

UC Berkeley

UC Berkeley Previously Published Works

Title

Observation of Synchronization between Two Quantum van der Pol Oscillators in Trapped Ions

Permalink

<https://escholarship.org/uc/item/1sj3s89m>

Journal

Physical Review X, 16(2)

ISSN

2160-3308

Authors

Liu, Jiarui

Wu, Qiming

Moore, Joel E

et al.

Publication Date

2026-04-01

DOI

10.1103/w1bm-wjl4

Copyright Information

This work is made available under the terms of a Creative Commons Attribution License, available at <https://creativecommons.org/licenses/by/4.0/>

Peer reviewed

Observation of Synchronization between Two Quantum van der Pol Oscillators in Trapped Ions

Jiarui Liu^{1,2,3,*} Qiming Wu^{1,2,3,*†} Joel E. Moore^{1,2,3}
Hartmut Haeffner^{1,2,3,‡} and Christopher W. Wächtler^{4,1,§}

¹Department of Physics, *University of California, Berkeley, California 94720, USA*

²Challenge Institute for Quantum Computation, *University of California, Berkeley, California 94720, USA*

³Lawrence Berkeley National Laboratory, *Berkeley, California 94720, USA*

⁴Instituto de Ciencia de Materiales de Madrid (ICMM-CSIC), *28049 Madrid, Spain*



(Received 17 November 2025; accepted 27 April 2026; published 29 June 2026)

Synchronization is a hallmark of collective behavior that emerges when nonlinear systems interact, spanning scales from mechanical oscillators to planetary orbits. As a universal phenomenon, it underpins the study of complex systems and has far-reaching technological implications. While classical synchronization has a long and rich history, it has not been observed experimentally between multiple quantum limit-cycle oscillators despite a decade of theoretical investigations. We realize synchronization between two quantum van der Pol oscillators by engineering dissipation in a mixed-isotope trapped-ion quantum simulator. The synchronized state is encoded in a fixed relative phase between the oscillators that is inaccessible to individual measurements and revealed only through joint readout of both oscillators, in stark contrast to the system in the (deterministic) classical limit where synchronization can be observed via individual phase measurements. We further show that the relative phase can be precisely controlled and that the chain of two oscillators can synchronize to an external field, suggesting applications in sensing. Our results provide a promising pathway for studying more complex synchronized quantum dynamics beyond two oscillators, where a theoretical treatment becomes increasingly challenging, and it remains to be understood whether genuinely quantum features persist in such cases.

DOI: [10.1103/w1bm-wjl4](https://doi.org/10.1103/w1bm-wjl4)

Subject Areas: Atomic and Molecular Physics,
Nonlinear Dynamics, Quantum Physics

I. INTRODUCTION

Self-sustained oscillations emerge when internal feedback mechanisms regulate energy input to compensate for dissipation [1–3]. These oscillations correspond to closed and attractive trajectories in phase space, so-called limit cycles. Because of their phase freedom, limit-cycle oscillators can entrain to an external drive or mutually synchronize [4–8], where the latter constitutes a genuine self-organization phenomenon arising from bidirectional interactions. Among classical models, the van der Pol (vdP) oscillator has served as the canonical example of a limit-cycle system: Originally introduced to describe nonlinear

electrical circuits in early radio technology [1], it has since provided the conceptual foundation for synchronization theory across diverse domains, from neuroscience to circadian rhythms [9,10]. It is therefore natural that efforts to formulate synchronization in the quantum regime start from the quantum vdP oscillator [11,12]. In the quantum regime [4–7,12–19], a limit cycle is defined as an attractive solution of the dissipative dynamics with a steady state exhibiting $U(1)$ symmetry. The simplest realization of such a quantum limit cycle is a harmonic oscillator with incoherent one-phonon gain and two-phonon loss processes, which together define the quantum vdP oscillator [11].

The quantized motional modes of trapped ions provide an excellent platform for realizing quantum limit-cycle oscillators and exploring their synchronized dynamics. Their long coherence times, precise controllability, and strong coupling to internal states [20–22] accommodate the implementation of quantum dynamics in bosonic degrees of freedom [23,24], which has enabled a wide range of applications including analog quantum simulation, quantum-enhanced metrology, and fault-tolerant quantum computing with bosonic codes [25–31]. However, the lack of direct observables for the motional states has

*These authors contributed equally to this work.

†Contact author: qiming.wu@berkeley.edu

‡Contact author: hhaeffner@berkeley.edu

§Contact author: cwwaechtler@icmm.csic.es

Published by the American Physical Society under the terms of the [Creative Commons Attribution 4.0 International](https://creativecommons.org/licenses/by/4.0/) license. Further distribution of this work must maintain attribution to the author(s) and the published article's title, journal citation, and DOI.

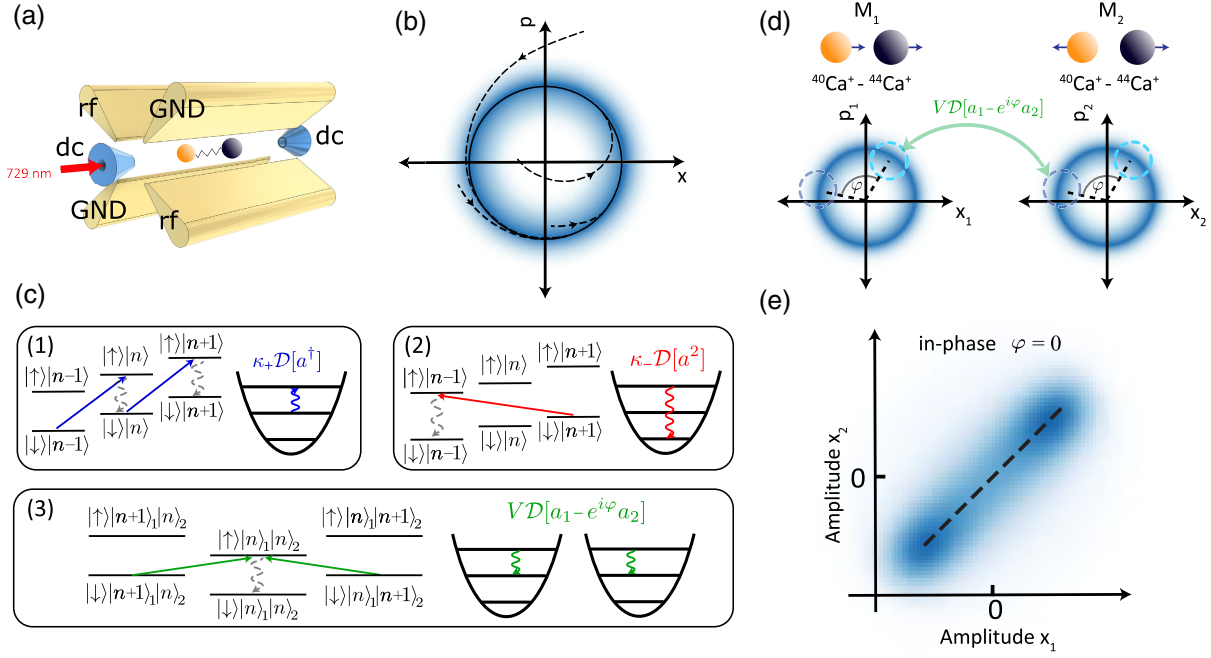


FIG. 1. Synchronization of quantum van der Pol oscillators with trapped ions. (a) Schematics of experimental setup with a $^{40}\text{Ca}^+ - ^{44}\text{Ca}^+$ ion crystal trapped in a linear rf Paul trap. The two axial motional modes are used for the synchronization experiment, where the state of the $^{40}\text{Ca}^+$ ion and the motional state of the ion crystal are controlled by a 729-nm laser light. (b) Illustration of the donut-shaped Wigner function of a quantum limit-cycle oscillator. Black dashed curves indicate classical trajectories attracted toward the classical limit cycle. (c) Experimental implementation of the corresponding dissipator to generate and synchronize two vdP oscillators. (1) Negative damping is realized by driving the blue sideband of M_i , followed by a qubit reset; (2) nonlinear damping is realized by driving the second-order red sideband of M_i , followed by a qubit reset; (3) collective dissipation is generated by simultaneous driving the red sidebands of M_1, M_2 with a phase difference φ , followed by another qubit reset. (d) The Wigner function of each mode M_1 and M_2 individually does not show any phase preference. Instead, the synchronized dynamics appears only as a stable relative phase relation between the two oscillators, which can be observed only by a joint measurement. (e) Joint probability distribution $P(x_1, x_2)$ for in-phase ($\varphi = 0$) synchronized oscillators. The black dashed line shows the corresponding classical trajectory.

traditionally limited the exploitation of quantum resources in the infinite-dimensional Hilbert space. Over the past two decades, however, substantial progress has been made in overcoming this challenge. Wigner function reconstruction was demonstrated in Refs. [32,33], and recent proposals and experimental realizations have introduced efficient benchmarking techniques for motional states [34–36]. These advancements, combined with reservoir engineering techniques with programmable dissipation and noise injection [7,37–41], enable the level of control and measurement required for synchronization experiments.

Here, we experimentally demonstrate synchronization between two quantum vdP oscillators encoded in the bosonic state of two motional modes of a two-ion crystal. The vdP oscillators are generated through a combination of two-phonon loss and one-phonon gain processes of the motional modes. We show that the steady states of the vdP oscillators are highly tunable from the near-classical to the quantum regime via tuning the gain-loss ratio, and the resulting limit-cycle behavior is revealed by tracking the motional-state dynamics via Wigner function reconstruction. We achieve synchronization via engineered dissipative coupling [42],

confirmed by reconstructing the joint probability distribution of the two-mode quantum states and supported by numerical simulations. We further show that synchronization is preserved over a range of coupling strengths and frequency differences between the oscillators. Finally, we probe phase diffusion during mutual synchronization by phase locking one oscillator to an external drive and measuring the Wigner functions of both modes at various detunings. Our results pave the way for investigating synchronized dynamics in networks of quantum limit cycles [43], harnessing engineered dissipation as a resource for complex quantum dynamics, and developing new applications in quantum metrology and quantum sensing [44,45].

II. SYNCHRONIZING QUANTUM VAN DER POL OSCILLATORS

A. Quantum vdP oscillators in trapped ions

To experimentally realize two coupled vdP oscillators, we employ the two axial motional modes, M_1 (in-phase) and M_2 (out-of-phase), of a $^{40}\text{Ca}^+ - ^{44}\text{Ca}^+$ linear two-ion crystal confined in an rf Paul trap [see Fig. 1(a)].

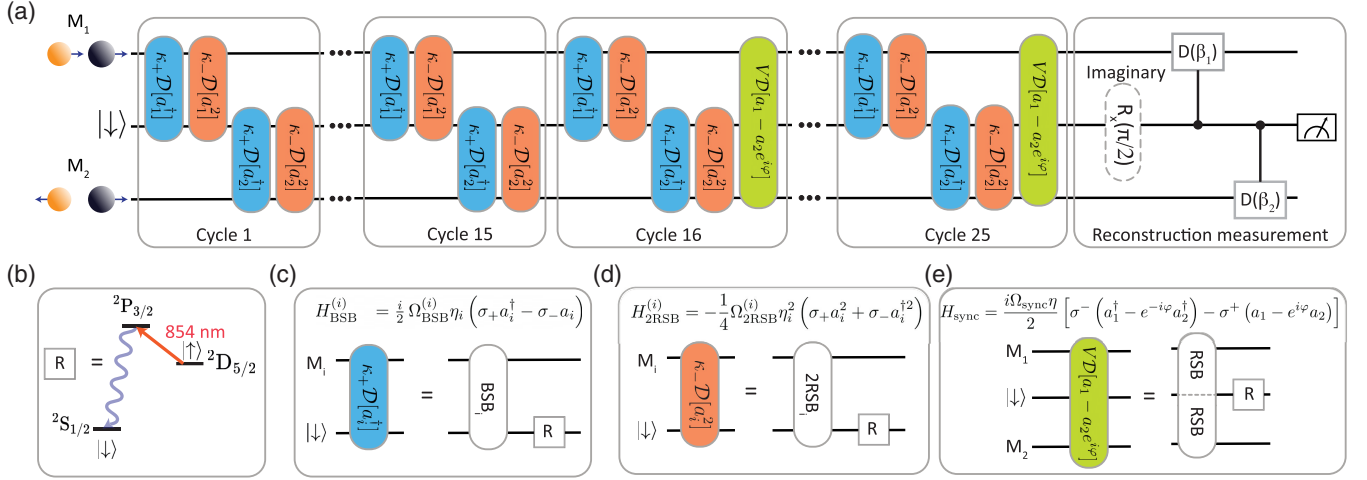


FIG. 2. The quantum circuit for synchronization between two quantum vdP oscillators and reconstruction of the motional states. (a) The circuit acts on one qubit and two motional modes, where the auxiliary qubit is to generate collective dissipation on the two vdP oscillators and to read out the two-mode state. Each cycle consists of a stroboscopic application of negative and nonlinear dampings on both modes [described in (c) and (d)]. We repeat the sequence 15 times to generate the two vdP oscillator states, with another 10 cycles including the dissipative coupling [described in (e)] to synchronize them. For the motional reconstruction, the controlled displacement in the σ_x basis (realized by SDF) maps the motional information onto the qubit. The imaginary part is read out with an additional $\pi/2$ qubit rotation along the x direction prior to state detection. (b) The qubit reset is realized with 854-nm repumping laser light to reset it to the $|\downarrow\rangle_z$ state. (c),(d) The negative (nonlinear) damping on M_i is realized by a coherent BSB (2RSB) drive followed by a qubit reset R . (e) Collective dissipation on the two motional modes is realized by simultaneously driving the RSB on M_1 and M_2 with a constant phase difference φ , followed by a qubit reset [see details in Appendix A].

The uncoupled collective motion is described by the Hamiltonian $H_0 = \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2$, where $\omega_{1,2}$ are the frequencies of mode $M_{1,2}$ and $a_{1,2}$ are the corresponding ladder operators. For convenience, we set $\hbar \equiv 1$ in the following discussions. Qubit-motion couplings are implemented via driving motional sidebands of the $4^2S_{1/2} \leftrightarrow 3^2D_{5/2}$ quadrupole transition of the $^{40}\text{Ca}^+$ ion. Because of the $^{40}\text{Ca}^+ - ^{44}\text{Ca}^+$ isotope shift of 5.34 GHz on this transition [46], the applied laser field does not couple to the internal state of the $^{44}\text{Ca}^+$ ion, resulting in Jaynes-Cummings-type interactions. Motional dissipation required for the limit-cycle dynamics of the quantum vdP oscillators is engineered by returning the $3^2D_{5/2}(m = -5/2)$ population to the $4^2S_{1/2}(m = -1/2)$ state via optical pumping, i.e., resetting the qubit [see Figs. 1(c) and 2(b)–2(d)]. Specifically, negative damping ($\mathcal{D}[a^\dagger]$) is implemented through coupling to the first-order blue sideband (BSB), which corresponds to transitions that add one phonon. Nonlinear damping ($\mathcal{D}[a^2]$) is realized by driving the second-order red sideband (2RSB), which corresponds to transitions that remove two phonons simultaneously. In both cases, the interaction is applied stroboscopically with a randomized phase and followed by resetting the qubit after each evolution step, ensuring effective damping in the individual motional modes.

To synchronize the two oscillators, we interleave layers of dissipative coupling ($\mathcal{D}[a_1 - a_2 e^{i\varphi}]$) with resonantly driving the first-order red sidebands (RSBs) on the two

modes simultaneously with a fixed phase difference φ , followed by qubit reset [see Fig. 2(e)]. Here, the auxiliary qubit is used to implement collective dissipation between the modes. The full sequence of individual negative damping, nonlinear damping, and dissipative coupling pulses is repeated for n cycles until the system reaches a steady state. The detailed quantum circuit is shown in Fig. 2(a). The resulting effective dynamics is described by the Lindblad master equation

$$\dot{\rho} = -i[\tilde{H}_0, \rho] + \kappa_+ \sum_{i=1}^2 \mathcal{D}[a_i^\dagger] \rho + \kappa_- \sum_{i=1}^2 \mathcal{D}[a_i^2] \rho + V \mathcal{D}[a_1 - a_2 e^{i\varphi}] \rho. \quad (1)$$

Here, $\tilde{H}_0 = \delta_1 a_1^\dagger a_1 + \delta_2 a_2^\dagger a_2$ with $\delta_{1,2}$ being the differences between motional frequencies $\omega_{1,2}$ and half the laser frequency splittings of the BSB and RSB drives, which set the rotating frame of the oscillators. The Lindblad dissipators $\mathcal{D}[L] \rho = L \rho L^\dagger - \{L^\dagger L, \rho\}/2$ describe negative damping with rate κ_+ , nonlinear damping with rate κ_- , and collective dissipation with strength V . Note that the ratio κ_-/κ_+ determines the average phonon number and, thus, the radius of the donut-shaped Wigner functions. Moreover, the phase φ appearing in Eq. (1) controls the phase of the dissipative coupling between the oscillators as illustrated in Fig. 1(c). The details of the experimental protocols can be found in Appendix A.

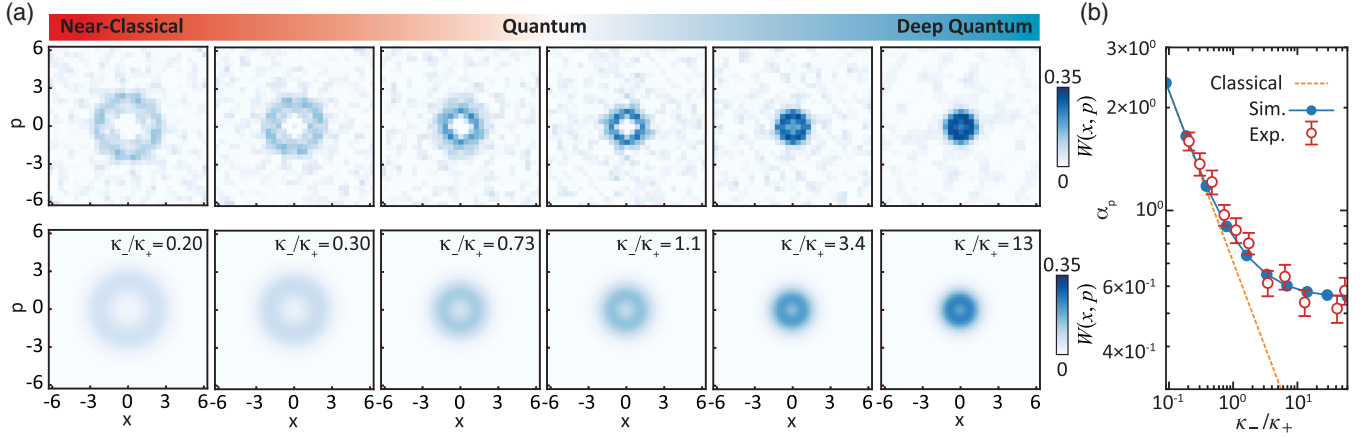


FIG. 3. Crossover of the vdP oscillator from the near-classical to the deep quantum regime. (a) The top row shows experimentally reconstructed Wigner functions of various vdP oscillator states κ_-/κ_+ . The bottom row shows the corresponding numerical simulation. (b) The radius α_p is shown as a function of the ratio κ_-/κ_+ on a logarithmic scale. Red dots are the experimental data, the blue solid curve is the numerical simulation using the Lindblad equation, and the orange dashed line represents the classical mean-field theory. The error bars represent the uncertainty arising from quantum projection noise. When κ_-/κ_+ increases, the oscillator becomes more quantum and deviates from the classical mean-field theory. During the experiment, we keep $\kappa_-/2\pi = 0.218$ kHz to be constant and change the strength of κ_+ by tuning the power of the BSB drive.

B. Experimental characterization of a single vdP oscillator

Before exploring synchronization, we first establish the control of a single quantum vdP oscillator using a single $^{40}\text{Ca}^+$ ion. First, we show that the steady state of the vdP oscillator can be precisely controlled by changing the relative strengths (κ_+ and κ_-), which affects the mean phonon number and the radius of the donut-shaped Wigner function. Previous research has classified different parameter regimes of a single vdP oscillator, i.e., the near-classical regime with $\kappa_-/\kappa_+ \lesssim 0.1$, the quantum regime with $\kappa_-/\kappa_+ \approx 1$, and the deep quantum regime with $\kappa_-/\kappa_+ \gtrsim 10$ [47]. In Fig. 3(a), we show the experimental reconstructed Wigner function of various vdP oscillator states from the near-classical to the deep quantum regime. When the negative damping κ_+ exceeds κ_- and the mean phonon number is large, classical mean-field theory predicts a limit cycle with a radius $\alpha_p = \sqrt{(x^2 + p^2)/2} = \sqrt{\kappa_+/2\kappa_-}$ in phase space, where α_p is defined as the most probable position of the vdP Wigner function (i.e., the radius of the donut). When the nonlinear damping κ_- exceeds κ_+ , the oscillator approaches the ground state upon increasing κ_- and α_p starts to deviate from the classical trajectory as shown in Fig. 3(b).

Second, we probe that the limit cycle is, in fact, the nonequilibrium steady-state (and thus attractive) solution of the dissipative dynamics, which is reached independently of initial conditions. To this end, we measure the time evolution of the Wigner function from a mixture of opposite coherent states $[\varrho = \frac{1}{2}(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|)]$ to the vdP oscillator state. In Fig. 4, we begin with the initial state of a mixture of two opposite coherent states at $t = 0$.

After applying the vdP interaction, the state gradually diffuses into a donut shape with the steady-state radius determined by the ratio of interaction strength κ_-/κ_+ .

Third, we show that the phase of a single vdP oscillator can be entrained to an external drive. An isolated vdP oscillator exhibits no intrinsic phase preference. When an external coherent drive $H = \Omega_d(ae^{i\varphi_d} + a^\dagger e^{-i\varphi_d})$ of sufficient strength is introduced, this rotational symmetry is broken and the oscillator's phase becomes entrained to that of the external drive [7, 11, 48, 49]. In practical experimental conditions, the phase of the external drive has a constant offset from the motional phase of the SDF ϕ_m for readout due to differences in the optical and electrical signal paths. In Fig. 5, we show the experimental results of a single vdP oscillator locked to the external drive with different phase preferences $\varphi_d = 0, \pi/2, \pi$. These results also serve as a reference for subsequent experiments on mutual synchronization under the drive (see Sec. IV).

C. Mutual synchronization in the near-classical and quantum regime

After characterizing the dynamics of a single vdP oscillator, we extend our study to the synchronization between two quantum vdP oscillators. To investigate quantum limit-cycle synchronization, we realize two quantum vdP oscillators and synchronize them via collective dissipation. We first implement it in the near-classical regime by tuning the ratio between the two damping processes to be $\kappa_-/\kappa_+ = 0.14$ [47], identical for each oscillator, resulting in a steady-state mean phonon number of $\bar{n} = 3.5$. The effective rates are implemented by adjusting the strengths and durations of the BSB and 2RSB

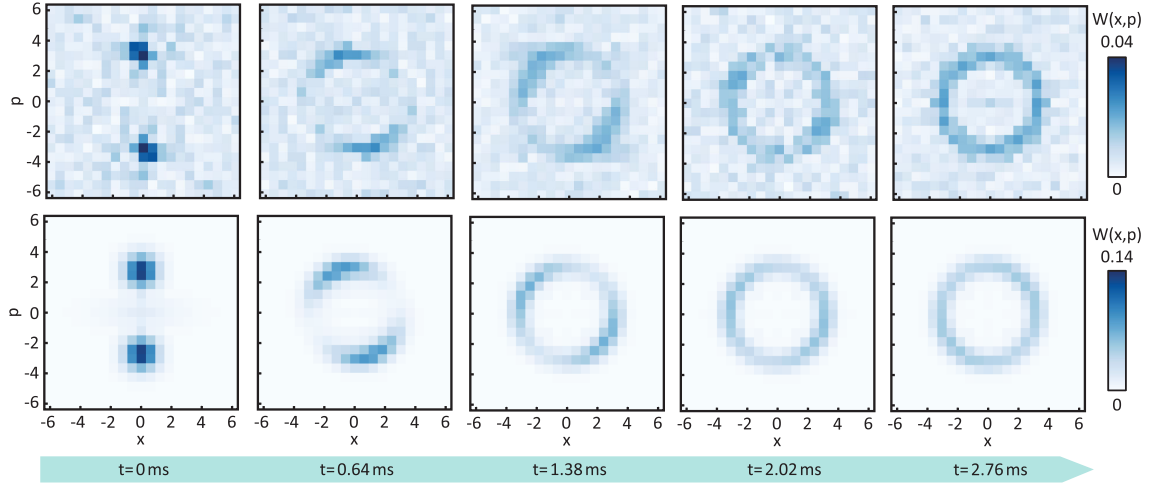


FIG. 4. Evolution toward the limit cycle of a single vdP oscillator. We create a mixture of opposite coherent states with $\alpha = 3$ at $t = 0$ ms and measure the time evolution to the vdP oscillator state with Wigner function reconstruction. The top row shows the experimental data, and the bottom row shows the numerical simulation. The parameters used in the experiment are $t_{\text{RSB}} = 48 \mu\text{s}$, $t_{\text{BSB}} = 24 \mu\text{s}$, $\Omega_{\text{BSB}}/2\pi = 0.06$ MHz, and $\Omega_{\text{RSB}}/2\pi = 0.1$ MHz.

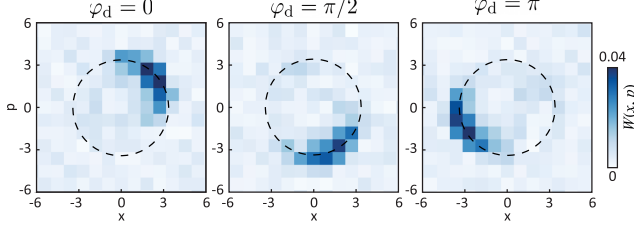


FIG. 5. A single vdP oscillator synchronized to an external drive with different phases $\varphi_d = 0, \pi/2, \pi$. The dashed circles indicate the size (radius α_p) of the vdP oscillator with no external drive.

interactions on each mode. The collective dissipative coupling has an effective rate $V_0/2\pi = 0.1$ kHz (see Appendix B). During the first 15 cycles, the dissipative coupling is not applied such that the two vdP oscillators reach their steady states independently and without phase preference. Synchronization is then introduced by applying ten additional cycles including the dissipative coupling with a fixed phase φ . By varying φ , we gain full control over the relative phase alignment between the two oscillators [see Fig. 6(b)].

After preparing the desired oscillator states, we perform a readout of the Wigner function for each individual mode. Figure 6(a) shows the reconstructed Wigner functions of each mode individually. Both oscillators exhibit similar phonon occupations, resulting in the donut-shaped Wigner function, which is characteristic of a quantum limit cycle. Because synchronization in our system occurs only in the relative phase, individual oscillators show no signatures of phase locking. This contrasts with externally driven quantum vdP oscillators, where synchronization remains locally observed through single-mode measurements [7]. Instead, the global

phase of each oscillator remains unpolarized, leading to isotropic Wigner functions. This shows that synchronization is not accessible through local measurements alone, since the phase of each oscillator cannot be individually resolved [see Figs. 1(d) and 1(e)]. In classical (deterministic) systems, by contrast, a joint readout is, in principle, not necessary. The phase of each classical oscillator can be measured separately, and synchronization can then be observed by comparing these phases. For the quantum oscillators, however, the relative phase is not locally accessible and revealed only through joint measurements.

To uncover the correlations between the two modes, both oscillators need to be coherently displaced simultaneously, which allows us to access the two-mode characteristic function [27]

$$\chi(\beta_1, \beta_2) = \langle \Psi | \mathcal{D}_1(\beta_1) \mathcal{D}_2(\beta_2) | \Psi \rangle, \quad (2)$$

where $\mathcal{D}_i(\beta_i) = \exp(\beta_i a_i^\dagger - \beta_i^* a_i)$ denotes the coherent displacement on M_i and $|\Psi\rangle$ is the two-mode state. The joint probability distribution for measuring the two modes along general quadrature axes $x_{\phi_j} = (a_j e^{-i\phi_j} + a_j^\dagger e^{i\phi_j})/\sqrt{2}$ is then obtained through the Fourier transform of the characteristic function:

$$P(x_{\phi_1}, x_{\phi_2}) = \iint \frac{d\beta_1 d\beta_2}{2\pi^2} e^{-i\sqrt{2}(\beta_1 x_{\phi_1} + \beta_2 x_{\phi_2})} \times \chi(i\beta_1 e^{i\phi_1}, i\beta_2 e^{i\phi_2}). \quad (3)$$

Here, $P(x_{\phi_1}, x_{\phi_2})$ denotes the joint distribution of rotated quadratures, which for $\phi_1 = \phi_2 = 0$ reduces to $P(x_1, x_2)$, and for $\phi_1 = 0, \phi_2 = \pi/2$ yields the joint position-momentum distribution $P(x_1, p_2)$. Experimentally, the real

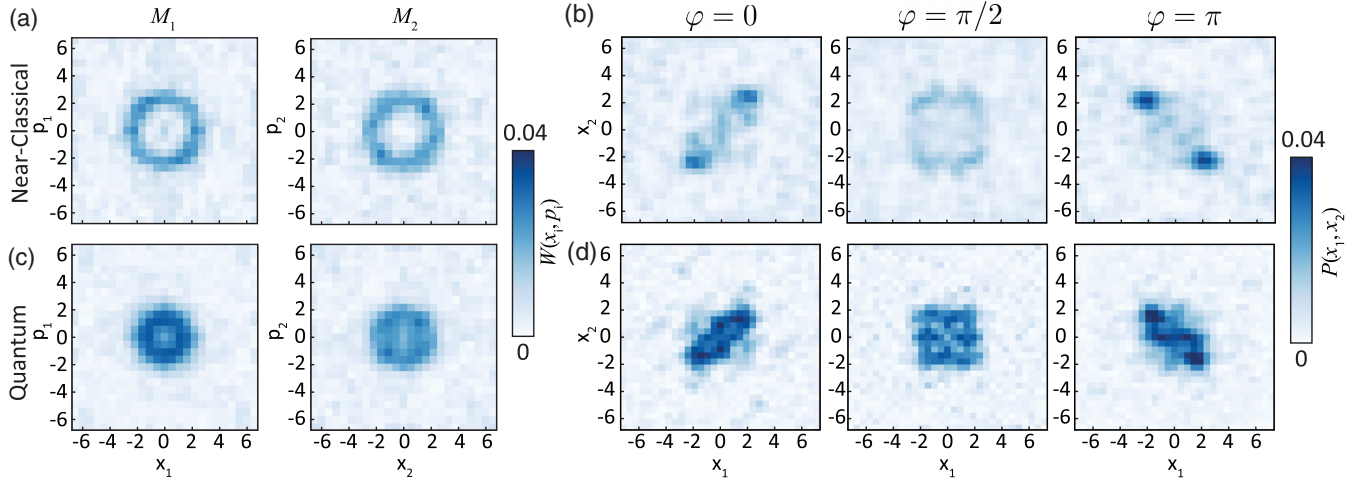


FIG. 6. Synchronized dynamics in the near-classical and quantum regime. (a),(b) Near-classical regime with $\kappa_-/\kappa_+ = 0.14$ and mean phonon number $\bar{n} = 3.5$. (a) Individually reconstructed Wigner functions of the two quantum vdP oscillators show the expected donut-shaped profile without a preferred phase. (b) The reconstructed joint probability distributions $P(x_1, x_2)$ for different synchronization phases φ . For $\varphi = 0$ and π , the two oscillators exhibit in-phase and antiphase synchronization, indicated by diagonal and antidiagonal features in the x_1 - x_2 plane. For $\varphi = \pi/2$, the distribution forms a circle, reflecting the relative phase lag between the oscillators. (c),(d) Quantum regime with $\kappa_-/\kappa_+ = 0.42$ and $\bar{n} = 1.4$. (c) Reconstructed Wigner functions show reduced limit-cycle amplitudes as the system approaches the ground state. (d) Despite the increased role of quantum noise, correlations in the x_1 - x_2 plane remain visible, indicating persistence of synchronization in the quantum regime. Wigner functions in (a) and (c) are measured at $\varphi = \pi$. Corresponding numerical simulations are provided in Appendix B, showing good agreement with experimental values.

part $\text{Re}[\chi]$ is extracted by applying two sequential spin-dependent displacements on $|\downarrow\rangle_z \otimes |\Psi\rangle$ and measuring the spin magnetization $\langle \sigma_z \rangle$, while the imaginary part $\text{Im}[\chi]$ is accessed via an additional $\pi/2$ qubit rotation prior to displacement (see Appendix A).

In Fig. 6(b), we show the joint probability distribution $P(x_1, x_2)$ for three different values of the phase φ . For $\varphi = 0$, the two vdP oscillators are (in-phase) synchronized. For $\varphi = \pi$, they are antiphase synchronized. In the joint probability distribution, this appears as structures along the diagonal and antidiagonal directions, respectively. When the phase difference is $\varphi = \pi/2$, we observe a circular shape in the $P(x_1, x_2)$ plot. This can be viewed as the simplest Lissajous figure, formed by two orthogonal oscillators of equal amplitude and a $\pi/2$ phase difference, resulting in a circular trajectory in the plane defined by their position axes (see Appendix E).

We next turn to the quantum regime, where synchronization should persist despite the dynamics of both oscillators being closer to their respective ground state and, thus, the increased role of quantum fluctuations due to quantum jumps and zero-point fluctuations. To reach this regime, we tune the effective dissipation rates by adjusting the duration of the stroboscopic steps, such that the ratio $\kappa_-/\kappa_+ = 0.42$, leading to steady-state occupation numbers $\bar{n} = 1.4$. The corresponding results are shown in Figs. 6(c) and 6(d). Strikingly, even at such low occupation, the system retains the features of limit-cycle behavior: The reconstructed Wigner functions exhibit a visible, albeit smaller, donut-shaped profile; cf. Fig. 6(c). Moreover, the joint probability

distributions continue to display the phase-dependent patterns for $\varphi = 0, \pi/2, \pi$, confirming that synchronization may persist into the quantum regime. Notably, even if only a few quanta per oscillator are present, synchronization between the two modes can be established.

III. STABILITY OF SYNCHRONIZATION TO DETUNING AND COUPLING

A central feature of classical synchronization is its robustness: Even nonidentical systems can adjust their rhythms and lock to a common frequency. This behavior manifests in the so-called Arnold tongue [1], a region in detuning-coupling space where synchronization persists. Analogous regions have been theoretically predicted for coupled quantum limit-cycle systems [14,42]. However, characterizing this behavior using full state reconstruction is experimentally demanding. To circumvent this challenge, we introduce mutual information extracted from the measurable quadrature distributions $P(x_{\phi_1}, x_{\phi_2})$ as a figure of merit, that retains sensitivity to synchronization (see Appendix E). A measure based on a single quadrature, however, depends on the relative phase difference φ . For example, at $\varphi = \pi/2$, $I[x_1 : x_2]$ vanishes due to symmetry [middle panel in Fig. 6(b)], while $I[x_1 : p_2]$ establishes that the two oscillators are actually synchronized. Conversely, at $\varphi = 0$, $I[x_1 : p_2]$ vanishes and the correlations are contained in $I[x_1 : x_2]$. For intermediate phases, both quantities contribute. To eliminate this dependence on φ , we combine the two, thereby defining a mutual information

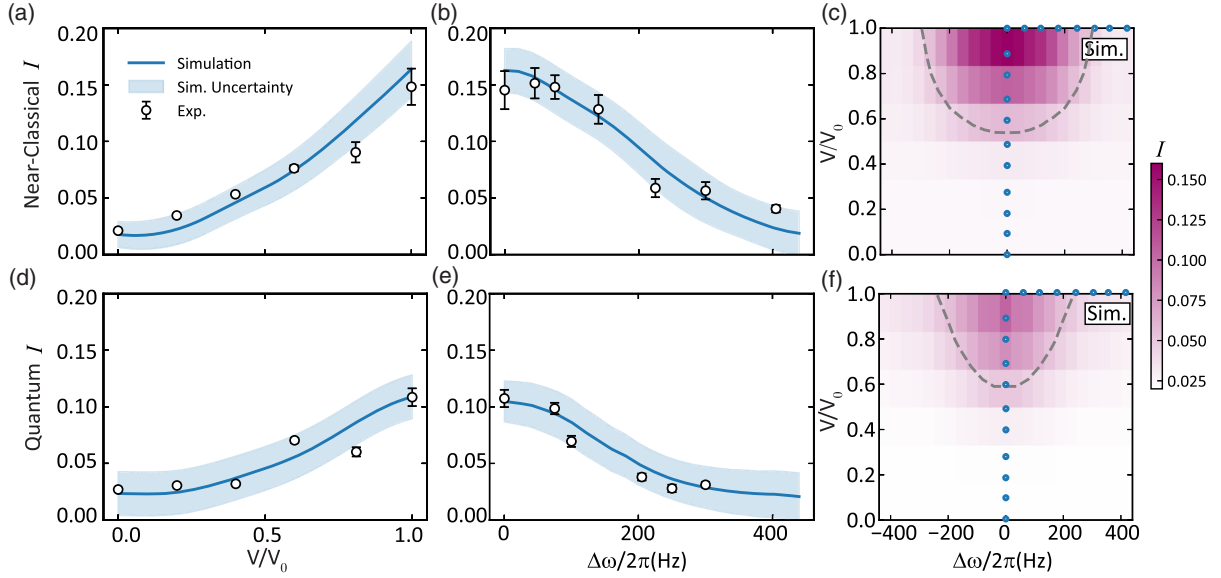


FIG. 7. Arnold tongue-like synchronization landscape. (a)–(c) Mutual information $\mathcal{I}$ as an indicator of synchronization between two quantum vdP oscillators in the near-classical regime. (a) At zero detuning, increasing the dissipative coupling strength enhances synchronization, reflected in higher mutual information. (b) At fixed coupling strength, increasing the detuning suppresses synchronization and, thus, decreases mutual information. V_0 is the coupling strength used in Fig. 6. (c) Simulation across the full coupling-detuning parameter space reveals the characteristic Arnold tongue-like structure. The gray dashed curve represents the contour for $\mathcal{I} = 0.05$. The blue dots represent the two cuts where the experimental data are taken. The simulations include a global offset accounting for readout noise. (d)–(f) The same as (a)–(c) but in the quantum regime. Here, quantum noise reduces the overall level of synchronization, leading to smaller values of mutual information, though the qualitative behavior persists. Black circles in (a),(b) and (d),(e) show the experimentally measured $\mathcal{I}$. Error bars are computed by Monte Carlo sampling of synthetic datasets based on the observed counts, thereby propagating quantum projection noise into the uncertainty of $\mathcal{I}$. Solid lines represent numerical simulations using experimental parameters, and the shaded regions indicate uncertainty from motional frequency drifts and Rabi frequency fluctuations (see Appendix C).

measure that is invariant under the relative phase:

$$\mathcal{I} = I[x_1 : x_2] + I[x_1 : p_2], \quad (4)$$

where $I[x_1 : x_2]$ and $I[x_1 : p_2]$ are computed from the respective distributions $P(x_1, x_2)$ and $P(x_1, p_2)$. This serves as our figure of merit for synchronization between the two oscillators.

Similarly to classical synchronization, increasing the dissipative coupling strength V enhances synchronization in the quantum regime. At the same time, compared to the near-classical regime, synchronization is generally weaker in the quantum regime due to enhanced quantum fluctuations that broaden the relative-phase distribution and reduce the achievable phase locking. This trend is evident in Figs. 7(a) and 7(d), which show $\mathcal{I}$ for zero frequency detuning between the oscillators in the near-classical and quantum regimes, respectively. Note that the noise of $P(x_1, x_2)$ and $P(x_1, p_2)$, stemming predominantly from quantum projection noise and Rabi frequency fluctuations from the readout, adds an offset to the mutual information, such that, even for zero coupling, $\mathcal{I}$ has a small finite value. Conversely, for fixed coupling strength, increasing the detuning $\Delta\omega = \delta_1 - \delta_2$ through changing the frequency

of M_2 progressively suppresses synchronization, as the frequency mismatch eventually becomes too large for the interaction to induce phase locking. This behavior is illustrated in Figs. 7(b) and 7(e) for a fixed effective coupling strength of $V/V_0 = 1.0$. To map out synchronization across the full coupling-detuning parameter space, we performed simulations spanning a wide range of parameters [see Figs. 7(c) and 7(f)]. The simulation results are plotted together with experimental data along the two representative cuts (indicated by the blue dots), including a global offset representing the measured contribution of the readout noise. The Arnold-tongue contour shows that synchronization of the two oscillators requires a finite coupling V . This arises from the requirement that dissipative interactions must overcome individual damping, which tends to erase the phase information.

IV. PHASE LOCKING AND DESYNCHRONIZATION UNDER AN EXTERNAL DRIVE

We augment the model in Eq. (1) with an additional electric field resonant with M_1 . This external drive explicitly breaks the underlying U(1) symmetry of the system and imposes a phase preference on the Wigner functions,

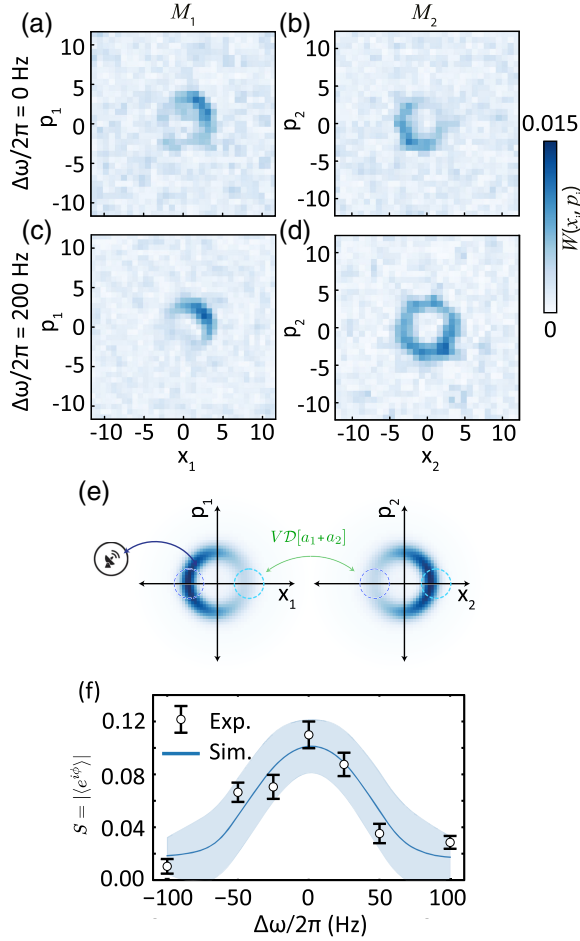


FIG. 8. Entrainment of mutual synchronization to an external drive. M_1 is driven by an external signal such that its phase aligns with the drive. The second mode M_2 is dissipatively locked to M_1 with a phase difference of $\varphi = \pi$. (a),(b) Reconstructed Wigner functions of M_1 and M_2 when the second oscillator is resonant with the external drive. (c),(d) The same as in (a),(b) but with the second oscillator detuned by 200 Hz. (e) Illustration of mutual synchronization between M_1 and M_2 with external drive. (f) Degree of phase localization of the second oscillator, quantified by the mean resultant length $S = |\langle e^{i\phi} \rangle|$ as a function of frequency detuning $\Delta\omega$ between the oscillator and the external signal. The black circles are experimentally measured S , with error bars representing the quantum projection noise. The blue curve is the numerical simulation with the shaded region indicating the uncertainty to motional frequency and Rabi frequency fluctuations.

determined by the drive phase, its amplitude, coupling strengths, and detunings. The resulting configuration can be viewed as a primary-secondary system: The externally driven mode M_1 locks to the drive, while the undriven mode M_2 synchronizes with M_1 through dissipative coupling. By varying the frequency of M_2 , we can now directly probe the desynchronization process by evaluating the local Wigner functions of each oscillator because of the externally imposed phase preference (as opposed to the previous case without external drive). Figures 8(a) and 8(b) show the

Wigner functions of both modes when the frequency of M_1 is tuned to match that of the external drive. In this regime, synchronization emerges, and both modes acquire well-defined phase preferences with the expected phase difference $\varphi = \pi$, set by the phase of the collective dissipation. As the frequency of M_2 increases, detuning causes it to desynchronize, while M_1 remains phase locked to the drive [Figs. 8(c) and 8(d)]. To quantify the degree of phase locking, we use the mean resultant length $S = |\langle e^{i\phi} \rangle|$ [47] of M_2 . Figure 8(f) shows S as a function of the detuning $\Delta\omega$ between M_1 and M_2 , which is maximized at zero detuning and decreases rapidly as the detuning increases and the two modes desynchronize with each other.

Synchronization in this primary-secondary configuration is related to a setup that has been used to enhance the resolution of a classical MEMS-based accelerometer [50], where a sensing resonator and a readout resonator are coupled unidirectionally. Although in our case the coupling between the two oscillators is reciprocal, we demonstrate that the externally applied drive frequency can nonetheless be estimated by measuring M_2 . This suggests opportunities for future work exploring sensing using quantum vdP oscillators [44,45,49,51] for enhanced resolution or noise suppression analogous to classical synchronized sensors [52].

V. CONCLUSIONS AND OUTLOOK

Unlike classical systems, synchronization between quantum systems must contend with the effects of quantum noise and measurement backaction. Here, we have realized the long-anticipated synchronization between two quantum van der Pol oscillators, enabled by engineered collective dissipation in a trapped-ion platform. The synchronized dynamics is encoded in a correlated joint measurement signal and hidden from local measurements, illustrating a uniquely quantum aspect of synchronization. Related theoretical work predicted that in the deep-quantum limit two coupled vdP oscillators can exhibit a weak “entanglement tongue” [42], similar to an Arnold tongue, yet defined via entanglement measures rather than phase locking. However, this phenomenon is conceptually distinct from synchronization (see Appendix E 4 for a detailed discussion) and worth investigating in future works.

The van der Pol oscillator has long served as the archetypal model of classical synchronization, providing the foundation for understanding more complex nonlinear networks. Our results establish its quantum counterpart as a practical and versatile building block, from which richer collective phenomena can be explored. Scaling to larger oscillator networks could uncover genuinely many-body effects beyond mean-field descriptions, such as quantum chimera states [53] or topologically protected synchronization [54], where classical simulations become intractable and quantum simulators provide a platform to explore these open-system dynamics at scale. Our approach is readily

scalable to larger networks of coupled van der Pol oscillators using ion chains. Notably, using a single $^{40}\text{Ca}^+$ ion and $N - 1$ $^{44}\text{Ca}^+$ ions [55] can unlock the control of $3N$ motional modes, opening the door to programmable oscillator networks. As in classical settings where phase-locked lasers boost output power and coupled sensors enhance sensitivity [52], analogous advantages may emerge in the quantum regime [17,56].

More broadly, the versatility of trapped ions in combining spin-spin, spin-boson, and boson-boson interactions [25,26,57] with programmable local and collective dissipation [37,38,58] offers a powerful setting for simulating nonequilibrium quantum matter. Our results thus stimulate further exploration of harnessing dissipation as a resource for both understanding and controlling complex quantum many-body dynamics [59–62].

ACKNOWLEDGMENTS

We thank Christoph Bruder, Zherui Chen, Tony Lee, Eric Lutz, Andrew Jayich, Ting Rei Tan, Juzar Thingna, and Baiyi Yu for helpful discussion and reading of the manuscript. We acknowledge the early contributions of Bingran You to the experimental setup, Thomas Lu to programming the arbitrary waveform generator, and Nils Citroth to modeling and simulation that informed design choices and interpretation in early experiments. This work was supported by the U.S. Department of Energy, Office of Science, Office of Basic Energy Sciences under Award No. DE-SC0023277. And we received some institutional support through the NSF-QLCI program through Grant No. 2016245. C.W.W. was supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation), Project No. 496502542 (WA 5170/1-1), and received funding from the European Union's Horizon Europe research and innovation program under the Marie Skłodowska-Curie Actions (MSCA) Grant Agreement No. 101149948.

Views and opinions expressed are, however, those of the author(s) only and do not necessarily reflect those of the European Union or the European Research Council.

DATA AVAILABILITY

The data that support the findings of this article are openly available [63].

APPENDIX A: EXPERIMENTAL PLATFORM AND PROTOCOLS

1. Mixed-isotope trapped-ion quantum simulator

We implement synchronization of two quantum vdP oscillators using the two axial motional modes M_1 and M_2 in a crystal of a $^{40}\text{Ca}^+$ and a $^{44}\text{Ca}^+$ ion. We use 397-nm light ($4^2S_{1/2} \leftrightarrow 4^2P_{1/2}$ transition) and 866-nm light

($4^2P_{1/2} \leftrightarrow 3^2D_{3/2}$ transition) to Doppler cool the $^{40}\text{Ca}^+$ ion while sympathetically cooling the $^{44}\text{Ca}^+$ ion. In addition, we add a sideband 800 MHz red-detuned from the $4^2S_{1/2} \leftrightarrow 4^2P_{1/2}$ transition of the $^{40}\text{Ca}^+$ ion to mitigate collision-induced melting of the mixed-isotope crystal [64]. The 866-nm light passes through an additional electro-optic modulator operated at 4.5 GHz to provide repumping on the $^{44}\text{Ca}^+$ ion to facilitate loading of the $^{44}\text{Ca}^+$ ion and maintaining the ion crystal. We perform single-shot readout of the qubit state on only the $^{40}\text{Ca}^+$ ion with a camera by shining the 397- and 866-nm light resonantly with the corresponding transition in $^{40}\text{Ca}^+$. From the camera image, the position of the $^{40}\text{Ca}^+$ in the ion crystal can be extracted to detect switching of the $^{40}\text{Ca}^+ - ^{44}\text{Ca}^+$ crystal orientation due to collisions with the background gas. If such an event is detected, we lower the trap rf power and raise it back again until the desired order is restored.

M_1 and M_2 are the in-phase and out-of-phase axial modes, whose frequencies are 522 and 906 kHz, respectively. After Doppler cooling, we perform sideband cooling on both modes to prepare them close to the ground state. The qubit states we use are $|\downarrow\rangle_z = 4^2S_{1/2}(m = -1/2)$ and $|\uparrow\rangle_z = 3^2D_{5/2}(m = -5/2)$. The qubit is initiated in the $|\downarrow\rangle_z$ state by optical pumping with σ_- polarized 397-nm light and 866-nm repumping light. The motional gain, loss, and dissipative coupling between the two oscillators are generated by driving the motional sidebands of the qubit transition of the $^{40}\text{Ca}^+$ ion $|\downarrow\rangle_z \leftrightarrow |\uparrow\rangle_z$ using a narrow-linewidth 729-nm laser locked to an ultralow expansion cavity, followed by qubit reset using 854-nm light resonant with $|\uparrow\rangle_z \leftrightarrow 4^2P_{3/2}(m = -3/2)$. The latter state mostly decays back to $|\downarrow\rangle_z$. The leakage out of the qubit manifold (from $4^2P_{3/2}$ to $3^2D_{3/2}$ process) is only 0.6% [65] and can be closed with a short optical pumping pulse once every five evolution cycles.

2. Dissipative and coherent quantum controls

The 729-nm light passes through a double-pass acousto-optic modulator (AOM), which is used to change the frequency of the light to match the qubit transition frequency with a constant offset of +80 MHz. It then passes through a single-pass AOM centered at -80 MHz before coupling to the ions, which enables control over dissipative and coherent operations on the sidebands. The single-pass AOM is driven by an arbitrary waveform generator (Spectrum M4i.6621-x8). Throughout the experiment, the phase of each rf signal is referenced to a common clock to ensure phase continuity. For implementing negative damping (via BSB) and nonlinear damping (via 2RSB), we tune the single-pass AOM to match the frequencies of the corresponding sidebands and randomize the phase of the qubit-motion coupling. In the dissipative coupling, the two RSBs are driven with a fixed phase difference φ , which sets the phase reference for the mutual

synchronization. To read out the motional state, we use the state-dependent force [66] whose phase defines the readout axis in phase space. For the phase locking to the external drive, an oscillating electric field at the same frequency of M_1 is applied to one of the trap electrodes to excite the ion motion. This drive is controlled by a separate channel of the same arbitrary waveform generator to ensure phase coherence with the optical modulation.

3. Readout of the motional states

Here, we discuss the reconstruction of the Wigner function of a single motional mode $W(x, p)$ and the reconstruction of the joint probability function $P(x_1, x_2)$ [$P(x_1, p_2)$] between two motional modes [reconstruction measurement in Fig. 2(a)]. To reconstruct the Wigner function of a single target motional state $|\psi_{\text{osc}}\rangle$, we read out the characteristic function by applying state-dependent forces (SDF) with different time durations and phases, which we couple to the RSB and BSB of the same mode simultaneously with phases ϕ_r and ϕ_b . In the readout process, we apply the SDF in the σ_x basis by keeping $\phi_b + \phi_r = \pi$ with Hamiltonian

$$H_{\text{SDF}} = \frac{\eta\Omega}{2} \sigma_x (a e^{i\phi_m} + a^\dagger e^{-i\phi_m}). \quad (\text{A1})$$

Here, η is the Lamb-Dicke parameter and Ω is the Rabi frequency of the SDF. The motional phase $\phi_m = (\phi_r - \phi_b)/2$ is controlled by the relative phase of the blue and red sideband. The initial qubit-motion state before applying the SDF is $|\psi(0)\rangle = |\psi_{\text{osc}}\rangle \otimes |\downarrow\rangle_z$. During time evolution with H_{SDF} , the motional state is coupled to the σ_x basis states, $|\uparrow\rangle_x$ and $|\downarrow\rangle_x$, displacing the oscillator toward opposite directions in phase space. The state after applying SDF for time t is

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}} [|\uparrow\rangle_x \mathcal{D}(\beta) |\psi_{\text{osc}}\rangle + |\downarrow\rangle_x \mathcal{D}(-\beta) |\psi_{\text{osc}}\rangle]. \quad (\text{A2})$$

Here, $\mathcal{D}(\beta) = \exp(\beta a^\dagger - \beta^* a)$ is the displacement operator with $\beta = (i/2)\eta\Omega t e^{i\phi_m}$. By measuring in the σ_z basis, we obtain the real part of the characteristic function

$$\langle \sigma_z \rangle = \text{Re}[\langle \psi_{\text{osc}} | \mathcal{D}(2\beta) | \psi_{\text{osc}} \rangle] = \text{Re}[\chi(2\beta)]. \quad (\text{A3})$$

To read out the imaginary part, we prepare the initial state in the σ_y basis $|\psi(0)\rangle = |\psi_{\text{osc}}\rangle \otimes |\downarrow\rangle_y$. The subsequent SDF drive and measurement in the qubit σ_z basis are the same as the readout of the real part. After obtaining the real and imaginary parts of the characteristic function, we then apply the two-dimensional Fourier transform and reconstruct the Wigner function

$$W(\alpha) = \frac{1}{\pi^2} \int d^2\beta \chi(2\beta) e^{2i(\beta_r \alpha_i - \beta_i \alpha_r)}. \quad (\text{A4})$$

In addition, quantum projection noise, state preparation, and other imperfections introduce a global offset of the qubit population. We estimate and remove the constant offset of the characteristic function by averaging over its marginal distribution [35]. After subtracting this offset, we apply zero padding to enhance the spatial resolution of the reconstructed Wigner function.

For reconstructing the joint probability distribution $P(x_1, x_2)$ [$P(x_1, p_2)$ or other combinations], we apply two subsequent SDFs on the two motional modes as shown in Eq. (2). Similar to the single-mode Wigner function, reading out the real part of the joint characteristic function requires starting in $|\downarrow\rangle_z$, while reading out the imaginary part requires starting in $|\downarrow\rangle_y$. By changing the motional phases on the SDFs on the two modes, we are able to control the readout axes of the two-mode joint probability distribution.

APPENDIX B: EXPERIMENTAL PARAMETERS AND CALIBRATION PROCEDURES

1. Experimental parameters for synchronizing two vdP oscillators

The synchronization experiment is conducted mainly in two parameter regimes, with vdP oscillators operating in the near-classical ($\bar{n} = 3.5$) and quantum ($\bar{n} = 1.4$) regime [47]. To generate the two vdP oscillators with similar occupation number, we tune the nonlinear and negative damping strengths such that κ_-/κ_+ are identical for both oscillators. Experimentally, for each of the sideband operations, we keep the same laser power and adjust the sideband drive time to account for the different Lamb-Dicke parameters of the two modes to achieve similar damping strength. For example, the duration of the blue sideband (BSB, $|\downarrow\rangle|n\rangle \leftrightarrow |\uparrow\rangle|n+1\rangle$ transition) drive is $\tau_{\text{BSB}} \propto 1/\eta_i$, while the duration of the second-order red sideband (2RSB, $|\downarrow\rangle|n\rangle \leftrightarrow |\uparrow\rangle|n-2\rangle$ transition) drive is $\tau_{\text{2RSB}} \propto 1/\eta_i^2$, where η_i is the Lamb-Dicke parameter for mode M_i of the $^{40}\text{Ca}^+$. In Table I, we list the experimental parameters used for the synchronization experiment in the two regimes: Lamb-Dicke parameters η_i for the two modes; corresponding carrier Rabi frequencies of the sideband operations Ω_{BSB} and Ω_{2RSB} to generate effective dampings, the duration $\tau_{\text{BSB}}, \tau_{\text{2RSB}}$ of applying the BSB and 2RSB drives, and the resulting effective rates κ_+, κ_- ; the corresponding carrier Rabi frequencies of simultaneous RSB drives Ω_{sync} on the two modes to generate the dissipative coupling with interaction time τ_{sync} and resulting effective coupling rate V_0 . The heating rate of in-phase mode M_1 is around 10 quanta/s, which is negligible compared to the engineered dissipative rates. The heating of the out-of-phase mode M_2 is strongly suppressed and can be neglected.

In Fig. 9, we show the numerical simulation using QUTIP [67] for synchronization between two vdP oscillators

TABLE I. The experimental parameters for synchronization of two vdP oscillators in the near-classical and quantum regimes.

	Near-classical		Quantum	
	M_1	M_2	M_1	M_2
η_i	0.094	0.072	0.094	0.072
$\Omega_{\text{BSB}}/2\pi$ (MHz)	0.12	0.12	0.11	0.11
$\Omega_{\text{RSB}}/2\pi$ (MHz)	0.22	0.22	0.22	0.22
$\tau_{\text{BSB}}(\mu\text{s})$	17.10	22.46	17.10	22.45
$\tau_{\text{RSB}}(\mu\text{s})$	48.77	84.06	97.54	168.12
$\kappa_+/2\pi$ (kHz)	0.12	0.12	0.10	0.10
$\kappa_-/2\pi$ (kHz)	0.017	0.017	0.042	0.042
$\Omega_{\text{sync}}/2\pi$ (MHz)	0.048	0.063	0.048	0.063
$\tau_{\text{sync}}(\mu\text{s})$	24		32	
$V_0/2\pi$ (kHz)	0.1		0.1	

in near-classical and quantum regimes using the parameters from Table I, which shows good agreement with the experimental results.

2. Frequency calibration of qubits and the motional modes

During data collection, we recalibrate the qubit frequency every 180 s to compensate for slow drifts in the reference cavity and magnetic field, and we recalibrate the motional frequencies of the two axial modes every 400 s.

Long-term stability of the motional frequencies is essential, as both the synchronization strength and the motional reconstructions are sensitive to frequency drift. For the motional frequency calibration, we simultaneously probe the RSBs and BSBs on both modes using low laser intensity and an interrogation time of approximately 2 ms

to reduce linewidth and laser-induced ac-Stark shifts [68]. To reduce the impact of qubit frequency fluctuations during the motional frequency calibration, we interleave the measurements by collecting one data point from BSB and one data point from RSB in a repeating cycle. The motional frequencies are then extracted from the difference between the BSB and RSB fit and updated for the subsequent experiments. Figure 10(a) shows the relative frequency drift of modes M_1 and M_2 over a two-hour interval after the trap has reached thermal equilibrium with its environment; the residual fluctuations are small, with standard deviations $\sigma_{M_1} = 21$ Hz and $\sigma_{M_2} = 30$ Hz. Remaining noise likely originates from the sources providing the end cap electrodes voltage, environmental electric-field noise, and uncertainties in the sideband frequency measurements. These frequency uncertainties are included in the numerical simulations of mutual synchronization and phase locking to the external drive.

By contrast, the motional mode frequencies drift much more rapidly immediately after loading the two-ion crystal [Fig. 10(b)]. During loading, the trap is operated at reduced rf power to facilitate trapping of the mixed-isotope chain; after switching to the higher rf power used for the synchronization measurements, resistive heating of the rf electrodes leads to thermal expansion, which increases the effective end cap separation and reduces the trap frequency. The frequency then relaxes toward a steady value on a timescale of one hour.

3. Phase control in mutual synchronization

With the precise frequency calibration of the motional modes, we then demonstrate the capability to accurately control the motional phases. We prepare the state in

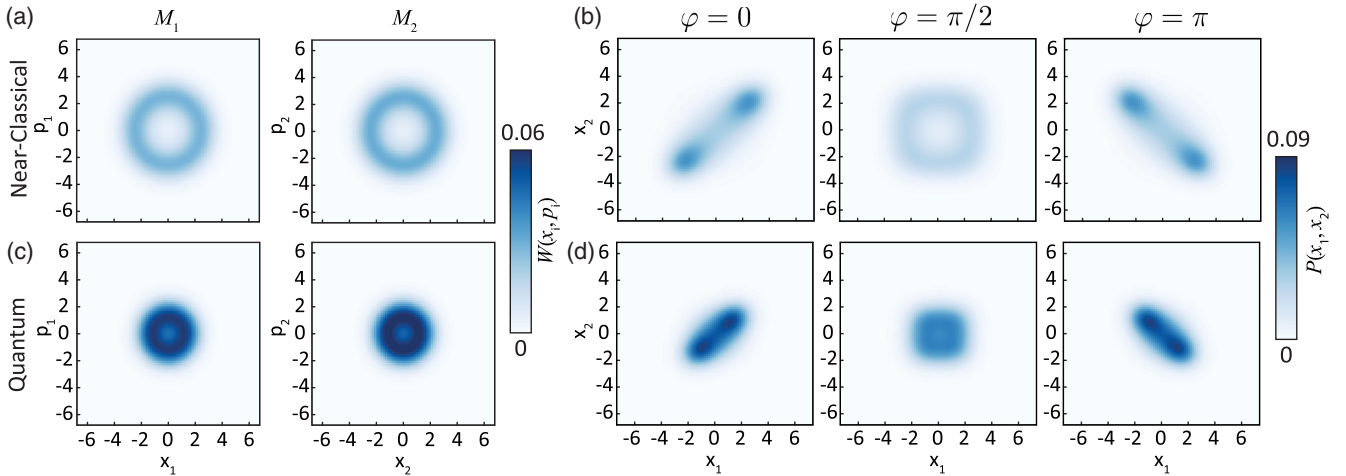


FIG. 9. Numerical simulation of synchronization between two vdP oscillators in the near-classical and quantum regimes. (a),(b) Near-classical regime with $\kappa_-/\kappa_+ = 0.14$ and mean phonon number $\bar{n} = 3.5$. (a) Simulated Wigner functions show the expected donut-shaped profiles without phase preference. (b) Joint probability distributions reproduce in-phase, circular, and antiphase correlations for $\varphi = 0, \pi/2$, and π , respectively. (c),(d) The same as (a),(b) but in the quantum regime with $\kappa_-/\kappa_+ = 0.42$ and $\bar{n} = 1.4$. These simulations correspond to the experimental results in Fig. 6.

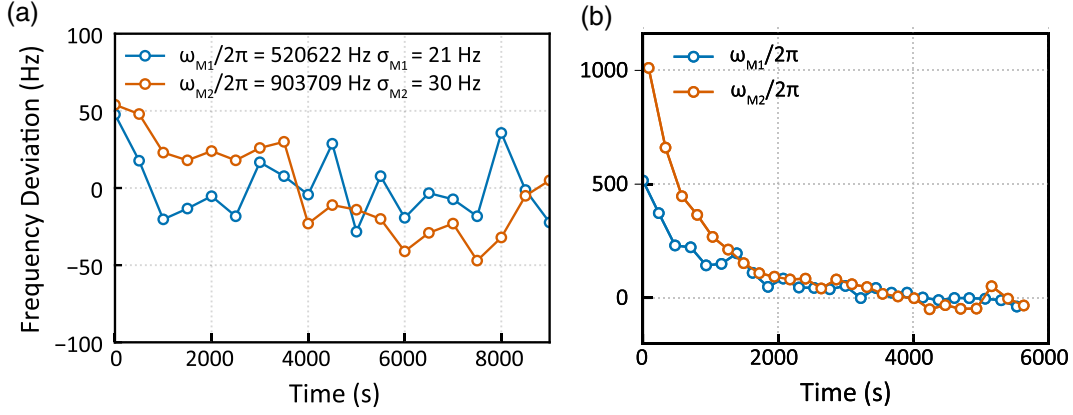


FIG. 10. (a) The measured relative frequencies of axial modes M_1 (blue) and M_2 (orange) during a typical data collection run. (b) The frequency relaxation process of the two modes upon switching from low rf power (for loading ions) to high rf power (for experiments) due to thermal expansion of the trap rf electrodes.

$|\psi\rangle = |0\rangle \otimes |\downarrow\rangle_z$ and drive the motional mode with the SDF $H_{\text{SDF}} = [(\eta\Omega)/2]\sigma_x(ae^{i\phi_m} + a^\dagger e^{-i\phi_m})$. The SDF entangles the qubit and motional degree of freedom, and the system evolves into a spin-motional cat state [66]. After resetting the qubit using repump light, the system is in a mixed state of two coherent states $\varrho_{\text{osc}} = \frac{1}{2}|\alpha\rangle\langle\alpha| + \frac{1}{2}|-\alpha\rangle\langle-\alpha|$, where $\alpha = \frac{1}{2}\eta\Omega te^{i\phi_m}$ is the displacement after applying the SDF for time t . Then, the motional mode is measured with the protocol described in the previous section on experimental protocols. In Fig. 11(a), we show the Wigner functions corresponding to the motional phases $\phi_m = -\pi/3, 0, \pi/4$, which align well with the expected results. We also illustrate the coherent phase control between the two motional modes M_1 and M_2 . Specifically, we apply the SDF to create the two-mode

cat state $|\psi\rangle = (1/\sqrt{2})(|\uparrow\rangle_x|\alpha_1, \alpha_2\rangle + |\downarrow\rangle_x|-\alpha_1, -\alpha_2\rangle)$, which after qubit reset is projected onto the mixed state $\varrho_{\text{osc}} = \frac{1}{2}[\varrho(\alpha_1, \alpha_2) + \varrho(-\alpha_1, -\alpha_2)]$. When the displacement α_1 of M_1 is along the position axis ($\phi_{m1} = \pi/2$) and the displacement of M_2 is also aligned with the position axis ($\phi_{m2} = 0, \pi$), the joint probability distribution $P(x_1, x_2)$ exhibits two lobes oriented along the diagonal and anti-diagonal directions [Fig. 11(b)].

4. Calibration of external drive

By driving the ion crystal with an oscillating electrical field applied on one of the end cap electrodes, the motional mode will be coherently excited [69], and the corresponding Hamiltonian is

$$H = \Omega_d(ae^{i\varphi_d} + a^\dagger e^{-i\varphi_d}). \quad (\text{B1})$$

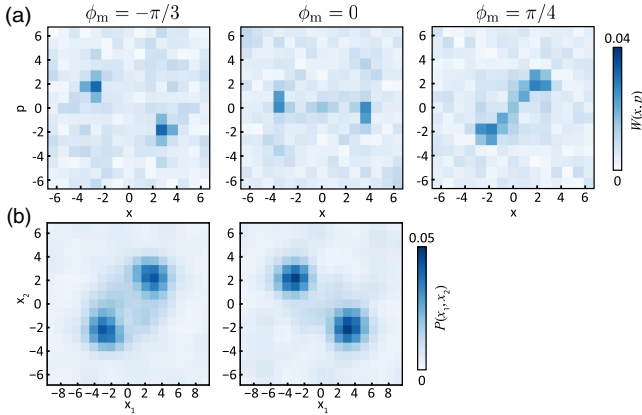


FIG. 11. (a) Wigner functions of $\varrho = \frac{1}{2}(|\alpha\rangle\langle\alpha| + |-\alpha\rangle\langle-\alpha|)$ with motional phase $\phi_m = -\pi/3, 0$, and $\pi/4$ with 60 μs SDF drive and 30 μs readout time. The corresponding carrier Rabi frequency of the SDF drive is $\Omega/2\pi = 0.077$ MHz. (b) Joint probability distribution $P(x_1, x_2)$ for the mixture of opposite two-mode coherent state on M_1 and M_2 ($\varrho = \frac{1}{2}[\varrho(\alpha_1, \alpha_2) + \varrho(-\alpha_1, -\alpha_2)]$) with a phase difference of 0 and π .

Here, Ω_d is the strength of external drive and φ_d is the phase of the external drive. With this additional external drive, the phase of the vdP oscillator will be entrained to the external drive. The resulting Wigner function breaks the radial symmetry and is attracted toward the drive axis (see Fig. 5). To calibrate the strength of the external drive, we apply the oscillating electric field resonant with a specific motional mode for different amounts of time and use the blue sideband flopping to probe the motional occupation. After the system evolved for time t , the displacement induced by the external drive is $\alpha = \Omega_d te^{i\varphi_d}$. To extract the amplitude of displacement $|\alpha|$, we fit the blue sideband Rabi flopping assuming a coherent displacement of a thermal state [33]. For the same external drive amplitude, the coupling strength for the axial mode of a single ion is found to be $\Omega_d/2\pi = 6.94$ kHz, while the coupling strength to the in-phase mode (M_1) of the $^{40}\text{Ca}^+ - ^{44}\text{Ca}^+$ ion crystal is found to be $\Omega_{d,M1}/2\pi = 9.80$ kHz (Fig. 12). Their ratio of 1.41 is consistent with the theoretical

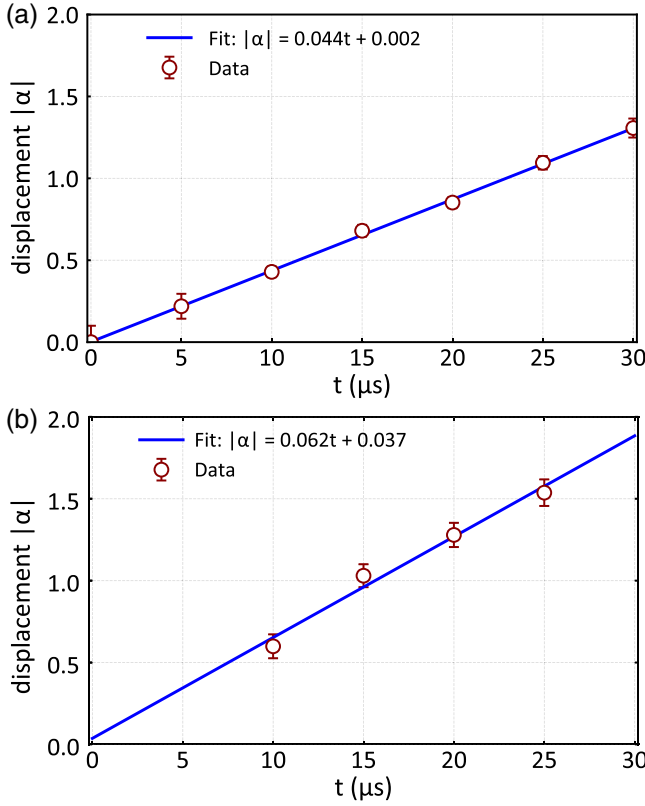


FIG. 12. Fitted displacement $|\alpha|$ from BSB flopping as a function of the time t for which we apply the external field with constant amplitude. Error bars represent uncertainty in the fitted displacement amplitude, obtained by fitting the experimental data to simulated BSB flopping of ions with displaced motional state. (a) Single $^{40}\text{Ca}^+$ ion axial mode. (b) $^{40}\text{Ca}^+ - ^{44}\text{Ca}^+$ in-phase mode (M_1).

calculation $\Omega_{d,M_1}/\Omega_d = 1.43$. Note that, for a pure $^{40}\text{Ca}^+ - ^{40}\text{Ca}^+$ ion crystal, the ratio is $\sqrt{2}$.

APPENDIX C: ERROR ANALYSIS IN STABILITY OF SYNCHRONIZATION

To further analyze the stability of synchronization to detuning and coupling strength, we perform numerical simulations that incorporate motional frequency drifts and Rabi frequency fluctuations from intensity noise and beam pointing errors. Specifically, we compute the mutual information as a function of frequency difference $\Delta\omega$ and coupling strength V : $\mathcal{I}(\Delta\omega, V)$. The coupling strength fluctuation δV and frequency fluctuation $\delta\omega$ contribute to uncertainty of mutual information. We calculate this uncertainty from the standard deviation of 20 numerical simulations using Gaussian distributed coupling strengths with standard deviation 2% and frequencies with standard deviation $\Delta\omega/2\pi = \sqrt{\sigma_1^2 + \sigma_2^2} = 37$ Hz. The estimation of the mutual information uncertainty interval $[\mathcal{I} - \Delta\mathcal{I}, \mathcal{I} + \Delta\mathcal{I}]$ is plotted as the shaded area in Figs. 7(a)–7(e). The error bar on the data

reflects quantum projection noise on $\mathcal{I}$, which is calculated by adding random quantum projection noise to the simulation and computing the standard deviation of 20 samples of noisy mutual information. We use a similar method for analyzing experimental imperfections and statistical errors of $S = |\langle e^{i\phi} \rangle|$ in Fig. 8(f).

APPENDIX D: EFFECTIVE INTERACTION GENERATED WITH SIDEBAND TRANSITION

1. Derivation of effective dissipation

To obtain the effective dynamics on the oscillators [Eq. (1)], we begin by deriving the effective Lindblad dissipator on the bosonic modes from a qubit-motion coupling Hamiltonian. The most general form of the qubit-motion coupling is

$$H = \Omega(\sigma_+ O + \sigma_- O^\dagger), \quad (\text{D1})$$

where O represents a certain bosonic operator (e.g., $a^\dagger$ for the BSB, a^2 for the 2RSB, and a for the RSB), Ω is the generalized Rabi frequency of the sideband transition, and σ_+ (σ_-) is the qubit-raising (-lowering) operator. At the beginning of each step, the qubit state is reset to $|\downarrow\rangle_z$, so that the qubit-motion density matrix takes the form

$$\rho = |\downarrow\rangle_z \langle\downarrow|_z \otimes \rho_{\text{osc}}. \quad (\text{D2})$$

After a coherent evolution under H for a small duration δt , followed by another qubit reset, the motional state is obtained by taking the partial trace of ρ over the qubit degree of freedom:

$$\rho_{\text{osc}}(\delta t) = \text{Tr}_s[e^{-i\hat{H}\delta t} \rho e^{i\hat{H}\delta t}] = \sum_{s=\downarrow, \uparrow} \langle s | e^{-i\hat{H}\delta t} \rho_{\text{osc}} e^{i\hat{H}\delta t} | s \rangle. \quad (\text{D3})$$

Expanding each term up to second order in δt and neglecting higher-order corrections, the $|\downarrow\rangle_z$ and $|\uparrow\rangle_z$ components of the trace are given by $\langle \downarrow | \rho(\delta t) | \downarrow \rangle = \rho_{\text{osc}} - \frac{1}{2}\Omega^2\delta t^2\{O^\dagger O, \rho_{\text{osc}}\}$ and $\langle \uparrow | \rho(\delta t) | \uparrow \rangle = \Omega^2\delta t^2\hat{O}\rho_{\text{osc}}\hat{O}^\dagger$. The density matrix of the motional degrees of freedom after a time δt is given by

$$\rho_{\text{osc}}(\delta t) \approx \rho_{\text{osc}} + \Omega^2\delta t^2\hat{O}\rho_{\text{osc}}\hat{O}^\dagger - \frac{1}{2}\Omega^2\delta t^2\{O^\dagger O, \rho_{\text{osc}}\}. \quad (\text{D4})$$

In the limit $\delta t \rightarrow 0$, the time derivative of the reduced density matrix is then given by

$$\dot{\rho}_{\text{osc}}(t) = \lim_{\delta t \rightarrow 0} \frac{\rho_{\text{osc}}(t + \delta t) - \rho_{\text{osc}}(t)}{\delta t} = \kappa \mathcal{D}[\hat{O}]\rho_{\text{osc}}(t), \quad (\text{D5})$$

where $\mathcal{D}[\hat{O}]$ denotes a collapse operator acting on the density matrix, i.e., $\mathcal{D}[\hat{O}] \rho = \hat{O} \rho \hat{O}^\dagger - \frac{1}{2} \{ \hat{O}^\dagger \hat{O}, \rho \}$. The resulting effective damping rate is then given by $\kappa = \Omega^2 \delta t$. We now recall that the Hamiltonians of the BSB and 2RSB transitions are [33], respectively,

$$H_{\text{BSB}}^{(i)} = \frac{i}{2} \Omega_{\text{BSB}}^{(i)} \eta_i (\sigma_+ a_i^\dagger e^{i\varphi_b} - \sigma_- a_i e^{-i\varphi_b}),$$

$$H_{\text{2RSB}}^{(i)} = -\frac{1}{4} \Omega_{\text{2RSB}}^{(i)} \eta_i^2 (\sigma_+ a_i^2 e^{i\varphi_r} + \sigma_- a_i^{\dagger 2} e^{-i\varphi_r}). \quad (\text{D6})$$

Here, $H_{\text{BSB}}^{(i)}$ and $H_{\text{2RSB}}^{(i)}$ denote the interaction Hamiltonians for the BSB and 2RSB transitions on mode M_i , respectively. $\Omega_{\text{BSB}}^{(i)}$ and $\Omega_{\text{2RSB}}^{(i)}$ are the corresponding Rabi frequencies, η_i is the Lamb-Dicke parameter, and φ_b (φ_r) denotes the phase of the BSB (2RSB) drive. The annihilation (creation) operators of motional mode M_i are denoted as a_i ($a_i^\dagger$).

Additionally, we define each cycle as the sum of all sideband interaction times

$$T = \sum_{i=1,2} (t_{\text{BSB}}^{(i)} + t_{\text{2RSB}}^{(i)}) + t_{\text{sync}}, \quad (\text{D7})$$

where the BSB and the 2RSB interaction times on M_i are $t_{\text{BSB}}^{(i)}$ and $t_{\text{2RSB}}^{(i)}$, respectively. Consequently, the negative damping $\mathcal{D}[a^\dagger]$ and nonlinear damping $\mathcal{D}[a^2]$ have effective rates $\kappa_+ = [(\Omega_{\text{BSB}}^2 \eta^2 t_{\text{BSB}}^2)/4T]$ and $\kappa_- = [(\Omega_{\text{2RSB}}^2 \eta^4 t_{\text{2RSB}}^2)/16T]$, respectively.

To realize the collective dissipation that enables the synchronization, we simultaneously drive the RSBs on both oscillators with a relative phase φ and equal strength and then perform a qubit reset. The Hamiltonian of the collective RSBs drive is

$$H_{\text{sync}} = i \frac{\Omega_{\text{sync}} \eta}{2} [\sigma^- (a_1^\dagger - e^{-i\varphi} a_2^\dagger) - \sigma^+ (a_1 - e^{i\varphi} a_2)], \quad (\text{D8})$$

corresponding to the dissipator $V \mathcal{D}[a_1 - e^{i\varphi} a_2]$. Here, for simplicity, we write the equal RSBs strength as $\Omega_{\text{sync}} \eta = \Omega_{\text{sync}}^{(1)} \eta_1 = \Omega_{\text{sync}}^{(2)} \eta_2$. Equation (D8) can be written in the form of general qubit-motion Hamiltonian [Eq. (D1)] with $\hat{O} = i(a_1 - e^{i\varphi} a_2)$ and $\Omega = \Omega_{\text{sync}} \eta/2$. Following the above derivation, the effective dissipation rate is $V = [(\Omega_{\text{sync}}^2 \eta^2 \tau_{\text{sync}}^2)/4T]$.

2. Frequency tuning between the two oscillators

To study the robustness of the synchronization against frequency mismatch between the two vdP oscillators, we tune the frequency difference $\Delta\omega = \delta_2 - \delta_1$ between the two oscillators by changing the frequency of the laser fields. Specifically, to set the frequency of mode M_i to δ_i ,

we tune the frequencies of the BSB, RSB, and 2RSB drives to $\omega_i + \delta_i$, $-(\omega_i + \delta_i)$, and $-2(\omega_i + \delta_i)$, respectively.

APPENDIX E: SYNCHRONIZATION INDICATORS AND DYNAMICS

1. Frequency and coupling strength tolerance of synchronization

As shown in Sec. III, synchronization persists over a certain range of frequency detuning and coupling strength. In general, synchronization becomes more pronounced for stronger coupling and smaller frequency mismatch. Figure 14 illustrates this trend where synchronization manifests itself as correlation of x_1 and x_2 quadrature for $\varphi = 0$ dissipative interaction. When the two oscillators are uncoupled ($V = 0$), $P(x_1, x_2)$ exhibits a square pattern (which does not change upon frequency detuning). In contrast, as the interaction strength V is increased, the diagonal pattern emerges, indicating the synchronization of the two oscillators.

2. Mutual information as indicator of synchronization

Mutual information quantifies the total amount of correlations between two subsystems. Since synchronization, as discussed here, also relies on correlations between the two vdP oscillators, one can expect mutual information to serve as a good measure of (quantum) synchronization [70]. Given the steady-state density matrix of the two oscillators ϱ_{12} , the mutual information is defined as

$$\mathcal{I}(\varrho_{12}) = S(\varrho_1) + S(\varrho_2) - S(\varrho_{12}), \quad (\text{E1})$$

where $S(\varrho) = -\text{Tr}(\varrho \log \varrho)$ denotes the von Neumann entropy and $\varrho_1 = \text{Tr}_2(\varrho_{12})$ is the reduced density matrix of M_1 (analogously for M_2). In Fig. 13(a), we show simulation results for $\mathcal{I}(\varrho_{12})$ in the steady state, plotted as a function of the coupling strength V and the frequency detuning between the two oscillators $\Delta\omega = \delta_1 - \delta_2$ [see also Eq. (1)]. The simulation clearly shows the Arnold tongue feature [1] that characterizes the synchronized regime and further shows a threshold in the coupling strength V below which synchronization is absent. Classically, the transition between synchronized and unsynchronized dynamics is a sharp boundary. In the presence of quantum fluctuations, however, the border of the Arnold tongue is smoothed [11,48]. Nevertheless, the mutual-information-based measure of synchronization remains a reliable indicator [70].

However, evaluating $\mathcal{I}(\varrho_{12})$ requires access to the full density matrix ϱ_{12} . A more experimentally accessible quantity that captures similar information can be derived from the joint probability distributions between the quadratures. In particular, the mutual information of the joint x_1 and x_2 quadratures given by

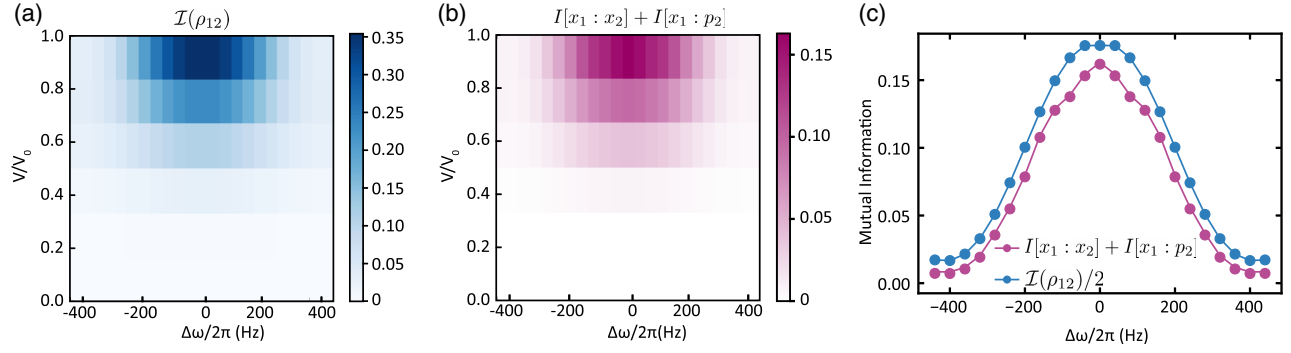


FIG. 13. (a) Mutual information between two motional modes as a function of coupling strength and frequency detuning. (b) Summation of mutual information between the quadratures x_1 - x_2 and x_1 - p_2 . (c) Comparison between $\mathcal{I}(\rho_{12})/2$ and $I[x_1 : x_2] + I[x_1 : p_2]$ as a function of frequency detuning.

$$I(x_1 : x_2) = \iint P(x_1, x_2) \log \left(\frac{P(x_1, x_2)}{P(x_1)P(x_2)} \right) dx_1 dx_2, \quad (\text{E2})$$

where $P(x_1, x_2) = \langle x_1, x_2 | \rho_{12} | x_1, x_2 \rangle$ and $P(x_i)$ denotes the marginal distribution for $i = 1, 2$. Similarly, one can calculate the mutual information $I(x_1 : p_2)$ from the joint distribution $P(x_1, p_2)$. The full mutual information would involve all quadratures, x and p , simultaneously, i.e., $P(x_1, p_1, x_2, p_2)$, which is experimentally quite demanding. However, the combination $\mathcal{I} = I(x_1 : x_2) + I(x_1 : p_2)$ (sufficient due to symmetry considerations) provides the necessary information to identify features analogous to the classical Arnold tongue. In Fig. 13(b), we show simulation results for $\mathcal{I}$, which clearly exhibits the same overall structure as $\mathcal{I}(\rho_{12})$. It thus serves as a sufficient indicator of synchronization in our setup. Figure 13(c) shows a cut at $V/V_0 = 1.0$ as the frequency difference between the two

oscillators increases, which demonstrates that $\mathcal{I}$ follows the same trend as $\mathcal{I}(\rho_{12})$ and thus signals the presence of an Arnold tongue.

3. Classical and quantum dynamics for different synchronization phases

In this section, we investigate how different synchronization phases affect the dynamics in the x_1 - x_2 plane in Figs. 6 and 9 by analyzing the mean-field equations. The classical dynamics corresponding to Eq. (1) is derived from the dynamics of $\alpha_i = \langle a_i \rangle$ by factorizing all expectation values (mean field). For simplicity, we consider equal oscillator frequencies, and the classical dynamics are given by the following equations:

$$\begin{aligned} \dot{\alpha}_1 &= \frac{\kappa_+}{2} \alpha_1 - \kappa_- |\alpha_1|^2 \alpha_1 - \frac{V}{2} (\alpha_1 - e^{i\varphi} \alpha_2), \\ \dot{\alpha}_2 &= \frac{\kappa_+}{2} \alpha_2 - \kappa_- |\alpha_2|^2 \alpha_2 - \frac{V}{2} (\alpha_2 - e^{-i\varphi} \alpha_1). \end{aligned} \quad (\text{E3})$$

The nonlinear system can be analyzed using linear stability analysis. To this end, we first identify the fixed points of Eq. (E3) and then analyze their stability by evaluating the Jacobian at those points. There are two sets of fixed points. The first is the trivial fixed point $\alpha_1 = \alpha_2 = 0$. The second set is given by $\alpha_1 = e^{i\varphi} \alpha_2 = e^{i(\varphi+\theta)} |\alpha|$, with positive amplitude $|\alpha| = \sqrt{\kappa_+ / (2\kappa_-)}$ and θ a free global phase. The Jacobian $\mathbf{J}(\bar{\alpha})$ evaluated at a fixed point $\bar{\alpha} = (\bar{\alpha}_1, \bar{\alpha}_2)$ is given by

$$\mathbf{J}(\bar{\alpha}) = \begin{pmatrix} \frac{\kappa_+}{2} - \kappa_- |\bar{\alpha}_1|^2 - \frac{V}{2} & e^{i\varphi} V \\ e^{-i\varphi} V & \frac{\kappa_+}{2} - \kappa_- |\bar{\alpha}_2|^2 - \frac{V}{2} \end{pmatrix}, \quad (\text{E4})$$

which is Hermitian and all the eigenvalues are real. A fixed point is linearly stable if all eigenvalues of the Jacobian evaluated at that point are negative.

For the trivial fixed point $\alpha_1 = \alpha_2 = 0$, the eigenvalues are $\lambda_1 = \kappa_+$ and $\lambda_2 = \kappa_+ - 2V$. Thus, for any positive κ_+ , at least one eigenvalue is positive, making this fixed point

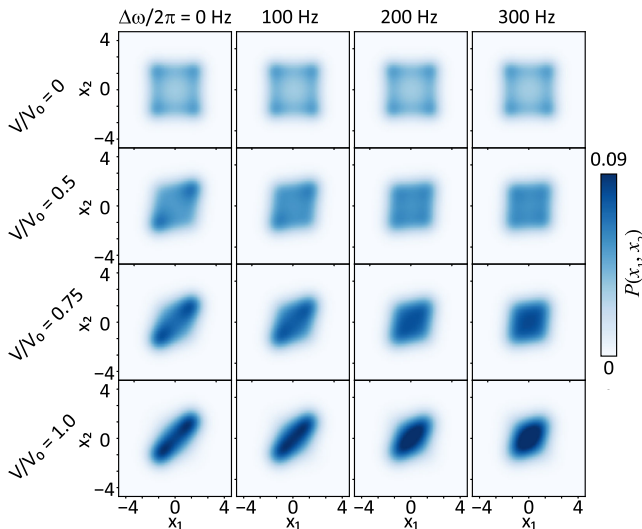


FIG. 14. Numerical simulation of the joint probability distribution $P(x_1, x_2)$ with $\varphi = 0$ in the dissipative coupling for different combinations of the coupling strength V and the frequency detuning $\Delta\omega/2\pi$. All other parameters are the same as the ones used in the quantum regime (Fig. 6).

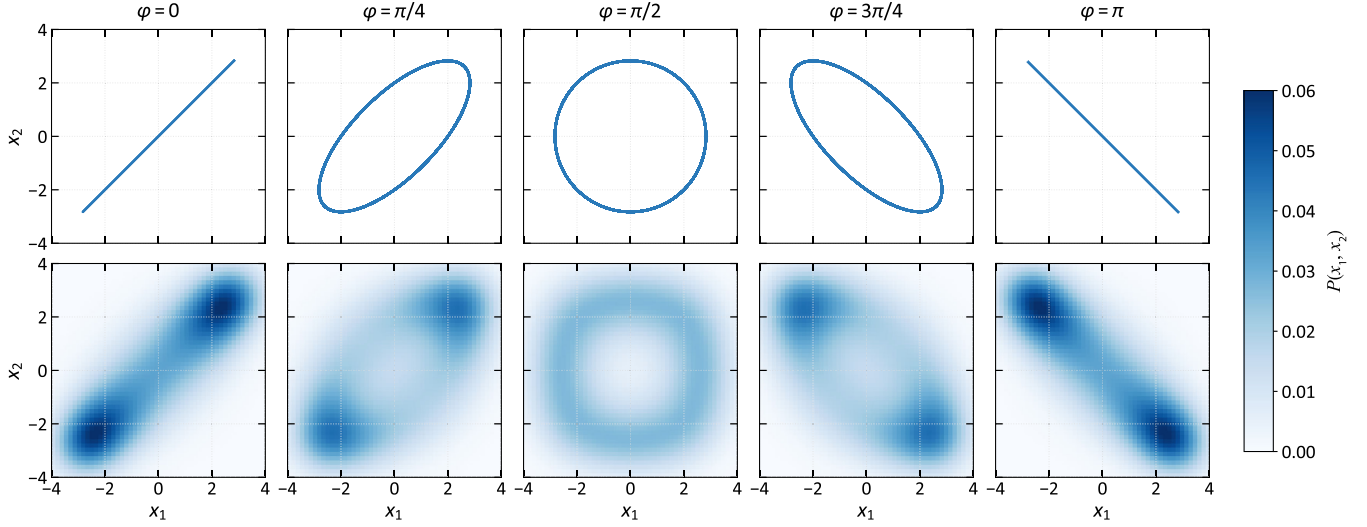


FIG. 15. Classical trajectory in x_1 - x_2 space (top row) and the joint probability distribution $P(x_1, x_2)$ between two oscillators (bottom row).

unstable. On the contrary, for the second fixed point, the eigenvalues become $\lambda_1 = -\kappa_+$ and $\lambda_2 = -\kappa_+ - 2V$, both of which are negative. Hence, this fixed point is stable, and nearby trajectories are attracted to it and eventually settle there in the long-time limit. In the original nonrotating frame, this corresponds to a synchronized solution of the form $\alpha_1(t) = R e^{-i(\omega t - \varphi)} = e^{i\varphi} \alpha_2(t)$, where we have set $\theta = 0$ for simplicity. Thus, at the mean-field level, the oscillators are synchronized with the same amplitude and a fixed phase difference determined by φ . As a result, changing the phase φ yields simple Lissajous figures in the x_1 - x_2 plane, where $x_i = (\alpha_i + \alpha_i^*)/\sqrt{2}$. These correspond to the simplest geometric figures: Since amplitude and frequency are equal and only the phase varies, we obtain a diagonal line for $\varphi = 0$, an ellipse for intermediate φ , and a full circle for $\varphi = \pi/2$. We show the classical trajectory and the corresponding quantum solution $P(x_1, x_2)$ for different φ in Fig. 15. Additionally, we compare these mean-field predictions with simulation results of the effective quantum model. Because of quantum noise, the numerical results exhibit a distribution rather than a single trajectory. However, the structure predicted by the classical mean-field model remains clearly visible.

4. Weak entanglement in the deep quantum regime

Beyond the experimental focused quantum and near-classical regime, theory predicts that in the deep quantum limit, an “entanglement tongue” can appear, similar to an Arnold tongue. In this limit, the oscillators no longer behave as genuine harmonic oscillators and are instead restricted to effectively two levels spanned by the Fock states $|0\rangle$ and $|1\rangle$. However, the predicted entanglement between the two oscillators is weak. In Ref. [11], a concurrence of at most 0.15 is reported, which is much smaller than the maximal value of 1 corresponding to a maximally entangled Bell state.

In our numerical simulation, we use logarithmic negativity $E_{\mathcal{N}}$ and normalized mutual information [Eq. (E1)] as the metrics of entanglement and synchronization between the two oscillators. The logarithmic negativity for the two oscillator state ρ_{M_1, M_2} is defined as [71]

$$E_{\mathcal{N}}(\rho_{M_1, M_2}) = \log_2 \|\rho_{M_2}^T\|_1, \quad (\text{E5})$$

where $\|X\|_1 = \text{Tr} \sqrt{X^\dagger X}$ and ρ_{M_2} is the reduced density matrix after tracing out the first oscillator M_1 . Figure 16 shows a comparison of logarithmic negativity and normalized mutual information as a function of mean phonon number. We can see that the logarithmic negativity remains small across the entire range and increases as the average occupation $\bar{n}$ of each oscillator is below 0.2

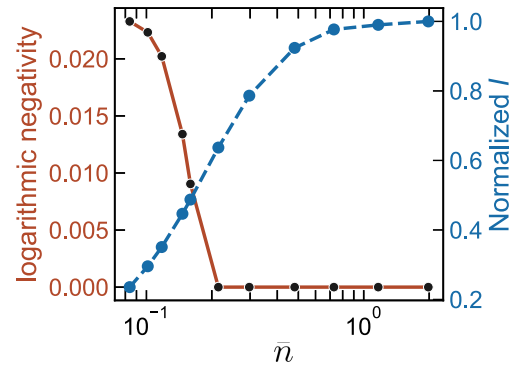


FIG. 16. Logarithmic negativity $E_{\mathcal{N}}$ (red curve) and normalized mutual information I (blue dashed curve) are shown on the left and the right scale of the figure for synchronized two oscillators as a function of their occupation number. We use experimental relevant parameters for the simulation, $\kappa_- = 2\pi \times 0.12$ kHz, $V = 2\pi \times 0.1$ kHz, and we use $\kappa_+ = 2\pi \times \{0, 0.001, 0.002, 0.004, 0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.4\}$ kHz for different mean phonon numbers.

(for comparison, in our experiment in the quantum regime, the average occupation of each oscillator is around 1.4).

On the other hand, synchronization itself is not restricted to the regime where entanglement between the two oscillators arises. Signatures of mutual synchronization persist over a broader range of oscillator occupations, including regimes with higher excitation numbers where entanglement is expected to be negligible. When the mutual information between two oscillators decreases as the system approaches ground state, the entanglement tends to increase. This suggests that entanglement is not a prerequisite for synchronization in our system; rather, it is a distinct form of quantum correlation that can arise only in the deep-quantum regime, where the synchronization metric of mutual information is reduced.

-
- [1] A. Pikovsky, M. Rosenblum, and J. Kurths, *Synchronization: A Universal Concept in Nonlinear Sciences* (Cambridge University Press, Cambridge, England, 2001).
 - [2] T. Herpich, J. Thingna, and M. Esposito, *Collective power: Minimal model for thermodynamics of nonequilibrium phase transitions*, *Phys. Rev. X* **8**, 031056 (2018).
 - [3] C. W. Wächter, P. Strasberg, S. H. L. Klapp, G. Schaller, and C. Jarzynski, *Stochastic thermodynamics of self-oscillations: the electron shuttle*, *New J. Phys.* **21**, 073009 (2019).
 - [4] A. W. Laskar, P. Adhikary, S. Mondal, P. Katiyar, S. Vinjanampathy, and S. Ghosh, *Observation of Quantum phase synchronization in spin-1 atoms*, *Phys. Rev. Lett.* **125**, 013601 (2020).
 - [5] M. Koppenhöfer, C. Bruder, and A. Roulet, *Quantum synchronization on the IBM Q system*, *Phys. Rev. Res.* **2**, 023026 (2020).
 - [6] L. Zhang, Z. Wang, Y. Wang, J. Zhang, Z. Wu, J. Jie, and Y. Lu, *Quantum synchronization of a single trapped-ion qubit*, *Phys. Rev. Res.* **5**, 033209 (2023).
 - [7] Y. Li, Z. Xie, X. Yang, Y. Li, X. Zhao, X. Cheng, X. Peng, J. Li, E. Lutz, Y. Lin *et al.*, *Experimental realization and synchronization of a quantum van der Pol oscillator*, *Sci. Adv.* **11**, eady5649 (2025).
 - [8] V. K. Vanag, L. Yang, M. Dolnik, A. M. Zhabotinsky, and I. R. Epstein, *Oscillatory cluster patterns in a homogeneous chemical system with global feedback*, *Nature (London)* **406**, 389 (2000).
 - [9] R. FitzHugh, *Impulses and physiological states in theoretical models of nerve membrane*, *Biophys. J.* **1**, 445 (1961).
 - [10] M. E. Jewett and R. E. Kronauer, *Refinement of limit cycle oscillator model of the effects of light on the Human Circadian Pacemaker*, *J. Theor. Biol.* **192**, 455 (1998).
 - [11] T. E. Lee and H. R. Sadeghpour, *Quantum synchronization of quantum van der Pol oscillators with trapped ions*, *Phys. Rev. Lett.* **111**, 234101 (2013).
 - [12] S. Walter, A. Nunnenkamp, and C. Bruder, *Quantum synchronization of two van der Pol oscillators*, *Ann. Phys. (Berlin)* **527**, 131 (2015).
 - [13] M. Xu, D. A. Tieri, E. C. Fine, J. K. Thompson, and M. J. Holland, *Synchronization of two ensembles of atoms*, *Phys. Rev. Lett.* **113**, 154101 (2014).
 - [14] A. Roulet and C. Bruder, *Quantum synchronization and entanglement generation*, *Phys. Rev. Lett.* **121**, 063601 (2018).
 - [15] A. Roulet and C. Bruder, *Synchronizing the smallest possible system*, *Phys. Rev. Lett.* **121**, 053601 (2018).
 - [16] F. Schmolke and E. Lutz, *Noise-induced quantum synchronization*, *Phys. Rev. Lett.* **129**, 250601 (2022).
 - [17] T. Murtadho, S. Vinjanampathy, and J. Thingna, *Cooperation and competition in synchronous open quantum systems*, *Phys. Rev. Lett.* **131**, 030401 (2023).
 - [18] V. R. Krithika, P. Solanki, S. Vinjanampathy, and T. S. Mahesh, *Observation of quantum phase synchronization in a nuclear-spin system*, *Phys. Rev. A* **105**, 062206 (2022).
 - [19] S. Sonar, M. Hajdušek, M. Mukherjee, R. Fazio, V. Vedral, S. Vinjanampathy, and L.-C. Kwek, *Squeezing enhances quantum synchronization*, *Phys. Rev. Lett.* **120**, 163601 (2018).
 - [20] J. Broz, B. You, S. Khan, H. Häffner, D. E. Kaplan, and S. Rajendran, *Test of causal nonlinear quantum mechanics by Ramsey interferometry with a trapped ion*, *Phys. Rev. Lett.* **130**, 200201 (2023).
 - [21] R. J. MacDonell, T. Navickas, T. F. Wohlers-Reichel, C. H. Valahu, A. D. Rao, M. J. Millican, M. A. Currington, M. J. Biercuk, T. R. Tan, C. Hempel, and I. Kassal, *Predicting molecular vibronic spectra using time-domain analog quantum simulation*, *Chem. Sci.* **14**, 9439 (2023).
 - [22] Q. Wu, Y. Shi, and J. Zhang, *Continuous Raman sideband cooling beyond the Lamb-Dicke regime in a trapped ion chain*, *Phys. Rev. Res.* **5**, 023022 (2023).
 - [23] D. Gottesman, A. Kitaev, and J. Preskill, *Encoding a qubit in an oscillator*, *Phys. Rev. A* **64**, 012310 (2001).
 - [24] S. Braunstein and P. van Loock, *Quantum information with continuous variables*, *Rev. Mod. Phys.* **77**, 513 (2005).
 - [25] J. Whitlow, Z. Jia, Y. Wang, C. Fang, J. Kim, and K. R. Brown, *Quantum simulation of conical intersections using trapped ions*, *Nat. Chem.* **15**, 1509 (2023).
 - [26] D. J. Gorman, B. Hemmerling, E. Megidish, S. A. Moeller, P. Schindler, M. Sarovar, and H. Häffner, *Engineering vibrationally assisted energy transfer in a trapped-ion quantum simulator*, *Phys. Rev. X* **8**, 011038 (2018).
 - [27] C. H. Valahu, V. C. Olaya-Agudelo, R. J. MacDonell, T. Navickas, A. D. Rao, M. J. Millican, J. B. Pérez-Sánchez, J. Yuen-Zhou, M. J. Biercuk, C. Hempel, T. R. Tan, and I. Kassal, *Direct observation of geometric-phase interference in dynamics around a conical intersection*, *Nat. Chem.* **15**, 1503 (2023).
 - [28] S. C. Burd, R. Srinivas, J. J. Bollinger, A. C. Wilson, D. J. Wineland, D. Leibfried, D. H. Slichter, and D. T. C. Allcock, *Quantum amplification of mechanical oscillator motion*, *Science* **364**, 1163 (2019).
 - [29] K. C. McCormick, J. Keller, S. C. Burd, D. J. Wineland, A. C. Wilson, and D. Leibfried, *Quantum-enhanced sensing of a single-ion mechanical oscillator*, *Nature (London)* **572**, 86 (2019).
 - [30] C. Flühmann, T. L. Nguyen, M. Marinelli, V. Negnevitsky, K. Mehta, and J. P. Home, *Encoding a qubit in a trapped-ion mechanical oscillator*, *Nature (London)* **566**, 513 (2019).

- [31] V. G. Matsos, C. Valahu, M. J. Millican, T. Navickas, X. C. Kolesnikow, M. J. Biercuk, and T. R. Tan, *Universal quantum gate set for Gottesman–Kitaev–Preskill logical qubits*, *Nat. Phys.* **21**, 1664 (2025).
- [32] D. Leibfried, D. M. Meekhof, B. E. King, C. Monroe, W. M. Itano, and D. J. Wineland, *Experimental determination of the motional quantum state of a trapped atom*, *Phys. Rev. Lett.* **77**, 4281 (1996).
- [33] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, *Quantum dynamics of single trapped ions*, *Rev. Mod. Phys.* **75**, 281 (2003).
- [34] V. Cimini, M. Barbieri, N. Treps, M. Walschaers, and V. Parigi, *Neural networks for detecting multimode Wigner negativity*, *Phys. Rev. Lett.* **125**, 160504 (2020).
- [35] C. Flühmann and J. P. Home, *Direct Characteristic-function tomography of quantum states of the trapped-ion motional oscillator*, *Phys. Rev. Lett.* **125**, 043602 (2020).
- [36] C. H. Valahu, T. Navickas, M. J. Biercuk, and T. R. Tan, *Benchmarking bosonic modes for quantum information with randomized displacements*, *PRX Quantum* **5**, 040337 (2024).
- [37] T. Behrle, T. L. Nguyen, F. Reiter, D. Baur, B. de Neeve, M. Stadler, M. Marinelli, F. Lancellotti, S. F. Yelin, and J. P. Home, *Phonon laser in the quantum regime*, *Phys. Rev. Lett.* **131**, 043605 (2023).
- [38] V. So, M. Duraisamy Suganthi, A. Menon, M. Zhu, R. Zhuravel, H. Pu, P. G. Wolynes, J. N. Onuchic, and G. Pagano, *Trapped-ion quantum simulation of electron transfer models with tunable dissipation*, *Sci. Adv.* **10**, eads8011 (2024).
- [39] J. F. Poyatos, J. I. Cirac, and P. Zoller, *Quantum reservoir engineering with laser cooled trapped ions*, *Phys. Rev. Lett.* **77**, 4728 (1996).
- [40] J. Roßnagel, S. T. Dawkins, K. N. Tolazzi, O. Abah, E. Lutz, F. Schmidt-Kaler, and K. Singer, *A single-atom heat engine*, *Science* **352**, 325 (2016).
- [41] P. M. Harrington, E. J. Mueller, and K. W. Murch, *Engineered dissipation for quantum information science*, *Nat. Rev. Phys.* **4**, 660 (2022).
- [42] T. E. Lee, C.-K. Chan, and S. Wang, *Entanglement tongue and quantum synchronization of disordered oscillators*, *Phys. Rev. E* **89**, 022913 (2014).
- [43] M. H. Matheny, J. Emenheiser, W. Fon, A. Chapman, A. Salova, M. Rohden, J. Li, M. Hudoba de Badyn, M. Pósfai, L. Duenas-Osorio *et al.*, *Exotic states in a simple network of nanoelectromechanical oscillators*, *Science* **363**, eaav7932 (2019).
- [44] C. Davis-Tilley, C. Teoh, and A. Armour, *Dynamics of many-body quantum synchronisation*, *New J. Phys.* **20**, 113002 (2018).
- [45] G. M. Vaidya, S. B. Jäger, and A. Shankar, *Quantum synchronization and dissipative quantum sensing*, *Phys. Rev. A* **111**, 012410 (2025).
- [46] F. W. Knollmann, A. N. Patel, and S. C. Doret, *Part-per-billion measurement of the $^{42}\text{S}_{1/2} \rightarrow ^{32}\text{D}_{5/2}$ electric-quadrupole-transition isotope shifts between $^{42,44,48}\text{Ca}^+$ and $^{40}\text{Ca}^+$* , *Phys. Rev. A* **100**, 022514 (2019).
- [47] W.-K. Mok, L.-C. Kwek, and H. Heimonen, *Synchronization boost with single-photon dissipation in the deep quantum regime*, *Phys. Rev. Res.* **2**, 033422 (2020).
- [48] S. Walter, A. Nunnenkamp, and C. Bruder, *Quantum synchronization of a driven self-sustained oscillator*, *Phys. Rev. Lett.* **112**, 094102 (2014).
- [49] S. Dutta and N. R. Cooper, *Critical response of a quantum van der Pol oscillator*, *Phys. Rev. Lett.* **123**, 250401 (2019).
- [50] L. Xu, S. Wang, Z. Jiang, and X. Wei, *Programmable synchronization enhanced MEMS resonant accelerometer*, *Microsyst. Nanoeng.* **6**, 63 (2020).
- [51] H. Wang, K. Jacobs, D. Fahey, Y. Hu, D. R. Englund, and M. E. Trusheim, *Exceptional sensitivity near the bistable transition point of a hybrid quantum system*, *Nat. Phys.* **22**, 577 (2026).
- [52] M. H. Matheny, M. Grau, L. G. Villanueva, R. B. Karabalin, M. C. Cross, and M. L. Roukes, *Phase synchronization of two anharmonic nanomechanical oscillators*, *Phys. Rev. Lett.* **112**, 014101 (2014).
- [53] V. M. Bastidas, I. Omelchenko, A. Zakharova, E. Schöll, and T. Brandes, *Quantum signatures of chimera states*, *Phys. Rev. E* **92**, 062924 (2015).
- [54] C. W. Wächtler and G. Platero, *Topological synchronization of quantum van der Pol oscillators*, *Phys. Rev. Res.* **5**, 023021 (2023).
- [55] G. Clos, D. Porras, U. Warring, and T. Schaetz, *Time-resolved observation of thermalization in an isolated quantum system*, *Phys. Rev. Lett.* **117**, 170401 (2016).
- [56] N. Jaseem, M. Hajdušek, V. Vedral, R. Fazio, L.-C. Kwek, and S. Vinjanampathy, *Quantum synchronization in nano-scale heat engines*, *Phys. Rev. E* **101**, 020201(R) (2020).
- [57] J. Zhang, G. Pagano, P. W. Hess, A. Kyprianidis, P. Becker, H. Kaplan, A. V. Gorshkov, Z.-X. Gong, and C. Monroe, *Observation of a many-body dynamical phase transition with a 53-qubit quantum simulator*, *Nature (London)* **551**, 601 (2017).
- [58] M. Malinowski, C. Zhang, V. Negnevitsky, I. Rojko, F. Reiter, T.-L. Nguyen, M. Stadler, D. Kienzler, K. K. Mehta, and J. P. Home, *Generation of a maximally entangled state using collective optical pumping*, *Phys. Rev. Lett.* **128**, 080503 (2022).
- [59] F. Verstraete, M. M. Wolf, and J. Ignacio Cirac, *Quantum computation and quantum-state engineering driven by dissipation*, *Nat. Phys.* **5**, 633 (2009).
- [60] H. P. Lüschen, P. Bordia, S. S. Hodgman, M. Schreiber, S. Sarkar, A. J. Daley, M. H. Fischer, E. Altman, I. Bloch, and U. Schneider, *Signatures of many-body localization in a controlled open quantum system*, *Phys. Rev. X* **7**, 011034 (2017).
- [61] C. Haack, N. A. Kamar, D. Paz, M. Maghrebi, and Z. Gong, *Probing non-equilibrium dissipative phase transitions with trapped-ion quantum simulators*, *arXiv:2311.06199*.
- [62] B. P. Marsh, R. M. Kroeze, S. Ganguli, S. Gopalakrishnan, J. Keeling, and B. L. Lev, *Entanglement and replica symmetry breaking in a driven-dissipative quantum spin glass*, *Phys. Rev. X* **14**, 011026 (2024).
- [63] J. Liu, Q. Wu, J. E. Moore, H. Haeffner, and C. W. Wächtler, *Data for: “Observation of synchronization between two quantum van der Pol oscillators in trapped ions”*, [10.5281/zenodo.19928791](https://zenodo.org/record/19928791) (2026).
- [64] B. You, Q. Wu, D. Miron, W. Ke, I. Monga, E. Saglamyurek, and H. Haeffner, *Temporally multiplexed*

- ion-photon quantum interface via fast ion-chain transport*, [arXiv:2405.10501](#).
- [65] C. Roos, *Controlling the quantum state of trapped ions*, Ph.D. thesis, Universität Innsbruck, 2000.
- [66] P. C. Haljan, K.-A. Brickman, L. Deslauriers, P. J. Lee, and C. Monroe, *Spin-dependent forces on trapped ions for phase-stable quantum gates and entangled states of spin and motion*, *Phys. Rev. Lett.* **94**, 153602 (2005).
- [67] J. Johansson, P. Nation, and F. Nori, *QUTIP: An open-source PYTHON framework for the dynamics of open quantum systems*, *Comput. Phys. Commun.* **183**, 1760 (2012).
- [68] H. Häffner, S. Gulde, M. Riebe, G. Lancaster, C. Becher, J. Eschner, F. Schmidt-Kaler, and R. Blatt, *Precision measurement and compensation of optical Stark shifts for an ion-trap quantum processor*, *Phys. Rev. Lett.* **90**, 143602 (2003).
- [69] D. M. Meekhof, C. Monroe, B. E. King, W. M. Itano, and D. J. Wineland, *Generation of nonclassical motional states of a trapped atom*, *Phys. Rev. Lett.* **76**, 1796 (1996).
- [70] V. Ameri, M. Eghbali-Arani, A. Mari, A. Farace, F. Kheirandish, V. Giovannetti, and R. Fazio, *Mutual information as an order parameter for quantum synchronization*, *Phys. Rev. A* **91**, 012301 (2015).
- [71] G. Vidal and R. F. Werner, *Computable measure of entanglement*, *Phys. Rev. A* **65**, 032314 (2002).