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SIGNAL PROCESSING FOR WIRELESS AD HOC  
COMMUNICATION NETWORKS

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# Maximizing the Throughput of Large Ad Hoc Wireless Networks

[A look at research issues]

**A**d hoc wireless networks can be quickly deployed to provide wireless communication services in areas without a pre-established infrastructure of base stations. Since delivering signals between sources and destinations is a primary function of ad hoc networks, the design and analysis of ad hoc networks are also a task of signal processing. In this article, we review some of the major results on maximum throughput of ad hoc networks. Both single-hop and multihop ad hoc networks are considered. In particular, we show examples of how to maximize the throughput of a large ad hoc network through a closed-loop analysis of two medium access control (MAC) schemes: synchronous array method (SAM) and asynchronous array method (AAM). Both omnidirectional antennas and directional antennas are examined. This study illustrates an importance of the interplays between signal processing, information theory, and networking protocols.

## OVERVIEW

### AD HOC WIRELESS NETWORK

This article studies ad hoc wireless networks as a communication network that consists of wireless nodes without the conventional infrastructure of base stations. Ad hoc networks can be quickly deployed in areas beyond the reach of existing infrastructure of communications. This property makes ad hoc networks useful for military, remote exploration, and disaster relief applications.

A primary function of ad hoc networks is to deliver signals from place to place. Hence, an ad hoc network can be viewed as a signal processor although it is much more complex than a conventional signal processor such as a linear filter. In recent years, signal processing researchers have begun to explore this field. We further demonstrate that fundamental

insights into ad hoc networks can be discovered from the signal processing perspective.

The research and development of ad hoc networks have been around for several decades. Many of these activities and results can be found in [19] and the references therein. Most of the research work in this area has been oriented towards implementations. There are many implementation examples of ad hoc networks. But there is not enough fundamental theory of ad hoc networks. In this article, we review some of the major results on the capacity of large ad hoc networks and share our own perspectives.

### CAPACITY OF AD HOC NETWORK

The capacity of a point-to-point communication channel is defined as the maximum number of bits per second that can be reliably transmitted over the channel. This notion was well established by Shannon more than 50 years ago and has been widely applied in the field of information theory and point-to-point communication systems. For cellular wireless networks, the concept of multiuser capacity measured in bits per second is also well understood. For a detailed introduction on this subject, see [31].

But the capacity of ad hoc networks is a much more complex subject. Ideally, a full description of the capacity of an ad hoc network should be defined as a multidimensional surface that encloses a multidimensional region spanned by all possible reliable data rates between all source-destination pairs. Pursuing such a full description of capacity does not seem feasible except for a small network. Even for a network of five nodes, only a partial description of the capacity is currently tractable as shown in [7].

However, it is useful to consider a maximum (uniform) data rate between all source-destination pairs in the network as it is natural to expect all nodes in the network to be evenly loaded in many applications. Table 1 summarizes three important units of capacity and their relationships with each other for a large network in a two-dimensional (2-D) space. Here,  $c$  is generally bounded as the number  $n$  of nodes in the network becomes large, and hence in terms of bits-hops/s/Hz/node, the capacity is invariant to a large  $n$ . In terms of both bits-hops/s/Hz/node and bits/s/Hz/node, the capacity is invariant to the dimension of the network space (one, two, or three dimensional). If it is a one-dimensional network, the scaling law in bits-meters/s/Hz/node is  $c\rho_1^{-1}$  where  $\rho_1$  is the number of nodes per meter. If it is a

three-dimensional (3-D) network, the scaling law in bits-meters/s/Hz/node is  $c\rho_3^{-1/3}$  where  $\rho_3$  is the number of nodes per cubic meter. We will only focus on 2-D networks.

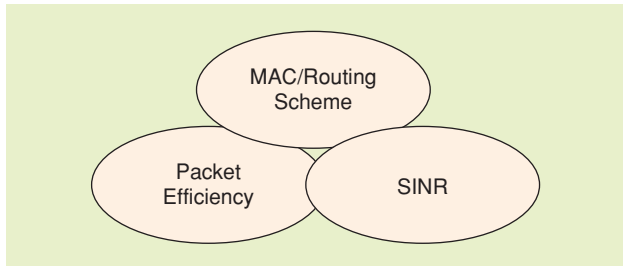
Researchers have explored the capacity (maximum throughput) of ad hoc networks in recent years. The most recognized and groundbreaking is the one by Gupta and Kumer [8]. The authors studied a large network of  $n$  nodes and presented a capacity scaling law of the form  $c_1\sqrt{A/n}$  in bits-meters/s/Hz/node, where  $n$  is the number of nodes optimally placed in a disk of area  $A$  and the routing/MAC scheme between source-destination pairs is also optimally designed. It is clear that this is the same form as  $c_1/\sqrt{\rho_2}$  in bits-meters/s/Hz/node. An upper bound on  $c_1$  was found under a restrictive condition in [8]. In a later paper by Xie and Kumar [33], a stronger result on the upper bound on  $c_1$  was established under a large attenuation model and a small attenuation model. Yet, a gap exists between the two models, where an upper bound on  $c_1$  is still not known. The scaling law  $c_1\sqrt{A/n}$  in bits-meters/s/Hz/node also applies to a network of fading channels as shown in [34] by Xue et al. Unless specified otherwise, only nonfading channels will be considered in this article.

We will provide a simple approach to derive the same scaling law  $c_1\sqrt{A/n}$  with bounded  $c_1$  assuming that the optimal topology of a network is a topology where all nodes are evenly distributed and also assuming that there is no (coding) cooperation between concurrent transmissions in different regions.

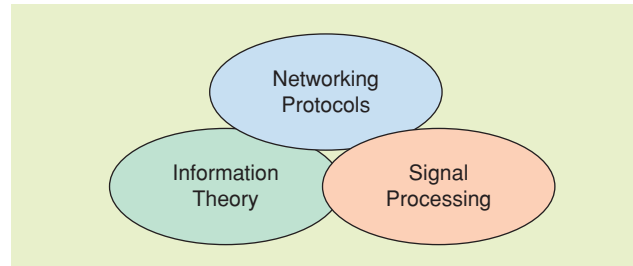
In [8], the authors also considered a random network where total  $n$  nodes are randomly placed in a disk of area  $A$ . When  $n$  becomes very large, the network capacity with a large probability is shown there to be  $c_2\sqrt{A/n}\log n$  in bits-meters/s/Hz/node. ("With a large probability" means that the probability approaches one as the number of nodes approaches infinity.) The technique by [8] has inspired many as in [1], [15], [17] and [29] for ad hoc networks assisted with base stations and that in [36] for ad hoc networks of nodes with directional antennas. Following a similar technique from [8], it is shown in [30] that for an ultra-wideband (UWB) random network where the transmission power is much more limited than the available bandwidth, the network capacity is in the form of  $c_3\sqrt{AP_T}(\sqrt{n}/\log n)^{\alpha-1}$  in bits-meters/s/node where  $P_T$  is the transmission power from each node and  $\alpha$  the path loss exponent. This result closed the gap between a lower bound and an upper bound previously given in [22]. For sensor networks with limited power but (relatively) unlimited bandwidth, the UWB network capacity scaling

**[TABLE 1] COMPARISON OF CAPACITY UNITS**

CAPACITY UNIT	SCALING LAW	MEANING
BITS-HOPS/S/Hz/NODE	$c$	THE NUMBER OF BITS TRANSMITTED FROM EACH NODE TO ITS NEXT-HOP NODE PER SECOND PER HERTZ, OR THE NUMBER OF BITS TIMES THE NUMBER OF HOPS PER SECOND PER HERTZ PER SOURCE NODE.
BITS/S/Hz/NODE	$c\frac{1}{h}$	THE NUMBER OF BITS TRANSMITTED FROM EACH SOURCE NODE TO ITS DESTINATION NODE PER SECOND PER HERTZ. HERE, $h$ IS THE AVERAGE NUMBER OF HOPS BETWEEN ANY SOURCE NODE AND ITS DESTINATION NODE.
BITS-METERS/S/Hz/NODE	$c\rho_2^{-1/2}$	THE NUMBER OF BITS TIMES THE NUMBER OF METERS TRAVELED FROM EACH SOURCE NODE TO ITS DESTINATION NODE PER SECOND PER HERTZ. HERE, $\rho_2$ IS THE NUMBER OF NODES PER SQUARE METER.



**[FIG1]** The maximum throughput of an ad hoc network should be determined by a closed-loop analysis of SINR, packet spectral efficiency and MAC/routing scheme.



**[FIG2]** A full-spectrum union for ad hoc communication network design and analysis.

law should be useful. For a review on sensor networks, see [24]. In the sequel, we will only discuss bandwidth-limited networks.

The exact value of  $c$  is clearly important in applications. (We will drop the subscript from  $c$  for convenience. But this constant may differ in different expressions.) But the value of  $c$  not only depends on physical conditions such as fading, path loss, topology, and property of antennas, but also the chosen routing and/or MAC scheme. We will show examples of how to maximize the exact values of  $c$  for a given network topology through a closed-loop analysis of signal-to-interference and noise ratio (SINR), packet spectral efficiency and a chosen MAC scheme (see Figure 1). This closed-loop analysis is in contrast to a conventional open-loop approach where the packet spectral efficiency (or equivalently, the threshold SINR) is prefixed. For example, in [8], the packet spectral efficiency is fixed at 1 b/s/Hz. The closed-loop analysis is an example of useful interplays between signal processing, information theory, and networking protocols (see Figure 2), which perhaps is a useful step going beyond the classical interplays between information theory and networking protocols [6]. Interferences are inherent in all wireless networks. Modeling and analysis of interferences are fundamentally signal processing. Signal processing should serve as a useful bridge between information theory and networking protocols for various network models.

### ROUTING

In ad hoc networks, routing is necessary to find routes and/or to forward packets between source-destination pairs. Routing has been a subject of research in the networking community for many years. The routing techniques can be broadly classified as proactive, reactive, and location based, although a more detailed classification can be found in [19]. A proactive technique requires each node in the network to periodically update its routing table and share it with the rest of the network. This technique is effective if the network topology does not change frequently. A reactive technique initiates route discovery only when a node needs to communicate with another. This technique has a much lower routing overhead than the proactive technique if the network topology changes frequently. But it may still cause significant flooding (i.e., a large volume of traffic) in the network each time routing is initiated. For a location-based technique, a source or relay node does not need to know an exact route to its destination before it forwards packets based

on location information [21]. This technique can be highly efficient if the destination node is always associated with a location (or direction) known to the source node and all the relay nodes. But a location-based technique can only successfully forward one packet during one packet transmission interval within a large subset of the network. If the subset surrounding a receiving node is not large enough, the probability of packet loss is large due to interference. A conventional control of the size of the subset is probabilistic, i.e., using a small transmission probability of a packet from each node.

However, no matter how efficient a routing technique is, a high mobility of nodes in a pure ad hoc network would reduce the network throughput significantly because of the networking overheads (e.g., the discovery of route, location and/or topology, and the sharing of the information). The loss of capacity due to high mobility is generally large. Only if the delay of packets (e.g., in the order of minutes, hours, or even days) is not a concern could the mobility of nodes improve the network throughput. In this case, the mechanical motion of nodes (as opposed to the propagation of electromagnetic waves) must be exploited for transportation of packets between source-destination pairs. However, this is not a case of interest in most practical situations.

### WIRELESS MESH NETWORKS

Because of the poor capacity of a pure ad hoc network, there is now a strong interest in wireless mesh networks. A wireless mesh network consists of wireless routers that are highly cooperative, of low mobility, and can serve as virtual base stations for mobile clients. Routing between mobile clients can be handled efficiently by wireless mesh network. Depending on applications, the nodes in a wireless mesh network can be positioned on a regular grid, and/or equipped with large power sources, antenna arrays and/or directional antennas.

There are many articles on wireless mesh networks, of which we mention [3], [4], [13], [14], and [32]. In [3], a broad overview of wireless mesh networks is provided, and many research issues are raised in a broad sense. (A wireless mesh network defined in [3] also includes the mobile clients.) Shown in [4] is a review of state-of-the-art implementations of wireless mesh networks, which are mostly tailored for practical engineers. In [13], the main objective is to demonstrate that the per-node capacity of a network with  $n$  nodes and a single access point scales as  $1/n$ .

This capacity scaling law is with respect to the spectrum allocated for the access point. Since all nodes share the same access point, the per-node share is then clearly in the order of  $1/n$ . The meaning of this capacity is different from that of the capacity with respect to the spectrum exclusively allocated for routing within the mesh network. The two papers [14] and [32] are more technical as discussed below.

In [14], the authors studied the achievable source-destination rates of a small network (15 nodes) by assuming that the data rate between any two nodes is available and independent of the activities at other neighboring nodes. This type of assumption is common in the networking literature. But a better model is available for ad hoc networks where the (effective) data rate between any two nodes generally depends on the schedules applied to their neighboring nodes.

The work by [32] took into account the mutual interferences among neighboring links in studying achievable source-destination rates. The physical model used in [32] is an improvement over that in [14]. A common feature between [32] and [14] is that a central processor is used to optimally combine in a time-sharing fashion a number of transmission scenarios to achieve desired source-destination rates. A transmission scenario is a given set of concurrent transmission pairs in the network. Using linear combination of transmission scenarios for capacity analysis is also applied in [7] and [23]. This approach is only feasible for a small network because the number of transmission scenarios in a network of  $n$  nodes grows unbearably rapidly as  $n$  increases [23].

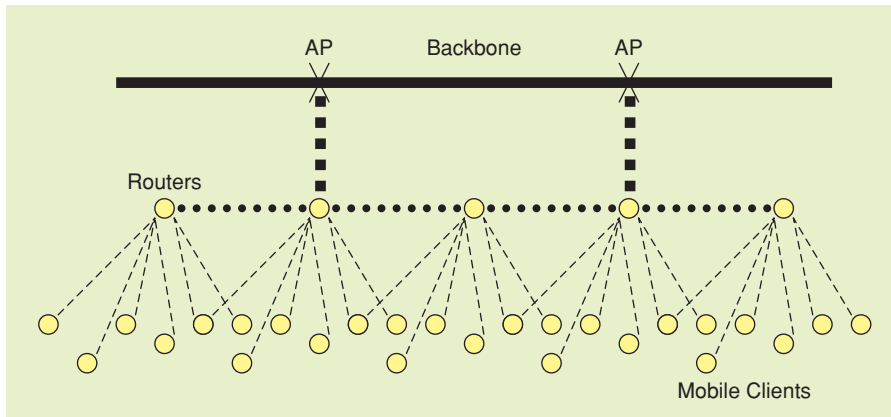
### LARGE WIRELESS MESH NETWORKS

To our knowledge, all prior research on wireless mesh networks has been for small networks. But there is a practical need for a large wireless mesh network when the communication among thousands of conventional mobile clients must be supported in the absence of conventional base stations in times of war, natural disaster, or temporary settlement in remote areas.

A common perception of a large network is that the network capacity decreases with the number of nodes  $n$  in the network. We know that this quantity  $n/A = \rho_2$  measures the node density, and the capacity  $c\sqrt{A/n}$  in bits-meters/s/Hz/node may remain constant no matter how large the number of nodes becomes. In terms of bits/s/Hz/node, the capacity has the form  $c/h$ . But the number of hops  $h$  required for a packet to travel from a source router to a destination router does not have to scale like  $\sqrt{n}$ . There are several ways to control the distance of source-destination pairs in a large network. A soft way is to charge high cost for long distance users. Alternatively, if possible, very high bandwidth (wireless or wireline) backbone access points are placed over the wireless mesh network. If there is one

backbone access point for every  $m$  wireless routers in a 2-D space, then the capacity of the wireless mesh network has the form  $c/\sqrt{m}$  in bits/s/Hz/node, which is still independent of  $n$ . (For a 3-D network, the capacity in bits/s/Hz/node would have the form  $c \times m^{-1/3}$ .) The spectrum used by the access points is however not included in this expression, which may belong to a difference class of resources (e.g., optical links are used for the access points and microwave links are used for wireless routers).

A three-tier architecture for large ad hoc networks is illustrated in Figure 3. The first (lowest) tier consists of mobile clients. The second tier is a wireless mesh network consisting of



**[FIG3] A three-tier architecture of large ad hoc networks. Tier 1 consists of mobile clients. Tier 2 consists of wireless routers—wireless mesh network. Tier 3 is the backbone. Each tier may be in a 2-D or 3-D space.**

wireless routers. Without the second tier, the mobile clients with conventional wireless devices cannot communicate. The third tier is the backbone with access points (AP) connected (with a broad bandwidth) to selected wireless routers. The third tier may not be necessary for the two lower tiers to operate, but can significantly increase the network capacity.

The capacity scaling laws shown before readily apply to the communications within the second tier although the meaning of “per Hertz” here is with respect to the frequency spectrum allocated to the second tier. The uplink and downlink between the second tier and the mobile clients should generally require a separate spectrum.

The capacity of second tier has the form  $c(1/h)$  in bits/s/Hz/node where  $h$  is primarily governed by the AP placement. Studying routing and/or MAC schemes to maximize  $c$  should be of great practical interest. For a large mesh network, little work has been done so far.

Routing for a wireless mesh network is an easier problem than for a pure ad hoc network. A conventional proactive routing technique can be used to discover the topology of a wireless mesh network.

### TOPOLOGY-BASED RESEARCH FOR AD HOC NETWORKS

The importance of topology for networking is like the importance of channel state information for point-to-point communications. Signal processing research for ad hoc networks has been largely topology based and mostly on cooperative relays.

These works include the three-nodes relay network [18], [26], broadcast-plus-multiple-access two-hop relay network [2], [11], and a single route of multihop parallel relays [5], [35]. The research on cooperative relays provide a cross-layer perspective complementary to the networking layer cooperative protocols as shown in [9] and [25]. Yet, there is a relative vacuum of signal processing research for large ad hoc networks. By a large network, we not only imply a large number of nodes or relays in the network but also a large number of concurrent cochannel routes/transmissions in the network. It is the concurrent cochannel routes/transmissions for multiple source-destination pairs that make it possible to achieve the maximum network throughput in bits-meters/s/Hz/node. Concurrent cochannel routes/transmissions unavoidably cause mutual interferences that are also schedule dependent. This poses a unique challenge of signal processing.

Topology has been used to evaluate the performance of given routing/MAC protocols. In [16] and [27], the ALOHA protocol was analyzed based on a square grid. But the dependency of the optimal spectral efficiency of a packet on the MAC protocol was not exploited. In [20], a multipacket reception model along with randomized time division multiplexing protocol was applied to a regular network, and a similar capacity scaling law was shown.

### THE REST OF THE ARTICLE

In the next section, we provide a tutorial on the principles behind the capacity scaling laws of large ad hoc networks. Both single-hop networks and multihop networks are considered. We show that for a small network (say, 20 nodes), single-hop schemes should be used because of its ease of routing and its high spectral efficiency at high transmission power. Only for a large network, multihop schemes are desirable. For a large multihop network, we show that the scaling law  $c\sqrt{A/n}$  with a bounded  $c$  in bits-meters/s/Hz/node results from a simple reason.

After a review of capacity scaling laws, we formulate and analyze two MAC schemes and compare their maximized throughputs in bits-hops/s/Hz/node for a large network of nodes located on a square grid. The two MAC schemes are SAM and AAM. SAM is highly scheduled and cooperative. AAM follows the principle of slotted ALOHA originated by Abramson [12]. For each method, both omnidirectional and directional antennas are treated. In all cases, an intrinsic relationship between the optimal packet spectral efficiency and the MAC scheme is exploited to maximize the network throughput. We will see a significant difference in the maximized throughput between the two schemes.

### CAPACITY SCALING LAWS

We now review the principles behind the capacity scaling laws of a large network. We consider two cases: single-hop schemes and multihop schemes.

#### SINGLE-HOP SCHEMES

If all nodes in a network can reach each other in a single hop, the network can be highly dynamic and there is no issue of routing.

This can be a very important advantage for many applications. But a MAC protocol must be established, which can be contention based or schedule based.

A common contention-based protocol is such that each node chooses to transmit randomly. If each node transmits with probability  $p$  in any time slot (such as in the slotted ALOHA protocol [12]), the probability for a node to transmit a packet successfully to another node is  $P = p(1-p)^{n-1}$ . Here, we assume that any collision of transmitted packets results in loss of packets, and any node that does not transmit is ready to receive. The probability  $P$  is maximized if  $p = 1/n$ , i.e.,  $P_{\max} = (1/n)(1 - (1/n))^{n-1}$ . Both  $(1/n)(1 - (1/n))^{n-1}$  and  $(1 - (1/n))^{n-1}$  are monotonically decreasing functions of  $n$ . For large  $n$ , we have  $P_{\max} \approx (1/ne)$  where  $e \hat{=} \lim_{n \rightarrow \infty} (1 + (1/n))^n \approx 2.718$ . If all nodes are randomly distributed inside a disk of area  $A$ , the average distance between two nodes is known to be:  $(128\sqrt{A}/45\pi^{3/2}) \approx 0.5\sqrt{A}$  [28]. Also, the information in a packet is upper-bounded by  $\log_2(1 + \text{SINR}_t)$  in bits/s/Hz where  $\text{SINR}_t$  is the threshold SINR used for packet detection. Therefore, the average data throughput of a large network in bits-meters/s/Hz/node is  $(\sqrt{A}/2en) \log_2(1 + \text{SINR}_t)$ .

A common schedule-based protocol is such that each node transmits a packet in a given orthogonal channel (in time and/or frequency). Given  $n$  nodes in the network, the average data throughput in bits-meters/s/Hz/node is simply  $(\sqrt{A}/2n) \log_2(1 + \text{SINR}_t)$ . This is  $e$  times better than the contention-based protocol. But the schedule-based protocol has to have a prior knowledge of all  $n$  active nodes, i.e., each node that wishes to transmit a packet must first receive an allocation of subchannel. This causes a networking overhead, and this overhead can be significant if the identities of the active nodes change frequently. There is no such a problem for the contention-based protocol.

The above schedule-based protocol may be called a single-input single-output (SISO) protocol. For a network that is relatively static, the SISO protocol can be replaced by the following multi-input, single-output (MISO) protocol. During each transmission interval, all nodes except one may cooperatively and simultaneously transmit their own packets to a receiving node. The maximum information received by the receiving node is

$$\log_2 \left( 1 + \frac{\sum_{i=1, i \neq j}^n P_T |h_{i,j}|^2}{\sigma^2} \right) = \log_2 \left( 1 + \frac{(n-1) P_T \sigma_j^2}{\sigma^2} \right)$$

in bits/s/Hz, where  $P_T$  is the transmitted power from each node,  $h_{i,j}$  is the channel gain from node  $i$  to node  $j$ ,  $\sigma^2$  is the noise variance, and

$$\sigma_j^2 = \frac{1}{n-1} \sum_{i=1, i \neq j}^n |h_{i,j}|^2.$$

Here, the receiving node needs to apply a successive interference cancellation (SIC) technique as explained next. Assume for convenience that during one transmission cycle, the receiving

node is indexed by  $j = n$  and the transmitting nodes are indexed by  $i = \{1, 2, \dots, n-1\}$ . Suppose that the packet transmitted from node  $i$  is encoded with the threshold SINR:

$$\text{SINR}_i = \frac{P_T |h_{i,n}|^2}{\sigma^2 + \sum_{l=i+1}^{n-1} P_T |h_{l,n}|^2}$$

with

$$\text{SINR}_{n-1} = \frac{P_T |h_{n-1,n}|^2}{\sigma^2}.$$

(Here, all transmitting nodes need to know the assigned threshold SINR or equivalently the assigned data rates.) The receiving node can then first decode the packet coming from node 1 and then remove the packet from the received signal before it decodes the packet coming from node 2. This SIC process continues until the packet coming from node  $n-1$  is decoded. One can now verify that the total amount of received information is

$$\sum_{i=1}^{n-1} \log_2 (1 + \text{SINR}_i) = \log_2 \left( 1 + \frac{(n-1) P_T \sigma_n^2}{\sigma^2} \right)$$

in bits/s/Hz. The SIC technique follows directly from the chain rule of mutual information between the  $n-1$  transmitted (independent) signals and the received signal. By ordering the transmitting nodes differently during different transmission cycles for the same receiver, a fair distribution of rates for the transmitting nodes can be achieved on average. If we assume that  $\sigma_j^2 = \sigma_h^2$  is independent of the receiving node index  $j$  (which should hold well in practice if  $n$  is large), then the maximum throughput of the MISO protocol, in bits-meters/s/Hz/node, is

$$C_{\text{MISO}} = \frac{\sqrt{A}}{2n} \log_2 \left( 1 + \frac{(n-1) P_T \sigma_h^2}{\sigma^2} \right).$$

It follows from Jensen's inequality that the maximum throughput of the MISO protocol is higher than that of the SISO protocol, i.e.,

$$C_{\text{MISO}} \geq \frac{\sqrt{A}}{2n} \frac{1}{n-1} \sum_{i=1}^{n-1} \log_2 \left( 1 + \frac{(n-1) P_T |h_i|^2}{\sigma^2} \right) = C_{\text{SISO}},$$

where the right-side term is the maximum throughput of the SISO protocol. The gap between the two throughputs is significant if the channel gains have a large dynamic range.

The MISO protocol is similar to the multiuser uplink scenario in cellular networks [31]. The difference here is that each node in the ad hoc network in turns serves as a common access point for other nodes.

However, all of the above single-hop schemes have a capacity that decreases like  $1/n$  as  $n$  increases. Therefore, the single-hop schemes are useful in practice only for small networks, and perhaps should be used for small networks.

## MULTIPLE-HOP SCHEMES

While the single-hop schemes described earlier have the advantage of simple routing, the capacity may become too low as the number  $n$  of nodes becomes large. Also, if the network is sparse, not all nodes may have enough power to reach each other by single hops. To improve the scaling law with respect to  $n$ , frequency reuse is necessary, i.e., there must be concurrent cochannel source-destination routes in the network.

We now derive the maximum (uniform) throughput scaling law assuming a topology where all nodes are evenly distributed and also assuming that there is no coding cooperation between concurrent transmissions from different regions. A topology of evenly distributed nodes is believed to lead to the maximum throughput.

When there are concurrent routes, there are concurrent transmissions (i.e., concurrent packet hops) at any given time. Then, without loss of generality, the network at any given time can be divided into  $m$  regions, and in each region there is a transmission of one hop. For maximum (uniform) throughput, the  $m$  regions should be evenly distributed. Then, the number of packets transmitted in the network per time slot is simply  $m P_D(m)$  where  $P_D(m)$  is the probability of packet detection and generally depends on  $m$ . But the average number of hops required for a (random) source-destination pair in the network is  $(1/2)\sqrt{m}$  for large  $m$ . Therefore, the source-destination throughput in packets per time slot for the whole network is  $2\sqrt{m} P_D(m)$ . It then follows that when  $n$  and  $m$  are large the throughput of the network in bits-meters/s/Hz/node is

$$\frac{\sqrt{A} \sqrt{m}}{n} \log_2 (1 + \text{SINR}_t) P_D(m) \leq \frac{\sqrt{A}}{\sqrt{2n}} \log_2 (1 + \text{SINR}_t).$$

Here,  $A$  is the area of the network,  $\sqrt{A}/2$  is the average distance of source-destination pair, and  $\log_2(1 + \text{SINR}_t)$  is the spectral efficiency of each packet in bits/s/Hz. The upper bound on the right side is due to the fact  $P_D(m) \leq 1$  and the constraint that each region at any time has at least two nodes or equivalently  $m \leq n/2$ . The above expression is equivalent to  $c\sqrt{A/n}$  with  $c \leq 1/\sqrt{2} \log_2(1 + \text{SINR}_t)$ . Note that the upper bound on  $c$  is not achievable unless  $\text{SINR}_t$  is small enough so that  $P_D(m)$  is close to one. Unlike the single-hop schemes, the capacity of a multihop scheme does not increase indefinitely as the transmitted power increases because  $P_D(m)$  decreases as  $\text{SINR}_t$  increases. Furthermore, there should be an optimal choice of  $\text{SINR}_t$  and  $m$  to maximize the actual network throughput  $\sqrt{A} (c\sqrt{m}/n) \log_2(1 + \text{SINR}_t) P_D(m)$  in bits-meters/s/Hz/node. The choice of  $m$  and  $\text{SINR}_t$  should be part of a routing/MAC scheme. This tradeoff will be illustrated later. (The above analysis and the resulting bound also apply to fading channels.)

Note that we have assumed a single omnidirectional antenna for each node in the network. If each node is equipped with directional antennas, the network throughput can be much higher. An analysis of this situation will be shown later. If each node is equipped with  $L$  antennas (in a cluttered environment),

there are effectively  $L$  parallel spatial channels between any two adjacent nodes. With the theory of MIMO channel capacity, we expect the network capacity to be upper bounded by  $L(\sqrt{2A}/\sqrt{\eta}) \log_2(1 + \text{SINR}_t)$  in bits-meters/s/Hz/node, where  $\text{SINR}_t$  is a reference SINR (at each receiving antenna) used for packet coding. Once again, the tightness of this upper bound depends on how close the actual (averaged) SINR at each receiving antenna is to  $\text{SINR}_t$ . The tightness of this upper bound also depends on the spread of the singular values of the MIMO channel matrix in addition to other variables. A more thorough analysis of a large network of MIMO links remains to be conducted.

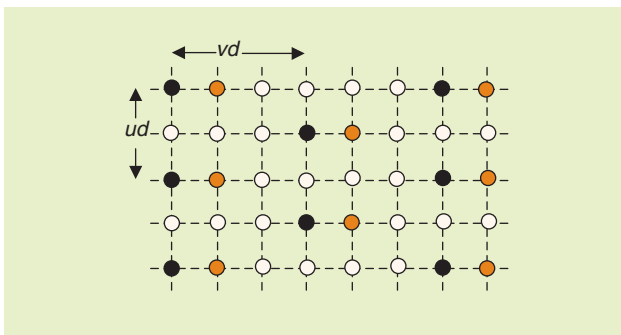
We note that although useful at a tutorial level, the analysis shown in this section cannot replace the more rigorous works as shown in [8], [33], and [34]. In the next two sections, we will present two specific examples of MAC schemes for a large network and the maximum throughputs achieved. The design and analysis of MAC schemes to maximize the throughput of a given large network are important tasks of signal processing.

### SYNCHRONOUS ARRAY METHOD

We now consider a network of nodes located on a square (or rectangular) grid as shown in Figure 4. We formulate the following MAC scheme called synchronous array method (SAM):

A session of data transmissions is divided into cycles. Each cycle is divided into four units. Each unit is preserved for data transmission in a given direction: west to east, south to north, east to west, or north to south. Each unit is further divided into subunits, and during each subunit, only a (predetermined) subset of nodes in the network transmits to its nearest neighboring nodes. (We consider four directions for the rectangular grid. For a hexagon topology, there would be six directions. This and other possible topologies are not considered here but are available in [10].)

This is a simple protocol and easy to implement. The nodes in each transmission subset use all available radio spectrum at



**[FIG4]** A network of nodes (wireless routers) positioned on a rectangular grid. Under the SAM scheme, data packets are transmitted from black nodes to their neighboring gray nodes simultaneously during a time interval. The spacing between two adjacent nodes is  $d$  meters. The vertical spacing of a transmission pair is  $ud$  meters, and the horizontal spacing of a transmission pair is  $vd$  meters. The optimal sparseness is  $(u, v) = (2, 3)$  for  $\alpha = 3, 4, 5$  and  $\epsilon = 1$ . For  $\epsilon \ll 1$ , the optimal sparseness is  $(u, v) = (1, 2)$ . Under the AAM scheme, each node transmits with probability  $p$  and receives with probability  $1-p$ .

the same time, which increases spectral reuse but also causes interference to each other. (If the spectrum is divided into sub-spectrums, the maximized throughput will be the same.) The network throughput is affected by both the sparseness and the distribution of nodes in each transmission subset. The transmission pairs at the center of the network are the most vulnerable to interferences. By assuming that the network is infinitely large, we can obtain the lower bound on the capacity of the transmission pairs at the network center. For this reason, we will assume that the network is infinite (although all equations to be shown can be easily specified for any finite network) and hence all transmission pairs are equivalent. The sparseness parameters  $u$  and  $v$ , as defined in Figure 4, are important under this scheme. We will find the optimal choice of  $u$  and  $v$  and the corresponding maximum value of the network throughput  $c_{\text{SAM}}$  in bits-hops/s/Hz/node under the SAM scheme.

For simple analysis, we will assume that each transmission pair is based on a single transmit antenna and a single receive antenna. But the optimal choice of  $u$  and  $v$  derived this way can be well applied to the case where each transmission pair uses multiple transmit/receive antennas. Also, the capacity of a network where each transmission pair is based on  $L$  transmit/receive antennas is expected to increase by the factor  $L$  although a more careful analysis is necessary to provide a more precise answer. Nonfading channels are assumed in the sequel.

The SINR at a receiving node can be expressed as

$$\begin{aligned} \text{SINR} &= \frac{P_T/d^\alpha}{\sigma^2 + \sum_i P_T/d_i^\alpha} \\ &= \frac{P_T/(\sigma^2 d^\alpha)}{1 + P_T/(\sigma^2 d^\alpha) \sum_i d^\alpha/d_i^\alpha} = \frac{1}{1/\text{SNR}_0 + \eta}, \end{aligned}$$

where  $P_T$  is the transmitted power from each transmitting node,  $d$  is the distance of a desired transmission pair,  $d_i$  is the distance from an interfering transmitter to the receiver,  $\text{SNR}_0 = P_T(\sigma^2 d^\alpha)^{-1}$  is a nominal signal to noise ratio,  $\eta = \sum_i d^\alpha/d_i^\alpha$  will be referred to as the interference factor that depends on the MAC scheme. The transmission subset and the sparseness of the transmission subset are both part of a MAC scheme. SINR is a monotonically increasing function of the transmission power  $P_T$ . Therefore, the network capacity under any given MAC scheme increases as  $P_T$  increases. But when  $P_T$  (or equivalently  $\text{SNR}_0$ ) is large enough, SINR is quickly saturated to its upper bound  $\text{SINR} = 1/\eta$  and hence the network capacity is maximized. In the sequel, this condition of large power will be assumed. We will also verify this assumption later.

Under SAM, the SINR at each receiving node is given by  $\text{SINR} = (1/\text{SNR}_0 + \eta_{\text{SAM}})^{-1}$ . With large  $P_T$ , we have  $\text{SINR} = 1/\eta_{\text{SAM}}$ . To complete the transmission of one data packet from each (and every) node in the network to its neighboring node in one direction, we need  $uv$  separate transmissions. The optimal spectral efficiency of each packet is governed by  $\log_2(1 + \text{SINR}) = \log_2(1 + 1/\eta_{\text{SAM}})$  in bits/s/Hz. Therefore, the network throughput in bits-hops/s/Hz/node is given by



$$c_{\text{SAM}} = \frac{1}{uv} \log_2 \left( 1 + \frac{1}{\eta_{\text{SAM}}} \right).$$

If the source-destination pairs on the square grid have long distances and are random, it can be shown that the (averaged) network throughput in bits-meters/s/Hz/node is  $(\pi/4)c_{\text{SAM}}/\sqrt{\rho}$  where  $\rho$  measures the node density [10]. We will assume that there are  $n$  nodes in every area  $A$ . Then, the throughput is also  $(\pi/4)c_{\text{SAM}}\sqrt{A/n}$  in bits-meters/s/Hz/node. In the sequel, we will mainly discuss  $c_{\text{SAM}}$ .

The expression of  $\eta_{\text{SAM}}$  depends on the distribution of the transmission subset as illustrated in Figure 4. The choice of the distribution is not unique. We can choose to make any two adjacent columns of transmission pairs to be maximally offset from each other (to maximize the distance between any two transmission pairs). Alternatively, we can choose to make any two adjacent transmission pairs along the line of sight of transmission to be maximally separated. But it turns out that both choices become the same when the sparseness parameters  $u$  and  $v$  are optimized. Based on the second distribution, the interference factor can be shown to be

$$\eta_{\text{SAM}} = \sum_{s=0}^1 \sum_{i=-\infty}^{\infty} \sum_{k=1}^{\infty} \left( \frac{\varepsilon_{u,i,k,s}^2}{\left( \sqrt{(ui + (-1)^s k)^2 + (vk + (-1)^s)^2} \right)^\alpha} \right) + 2 \sum_{i=1}^{\infty} \frac{\varepsilon^2}{\left( \sqrt{(ui)^2 + 1} \right)^\alpha},$$

where  $(ui + (-1)^s k, vk + (-1)^s)$  denotes the location of an interferer with respect to a receiver at  $(0, 0)$ . The terms associated with  $s = 0$  are due to the interferers from the (left) half plane opposite to the direction of data transmission. The terms associated with  $s = 1$  are due to the interferers from the (right) half plane in the direction of data transmission. The last terms are due to the interferers along the vertical column of the desired transmitter. Here, we also assume the use of directional antennas with  $\varepsilon$  ( $0 < \varepsilon \leq 1$ ) denoting the power gain of the directional antennas along a non-line-of-sight (NLOS). (In practice, the directional antennas on each node may consist of four sets of directional antennas pointing to four directions, and the switching between directions is electronically done.) The power gain becomes  $\varepsilon^2$  when neither the transmitter nor the receiver is pointing to each other in the line-of-sight. We also have

$$\varepsilon_{u,i,k,s}^2 = \begin{cases} 1, & ui + k = 0, s = 0 \\ \varepsilon^2, & \text{otherwise.} \end{cases}$$

If omnidirectional antennas are used, set  $\varepsilon = 1$ .

The above model for directional antennas is reasonable for the nodes within a short range from the receiver (see Figure 4). For a node far away from the receiver, a NLOS could be very close to the LOS. But their effect would be small. Other models of directional antennas should be considered in further research.

It can be shown that

$$\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{1}{(\sqrt{i^2 + j^2})^\alpha} < \infty$$

if and only if  $\alpha > 2$ . Then, it follows that  $\eta_{\text{SAM}} < \infty$  if and only if  $\alpha > 2$ . This means that even for an infinitely large network, the interference factor can be made small enough by choosing large enough  $u$  and  $v$  if and only if  $\alpha > 2$ . The condition  $\alpha > 2$  is always met unless in the free space. For most terrain environment,  $\alpha$  can be anywhere between two and five.

We can now maximize  $c_{\text{SAM}}$  over the choices of  $u$  and  $v$  for any given  $\alpha$  and  $\varepsilon$ . The results are shown in Table 2. We see that  $(u, v)_{\text{opt}} = (2, 3)$  remains optimal for  $\alpha = 3, 4, 5$  under  $\varepsilon = 1$  (omnidirectional antennas). It is somewhat surprising that  $(u, v)_{\text{opt}} = (1, 2)$  for  $\varepsilon = 0.1$  or lower. This means that with directional antennas, all transmission pairs should be located immediately next to each other (even along the line of sight) and every node in the network should be either a transmitter or a receiver at any time. Since  $\eta_{\text{SAM}} = 1/(2^{uc_{\text{SAM}}} - 1)$ , one can easily verify that this assumption  $1/\text{SNR}_0 + \eta_{\text{SAM}} \approx \eta_{\text{SAM}}$  is well satisfied by a moderate  $\text{SNR}_0$ . For example, when  $(u, v) = (2, 3)$ ,  $\varepsilon = 1$  and  $\alpha = 3$ , we have  $\eta_{\text{SAM}} = 0.6813$  and equivalently  $1/\eta_{\text{SAM}} \approx 1.7$  db. Also note that when  $(u, v) = (2, 3)$ ,  $\varepsilon = 1$  and  $\alpha = 3$ , the network throughput is  $(\pi/4) \times 0.2172\sqrt{A/n} = 0.1706\sqrt{A/n}$  in bits-meters/s/Hz/node, which is five times larger than  $0.033\sqrt{A/n}$  that is a lower bound of capacity from [8] using  $\text{SINR}_t = 1/0.6813$  and  $\alpha = 3$ .

The dominant interferers are always the transmitters that are the nearest to the receiver. If the network is finite, the receiver at the center of the network receives the worst interference. Table 3 shows how the throughput at the center of a network changes with the network size assuming  $\alpha = 3$  and  $(u, v) = (2, 3)$ .

Although it is highly likely that one can develop a new routing/MAC protocol to improve the  $c_{\text{opt}}$  value, it is not an obvious task and further research is needed. In the next section, we

**[TABLE 2] SHOWN HERE ARE THE OPTIMIZED NETWORK THROUGHPUT (IN BITS-HOPS/S/HZ/NODE) AND THE OPTIMAL SPARSNESS PARAMETERS  $U$  AND  $V$  UNDER THE SAM SCHEME. THE PARAMETER  $\alpha$  IS THE PATH LOSS EXPONENT, AND  $\varepsilon$  IS THE POWER GAIN OF DIRECTIONAL ANTENNAS ALONG NLOS. IN BIT-METERS/S/HZ/NODE, THE THROUGHPUT IS  $(\pi/4)c_{\text{SAM}}\sqrt{A/n}$  WHERE  $n/A$  MEASURES THE NODE DENSITY. A NETWORK OF  $201 \times 201$  NODES WAS CONSIDERED IN THE COMPUTATION.**

$c_{\text{SAM,opt}}, (u, v)_{\text{opt}}$	$\varepsilon = 1$	$\varepsilon = 0.1$	$\varepsilon = 0.01$
$\alpha = 3$	0.2184, (2,3)	1.3191, (1,2)	1.4539, (1,2)
$\alpha = 4$	0.4208, (2,3)	1.8772, (1,2)	2.1073, (1,2)
$\alpha = 5$	0.6210, (2,3)	2.3346, (1,2)	2.7388, (1,2)

**[TABLE 3] THE THROUGHPUT IN BITS-HOPS/S/HZ OF A NODE AT THE CENTER OF THE NETWORK VERSUS THE NETWORK SIZE ASSUMING  $\alpha = 3$  AND  $(u, v) = (2, 3)$ .**

NETWORK SIZE	$11 \times 11$	$21 \times 21$	$41 \times 41$	$81 \times 81$	$161 \times 161$
$c_{\text{SAM,opt}}$	0.2572	0.2370	0.2263	0.2213	0.2188

study the AAM to illustrate how the value of  $c_{\text{opt}}$  can be affected by different MAC protocols.

### ASYNCHRONOUS ARRAY METHOD

The AAM follows the same principle of the slotted ALOHA random access protocol originated by Abramson [12]. We will discuss it in two different cases: omnidirectional and directional antennas.

#### OMNIDIRECTIONAL ANTENNAS

The AAM is as follows: During each transmission cycle, each node randomly decides to be either a transmitter or a receiver. A node becomes a transmitter with probability  $p$  or a receiver with probability  $1 - p$ . If a transmitting node has a packet to transmit, it will transmit it to one of the nearest neighbors according to the final destination of the packet.

To study the maximum throughput under this scheme, we need to refine the scheme as follows: Each node has packets waiting for transmission in all directions. Based on its final destination, each packet in a node is given a label indicating its next-hop node (east, north, west or south). If a node receives a packet not intended for this node, the packet is dropped. When a node is in a receiving mode, it tries to decode a packet from each of the four directions using the same signal.

Under AAM, the probability that a node is in the receiving mode and its west neighbor is transmitting a packet to this node is  $(1/4)p(1 - p)$ . Then, the information in bits/s/Hz received by any node from its west neighbor is

$$\frac{1}{4}p(1 - p) \Pr(\eta_{\text{AAM}} \leq \eta_s) \log_2 \left( 1 + \frac{1}{\eta_s} \right)$$

where  $1/\eta_s$  is the reference SINR used for encoding each packet. Since a receiving node attempts to decode a packet from each of the four directions and all four directions have the same statistical property, the total information in bits/s/Hz received by each receiving node is

$$c_{\text{AAM}} = p(1 - p) \Pr(\eta_{\text{AAM}} \leq \eta_s) \log_2 \left( 1 + \frac{1}{\eta_s} \right).$$

This information also represents the throughput of a large network in bits-hops/s/Hz/node. Like SAM, the throughput of AAM in terms of bits-meters/s/Hz/node is simply  $(\pi/4)c_{\text{AAM}}\sqrt{A/n}$ .

**[TABLE 4] SHOWN HERE IS THE OPTIMIZED NETWORK THROUGHOUT IN BITS-HOPS/S/HZ/NODE BY USING AAM (SLOTTED ALOHA) ON A NETWORK OF  $201 \times 201$  NODES. ONE RECEIVING ANTENNA IS USED FOR  $\varepsilon = 1$ , AND FOUR (DIRECTIONAL) RECEIVING ANTENNAS ARE USED FOR  $\varepsilon = 0.1$  AND  $\varepsilon = 0.01$ .**

$c_{\text{AAM,opt}}, p_{\text{opt}}, \eta_{s,\text{opt}}$	$\varepsilon = 1$	$\varepsilon = 0.1$	$\varepsilon = 0.01$
$\alpha = 3$	0.0685, 0.184, 2.048	0.6194, 0.320, 0.072	0.7656, 0.500, 0.048
$\alpha = 4$	0.1091 0.130, 0.440	0.6892, 0.340, 0.040	0.8913, 0.510, 0.028
$\alpha = 5$	0.1486, 0.140, 0.270	0.7561, 0.400, 0.040	0.9198, 0.500, 0.016

The interference factor of AAM has the following form:

$$\eta_{\text{AAM}} = \sum_{(i,j) \in S} a_{i,j} x_{i,j} \text{ with } a_{i,j} = \frac{1}{(\sqrt{i^2 + j^2})^\alpha}.$$

The set  $S$  denotes all nodes except the receiver at  $(i, j) = (0, 0)$  and the transmitter at  $(i, j) = (-1, 0)$  without loss of generality. And  $x_{i,j}$  is a binary (interference) random variable, i.e.,  $x_{i,j} = 1$  indicates that the node at  $(i, j)$  is transmitting, and  $x_{i,j} = 0$  indicates that the node at  $(i, j)$  is receiving. Then,  $\Pr(x_{i,j} = 1) = p$ , and  $\Pr(x_{i,j} = 0) = 1 - p$ . Unlike  $\eta_{\text{SAM}}$ ,  $\eta_{\text{AAM}}$  is random.

But unlike  $c_{\text{SAM}}$  that depends on  $u$  and  $v$ ,  $c_{\text{AAM}}$  depends on  $p$  and  $\eta_s$ . We now need to find a more explicit expression of  $\Pr(\eta_{\text{AAM}} \leq \eta_s)$  in terms of  $p$  and  $\eta_s$ . Note that  $\eta_{\text{AAM}}$  is a weighted sum of independent, identically distributed, binary random variables  $a_{i,j}x_{i,j}$ . Hence, the probability density function (PDF) of  $\eta_{\text{AAM}}$  is  $f_{\eta_{\text{AAM}}}(x) = \otimes_{(i,j) \in S} f_{\eta_{i,j}}(x)$  where  $\otimes$  denotes a series of convolutions and  $f_{\eta_{i,j}}(x) = (1 - p)\delta(x) + p\delta(x - a_{i,j})$ . But the continuous-variable delta functions cannot be directly stored or computed by computers. We need an alternative form of  $f_{\eta_{\text{AAM}}}(x)$ . We know that

$$\sum_{(i,j) \in S} a_{i,j} < \infty$$

if and only if  $\alpha > 2$ . Then, there is a finite value  $T$  such that

$$\sum_{(i,j) \in S} a_{i,j} < T$$

if  $\alpha > 2$ , and hence  $f_{\eta_{\text{AAM}}}(x)$  is nonzero only when  $0 \leq x \leq T$ . So, we can apply the Fourier series expansion of  $f_{\eta_{\text{AAM}}}(x)$  as follows

$$f_{\eta_{\text{AAM}}}(x) = \sum_{k=-\infty}^{\infty} g_k \exp\left(j \frac{2\pi k}{T} x\right),$$

where

$$g_k = \frac{1}{T} \prod_{(i,j) \in S} \left( 1 - p + p \exp\left(-j \frac{2\pi k}{T} a_{i,j}\right) \right).$$

The Fourier series coefficient  $g_k$  should become negligible when  $|k|$  is large. Then, it follows that

$$\begin{aligned} \Pr(\eta_{\text{AAM}} \leq \eta_s) &= \int_0^{\eta_s} f_{\eta_{\text{AAM}}}(x) dx \\ &= \sum_{k=-\infty}^{\infty} g_k \frac{T}{j2\pi k} \left( \exp\left(j \frac{2\pi k}{T} \eta_s\right) - 1 \right). \end{aligned}$$

This is a desired form of  $\Pr(\eta_{\text{AAM}} \leq \eta_s)$  in terms of  $p$  and  $\eta_s$ . We can now maximize  $c_{\text{AAM}}$  over  $\eta_s$  and  $p$ . The result is shown in Table 4 under  $\varepsilon = 1$ .

It is interesting to know that the optimal network throughput is achieved when coding at each node is done under such a low target SINR:  $1/\eta_s \approx -3$  dB, which is lower than that for SAM. At this target SINR, the maximal information in each packet can only be 0.57 b/s/Hz. This is in contrast to a conventional practice of 1 b/s/Hz, e.g., see [8] and [13].

## DIRECTIONAL ANTENNAS

With directional antennas, we need to add the following refinement: When a node is in its transmitting mode, it randomly steers its transmit beam towards one of the four directions. A transmitting node only transmits a packet intended for the direction to which its beam is steered. When a node is in its receiving mode, it uses four directional antennas to receive signals from each of the four directions. The signals from all four directions are separately used to detect packets intended for this receiving node.

The AAM scheme is slightly more costly than the SAM since four directional receiving antennas of each receiving node are used concurrently. If only one directional receiving antenna is allowed at each receiving node, the network throughput would be reduced by the factor 4 (assuming that  $\varepsilon$  is small).

A similar analysis yields the same general expression of the network throughput  $c_{\text{AAM}} = p(1-p) \Pr(\eta_{\text{AAM}} \leq \eta_s) \log_2(1 + \eta_s)$  as obtained previously for the omnidirectional case. However, the expression of the interference factor  $\eta_{\text{AAM}}$  is now different:

$$\eta_{\text{AAM}} = \sum_{(i,j) \in S_k, k=\{1,2,3,4,5\}} a_{i,j} x_{k,i,j}$$

where  $S_1 = \{i \leq -2, j = 0\}$ ,  $S_2 = \{i \geq 1, j = 0\}$ ,  $S_3 = \{i = 0, j \geq 1\}$ ,  $S_4 = \{i = 0, j \leq -1\}$ , and  $S_5 = \{i \neq 0, j \neq 0\}$ . Furthermore,

$$x_{1,i,j} = \begin{cases} 1 & \Pr = p/4 \\ \varepsilon & \Pr = 3p/4 \\ 0 & \Pr = 1-p \end{cases}; \quad x_{2,i,j} = \begin{cases} \varepsilon & \Pr = p/4 \\ \varepsilon^2 & \Pr = 3p/4 \\ 0 & \Pr = 1-p \end{cases};$$

$$x_{5,i,j} = \begin{cases} \varepsilon^2 & \Pr = p \\ 0 & \Pr = 1-p \end{cases}$$

and  $x_{3,i,j}$  and  $x_{4,i,j}$  have the same probability distribution as  $x_{2,i,j}$ . The PDF of  $\eta_{\text{AAM}}$  is then

$$f_{\eta_{\text{AAM}}}(x) = \otimes_{(i,j) \in S_k, k=\{1,2,3,4,5\}} f_{\eta_{k,i,j}}(x),$$

where

$$f_{\eta_{1,i,j}}(x) = (1-p)\delta(x) + \frac{3p}{4}\delta(x - \varepsilon a_{i,j}) + \frac{p}{4}\delta(x - a_{i,j})$$

$$f_{\eta_{2,i,j}}(x) = f_{\eta_{3,i,j}}(x) = f_{\eta_{4,i,j}}(x) = (1-p)\delta(x) + \frac{3p}{4}\delta(x - \varepsilon^2 a_{i,j}) + \frac{p}{4}\delta(x - \varepsilon a_{i,j}),$$

and  $f_{\eta_{5,i,j}}(x) = (1-p)\delta(x) + p\delta(x - \varepsilon^2 a_{i,j})$ . Equivalently, by using the Fourier series expansion, we have

$$f_{\eta_{\text{AAM}}}(x) = \sum_{k=-\infty}^{\infty} g_k \exp\left(j \frac{2\pi k}{T} x\right),$$

where

$$g_k = \frac{1}{T} \left[ \prod_{(i,j) \in S_1} \left(1 - p + \frac{3p}{4} z_{k,i,j}^\varepsilon + \frac{p}{4} z_{k,i,j}\right) \right] \\ \times \left[ \prod_{(i,j) \in S_2 \cup S_3 \cup S_4} \left(1 - p + \frac{3p}{4} z_{k,i,j}^{\varepsilon^2} + \frac{p}{4} z_{k,i,j}^\varepsilon\right) \right] \\ \times \left[ \prod_{(i,j) \in S_5} \left(1 - p + p z_{k,i,j}^{\varepsilon^2}\right) \right]$$

and

$$z_{k,i,j} = \exp\left(-j \frac{2\pi k}{T} a_{i,j}\right).$$

When  $\varepsilon = 1$ , the above expressions reduce to the same expression for the omnidirectional case. The values of the optimized  $c_{\text{AAM}}$  for three choices of  $\varepsilon$  are shown in Table 4. Like  $c_{\text{SAM}}$ ,  $c_{\text{AAM}}$  increases when  $\alpha$  increases and/or  $\varepsilon$  decreases.

Shown in Table 5 are ratios of the SAM throughput over the AAM throughput. The SAM is two to four times better than the AAM.

## CONCLUSIONS

Large ad hoc networks are important for military as well as civilian applications. There are many fundamental research issues spanning the fields of signal processing, information theory, and networking protocol. We have reviewed the capacity scaling laws of large ad hoc networks. We have also presented a closed-loop analysis of signal to interference and noise ratio, packet spectral efficiency, and MAC schemes to yield a useful insight into maximizing the throughput of a large ad hoc network of known topology. There are many more research issues yet to be investigated: What is the best MAC scheme for a given topology? How does fading affect the best MAC scheme? Is the concept of multiuser diversity and/or fading channel diversity important for a large ad hoc network? What is the best MAC scheme for a large network of MIMO links? Is network coding useful for a large ad hoc network? How should the channel state information be estimated and utilized in a large ad hoc network? How should a MAC scheme be made adaptive to desired source-destination rates in a large ad hoc network? To answer these questions, interplays between signal processing, information theory and networking protocols seem essential.

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**[TABLE 5] THROUGHPUT RATIO OF THE SAM SCHEME OVER THE AAM (SLOTTED ALOHA) SCHEME. THE SAM USES ONE RECEIVING ANTENNA FOR ALL  $\varepsilon$ . THE AAM USES FOUR (DIRECTIONAL) RECEIVING ANTENNAS FOR  $\varepsilon = 0.1$  AND  $\varepsilon = 0.01$ .**

$c_{\text{SAM,opt}}/c_{\text{AAM,opt}}$	$\varepsilon = 1$	$\varepsilon = 0.1$	$\varepsilon = 0.01$
$\alpha = 3$	3.188	2.130	1.899
$\alpha = 4$	3.857	2.724	2.364
$\alpha = 5$	4.179	3.088	2.978

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