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ABSTRACT

It is argued that the pure bootstrap idea is physically incompatible with the closure property of conventional quantum theory; a physical meaning for locality would then be precluded in a bootstrap regime.

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The "nuclear democracy" hypothesis, proposing all hadrons as equivalently composite, is gaining wide acceptance, but one cannot be sure that democracy necessarily requires a true bootstrap dynamics. It is often conjectured that beneath nuclear matter lies a basic field, unconnected with special hadrons and obeying a simple "master equation" of motion. Lagrangian models do indeed indicate a tendency for hadrons sharing the quantum numbers of such a master field to exhibit "aristocratic" properties, such as not lying on a Regge trajectory, but no general demonstration ever has been given that a master equation precludes nuclear democracy. A master field would be inaccessible to direct measurement, so one might despair of ever verifying its actuality, but Gell-Mann¹ (while eschewing explicit reference to a master equation) has observed that an underlying local structure for strong interactions should manifest itself through corresponding commutation relations for physically-measurable currents. Adler,² furthermore, has proposed sum rules for observable cross sections to test Gell-Mann's commutation relations. These sum rules, a special case of a general category formulated by Fubini, Furlan and Rossetti,³ depend on the "closure" aspect of conventional quantum theory, i.e., the existence of a "physical Hilbert space" of states with respect to which the product of two physically meaningful operators may be defined. In pure S-matrix theory, on the other hand, there are no operators besides the scattering matrix itself and correspondingly no general concept of closure. One sees therein no necessity for sum rules of the Fubini-Adler type. In

this note we discuss the possibility that such sum rules may allow confrontation between an underlying local space-time structure for strong interactions and a true bootstrap. The pure bootstrap idea, we suggest, may be incompatible with closure.

To avoid confusion, it must immediately be recognized that S-matrix unitarity implies "on-shell" closure within sub-sets of states sharing a common value of energy-momentum. Commutation relations for conserved quantities like total charge (or isospin) are correspondingly not being called into question. These relations do not involve the full Hilbert space of conventional quantum theory and amount to no more than a statement of the symmetry underlying the conservation law. Commutation relations for charge or current densities, however, require that "off-shell" closure have a meaning. It is only sum rules based on off-shell closure that are being questioned here.

Our basic argument is that conventional quantum-mechanical closure of the full physical Hilbert space implies a denumerable set of degrees of freedom for the dynamical system. We here use the term "degrees of freedom" to describe a "minimum" set of commuting operators through repeated applications of which the entire Hilbert space may be spanned. The degrees of freedom may be infinite and may be continuous, but we believe closure to require a sense in which a minimal set can be identified. A fundamental field (or set of fields) provides such a sense; the fact that the manifestations of such a field may include an infinite number of composite particles should not be confused with the denumerability of its degrees of freedom, a characteristic of any

system described by conventional quantum mechanics. Conversely, we believe such denumerability to be physically inseparable from the general concept of closure.

We recognize the old-fashioned character of the above viewpoint and are aware that modern approaches to axiomatic field theory avoid mentioning the "degrees of freedom" concept, but without it the notion of closure (or completeness) of the physical Hilbert space seems to us unnatural. That is to say, we associate the "size" of the physical Hilbert space with the number of underlying degrees of freedom. If the number of degrees of freedom is increased, the "size" of the space increases and conversely. In this way of thinking, the notion of "spanning the space" is tied to the "size" of the space and thus to the number of degrees of freedom.

Now the essence of the pure bootstrap idea is to abandon meaning for "dynamical degrees of freedom." This point tends to be overlooked because of the evident freedom, bootstrap or no, enjoyed by the experimenter in setting up initial configurations of stable particles. It is far from obvious, however, that the experimenter necessarily possesses the capacity to set up a "quantum state" in the sense of Dirac, where a meaning is attached to subsequent state-evolution from one instant of time to another. The capabilities of physical measurement require meaning only for free-particle configurations, and it begs the question to assume that the flexibility available to a nuclear experimenter corresponds to the "degrees of freedom" of a conventional quantum system.

Why do we assert incompatibility between the "degrees of freedom" concept and the bootstrap? It is because the bootstrap hypothesis postulates a dynamical determination of the experimenter's alternatives. For example, one can imagine the sole consistent solution of the bootstrap conditions to yield a single stable hadron of zero spin. There would then be no freedom to vary such quantities as baryon number, isotopic spin or strangeness. In the real world there happen to be several different stable hadrons, and conservation laws giving a meaning to B , I and S ; but if the very existence of these quantities is a consequence of dynamics, they are not "degrees of freedom." To generalize: the experimenter has a certain choice of initial particles, but in a bootstrap regime the set from which he makes his choice represents no freedom at all. We see no conceivable sense in which an "initial state" could be specified without foreknowledge of the solution to the bootstrap conditions.

Because the "degrees of freedom" notion never rears its head within the S-matrix framework, this structure is suitable to formulation of the bootstrap. Confusion, indeed, arises when unitarity sums are truncated in approximate S-matrix calculations; certain particles are thereby in effect given an "elementary" status and provide a basis for "counting." As shown below in an example, this phenomenon associated with approximation has important utility in special situations where unusual mass ratios occur. In such special situations one may speak of "approximate closure" and correspondingly of "approximate locality," the goodness of the approximation being governed by the mass ratios;

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but no basis is provided by S-matrix theory for any general notion of closure.

It is often argued that analytic S-matrix theory⁴ must be equivalent to local field theory because analyticity is unmotivated except in terms of locality. It is true that historically the idea of an analytic S-matrix arose from local field theory, but no demonstration has been given that local fields may be constructed from analytic S-matrix elements.⁵ Our suggestion here is that if the pure bootstrap idea is correct, no such construction will be possible. To summarize our position: Locality requires a meaning for products of fields, which in turn requires off-shell closure and thus a countable set of degrees of freedom. The essence of the bootstrap notion in foregoing any master equation is that there are no degrees of freedom! More accurately, the "degrees of freedom" concept is inadmissible.

The author has found the above characterization of the bootstrap to be greeted by two types of disbelieving reactions. The first is that by foregoing a physical Hilbert space established principles of quantum mechanics are abandoned. Certain principles indeed are abandoned, but no experimental successes of quantum theory are undermined. The S matrix permits (in fact, requires) approximate particle localization,⁶ on a scale larger than the particle Compton wavelength, and the description of long-range interactions by conventional quantum theory remains unquestioned. It seems possible, in particular, to construct an approximately-local nonrelativistic wave function satisfying a Schrödinger equation, starting from an analytic S matrix. This

important circumstance has apparently not yet been systematically explored, but the work of Charap and Fubini and of Martin and Targonski⁷ shows how the Mandelstam representation for a two-particle scattering amplitude permits the definition of a function which becomes asymptotically identifiable with an ordinary wave function as the particle masses become heavier and heavier (with momentum-transfer singularities held fixed). This possibility has confused the above-discussed question concerning the construction of local fields. The crucial distinction is that particle localization down to the Compton wavelength forms an adequate basis for nonrelativistic quantum theory but not for a more general quantum theory which conforms to the principles of special relativity.

A borderline situation arises in the use of conventional quantum theory for "classical" nuclear physics. The key here lies in the exceptionally small mass of the pion, whose Compton wavelength provides the scale of spatial dimensions for the nonrelativistic physics of hadrons with baryon number greater than zero. All such hadrons have small Compton wavelengths on this scale and thus can be approximately localized.⁸

The second adverse reaction to our position argues that since S-matrix theory employs superposition of different energy-momentum values, and since unitarity implies a Hilbert space for each individual value thereof, it is inevitable that there should exist a complete Hilbert space.⁹ This argument, however, only shows that a formal Hilbert space can be constructed. Since the S matrix lacks elements connecting

different values of total energy-momentum, there is no compelling reason to expect physical usefulness of this formal space for defining off-shell operator products.

To see the connection between approximate localization and "closure," consider the deuteron. If the unitarity condition for strangeness 0 and baryon-number 2 is truncated so as to include only the two-nucleon channel, and if nonrelativistic kinematics is employed, it has been shown that to order m_π^2/m_N^2 properties of the analytic S matrix allow the construction of a local two-nucleon wave function satisfying a Schrödinger equation.⁷ There is then a corresponding closure relation which allows the derivation of sum rules. For example the total cross section for the scattering of "low energy" electrons by a deuteron, summed over both elastic and inelastic final states, can be shown to approach the cross section of a free point-charge proton.¹⁰ This evidently is only an approximate sum rule because, among other considerations, the proton is only approximately a point charge. If the electron energy is increased sufficiently, the sum rule becomes inaccurate for large momentum transfers. The truncation of the unitarity sum is now manifestly invalid (reflecting, e.g., the fact that the proton has structure) and the justification for the two-nucleon wave function disappears. If there appeared another broad energy region in which a definite number of channels were effective, one might conceivably define an appropriate multi-channel multi-particle wave function and derive a corresponding approximate sum rule. The accuracy of this

rule would, however, deteriorate when further channels became important.

The Fubini-Adler sum rules based on the Gell-Mann commutation relations are supposed to have a more general basis than the above-described approximate sum rule, but as derived, we believe them to rest on the denumerability of the underlying degrees of freedom. It is possible that a careful study of Adler's derivation will reveal loopholes which permit violation of his sum rules without abandonment of locality. Accurate experimental verification of the sum rules, however, would in our opinion constitute a severe blow to the original bootstrap concept.

A final remark may head-off confusion about this paper's message, which is to underline the conflict between a "minimal" set of operators and the bootstrap. It is of course always possible to define "off-shell" operators connecting states of different momentum-energy in the formal Hilbert space. Certain operators of this kind, such as the charge density, may even have direct physical significance. Nevertheless, we suggest that the locality of any operator has meaning only if a "minimal" set of operators exists.¹¹ The bootstrap notion, by foregoing such a minimal set, may preclude locality.

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FOOTNOTES AND REFERENCES

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