Assets, Cash flow, Adv, Gr, time dummies time dummies</td>
<td>2.5-3.0 (.8)</td>
<td>0.48 (.02)</td>
<td>By year also</td>
</tr>
<tr>
<td>Hall 1993a</td>
<td>US</td>
<td>1972-90</td>
<td>Linear (Q)</td>
<td></td>
<td>2.0-10.0</td>
<td>0.5-2.0</td>
<td>By year; LAD; absolute coeff</td>
</tr>
<tr>
<td>Hall 1993b</td>
<td>US semi-</td>
<td>1977-90</td>
<td>Linear (Q)</td>
<td>Rivals R&amp;D and patents</td>
<td>0.82 (0.2)</td>
<td>0.38 (0.2)</td>
<td>Patent stock</td>
</tr>
<tr>
<td>Johnson, Pazderka 1993</td>
<td>US</td>
<td>1972-82</td>
<td>Linear (V)</td>
<td>Time dummies, Assets, Mkt share</td>
<td>1.93 (.93)</td>
<td></td>
<td>Innovation counts</td>
</tr>
<tr>
<td>Thompson 1993</td>
<td>US</td>
<td>1989-95</td>
<td>Linear (V)</td>
<td>Assets, Debt, Grow, Mkt share, investment, Cashflow, time dummies, Mills ratio</td>
<td>2.5 (1.5)</td>
<td>insig.</td>
<td>Selection correction; by year</td>
</tr>
</tbody>
</table>
### Table 2

"Horse-race" Regressions comparing R&D, Patents and Citations

U.S. Manufacturing Firm Sample (Cleaned) - 1976-92

Nonlinear Model with Dependent Variable = log Tobin’s q

<table>
<thead>
<tr>
<th></th>
<th>All Firms</th>
<th>R&amp;D-Doing Firms</th>
<th>Patenting Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Period: 1976-1984</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of observations</td>
<td>17,111</td>
<td>10,761</td>
<td>10,509</td>
</tr>
<tr>
<td>R&amp;D Stock/Assets R(D&amp;D=0)</td>
<td>1.736 (.069)</td>
<td>1.741 (.070)</td>
<td>1.754 (.082)</td>
</tr>
<tr>
<td>Patent Stock/Assets</td>
<td>.029 (.014)</td>
<td>-</td>
<td>.024 (.017)</td>
</tr>
<tr>
<td>Cite Stock/Assets D(Pats=0)</td>
<td>0.607 (.042)</td>
<td>0.493 (.042)</td>
<td>0.599 (.041)</td>
</tr>
<tr>
<td></td>
<td>0.108 (.006)</td>
<td>0.087 (.006)</td>
<td>0.107 (.006)</td>
</tr>
<tr>
<td></td>
<td>0.120 (.013)</td>
<td>0.252 (.019)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.144 (.013)</td>
<td>0.282 (.018)</td>
<td>-</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.228</td>
<td>0.249</td>
<td>0.231</td>
</tr>
<tr>
<td>Std. Err.</td>
<td>0.669</td>
<td>0.683</td>
<td>0.644</td>
</tr>
<tr>
<td>Ratio of Total Pats or Cites to Total R&amp;D ($1980M)</td>
<td>0.566</td>
<td>0.550</td>
<td>0.619</td>
</tr>
<tr>
<td>Coefficient scaled by ratio of totals</td>
<td>1.736</td>
<td>1.741</td>
<td>1.754</td>
</tr>
</tbody>
</table>

|                           | Period: 1985-1992 |                  |                |
| Number of observations    | 15,605    | 10,432          | 9,718           |
| R&D Stock/Assets R(D&D=0) | 0.547 (.027) | 0.560 (.027)   | 0.563 (.033)   |
| Patent Stock/Assets       | 0.710 (.049)  | -               | 0.711 (.049)   |
| Cite Stock/Assets D(Pats=0) | 0.004 (.015)  | -               | 0.060 (.015)   |
|                           | 0.080 (.005)  | 0.074 (.005)   | 0.080 (.005)   |
|                           | 0.144 (.014)  | 0.279 (.020)   | -              |
|                           | 0.148 (.014)  | 0.291 (.019)   | -              |
| R-squared                 | 0.116      | 0.123           | 0.121           |
| Std. Err.                 | 0.748      | 0.764           | 0.729           |
| Ratio of Total Pats or Cites to Total R&D ($1987M) | 0.331 | 0.324 | 0.352 |
| Coefficient scaled by ratio of totals | 0.547 | 0.560 | 0.563 |

Heteroskedastic-consistent standard errors.
All equations have a complete set of year dummies.
Stocks are computed using 15 percent annual depreciation rate.

Tab2
Table 3

Effect of Adding Patents and Citations to R&D Regression
U.S. Manufacturing Firms (Cleaned Sample) - 1979-88 - 19,628 firm-years
Nonlinear Model with Dependent Variable = logarithm of Tobin's q

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>(1) K</th>
<th>(2) K with P/K</th>
<th>(3) P/K and C/P</th>
<th>(4) K with C/K</th>
</tr>
</thead>
<tbody>
<tr>
<td>R&amp;D Stock(K)/ Assets</td>
<td>1.112 (.043)</td>
<td>1.139 (.045)</td>
<td>1.189 (.052)</td>
<td>1.159 (.046)</td>
</tr>
<tr>
<td>D(R&amp;D=0)</td>
<td>0.004 (.013)</td>
<td>-.011 (.015)</td>
<td>-.001 (.015)</td>
<td>0.012 (.015)</td>
</tr>
<tr>
<td>Pat Stock/K</td>
<td></td>
<td>0.0252 (.0061)</td>
<td>0.0299 (.0075)</td>
<td></td>
</tr>
<tr>
<td>Cite Stk/Pat Stk</td>
<td></td>
<td>0.0494 (.0087)</td>
<td>0.0494 (.0037)</td>
<td>0.0095 (.0102)</td>
</tr>
<tr>
<td>Cite Stock/K</td>
<td></td>
<td>0.116 (.011)</td>
<td>0.347 (.019)</td>
<td>0.136 (.012)</td>
</tr>
<tr>
<td>D(Pats=0)</td>
<td></td>
<td></td>
<td>0.347 (.019)</td>
<td>0.136 (.012)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.206</td>
<td>0.211</td>
<td>0.228</td>
<td>0.216</td>
</tr>
<tr>
<td>Standard error</td>
<td>0.710</td>
<td>0.708</td>
<td>0.701</td>
<td>0.706</td>
</tr>
</tbody>
</table>

Heteroskedastic-consistent standard errors in parentheses.
All equations include year dummies.

Citation stocks are patent stocks weighted by all the cites they received before 1994 plus an estimate of post-1993 cites, depreciated as of the patent date.
Table 4  
Splitting Total Citation Stock into Past and Future  
U.S. Manufacturing Firms (Cleaned Sample) - 1979-88 - 19,628 firm-years  
Nonlinear Model with Dependent Variable = logarithm of Tobin's q  

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Cite/Assets</th>
<th>P/K and C/P</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>R&amp;D Stock(K)/Assets D(K=0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cite Stock/A</td>
<td>0.117 (.006)</td>
<td></td>
</tr>
<tr>
<td>Past Cite Stk/A</td>
<td></td>
<td>-0.056 (.019)</td>
</tr>
<tr>
<td>Future Cite Stk/A</td>
<td></td>
<td>0.162 (.008)</td>
</tr>
<tr>
<td>Pred. Cite Stk/A</td>
<td></td>
<td>0.106 (.005)</td>
</tr>
<tr>
<td>Unpred. Cite Stk/A</td>
<td></td>
<td>0.159 (.008)</td>
</tr>
<tr>
<td>Pat Stock/K</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cite Stk/Pat Stk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Past Cite Stk/P Stk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Future Cite Stk/P Stk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pred. Cite Stk/P Stk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unpred. Cite Stk/P Stk</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D(P=0)</td>
<td>0.204 (.012)</td>
<td>0.202 (.012)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.140</td>
<td>0.145</td>
</tr>
<tr>
<td>Standard error</td>
<td>0.740</td>
<td>0.737</td>
</tr>
</tbody>
</table>

Heteroskedastic-consistent standard errors in parentheses.  
All equations include year dummies.  
Stocks are computed using 15 percent annual depreciation rate.  
Past citation stocks are stocks of citations that have already occurred as of the valuation date, depreciated as of the patent date.  
Future citation stocks are the difference between citation stocks and past citation stocks.  
Pred. and unpred. citation stocks are the orthogonal decomposition of citation stocks into the piece predictable from the past and the residual.
<table>
<thead>
<tr>
<th>Indep Variable</th>
<th>All Firms (19,706 obs)</th>
<th>R&amp;D-Doing Firms (12,771 obs)</th>
<th>Patenting Firms (12,119 obs)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>K and C/P</td>
<td>K, P/K, C/P</td>
<td>K and C/P</td>
</tr>
<tr>
<td>R&amp;D Stock (K)/Assets</td>
<td>0.910 (.039)</td>
<td>0.943 (.041)</td>
<td>0.961 (.041)</td>
</tr>
<tr>
<td>D(R&amp;D=0)</td>
<td>-0.054 (.013)</td>
<td>-0.034 (.014)</td>
<td>--</td>
</tr>
<tr>
<td>Pat Stock/A</td>
<td>--</td>
<td>0.620 (.039)</td>
<td>0.520 (.039)</td>
</tr>
<tr>
<td>Pat Stock/K</td>
<td>0.196 (.016)</td>
<td>0.207 (.016)</td>
<td>0.246 (.022)</td>
</tr>
<tr>
<td>D(Pats=0)</td>
<td>0.207 (.016)</td>
<td>0.302 (.017)</td>
<td>0.270 (.023)</td>
</tr>
<tr>
<td></td>
<td>0.620 (.039)</td>
<td>0.520 (.039)</td>
<td>0.616 (.039)</td>
</tr>
<tr>
<td></td>
<td>0.022 (.005)</td>
<td>0.024 (.006)</td>
<td>0.025 (.006)</td>
</tr>
<tr>
<td></td>
<td>0.302 (.017)</td>
<td>0.460 (.024)</td>
<td>--</td>
</tr>
<tr>
<td>4-6 Cites per Patent</td>
<td>-0.012 (.017)</td>
<td>-0.019 (.022)</td>
<td>-0.004 (.017)</td>
</tr>
<tr>
<td>(3,211 observations)</td>
<td>-0.013 (.017)</td>
<td>-0.058 (.023)</td>
<td>-0.003 (.017)</td>
</tr>
<tr>
<td>6-10 Cites per Patent</td>
<td>0.095 (.017)</td>
<td>0.085 (.021)</td>
<td>0.112 (.017)</td>
</tr>
<tr>
<td>(3,900 observations)</td>
<td>0.094 (.017)</td>
<td>0.086 (.021)</td>
<td>0.112 (.017)</td>
</tr>
<tr>
<td>10-20 Cites per Patent</td>
<td>0.346 (.023)</td>
<td>0.357 (.027)</td>
<td>0.375 (.024)</td>
</tr>
<tr>
<td>(1,853 observations)</td>
<td>0.344 (.023)</td>
<td>0.357 (.027)</td>
<td>0.372 (.023)</td>
</tr>
<tr>
<td>&gt;20 Cites per Patent</td>
<td>0.534 (.043)</td>
<td>0.583 (.047)</td>
<td>0.569 (.043)</td>
</tr>
<tr>
<td>(508 observations)</td>
<td>0.536 (.043)</td>
<td>0.590 (.047)</td>
<td>0.570 (.043)</td>
</tr>
<tr>
<td></td>
<td>0.811 (.041)</td>
<td>0.835 (.044)</td>
<td>0.819 (.041)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.232</td>
<td>0.245</td>
<td>0.254</td>
</tr>
<tr>
<td>Standard error</td>
<td>0.703</td>
<td>0.710</td>
<td>0.671</td>
</tr>
</tbody>
</table>

Heteroskedastic-consistent standard errors in parentheses.
All equations include year dummies.
Stock are computed using 15 percent annual depreciation rate.
Citation stocks are patent stocks weighted by all the cites they received before 1994 plus an estimate of post-1993 cites, depreciated as of the patent date.
The left-out category is fewer than 4 cites per patent (2,478 observations) and there are 7,479 observations with no patents or cites.
Figure 1
US Manufacturing - Cleaned Sample - 4,846 Firms

Percentage of Firms

Year


- % with R&D Stock
- % with Patent Stocks
- % with Cite-wtd Patents
- % with Cite Stocks
Figure 2

Citation Counts before and after Truncation Correction

- Total Cites per Patent (raw)
- Total Cites per Patent (corrected)
Figure 3
U.S. Manufacturing Sector - 4,846 Firms

Cites per Millions $1987

Patents per Millions $1987

Year

Cites per real R&D
Corrected cites per real R&D
Patents per real R&D
Figure 4
Citation Distribution

Citation Distribution - More than 100 Citations

Citation Count through 1995

Number of Patents

Citation Count through 1995

0 1 14 21 28 35 42 49 56 63 70 77 84 91 98 105 112 124 132 140 155 175 221
Figure 5a
R&D Performing Firms - R-Squared from Tobin's Q Equation

R-squared vs. Year

- R&D Stock/Assets
- Pat Stock/Assets
- Cite-wtd Pat Stock/Assets
- Past Cite-wtd Stock/Assets
Figure 5b
Patenting Firms Only - R-Squared from Tobin's Q Equation
Figure 6
R&D Performing Firms - Splitting Citations Stocks into Past and Future

Graph showing the relative shadow value of future and past cite stocks from 1973 to 1993. The graph compares the relative shadow values of future cite stock, past cite stock, and total cite stock over the years.
Figure 7
R&D Performing Firms - Splitting Citation Stocks into Predictable and Unpredictable Components

Relative Shadow Value

Year


Unpredictable Future Cites
Predictable Cite Stock
Total Cite Stock
7. Appendix A: Data Description

The data we use are drawn from the Compustat files and from files produced by the United States Patent Office. We have included all the firms in the manufacturing sector (SIC 2000-3999) between 1976 and 1995 in a large unbalanced panel (approximately 4800 firms). The firms are all publicly traded on the New York, American, and regional stock exchanges, or traded Over-the-Counter on NASDAQ. For details on data construction, see the documentation in Hall (1990), although we have drawn a new sample from a larger dataset than the file described in that document.

The main Compustat variables that we use are the market value of the firm at the close of the year, the book value of the physical assets, and the book value of the R&D investment. The market value is defined as the sum of the value of the common stock, the value of the preferred stock (the preferred dividends capitalized at the preferred dividend rate for medium risk companies given by Moody’s), the value of the long-term debt adjusted for inflation, and the value of short-term debt net of assets. The book value is the sum of the net plant and equipment (adjusted for inflation), the inventory (adjusted for inflation), and the investments in unconsolidated subsidiaries, intangibles, and others (all adjusted for inflation). Note that these intangibles are normally the good will and excess of market over book from acquisitions, and do not include the R&D investment of the current firm, although they may include some value for the results of R&D by firms that have been acquired by the current firm. The R&D capital stock is constructed using a declining balance formula and the past history of R&D spending with a 15 percent depreciation rate.

The patents data have been cleaned and aggregated to the patent assignee level at the Regional Economics Institute, Case Western Reserve University. We have matched the patent assignee names with the names of the Compustat firms and the names of their subsidiaries in the Who Owns Whom Directory of Corporate Affiliations as of 1989 in order to assign patents to each firm. In order to ensure that we picked up all important subsidiaries, we also tried to positively identify the unmatched patenting organizations that had more than 50 patents during the period to ensure that we had not missed any subsidiaries. A spot check of firms in the semiconductor industry, which is an industry with lots of new entry during the period, suggests that our total patent numbers are fairly accurate for the period 1975-1995, but that they are an undercount in the case of some firms.
(averaging about 5-15% under).\textsuperscript{18}

\textsuperscript{18}See Hall and Ham Ziedonis (1999).
8. Appendix B: Highly Cited Patents
Citations to Patent on Previous Page
9. Appendix C: Citation-weighted patent stocks

Define

\[ C(t, s) = \text{number of cites received at time } s \text{ to patents applied for at time } t \]  \hspace{1cm} (9.1)

and

\[ C(t) = \sum_{s=t}^{T} C(t, s) = \text{total number of cites to patents applied for at time } t \]  \hspace{1cm} (9.2)

Note that in our case \( T = 1995 \), so the measure of the total number of cites is truncated for all patents, and badly truncated for patents post 1985 or so.

Assume a single depreciation rate for the "private value" of a patent, \( \delta \). Then the simplest citation-weighted patent stock treats each patent as if it is worth the number of citations it ever receives and sums these citations over all the patents applied for in a given year to obtain a measure for the increment to the stock of knowledge at time \( t \) equal to \( C(t) \). Using the standard declining balance formula, the stock of knowledge itself is defined by the following equation:

\[
K^C(t) = (1 - \delta)K^C(t - 1) + C(t) \\
= C(t, t) + (1 - \delta)C(t - 1, t - 1) + (1 - \delta)^2C(t - 2, t - 2) + \ldots \\
C(t, t + 1) + (1 - \delta)C(t - 1, t) + (1 - \delta)^2C(t - 2, t - 1) + \ldots \\
C(t, t + 2) + (1 - \delta)C(t - 1, t + 1) + (1 - \delta)^2C(t - 2, t) + \ldots \\
+ \ldots
\]  \hspace{1cm} (9.3)

Note that in the above equation, the knowledge associated with a patent is assumed to be created at the time the patent is applied for, and future citations are depreciated as though they occurred at that time. Note also that when we put this stock into a valuation equation, we are implicitly assuming that the market knows the "true" value of the innovation, which is only revealed later to us via the citations.

An alternative possibility is to use citation weights that depend only on citations to the patents that have already occurred, and to depreciate the citations as of the date when they occur, rather than the date of the original patent application:
\[ K^{PC}(t) = (1 - \delta)K^{PC}(t-1) + C(t, t) + C(t-1, t) + C(t-2, t) + \ldots \]  

(9.4)

\[ = C(t, t) + (1 - \delta)C(t-1, t-1) + (1 - \delta)^2C(t-2, t-2) + \ldots \]
\[ C(t-1, t) + (1 - \delta)C(t-2, t-1) + (1 - \delta)^2C(t-3, t-2) + \ldots \]
\[ C(t-2, t) + (1 - \delta)C(t-3, t-1) + (1 - \delta)^2C(t-4, t-2) + \ldots \]
\[ + \ldots \]

Comparing equations (9.3) and (9.4) we can see that they differ not only because one is forward-looking (it counts citations at time \( t \) that will not be made until later than \( t \)), but also because the pattern of depreciation is different. This suggests an alternative to equation (9.4), shown below, that uses only past citations, but depreciates them as of the patent date rather than at the citation date:

\[ K^{PPC}(t) = (1 - \delta)K^{PPC}(t) + \sum_{s=0}^{t-1} (1 - \delta)^s C(t-s, t) \]  

(9.5)

\[ = C(t, t) + (1 - \delta)C(t-1, t-1) + (1 - \delta)^2C(t-2, t-2) + \ldots \]
\[ +(1 - \delta)C(t-1, t) + (1 - \delta)^2C(t-2, t-1) + \ldots \]
\[ +(1 - \delta)^2C(t-2, t) + \ldots \]
\[ + \ldots \]

That is, the innovation is assumed to be valuable when it is made, but we don’t learn about it until the citations happen. In the previous equation (9.4), the assumption is that the innovation becomes more valuable each time a citation is made. The advantage of the formulation in (9.5) is that it is nested within (9.3):

\[ K^C(t) = K^{PPC}(t) + C(t, t+1) + C(t, t+2) + (1 - \delta)C(t-1, t+1) \]
\[ + C(t, t+3) + (1 - \delta)C(t-1, t+2) + (1 - \delta)^2C(t-2, t+1) + \ldots \]
\[ = K^{PPC}(t) + \sum_{s=1}^{T} \sum_{j=0}^{s-1} (1 - \delta)^j C(t-j, t+s-j) \]
\[ = K^{PPC}(t) + K^{FPC}(t) \]  

(9.6)
Thus this formulation will allow us to separate the total citation weighted stock of patents at time \( t \) into two components, one that contains only citation information prior to \( t \), and one which contains only citation information at \( t+1 \) and later. However, it remains unclear whether we have chosen an appropriate depreciation structure.

10. Appendix D: Truncation Bias

We face at least two types of truncation bias in our measures: bias in our patent measures due to the fact that we have patent data only for granting years between 1967 and 1996 and bias in our citation measures because there is a long citation lag and therefore we will not have observed many citations for patents granted during the later part of our sample. The first bias is fairly straightforward and easy to correct for because the application grant lag is fairly stable and has a median of only about two years. The second is both more complex and of substantive interest, since it is unclear whether our forecast of future citations should agree with that of the market. Therefore we may wish to investigate the relationship between current citation stock and future realized citations in somewhat more detail.

In this section of the paper we outline the steps we take to correct for the first type of bias (due to the application-grant lag). Then we describe the measures of citation stocks that we construct for use in our regressions.

10.1. Patent truncation

Figure D.1 shows the average distribution of the lag between application and grant for all U.S. patents issued during the past four decades. The distributions are quite similar across decades, although there does seem to be a net reduction in the lag between the 1960s and the later periods.\(^{19}\) Except for the 1960s, 95 percent of patent applications that are eventually granted will be granted by year 3, and 99 percent by year 5. Our measure of patents in a year is the number applied for that are ultimately granted, so our goal is to adjust the granted-application count at both ends of our sample. The fact that the median lag is short means that in making our adjustment we cannot go back before an application data of 1964 (because we only have grants made in 1967 and later)

\(^{19}\)The data for the 1990s are based only on the first half of the period and therefore longer issue lags will be truncated, implying that the apparent shortening of the issue lag may be an artifact.
and that we will not be able to adjust the patent counts beyond 1993 (because we only see about half the patents applied for in 1994 due to the grant lags). We describe these adjustments in more detail below.

At the beginning of the sample (1967) we observe some of the patents applied for in 1964-1966, but not the ones that were granted so quickly that their grant date is before 1967. This suggests that we might be able to correct our application counts for 1964-1966 (that is, fill in for lags 0 to 3) using weights drawn from the distribution for the 1960s. At the end of the sample, the opposite happens: there are patents applied for between about 1991 and 1996 that are still pending; some of them will be granted eventually, meaning that our counts of successful patent applications for those years are too small. Again, we can scale up the numbers we do have using the grant-lag distribution. Therefore we will compute the following two adjustments to our patent counts:

\[
\tilde{P}_t = \frac{P_t}{\sum_{s=67-t}^{M} w_s} \quad 64 < t < 67
\]

\[
\tilde{P}_t = \frac{P_t}{\sum_{s=94-t}^{94} w_s} \quad 91 < t < 94
\]

where \(P_t\) is the number of patent applications at time \(t\), \(M\) is the maximum issue lag (assumed to be equal to about 10), and the weights \(w_s\) are weights constructed from the average lag distributions shown in Figure D.1. In Table D.1 we show the weighting factors we will use (the inverse of the expressions in equations (10.1)). Note the edge effects, which imply that the 1996 data will not be usable, and that the 1964 and 1995 data will have more variance due to estimation error.\(^{20}\)

\[\text{Table D.1 about here}\]

\(^{20}\)For this reason, we make no attempt to use data later than 1993 in the body of the paper (although we do use that data to construct stocks of future citations). The 1964 data is used only to the extent that it enters into the stocks of patents and citations that we construct. The first stock in our regressions is dated around 1973, so the measurement error effect should be quite small (recall that the counts are being depreciated by 15 percent per year).
10.2. Citation truncation

As discussed in the text, we seek citation-based measures of patent quality that are comparable across patents. The number of citations that we observe for any given patent may depend on many things. These include:

1. Technological field;

2. Grant date or "vintage" of the patent;

3. The total number of years for which we have data on the patent’s citations.

The first two factors may well contain artificial influences due to variations in patent and citation practice across technologies and across time. For example, the number of citations made by each patent rose over much of this period, possibly due to changes in the ease of identifying prior art due to computer technology. Arguably, this "citation inflation" means that a typical citation given in the 1990s is less significant than the typical citation given in the 1970s. Similarly, there may be variations in norms of citation behavior that make a typical citation in a particular technological field more or less significant than the average. In the absence, however, of some external information about these artifactual variations, the only way one could deal with them would be to "take out" time and technology effects, by subtracting from the observed citation counts mean field and time effects. The problem with such an approach is that there is likely to be variation over time and across fields in the true importance of the typical patent. Taking out these effects would drastically reduce the variance in the data, probably throwing out a good part of the baby with the bathwater. Therefore in this paper we choose not to make any correction for technology field or secular citation trends.\footnote{One might believe that secular trends associated with the year of the citing patent are associated with changes in citation practice, and hence artifactual, while secular trends associated with the cited year represent movements in the true average importance of new inventions. Separating the two, while simultaneously estimating the citation-lag distribution, requires strong functional form assumptions. See Caballero and Jaffe, 1993.}

We do, however, adjust for truncation due to the number of years of citations we actually observe, by fitting a model of the citation lag distribution introduced by Caballero and Jaffe (1993)\footnote{To the extent that the variation across time and fields is artifactual, it represents measurement error in the citation variables that reduces their explanatory power and biases their coefficients. One could construct the citation stocks with and without purging citations of field and time effects; comparing the results might yield some insight as to the fraction of field and time variance that is real and the fraction that is artifactual.}
and developed by Jaffe and Trajtenberg (1996). This model allows the average level of citing activity to vary across fields and times (either because of real or artifactual forces), and thereby estimates a citation lag distribution that is purged of these variations.\textsuperscript{23}

A patent can be cited at any time after it is issued. Although the majority of citations happen in the first 10 years of a patent’s life, there is a long tail of citations that can occur into the infinite future.\textsuperscript{24} Figure D.2 shows the number of citations received at each lag for patents in our sample applied for at different dates between 1965 and 1993. Because our citation data only go from 1976 until 1996, our citations are truncated at both ends: for patents applied for in 1965, we have only the citations at lag 10 and later, whereas for patents issued in 1993, our citations are abruptly truncated at lag 3. Thus we need to adjust for both these truncation biases.

[Figure D.2 about here]

In addition, as Jaffe, Trajtenberg, and Henderson (1993) have shown, citations vary considerably by technological fields. To adjust for this fact, we have grouped the 436 3-digit patent classes in our data into 6 major technological fields: drugs and medical, chemical exc. drugs, electrical and electronics, computers and communication, mechanical, and all other.\textsuperscript{25} Our estimated citation model will include these effects, in order to prevent our estimate of the citation-lag distribution from being distorted by the combination of differences in citation practices across fields and changes in the field mix of patents over time.

We define our dependent variable to be the following:

\[ C_{t,j,s} = \Pr(\text{cite at lag } s|\text{patent applied for at } t \text{ in field } j) \] (10.2)

\textsuperscript{23}The unbalanced nature of the patent data makes it important to control for field and time effects. For example, the only observations that we have on the longest lags are produced by citations made in years at the end of the data period. If the citation-lag distribution were estimated without controlling for the secular rise in citation rates, the estimated tail of the lag distribution would be too thick, because we only observe long lags from high-citation years.

\textsuperscript{24}Because we date patents by date of application, and grant lags are somewhat variable, we occasionally observe citations with negative citation lags, i.e. the citing patent has an application date before the cited patent. For example, there is one patent in our dataset (#1,420,995, belonging to Calgon Corporation) that was issued in 1992, but had originally been applied for in 1921 and then continued. This patent cited another patent that had been issued in 1979, 13 years before Calgon’s patent actually issued, but 58 years after the citing patent (Calgon’s) had been applied for. Such anomalies are rare, and we have chosen to set the cite lag to zero in these cases.

\textsuperscript{25}This classification was performed by Gal Steinberg and Manuel Trajtenberg (Tel Aviv University) and is available from the present authors on request.
where \( j \) is the one digit field of the patent, \( t \) is the year in which the patent was applied for, and \( s \) is equal to \( t' - t \), where \( t' \) is the application year of the citing patent. A problem one confronts when trying to model the citation lag distribution is the fact that we cannot allow both the citing and cited year effects to change freely over time, because some years (notably those post-1996) are never observed, so their effects will not be identified. We use a slightly more flexible version of the model of Jaffe and Trajtenberg (1996) to solve this problem. Our model of the citation probability is then the following:

\[
C_{t,j,s} = \beta_0 \alpha_{\tau(t)} \gamma_{t+s} \beta_j \exp(-\beta_1 \beta_j^S s)(1 - \exp(-\beta_2 \beta_j^D s))
\]

where \( \tau(t) \) maps the years into five-year cohorts (see the table of results for details). As in Jaffe-Trajtenberg, this model constrains the citation-lag distribution to have the same shape for patents of any year, but allows the shape and level of the distribution to be different for each field and it allows the overall level to vary by 5-year patenting interval and by citation year. The only difference from their model is that in our most general version we have allowed both the diffusion process (the \( \beta_2 \beta_j^D \) term) and the obsolescence process (the \( \beta_1 \beta_j^S \) term) to vary by technology field, whereas they held the diffusion constant across field due to difficulties with identification.

Model (10.3) was fit to citation data aggregated by field (6 fields), application year (1963-1993), and citation year (1976-1994), yielding 2616 observations, each of which is the ratio of citations made with lag \( s \) to patents in field \( j \) applied for in the application year \( t \). The results are shown in Table D.2. There are three sets of columns, the first for a model where diffusion and obsolescence effects do not vary across field (\( \beta_j^S = 1 \forall j = 1, \ldots, 5 \) and \( \beta_j^D = 1 \forall j = 1, \ldots, 5 \)), the second for a model where diffusion is the same for all technology fields (\( \beta_j^D = 1 \forall j = 1, \ldots, 5 \)), and the third for a model where both diffusion and obsolescence are allowed to vary over technology field. Although allowing diffusion to vary over technology fields produces only a marginal improvement in the R-squared, it is clear from the coefficient estimates that there are substantial differences among fields.
The J-T model allows for both cited and citing year effects. Although the cited year effects do not vary much over the sample, the citing year effects show a substantial growth rate, consistent with the "citation inflation" mentioned earlier, plus the increase in the rate of patenting, which generates more citations made each year. Figure D.3 shows the estimated citation lag distribution purged of the fact that the probability of citation is increasing over time for all patents, and that patents in a given cohort may be cited more or less, based on the complete model in the last column of Table D.2. The vertical axis is the relative citation probability; the area under each of the curves is the estimated relative overall citation intensity for a given field; the left-right position of the curve indicates the "speed" of citation in a given field. The most cited patents are those in computers and communications, followed by drug and medical patents. The average citation lag is notably longer for drugs and medical, and the diffusion rate substantially slower for patents in the "other" category. The modal lag ranges from 2.6 years for mechanical to 5.3 years for other.

We will use the results for the first model (shown on Figure D.3 as column 1 estimates) to estimate the unobserved citation probabilities, since we do not want to purge the data of field effects. We are missing citations made before 1976 to patents issued between 1963 and 1975, and citations made after 1996 for patents issued between 1963 and 1995 (that is, we are missing one year (1996) for patents issued in 1965, two years (1996, 1997) for patents issued in 1966, and 30 years (1997-2026) for patents issued in 1996. Our model allows us to predict the number of these missing citations. Define the fitted lag distribution in the following way:

\[ D_s = \exp(-\beta_1 s)(1 - \exp(-\beta_2 s)) \quad s = 0, \ldots, 30 \]

Then the predicted number of citations at lag s for a patent issued at time t, when we do not observe citations beyond lag S=T-t is the following:

\[ C_{t,s} = D_s \frac{\sum_{j=0}^{S} C_{t,j} \times \sum_{j=0}^{S} D_j}{\sum_{j=0}^{S} D_j} \]

For example, the predicted number of citations in the year 2000 to a patent issued in 1993 is the following (when we observe citations only through 1996):
There is one problem with this procedure in the later years: because count data has a skew distribution and is bounded below by zero, when zero citations are observed (due to a short period over which to observe them), the expected number of citations that will eventually be observed conditional on this fact is not zero, but positive. We correct for this fact by replacing \( \frac{\sum_{j=0}^{S} C_{t,j}}{\sum_{j=0}^{S} D_{j}} \) in the equation above by the following quantity, derived empirically from our data in 1975 (where we see 20 years of citations):

\[
E[\sum_{j=0}^{20} C_{t,j} | \sum_{j=0}^{S} C_{t,j} = 0]
\]

That is, we predict the total number of citations that will be observed in the 20 years given that we observed zero citations in the first \( S \) years, and then distribute these citations according our citation lag distribution.

To derive the estimated total (20-year) citations for any patent, we simply sum the observed citations from the observed years and the predicted citations based on the above methodology for the unobserved years. Totals based on these "corrected" citation counts are used as indicated in Figures 2 and 3, and were used to construct the citation stocks used in the regressions.
### Table D.1
Weighting Factors for Patent Applications

<table>
<thead>
<tr>
<th>Year</th>
<th>Inverse Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1964</td>
<td>2.119</td>
</tr>
<tr>
<td>1965</td>
<td>1.229</td>
</tr>
<tr>
<td>1966</td>
<td>1.063</td>
</tr>
<tr>
<td>1967</td>
<td>1.000</td>
</tr>
<tr>
<td>1968-89</td>
<td>1.000</td>
</tr>
<tr>
<td>1990</td>
<td>1.000</td>
</tr>
<tr>
<td>1991</td>
<td>1.003</td>
</tr>
<tr>
<td>1992</td>
<td>1.009</td>
</tr>
<tr>
<td>1993</td>
<td>1.034</td>
</tr>
<tr>
<td>1994</td>
<td>1.166</td>
</tr>
<tr>
<td>1995</td>
<td>2.230</td>
</tr>
<tr>
<td>1996</td>
<td>37.461</td>
</tr>
<tr>
<td>Tech Field Effects (base=other)</td>
<td>Coefficient</td>
</tr>
<tr>
<td>--------------------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>Chemicals excl Drugs</td>
<td>0.8788</td>
</tr>
<tr>
<td>Computers &amp; Communication Equip.</td>
<td>1.6150</td>
</tr>
<tr>
<td>Drugs &amp; Medical</td>
<td>1.3714</td>
</tr>
<tr>
<td>Electrical &amp; Electronics</td>
<td>1.1146</td>
</tr>
<tr>
<td>Mechanical</td>
<td>0.9011</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Citing Year Effects (base=1976)</th>
<th>Coefficient</th>
<th>S.E.</th>
<th>Coefficient</th>
<th>S.E.</th>
<th>Coefficient</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1977</td>
<td>0.8920</td>
<td>0.0279</td>
<td>0.8851</td>
<td>0.0220</td>
<td>1.0077</td>
<td>0.0251</td>
</tr>
<tr>
<td>1978</td>
<td>0.9961</td>
<td>0.0304</td>
<td>0.9902</td>
<td>0.0240</td>
<td>1.0936</td>
<td>0.0264</td>
</tr>
<tr>
<td>1979</td>
<td>1.0821</td>
<td>0.0330</td>
<td>1.0764</td>
<td>0.0261</td>
<td>1.1540</td>
<td>0.0277</td>
</tr>
<tr>
<td>1980</td>
<td>1.1507</td>
<td>0.0357</td>
<td>1.1468</td>
<td>0.0281</td>
<td>1.1959</td>
<td>0.0289</td>
</tr>
<tr>
<td>1981</td>
<td>1.1751</td>
<td>0.0375</td>
<td>1.1727</td>
<td>0.0296</td>
<td>1.1927</td>
<td>0.0295</td>
</tr>
<tr>
<td>1982</td>
<td>1.2540</td>
<td>0.0408</td>
<td>1.2508</td>
<td>0.0322</td>
<td>1.2390</td>
<td>0.0312</td>
</tr>
<tr>
<td>1983</td>
<td>1.2530</td>
<td>0.0426</td>
<td>1.2526</td>
<td>0.0337</td>
<td>1.2051</td>
<td>0.0317</td>
</tr>
<tr>
<td>1984</td>
<td>1.3981</td>
<td>0.0482</td>
<td>1.3956</td>
<td>0.0381</td>
<td>1.3048</td>
<td>0.0349</td>
</tr>
<tr>
<td>1985</td>
<td>1.5637</td>
<td>0.0548</td>
<td>1.5618</td>
<td>0.0434</td>
<td>1.4213</td>
<td>0.0388</td>
</tr>
<tr>
<td>1986</td>
<td>1.7444</td>
<td>0.0622</td>
<td>1.7447</td>
<td>0.0494</td>
<td>1.5508</td>
<td>0.0433</td>
</tr>
<tr>
<td>1987</td>
<td>1.9992</td>
<td>0.0727</td>
<td>2.0011</td>
<td>0.0578</td>
<td>1.7328</td>
<td>0.0497</td>
</tr>
<tr>
<td>1988</td>
<td>2.2563</td>
<td>0.0844</td>
<td>2.2648</td>
<td>0.0674</td>
<td>1.9086</td>
<td>0.0566</td>
</tr>
<tr>
<td>1989</td>
<td>2.4626</td>
<td>0.0954</td>
<td>2.4684</td>
<td>0.0761</td>
<td>2.0245</td>
<td>0.0625</td>
</tr>
<tr>
<td>1990</td>
<td>2.5883</td>
<td>0.1038</td>
<td>2.5991</td>
<td>0.0831</td>
<td>2.0780</td>
<td>0.0667</td>
</tr>
<tr>
<td>1991</td>
<td>2.6691</td>
<td>0.1106</td>
<td>2.6823</td>
<td>0.0886</td>
<td>2.0976</td>
<td>0.0698</td>
</tr>
<tr>
<td>1992</td>
<td>2.8730</td>
<td>0.1227</td>
<td>2.8921</td>
<td>0.0966</td>
<td>2.2111</td>
<td>0.0762</td>
</tr>
<tr>
<td>1993</td>
<td>2.7362</td>
<td>0.1219</td>
<td>2.7587</td>
<td>0.0961</td>
<td>2.0596</td>
<td>0.0741</td>
</tr>
<tr>
<td>1994</td>
<td>1.2983</td>
<td>0.0667</td>
<td>1.3108</td>
<td>0.0538</td>
<td>0.9545</td>
<td>0.0389</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cited Year Effects (base=1963-65)</th>
<th>Coefficient</th>
<th>S.E.</th>
<th>Coefficient</th>
<th>S.E.</th>
<th>Coefficient</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966-70</td>
<td>0.6736</td>
<td>0.0139</td>
<td>0.6723</td>
<td>0.0110</td>
<td>0.8749</td>
<td>0.0174</td>
</tr>
<tr>
<td>1971-75</td>
<td>0.6352</td>
<td>0.0125</td>
<td>0.6283</td>
<td>0.0104</td>
<td>0.9295</td>
<td>0.0220</td>
</tr>
<tr>
<td>1976-80</td>
<td>0.5687</td>
<td>0.0144</td>
<td>0.5594</td>
<td>0.0120</td>
<td>0.9369</td>
<td>0.0284</td>
</tr>
<tr>
<td>1981-85</td>
<td>0.4959</td>
<td>0.0159</td>
<td>0.4860</td>
<td>0.0133</td>
<td>0.9300</td>
<td>0.0352</td>
</tr>
<tr>
<td>1986-90</td>
<td>0.4275</td>
<td>0.0173</td>
<td>0.4152</td>
<td>0.0142</td>
<td>0.8935</td>
<td>0.0403</td>
</tr>
<tr>
<td>1991-93</td>
<td>0.3332</td>
<td>0.0188</td>
<td>0.3174</td>
<td>0.0148</td>
<td>0.7435</td>
<td>0.0410</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Beta1: Obsolescence by Technology Field</th>
<th>Coefficient</th>
<th>S.E.</th>
<th>Coefficient</th>
<th>S.E.</th>
<th>Coefficient</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemicals excl Drugs</td>
<td>1.0249</td>
<td>0.0204</td>
<td>0.7022</td>
<td>0.0248</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computers &amp; Communication Equip.</td>
<td>1.2731</td>
<td>0.0185</td>
<td>1.1015</td>
<td>0.0268</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drugs &amp; Medical</td>
<td>0.7796</td>
<td>0.0140</td>
<td>0.4559</td>
<td>0.0167</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electrical &amp; Electronics</td>
<td>1.1558</td>
<td>0.0197</td>
<td>0.7960</td>
<td>0.0223</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mechanical</td>
<td>1.0125</td>
<td>0.0199</td>
<td>0.6251</td>
<td>0.0214</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beta1 (Base=Other)</td>
<td>0.1229</td>
<td>0.0018</td>
<td>0.1188</td>
<td>0.0017</td>
<td>0.1250</td>
<td>0.0017</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Beta2: Diffusion by Technology Field</th>
<th>Coefficient</th>
<th>S.E.</th>
<th>Coefficient</th>
<th>S.E.</th>
<th>Coefficient</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemicals excl Drugs</td>
<td>3.5478</td>
<td>0.2699</td>
<td>2.2286</td>
<td>0.1406</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Computers &amp; Communication Equip.</td>
<td>3.9224</td>
<td>0.2342</td>
<td>3.8451</td>
<td>0.2489</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drugs &amp; Medical</td>
<td>4.7339</td>
<td>0.3507</td>
<td>4.7339</td>
<td>0.3507</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Electrical &amp; Electronics</td>
<td>0.4801</td>
<td>0.0132</td>
<td>0.4608</td>
<td>0.0104</td>
<td>0.1599</td>
<td>0.0063</td>
</tr>
<tr>
<td>Mechanical</td>
<td>0.904</td>
<td>0.939</td>
<td>0.949</td>
<td>0.0430</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.0591</td>
<td>0.0472</td>
<td>0.0430</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TABLE D.2

Estimation of Citation Probability based on Jaffe-Trajtenberg Model
2616 Observations = 6*(14*19 + (18*19)/2 - 1)
Dep. Var.=No. of Citations by Citing Year Patents to Patents in Cited Year & Field
Figure D.1
Application-Grant Lag Distribution
Figure D.2
Empirical Citation Lag Distribution by Year
HJT Sample of Manufacturing Firms
Figure D.3
Citation Lag Distribution (1976-1994)
JT Methodology with Field-Specific Diffusion and Obsolescence
Figure D.4
Citation Lag Distribution by Cited Year