

# UCLA

## UCLA Previously Published Works

### Title

Beam on Nonlinear Winkler Foundation and Modified Neutral Plane Solution for Calculating Downdrag Settlement

### Permalink

<https://escholarship.org/uc/item/1sk9q12b>

### Journal

Journal of Geotechnical and Geoenvironmental Engineering, 139(9)

### ISSN

1090-0241

### Authors

Wang, Rui  
Brandenberg, Scott J

### Publication Date

2013-09-01

### DOI

10.1061/(asce)gt.1943-5606.0000888

Peer reviewed

This version is the authors' final copy. The typeset version is under copyright and can be downloaded at the link below.

[http://dx.doi.org/10.1061/\(ASCE\)GT.1943-5606.0000888](http://dx.doi.org/10.1061/(ASCE)GT.1943-5606.0000888)

1 **Beam on nonlinear Winkler foundation and modified neutral plane solution for calculating**  
2 **downdrag settlement**

3 Rui Wang<sup>1</sup> and Scott J. Brandenburg<sup>2</sup>, M.ASCE.

4 **Abstract**

5 Since the work of Fellenius (1972), the neutral plane solution has been widely used to estimate  
6 downdrag settlements and drag loads mobilized in piles in consolidating soil profiles. Pile  
7 settlement is typically assumed equal to soil settlement at the neutral plane depth computed  
8 based on effective stress conditions at the end of consolidation. This paper demonstrates that, in  
9 general, pile settlement is not equal to soil settlement at the neutral plane depth; rather it is the  
10 relative velocity between the pile and soil that is zero at the neutral plane depth. A beam on  
11 nonlinear Winkler foundation (BNWF) solution, in which the shaft friction capacity is updated as  
12 consolidation progresses, is utilized to demonstrate that pile settlement is not equal to soil  
13 settlement at the neutral plane depth because the neutral plane depth evolves as consolidation  
14 progresses. The BNWF solution also shows that pile settlement depends on drainage conditions,  
15 with more settlement occurring when consolidation occurs first near the top of the consolidating  
16 soil layer, and less settlement occurring when consolidation initiates at the bottom. A modified  
17 neutral plane solution that is amenable to hand calculation is formulated to account for the  
18 evolution of neutral plane depth on pile settlement. Finally, the proposed BNWF and modified  
19 neutral plane solutions are compared with measurements of downdrag settlement from a  
20 centrifuge test program. The proposed methods produced more accurate estimates of pile  
21 settlement than the traditional neutral plane solution.

---

<sup>1</sup> PhD Candidate, State Key Laboratory of Hydrosience and Engineering, Department of Hydraulic Engineering, Tsinghua University, Beijing, China

<sup>2</sup> Associate Professor and Vice Chair, Department of Civil and Environmental Engineering, University of California, Los Angeles 90095-1593, email: sjbrandenberg@ucla.edu (corresponding author)

## 22 **Introduction**

23 Pile foundations embedded in soil profiles that settle due to surcharge loading, ground water  
24 level drop, liquefaction, etc, are subject to increased axial loads (i.e., drag load) and/or pile head  
25 settlements (i.e., downdrag). Consolidation-induced downdrag and drag load have been the topic  
26 of numerous field studies utilizing instrumented piles (e.g., Bjerrum et al. 1969; Endo et al. 1969;  
27 Fellenius 1972, 1984; Poulos and Davis 1980). Based on field observations Fellenius (1972)  
28 developed the neutral plane solution, NPS, where the neutral plane is the depth of maximum  
29 axial load marking the transition between downward shaft friction and upward shaft friction. The  
30 neutral plane depth is typically computed by summing axial loads from the top down and from  
31 the bottom up, and by force equilibrium the neutral plane lies at the intersection of these two  
32 lines as shown in Fig. 1. Typically the shaft friction capacity,  $f_s$ , is assumed to be mobilized along  
33 the full length of the pile because small relative displacements between soil and pile are required  
34 to mobilize  $f_s$ . Based on the observation that shaft friction is mobilized in an upward direction  
35 when the pile settles more than the soil, and in a downward direction when the soil settles more  
36 than the pile, Fellenius (1972) postulated that the soil settlement and the pile settlement are  
37 identical at the neutral plane, and this approach has been widely used to calculate downdrag  
38 settlement.

39 Although the neutral plane concept has contributed significantly to our understanding of piles  
40 in settling ground, several assumptions made in its typical application may deviate from actual  
41 loading conditions. First, as soil expels pore water during consolidation the effective stress  
42 increases, thereby resulting in time- and depth-varying  $f_s$  and time-varying neutral plane depth.  
43 Second, shaft friction exhibits elasto-plasticity such that relative displacements between a pile  
44 and soil may be small enough to mobilize only a portion of the ultimate shaft friction capacity,

45 whereas full mobilization (i.e., rigid-plastic response) is typically assumed. Third, tip resistance  
46 is often assumed to be constant whereas in reality it depends on pile tip settlement.

47 To address these assumptions, a number of studies have approached the downdrag problem  
48 using continuum numerical solutions (e.g., Esmail 1996; Lee and Ng 2004; Jeong et al. 2004;  
49 Hanna and Sharif 2006; Sun and Yan 2010). However, the interface between the soil and pile  
50 requires careful selection of contact elements, and the complexity of the three dimensional  
51 continuum solutions renders them poorly suited to routine use. Due to the computational  
52 complexity of modelling a soil continuum, other researchers have adopted a beam on nonlinear  
53 Winker foundation (BNWF) approach to the neutral plane problem in which t-z elements model  
54 soil-pile interaction and a beam-column models the pile (e.g., Wong and Teh 1995; Kim and  
55 Mission 2009). However, the properties of the interaction elements are typically time-invariant  
56 and therefore the evolution of shaft friction capacity and neutral plane depth with time is not  
57 modelled. Wong and Teh (1995) acknowledge this problem, and suggest using effective stress  
58 conditions at the time when downdrag is to be computed (often the end of primary consolidation)  
59 to define properties of the t-z materials. However, Boulanger and Brandenburg (2004)  
60 demonstrated that accounting for the evolution of shaft friction capacity and the associated  
61 changes in neutral plane depth can result in significant differences in estimated downdrag  
62 settlement.

### 63 **Beam on Nonlinear Winkler Foundation Solution of Downdrag Problem**

64 A schematic of a BNWF approach that removes many of the assumptions in the traditional  
65 NPS is shown in Fig. 2. The solution utilizes the TzLiq1 material model implemented in  
66 OpenSees (McKenna and Fenves 2001) along the length of the pile to model shaft friction, and  
67 beam column elements for the structural properties of the pile. End bearing in the BNWF

68 analysis can be modeled in two different ways: (i) a Q-z element (e.g., QzSimple1 in OpenSees)  
69 can be used at the pile tip to capture variation in end bearing load with pile tip settlement, or (ii)  
70 an upward force may be applied at the pile tip to represent a constant end bearing resistance. A  
71 load may also be applied to the pile head. The TzLiq1 and QzSimple1 materials adopt a  
72 nonlinear plasticity formulation such that the backbone load transfer behavior closely matches  
73 published relations [Reese and O'Neill (1988) or Mosher (1984) for t-z behavior; Reese and  
74 O'Neill (1988) or Vijayvergiya (1977) for Q-z behavior. A complete description of the material  
75 model equations is beyond the scope of this paper, but can be found in Boulanger et al. (2003),  
76 and in the OpenSees documentation. The TzLiq1 material was implemented in OpenSees with  
77 the specific intention of modeling piles in liquefiable soils (hence its name), but it is equally well  
78 suited for modeling downdrag problems resulting from more traditional consolidation  
79 mechanisms.

80 The key feature that makes the TzLiq1 materials amenable to consolidation analysis is the  
81 relation between  $f_s$  (also called  $t_{ult}$  or  $t_u$  in the literature) and vertical effective stress in the soil,  
82  $\sigma_v'$ . The TzLiq1 material assumes that  $f_s$  varies linearly with  $\sigma_v'$ , and is zero when  $\sigma_v'$  is zero.  
83 This is an important improvement upon previous analysis approaches that utilized constant  $f_s$ ,  
84 regardless of consolidation condition. The analysis proceeds by computing values of  $f_s$  at each  
85 node along the pile based on the initial effective stress condition and soil-pile interface friction  
86 angle. Subsequently, time- and depth-dependent values of  $\sigma_v'$  and soil settlement,  $S_z$ , are input to  
87 the free-ends of the t-z elements, and the  $f_s$  values are updated to be compatible with  $\sigma_v'$  at each  
88 increment.

89 A simple example problem consisting of a 20m long reinforced concrete pile embedded in a  
90 layer of clay (Fig. 3) is selected to demonstrate the BNWF downdrag solution, and for

91 comparison with the traditional NPS. The uniform clay layer has a saturated unit weight of  
92  $20\text{kN/m}^3$ , initial void ratio of 0.8, and coefficient of compressibility ( $m_v$ ) of  $2.22 \times 10^{-4} \text{kPa}^{-1}$ . A  
93  $150\text{kPa}$  surcharge was applied at the surface of the clay layer, resulting in a uniform vertical  
94 strain of 3.3%, and an ultimate surface settlement of  $0.67\text{m}$ .

95 The square pile with  $0.4\text{m}$  side length ( $B$ ) was modeled using elastic beam column elements  
96 with Young's modulus of  $40\text{GPa}$  (consistent with typical reinforced concrete). The pile was  
97 discretized into 100 elements (101 nodes) evenly distributed along its length. The soil-pile  
98 interface friction angle  $\delta$  was set as  $28^\circ$  and the at-rest earth pressure coefficient  $K_0$  was set as 0.5.  
99 The ultimate soil-pile interface friction was calculated as  $f_s = \sigma_v' K_0 \tan \delta$ . The load transfer  
100 behavior followed Reese and O'Neill's (1988) relation for clay, and the value of  $z_{50}$  (i.e., the  
101 displacement at which half of the ultimate shaft friction is mobilized) was set to  $0.0002\text{m}$ . The  
102 resulting load transfer curve is fairly stiff, and is consistent with empirical observations that  
103 ultimate shaft friction is mobilized at small relative displacements on the order of millimeters. At  
104 the tip of the pile, a constant upward load of  $144\text{ kN}$  was imposed to simulate full development  
105 of the undrained tip resistance during downdrag. A constant upward load was selected instead of  
106 a Q-z element at the pile tip to facilitate a direct comparison with the traditional neutral plane  
107 solution. The geotechnical capacity of the pile can be calculated through the sum of fully  
108 mobilized upward shaft friction and tip resistance, which comes up to  $995\text{kN}$ . The solution was  
109 computed for various values of pile head load within the geotechnical capacity prior to  
110 consolidation, ranging from  $144\text{ kN}$  to  $900\text{ kN}$ . The example pile has a rather low end bearing  
111 resistance. For design, piles are often founded in more competent strata to provide higher end  
112 bearing resistance. In such cases, the neutral plane may be near the pile tip, which would reduce  
113 or eliminate downdrag settlement.

114 The example problem was solved using three different types of drainage conditions: drainage  
 115 through both the top and bottom of the clay layer (double drainage, DD), single drainage through  
 116 the top (SDtop), and single drainage through the bottom (SDbottom). The consolidation solution  
 117 followed the Fourier series expansion of Terzaghi's one dimensional consolidation theory:

$$118 \quad u(z,t) = \frac{4p}{\pi} \sum_{n=0}^{\infty} \frac{1}{2n+1} \sin \frac{(2n+1)\pi z}{2H} \exp \left( -(2n+1)^2 \left( \frac{\pi^2}{4} \right) T_v \right) \quad (1)$$

119 where  $u(z,t)$  is the excess pore pressure at depth  $z$  and time  $t$ ,  $H$  is drainage path length, and  $T_v$  is  
 120 the time factor defined as  $T_v = \frac{C_v}{H^2} t$ . Isochrones of the consolidation ratio,  $U_z$ , computed from  
 121 Eq. 1 are shown in Fig. 4, and are also available in many soil mechanics text books (e.g., Holtz,  
 122 Kovacs, and Sheahan 2011). Time- and depth-dependent values of vertical effective stress,  $\sigma_v'(z,t)$   
 123 for the free end of the t-z elements were computed as  $\sigma_v'(z,t) = \sigma_{vf}'(z) - u(z,t)$ , where  $\sigma_{vf}'(z)$  is the  
 124 final vertical effective stress after consolidation at depth  $z$ . Utilizing Terzaghi's 1-D consolidation  
 125 theory inherently neglects excess pore pressures caused by pile installation, and changes to soil  
 126 permeability and compressibility during consolidation.

127 The soil settlement  $S_{soil}(z,t)$  in the clay layer was acquired by integrating the vertical strain in  
 128 the soil profile as the clay consolidates. Isochrones of the dimensionless settlement ratio  
 129 computed by integrating  $U_z$  with depth are also shown in Fig. 4 based on the assumptions that  
 130 double drainage boundary conditions apply, and settlement is zero at the bottom of the  
 131 consolidating layer. Soil settlement profiles at a desired time can be computed by multiplying the  
 132 appropriate settlement ratio by the ultimate surface settlement. Settlement ratio isochrones for  
 133 single drainage conditions are not presented herein for brevity, but can easily be obtained using  
 134 the methods described earlier.



135 The computed time- and depth-dependent values of  $\sigma_v'(z,t)$  and  $S_{soil}(z,t)$  were imposed on the  
136 free ends of the TzLiq1 elements, and solutions of pile settlement were computed using  
137 OpenSees. The UpdateMaterialStage command was utilized prior to the first load increment to  
138 initialize the TzLiq1 materials so that the initial capacities were tied to the initial effective stress  
139 values. Subsequently, the capacities were updated as the effective stresses increased during  
140 consolidation. Penalty constraints were used to enforce the imposed displacement boundary  
141 conditions, and convergence was based on the norm of the displacement residuals (i.e.,  
142 NormDispIncr in OpenSees) with the tolerance set to  $10^{-8}$ . A Newton-Raphson algorithm was  
143 used to iterate on an equilibrium displacement field for each loading increment. Solutions were  
144 computed using 800 increments to reach an average degree of consolidation beginning at 0% and  
145 ending at 99.9%, and an automatic substepping algorithm was utilized to reduce the step size  
146 when convergence did not occur in 25 Newton-Raphson iterations.

#### 147 **BNWF Computation Results**

148 Figs. 5 to 7 show the soil settlement, effective stress, soil-pile friction, and axial pile load  
149 distributions at four different average degrees of consolidation (25%, 50%, 75% and 99.9%) for a  
150 pile head load of  $Q_d=445\text{kN}$ . The depth of the neutral plane is clearly evident at the abrupt  
151 transition from negative to positive friction, and also at the depth of the maximum axial load.  
152 The profiles in Figs. 5-7 are identical at the end of primary consolidation, but differences in the  
153 profiles arise at intermediate degrees of consolidation.

154 In the double drainage case, effective stress initially builds up at both the top and bottom of  
155 the clay layer, causing soil strain and increase in soil-pile friction to be more prominent at the top  
156 and bottom. The increase in friction at the top serves to partially offset the increase in friction at  
157 the bottom, and the depth of the neutral plane remains nearly constant at slightly deeper than

158 10m as consolidation evolves. On the other hand, for the case with single drainage through the  
159 top the friction increases more quickly at the top of the pile, which shifts the neutral plane  
160 upward. As consolidation progresses, friction increases with depth along the pile and the neutral  
161 plane shifts downward to its final equilibrium depth at the end of consolidation. Conversely,  
162 when single drainage occurs through the bottom the friction increases first at the bottom of the  
163 pile, which shifts the neutral plane downward, and it progresses upward to its final equilibrium  
164 position at the end of consolidation.

165 The depth to the neutral plane, and pile settlement at the neutral plane depth are plotted  
166 versus average degree of consolidation in Fig. 8. The pile was essentially rigid (elastic  
167 compression was only a fraction of a millimeter at the end of consolidation), so Fig. 8 can be  
168 interpreted as pile head settlement. For the double-drained case, the pile settlement increases  
169 approximately linearly with average degree of consolidation, reaching a final value of 0.306m.  
170 For the SDtop case, the initial incremental soil strains occur first near the surface such that soil  
171 settlement is nearly zero below the neutral plane depth, which causes a very slow initial pile  
172 settlement rate. However, with time, the neutral plane shifts upward as the downdrag stresses  
173 increase near the pile head, soil strains shift downward as consolidation progresses, and the pile  
174 settlement rate increases quickly. The pile settlement at the end of consolidation is 0.350m,  
175 which is 14% larger than the double-drained case. For the SDbottom case, the pile initially settles  
176 quickly because incremental soil strains are largest deep in the profile, below the neutral plane  
177 depth. However with time the incremental soil strains move upward, resulting in a reduction in  
178 pile settlement rate. The final pile settlement reaches 0.262m, which is 14% less than the double-  
179 drained case. The traditional NPS claims that the pile settlement is equal to the soil settlement at  
180 the depth of the neutral plane at the end of consolidation, which is 0.310m for the example

181 problem. This value is close to the double-drained case, but differs from the SD<sub>top</sub> and  
182 SD<sub>bottom</sub> cases by  $\pm 14\%$ , which is a non-negligible amount.

183 Having investigated the effect of drainage conditions on the settlement of piles in  
184 consolidating soil, we now turn our attention to the influence of pile head loading. Using the  
185 same procedures mentioned above, the settlement of single piles subjected to varying head loads  
186 within their geotechnical capacity were calculated through both the BNWF method and  
187 traditional NPS under the three drainage conditions (Fig. 9). For all four solutions, the pile  
188 settlement increased as the pile head load increased because the head load shifted the neutral  
189 plane upward in the soil profile. The traditional NPS solution does not match any of the BNWF  
190 cases, though it corresponds more closely with the double drainage case than with the single-  
191 drainage cases.

### 192 **Fundamental Error in Traditional Neutral Plane Solution**

193 The BNWF solution of the example problem illustrates a fundamental error in the manner in  
194 which the NPS is typically utilized to estimate downdrag settlement. The fundamental error is  
195 that the neutral plane solution assumes that the pile settlement and soil settlement are equal at the  
196 depth of the neutral plane. However, it is the relative velocity, not the relative displacement that  
197 must be zero at the neutral plane depth. Consider the elastic perfectly-plastic material response  
198 shown in Fig. 10. The neutral plane is defined as the position along the pile where shaft friction  
199 transitions from upward to downward, and is therefore zero. The load transfer curve in Fig. 10  
200 illustrates two different points on where shaft friction is equal to zero, but they are associated  
201 with different amounts of displacement. This clearly establishes that relative displacement  
202 between pile and soil is not necessarily equal to zero at the depth where shaft friction is zero.

203 The kinematic condition describing relative movement between soil and pile at the neutral  
 204 plane depth can be easily defined by traditional one-dimensional rate independent plasticity  
 205 theory. The yield function is defined as  $f = |Friction| - f_s$ , and the Kuhn-Tucker complementary  
 206 conditions require that  $\dot{z}_p \text{sign}(Friction) \cdot f = 0$ , where  $\dot{z}_p$  is the plastic displacement rate (e.g.,  
 207 Simo and Hughes 1998). In the elastic region where  $f < 0$ , the Kuhn-Tucker conditions dictate that  
 208  $\dot{z}_p = 0$ , whereas in the plastic region where  $f = 0$ , the Kuhn-Tucker conditions dictate that  $\dot{z}_p \neq 0$ .  
 209 Extending these plasticity concepts to the neutral plane solution, the neutral plane is defined as  
 210 the depth where shaft friction is zero, which corresponds to the elastic region where  $f < 0$ .  
 211 Therefore  $\dot{z}_p = 0$  at the neutral plane based on the Kuhn-Tucker complementary conditions. One-  
 212 dimensional rate independent plasticity theory dictates that it is the relative plastic displacement  
 213 **rate** between the soil and pile,  $\dot{z}_p$ , and not the relative displacement,  $z_p$ , that must be zero at the  
 214 neutral plane depth. Note that the condition when  $\dot{z}_p = 0$  and  $f = 0$  also satisfies the Kuhn-Tucker  
 215 complementary conditions. Therefore,  $\dot{z}_p = 0$  does not necessarily indicate a condition of zero  
 216 friction (e.g., consider the end of consolidation condition where soil and pile are not settling, but  
 217 shaft friction is nevertheless mobilized along the pile). However, when friction is equal to zero,  
 218  $\dot{z}_p$  must be zero as well.

219 Considering that the relative velocity must be zero at the neutral plane depth, pile settlement  
 220 can be computed as the integral of soil settlement velocity,  $V_{soil}$ , at the neutral plane depth over  
 221 time:

$$222 \quad S_{pile}(z_{np}(t)) = \int_0^t V_{soil}(z_{np}(t), t) dt \quad (2)$$

223 where  $z_{np}(t)$  is the depth of the neutral plane at time  $t$ , and  $V_{soil}(z, t) = \frac{\partial S_{soil}(z, t)}{\partial t}$ ,  $S_{soil}(z, t)$  is the  
 224 soil settlement at depth  $z$  and time  $t$ . For the special case where  $z_{np}(t)$  is constant, the soil

225 settlement would be equal to the pile settlement at the neutral plane depth. However, if  $z_{np}(t)$  is  
226 not constant, the pile settlement will, in general, be different than the soil settlement at the  
227 neutral plane depth, and will depend on the evolution of the neutral plane depth over time. For  
228 typical consolidation problems, the neutral plane depth will change with time because the  
229 effective stresses at the soil-pile interface will change as consolidation evolves. The traditional  
230 NPS utilizes the end-of-consolidation neutral plane depth and does not account for the evolution  
231 of neutral plane depth over time, and computes an erroneous settlement as a result. On the other  
232 hand, the BNWF solution inherently includes shifting of the neutral plane depth due to  
233 discretization of time, the link between t-z properties and consolidation stress, and enforcement  
234 of force equilibrium in each increment.

235

### 236 **Modified Neutral Plane Solution**

237 Although the BNWF method correctly captures the evolution of neutral plane depth over  
238 time, and its influence on pile settlement, performing such a BNWF analysis is currently beyond  
239 the capabilities of software commonly used in geotechnical design. Therefore we now turn our  
240 attention to formulating a simple modification to the neutral plane solution that is amenable to  
241 spreadsheet calculation. The steps of the modified neutral plane solution are summarized in the  
242 flow chart in Fig. 11. The first step involves discretizing time into convenient intervals for  
243 solving the consolidation problem. Times should be selected to correspond to reasonably  
244 consistent average degrees of consolidation (e.g., times corresponding to  $U_{ave} = 0\%$ ,  $25\%$ ,  $50\%$ ,  
245  $75\%$ , and  $100\%$  might be selected if five time steps are desired). Second, profiles of excess pore  
246 pressure and vertical strain are computed at each time using consolidation theory, and the  
247 settlement profile  $S_{soil}(z_{np}(t_i), t_i)$  is computed by integrating the vertical strain profile from the

248 bottom up (e.g., see Fig. 4). Third, the depth of the neutral plane is solved at each time interval in  
 249 the traditional manner originally suggested by Fellenius (1972) in which forces are summed from  
 250 the top down and bottom up, and the neutral plane depth lies at the intersection of the two lines.  
 251 However, the shaft friction values must be based on the current effective stress at a particular  
 252 depth based on the consolidation solution from step 2. The variation in shaft friction during  
 253 consolidation is precisely why the neutral plane shifts with time, and is why the traditional NPS  
 254 incorrectly predicts pile settlement. Fourth, the pile settlement at the neutral plane depth is  
 255 computed by integrating soil settlement velocity at the neutral plane depth over time. Numerical  
 256 discretization of time transforms the integral of velocity into a difference in incremental  
 257 displacements. Hence, the pile settlement for a particular time step, n, can be computed using the  
 258 forward Euler integration method in Eq. 3:

$$259 \quad S_{pile}(z_{np}) = \sum_{i=1}^n \left[ S_{soil}(z_{np}(t_{i+1}), t_{i+1}) - S_{soil}(z_{np}(t_{i+1}), t_i) \right] \quad (3)$$

260 For cases where elastic deformation of the pile is anticipated to be significant, axial strains  
 261 must be integrated over the pile length to compute the contribution of pile shortening to head  
 262 settlement. Furthermore, if a load-transfer curve (i.e., a Q-z relationship) is utilized rather than a  
 263 constant specified tip resistance, iteration is required to obtain a tip resistance that is compatible  
 264 with the current pile tip settlement.

265 The example problem presented in Figs. 5 through 7 was also analyzed using the modified  
 266 NPS using various numbers of time steps (3, 5, and 33). The time steps were chosen to be at  
 267 constant intervals of average degree of consolidation. Fig. 12 compares the BNWF method and  
 268 the modified neutral plane method. The modified NPS accuracy increases as the number of time  
 269 steps increases. The small differences between the modified NPS with 33 time steps and the

270 BNWF solution are likely attributed to differences in time discretization (800 time steps  
271 compared with 33) and elasto-plasticity of the t-z materials in the BNWF solution compared with  
272 the assumption of rigid plasticity in the modified NPS. Using a modest number of 5 time steps  
273 provides reasonable solutions for all three cases, and is reasonably approachable in a spreadsheet  
274 calculation.

### 275 **Comparison with experimental data**

276 To further validate the BNWF approach and modified NPS, a model pile from a centrifuge  
277 test by Lam et al. (2009) is analyzed. The centrifuge test was conducted at the Geotechnical  
278 Centrifuge Facilities at the Hong Kong University of Science and Technology to investigate axial  
279 load effects on piles in consolidating ground. The test program involved multiple pile  
280 foundations, but only one single pile test (test no. 1 in their paper) is analyzed here. The  
281 centrifugal acceleration was 60g and results are presented in prototype dimensions.

282 An instrumented tubular aluminium pile with an outer diameter ( $D$ ) of 1.2m and wall  
283 thickness ( $t_{wall}$ ) of 9cm was installed in an 18m thick layer of clay (Speswhite China clay)  
284 consolidated to a vertical effective stress of 80kPa before pile installation and spin up (Fig. 13).  
285 The clay rested atop a dense Leighton Buzzard sand layer that provided free drainage to the clay,  
286 another layer of sand atop the clay layer, resulting in a double-drained condition. The pile tip was  
287 1.2m above the bottom of the clay layer. The top sand layer provided a surcharge of 45kPa,  
288 resulting in a measured 10 kN drag load on the pile from the sand. No load was applied to the  
289 pile head. The saturated unit weight, at-rest earth pressure coefficient, and initial void ratio of the  
290 clay were specified to be 16.3kN/m<sup>3</sup>, 0.58, and 1.602 respectively. The coefficient of  
291 consolidation  $c_v$  was back calculated to be  $5 \times 10^{-7}$  m<sup>2</sup>/s from the distributions of excess pore  
292 pressure measured in the test using Terzaghi's one dimensional consolidation theory. Isochrones

293 of predicted and measured excess pore pressure plotted in Fig. 13 show good agreement. The  
294 coefficient of compressibility  $m_v = 3.63 \times 10^{-7} \text{Pa}^{-1}$  was back calculated based on the measured soil  
295 surface settlement of 654mm.

296 In the BNWF simulations, the backbone of Reese and O'Neill's (1988) load transfer curve  
297 was used for the t-z elements. The soil-pile interface friction angle was estimated from the  
298 distribution of dragload after consolidation to be  $24^\circ$ . The value of  $z_{50}$  (displacement at which 50%  
299 of ultimate resistance is mobilized) was set to be 0.0005m, such that 99% of the shaft friction  
300 was mobilized at around 4~5mm.

301 A Q-z element was attached to the tip of the pile to model end bearing resistance. End  
302 bearing is a bit complicated for this problem because (i) it is unclear whether undrained or  
303 drained end bearing resistance would apply for the slow loading conditions induced during  
304 downdrag, and (ii) end bearing resistance would be anticipated to increase over time as the clay  
305 near the tip of the pile consolidates. Regarding (i), the test data can be used to provide some  
306 guidance since drained tip resistance is typically significantly larger than undrained tip resistance.  
307 Lam et al (2009) stated that prior to spin up, the soil was preloaded to 80kPa using a hydraulic  
308 press, resulting in an estimated undrained shear strength  $s_u$  of 17.6kPa prior to swelling of the  
309 clay, giving a strength ratio  $\frac{s_u}{\sigma_v'} = 0.22$ . Invoking concepts of normalization of undrained shear  
310 strength with consolidation stress and overconsolidation ratio (e.g., Ladd 1991), the undrained  
311 shear strength at the tip of the pile was estimated as  $s_u = 0.22 \cdot \sigma_v' \cdot OCR^{0.8}$ . The undrained shear  
312 strength prior to spin-up was estimated to be 9kPa, and the final undrained shear strength at the  
313 end of reconsolidation was estimated to be 35kPa based on the effective stress profiles in Fig. 14.  
314 Computing tip resistance as  $Q_t = 10s_u A$  the initial and final tip resistance came to 100kN and



315 400kN, respectively. At the end of consolidation, when the pile had settled significantly and  
316 clearly mobilized the ultimate tip resistance, the axial load at the tip of the pile was quite close to  
317 400kN based on extrapolation from the deepest strain gauge measurement (Fig. 14). Although a  
318 bearing factor of 9 is commonly used for undrained tip resistance, many researchers suggest that  
319 it is too low and suggest a higher value ranging from about 9 to 12 (e.g., Salgado 2008), so the  
320 fact that a bearing factor of 10 agreed well with the data is not surprising. On the other hand, the  
321 drained bearing capacity would be significantly larger than the measurements [e.g., over 1300  
322 kN is estimated using Meyerhof's (1976) bearing factors for a friction angle of only 20°]. Hence,  
323 we conclude that undrained tip resistance was mobilized during downdrag.

324 A Q-z element was attached to the pile tip, and the capacity of the element was increased  
325 from 100 kN to 400 kN in proportion to degree of consolidation at the pile tip elevation during  
326 consolidation. The  $z_{50}$  value was set to 0.012m such that the ultimate load is mobilized at  
327 approximately 8% of the pile diameter, which is consistent with the range presented by Reese  
328 and O'Neill (1988).

329 Fig. 14 shows the soil and pile responses at different average degrees of consolidation from  
330 the BNWF solution along with the final axial load distribution measured during the test. The  
331 final axial load distribution matches the centrifuge test data reasonably well. The final pile head  
332 settlement was estimated to be 0.194m (Fig. 15), which corresponds well with the measured  
333 settlement of 0.206m (-6% error).

334 In addition to the BNWF solution, the settlement was computed using the modified NPS with  
335 time discretization at  $U_{ave} = 0\%$ , 25%, 50%, 75% and 100%. Iteration was used to match the  
336 properties of the same Q-z relation used in the BNWF solution. The final pile settlement using  
337 the modified NPS method was 0.208m, which is also very close to the measured settlement (+1%

338 error). On the other hand, the traditional NPS method predicts the pile settlement to be 0.277m  
339 (+34% error), which is significantly larger than the measured value and the values computed  
340 from the BNWF method and modified NPS method. The overprediction of the traditional NPS  
341 method is expected because the neutral plane begins near the tip of the pile and transitions  
342 upward as consolidation progresses. Using the final neutral plane position in the traditional NPS  
343 method therefore over-estimates pile settlement.

#### 344 **Conclusions**

345 Pile settlement is typically assumed equal to soil settlement at the depth of the neutral plane,  
346 but this is a false inference; rather, the pile velocity is equal to the soil velocity at the neutral  
347 plane depth. This fact is supported by fundamental equations from one-dimensional rate  
348 independent plasticity theory. Pile displacement must be computed as the integral of soil  
349 settlement velocity at the neutral plane depth over time. If the neutral plane depth changes during  
350 consolidation (it typically does because interface friction depends on consolidation condition),  
351 the traditional neutral plane solution produces an inaccurate estimate of pile settlement. If the  
352 neutral plane depth is constant during consolidation, the traditional neutral plane solution is  
353 accurate.

354 An innovative new beam on nonlinear Winkler foundation approach was presented in which  
355 the shaft friction capacity evolves as effective stresses increase during consolidation. The new  
356 BNWF method clearly demonstrated the fundamental mechanisms involved in time-varying load  
357 transfer between pile and consolidating soil, and showed that settlements from the traditional  
358 neutral plane solution are generally inaccurate. A modified neutral plane solution that is  
359 amenable to spreadsheet calculation was formulated to account for evolution of the neutral plane  
360 depth over time, and provided reasonable agreement with the BNWF solutions.

361 When end-of-consolidation effective stress conditions are used to compute  $z_{np}$  in the  
362 traditional NPS, settlement will be under-predicted if  $z_{np}$  moves higher than the final  $z_{np}$  during  
363 consolidation, and over-predicted if  $z_{np}$  moves lower than the final  $z_{np}$ . The explanation is that  
364 settlement decreases with depth, so contributions to settlement at depths shallower than the final  
365  $z_{np}$  tend to increase pile settlement compared with the traditional NPS. Although the evolution of  
366 the neutral plane depth affects downdrag settlement, it has no influence on the maximum  
367 dragload mobilized in the pile, which occurs at the end of primary consolidation.

### 368 **Acknowledgments**

369 The authors would like to thank the research group at Hong Kong University of Science and  
370 Technology for having performed high quality centrifuge tests and for publishing the data that  
371 was directly relevant to this paper. The authors would also like to thank National Natural Science  
372 Foundation of China (No. 51079074 and 51038007) and the Chinese Scholarship Council for  
373 providing the funding that enabled Rui Wang to visit UCLA to do the work presented herein.  
374 This material is based upon research performed in a renovated collaboratory by the National  
375 Science Foundation under Grant No. 0963183, which is an award funded under the American  
376 Recovery and Reinvestment Act of 2009 (ARRA).

377

### 378 **References**

379 Bjerrum L., Johannessen, I. J., Eide, O. (1969). "Reduction of negative skin friction on steel piles  
380 to rock". Proceedings 7th International Conference on Soil Mechanics and Foundation  
381 Engineering, Mexico City, August 25 - 29, Vol. 2, 27-34.

382 Boulanger, R.W., Brandenburg, S.J. (2004). "Neutral plane solution for liquefaction-induced  
383 downdrag on vertical piles". Proceedings ASCE Geo-Trans Conference, Los Angeles,  
384 California, 27-31.

385 Boulanger, R.W., Kutter, B.L., Brandenburg, S.J., Singh, P., and Chang, D. (2003). Pile  
386 Foundations in Liquefied and Laterally Spreading Ground during Earthquakes: Centrifuge  
387 Experiments and Analyses. Report No. UCD/CGM-03/01, Center for Geotechnical Modeling,  
388 Department of Civil Engineering, University of California, Davis, CA, 205 pp.

389 Endo M., Minou A., Kawasaki T., Shibata T. (1969). "Negative skin friction acting on steel piles  
390 in clay". Proc. 8th International Conference on Soil Mechanics and Foundation Engineering,  
391 Mexico City, August 25 - 29, Vol. 2, 85-92.

392 Esmail H. (1996). "Neutral plane of single piles in clay subjected to surcharge loading". M.S.  
393 thesis, Concordia University, Quebec, Canada.

394 Fellenius, B. H. (1972). "Downdrag on long piles in clay due to negative skin friction". Canadian  
395 Geotechnical Journal, 9(4), 323-337.

396 Fellenius, B. H., (1984). "Negative skin friction and settlement of piles". Proceedings of the  
397 Second International Seminar, Pile Foundations, Nanyang Technological Institute, Singapore,  
398 1-12.

399 Hanna A. M., Sharif A. (2006). "Drag force on single piles in clay subjected to surcharge  
400 loading". International Journal of Geomechanics, 6(2), 89-96.

401 Jeong S., Lee J., Lee C. J. (2004). "Slip effect at the pile-soil interface on dragload". Computer  
402 and Geotechnics 31, 115-126.

403 Holtz R. D., Kovacs W. D., and Sheahan T. C. (2011). An Introduction to Geotechnical  
404 Engineering, 2nd Edition, Pearson Prentice-Hall, Upper Saddle River, New Jersey.

405 Kim H., Mission J. L. C. (2011). "Development of negative skin friction on single piles:  
406 uncoupled analysis based on nonlinear consolidation theory with finite strain and the load-  
407 transfer method". Canadian Geotechnical Journal, 48(6), 905-914.

408 Ladd, C.C. (1991). "Stability Evaluation during Staged Construction." Journal of Geotechnical  
409 and Geoenvironmental Engineering. 117(4), 540-615.

410 Lam S. Y., Ng, C. W. W., Leung C. F., Chan, S. H. (2009). "Centrifuge and numerical modeling  
411 of axial load effects on piles in consolidating ground". Canadian Geotechnical Journal, 46: 10-  
412 24.

413 Lee C. J., Ng W. W. (2004). "Development of down-drag on piles and pile groups in  
414 consolidating soil". Journal of Geotechnical and Geoenvironmental Engineering, 130 (9),  
415 905-914.

416 McKenna F., and Fenves G. L. (2001). "OpenSees Manual." PEER Center,  
417 <http://OpenSees.berkeley.edu>.

418 Mosher R. L. (1984). "Load Transfer Criteria for Numerical Analysis of Axial Loaded Piles in  
419 Sand." US Army Engineering Waterways Experimental Station, Automatic Data Processing  
420 Center, Vicksburg, Mississippi, January.

421 Poulos H. G., and Davis E. H. (1980). Pile foundation analysis and design, Wiley, New York.

422 Reese, L. C. and O'Neill M. W. (1988). "Drilled Shafts: Construction Procedures and Design  
423 Methods." Report No. FHWA-HI-88-042, U.S. Department of Transportation, Federal  
424 Highway Administration, Office of Implementation, McLean, Virginia.

425 Salgado R. (2008). *The Engineering of Foundations*. 882p. McGraw Hill, New York.

426 Simo J. C. and Hughes T. J. R. (1998) Computational Inelasticity. Springer, Berlin Heidelberg  
427 New York.

428 Sun T. K., Yan W. M. (2010). "Development of neutral plane on a pile in a consolidating ground".  
429 Proceedings of the 2nd International Symposium on Computational Mechanics, Hong Kong,  
430 Vol. 1233, 1594-1599.

431 Terzaghi K. (1925). *Erdbaumechanik auf Bodenphysikalischer Grundlage*, Franz Deuticke,  
432 Leipzig und Wein, 399p.; *Structure and Volume of Voids of Soils*, 10-13 (translated by A.  
433 Casagrande) in Terzaghi (1960).

434 Terzaghi K. (1960). *From Theory to Practice in Soil Mechanics*, Wiley, New York, 425.

435 Vijayvergiya V. N. (1977). "Load-Movement Characteristics of Piles." Proceedings, Ports 77  
436 Conference, American Society of Civil Engineers, Long Beach, California, March.

437 Wong K. S. and Teh C. I. (1995). "Negative skin friction on piles in layered soil deposits".  
438 *Journal of Geotechnical Engineering*, 121 (6), 457-465.

Figure  
[Click here to download high resolution image](#)

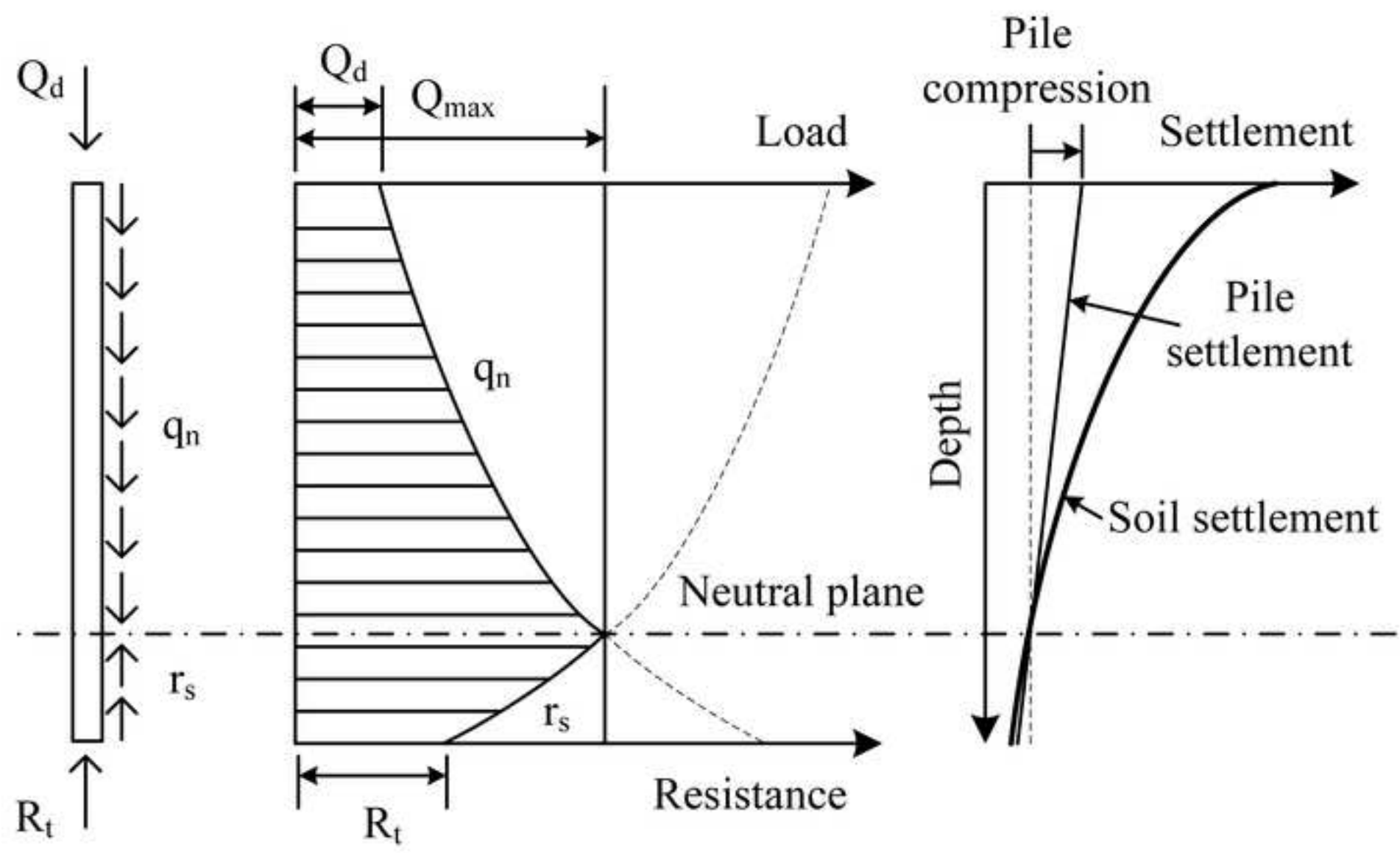


Figure 2  
[Click here to download high resolution image](#)

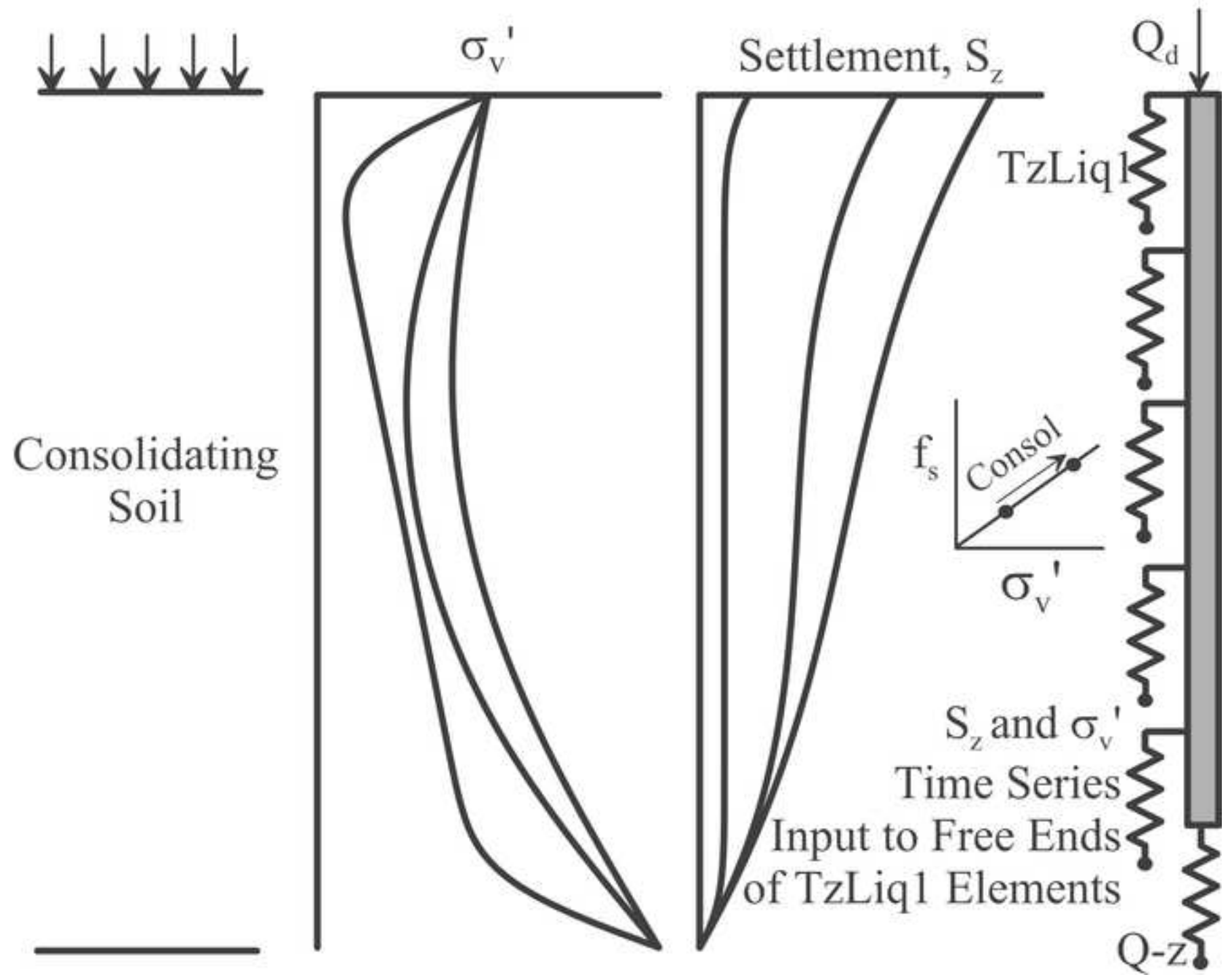




Figure 3  
[Click here to download high resolution image](#)

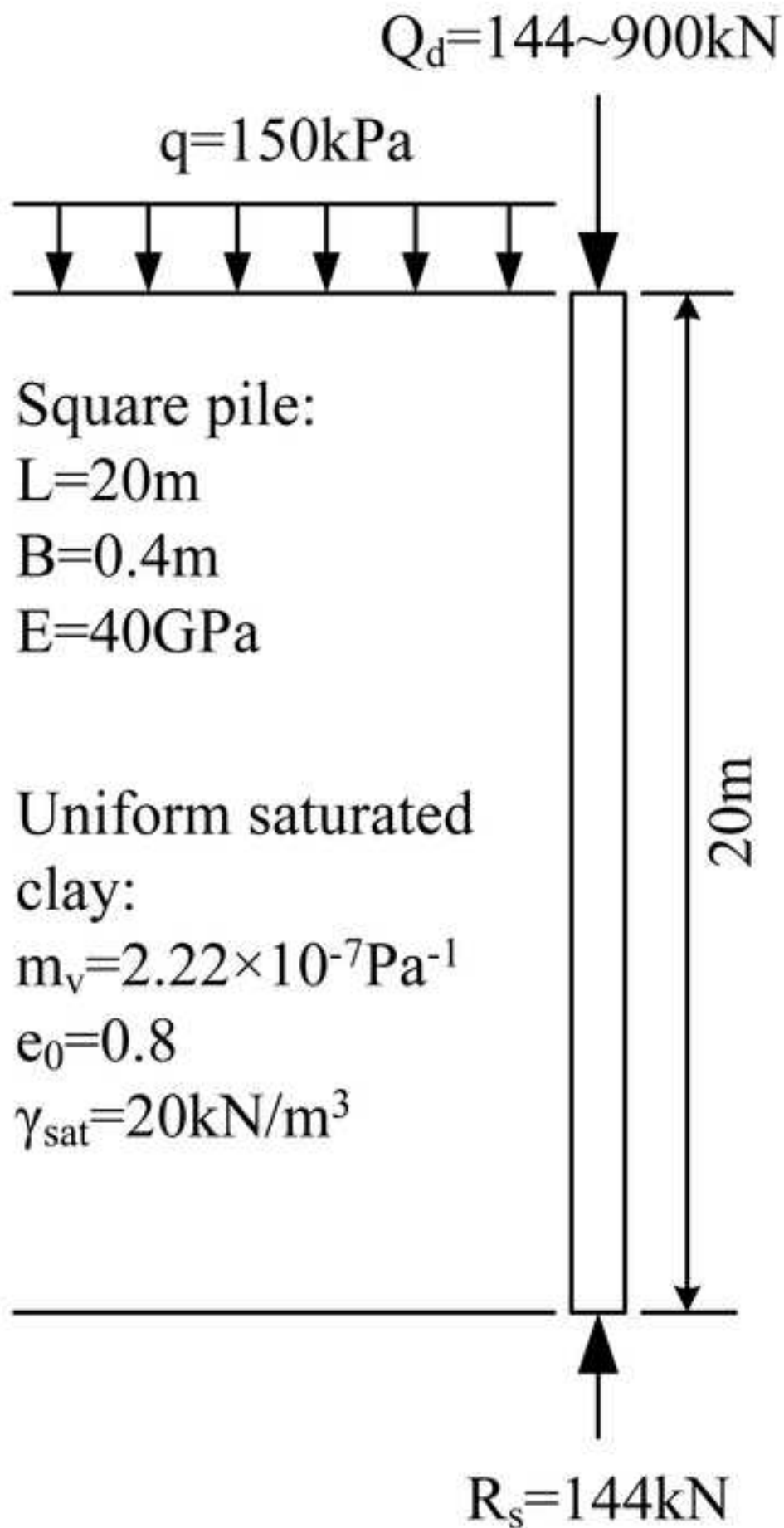


Figure 4  
[Click here to download high resolution image](#)

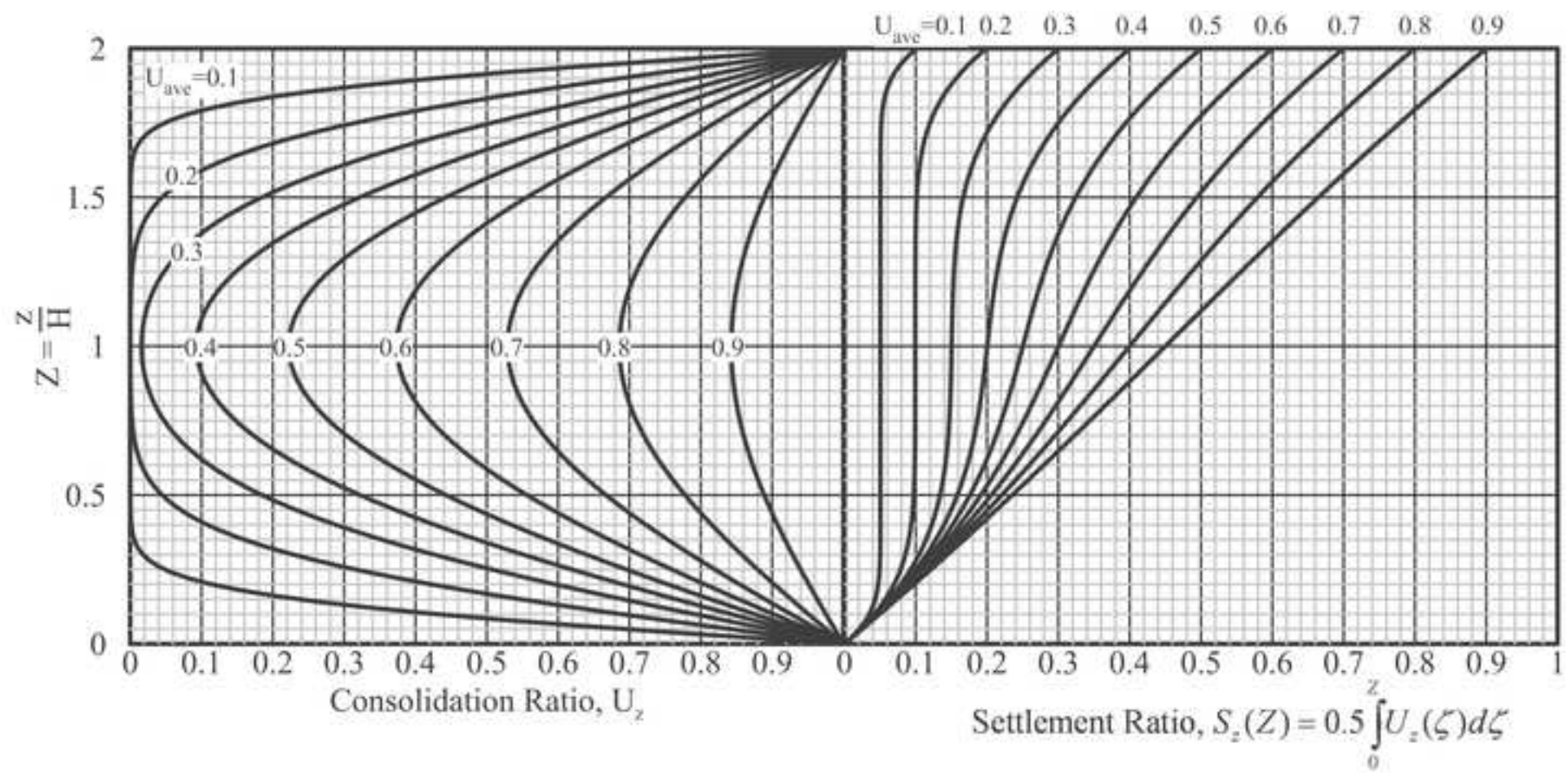


Figure 5  
[Click here to download high resolution image](#)

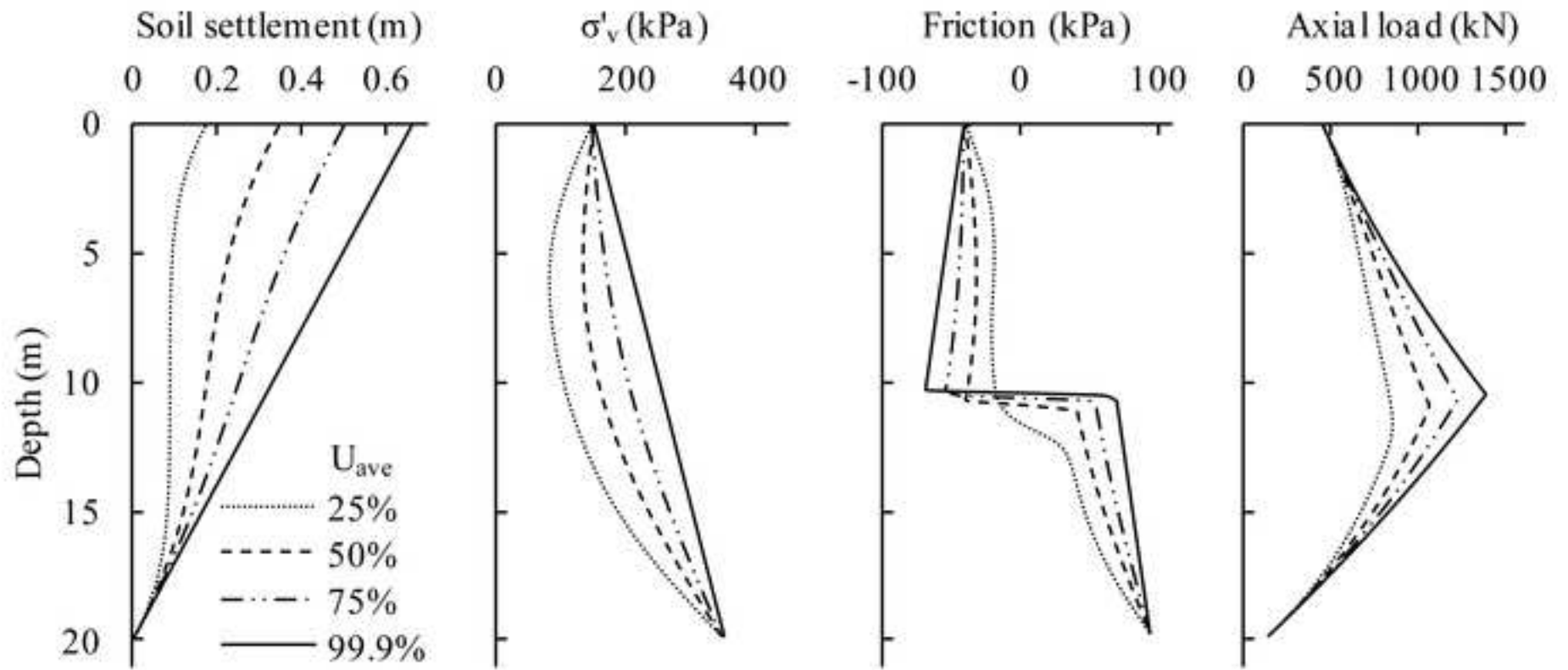


Figure 6  
[Click here to download high resolution image](#)

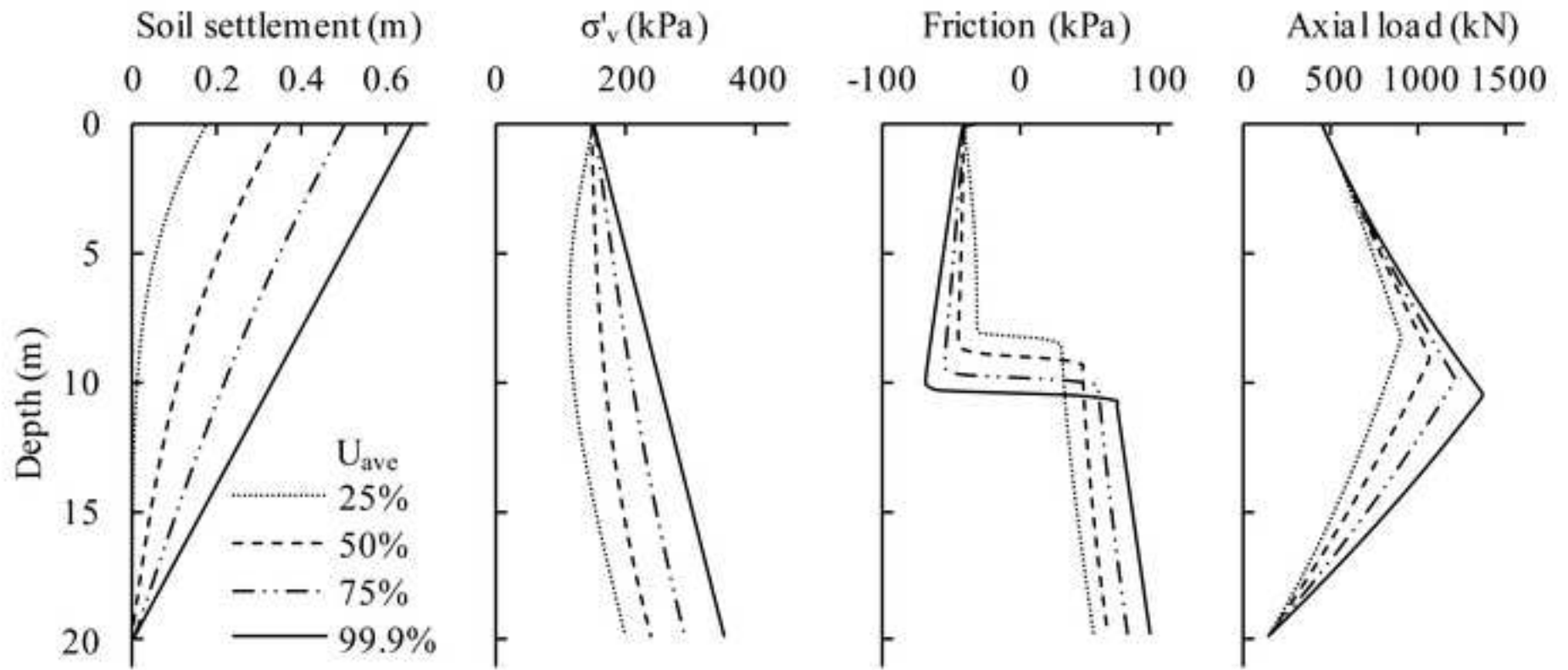


Figure 7  
[Click here to download high resolution image](#)

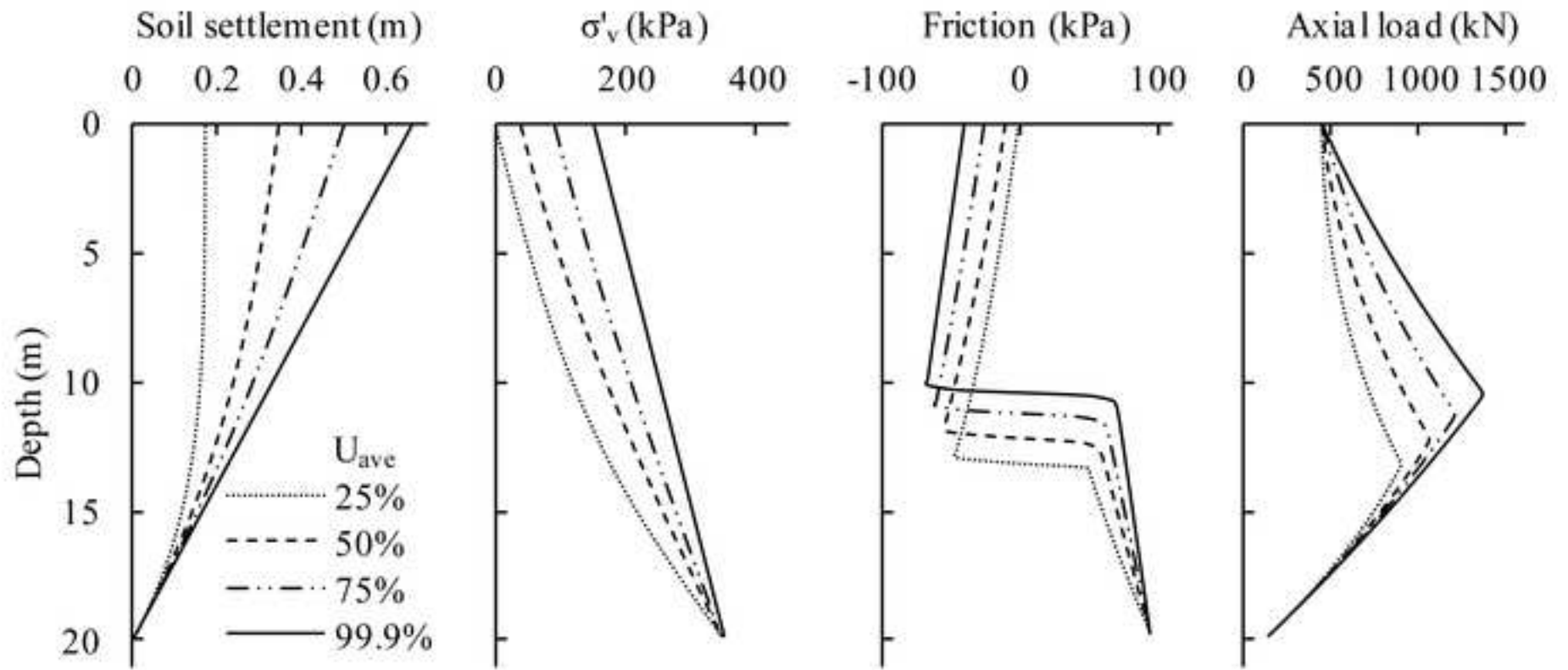


Figure 8

[Click here to download high resolution image](#)

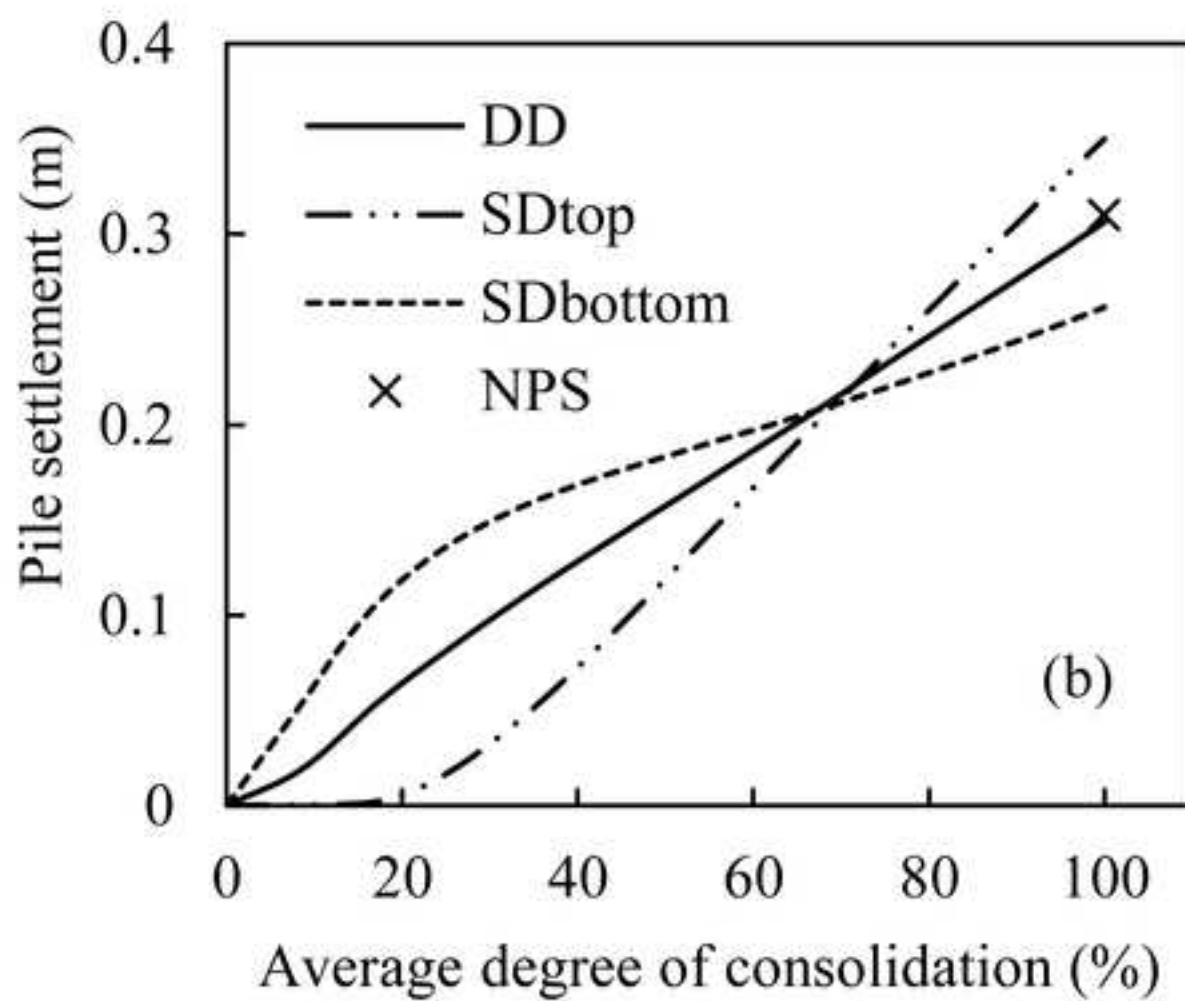
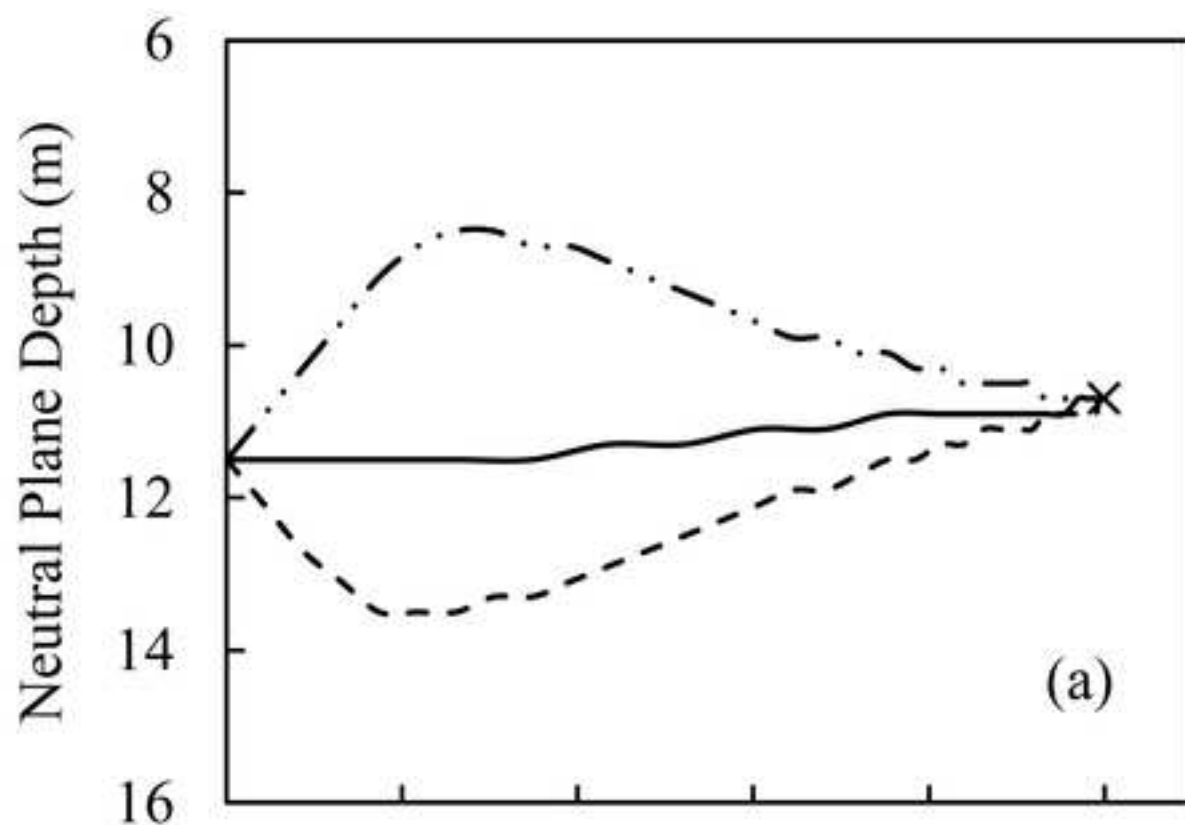




Figure 9  
[Click here to download high resolution image](#)

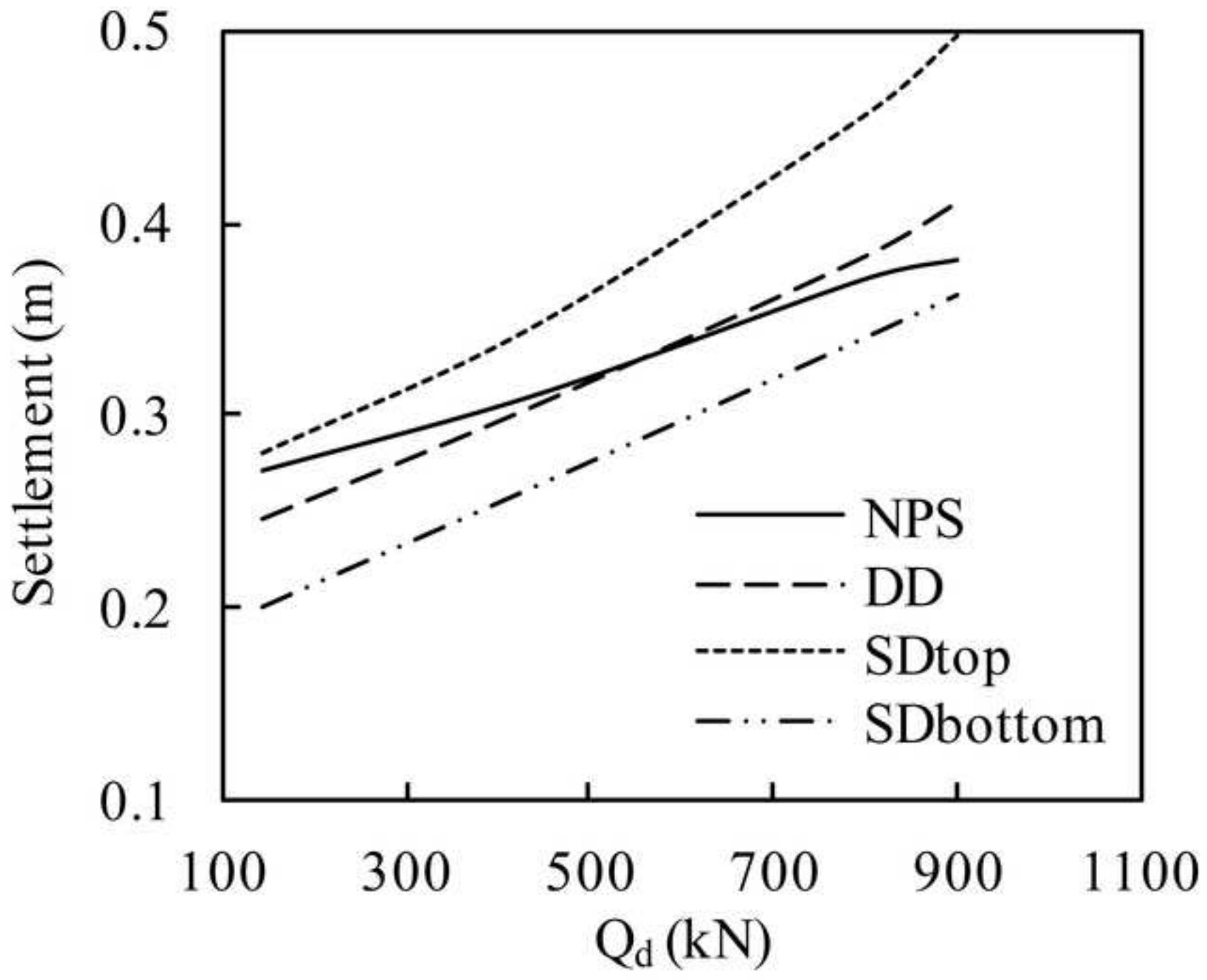


Figure 10  
[Click here to download high resolution image](#)

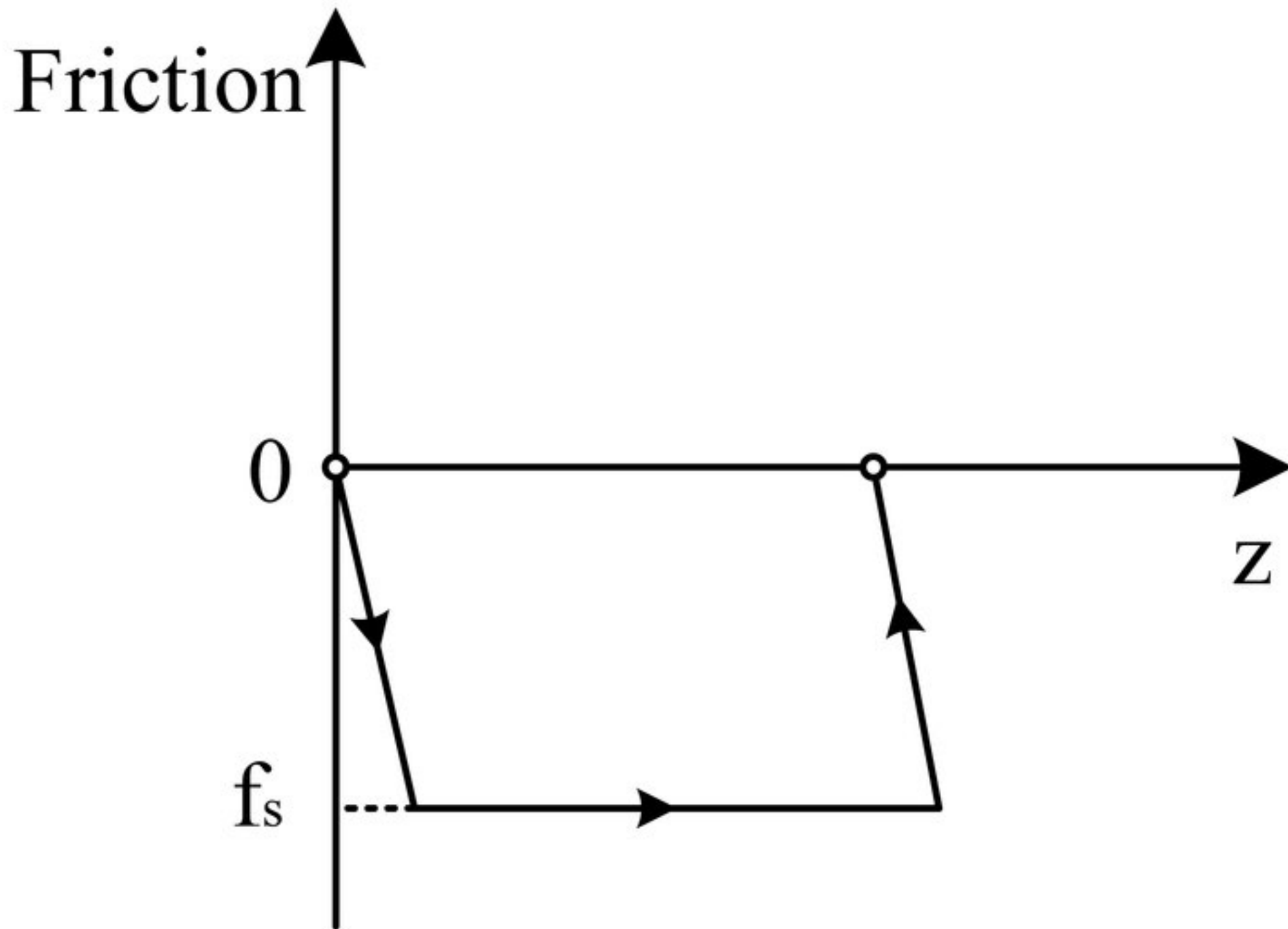




Figure 11  
[Click here to download high resolution image](#)

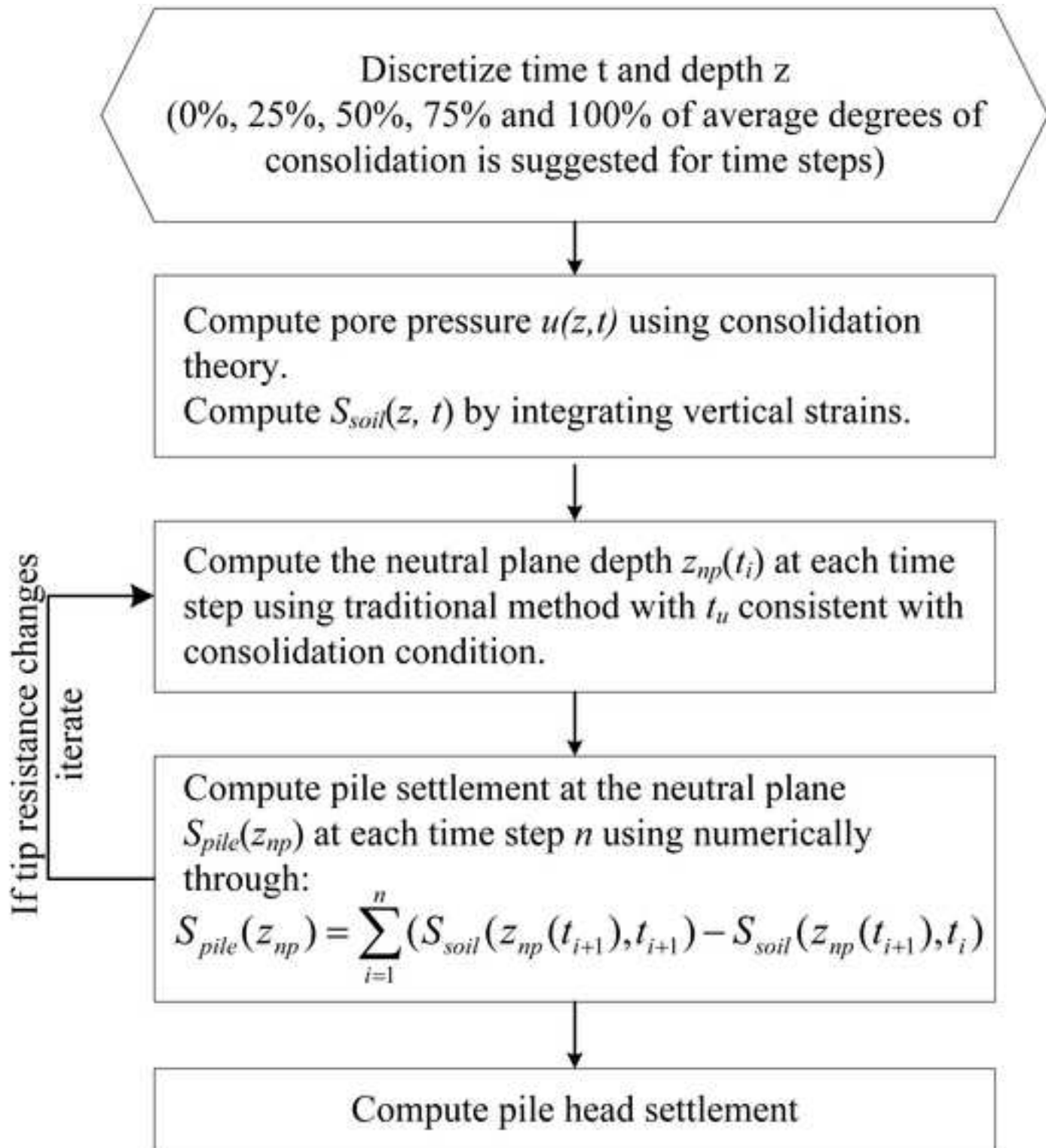


Figure 12

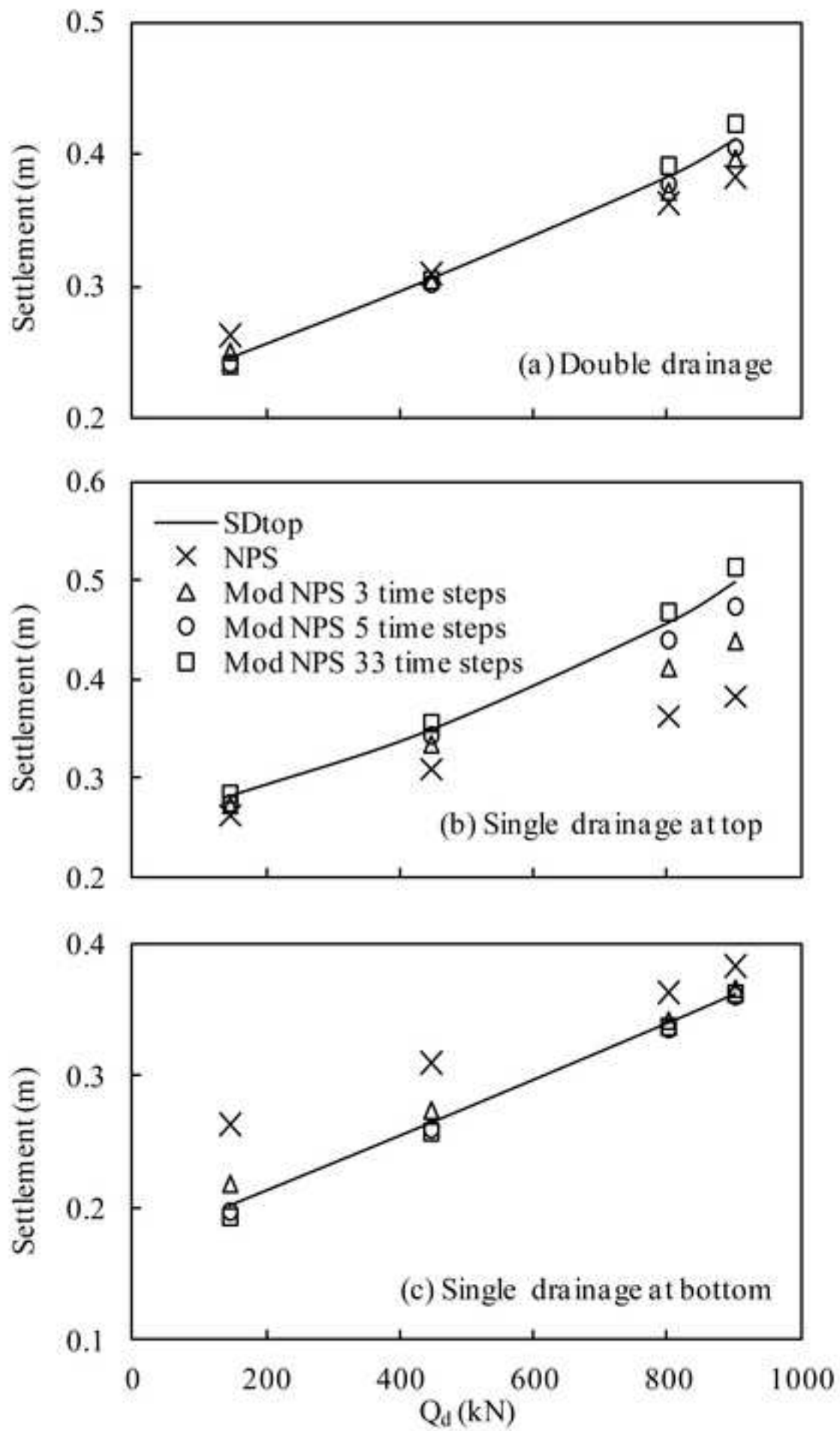
[Click here to download high resolution image](#)

Figure 13  
[Click here to download high resolution image](#)

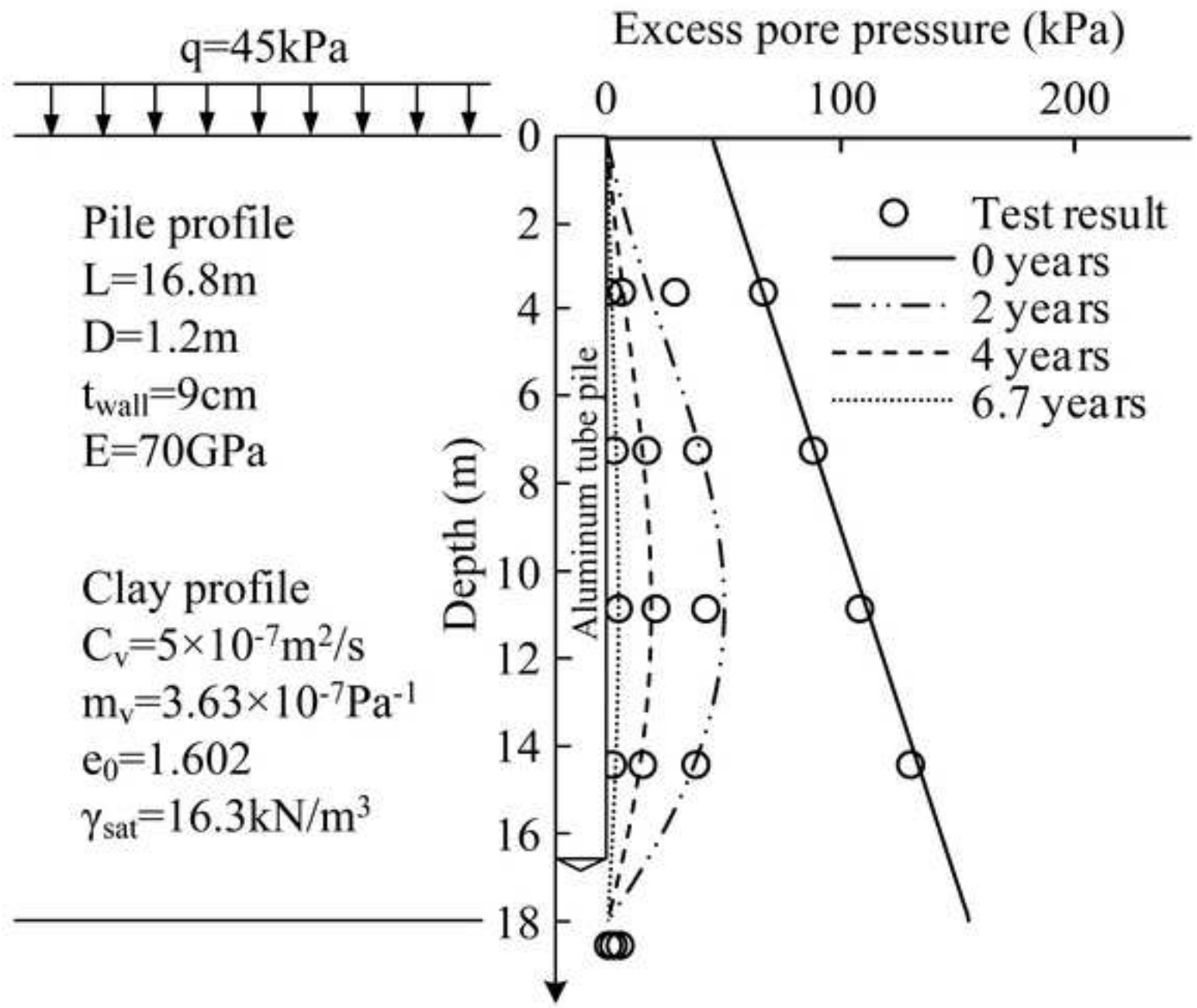


Figure 14  
[Click here to download high resolution image](#)

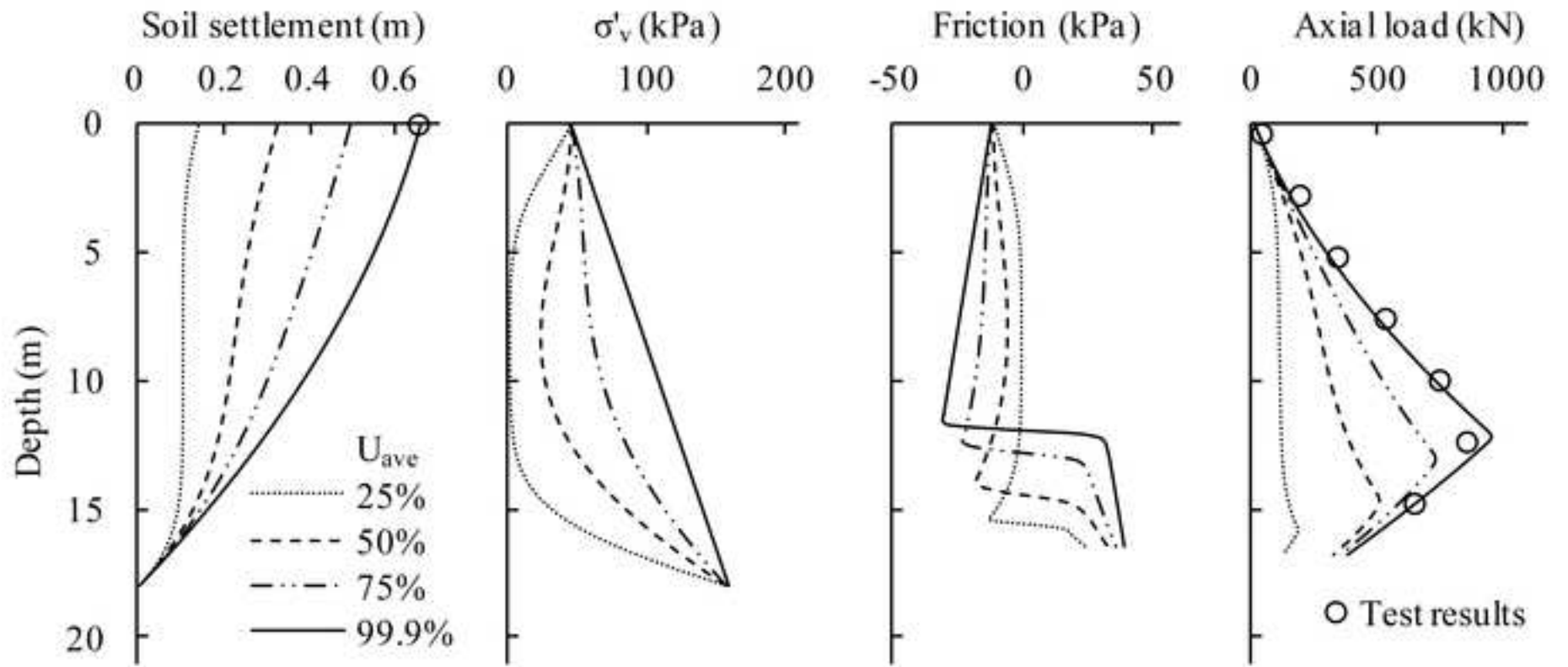
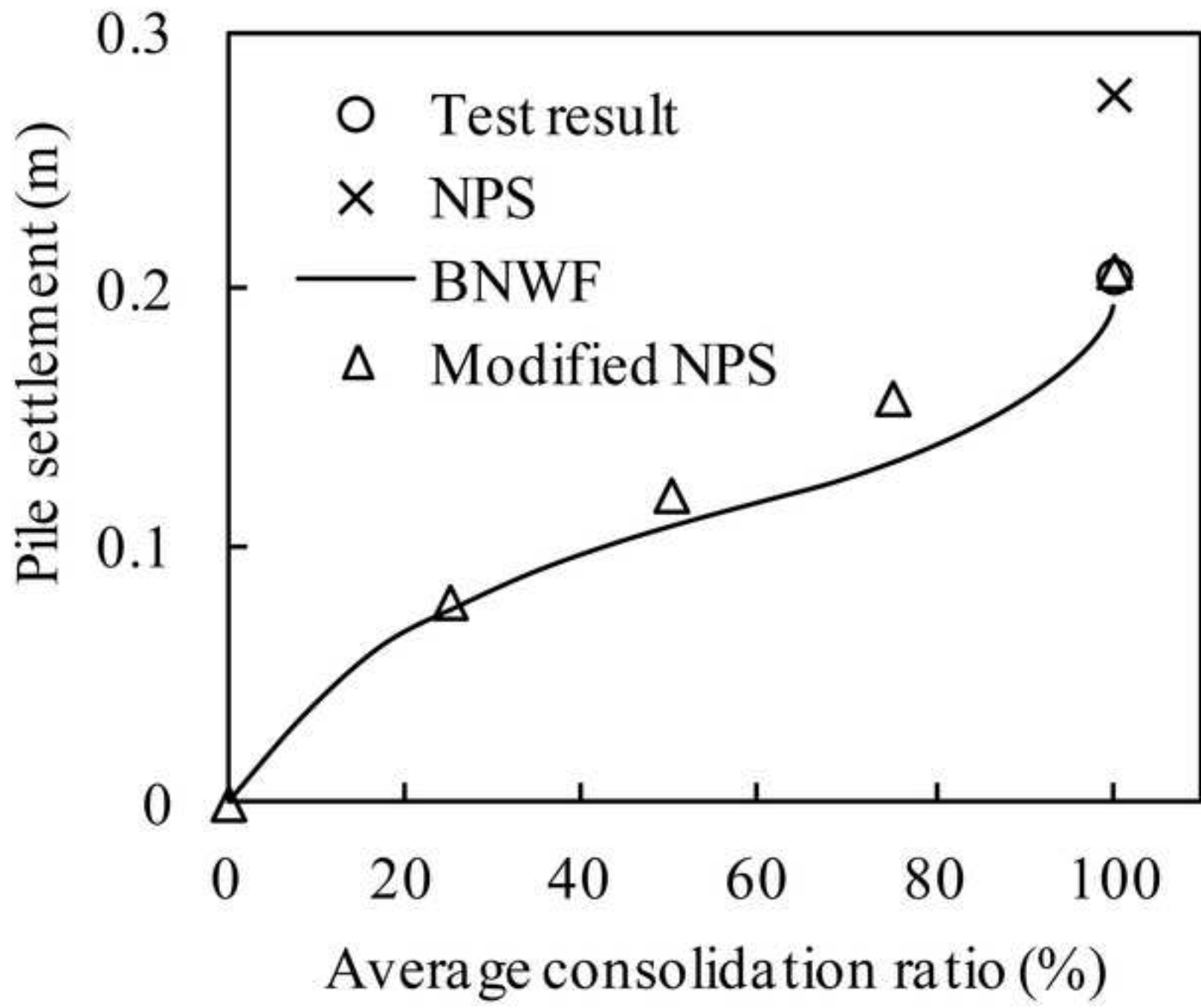


Figure 15  
[Click here to download high resolution image](#)



## List of figures

**Fig. 1.** Illustration of the neutral plane solution (after Fellenius 1984)

**Fig. 2.** Schematic of BNWF method using TzLiq1 material.

**Fig. 3.** Basic pile and soil setup for the example analyses

**Fig. 4.** Pore pressure and settlement isochrones from Terzaghi's one dimensional consolidation theory.

**Fig. 5.** Soil and pile responses at different average degrees of consolidation in a double drainage soil profile.  $Q_d=445\text{kN}$ .

**Fig. 6.** Soil and pile responses at different average degrees of consolidation in a single drainage through the top soil profile.  $Q_d=445\text{kN}$ .

**Fig. 7.** Soil and pile responses at different average degrees of consolidation in a single drainage through the bottom soil profile.  $Q_d=445\text{kN}$ .

**Fig. 8.** Neutral plane depth and pile settlement histories.  $Q_d=445\text{kN}$ .

**Fig. 9.** Pile settlements versus pile head load in the neutral plane solution compared with BNWF results under different drainage conditions

**Fig. 10.** Force-displacement behavior between soil and pile considering elasto-plasticity

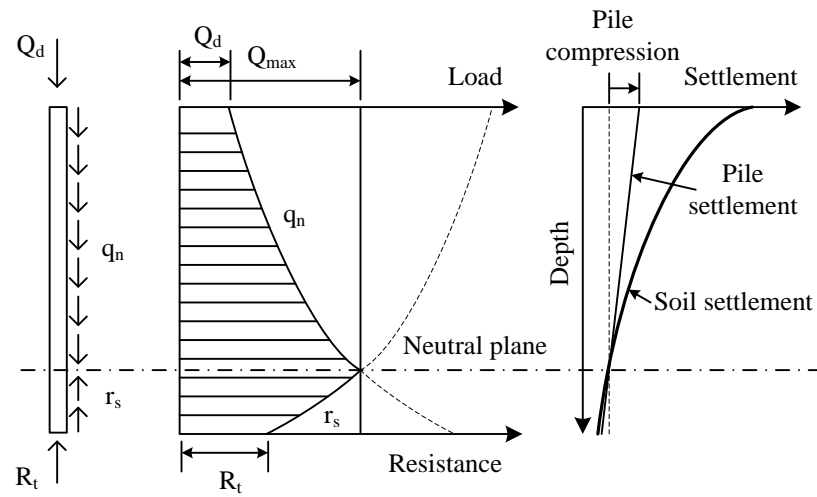
**Fig. 11.** Flow chart of the proposed modified neutral plane method

**Fig. 12.** Comparison between settlements calculated by the BNWF method and the modified neutral plane method using different number of intervals (including the conventional neutral plane solution).

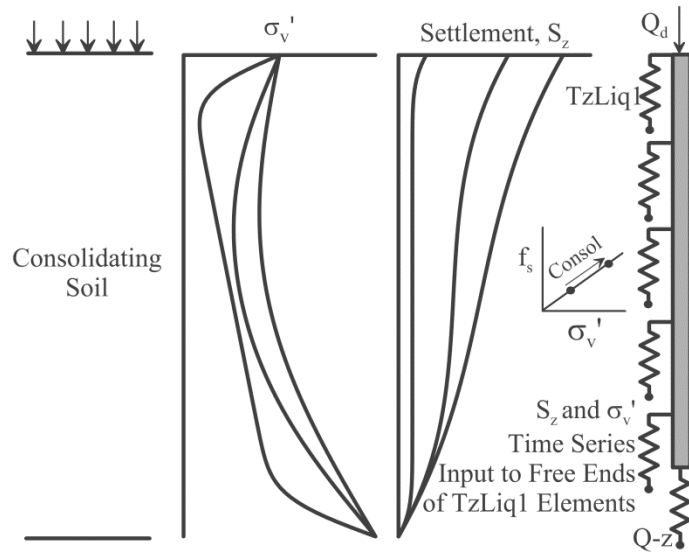
**Fig. 13.** Excess pore pressure isochrones from Terzaghi's one dimensional consolidation theory compared with test data

**Fig. 14.** Soil and pile responses at different average degrees of consolidation compared with centrifuge test data

**Fig. 15.** Pile settlement results from test data and different calculation methods

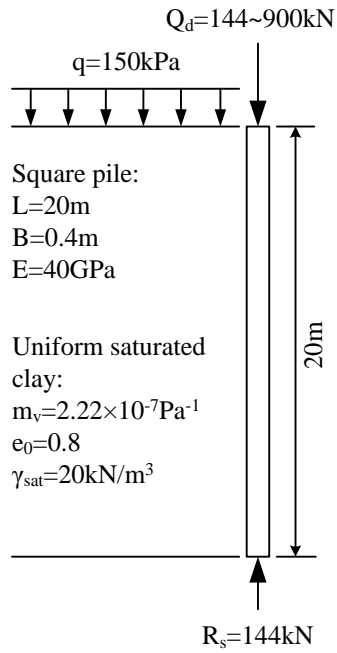


**Fig. 1.** Illustration of the neutral plane solution (after Fellenius 1984)

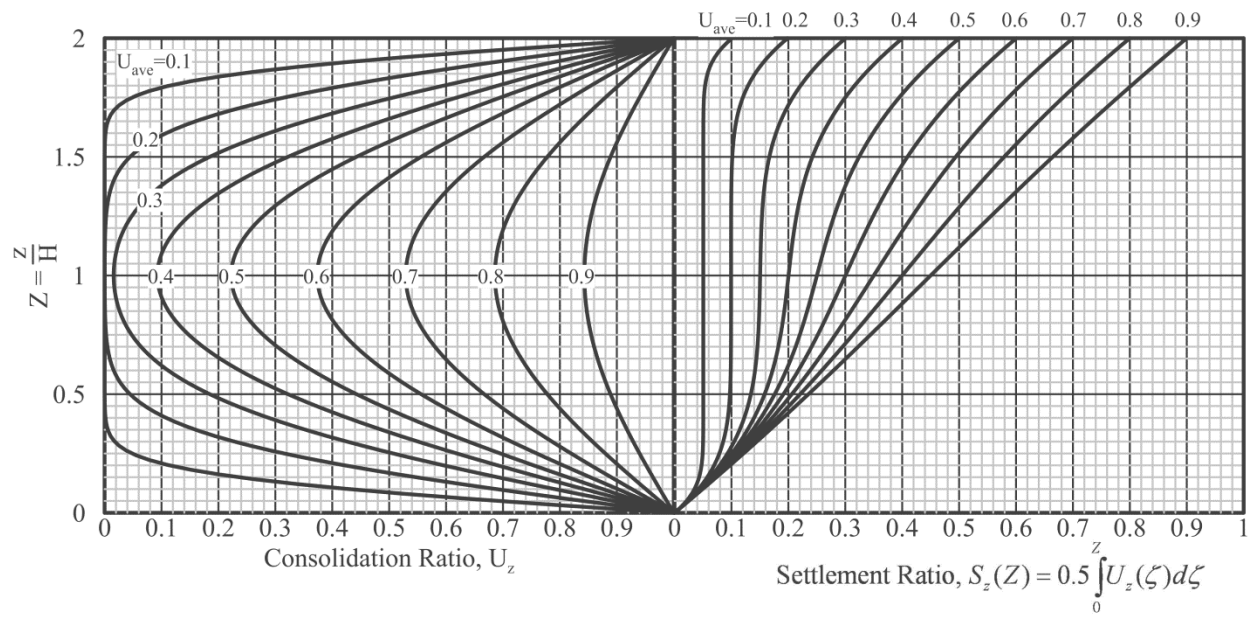


**Fig. 2.** Schematic of BNWF method using TzLiq1 material.

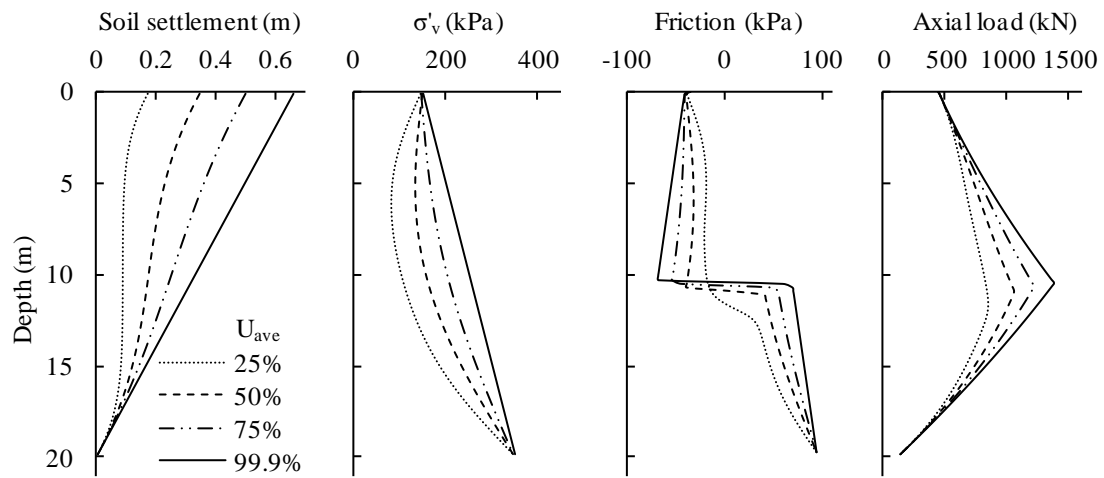




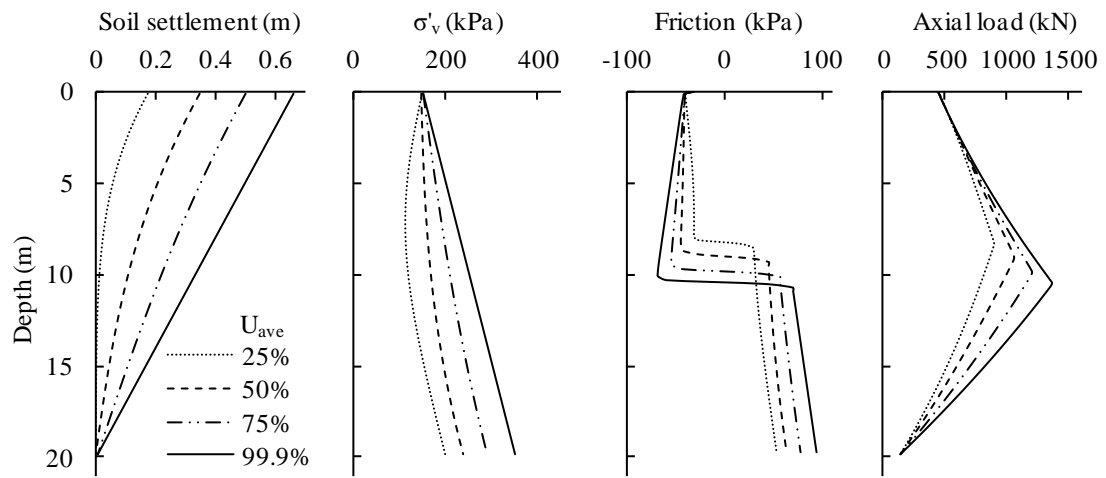
**Fig. 3.** Basic pile and soil setup for the example analyses



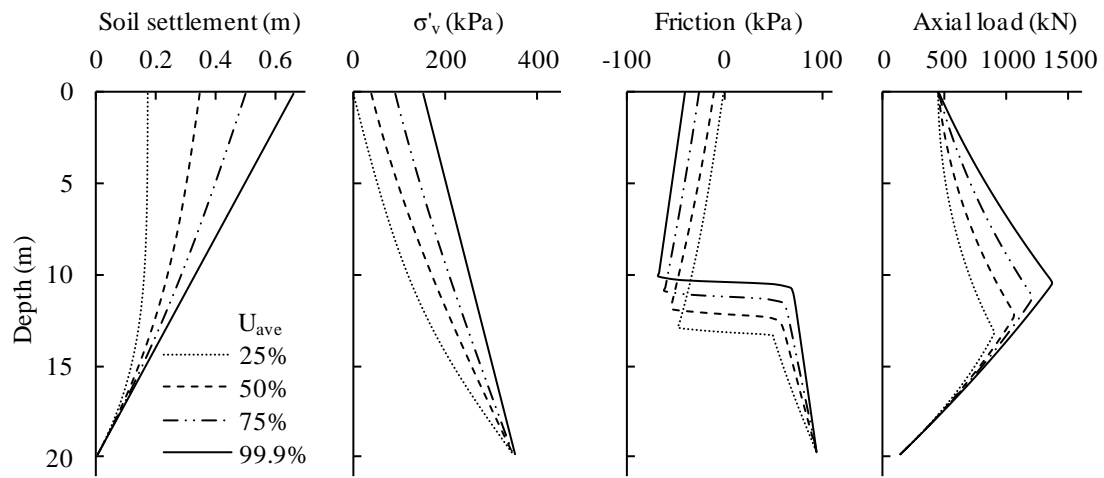
**Fig. 4.** Pore pressure and settlement isochrones from Terzaghi's one dimensional consolidation theory.



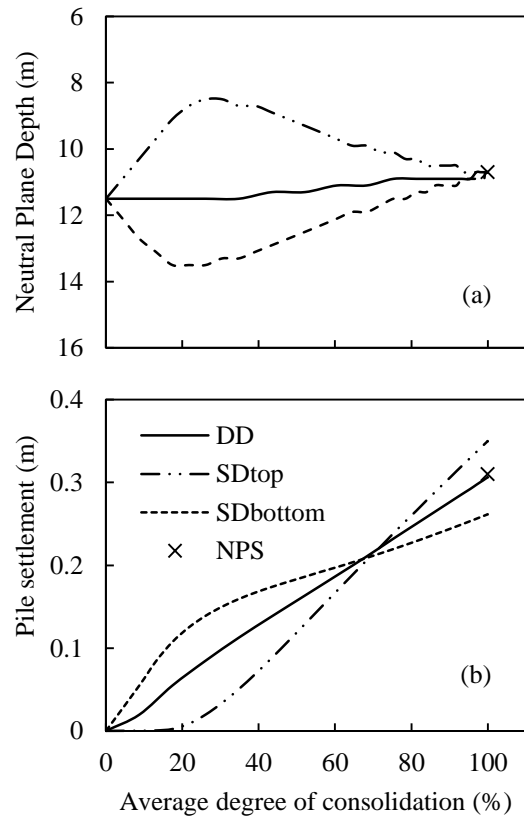
**Fig. 5.** Soil and pile responses at different average degrees of consolidation in a double drainage soil profile.  
 $Q_u=445\text{kN}$ .



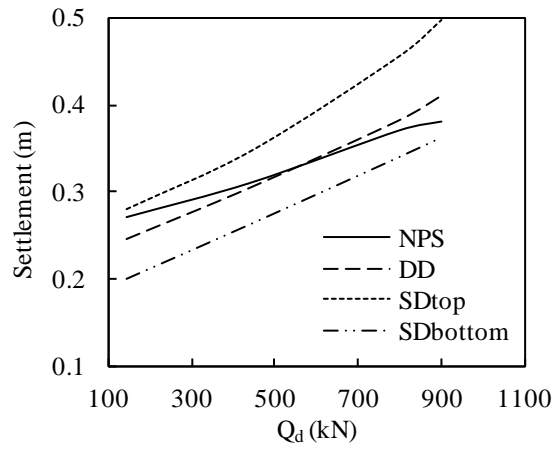
**Fig. 6.** Soil and pile responses at different average degrees of consolidation in a single drainage through the top soil profile.  $Q_d=445\text{kN}$ .



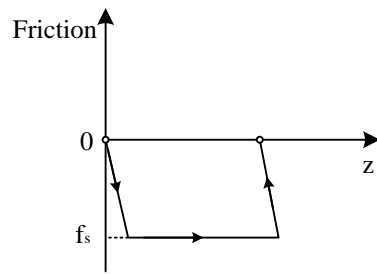
**Fig. 7.** Soil and pile responses at different average degrees of consolidation in a single drainage through the bottom soil profile.  $Q_d=445\text{kN}$ .



**Fig. 8.** Neutral plane depth and pile settlement histories.  $Q_d=445\text{kN}$ . (a) Neutral plane depth versus average degree of consolidation. (b) Pile settlement versus average degree of consolidation compared with conventional neutral plane solution

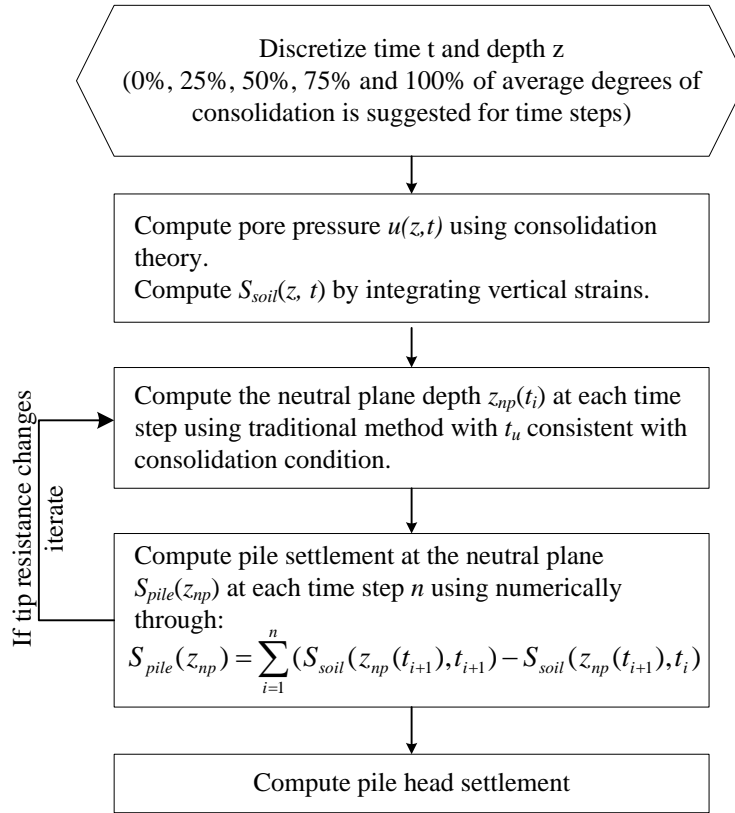


**Fig. 9.** Pile settlements versus pile head load in the neutral plane solution compared with BNWF results under different drainage conditions

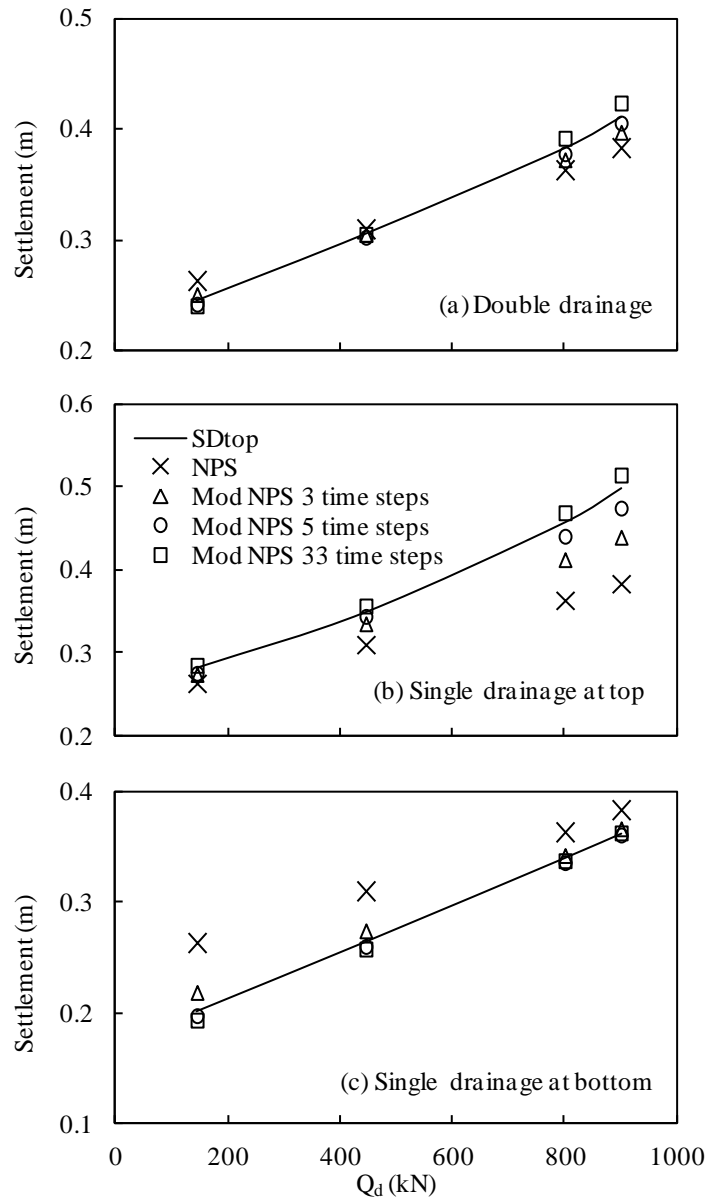


**Fig. 10.** Force-displacement behavior between soil and pile considering elasto-plasticity

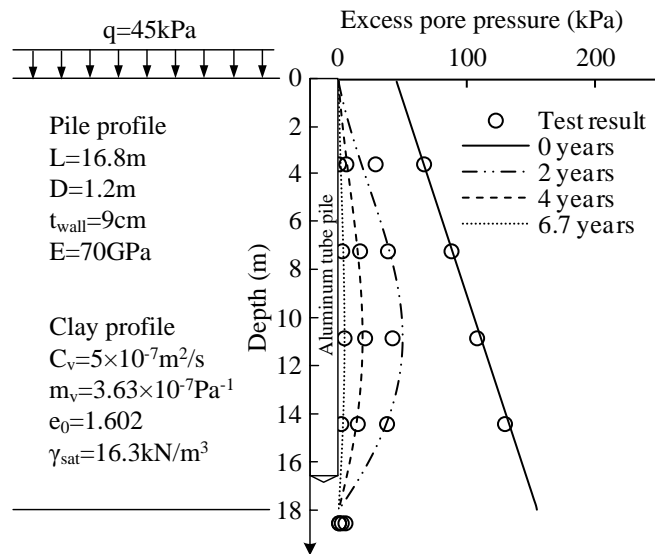




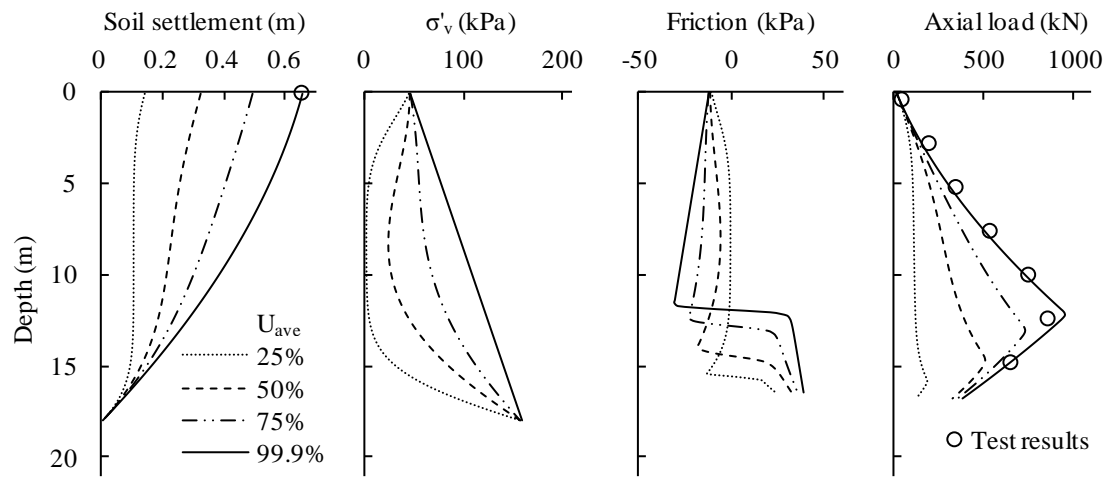
**Fig. 11.** Flow chart of the proposed modified neutral plane method



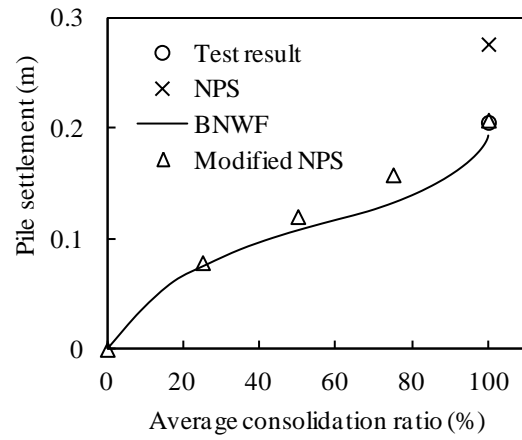
**Fig. 12.** Comparison between settlements calculated by the BNWF method and the modified neutral plane method using different number of intervals (including the conventional neutral plane solution).



**Fig. 13.** Excess pore pressure isochrones from Terzaghi's one dimensional consolidation theory compared with test data (test data from Lam et al., 2009)



**Fig. 14.** Soil and pile responses at different average degrees of consolidation compared with centrifuge test data (test data from Lam et al., 2009)



**Fig. 15.** Pile settlement results from test data and different calculation methods (test data from Lam et al., 2009)