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GEOMETRICAL CONSTRAINTS IN QUANTUM MECHANICS AND COSMOLOGY*

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In an earlier note (1), we examined the manner in which a set of geometrical constraints can be used to give closed cosmological solutions to Einstein's field equations. These constraints are expressed as a set of quantities or physical variables which are defined uniquely in terms of universal constants (1, 2, 3) and are termed quantized variables (1).

We also presented a set of canonically conjugate relations of these variables (4). It was demonstrated that the quantized variables have operator representations (5), and a generalized Schrödinger equation was developed.

Also introduced in ref. (5), is a multidimensional space called the Descartes space, in terms of which we can formulate a set of geometrical constraints. The dimensions of this multidimensional geometry was given in terms of invariant unit dimensions, called the quantal units (1). The metric for the multidimensional Descartes space is termed the generalized Minkowski metric (6).

In this note, we demonstrate the manner in which the quantized variable geometrical constraints act in the "domain" of quantum mechanics and in that of relativity, and the relationship of these two formalisms. What is meant by "quanta" is also discussed.

Some comments and references are made about relativistic quantum field theories, such as the Dirac theory, and the second quantization formulation of our theory. Also some of the recent "quantum gravitational" theories are discussed in the context of our formalism. The quantal units relevant here are: Planck's length (2) or the Wheeler "wormhole" length (3), $\ell = (G\hbar/c^3)^{1/2}$; time, $t = (G\hbar/c^5)^{1/2}$; momentum, $p = (c^3\hbar/G)^{1/2}$; and energy, $E = (c^5\hbar/G)^{1/2}$. In these equations, $\hbar$, G and c denote Planck's constant, the universal gravitational constant, and the velocity of light, respectively. Other quantal units are given in Table I of ref. (6).

These quantities can represent dynamical quantities or variables as well as quantal unit "measuring rods," and in the form of variables, are termed quantized variables. That is, each quantal unit has an associated quantized variable (4, 5).

In relation to our terminology, quantal units or quantized variables we define two distinct quantization procedures, primary in terms of the quantized variables and secondary, or the usual quantization procedure (4). The essential distinguishing characteristic between these two procedures is "scale" (7, 8); but in both procedures there exists a set of canonically conjugate variables, termed the generalized Heisenberg relations. It can be shown that, in fact, these two procedures give equivalent results (4). In refs. (4 and 8) quantized variables are used to develop new Heisenberg relations.

It is also possible to form a set of pair relations as relativistic invariant "four-vectors" (6). We will develop the relationship between the canonically conjugate formalism of the quantized variables and the set of invariant relations that make up the generalized Minkowski metric.

The set of canonically conjugate relations, are shown in fig. 1. We have the two usual relations, $(x, p) \geq \hbar$ and $(E, t) \geq \hbar$, and four new relations (9). We shall denote possible pair representations as the generalized pair $(\rho_{i\epsilon}, \nu_{i\epsilon})$, where the index i runs 1 to 3 for vector variables and i = 1 for scalar variables. In fig. 1, we consider only one component of each vectors; for example, $x = x_1 = x_2 = x_3$.

In this notation, for the pair $(\rho_\epsilon, \nu_\epsilon)$, if $\epsilon = 1$, have the pair $(x, p) \geq \hbar$; and for $\epsilon = 2$, we have the pair $(t, E) \geq \hbar$. The four new relations (4) are for $\epsilon = 3$, $(x, E) \geq c\hbar$; $\epsilon = 4$, $(p, t) \geq \hbar/c$; $\epsilon = 5$, $(x, t) \geq \hbar/F$; and $\epsilon = 6$, $(p, E) \geq \hbar F$, where F is the quantal force, $F = c^4/G$. Extensive literature exists which discusses the interpretation of the scalar Heisenberg relation, $(E, t) \geq \hbar$. Time operators have been presented by several authors (5, 10, 11). J. H. Eberly and L. P. S. Singh (7) develop an unambiguous and non-singular statement of the energy-time uncertainty relation. We develop a time operator (as well as a space operator), in conjunction with the development of the generalized Schrödinger equation (5), which V. S. Olkhousky and E. Recami also discuss (12). Eberly and Singh use the density matrix formalism to develop time operators and their uncertainty principle with Hamiltonian operators.

We can form an invariant "line element," in terms of a universal constant or combinations of universal constants, between any two variables, $\mu_{i\kappa}$ and $\eta_{i\kappa}$, where again the index i runs 1 to 3 for vector variables and i = 1 for scalar variables and the index κ runs 1 to n (n is defined as the dimensionality of the Descartes space). Considering one component of vector quantities, for example $x = x_1$ and $p = p_1$, we can define a Descartes four-space as $\{x_\kappa\} = \{x, t, p, E\}$. For example, we have the usual invariant relation

$$s_1^2 = x^2 - c^2t^2 \tag{1}$$

for one component of x; for an isotropic subspace $x = x_1 = x_2 = x_3$ (1) and for metrical signature (+,+,+, —). There are six invariant "line-elements" for a Descartes four-space (6).

We will define a generalized four-vector invariant. The usual definition of a four-vector, for invariance relations, is in terms of a spatial vector quantity and a temporal scalar quantity which form an invariant variable pair relation in terms of the invariance of the universal constant, c. The usual cases are eq. (1) and,

$$s_2^2 = p^2 - \frac{1}{c^2} E^2 . \tag{2}$$

Again we consider one-component of the momentum vector only: $p = p_1 = p_2 = p_3$. We show the six invariant relations for a Descartes four-space in table I. We have the usual relations for $\delta = 1$ and $\delta = 2$ and for $\delta = 3$ we have one of the new relations in terms of the invariance of F,

$$s_{\delta=3}^2 = x^2 - \frac{1}{F^2} E^2 . \tag{3}$$

The quantal force is uniquely expressed in terms of the universal constants c and G as $F = c^4/G$ (1); thus the invariance of the expression in eq. (3) is dependent on the invariance of the universal constants c and G.

For $\delta = 5$ in table I, we have a six-subspace for the (x, p) variable pair and for $\delta = 6$, we have a two-subspace for the (t, E) variable pair. Using the one component forms, as in eq. (1) and (2), each subspace is then a two space.

The generalized invariant four-vector (or six-or two-vector) can be formed in terms of any two variables in terms of the invariance of any of the universal constants, c, G and $\hbar$ or combinations of them. Using one-component of vector quantities then the set of generalized invariant relations are generalized two-sub-spaces.

A generalized invariant expression for a multidimensional Descartes space is given in ref. (6), both for the four-space and for a ten-space in terms of one-component vector and scalar coordinates, $\{x_\kappa\} = \{x, t, p, E, m, F, c, a, P, L\}$, where m is mass, a is acceleration, P is power and L is angular momentum, other quantities are defined previously. In ref. (8) a higher order Descartes space, of as many as thirty dimensions is presented which includes electromagnetic and thermodynamic coordinates (13, 14).

The generalized form of an invariant-variable pair is

$$s_\delta^2 = \mu_{i\kappa}^2 + m_{\lambda\kappa}\,\eta_{i\kappa}^2 \tag{4}$$

for any two variables $\mu_{i\kappa}$ and $\eta_{i\kappa}$, where λ and κ run from 1 to n (the dimensionality of the Descartes space being considered) and the index δ runs 1 to I (I is the number of invariant relations for a Descartes space of n dimensions). As before i runs 1 to 3 for vectors and i = 1 for scalars. The invariance relation between any two variables is expressed in terms of the metrical elements $m_{\lambda\kappa}$ which are expressed in terms of universal constants or combinations of universal constants. The constant elements from a non-diagonal matrix termed the generalized Minkowski metric, M (6).

A generalized invariant, for all n dimensions of the space, can be expressed in terms of the diagonal form of the Minkowski metric. The diagonal form of M is an analytic expression but is not a simple, easily usable form (15). In forming the invariants for a particular Descartes space, of dimensionality n , the maximum number of invariants that can be formed is given by the following Lemma:

Given a Descartes space of dimensionality n , the number of invariants that can be formed for that space is equal to the number of invariants of the Descartes space of one less dimensionality (n - 1), plus the number of dimensions of the n - 1 dimensional space.

Let I_n be the maximum number of invariants for a Descartes space of n dimensions and let $I_{n'}$ be the maximum number of invariants for a Descartes space of $n' \equiv n - 1$ dimensions, the

$$I_n = I_{n'} + n' . \tag{5}$$

A rigorous proof of this new Lemma is given in ref. (8).

In developing the generalized Heisenberg relations and generalized invariance in a multidimensional geometry in which each dimensioned physical variable is considered on an equal footing, we see that physical variables can be paired in uncertainty relations, such as those in fig. 1. Also paired variables can form invariant relations in terms of universal constants, as in table I. The relationship between these two pair relationships, as given in more detail in refs. (4) and (6), is presented in table II. For example the index $\epsilon = 1$ to denote the Heisenberg pair (x, p); the index $\delta = 5$, denotes the same pair (x, p) in an invariant relation. In the notation in table II, $\epsilon = \delta = 3$ and $\epsilon = \delta = 4$ both denote the same variable pair related in a Heisenberg relation and as a relativistic invariant. In this manner and with the assumption of "equal footing" of physical variables, we see a way in which quantum mechanics and relativitistic invariance can be tied together by a geometrical model of the manifold.

Note that all invariant expressions in this note are special relativistic invariants. General relativistic invariants are discussed in ref. (6) as are some light cone relations for the generalized special relativistic invariants. In ref. (1), the quantized variable geometrical constraints are applied in general relativity and closed cosmological solutions are found for Einstein's field equations. Experimental evidence for closed cosmologies are sited in refs. (1 and 8).

Unifying quantum mechanics and general relativity is one thing, it is quite another to demonstrate the " common roots" of the formalism. A relativistic quantum field theory, or a " quantized" gravitational theory, may relate to an aspect of a unifying theory; but it cannot by itself be considered a fundamental unifying theory. However, quantized gravitional theories may shed some light on the path toward a fundamental relation between quantum and relativistic physics. For example, the problems of reconciling non-linearities in general relativistic fields and the superposition principle in quantum mechanics are well pointed out by some of the workers in quantized gravitational theories. For example B. S. De Witt (17, 18) in an extensive treatise, discuss some of the difficulties in " synthesizing" nonlinearities of the metric tensor and the quantum superposition principle. Attempts have been made to develop a linear theory of the massless, spin 2 field (19). Also, some of the tie-in to the " roots" of the canonical formalism is fundamental to a unifying of quantum mechanics and relativity (8).

Since both quantum mechanics (primarily relating to micro-phenomona) and relativity (relating to macro-phenomona) are so successful in elegantly describing physical phenomona, a unifying aspects for these fields of physics must necessarily involve the interrelation of diverse aspects of reality.

It also seems apparent that since every aspect of reality depends on every other aspect, as expressed clearly by H. Stapp (20): "Every part of the universe depends on every other part," the bootstrap model of elementary particles developed by G. Chew (21) is another statement of this proposition, in elementary particle physics.

A unified theory of reality which would intimately bring together quantum and relativistic phenomona must necessarily be complete. The concept of completeness in quantum theory is discussed in the "classical" paper by A. Einstein, B. Podolski and N. Rosen (22) and subsequent papers inspired by this work.

The quantal units as dimensions, which are expressable uniquely in terms of universal constants, are manifest in quantum mechanics and special relativistic invariants and general relativity, as well as thermodynamics (8), and electromagnetic theory (8, 14). According to B. N. Taylor, W. H. Parker, and D. W. Langenberg (23), the "universal constants are an important link in the chain of physical theory which binds all the diverse branches of physics together."

In 1921, Einstein discussed a fundamental aspect of reality: "It was formerly believed that if all material things disappeared out of the universe, time and space would be left, according to the relativity theory, however, time and space disappear together with the things" (24).

In a Descartes space, should any one-dimensional physics variable (space or time or matter or energy or velocity) disappear, then all the rest would also disappear.

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FOOTNOTES AND REFERENCES

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[where E is the quantal energy $E = (c^5\hbar/G)^{1/2}$] and the quantal entropy, S = k [where k is the Boltzmann constant], to the constraints for the early universe is given in this reference. Also, E. R. Harrison, Rev. Mod. Phys.39, 862 (1967) discusses a similar constraint system in the early universe.

Geometrical constraints, expressed in quantal unit form, hold for the early universe and throughout its dynamic evolution (1,8) the quantal temperature and entropy are considered as dimensions for a multi-dimensional Descartes space.

14. E. A. RAUSCHER, LBL-2150 November 1973, Electromagnetic quantal units are utilized in phenomona related to when the electromagnetic field predominates. Some new electromagnets equations are devloped. Some are compared equivalent to those in N. D. Sen Gupta's Phys. Rev. 8D, 1940 (1973).

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Invariant, S_k	Variable pair μ_k,	Component of the Minkowski metric
$S_\delta^2 = 1$	$\mu_1 = x, \eta_1 = t$	c^2
$S_\delta^2 = 2$	$\mu_2 = p, \eta_2 = E$	$c^{\frac{1}{2}}$
$S_\delta^2 = 3$	$\mu_3 = x, \eta_3 = E$	$\frac{1}{F^2}$
$S_\delta^2 = 4$	$\mu_4 = p, \eta_4 = t$	F^2
$S_\delta^2 = 5$	$\mu_5 = x, \eta_5 = p$	$\frac{c^2}{F^2}$
$S_\delta^2 = 6$	$\mu_6 = t, \eta_6 = E$	c^2F^2

Table I. Pair-variable relations as invariants in terms of the elements of the generalized Minkowski metric. Six such invariant pair relations can be formed for a four-dimensional Descartes space.

Generalized Heisenberg Pairs	Generalized invariant pair variables	
$\epsilon = 1$, (x, p)	$\delta = 1$, $s_\delta = 1$	(x, t)
$\epsilon = 2$, (t, E)	$\delta = 2$, $s_\delta = 2$	(p, E)
$\epsilon = 3$ (x, E)	$\delta = 3$, $s_\delta = 3$	(x, E)
$\epsilon = 4$, (p, t)	$\delta = 4$, $s_\delta = 4$	(p, t)
$\epsilon = 5$, (x, t)	$\delta = 5$, $s_\delta = 5$	(x, p)
$\epsilon = 6$, (p, E)	$\delta = 6$, $s_\delta = 6$	(t, E)

Table II. We can represent the relation between a pair of physical variables in two different formalisms. We have the uncertainty relation between two variables $(p_\epsilon, \nu_\epsilon)$ where the index ϵ denotes a particular variable pair. We can also represent an invariant relation between two variables as $(\mu_\kappa, \eta_\kappa)$ and the index δ denotes a particular invariant relation. For $\epsilon = 1$ and 2 we have the usual quantum mechanical relation and for $\delta = 1$ and 2 we have the usual invariant four-vector relations. Table II depicts the manner in which these two representations of paired variables relate to each other.

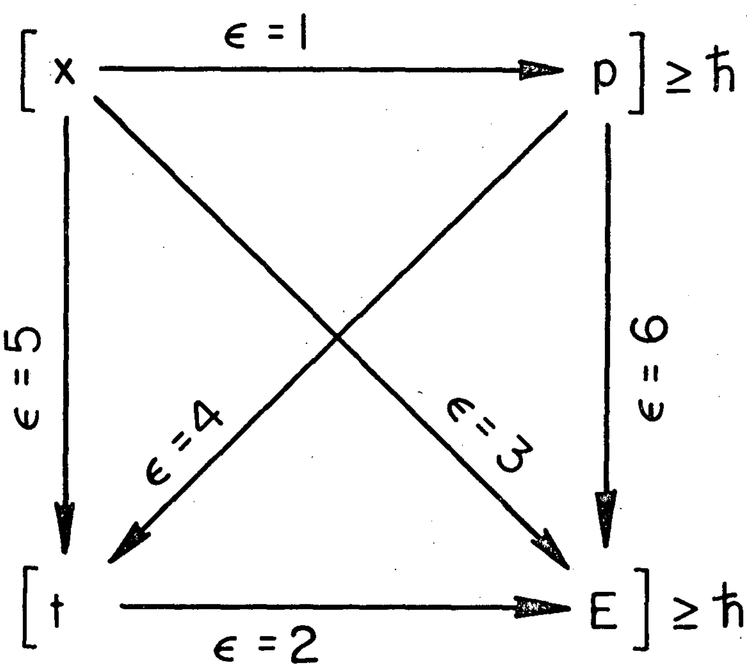

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Fig. 1. Pair variable relations represented schematically as the generalized Heisenberg Relations, where ϵ denotes a particular variable pair; for example, $\epsilon = 4$ denotes the pair $(\rho_{\epsilon = 4}, \nu_{\epsilon = 4}) = (p, t) \geqslant \hbar/c$.

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