UC Santa Barbara
Ted Bergstrom Papers

Title
Interrelated Consumer Preference and Voluntary Exchange

Permalink
https://escholarship.org/uc/item/1rv2c08k

Author
Bergstrom, Ted

Publication Date
1971-03-01

Peer reviewed

eScholarship.org Powered by the California Digital Library
University of California

INTERRELATED CONSUMER PREFERENCE

AND VOLUNTARY EXCHANGE

by

Theodore C. Bergstrom*
Washington University

It is traditional in the theory of economic choice to assume that an individual economic agent chooses his economic activities in such a way to maximize a preference ordering which depends only on his own consumption. This assumption, while an immensely useful simplification, does cause some embarrassment when economists desire to explain such phenomena as gifts and inheritances, compulsory schooling, forced savings plans such as Social Security, and even the often discussed notion of a "household".

The first section of this paper suggests some weaker assumptions on the nature of individualism which preserve much of the individualistic flavor of Western economics, yet allow one to consider some of the effects of interrelated consumer preferences. These assumptions provide a useful taxonomy for the analysis of consumer interrelationships.

In the second section we define an exchange equilibrium which allows mutually voluntary bilateral gifts between interrelated pairs of consumers. It is demonstrated that if all interrelatedness is between "monogamous" pairs of consumers who are "benevolent" but somewhat "selfish", then equilibrium exists and is Pareto optimal. These results may provide a useful starting point for a theory of the "household".

Section I. Interrelated Consumer Preferences

There are assumed to be m commodities and n consumers. Let N be the set of all consumers. Define the consumption set, X_i, of Consumer i so that X_i is that subset of Euclidean m space which consists of all commodity bundles which could be consumed by i. Define the allocation set, X, so that $X \equiv \prod_{i \in N} X_i$. An allocation is a point $u \in X$ where $u = (u_1, \ldots, u_n)$ and u_i is the commodity bundle allocated to Consumer i.

Each consumer, $i \in N$, is assumed to have a complete preordering, R_i, defined on X. This is called the preference ordering of i.[1] The relations, P_i and I_i, are defined in the usual way.

Two consumers are said to be interrelated if at least one of them expresses strict preference between some pair of allocations which differ only in what the other receives.

Definition 1. Consumer i is related to Consumer j if there exists a pair of allocations u and v in X such that $u_k = v_k$ for all $k \neq j$ and uP_iv. Consumers i and j are interrelated if either i is related to j, or j is related to i.

Definition 2. Preferences of Consumer i are separable with respect to Consumer j if for all allocations, u and v, such that $u_k = v_k$ for every $k \neq j$ and all allocations u' and v' such that $u'_k = v'_k$ for every $k \neq j$ and such that $u'_j = u_j$ and $v'_j = v_j$, it is implied that uR_iv if and only if $u'R_iv'$. Preferences of i are separable between individuals if preferences of i are separable with respect to every consumer.

This is the notion of separability familiar in consumer theory. It can be shown that if preferences of i are separable between individuals and representable by a continuous utility function, $F_i(x)$, then $F_i(x)$ can be written as $F_i(g_{i1}(x_1),\ldots,g_{in}(x_n))$. Separability between individuals rules out such Veblenesque effects as the desire to imitate the consumption of others or desire for a commodity solely because of its scarcity.

When preferences are separable between individuals, one can define a private preference ordering, $\gtrsim_i$, on the consumption set, X_i, of each individual. In particular if uR_iv for some pair of allocations, u and v, which differ only in the consumption bundles allocated to i, it will be said that $u_i \gtrsim_i v_i$. It is easily verified that if preferences are separable between individuals, then the relation, $\gtrsim_i$, is a complete quasi-ordering on X_i.

Definition 3. The private preference ordering, $\gtrsim_i$, is defined as follows. If u and v are allocations such that $u_j = v_j$ for all $j \neq i$, then $u_i \gtrsim_i v_i$ if and only if uR_iv. The relations of strict private preference, $>_i$, and private indifference, $\sim_i$, are defined in the natural way.

Definition 4. Consumer i is nonmalevolently related to j if preferences of i and j are separable between individuals and if for every pair of allocations u and v in X such that $u_j \gtrsim_j v_j$ and such that $u_k = v_k$ for all $k \neq j$, it is implied that uR_iv.

If Consumer i is nonmalevolently related to j then for any two allocations, u and v, which contain the same bundles for everyone except j, if j privately prefers his bundle in u to his bundle in v then i will consider u at least as good as v. Nonmalevolence rules out the possibility that i disagrees with j about what kinds of goods j should consume.

Definition 5. Consumer i is benevolently related to j if preferences of consumers i and j are separable between individuals and if for all allocations, u and v, such that $u_k = v_k$ for $k \neq j$; $u_j >_j v_j$ if and only if uP_iv.

It can be shown that if preferences of i are separable between individuals and representable by a continuous utility function and if consumer i is nonmalevolently (benevolently) related to consumer j, then the utility function of i can be represented by a function $F_i(g_1(x_1),\ldots,g_n(x_n))$, where $g_j(.)$ represents the private preferences, $\gtrsim_j$, of j and F_i is a nondecreasing (increasing) function of $g_j(.)$.

Definition 6. Consumer i is locally nonsatiated on X_j if for all $u \varepsilon X$, in every open neighborhood of u_j in X_j there is an $x_j \varepsilon X_j$ such that vP_iu where v is an allocation such that $v_k = u_k$ for $k \neq j$ and $v_j = x_j$.

Remark. If i has transitive and continuous preferences and is locally nonsatiated on X_j, then i is benevolently related to j if and only if i is nonmalevolently related to j.

Proof: The only way in which a nonmalevolently related i could not be benevolently related to j would be if for some $u \varepsilon X$ and some $v_j \varepsilon X_j$, it is true that $u_j >_j v_j$ and $(u_1 \ldots u_j \ldots u_n)I_i(u_1 \ldots v_j \ldots u_n)$. Continuity of R_j in the product topology implies continuity of $\gtrsim_j$. Hence if the $u_j >_j v_j$, there is some open neighborhood, $N(v_j)$, of v_j in X_j such that $x_j \varepsilon N(v_j)$ implies that $u_j >_j x_j$. By local nonsatiation of i on X_j, there is some $\bar{x}_j \varepsilon N(v_j)$ such that $(u_1 \ldots \bar{x}_j \ldots u_n) P_i(u_1 \ldots u_j \ldots u_n)$. But $u_j >_j \bar{x}_j \varepsilon N(v_j)$. This contradicts nonmalevolence. Therefore nonmalevolence implies benevolence. That benevolence implies nonmalevolence follows trivially from the definitions. The remark is now proved.

Definition 7. Consumer i is egocentric with respect to consumer j if preferences of i are separable between individuals and if for any pair of allocations, u and v, such that $u_k = v_k$ where $k \neq i$ and $k \neq j$; if uR_jv and $u_i \gtrsim_i v_i$ then uR_iv.

Remark. If consumer i is egocentric with respect to consumer j, he is nonmalevolent with respect to j.

Proof: Simply apply the definition of egocentricity where $u_i = v_i$.

Theorem 1.

(a) Consumer i is egocentric with respect to consumer j if and only if for all allocations, u and v, such that $u_k = v_k$ where $k \neq i$ and $k \neq j$; if $vP_i u$ and $uR_j v$ then $v_i >_i u_i$.

(b) If consumer i is egocentric with respect to j and j is nonmalevolent with respect to i, and if u and v are allocations such that $vP_i u$ and $uR_j v$, then $u_j \geq_j v_j$ and $v_i >_i u_i$.

Proof:

To prove Part (a), simply observe that since the relation, R_i, is complete, the statement, "If $uR_j v$ and $u_i \geq_i v_i$ then $uR_i v$" is logically equivalent to the statement "If $uR_j v$ and $vP_i u$ then $v_i >_i u_i$." Therefore the statement in Theorem 1(a) is equivalent to the definition of egocentricity.

Consider two allocations u and v such that $u_k = v_k$ where $k \neq i$ and $k \neq j$ and such that $vP_i u$ and $uR_j v$. Part (a) of Theorem 1 implies that $v_i >_i u_i$. If consumer j is nonmalevolent, it is easily shown that if $v_j >_j u_j$ then $vP_j u$. But by assumption, $uR_j v$. Therefore $u_j \geq_j v_j$. This proves Part (b). QED

One's intuition about the nature of egocentricity is aided by Theorem 1. If two persons are egocentric with respect to each other, then whenever they disagree about the relative merits of two allocations, the disagreement is such that each person prefers the allocation in which his own bundle ranks higher in his private preference, $\geq_i$. If two persons disagree because each wants the other to have the better part, then they violate the assumptions of egocentricity. (A delightful fictional account of a pair of non-egocentric individuals is found in O. Henry's short story, The Gift of the Magi.)

To consider an example, suppose there were only one commodity and two persons, an egocentric rich man and an egocentric poor man. If each prefers to have more of the commodity rather than less, given the consumption of the other, then the rich man might give to the poor man, but one would never find that the rich man wants to change places with the poor man while the poor man is unwilling to do so.

This can be illustrated by the figure below. There is just one good to be allocated between two consumers. Each consumer privately prefers more of the good. Both are benevolent. Possible allocations are represented by points on the line 0 - 1. The distance to the left of any point measures the amount consumed by A. The distance to the right

Figure 1: Egocentric preference

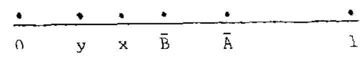
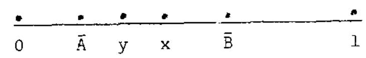


Figure 2: Non-egocentric preference



of the point measures the amount consumed by B. Consumer A prefers the point $\bar{A}$ to all other allocations on the line. Consumer B prefers $\bar{B}$ to all other allocations on the line. Each ranks other allocations inversely with their distance from his favorite point. We show that when $\bar{A}$ and $\bar{B}$ are located as in Figure 1, preferences are egocentric. Suppose $xR_B y$ and $x_A >_A y_A$. Then x must be to the right of y. The only way this can happen when $xR_B y$ is if both points are to the left of $\bar{B}$. But if this is the case, both points are also to the left of $\bar{A}$ and therefore, $xR_A y$. Consumer A must be egocentric with respect to B. Likewise one can show that B is egocentric with respect to A. To see that preferences are not egocentric if $\bar{A}$ and $\bar{B}$ are located as in Figure 2, consider points x and y between $\bar{A}$ and $\bar{B}$ such that x is to the right of y. Then $xR_B y$ and $x_A >_A y_A$ but $yP_A x$, which could not happen if A were egocentric with respect to B.

It is sometimes useful to have a slightly stronger version of egocentricity.

Definition 8. Consumer i is strongly egocentric with respect to j if i is egocentric with respect to j and if in addition, for any allocations, u and v, such that $u_k = v_k$ where $k \neq i$ and $k \neq j$; $uR_j v$ and $u_i >_i v_i$ imply that $uP_i v$.

It is not in general true that egocentricity, even with continuity of preferences, implies strong egocentricity. If, for example, two consumers are in complete agreement about the relative merits of every pair of allocations, then they are egocentric with respect to each other but neither need be strongly egocentric with respect to the other.

It turns out that if preferences are egocentric and continuous and if for any allocation, u, there is some allocation, w, close to u (in a sense defined below) such that Consumers i and j disagree about whether u is better than w, then preferences are strongly egocentric. These notions are made precise below.

Definition 9. There is local conflict of interest between Consumers i and j, if in every open neighborhood of every allocation, u, there is an allocation, w, such that $w_k = u_k$ where $k \neq i$ and $k \neq j$, and such that $wP_j u$ and $uP_i w$.

Theorem 2.
If preferences of Consumer i are continuous and egocentric with respect to Consumer j, and if there is local conflict of interest between i and j, then preferences of i are strongly egocentric with respect to j.
Proof:
Suppose that u and v are allocations such that $u_k = v_k$ where $k \neq i$ and $k \neq j$ and suppose that $uR_j v$ and $u_i >_i v_i$. Since preferences of i are continuous, there is an open neighborhood, N(u), of u, such that if $w \in N(u)$ then $w_i >_i v_i$. Since there is local conflict of interest, there is some $w \in N(u)$ such that $wP_j u$ and $uP_i w$. Since $w \in N(u)$, $w_i >_i v_i$. Since $wP_j u$ and $uR_j v$, $wR_j v$. Egocentricity therefore implies that $wR_i v$. But $uP_i w$. Therefore $uP_i v$.
QED

The following theorem states some properties of strongly egocentric preferences.

Theorem 3.
If preferences of Consumer j are nonmalevolent with respect to Consumer i then the following statements are logically equivalent.

(a) Consumer i is strongly egocentric with respect to Consumer j.

(b) For any allocations, u and v, such that $u_k = v_k$ where $k \neq i$ and $k \neq j$; if $vR_i u$ and $uR_j v$, then $v_i \geq_i u_i$ and $u_j \geq_j v_j$.

(c) Consumer j is strongly egocentric with respect to Consumer i.

Proof:
Completeness of R_i implies that the statement, "If $uR_j v$ and $u_i >_i v_i$ then $uP_i v$", is logically equivalent to the statement, "If $vR_i u$ and $uR_j v$ then $v_i \geq_i u_i$." Nonmalevolence of j toward i implies that if $vR_i u$, $uR_j v$, and $v_i \geq_i u_i$ then $u_j \geq_j v_j$. The equivalence of Statements (a) and (b) follows immediately.

Since Statement (b) is symmetric with respect to i and j, it is immediate that Statement (c) is also equivalent to Statements (a) and (b).
QED

Section II. Households and Competitive Gift Equilibrium

It is well known that a competitive equilibrium as ordinarily defined will not in general be Pareto optimal if there is consumer interrelatedness. Where all consumer interrelatedness is nonmalevolent, it is of interest to consider an exchange equilibrium in which bilateral gifts are allowed. Such an equilibrium will be defined below.

Definition 10. A household is a set of interrelated consumers such that each member of the set is interrelated with some other member and no member of the set is interrelated with any consumer not in the set. (The definition of interrelatedness is such that an individual is interrelated with himself. Therefore the set containing a completely "selfish" individual is a single member household.)

Definition 11. If each member, i, of a household, H, commands an initial vector, w_i, of commodity holdings, then the allocation, x, is a household equilibrium for H at the prices, p, if $p \sum_{i \in H} x_i \leq p \sum_{i \in H} w_i$ and if for every $i \in H$ and for every allocation, x', such that $p \sum_{i \in H} x'_i \leq p \sum_{i \in H} w_i$; if $x' P_i x$ then $px'_i > pw_i$.

If an allocation is a household equilibrium at the prices, p, then it must be that the total value of goods consumed by members of the household does not exceed the total value of the initial holdings of household members. If some member of the household prefers another allocation to a household equilibrium, it must be either that the total cost of goods consumed by members of the household exceeds total household wealth or that the bundle preferred by that member costs more than the value of his personal initial holdings. It is possible that in equilibrium, some members of the household consume bundles of higher value than their holdings. If this is to happen, other members of the household must voluntarily consume bundles of lower value than their initial holdings.

Definition 12. A competitive gift equilibrium (c.g.e.) for an exchange economy is an allocation $\bar{x}$ and a set of prices, $\bar{p}$, such that:

(a) The allocation $\bar{x}$ is a household equilibrium at rices, $\bar{p}$, for every household.

(b) $\sum_{i \varepsilon N} \bar{x}_i = \sum_{i \varepsilon N} w_i$, where N is the set of all consumers.[2]

Even if all consumer interrelatedness is nonmalevolent, competitive gift equilibrium does not necessarily exist. n fact there may be households for which there is no house-old equilibrium at any price vector. Furthermore, a c.g.e. eed not be Pareto optimal. (See Bergstrom [2]) A special ase of interrelated preferences for which a c.g.e. exists nd is Pareto optimal will be presented below.

Theorem 4.

Let the allocation, x, be a household equilibrium for ousehold, H, at prices, p. Suppose that for all $i \varepsilon H$, pref-ences of i are locally nonsatiated on X_i. Then if $x' R_i x$ or all $i \varepsilon H$, it must be that $p \sum_{i \varepsilon H} x'_i \geqq \sum_{i \varepsilon H} w_i$.

oof:

Suppose that $x' R_i x$ for all $i \varepsilon H$ and that $p \sum_{i \varepsilon H} x'_i < \sum_{i \varepsilon H} w_i$. Then for some $j \varepsilon H$, $p x'_j < p w_j$. Since j is locally onsatiated on X_j there is an $\overset{o}{x}_j$ near x'_j such that $p \overset{o}{x}_j < p w_j$, $\overset{o}{x}_j + \sum_{\substack{i \varepsilon H \\ i \neq j}} x'_i < p \sum_{i \varepsilon H} w_i$, and such that $\hat{x} P_j x'$ where $\hat{x}$ is an al-ocation such that $\hat{x}_j = \overset{o}{x}_j$ and $\hat{x}_k = x'_k$ for all consumers, $\neq j$. Since $s' R_j x$, it follows that $\hat{x} P_j x$. But this contra-cts the assertion that x is a c.g.e. Therefore if $x' R_i x$ r all $i \varepsilon H$ then $p \sum_{i \varepsilon H} x'_i \geqq p \sum_{i \varepsilon H} w_i$. QED

An easy consequence of Theorem 4 is that if x is a house-ld equilibrium for household, H, at prices, p, then $p \sum_{i \varepsilon H} x_i = \sum_{\varepsilon H} w_i$.

In the remainder of this section, attention will be con-ned to the case where no consumer is interrelated with more an one other consumer. In this case, no household consists more than two consumers. Theorem 5 states a property of uilibrium for a household which contains two benevolent and ocentric members.

Theorem 5.

Let H be a household with exactly two members, A and B. sume that consumers A and B are benevolent and egocentric th respect to each other, that preferences of A are locally nonsatiated on X_A and that preferences of B are locally non-satiated on X_B. Let x be a household equilibrium for H at prices, p. If $x' P_A x$ and $x' R_B x$, or if $x' R_A x$ and $x' P_B x$, then $p(x'_A + x'_B) > p(w_A + w_B)$.

Proof:

Suppose that $x' P_A x$, $x' R_B x$, and that $p(x'_A + x'_B) \leqq p(w_A + w_B)$. Since x is a household equilibrium, it must be that $p x'_A > p w_A$. Therefore $p x'_B < p w_B$. But, since x is a household equilibrium, this implies that $x R_B x'$. Since con-sumers A and B are egocentric with respect to each other, it follows from Theorem 1b that $x'_A >_A x_A$ and that $x_B \gtrsim_B x'_B$.

If $p x'_B < p x_B$ then, since B is locally nonsatiated on X_B, there is an $\overset{o}{x}_B$ near x'_B such that $p \overset{o}{x}_B < p x_B$, $p(x_A + \overset{o}{x}_B) < p(w_A + w_B)$, and such that $\hat{x} P_B x$ where $\hat{x}$ is an allocation in which $\hat{x}_B = \overset{o}{x}_B$, and $\hat{x}_i = x_i$ for all consumers, i, other than B. But this cannot be if x is a c.g.e. Therefore $p x'_B \geqq p w_B$. Suppose that $p x'_A \leqq p x_A$. Let $\hat{x}$ be an allocation such that $\hat{x}_A = x'_A$ and such that $\hat{x}_i = x_i$ for all consumers, i, other than A. Then $p(\hat{x}_A + \hat{x}_B) \leqq p(w_A + w_B)$ and $p \hat{x}_B \leqq p w_B$. But since Consumer B is benevolent toward A, and since $\hat{x}_A = x'_A >_A x_A$ and $\hat{x}_B = x_B$, it must be that $\hat{x} P_B x$. This cannot be, since x is a c.g.e. Therefore $p x'_A > p x_A$. It follows that $p(x'_A + x'_B) > p(x_A + x_B)$. Since x is a household equili-brium, $p(x_A + x_B) = p(w_A + w_B)$. Therefore $p(x'_A + x'_B) > p(w_A + w_B)$.

If $x' R_A x$ and $x' P_B x$, an analogous proof shows that $p(x'_A + x'_B) > p(w_A + w_B)$. QED

Observe that in Theorem 5 it is assumed that both con-sumers are benevolent. It might seem plausible that contin-uity, interrelatedness, and egocentricity (which implies non-malevolence) would be sufficient for the conclusion of Theorem 5. This turns out not to be the case unless it is also as-sumed that there is some point in the consumption set of each consumer which is cheaper at the equilibrium prices than his equilibrium consumption.

Theorem 5 may be helpful for interpreting the meaning of a household equilibrium for two-member households. If x is a household equilibrium at prices, p, then Theorem 5 implies that for any allocation, x', such that $p(x'_A + x'_B) \leq p(w_A + w_B)$; if $x'P_Ax$ then $px'_A > 0$ and xP_Bx'. Thus, if the total cost to the household of allocation x' does not exceed the total wealth of the household and if Consumer A prefers x' to x then it must be that the allocation x' is achieved only if Consumer B makes a gift of value, px'_A, to Consumer A. But B prefers not to make such a gift.

Theorem 6.

If no household contains more than two consumers, if members of each household are egocentric and benevolent with respect to each other, and if preferences of each consumer, i, are locally nonsatiated on X_i, then a competitive gift equilibrium is Pareto optimal.

Proof:

Suppose that $\bar{x}$ is a c.g.e. and suppose that x is Pareto superior to $\bar{x}$. Then $\bar{x}$ is a household equilibrium for every household at prices, $\bar{p}$. If a consumer belongs to a single member household, this means that at $\bar{x}$ he is maximizing his preferences subject to an ordinary private competitive budget constraint. Just as in the traditional proof of the optimality of competitive equilibrium, it must be that when i is the only member of a household, if $xR_i\bar{x}$ then $\bar{p}x_i \geq \bar{p}w_i$ and if $xP_i\bar{x}$ then $\bar{p}x_i > \bar{p}w_i$.

Since $xR_i\bar{x}$ for all $i \in N$ and since $xP_j\bar{x}$ for some $j \in N$, it follows from Theorem 4 that $\bar{p}\sum_{i\in H} x_i \geq \bar{p}\sum_{i\in H} w_i$ for every household, H. From Theorem 5, it follows that for some household, H, $\bar{p}\sum_{i\in H} x_i > \bar{p}\sum_{i\in H} w_i$. Therefore $\bar{p}\sum_{i\in N} x_i > \bar{p}\sum_{i\in N} w_i$. This cannot be if x is a feasible allocation. Therefore $\bar{x}$ is Pareto optimal.

QED

It is, of course, not very useful to know that a completitive gift equilibrium if Pareto optimal unless we also know that such an equilibrium exists. In the usual proofs of the existence of competitive equilibrium, the crucial step is to show that there is a nonempty, upper-semi-continuous aggregate demand correspondence. Theorem 8 states conditions under which there if such a correspondence when there if pairwise consumer interrelatedness. With the use of Theorem 8, it is a quite mechanical exercise to adapt any of the familiar proofs of Arrow and Debreu [1], Debreu [3], McKenzie [4] or Rader [5], to prove the existence of a competitive gift equilibrium.

Definition 13. The joint household demand correspondence, f_H, of household, H, is a correspondence $f_H: E^m \to \sum_{i\in H} X_i$ such that for $p \in E^m$, $f_H(p) = \{\sum_{i\in H} x_i \mid x$ is a household equilibrium for H at prices, p$\}$. Let $\mathcal{H}$ be the set of all households. Then the aggregate demand correspondence, f, is a correspondence such that for $p \in E^m$, $f(p) = \sum_{H\in\mathcal{H}} f_H(p)$.

The set, $f_H(p)$, will be nonempty if at prices, p, there is a household equilibrium for H. If consumers are not egocentric, $f_H(p)$ may be empty at some prices. In the example illustrated in Figure 2 of Section I, if the initial allocation of wealth is represented by a point between $\bar{A}$ and $\bar{B}$, then there is no household equilibrium when the price of the good is positive. The only point which A likes at least as well as every x such that $px_A \leq pw_A$ and $p(x_A + x_B) \leq p(w_A + w_B)$ is $\bar{A}$. The only point which B likes at least as well as every x such that $px_B \leq pw_B$ and $p(x_A + x_B) \leq p(w_A + w_B)$ is $\bar{B}$.

Theorem 7.

If household, H, has exactly two members, A and B, with compact, convex consumption sets X_A and X_B respectively, if preferences of A and B are weakly convex, continuous and egocentric and if preferences of A and B are locally nonsatiated on X_A and X_B respectively, then for any price vector, p, such that $px_A \leq pw_A$ for some $x_A \in X_A$ and such that $px_B \leq pw_B$ for some $x_B \in X_B$, the set $f_H(p)$ is nonempty.

Proof:

The set $f_H(p)$ will be nonempty if there exists a household equilibrium for H at prices, p. Since Consumers A and B are interrelated with no consumers outside the household, the level of preference of A and B depends only on what is consumed by A and B. One can therefore treat preferences of A and B as if they were defined only on (X_A, X_B). Let $x^A(p) \equiv (x^A_A(p), x^A_B(p))$ be an allocation such that $x^A(p)$ maximizes R_A on the set $\{(x_A, x_B) \mid p(x_A + x_B) \leq p(w_A + w_B)\}$. Let $x^B(p) \equiv (x^B_A(p), x^B_B(p))$ be an allocation such that $x^B(p)$ maximizes R_B on $\{(x_A, x_B) \mid p(x_A + x_B) \leq p(w_A + w_B)\}$.

Suppose that $px^A_A(p) < pw_A$. It will be shown that in this case $x^A(p)$ is a household equilibrium at prices, p. Consider (x_A', x_B') such that $p(x_A' + x_B') \leq p(w_A + w_B)$ and $(x_A', x_B')\ P_B(x^A_B(p), x^A_B(p))$. The definition of $x^A(p)$ implies that $x^A(p)\ R_A(x_A', x_B')$. Since preferences are egocentric, it must be that $x_B' >_B x_B^A(p)$. If $px_B' < px_B^A(p)$, then continuity of preferences together with Assumption 5, ensure that for some $\mathring{x}_B$ near x_B', $\mathring{x}_B >_B x_B^A(p)$ and $p\mathring{x}_B < px_B^A(p)$. But since A is nonmalevolent, $(x_A^A(p), \mathring{x}_B) R_A (x_A^A(p), x_B^A(p))$. Since A is locally nonsatiated on X_A, there is an $\mathring{x}_A$ near $x_A^A(p)$ such that $(\mathring{x}_A, \mathring{x}_B) P_A (x_A^A(p), x_B^A(p))$ and such that $p(\mathring{x}_A + \mathring{x}_B) < p(w_A + w_B)$. This contradicts the definition of $x^A(p)$. Therefore if $(x_A', x_B') P_B x^A(p)$ and if $p(x_A' + x_B') \leqq p(w_A + w_B)$ then $px_B' \geqq px_B^A(p)$. But since $px_A^A(p) < pw_A$, $px_B^A(p) > pw_B$. Therefore $px_B' > pw_B$. It follows that when $px_A^A(p) < pw_A$, $x^A(p)$ is a household equilibrium at prices, p. An analogous proof shows that if $px_B^B(p) < pw_B$, then $x^B(p)$ is a household equilibrium at prices, p.

The remaining case is where $px_A^A(p) \geqq pw_A$ and $px_B^B(p) \geqq pw_B$. Consider an allocation, $(\bar{x}_A, \bar{x}_B)$, such that $\bar{x}_A$ maximizes $\gtrsim_A$ on $\{x_A | px_A \leqq pw_A\}$ and $\bar{x}_B$ maximizes $\gtrsim_B$ on $\{x_B | px_B \leqq pw_B\}$. Suppose that $(x_A, x_B) P_A (\bar{x}_A, \bar{x}_B)$, $p(x_A + x_B) \leqq p(w_A + w_D)$ and $px_A \leqq pw_A$. Then $x^A(p) R_A (x_A, x_B)$. Since preferences are weakly convex, if $(x_A(\lambda), x_B(\lambda)) = \lambda(x_A^A(p), x_B^A(p)) + (1-\lambda)(x_A, x_B)$ for λ such that $0 \leqq \lambda \leqq 1$, then $(x_A(\lambda), x_B(\lambda)) R_A (x_A, x_B) P_A (\bar{x}_A, \bar{x}_B)$. But $px_A^A(p) \geqq pw_A$ and $px_A \leqq pw_a$. Therefore for some $\hat{\lambda}$ such that $0 \leqq \hat{\lambda} \leqq 1$, $px_A(\lambda) = pw_A$ and $px_B(\hat{\lambda}) \leqq pw_B$. But this implies that $\bar{x}_A \gtrsim_A x_A(\hat{\lambda})$ and $\bar{x}_B \gtrsim_B x_B(\hat{\lambda})$. Nonmalevolence implies that $(\bar{x}_A, \bar{x}_B) R_A (x_A(\lambda), x_B(\lambda))$. But this is a contradiction. It follows that if $p(x_A + x_B) \leqq p(w_A + w_B)$ and if $px_B \leqq pw_A$, then $(\bar{x}_A, \bar{x}_B) R_A (x_A, x_B)$. An analogous proof shows that if $p(x_A + x_B) \leqq p(w_A + w_B)$ and if $px_B \leqq pw_B$, then $(\bar{x}_A, \bar{x}_B) R_B (x_A, x_B)$. Therefore $(\bar{x}_A, \bar{x}_B)$ is a household equilibrium at prices, p. QED

Theorem 8.

If the following assumptions hold:

(1) For all $i \epsilon N$, X_i is a convex, compact subset of Euclidean m space.

(2) For all $i \epsilon N$, R_i is a weakly convex, continuous quasi-ordering on $X = \prod_{i \epsilon N} X_i$.

(3) For all $i \epsilon N$, Consumer i is locally nonsatiated on X_i and preferences of Consumer i are egocentric with respect to every other consumer.

(4) No household contains more than two consumers.

(5) The set of prices, P, is a closed convex set not containing zero, such that for all $i \epsilon N$, and for all $p \epsilon P$, there exists an $x_i \epsilon X_i$ such that $px_i < pw_i$.

then the aggregate demand correspondence $f : P \to \sum_{i \epsilon N} X_i$ is upper semi-continuous and for every $p \epsilon P$, $f(p)$ is a nonempty convex set.

Proof:

Since X_i is compact and R_i is continuous for all $i \epsilon N$, it must be that if H has only one member then $f_H(p)$ is nonempty for all $p \epsilon P$. According to Theorem 7, if H has two members, $f_H(p)$ is nonempty for all $p \epsilon P$.

If a household has only one member, one can immediately apply the method used by Debreu [3, p. 63] to show that f_H is upper semi-continuous. Suppose that a household has two members, A and B.

Let $g_A(p) = \{x | p(x_A + x_B) \leqq p(w_A + w_B)$ and xR_Ax' if $p(x_A' + x_B') \leqq p(w_A + w_B)$ and $px_A' \leqq pw_A\}$. Let $g_B(p) = \{x | p(x_A + x_B) \leqq p(w_A + w_B)$ and xR_Bx' if $p(x_A' + x_B') \leqq p(w_A + w_B)$ and $px_B' \leqq pw_B\}$. Assumption 5 and the continuity of R_A and R_B ensure that $g_A(p)$ and $g_B(p)$ are upper semi-continuous correspondences. Since $f_H(p) = g_A(p) \cap g_B(p)$, f_H is also u.s.c.

Weak convexity of preferences guarantees that $f_H(p)$ is a convex set for every $p \epsilon P$. Since $f(p) = \sum_{H \epsilon \mathcal{H}} f_H(p)$, f is u.s.c. and has nonempty convex image sets for every $p \epsilon P$. QED

The final theorem and its corollary are counterparts to the "Second Optimality Theorem of Welfare Economics". It is shown that when consumer interrelatedness is restricted to monogamous pairs and is egocentric that a large class of Pareto optima can be sustained as competitive gift equilibria.

Theorem 9.

If $\bar{x}$ is a Pareto optimal allocation for an exchange economy with an aggregate vector of commodity holdings, w, and if

(1) For all $i \epsilon N$, R_i is a convex quasi-ordering on X_i and Consumer i is locally nonsatiated on X_i.

(2) No household contains more than two consumers.
(3) All consumer interrelatedness is egocentric.

then there exists a price vector $\bar{p}$ and budgets, $(b_1,\ldots,b_n)$, such that: if i belongs to a single member household and if $xP_i\bar{x}_i$ then $px_i \geq b_i$; if A and B belong to a two member household then for x such that $p(x_A + x_B) < b_A + b_B$, if $xP_A\bar{x}$ then $px_A \geq b_A$, and if $xP_B\bar{x}$ then $px_B \geq b_B$.

Corollary.[3]

If the assumptions of Theorem 9 are true, if preferences are continuous, and if for every $i\varepsilon N$, there exists an $\overset{o}{x}_i \varepsilon X_i$ such that $\bar{p}\overset{o}{x}_i < \bar{p}\bar{x}_i$ where $\bar{p}$ is a price vector satisfying Theorem 8, then $\bar{x}$ is a competitive gift equilibrium at prices $\bar{p}$ where for all $i\varepsilon N$, consumer i is given a budget, $b_i = \bar{p}\bar{x}_i$.

Proof of Theorem 9:

Since preferences are convex, the set, $V\equiv\{\sum_{i\varepsilon N} x_i | x$ is Pareto superior to $\bar{x}\}$ is convex. Since $\bar{x}$ is Pareto optimal, $w\notin V$. Minkowski's separation theorem implies that there exists a (non-zero) $\bar{p}$ such that if $\sum_{i\varepsilon N} x_i \varepsilon V$ then $\bar{p}\sum_{i\varepsilon N} x_i \geq \bar{p}w = \bar{p}\sum_{i\varepsilon N} \bar{x}_i$. It is easily shown, since preferences are nonmalevolent, that for all $i\varepsilon N$, if $x_i >_i \bar{x}_i$ then $\bar{p}x_i \geq \bar{p}\bar{x}_i$. If consumer i belongs to a single member household it is immediate that if $xP_i\bar{x}$, then $\bar{p}x_i \geq \bar{p}\bar{x}_i$.

Suppose that A and B belong to the same two member household. Since A and B are interrelated with no consumers outside the household, it follows that if $\bar{p}(x_A + x_B) < \bar{p}(\bar{x}_A + \bar{x}_B)$ then $\bar{x}P_Ax$ or $\bar{x}P_Bx$. Suppose $p(x_A + x_B) < \bar{p}(\bar{x}_A + \bar{x}_B)$ and $xP_A\bar{x}$. Then $\bar{x}P_Bx$. Since preferences are egocentric $x_A > \bar{x}_A$. But it was shown above that this implies that $\bar{p}x_A \geq \bar{p}\bar{x}_A$. Likewise if $p(x_A + x_B) < \bar{p}(\bar{x}_A + \bar{x}_B)$ and $xP_B\bar{x}$, it must be that $\bar{p}x_B \geq \bar{p}\bar{x}_B$. Therefore if each consumer $i\varepsilon N$, is given a budget $b_i = \bar{p}\bar{x}_i$, the conclusion of the theorem holds. QED

Proof of Corollary:

If consumer i belongs to a single member household, the proof that $\bar{x}$ is a household equilibrium at $\bar{p}$ for his household is the same as that offered in Debreu [3, p. 96].

Suppose that A and B are members of a two person household and that $\bar{p}$ is a price vector satisfying Theorem 9. Suppose that $\bar{p}(x_A + x_B) \leq \bar{p}(\bar{x}_A + \bar{x}_B)$, $\bar{p}x_A \leq \bar{p}\bar{x}_A$ and $xP_A\bar{x}$. By assumption there is an $\overset{o}{x}_A \varepsilon X_A$ such that $\bar{p}\overset{o}{x}_A < \bar{p}\bar{x}_A$. Consider the allocation, $x(\lambda)$ where $x_B(\lambda) = x_B$ and $x_A(\lambda) = \lambda x_A + (1-\lambda)\overset{o}{x}_A$. Continuity implies that for λ close to one, $x(\lambda)P_A\bar{x}$. But $\bar{p}x_A(\lambda) < \bar{p}\bar{x}_A$ and $\bar{p}(x_A(\lambda) + x_B(\lambda)) < \bar{p}(\bar{x}_A + \bar{x}_B)$. This contradicts Theorem 8. Therefore if $\bar{p}(x_A + x_B) \leq \bar{p}(\bar{x}_A + \bar{x}_B)$ and $\bar{p}x_A \leq \bar{p}\bar{x}_A$ then $\bar{x}R_Ax$. An analogous statement can be made for B. The corollary follows immediately. QED

Conclusion:

The purposes of this paper have been twofold. Restrictions on the nature of consumer interrelatedness such as separability between individuals, nonmalevolence, and egocentricity (in decreasing order of generality) are suggested. The relations between these assumptions are explored, as are some of their implications for individual behavior. Perhaps the most interesting and least intuitively understandable of these assumptions is that of egocentricity which limits the degree of generosity of our consumers. Theorems 1, 2, and 3 help to clarify the meaning of this assumption.

It is established that a competitive gift equilibrium can be shown to exist when the only consumer interrelatedness is between pairs of egocentric consumers. In this case there is also a two way correspondence between the set of competitive gift equilibria and the set of Pareto optima.

Things become more difficult (and perhaps more interesting) when preferences are less monogamous. An allocation mechanism which is Pareto efficient when there are large networks of interrelated consumers is discussed in Bergstrom [2].

FOOTNOTES

[1] Implicit in the interpretation of R_i as the preference relation of consumer i is that his ranking of situations depends only on the quantities of commodities consumed by each consumer. If there is more than one way in which the same commodity bundle can be used by a given consumer, the traditional theory is salvaged by arguing that the R_i's are derived from the solutions of the underlying problem of how best to use each commodity bundle. This underlying problem is solved unambiguously by each consumer and the resultant activities are compared. If there is consumer interrelatedness, and if two consumers disagree about how one of them should consume a particular bundle, more care is required in the interpretation of R_i.

[2] Here and in the sequel we deal only with an exchange economy. This theorem and subsequent theorems can be extended to a production economy of the sort described by Debreu [3]. Attention is restricted here to an exchange economy solely as a matter of notational convenience.

[3] An alternative and perhaps more satisfactory theorem on the correspondence between Pareto optima and competitive equilibrium is proved by Rader [4]. Rader's assumptions and method of proof can be readily adapted to this problem.

REFERENCES

* I owe a debt of thanks to Professor Trout Rader of Washington University for many helpful comments and suggestions.

[1] Arrow, K.J., and G. Debreu, "Existence of an Equilibrium for a Competitive Economy", Econometrica XXII, (July, 1954), 265-90.

[2] Bergstrom, T., "A 'Scandinavian Consensus Solution' for Efficient Income Distribution among Benevolent Consumers", (mimeographed).

[3] Debreu, G., Theory of Value (New York, John Wiley & Sons, 1959).

[4] McKenzie, L.W., "On the Existence of General Equilibrium: for a Competitive Market", Econometrica XXVII, (January, 1959) 54-57.

[5] Rader, J.T., "Pairwise Optimality and Non-Competitive Behavior," in Papers in Quantitative Economics, edited by J. P. Quirk and A. M. Zarley (Lawrence, Kansas, University of Kansas Press, 1968) 101-128.

tion-to-capacity functions.