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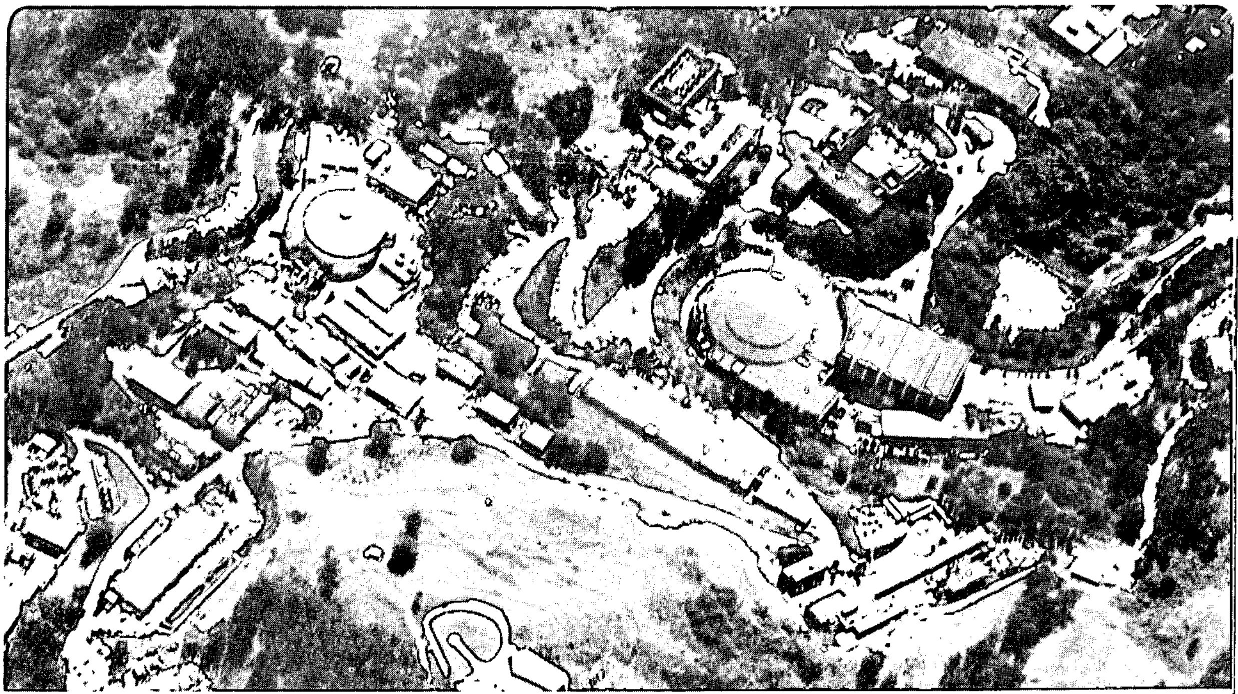

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Neutral-Kaon Regeneration Probabilities At Asymmetric ϕ-Factory Energies.*

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ABSTRACT

Theoretical estimates of neutral-kaon regeneration amplitudes and probabilities are given for various materials and kaon momenta between 0.35 and 2.6 GeV/c. These estimates can be used, in particular, to optimize experimental conditions at asymmetric ϕ-factories when regeneration of neutral kaons is involved. As an example, this data is used to find best parameters for a test of non-local interferences in quantum mechanics.

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Contents

1 Introduction.

Asymmetric ϕ-factories have been proposed a long time ago [1], and interest in them has been revived recently [2]. One advantage of ϕ-factories over symmetric ones is that regeneration of neutral kaons can be made in larger quantities, because kaon momenta are higher and short-lived kaons K_S's travel farther inside of a piece of material interposed on their path. To help design any experiment involving kaon regeneration, this paper gives theoretical estimates of kaon-regeneration amplitudes and probabilities in various materials at kaon momenta between 0.35 and 2.6 GeV/c. This paper also gives an example of how to use these estimates to optimize a test of quantum mechanics.

An asymmetric ϕ-factory is an e^+e^-–collider with two beams of different momenta and an energy in the center of mass equal to the mass of the ϕ-meson, 1020 MeV/c^2, [3]. Systems of two neutral kaons in a state of negative charge conjugation ($C = -$) are produced with a cross section of about 1.3 μb. Let E^+ be the highest of the two beam energies. The momentum p_{kaon} of the kaons is related to E^+,

$$p_{\text{kaon}} \approx \frac{1}{2}\left(E^+ - \frac{0.26\ \text{GeV}^2}{E^+}\right) \approx \frac{E^+}{2}\ . \tag{1}$$

It is possible to build an asymmetric ϕ-factory as an add on to an intense light-source, [1] or to an asymmetric B-factory, [4]. Such light-source or B-factory would already have an electron or a positon beam of energy probably between 0.7 and 5 GeV and that beam could be used in the asymmetric ϕ-factory as the beam of highest energy E^+. Then to optimize experimental conditions of kaon regeneration at such ϕ-factories, one needs estimates of kaon regeneration probabilities in the momentum range considered here, 0.35 to 2.6 GeV/c. Of course these estimates may be approximate but they should be given for a wide variety of materials.

2 Regeneration Probabilities.

Let σ_{tot} be the total cross section to K_L's of the atoms of a material, ν the atomic density, and ℓ the thickness of the regenerator. Neglecting the decay probability of the long-lived kaon K_L, one can compute the K_L-attenuation factor due to nuclear absorption and scattering in the regenerator, i.e. the probability p_{thru} for a K_L not to interact in the material :

$$p_{\text{thru}} = e^{-\nu\ell\sigma_{\text{tot}}} \ . \tag{2}$$

For K_S's, decay probabilities cannot be neglected neither inside nor outside of the material. The average decay length is

$$\lambda_S = 5.4 \ \text{cm}/(\text{GeV/c}) \quad p_{\text{kaon}} \ . \tag{3}$$

In the regenerator, the K_S-attenuation factor is $p_{\text{thru}} \ e^{-\ell/\lambda_S}$.

Knowing the difference Δf between forward scattering amplitudes of K^0 and $\overline{K^0}$ by the atoms, the mean lifetime τ_S of the K_S, the kaon mass m, the K_L–K_S mass difference Δm, and the time t taken by the kaon in its own rest frame to traverse the regenerator, one can predict the probability p_{regen} for a kaon to be coherently regenerated [5] :

$$\begin{aligned} p_{\text{regen}} &= |\rho|^2 \, p_{\text{thru}} \ , && (4) \\ \text{where} \qquad \rho &= \frac{\pi\nu}{\frac{1}{2\tau_S} - i\Delta m} \, \frac{\Delta f}{m} \, \kappa \ , && (5) \\ \text{and} \qquad \kappa &= 1 - e^{\left(-\frac{1}{2\tau_S} + i\Delta m\right)t} && (6) \end{aligned}$$

A computation of scattering amplitudes based on an optical model, [6], and charge symmetry was made for various elements, using measured protons and neutrons total cross sections to K^+ and K^-, [7], and nuclear distributions of nucleons given in Ref. [8]. The total absorption cross section σ_{tot} for K_L's (which is the same as for K_S's), the real part and the imaginary parts of Δf in various elements at various kaon momenta have been computed and are given in Tables 1 to 10. The data for hydrogen and nitrogen have been included, for the case where these elements would be components of a chemical compound used in the regenerator.

As a function of the regenerator thickness, the number of regenerated kaons first increases, then reaches a maximum [5] because of K_S decay and absorption in the material. For each kaon momentum and each one of the elements of interest, Tables 1 to 10 also show the value of that thickness ℓ along the path of the kaon that gives maximum regeneration, the corresponding regeneration probability p_{regen} given by Eq. (4), and the K_L-absorption factor p_{thru} given by Eq. (2). For all elements including hydrogen and nitrogen, the atomic density of the solid state was assumed.

Another important parameter in the design of an experiment involving coherent regeneration is the amount of background due to incoherent regeneration (i.e. regeneration associated with a nucleus recoiling in the material). In general that background is distinguishable from the signal (i.e. coherent regeneration) because, in the case of incoherent regeneration, there is an angle θ between directions of incident and outgoing kaons. However, because of limited experimental resolution $\delta\theta$ in the determination of the angle θ, background and signal cannot be distinguished in very forward directions.

There is a cone of ambiguity of solid angle

$$\Omega = \pi \, \delta\theta^2 \, . \tag{7}$$

An expression for the amount of incoherent background per unit of solid angle can be found in Ref. [5]. From that expression it is easy to compute the ratio of the amount of background that does not get discarded by a cut in θ to the signal due to the coherent regeneration. That ratio is the product of the solid angle Ω of that cone of ambiguity by a coefficient B.

$$n_{LL}^{\mathrm{bckg}} = \Omega \, B \, ; \tag{8}$$

$$B \approx 6 \; 10^{21} \; \frac{p_{\mathrm{kaon}}(1 - e^{-t/\tau_S})}{\nu \, |\kappa|^2} \, , \tag{9}$$

where κ is the quantity given by Eq. (6). The value of the coefficient B is given in the last column of Tables 1 to 10 in % per millisteradian.

3 A Test of Non-Local Destructive Interferences.

Hereafter, as an illustration, Tables 1 to 10 are used to determine the best parameters of a test of non-local interferences of the type suggested in Ref. [9]. The relationship between the two-neutral-kaon system produced by ϕ-decay and the EPR-paradox, [10], is well known [11], and tests of non-locality related to that paradox can be done in experiments involving kaon regeneration at an asymmetric ϕ-factory [9][12].

In the asymmetric factory, both neutral kaons are moving in the direction of the beam of the highest energy, one kaon slightly above the beam plane (the "up" kaon) and the other kaon below that plane (the "down" kaon), as shown on Fig. 1. Two regenerators are involved, one on the path of the

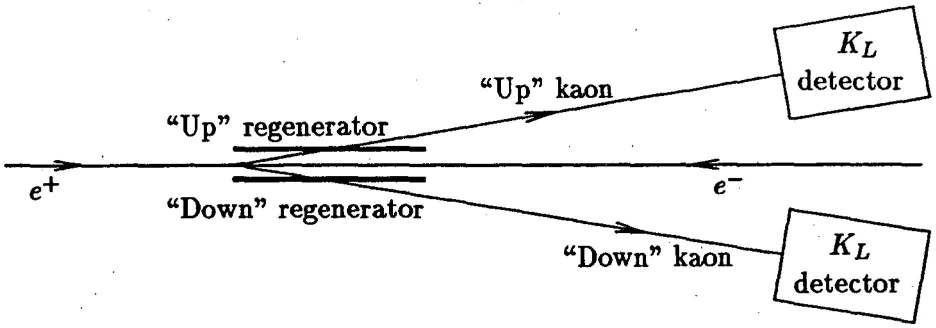

Figure 1: *Setup for the destructive interference test.*

"up" kaon and one on the "down" path. On Fig. 1, the two regenerators are disposed more conveniently from an experimental point of view than in Ref. [9]. They are disposed along a parallel to the beam line at some distance r, keeping out of the beam which has to be cleared to allow accelerator operations. However what we refer to as the regenerator thickness ℓ is not the thickness normal to the beam line, Δr, but the distance traveled inside of the material by the kaon along its path. Let Θ be the angle at which a K_S is emitted with respect to the beam line.

$$\ell = \frac{\Delta r}{\sin\Theta} . \tag{10}$$

For our estimate, for sake of simplicity, we will use an average of $\sin\Theta$ in Eq. (10).

First, the test consists of checking that, without regenerators, there are only events of types $K_L K_S$ and $K_S K_L$, using the first symbol in the event type to refer to the state of the "up" kaon and the second symbol to the state of the "down" kaon. Then, with only the "up" regenerator in place, one checks that the number of $K_L K_L$ events is equal to the predicted number of "up" K_S's (from systems born $K_S K_L$) regenerated into K_L in the "up" regenerator. Then, similarly, with only the "down" regenerator on, one checks that the number of $K_L K_L$ events is equal to what is predicted from "down" $K_S \rightarrow K_L$ regeneration. Finally, with both regenerators in place, the test requires the measurement of the number n_{LL} of $K_L K_L$ events. In that final setup, there will be K_S's regenerated into K_L in each of the two regenerators while the associated K_L traverses the other regenerator unperturbed, but quantum mechanics predicts also a non-local interference effect between these two regeneration processes of two different kaons in two distant regenerators. If, in the computation, one ignores the interference term, the number of $K_L K_L$ events is

$$n_{LL}^{\text{no int}} = N(r)\, p_{\text{regen}}\, p_{\text{thru}} \ , \tag{11}$$

where $N(r)$ is the number of systems $K_L K_S$ or $K_S K_L$, where the K_S reaches the location of the regenerators at the distance r from the beam line, Fig. 1.

Let us call n_{LL}^{int} the contribution to n_{LL} of the interference term,

$$n_{LL} = n_{LL}^{\text{no int}} + n_{LL}^{\text{int}} \ . \tag{12}$$

Quantum mechanics predicts this interference effect to be 100% destructive, i.e. n_{LL}^{int} to be a negative number of events

$$n_{LL}^{\text{int}} = -n_{LL}^{\text{no int}} = -N(r)\, p_{\text{regen}}\, p_{\text{thru}} \ . \tag{13}$$

From Eqs. (12) and (13), one expects n_{LL} to be zero. Any deviation from zero is a violation of quantum theory. Such violation could be expressed quantitatively as

$$\epsilon = \frac{n_{LL}}{n_{LL}^{\text{no int}}} = \frac{n_{LL}}{N(r)\, p_{\text{regen}} p_{\text{thru}}} \ . \tag{14}$$

4 Optimization of the Test Parameters.

To detect the smallest possible violation ϵ for a given integrated luminosity, the product $p_{\rm regen}\ p_{\rm thru}$ should be maximized. The optimum kaon momentum could be found by looking at the values of $p_{\rm regen}$ and $p_{\rm thru}$ of Tables 1 to 10. The maximum value of the product $p_{\rm regen}\ p_{\rm thru}$ is

$$p_{\rm regen}\ p_{\rm thru} = 2\ 10^{-3} \tag{15}$$

for a kaon beam of 0.7 GeV/c and a regenerator 9 cm long made of copper. According to Eq. (1), that kaon momentum could be obtained in a ϕ-factory where the highest of the two beam energies is about

$$E^+ = 1.6 \text{ GeV} , \tag{16}$$

and the lowest beam energy about 160 MeV.

For these beam conditions, let us determine the parameters of an experiment that could detect a violation ϵ as little as 0.01. The actual measured number $n_{LL}^{\rm meas}$ of $K_L K_L$ events is the sum of the contributions of the signal n_{LL} (coherent regeneration) due to the reduction of the interference term by the factor $1-\epsilon$, and of the incoherent regeneration $n_{LL}^{\rm bckg}$, given by Eq. (8), and which is not subjected to a non-local interference effect :

$$n_{LL}^{\rm meas} = n_{LL} + n_{LL}^{\rm bckg} = \epsilon\ n_{LL}^{\rm no\ int} + \Omega\ B\ n_{LL}^{\rm no\ int} , \tag{17}$$

$$\epsilon = \frac{n_{LL}^{\rm meas}}{n_{LL}^{\rm no\ int}} - \Omega B , \tag{18}$$

where B is the coefficient shown in Table 8, i.e. 5%/mstr.

One contribution to the systematic error on ϵ comes from the uncertainty on the solid angle Ω of the cone of ambiguity. Assuming Ω is known with a relative error of 50% or less, a violation of ϵ of 0.01 can be detected if

$$\Omega B \approx 0.01 , \tag{19}$$

$$\text{i.e.} \qquad \Omega = \frac{0.01}{B} = 0.2 \text{ mstr} . \tag{20}$$

Thus, using Eq. (7), we need to measure the angle θ between incident and emerging kaon on the regenerator with an accuracy equal to or better than

$$\delta\theta = \sqrt{\frac{0.2 \text{ mstr}}{\pi}} = 8 \text{ milliradian} . \tag{21}$$

As to statistical errors, we want to insure a minimum two-standard-deviation significance of the measurement if ϵ is equal to or larger than 0.01. For that purpose, the one-standard-deviation statistical error on n_{LL}^{meas} should be smaller than half the signal, i.e. $\epsilon\ n_{LL}^{\text{no int}}$. Using Eq. (17), one needs

$$\sqrt{(\epsilon + \Omega B)\ n_{LL}^{\text{no int}}} < \frac{1}{2}\ \epsilon\ n_{LL}^{\text{no int}} . \tag{22}$$

If Ω is equal to or smaller than the value defined in Eq. (19), Eq. (22) can satisfied for any ϵ larger than 0.01 by making

$$n_{LL}^{\text{no int}} = 1000 \text{ events} , \tag{23}$$

i.e., using Eqs. (11) and (15),

$$N(r) = 500\ 000 \text{ events} . \tag{24}$$

$N(r)$ is the number of events where the K_S reaches one of the two regenerators shown on Fig. 1. It depends on the distance r of the regenerators to the beam line, which is hopefully small but is limited by the characteristics of the accelerator. To pursue the calculation, we will use the figure

$$r = 1 \text{ cm} . \tag{25}$$

To reach a regenerator, K_S emitted at an angle Θ from the beam line have to travel a distance 1 cm/sin Θ. From Eq. (3) and from invariance of transverse momenta in Lorentz transformations, the attenuation due to the K_S decay can be computed as

$$e^{-1\ \mathrm{cm}/(\lambda_S \sin\Theta)} = e^{-1.68\ /\sin\Theta^*} \approx \frac{1}{10}\,, \tag{26}$$

where Θ^* is the angle of the K_S with respect to the beam direction in the ϕ restframe; $\sin\Theta^*$ is about 0.85 in average.

It follows that about 5 million $\phi \rightarrow$neutral kaons are needed, i.e. an integrated luminosity of about 4 pb^{-1}. Such integrated luminosity could be reached in one effective month ($\approx 10^6$ seconds) of running time with an average luminosity of 4 10^{30} $\mathrm{cm}^{-2}\mathrm{s}^{-1}$.

Note that, using the present data, we reduced the value of the integrated luminosity needed for the test by a factor of more than 30 below the values of luminosity considered to be necessary in Ref. [4].

5 Conclusions.

Regeneration amplitudes and probabilities have been calculated for kaon momenta of 0.35 to 2.6 GeV/c. They are summarized in Table 1 to 10. They can be used in any experiment involving kaon regeneration in this momentum region.

An example of possible use of those tables has been given and shows that a test of non-local interferences could be pursued with a sensitivity to violations as small as 1% in reduction of the interference term, at a ϕ-factory with a beam of 1.6 GeV colliding with a beam of 160 MeV, a month of running time at an average luminosity of 4 10^{30}, and a resolution on the measurement of the angle θ better or equal to 8 milliradian.

If there is no opportunity to build a ϕ-factory such that the highest energy of the two beams is 1.6 GeV, Table 1 to 10 can still be used to find the optimum conditions with other kaon momenta. Data has been given for various elements to handle cases where the construction of the machine requires that these elements lie in the path of the neutral kaons.

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p_{kaon} (GeV/c)	σ_{tot} (mb)	$\Re(\Delta f)$ (Fermi)	$\Im(\Delta f)$ (Fermi)	ℓ (cm)	p_{regen} $\times 1000$	p_{thru} %	B %/mstr
0.35	26.	-0.26	-0.41	9.0	0.00	0.99	3.8
0.40	25.	-0.30	-0.42	10.0	0.00	0.99	4.3
0.50	21.	-0.35	-0.30	13.0	0.00	0.99	5.4
0.60	21.	-0.48	-0.34	15.0	0.00	0.99	6.5
0.70	23.	-0.76	-0.40	18.0	0.00	0.98	7.6
0.80	24.	-0.78	-0.42	20.0	0.00	0.98	8.7
0.90	26.	-0.88	-0.65	22.0	0.01	0.97	9.8
1.00	30.	-0.31	-0.85	24.0	0.00	0.97	10.9
1.10	27.	-0.23	-0.55	28.0	0.00	0.97	12.0
1.20	26.	-0.40	-0.50	30.0	0.00	0.96	13.0
1.30	24.	-0.50	-0.57	32.0	0.00	0.97	14.1
1.40	23.	-0.55	-0.51	34.0	0.00	0.96	15.2
1.50	23.	-0.55	-0.46	38.0	0.00	0.96	16.3
1.60	22.	-0.51	-0.42	40.0	0.00	0.96	17.4
1.80	21.	-0.60	-0.36	44.0	0.00	0.96	19.6
2.00	20.	-0.66	-0.26	48.0	0.00	0.96	21.7
2.20	20.	-0.64	-0.20	48.0	0.00	0.96	24.1
2.40	19.	-0.69	-0.16	48.0	0.00	0.96	26.6
2.60	18.	-0.69	-0.07	48.0	0.00	0.96	29.5

Table 1: HYDROGEN – Kaon momentum p_{kaon}, total cross section σ_{tot}, real and imaginary parts of the regeneration amplitude Δf; optimized regenerator thickness ℓ, corresponding regeneration probability p_{regen}, and absorption factor p_{thru}; percentage of background due to incoherent background.

p_{kaon} (GeV/c)	σ_{tot} (mb)	$\Re(\Delta f)$ (Fermi)	$\Im(\Delta f)$ (Fermi)	ℓ (cm)	p_{regen} ×1000	p_{thru} %	B %/mstr
0.35	248.	-3.33	-3.34	7.0	0.70	0.81	1.4
0.40	241.	-2.97	-3.67	8.0	0.68	0.79	1.6
0.50	190.	-3.48	-2.90	10.0	0.63	0.79	2.1
0.60	181.	-4.39	-3.14	12.0	0.86	0.76	2.5
0.70	181.	-5.70	-3.87	14.0	1.34	0.73	2.9
0.80	193.	-5.64	-4.63	15.0	1.41	0.70	3.3
0.90	207.	-5.66	-5.74	16.0	1.60	0.66	3.8
1.00	219.	-3.90	-6.22	18.0	1.23	0.61	4.2
1.10	206.	-2.16	-5.15	19.0	0.70	0.62	4.7
1.20	196.	-2.77	-4.15	20.0	0.55	0.62	5.2
1.30	182.	-3.81	-3.93	22.0	0.66	0.61	5.6
1.40	182.	-4.76	-3.94	24.0	0.81	0.58	6.0
1.50	183.	-5.19	-4.48	24.0	0.96	0.58	6.6
1.60	179.	-4.92	-4.39	26.0	0.86	0.56	7.0
1.80	177.	-4.79	-4.67	28.0	0.83	0.54	8.1
2.00	175.	-5.33	-5.02	30.0	0.94	0.52	9.1
2.20	167.	-5.61	-4.31	32.0	0.85	0.52	10.2
2.40	164.	-5.80	-4.30	34.0	0.85	0.50	11.3
2.60	161.	-6.06	-4.10	36.0	0.83	0.49	12.4

Table 2: BERYLLIUM – Kaon momentum p_{kaon}, total cross section σ_{tot}, real and imaginary parts of the regeneration amplitude Δf; optimized regenerator thickness ℓ, corresponding regeneration probability p_{regen}, and absorption factor p_{thru}; percentage of background due to incoherent background.

p_{kaon} (GeV/c)	σ_{tot} (mb)	$\Re(\Delta f)$ (Fermi)	$\Im(\Delta f)$ (Fermi)	ℓ (cm)	p_{regen} ×1000	p_{thru} %	B %/mstr
0.35	300.	-3.75	-3.61	7.0	1.26	0.71	1.1
0.40	291.	-3.41	-3.99	8.0	1.24	0.69	1.3
0.50	235.	-4.00	-3.30	10.0	1.20	0.69	1.6
0.60	228.	-5.02	-3.69	11.0	1.64	0.67	1.9
0.70	231.	-6.49	-4.65	12.0	2.50	0.64	2.3
0.80	243.	-6.37	-5.43	14.0	2.51	0.58	2.6
0.90	258.	-6.32	-6.72	14.0	2.76	0.56	3.1
1.00	269.	-4.20	-7.08	15.0	2.01	0.52	3.5
1.10	255.	-2.47	-5.78	16.0	1.14	0.52	3.9
1.20	245.	-3.26	-4.72	18.0	0.92	0.49	4.2
1.30	229.	-4.48	-4.58	19.0	1.14	0.50	4.6
1.40	228.	-5.51	-4.56	20.0	1.34	0.48	5.1
1.50	229.	-5.97	-5.11	20.0	1.53	0.48	5.6
1.60	225.	-5.69	-4.98	22.0	1.38	0.45	5.9
1.80	221.	-5.64	-5.24	24.0	1.31	0.43	6.8
2.00	219.	-6.28	-5.55	24.0	1.43	0.43	8.0
2.20	210.	-6.60	-4.76	26.0	1.29	0.42	8.9
2.40	206.	-6.87	-4.72	28.0	1.27	0.40	9.8
2.60	202.	-7.16	-4.45	30.0	1.22	0.38	10.7

Table 3: CARBON – Kaon momentum p_{kaon}, total cross section σ_{tot}, real and imaginary parts of the regeneration amplitude Δf; optimized regenerator thickness ℓ, corresponding regeneration probability p_{regen}, and absorption factor p_{thru}; percentage of background due to incoherent background.

p_{kaon} (GeV/c)	σ_{tot} (mb)	$\Re(\Delta f)$ (Fermi)	$\Im(\Delta f)$ (Fermi)	ℓ (cm)	p_{regen} ×1000	p_{thru} %	B %/mstr
0.35	338.	-4.1	-3.8	8.0	0.14	0.89	4.0
0.40	327.	-3.8	-4.3	9.0	0.15	0.88	4.5
0.50	268.	-4.4	-3.6	11.0	0.15	0.88	5.7
0.60	259.	-5.5	-4.1	14.0	0.21	0.85	6.8
0.70	263.	-7.1	-5.2	15.0	0.33	0.84	8.0
0.80	276.	-6.9	-6.0	17.0	0.35	0.81	9.1
0.90	292.	-6.8	-7.4	19.0	0.40	0.78	10.3
1.00	303.	-4.6	-7.7	20.0	0.31	0.76	11.5
1.10	288.	-2.7	-6.3	22.0	0.18	0.75	12.7
1.20	277.	-3.6	-5.2	24.0	0.15	0.74	13.8
1.30	260.	-5.0	-5.0	26.0	0.18	0.74	15.0
1.40	259.	-6.1	-5.0	28.0	0.22	0.72	16.1
1.50	260.	-6.6	-5.6	30.0	0.26	0.71	17.3
1.60	255.	-6.3	-5.5	32.0	0.24	0.70	18.4
1.80	252.	-6.2	-5.8	34.0	0.24	0.68	21.0
2.00	249.	-7.0	-6.1	38.0	0.27	0.66	23.3
2.20	239.	-7.3	-5.2	40.0	0.26	0.65	26.0
2.40	234.	-7.6	-5.2	44.0	0.26	0.63	28.3
2.60	231.	-8.0	-4.9	46.0	0.26	0.63	31.0

Table 4: NITROGEN – Kaon momentum p_{kaon}, total cross section σ_{tot}, real and imaginary parts of the regeneration amplitude Δf; optimized regenerator thickness ℓ, corresponding regeneration probability p_{regen}, and absorption factor p_{thru}; percentage of background due to incoherent background.

p_{kaon} (GeV/c)	σ_{tot} (mb)	$\Re(\Delta f)$ (Fermi)	$\Im(\Delta f)$ (Fermi)	ℓ (cm)	p_{regen} $\times 1000$	p_{thru} %	B %/mstr
0.35	593.	-6.8	-5.5	7.0	0.55	0.78	3.0
0.40	572.	-6.3	-6.3	8.0	0.55	0.76	3.4
0.50	477.	-7.4	-5.6	10.0	0.59	0.75	4.2
0.60	463.	-9.0	-6.6	12.0	0.82	0.72	5.1
0.70	469.	-11.4	-8.6	13.0	1.26	0.69	6.0
0.80	487.	-11.0	-10.0	15.0	1.27	0.64	6.9
0.90	510.	-10.6	-12.0	16.0	1.38	0.61	7.9
1.00	526.	-7.3	-12.3	17.0	1.02	0.58	8.9
1.10	504.	-4.5	-10.1	18.0	0.60	0.58	9.9
1.20	487.	-5.9	-8.2	20.0	0.49	0.56	10.8
1.30	460.	-8.2	-8.1	22.0	0.61	0.54	11.6
1.40	460.	-10.0	-8.1	22.0	0.74	0.54	12.9
1.50	461.	-10.8	-9.1	24.0	0.85	0.51	13.7
1.60	453.	-10.4	-8.9	24.0	0.78	0.52	15.0
1.80	447.	-10.3	-9.4	26.0	0.75	0.50	17.2
2.00	444.	-11.5	-10.0	28.0	0.84	0.47	19.5
2.20	427.	-12.2	-8.7	30.0	0.77	0.46	21.7
2.40	419.	-12.8	-8.6	32.0	0.77	0.45	24.0
2.60	413.	-13.4	-8.2	34.0	0.76	0.43	26.3

Table 5: ALUMINUM – Kaon momentum p_{kaon}, total cross section σ_{tot}, real and imaginary parts of the regeneration amplitude Δf; optimized regenerator thickness ℓ, corresponding regeneration probability p_{regen}, and absorption factor p_{thru}; percentage of background due to incoherent background.

p_{kaon} (GeV/c)	σ_{tot} (mb)	$\Re(\Delta f)$ (Fermi)	$\Im(\Delta f)$ (Fermi)	ℓ (cm)	p_{regen} $\times 1000$	p_{thru} %	B %/mstr
0.35	617.	-7.0	-5.9	7.0	0.43	0.81	3.6
0.40	596.	-6.5	-6.7	8.0	0.44	0.79	4.1
0.50	498.	-7.7	-6.0	10.0	0.47	0.78	5.1
0.60	485.	-9.4	-7.0	12.0	0.65	0.75	6.1
0.70	493.	-11.9	-9.1	14.0	1.01	0.71	7.1
0.80	511.	-11.5	-10.5	15.0	1.02	0.68	8.3
0.90	536.	-11.1	-12.7	16.0	1.12	0.65	9.5
1.00	553.	-7.5	-12.9	17.0	0.83	0.62	10.7
1.10	529.	-4.7	-10.5	19.0	0.48	0.60	11.7
1.20	511.	-6.2	-8.6	20.0	0.40	0.60	12.9
1.30	483.	-8.6	-8.5	22.0	0.51	0.59	13.9
1.40	482.	-10.5	-8.5	24.0	0.61	0.56	14.9
1.50	484.	-11.3	-9.5	24.0	0.70	0.56	16.4
1.60	475.	-10.9	-9.3	26.0	0.64	0.54	17.4
1.80	468.	-10.9	-9.8	28.0	0.62	0.52	20.0
2.00	465.	-12.1	-10.4	30.0	0.70	0.50	22.6
2.20	447.	-12.8	-8.9	32.0	0.64	0.49	25.2
2.40	439.	-13.4	-8.9	34.0	0.65	0.47	27.9
2.60	432.	-14.0	-8.4	36.0	0.63	0.46	30.6

Table 6: SILICON – Kaon momentum p_{kaon}, total cross section σ_{tot}, real and imaginary parts of the regeneration amplitude Δf; optimized regenerator thickness ℓ, corresponding regeneration probability p_{regen}, and absorption factor p_{thru}; percentage of background due to incoherent background.

p_{kaon} (GeV/c)	σ_{tot} (mb)	$\Re(\Delta f)$ (Fermi)	$\Im(\Delta f)$ (Fermi)	ℓ (cm)	p_{regen} $\times 1000$	p_{thru} %	B %/mstr
0.35	1054.	-10.7	-7.2	6.0	1.75	0.57	2.1
0.40	1013.	-10.1	-8.6	7.0	1.75	0.54	2.4
0.50	868.	-12.0	-8.5	8.0	2.06	0.54	3.1
0.60	845.	-14.4	-10.5	9.0	2.77	0.51	3.9
0.70	856.	-17.7	-14.1	10.0	3.96	0.47	4.6
0.80	879.	-16.7	-16.1	11.0	3.67	0.43	5.4
0.90	909.	-15.7	-19.1	11.0	3.67	0.41	6.5
1.00	928.	-10.9	-18.8	12.0	2.53	0.37	7.3
1.10	896.	-7.3	-15.5	12.0	1.48	0.39	8.6
1.20	873.	-9.6	-12.7	13.0	1.20	0.37	9.4
1.30	833.	-13.3	-12.6	14.0	1.54	0.36	10.2
1.40	833.	-16.1	-12.7	14.0	1.78	0.36	11.6
1.50	836.	-17.4	-14.3	15.0	2.00	0.33	12.4
1.60	822.	-16.9	-14.0	15.0	1.79	0.34	13.9
1.80	813.	-16.9	-14.8	16.0	1.66	0.32	16.3
2.00	807.	-18.8	-15.8	17.0	1.78	0.30	18.8
2.20	781.	-20.2	-13.7	18.0	1.62	0.29	21.4
2.40	769.	-21.1	-13.7	19.0	1.57	0.28	24.0
2.60	758.	-22.2	-13.0	20.0	1.50	0.26	26.6

Table 7: IRON – Kaon momentum p_{kaon}, total cross section σ_{tot}, real and imaginary parts of the regeneration amplitude Δf; optimized regenerator thickness ℓ, corresponding regeneration probability p_{regen}, and absorption factor p_{thru}; percentage of background due to incoherent background.

$p_{\rm kaon}$ (GeV/c)	$\sigma_{\rm tot}$ (mb)	$\Re(\Delta f)$ (Fermi)	$\Im(\Delta f)$ (Fermi)	ℓ (cm)	$p_{\rm regen}$ ×1000	$p_{\rm thru}$ %	B %/mstr
0.35	1235.	-12.5	-8.3	6.0	2.10	0.53	2.2
0.40	1187.	-11.9	-10.0	6.0	2.11	0.54	2.6
0.50	1013.	-14.0	-9.8	8.0	2.47	0.50	3.2
0.60	983.	-17.0	-12.1	9.0	3.29	0.47	3.9
0.70	991.	-20.8	-16.3	9.0	4.67	0.46	5.0
0.80	1019.	-19.6	-18.7	10.0	4.31	0.42	5.8
0.90	1055.	-18.5	-22.1	10.0	4.25	0.40	7.1
1.00	1078.	-13.1	-22.0	11.0	2.95	0.36	7.9
1.10	1041.	-8.6	-18.2	12.0	1.72	0.34	8.8
1.20	1013.	-11.2	-14.9	12.0	1.37	0.35	10.2
1.30	966.	-15.5	-14.6	13.0	1.74	0.34	11.0
1.40	967.	-18.8	-14.8	14.0	2.03	0.31	11.9
1.50	971.	-20.4	-16.8	14.0	2.27	0.31	13.4
1.60	955.	-19.8	-16.4	14.0	2.04	0.32	15.1
1.80	945.	-19.7	-17.5	15.0	1.87	0.30	17.6
2.00	939.	-22.0	-18.8	16.0	1.99	0.27	20.3
2.20	908.	-23.5	-16.3	17.0	1.81	0.27	23.0
2.40	894.	-24.6	-16.4	17.0	1.75	0.27	27.0
2.60	882.	-25.9	-15.6	18.0	1.67	0.25	29.8

Table 8: COPPER – Kaon momentum $p_{\rm kaon}$, total cross section $\sigma_{\rm tot}$, real and imaginary parts of the regeneration amplitude Δf; optimized regenerator thickness ℓ, corresponding regeneration probability $p_{\rm regen}$, and absorption factor $p_{\rm thru}$; percentage of background due to incoherent background.

p_{kaon} (GeV/c)	σ_{tot} (mb)	$\Re(\Delta f)$ (Fermi)	$\Im(\Delta f)$ (Fermi)	ℓ (cm)	p_{regen} $\times 1000$	p_{thru} %	B %/mstr
0.35	2987.	-23.0	-8.2	4.5	1.99	0.43	3.4
0.40	2855.	-23.1	-11.9	5.0	2.08	0.41	4.0
0.50	2513.	-28.4	-14.8	6.0	2.90	0.39	5.1
0.60	2428.	-33.6	-21.0	7.0	3.83	0.34	6.2
0.70	2416.	-39.6	-30.6	7.0	5.24	0.34	8.1
0.80	2442.	-36.0	-35.5	7.0	4.53	0.34	10.3
0.90	2487.	-32.8	-40.7	8.0	4.15	0.28	11.4
1.00	2520.	-24.6	-39.4	8.0	2.83	0.28	13.8
1.10	2462.	-17.7	-33.3	8.0	1.70	0.29	16.5
1.20	2424.	-22.2	-26.9	9.0	1.32	0.25	17.5
1.30	2343.	-31.0	-26.4	9.0	1.69	0.26	20.3
1.40	2350.	-37.3	-27.3	9.0	1.96	0.26	23.3
1.50	2359.	-40.5	-31.5	10.0	2.18	0.23	24.2
1.60	2329.	-39.9	-31.1	10.0	1.97	0.23	27.4
1.80	2312.	-39.6	-33.3	10.0	1.76	0.23	34.2
2.00	2302.	-44.2	-36.7	10.0	1.86	0.23	41.9
2.20	2243.	-48.2	-32.3	11.0	1.71	0.21	46.1
2.40	2218.	-50.6	-32.8	11.0	1.63	0.21	54.6
2.60	2196.	-53.5	-31.7	11.0	1.56	0.22	63.8

Table 9: TUNGSTEN – Kaon momentum p_{kaon}, total cross section σ_{tot}, real and imaginary parts of the regeneration amplitude Δf; optimized regenerator thickness ℓ, corresponding regeneration probability p_{regen}, and absorption factor p_{thru}; percentage of background due to incoherent background.

p_{kaon} (GeV/c)	σ_{tot} (mb)	$\Re(\Delta f)$ (Fermi)	$\Im(\Delta f)$ (Fermi)	ℓ (cm)	p_{regen} $\times 1000$	p_{thru} %	B %/mstr
0.35	3207.	-22.9	-7.2	6.0	0.79	0.53	5.7
0.40	3069.	-23.3	-10.9	6.0	0.86	0.55	6.9
0.50	2713.	-29.2	-14.2	8.0	1.27	0.49	8.3
0.60	2619.	-34.5	-20.8	9.0	1.78	0.46	10.3
0.70	2600.	-40.6	-30.7	9.0	2.53	0.46	13.1
0.80	2621.	-36.8	-35.8	10.0	2.29	0.42	15.2
0.90	2664.	-33.4	-41.1	11.0	2.16	0.38	17.4
1.00	2699.	-25.3	-39.8	11.0	1.53	0.38	20.7
1.10	2641.	-18.3	-33.8	12.0	0.94	0.35	22.9
1.20	2604.	-22.8	-27.2	12.0	0.75	0.36	26.7
1.30	2523.	-31.9	-26.7	13.0	0.97	0.34	28.9
1.40	2531.	-38.3	-27.7	14.0	1.15	0.31	31.1
1.50	2541.	-41.6	-31.9	14.0	1.30	0.31	35.1
1.60	2509.	-41.1	-31.5	14.0	1.20	0.31	39.4
1.80	2492.	-40.8	-33.9	15.0	1.10	0.29	46.1
2.00	2482.	-45.6	-37.4	16.0	1.19	0.27	53.0
2.20	2423.	-49.7	-33.0	17.0	1.12	0.26	60.1
2.40	2397.	-52.2	-33.5	17.0	1.09	0.26	70.6
2.60	2374.	-55.3	-32.4	18.0	1.06	0.25	78.0

Table 10: LEAD – Kaon momentum p_{kaon}, total cross section σ_{tot}, real and imaginary parts of the regeneration amplitude Δf; optimized regenerator thickness ℓ, corresponding regeneration probability p_{regen}, and absorption factor p_{thru}; percentage of background due to incoherent background.

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