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AN EXTENSION OF THE S.I. UNIT SYSTEM FOR USE IN PHYSICS

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ABSTRACT

At present there are two major unit systems in use in physics: the cgs-Gaussian system of "scientific" units and the Systême International (SI) of "practical" units. It is shown that a single unified unit system suitable for both scientific and practical use can be achieved by adding some auxiliary "scientific" electromagnetic units to the SI. These auxiliary electromagnetic units would be used in research areas such as radiation theory, quantum mechanics, etc., for which the ordinary SI electromagnetic units are inappropriate. The definitions of the proposed auxiliary units are symmetric in form and are easily remembered. They are chosen so that the factors ε_0 and μ_0 do not appear in equations written in auxiliary units. Also, to convert magnitudes in auxiliary units to SI magnitudes, or vice versa, only one unit conversion factor k_0 need be remembered ($k_0 \approx 900$ SI units).

I. THE UNITS IMPASSE

The hope that the Systême International d'Unités would lead to unification and simplification of scientific and engineering measurements has been disappointed in at least one area. Many physicists, in particular theorists, have refused to adopt the Systême International (SI) in their research work and publications. Thus we find, for example, a set of introductory physics texts,[1] and texts in electrodynamics[2] and quantum mechanics[3,4] using the cgs-Gaussian unit system ten to twenty years after the standarization and official adoption of the Giorgi system.[5]

While habit and simple inertia may be partly responsible, there is no doubt that one principal stumbling block to the universal adoption of the Systême International is the inelegance of the Giorgi system of electromagnetic units, particularly when applied to physics problems such as the electromagnetic field relations in empty space. In a discussion of the emission of radiation by a single atom, for example, factors ε_0 and μ_0 appear which convey no physical information. They are simply factors of unit conversion. These factors are viewed by many physicists as an unnecessary bother, one which they can easily avoid by retaining the cgs unit system.

On the other hand, the simultaneous existence of two complete systems of units, cgs and SI, is confusing and unsatisfactory, particularly for undergraduates meeting physics for the first time. Of course there should be, and always will be, special units for special areas of physics, such as the parsec, angstrom, field theory units, barn, etc. But an agreement among physicists on one system which would be used in teaching and to which all other units would be universally referred would undoubtedly help beginners approach the study of physics more securely. Requiring students to learn not only two complete systems of electromagnetic units, but also two complete systems of mechanical units and the conversion factors between (joules to ergs, newtons to dynes, etc.), seems an overly drastic solution to the inelegance of the Giorgi electromagnetic unit system. The advantages of a universally accepted system are so obvious that the refusal of so many physicists to adopt the SI must be considered strong evidence of its inappropriateness and unacceptability in their specialties.

Progress toward unification of scientific measurements seems to have reached an impasse, one which threatens to become permanent. To be useful in escaping it, a proposed unit system modification:

(1) Must not be yet another unit system. We do not need more unit systems, we need fewer, preferably only one.

(2) Must, if possible, serve the needs both of the quantum theorist and the electrical engineer. Both have demonstrated that neither will compromise for the convenience of the other.

(3) Must be easy to remember and convenient to use.

(4) Must be pedagogically sound. No one in the present generation of practicing physicists can be expected to change his accustomed system of units, the system in which he has become familiar with the standard magnitudes of his specialty. Thus, a modification will be accepted only if it eases the task of initiating the next generation of researchers.

I justify the presentation of yet another paper on the units question by noting that many of my colleagues appear to be unaware that such a simple units transformation as the one proposed here is even possible. It will perhaps be of some value to remind the physics community that the distinction between "scientific" and "practical" units is by no means necessary or absolute.

2. PROPOSAL

The SI unit system, despite opposition to it, seems at present the only one which has any chance at all of becoming universally accepted. We propose here a modification of the SI which would seem to meet most of the rational objections to it by physicists. It is essentially a modification by addition. Instead of having two complete unit systems, the one "scientific" (cgs-Gaussian) and the other "practical" (SI), a set of auxiliary "scientific" electromagnetic quantities is added to the "practical" SI unit system. These auxiliary quantities can be defined in a standard way in terms of the SI units already in existence. They can be designed for the convenience of physicists in much the same spirit as the physiological SI radiation units (candela, etc.) are designed for the convenience of illumination engineers. The objective is an augmented SI which is acceptable to the physics community. The auxiliary quantities (which we denote here by the same letter as the corresponding SI unit, but written in script instead of roman lettering) are chosen to give the free field electromagnetic equations a suitably elegant and physically meaningful form. In particular, the factors ε_0 and μ_0 do not appear at all when they are used. Fortunately, such auxiliary quantities can be defined in a way which is quite simple and easy to remember. Only one unit conversion factor, which we denote k_0, is needed. By fortunate accident, its approximate numerical value is an easily remembered number. It is $k_0 \equiv \mu_0^{-\frac{1}{2}} \cong 900$ SI units to one percent accuracy. Aside from this one number k_0, the numerical values of ε_0 and μ_0 are not used, and do not need to be remembered.

A great advantage of the proposed scheme of auxiliary quantities is that the mechanical SI units, such as the meter, joule, etc., are retained. There are no auxiliary mechanical quantities. Also, those electrical SI units such as inductance, capacitance, etc., which are of little use in a large class of problems in theoretical and quantum physics, are not given corresponding auxiliary quantities. (Auxiliary quantities related to inductance, etc., can be defined, of course. But there is little point in doing so.) We want to emphasize that we are not proposing yet another complete unit system. By retaining the SI, and augmenting it with auxiliary quantities which can be used comfortably by a large group of researchers in physics, we hope to satisfy the first two criteria above, unification and versatility.

Since criterion 4 above, pedagogical usefullness, is perhaps the most important of the four, we choose to present the details of the scheme of auxiliary quantities as it would be presented in, say, an advanced undergraduate electrodynamics text book.

To begin, the students would be introduced to electrostatics and magnetostatics in terms of the usual SI units. They would learn the usual equations

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\,\frac{Q\,\hat{r}}{r^2} \qquad\qquad \vec{B} = \frac{\mu_0}{4\pi}\,\frac{\vec{I}\times\hat{r}}{r^2} \qquad (1\text{-a,b})$$

and the definitions of the macroscopic field quantities D, P, M, and H. The concepts of inductance, capacitance, resistance, flux, etc., would also be introduced in the first section, along with the energy relations for static and quasi-static fields. The first section of the text, presented entirely in standard SI units, could conclude with the usual plausability arguments leading to the Maxwell equations.

The second section of the text could begin by re-writing the Maxwell equations in the absence of material bodies (eg., with P = 0 and M = 0) in the standard SI form, as shown in the second column of Table III below. The empty space wave equations would then be derived, and $(\varepsilon_0\mu_0)^{-\frac{1}{2}}$ identified with c, the velocity of light.

The students would then be told that the SI electromagnetic units, which had been used up to that point in the text, are particularly designed for problems such as currents in wires, electric circuits, etc., for which it is important to have current, inductance, capacitance, resistance measured in units of practical size. The conversion factors ε_0 and μ_0 were in fact introduced to achieve these reasonable unit magnitudes. But for problems such as the emission of radiation, relativity theory, energy relations of fields in space, these SI electromagnetic units are rather awkward. Therefore, for the remainder of the book, all equations will be written, not in terms of the standard SI units, but in terms of auxiliary quantities which are more convenient to use. For example, the auxiliary quantities will be chosen to eliminate ε_0 and μ_0 from all equations, including those of field energy and momentum conservation. Moreover, in regions outside material bodies (where P and M vanish), the equations $\mathcal{E} = \mathcal{D}$ and $\mathcal{B} = \mathcal{H}$ will hold identically, both in units and magnitude. Also the units of $\mathcal{E}$ and $\mathcal{B}$ will be the same, with $\mathcal{E}^2$ and $\mathcal{B}^2$ having units of energy density. The factor c will appear in equations almost always in the form of a multiplier for dt, making the differential field equations easy to remember.

The definitions of the auxiliary quantities which accomplish this simplification would then be derived. The students would be asked simply to re-write eqs. (1-a,b) with ε_0 and μ_0 each broken into two square-root factors

$$\sqrt{\varepsilon_0}\,\vec{E} = \frac{1}{4\pi}\,\frac{(Q/\sqrt{\varepsilon_0})\,\hat{r}}{r^2} \qquad \frac{1}{\sqrt{\mu_0}}\,\vec{B} = \frac{1}{4\pi}\,\frac{\sqrt{\mu_0}\,\vec{I}\times\hat{r}}{r^2} \qquad (2\text{-a.b})$$

One can then read off the required definitions,

$$\mathcal{E} = \sqrt{\varepsilon_0}\;E \qquad \mathcal{B} = \frac{1}{\sqrt{\mu_0}}\;B \qquad (3\text{-a.b})$$

$$\mathcal{Q} = \frac{1}{\sqrt{\varepsilon_0}}\;Q \qquad \mathcal{I} = \sqrt{\mu_0}\;I \qquad (3\text{-c.d})$$

which define the auxiliary quantities, *E*, *B*, *Q*, *I*. As shown in Table I, all other auxiliary quantities are related to, and are defined in the same way as, one of these four. Thus ρ_x, *D*, *P* are defined like *Q*, and *J*, *H*, *M* like *I*, etc., as shown in Table I. The students would be asked to make for themselves the substitutions of eqs.(3), and ρ and J from Table I, into the SI form of the Maxwell equations. The result would be the Maxwell equations expressed entirely in terms of auxiliary quantities, as shown in the third column of Table III below. They could also be asked to repeat the derivation of the empty space wave equations in terms of the auxiliary quantities, to see that these equations preserve their form and c its meaning as the velocity of light.

The students would be asked to try to remember the definitions in eqs. (3). But they should realize that if they do forget them, they can always go back to eqns. (1-a,b) which they presumably do know, and repeat the two step derivation of eqs. (3). The elementary SI equations themselves thus serve as a handy mnemonic to remember the definitions of the auxiliary quantities.

The numerical values of ε_0, μ_0, $\sqrt{\varepsilon_0}$, and $\sqrt{\mu_0}$ do not need to be taught or remembered. Rather, the text would define the conversion factor k_0 and use the just-derived relation $c = (\varepsilon_0\, \mu_0)^{-\frac{1}{2}}$ to write

$$\frac{1}{\sqrt{\mu_0}} \equiv k_0 \qquad\qquad \sqrt{\varepsilon_0} = \frac{k_0}{c} \qquad\qquad (4\text{-a,b})$$

where $k_0 \equiv \mu_0^{-\frac{1}{2}} \cong 900$ SI units to one-percent accuracy and $c \cong 3 \times 10^8$ m/s. (Approximate values of k_0 and c to various levels of accuracy are given in Table II.) The students would thus need to memorize outright only the definition and approximate value of k_0.

But, actually, not even equations (3) and (4) need to be memorized. When written in terms of k_0 and c, the definitions eqs. (3) take the symmetric and easily remembered form

$$\mathcal{E} = \frac{k_0}{c} E \qquad \mathcal{B} = k_0 B \qquad (5\text{-a,b})$$

$$\mathcal{Q} = \frac{c}{k_0} Q \qquad \mathcal{I} = \frac{1}{k_0} I \qquad (5\text{-c,d})$$

in which the $\mathcal{E}$ and $\mathcal{B}$ definitions differ only by the divisor c in the former. Many students would find it easier simply to memorize eqs. (5) and the value k_0, rather than remembering eqs. (3) and (4). Although these students should be able to derive eqs. (3) in order to feel secure with the auxiliary quantities, they will in fact not use eqs. (3) very much in problem solving or research. This is because problems and problem areas will in general be treated either entirely in terms of standard SI units or entirely in terms of auxiliary quantities. Thus the only use of the auxiliary definitions will be in numerical evaluations of auxiliary quantities at the beginnings and ends of calculations, in which the data are given in SI units and the final numerical result are to be standard SI magnitudes. Such numerical evaluations can be made easily using only eqs. (5) and the values of k_0 and c. We note that the four eqs. (5) and the value of k_0 should pose much less of a burden on the memory than that posed, for example, by the many conversion factors between cgs-Gaussian and SI units.

After introduction of the auxiliary quantities, the remainder of the text can then use them exclusively, with reference back to SI units only when explicit magnitudes are discussed. Thus Maxwell's equations, the energy and momentum relations of electromagnetic fields, wave propagation, emission and absorption of radiation by charge systems, the covariant formulation of electromagnetism, etc., would all be treated entirely in terms of auxiliary quantities. The standard SI forms of equations in these areas would not even be quoted. Equations in advanced theory would be learned and remembered in their simplest and most physically meaningful forms, as relations among the auxiliary quantities. If at any point in the text the students are made curious or perturbed by this procedure, they can always use the easily derived eqs. (3) to return any expression to the standard SI form. Thus the students will be re-assured that they are not being asked to desert the familiar SI unit system, but only to use some new quantities which make the presentation of electromagnetic theory more elegant, symmetric, and easy to remember.

Although it may be an arguable point, it seems to us more in keeping with the ideal of a single unified unit system to urge the students not to take the actual magnitudes of the auxiliary quantities too seriously. One should consider the auxiliary quantities just as stand-ins for the real SI units which express the real physical magnitudes. Also, although the auxiliary quantities do have units which can be derived and quoted, the students should probably be urged not to write these units out in detail. Thus instead of $\mathcal{B} = 900(\mathrm{joule/m^3})^{\frac{1}{2}}$, one should write $\mathcal{B} = 900$ ASI units. (ASI is an abbreviation for "Auxiliary Systême International"). Then $\mathcal{B} = k_o\,B = 900$ ASI units leads to $B \cong 1\ \mathrm{weber/m^2}$, which is the real measure of the magnitude involved. On the other hand, the emphasis on SI magnitudes should not prevent, for example, presentation of the fact that $\frac{1}{2}\mathcal{B}^2$ is an energy density and hence that the magnitude of $\mathcal{B}$ has in that sense an absolute significance.

Table III presents several standard electromagnetic equations in both SI and auxiliary form. One can go back and forth between these forms by the use of eqs. (3).

For completeness, Table IV presents possible definitions of auxiliary quantities which should probably not be emphasized in teaching because the ordinary SI units are preferable for problem areas involving these quantities. The only possible exception is the conductivity σ_x which could be useful when using auxiliary quantities in the physics of plasmas. The relations in Tables I and IV, taken all together, define an internally consistent MKS Heaviside-Lorentz unit system of auxiliary quantities. But, as we have emphasized above, we are not proposing the adoption of such a complete set of auxiliary quantities as yet another unit system. We propose that, by using the standard SI units in some problem areas and the selected auxiliary quantities of Table I in other problem areas, and by continuing to quote all physical magnitudes in standard SI units, one gains the simplicity of Heaviside-Lorentz field equations and energy relations without losing the universality of the Système International.

3. CONCLUSION

The subject of electromagnetic units is so worked over that almost any idea is likely to have been proposed before by someone. We have not seen the particular substitution of eqs. (3) before, but the general idea of simple substitutions to convert between unit systems is an old one. For example, Panofsky and Phillips[6] give simple rules for conversion of SI equations to Gaussian ones. Besides eqs. (3) themselves, the novel parts of our proposal seem to be the mnemonic trick of using the familiar eqs. (1) to remember eqs. (3), and use of $k_0 \cong 900$ SI units and $c \cong 3 \times 10^8$ m/s instead of $\sqrt{\varepsilon_0}$ and $\sqrt{\mu_0}$ in numerical evaluations, as in eqs. (5).

Also, the idea of adding to the SI a set of auxiliary quantities with standard definitions in order to satisfy the needs of the physics community has not, to the best of our knowledge, been proposed before. The candela, lumen, etc., were tacked on to the SI to meet the needs of illumination engineers, but no one seems to have proposed doing a similar favor for physicists.

Students exposed to a course of study such as the one outlined above should emerge with the feeling that no necessary conflict exists between "practical" and "scientific" unit systems. Such a feeling might eventually lead to the adoption by the physics community of a Système International which had been suitably modified, in a way such as the one we suggest above, to meet the special needs of physicists.

APPENDIX: AN ALTERNATE DEFINITION OF CURRENT

The observant critic will have noticed that the auxiliary quantities form what is called a mixed unit system, in which J is a "magnetic" unit and ρ_x is an "electrostatic" unit. That is, we have treated J primarily as the source of the magnetic induction field B. This choice is largely a matter of taste, and we realize that most modern non-SI texts use an electrostatic definition of both current and charge.

The advantages of our choice are that the factor 1/c multiplying J is absent from such equations as $\Box^2 \vec{A} = \vec{J}$, $\vec{F} = \rho_x\vec{E} + \vec{J} \times \vec{B}$ and $\vec{\nabla} \times \vec{B} = \vec{J} + \frac{1}{c} \frac{\partial\vec{E}}{\partial t}$. Also $(\vec{J}, \rho_x)$ forms a four-vector like $(\vec{A}, V)$, and the continuity equation $\vec{\nabla} \cdot \vec{J} + \frac{1}{c} \frac{\partial\rho_x}{\partial t} = 0$ has a "c" multiplying the "dt" in just the familiar way.

The disadvantages of our choice are the necessity to remember that J is a special quantity, defined by $J = \rho_x \frac{v}{c}$ and not $J = \rho_x v$ as one might expect. Thus J and ρ_x have the same units, which may bother some people. Also, Ohm's law $J = \frac{1}{c} \rho_x E$ contains a "c" factor, as does the Joule heating formula $cJ \cdot E = \sigma_x E^2 = \frac{(cJ)^2}{\sigma_x}$. These latter objections can be overcome somewhat by simply noting that Ohm's law and Joule heating always involve the "electrostatic" current cJ.

We feel that, particularly for radiation theory, quantum theory, and covariant field expressions, our choice is preferable. Also, the definitions in eq. (3) follow completely symmetrically from eqs. (2), which may have some pedagogical value. But, an electrostatic definition of current is also possible. We denote it below by $J^{(\text{elec})}$ and $I^{(\text{elec})}$. The only change in definitions of auxiliary quantities is that the last of eqs. (3), eq. (3-d), is dropped. The definition of current $I^{(elec)}$ is not read directly from eq. (2-b). Instead, in Table I, the definitions $I^{(elec)} = \frac{1}{\sqrt{\varepsilon_0}} I$ and $J^{(elec)} = \frac{1}{\sqrt{\varepsilon_0}} J$ are added to the class of "auxiliary quantities defined like Q". Also, I and J are removed from the class "auxiliary quantities defined like I" and the class has its name changed to "auxiliary quantities defined inverse to B". Otherwise, Table I is unchanged. In Table III, the only change is to substitute $J = \frac{1}{c} J^{(elec)}$ everywhere that J appears, and to cancel some redundant "c" factors which then appear. The only change in Table IV is that the auxiliary inductance becomes $L^{(elec)} = \varepsilon_0 L$, and $J = \frac{1}{c} J^{(elec)}$ and $I = \frac{1}{c} I^{(elec)}$ are substituted as in Table III.

Needless to say, the important point is not whether J or $J^{(elec)}$ is used, but that one or the other choice be universally and officially adopted.

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TABLE I

Definitions of Auxiliary quantities

Class	Name of unit	S.I. symbol	S.I. units	Auxiliary symbol	Auxiliary quantity definition	Rule for numerical evaluation
Auxiliary quantities are defined like *E*	Electric field	E	volts/m	*E*	$E = \sqrt{\varepsilon_0}\, \mathrm{E}$	$E = \frac{k_0}{c}\, \mathrm{E}$
	Electric potential	V	volt	*V*	$V = \sqrt{\varepsilon_0}\, \mathrm{V}$	$V = \frac{k_0}{c}\, \mathrm{V}$
Auxiliary quantities are defined like *Q*	Charge	Q	coulomb	*Q*	$Q = \frac{1}{\sqrt{\varepsilon_0}}\, \mathrm{Q}$	$Q = \frac{c}{k_0}\, \mathrm{Q}$
	Charge density	ρ	coulombs/m^3	ρ_x	$\rho_x = \frac{1}{\sqrt{\varepsilon_0}}\, \rho$	$\rho_x = \frac{c}{k_0}\, \rho$
	Electric displacement	D	coulombs/m^2	*D*	$D = \frac{1}{\sqrt{\varepsilon_0}}\, \mathrm{D}$	$D = \frac{c}{k_0}\, \mathrm{D}$
	Polarization	P	coulombs/m^2	*P*	$P = \frac{1}{\sqrt{\varepsilon_0}}\, \mathrm{P}$	$P = \frac{c}{k_0}\, \mathrm{P}$
Auxiliary quantities are defined like *B*	Magnetic induction	B	webers/m^2	*B*	$B = \frac{1}{\sqrt{\mu_0}}\, \mathrm{B}$	$B = k_0\, \mathrm{B}$
	Vector potential	A	amperes/m^2	*A*	$A = \frac{1}{\sqrt{\mu_0}}\, \mathrm{A}$	$A = k_0\, \mathrm{A}$
	Magnetic flux	Φ	weber	Φ_x	$\Phi_x = \frac{1}{\sqrt{\mu_0}}\, \Phi$	$\Phi_x = k_0\, \Phi$
Auxiliary quantities are defined like *I*	Current	I	ampere	*I*	$I = \sqrt{\mu_0}\, \mathrm{I}$	$I = \frac{1}{k_0}\, \mathrm{I}$
	Current density	J	amperes/m^2	*J*	$J = \sqrt{\mu_0}\, \mathrm{J}$	$J = \frac{1}{k_0}\, \mathrm{J}$
	Magnetic field	H	amperes/m	*H*	$H = \sqrt{\mu_0}\, \mathrm{H}$	$H = \frac{1}{k_0}\, \mathrm{H}$
	Magnetization	M	amperes/m	*M*	$M = \sqrt{\mu_0}\, \mathrm{M}$	$M = \frac{1}{k_0}\, \mathrm{M}$

N.B. The subscript "x" is used for Greek-letter auxiliary symbols.

Table II

Approximate values of the conversion factors to various levels of accuracy

Quantity	Approximate value to 1% accuracy (blackboard accuracy)	Approximate value to 100 ppm accuracy	Approximate value to .004 ppm accuracy[7]
k_o	900	892	892.062058
c	3×10^8	2.998×10^8	299792458.

N.B. The exact value of k_o is $k_o = (4\pi \times 10^{-7})^{-1/2}$.

Table III

Some equations of electromagnetic theory written both in standard SI units and in terms of auxiliary quantities

Comment	Equation in S.I. units	Same equations in terms of auxiliary quantities
Maxwell's equations	$\vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_0} \rho$	$\vec{\nabla} \cdot \vec{\mathcal{E}} = \rho_x$
	$\vec{\nabla} \cdot \vec{B} = 0$	$\vec{\nabla} \cdot \vec{\mathcal{B}} = 0$
	$\vec{\nabla} \cdot \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\vec{\nabla} \cdot \vec{\mathcal{E}} = -\frac{1}{c} \frac{\partial \vec{\mathcal{B}}}{\partial t}$
	$\frac{1}{\mu_0} \vec{\nabla} \times \vec{B} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$	$\vec{\nabla} \times \vec{\mathcal{B}} = \vec{\mathcal{J}} + \frac{1}{c} \frac{\partial \vec{\mathcal{E}}}{\partial t}$
Empty space wave equations	$(-\nabla^2 + \varepsilon_0\mu_0 \frac{\partial^2}{\partial t^2})\vec{E} = 0$	$(-\nabla^2 + \frac{1}{c^2} \frac{\partial^2}{\partial t^2})\vec{\mathcal{E}} = 0$
	(Same for B)	(Same for $\mathcal{B}$)
Continuity equation	$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$	$\vec{\nabla} . \vec{\mathcal{J}} + \frac{1}{c} \frac{\partial \rho_x}{\partial t} = 0$
Lorentz volume force, F	$\vec{F} = \rho(\vec{E} + \vec{v} \times \vec{B})$	$\vec{F} = \rho_x(\vec{\mathcal{E}} + \frac{1}{c} \vec{v} \times \vec{\mathcal{B}})$
	$= \rho \vec{E} + \vec{J} \times \vec{B}$	$= \rho_x \vec{\mathcal{E}} + \vec{\mathcal{J}} \times \vec{\mathcal{B}}$
Energy density of field, U	$U = \frac{\varepsilon_0 E^2 + \frac{1}{\mu_0} B^2}{2}$	$U = \frac{\mathcal{E}^2 + \mathcal{B}^2}{2}$
Poynting vector, N	$\vec{N} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$	$\vec{N} = c(\vec{\mathcal{E}} \times \vec{\mathcal{B}})$
Maxwell stress tensor, T	$\overleftrightarrow{T} = \varepsilon_0 \vec{E}\vec{E} - \frac{1}{2}\varepsilon_0 E^2 \overleftrightarrow{\delta}$	$\overleftrightarrow{T} = \vec{\mathcal{E}}\vec{\mathcal{E}} - \frac{1}{2} \mathcal{E}^2 \overleftrightarrow{\delta}$
	$+ \frac{1}{\mu_0} \vec{B}\vec{B} - \frac{1}{2} \frac{1}{\mu_0} B^2 \overleftrightarrow{\delta}$	$+ \vec{\mathcal{B}}\vec{\mathcal{B}} - \frac{1}{2} \mathcal{B}^2 \overleftrightarrow{\delta}$
Lorentz condition	$\vec{\nabla} \cdot \vec{A} + \frac{\partial V}{\partial t} = 0$	$\vec{\nabla} \cdot \vec{\mathcal{A}} + \frac{1}{c} \frac{\partial \mathcal{V}}{\partial t} = 0$
Potentials from sources (Lorentz gauge)	$\Box^2 \vec{A} = \mu_0 \vec{J}$	$\Box^2 \vec{\mathcal{A}} = \vec{\mathcal{J}}$
	$\Box^2 V = \frac{1}{\varepsilon_0} \rho$	$\Box^2 \mathcal{V} = \rho_x$
Fields from potentials	$\vec{B} = \vec{\nabla} \times \vec{A}$	$\vec{\mathcal{B}} = \vec{\nabla} \times \vec{\mathcal{A}}$
	$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$	$\vec{\mathcal{E}} = -\vec{\nabla} \mathcal{V} - \frac{1}{c} \frac{\partial \vec{\mathcal{A}}}{\partial t}$
Macroscopic fields	$E = \frac{1}{\varepsilon_0} D - \frac{1}{\varepsilon_0} P$	$\mathcal{E} = \mathcal{D} - \mathcal{P}$
	$B = \mu_0 H + \mu_0 M$	$\mathcal{B} = \mathcal{H} + \mathcal{M}$
Conservation of energy	$\vec{\nabla} \cdot \vec{N} + \frac{\partial U}{\partial t} = \vec{J} \cdot \vec{E}$	$\vec{\nabla} \cdot \vec{N} + \frac{\partial U}{\partial t} = c\vec{\mathcal{J}} \cdot \vec{\mathcal{E}}$

Table IV

Auxiliary quantities and relations which are
not useful, and which are given here only for completeness

Name of unit	S.I. symbol	S.I. units	Auxiliary symbol	Auxiliary quantity definition	Rule for numerical evaluation
Inductance	L	henry	L	$L = \frac{1}{\mu_0}\mathrm{L}$	$L = k_0^2\,\mathrm{L}$
Capacitance	C	farad	C	$C = \frac{1}{\varepsilon_0}\mathrm{C}$	$C = \left(\frac{c}{k_0}\right)^2 \mathrm{C}$
Resistance	R	ohm	R	$R = \varepsilon_0\,\mathrm{R}$	$R = \left(\frac{k_0}{c}\right)^2 \mathrm{R}$
Conductivity	σ	$(\text{m-ohm})^{-1}$	σ_x	$\sigma_x = \frac{1}{\varepsilon_0}\sigma$	$\sigma_x = \left(\frac{c}{k_0}\right)^2 \sigma$

Comment	Equation in S.I. units	Same equation in terms of auxiliary quantities
Electrostatic energy U_e	$U_e = \frac{1}{2}\frac{Q^2}{C}$	$U_e = \frac{1}{2}\frac{Q^2}{C}$
Magnetostatic energy U_m	$U_m = \frac{1}{2}\,L\,I^2$	$U_m = \frac{1}{2}\,L\,I^2$
Magnetic flux of current distribution	$\Phi = L\,I$	$\Phi_x = L\,I$
Induced e.m.f.	$V = -L\frac{dI}{dt}$	$V = -\frac{1}{c}\,L\frac{dI}{dt}$
Ohm's law	$J = \sigma E$	$J = \frac{1}{c}\,\sigma_x E$
Joule heating rate W in conducting medium	$W = J \cdot E = \sigma E^2 = \frac{J^2}{\sigma}$	$W = c\,J \cdot E = \sigma_x E^2 = \frac{(cJ)^2}{\sigma_x}$

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