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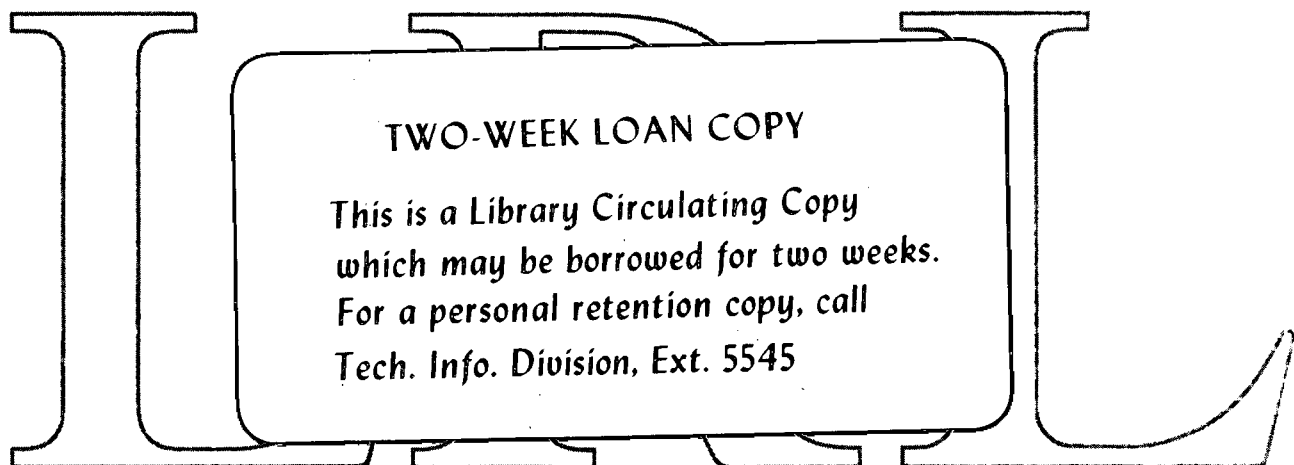Brunnader, H.
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A SIMPLE METHOD FOR INVESTIGATING THE PARENTAGE OF STATES
USING TWO-NUCLEON TRANSFER REACTIONS*

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A promising method is described for investigating the parentage of nuclear states. It involves the simultaneous observation of (p,t) and $(p,^3He)$ reactions on 0^+ $(T > 0)$ targets producing analogue final states with the same isospin as the target. Experimental results for $20 \leq A \leq 36$ $(T = 1)$ reveal a number of states with apparently very simple parentage for which J^2 "paired" pick-up predominates.

The simultaneous observation of (p,t) and $(p,^3He)$ reactions on a target with isospin T_i has been used for some time[1-5] to locate and identify analogue final states with isospin $T_f = T_i + 1$. The signature of a pair of such states is that the angular distributions of the corresponding tritons and ^3He particles have the same shape, and their magnitudes are related by a simple calculable factor. In this letter we shall first examine what is now a fairly large body of relevant experimental data, and use it as a means of establishing the validity of the approximations made in calculating the cross section ratio. We will then use the same approximations to calculate ratios for analogue final states with $T_f = T_i$. In certain cases the same experimental techniques can be used not only to identify such states, but also to determine information regarding their structure. Experimental data presented on such states in sd-shell nuclei

indicate a striking simplicity in their parentage. Evidently the experimental

method described provides a new and useful spectroscopic tool for investigating

the parentage of nuclear states.

An expression for the differential cross section for the two-nucleon

pick-up reaction A(a,b)B can be derived using the distorted wave Born ap-

proximation (DWBA). If spin-orbit forces are neglected, the result obtained

is:[6,7]

$$\frac{d\sigma}{d\Omega} \alpha \frac{k_b}{k_a} \sum_{\Lambda}^{LSJT} b_{ST}^2 (T_B N_B T \ N | T_A N_A)^2 [D(S,T)]^2 \left| \sum^N G_{NLSJT} B_{\Lambda}^{LN} (\underset{\sim}{k}_a, \underset{\sim}{k}_b) \right|^2 \quad (1)$$

Here b_{ST}^2 is a spectroscopic factor for the light particles a and b; it

equals $1/2(\delta_{S0}\delta_{T1} + \delta_{S1}\delta_{T0})$ for $(p, {}^3He)$ and $\delta_{S0}\delta_{T1}$ for (p,t). The func-

tion D(S,T) depends[7,8] upon the strength of the spin and isospin exchange

terms in the interaction potential. The term B_{Λ}^{LN} contains distorted waves

and the form factor of the transferred nucleons, while G_{NLSJT} contains all

the information on nuclear structure. The latter is written:

$$G_{NLSJT} = \sum^{\rho_1 \rho_2} (2-\delta_{\rho_1 \rho_2})^{1/2} \Omega_n \ \mathscr{S}_{AB}^{1/2} (\rho_1 \rho_2; LSJT) \ \langle nONL:L | n_1 \ell_1 n_2 \ell_2 : L \rangle \quad (2)$$

Here the summation is over all possible configurations $\rho_i [\equiv (n_i \ell_i J_i)]$ for the

transferred particles; Ω_n is the overlap of the relative motion of these

particles in the target with their relative motion in the triton (or ${}^3He$

particle); and $\langle | \rangle$ is a Moshinsky bracket.[9] The spectroscopic amplitude,

$\mathscr{S}_{AB}^{1/2}$, is an overlap integral which is a measure of the probability that the

nucleons which are common to the initial and final nuclear states will have

identical configurations in both and that the extra nucleons in A are in

shell-model states ρ_1 and ρ_2.

For those cases in which (p,t) and $(p,{}^3He)$ reactions on the same

target produce analogue final states with $T_f = T_i + 1$, the sum over S and T

in Eq. (1) is restricted to a single term (S = 0; T = 1) and all factors in

Eq. (2) except Ω_n are the same for both reactions. The ratio of their dif-

ferential cross sections may then be written (with $G_{NLJ} \equiv \sum_n \Omega_n A_{NLJ}$) as:

$$R \equiv \frac{\frac{d\sigma}{d\Omega}(p,t)}{\frac{d\sigma}{d\Omega}(p,^3He)} = \frac{k_t}{k_{3_{He}}} \cdot \frac{2}{(2T_f-1)} \cdot \left\{ \frac{\sum_\Lambda^{LJ} \left| \sum^{N\rho_1\rho_2} A_{NLJ}\left[\Omega_n B_\Lambda^{LN}\right]_{(p,t)} \right|^2}{\sum_\Lambda^{LJ} \left| \sum^{N\rho_1\rho_2} A_{NLJ}\left[\Omega_n B_\Lambda^{LN}\right]_{(p,^3He)} \right|^2} \right\} \quad (3)$$

The numerator and denominator of the bracketed factor on the right hand side

of Eq. (3) differ principally inasmuch as the mass-3 wave functions and the

reaction Q-values are different. The effects of both can be estimated. If

the target nucleus has $A \leq 60$, then the difference in the mass-3 wave func-

tions affects R by <2%; and by choosing a suitably high bombarding energy

($E_p = 45$ MeV) the effect of the different Q-values ($2.5 \leq \Delta Q \leq 8.5$) on R is

$\lesssim 5\%$. Thus, to a good approximation the bracketed factor may be assumed equal

to one.

Experimental cross-section ratios ($15° \leq \theta_{cm} \leq 45°$) to such analogue

states for a number of nuclei with $14 \leq A \leq 40$ (and $1 \leq T_f \leq 3$) are listed in

Table 1 together with the values of R calculated from the first two factors

in Eq. (3). The agreement is uniformly excellent and justifies neglecting the

third factor in that equation.

What has not been recognized previously is that by making the same

approximations for any 0^+ target a simple result can also be derived for the

cross-section ratios corresponding to certain analogue final states with $T_f = T_i$.

Then, only transitions to natural-parity levels ($\pi = (-)^J$) will be excited in

both (p,t) and (p,^{3}He) reactions, and each will be characterized by a single

L-value, namely that for which L = J. Two general cases will be considered:

(i) If both picked-up nucleons come from the same shell (i.e. referring to Eq. (2), $\rho_1 \equiv \rho_2$ for all terms in the sum) then, in order to produce a natural parity state, J must be even and the total isospin of the transferred nucleons is restricted to T = 1 in order to preserve the anti-symmetry of their wave function. This is the same restriction, though for a different reason, as that which led to the derivation of Eq. (3). The subsequent approximation which was used to reduce that equation can again be used with the result:

$$R = \frac{k_t}{k_{^3He}} \cdot \frac{2}{T_f} \quad . \tag{4}$$

(ii) If, on the other hand, the two picked-up nucleons come from different shells (i.e., $\rho_1 \neq \rho_2$ for at least one term in the sum in Eq. (2)) then T = 0 and T = 1 are both permitted in the (p,^{3}He) reaction. Thus, any mixture of both "paired" and "unpaired" particles or combinations of different "unpaired" particles involved in the transfer would in general result in a ratio less than that given by Eq. (4).

As a quantitative example, consider the transfer of specific unpaired particles characterized by ρ_1 and ρ_2. Using algebraic expressions[7] for the spectroscopic amplitudes $\mathscr{S}_{AB}^{1/2}(\rho_1\rho_2; JT)$ and invoking a variant of the same approximation, the cross-section ratio can be expressed in the form:

$$R = \frac{k_t}{k_{^3He}} \cdot \frac{2}{T_f} \cdot \left[1 + \frac{1}{3} \frac{\lambda^2}{\tau^2} \frac{(T_f + 1)^2}{L(L+1)} \right]^{-1} \quad ,$$

where $\lambda = [\ell_1(\ell_1+1) - \ell_2(\ell_2+1)] - [j_1(j_1+1) - j_2(j_2+1)]$,

and $\tau = [t_1(t_1+1) - t_2(t_2+1)] - [t_1'(t_1'+1) - t_2'(t_2'+1)]$. $\tag{5}$

The factor 1/3 is the approximate value for $[D(1,0)/D(0,1)]^2$ determined in

Refs. 8 and 10; also t_i and $t_i'(i = 1,2)$ are the initial and final isospins

of the shells from which the nucleons are picked up.[7] In the   sd-shell the

ratio calculated from Eq. (5) would typically be less than the "paired"

value by $\sim$30%.

To summarize, the angular distributions of (p,t) and (p,^{3}He) reactions

on 0^+ nuclei leading to analogue final states with $T_f = T_i$ should be identical

in shape and their relative magnitudes should acquire a <u>maximum</u> value when

the wave function of the final state differs from that of the target ground

state by the removal of paired nucleons. It should be emphasized that in

principle any number of shells can be involved, and that there is no restric-

tion on the complexity of the wave functions of the initial and final states.

These transitions can be discerned since any unpaired particles involved in

the transfer would reduce the magnitude ratio.

To provide a preliminary experimental investigation of this theory,

we have examined the (p,t) and (p,^{3}He) reactions on even-even T = 1 target

nuclei in the region $22 \leq A \leq 38$. The experiments were carried out using the

external 45.0 MeV proton beam of the Berkeley 88-inch cyclotron. Reaction

products were detected and identified using a solid-state counter telescope;

spectra of tritons and ^{3}He particles were recorded simultaneously. A detailed

description of the apparatus is given elsewhere.[4]

Five targets were bombarded: ^{22}Ne, ^{26}Mg, ^{30}Si, ^{34}S, and ^{38}Ar. As an

example of the experimental results, triton and ^{3}He spectra from the ^{22}Ne

target are shown in Fig. 1. As with the other targets, angular distributions

were extracted for all statistically significant peaks whose energies might

correspond to pairs of T = 1 final states; the angular range covered was

typically $15° \leq \theta_{cm} \leq 45°$. If the angular distributions for two of these
states had the same shape, then they were positively identified as being T = 1
analogues. In mass 20, two pairs were observed and these have been marked in
the figure. A list of such states covering all targets investigated is given
in Table 2 together with cross-section ratios extracted from the data. The
top half of the table includes only the lowest natural-parity T = 1 state in
each nucleus, while in the bottom half a number of excited T = 1 states are
shown. Those energies for which error bars are given were determined from this
work; the others were taken from Ref. 11.

Calculated values for the cross-section ratios, assuming the picked-up
nucleons are "paired", comprise the last column of Table 2. Those cases for
which the experimental ratio is significantly less than the calculated one must
involve some "unpaired" pick-up; then the calculated ratio in the table is
bracketed. Two observations can be made immediately from the bottom half of
the table. First, the two pairs of 0^+ levels, which could only be produced by
paired pick-up, do indeed show the maximum ratio; and second, the (3^-) levels,
which could not involve paired pick-up, give a reduced value as expected.

The most striking result, however, appears in the top half of the
table. Here, three transitions are indicated as being dominated by the trans-
fer of paired nucleons. These transitions are just the ones which would have
been predicted from the simplest shell model interpretation as being $j^n \rightarrow j^{n-2}$;
the remaining transitions would have been predicted as crossing subshells with
unpaired transfer. There can be no doubt that the wave functions of the
states involved in these reactions are vastly more complex than is indicated
by such a simple model, so it is therefore all the more surprising that at
least in the case of paired transfer the parentage remains simple. These

conclusions are further confirmed by results of the reactions $^{22}Ne(d,\alpha)^{20}F$ and $^{38}Ar(d,\alpha)^{36}Cl$. Since the (d,α) reaction must transfer $T = 0$, paired pick-up to natural parity states is forbidden. Three of the five transitions listed in Table 2 for these target nuclei were characterized as being "paired"; as expected, all were observed to be weak ($\leq 8\%$ of the strongest transition).

The usefulness of the experimental method as a general spectroscopic tool is certainly not limited to the mass region discussed here. The approximations are widely applicable and, although Eqs. (4) and (5) indicate that R decreases with increasing T, the percentage difference between "paired" and "unpaired" transfer becomes greater. Thus with larger A (and T) the sensitivity of the method will be increased. The striking results already obtained provide useful restrictions for future model calculations, and invite more general and detailed experimental investigations.

FOOTNOTES AND REFERENCES

*This work performed under the auspices of the U. S. Atomic Energy Commission.

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Table 1. Experimental and calculated relative cross-sections $\frac{d\sigma}{d\Omega}$ (p,t)$\Big/\frac{d\sigma}{d\Omega}$ (p,^{3}He) for states with $T_f = T_i + 1$.

Final States J^π, T_f	Allowed L-value(s)	Target nucleus	Cross-section ratio R(exp)	R(calc)	Ref.
$0^+,1$	0	^{16}O	2.19±0.22	1.88	2
$0^+,1$	0	^{36}Ar	1.92±0.19	1.80	5
$2^+,1$	2	^{36}Ar	1.54±0.20	1.78	5
$3/2^+,3/2$	0,2	^{21}Ne	1.05±0.10	0.93	4
$5/2^+,3/2$	0,2,4	^{25}Mg	0.85±0.09	0.92	4
$5/2^+,3/2$	0,2,4	^{27}Al	0.89±0.09	0.90	3
$5/2^+,3/2$	2	^{31}P	0.71±0.11	0.88	3
$0^+,2$	0	^{22}Ne	0.70±0.09	0.62	1,4
$0^+,2$	0	^{26}Mg	0.61±0.06	0.61	4
$0^+,2$	0	^{30}Si	0.54±0.10	0.60	5
$0^+,2$	0	^{34}S	0.66±0.06	0.60	5
$0^+,2$	0	^{38}Ar	0.62±0.07	0.60	5
$0^+,2$	0	^{42}Ca	0.60±0.05	0.60	5
$0^+,3$	0	^{40}Ar	0.36±0.04	0.35	5

Table 2. Experimental and calculated relative cross-sections
$\frac{d\sigma}{d\Omega}$ (p,t)/$\frac{d\sigma}{d\Omega}$ (p,^{3}He) for states with $T_f = T_i$.

Final states			Target nucleus	Cross-section ratio	
nucleus	E_x	J^π,T		R(exp)	R(j^2;calc)
^{20}Ne	10.275	2^+,1	^{22}Ne	2.00±0.20	1.88
^{20}F	g.s.				
^{24}Mg	9.517	4^+,1	^{26}Mg	2.50±0.30	1.86
^{24}Na	g.s.				
^{28}Si	9.379	2^+,1	^{30}Si	1.15±0.10[a]	[1.84]
^{28}Al	0.031				
^{32}S	7.005	2^+,1	^{34}S	1.20±0.30	[1.82]
^{32}P	0.078				
^{36}Ar	6.612	2^+,1	^{38}Ar	1.90±0.20	1.81
^{36}Cl	g.s.				
^{20}Ne	12.25±0.03	(3^-),1[b]	^{22}Ne	1.40±0.15	c
^{20}F	1.851				
^{28}Si	10.70±0.03	0^+,1[b,d]	^{30}Si	1.85±0.20	1.84
^{28}Al	1.35±0.03				
^{28}Si	10.909	(2^+),1[b]	^{30}Si	1.80±0.20	1.83
^{28}Al	1.633				
^{36}Ar	8.55±0.03	2^+,1	^{38}Ar	1.45±0.20	[1.81]
^{36}Cl	1.949				
^{36}Ar	9.70±0.03	0^+,1[c]	^{38}Ar	2.20±0.70	1.80
^{36}Cl	3.12±0.10				

(continued)

Table 2. Continued.

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^{a}The ground state of ^{28}Al was not resolved from its 2^+ state, so the quoted

experimental ratio is a lower limit. For various reasons (e.g., pure L = 2

angular distribution) it seems unlikely that this ratio will approach the

"paired" value.

^{b}These spin-parities were determined from this work by fitting experimental

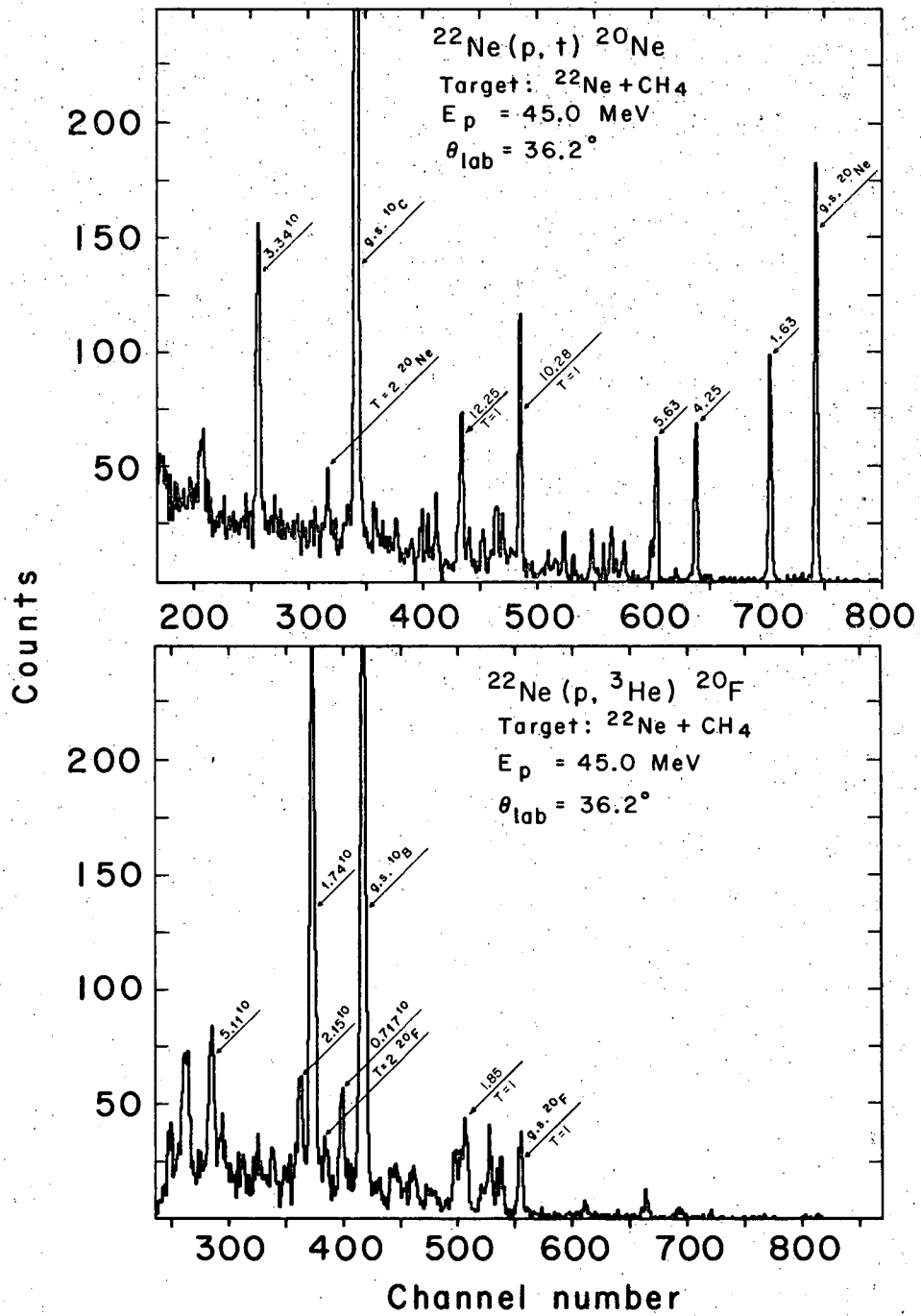angular distributions with DWBA calculations.

^{c}As a simple example of an "unpaired" ratio for this transition, pick-up from

the $p_{1/2}$ and $d_{5/2}$ shells yields a value for R of 1.30.

^{d}A level is known[11] in ^{28}Al at 1.372 MeV which is certainly 1^+; the strong

0^+ observed by us presumably indicates the presence of a doublet.

FIGURE CAPTION

Fig. 1. Energy spectra from the reactions ^{22}Ne$(p,t)^{20}$Ne and ^{22}Ne$(p,^{3}$He$)^{20}$F.

The target was a 50:50 mixture of neon and methane, the neon being 92%

enriched in ^{22}Ne. The methane was used to provide an energy calibration.

Fig. 1

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