Title
Essays on Asset Liquidity and Its Macroeconomic Effects

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Publication Date
2021

Peer reviewed|Thesis/dissertation

Essays on Asset Liquidity and Its Macroeconomic Effects

By

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DISSERTATION

Submitted in partial satisfaction of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

Economics

in the

OFFICE OF GRADUATE STUDIES

of the

UNIVERSITY OF CALIFORNIA

DAVIS

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2021

Abstract

The financial sector influences the macroeconomy in many aspects. Monetary policy affects firms' external financing and investment decisions through its impact on credit costs in financial markets. Institutional reform in financial markets can affect this monetary policy transmission. Safe assets are an important part of an economy. They are demanded by agents who look for a safe store of value, and how safe they are and how large their supply is affect the liquidity property of those assets and welfare of the economy. Financial crises, like the Great Recession, are not only costly in terms of output and investment in the short run, but they also cause productivity slowdown in the longer horizon through their impact on research and development (R&D). This dissertation contributes to the understanding of these issues.

Chapter One studies how monetary policy affects credit costs in financial markets and firms' external financing decisions. I study this transmission mechanism of monetary policy in a general equilibrium macroeconomic model where firms issue corporate bonds or obtain bank loans, and corporate bonds are not just stores of value but also serve a liquidity role. The model shows that an increase in the nominal policy rate can lower the borrowing cost in the corporate bond market, while increasing that in the bank loan market, and I provide

empirical evidence that supports this result. The model also predicts that a higher nominal policy rate induces firms to substitute corporate bonds for bank loans, which is supported by the existing empirical evidence. In the model, the Friedman rule is suboptimal so that keeping the cost of holding liquidity at a positive level is socially optimal. The optimal policy rate is an increasing function of the degree of corporate bond liquidity.

Chapter Two, joint work with Athanasios Geromichalos and Lucas Herrenbrueck, studies the relationship between an asset's safety and liquidity. Recently, a lot of attention has been paid to the role "safe and liquid assets" play in the macroeconomy. Many economists take as given that safer assets will also be more liquid, and some go a step further by practically using the two terms as synonyms. However, they are not synonyms: safety refers to the probability that the (issuer of the) asset will pay the promised cash flow, and liquidity refers to the ease with which an asset can be sold if needed. Mixing up these terms can lead to confusion and wrong policy recommendations. In this chapter, we build a multi-asset model in which an asset's safety and liquidity are well-defined and distinct from one another, and examine the relationship between an asset's safety and liquidity in general equilibrium. We show that the commonly held belief that "safety implies liquidity" is generally justified, but there may be exceptions. We then describe the conditions under which a relatively riskier asset can be more liquid than its safe(r) counterparts. Finally, we use our model to rationalize the puzzling observation that AAA corporate bonds are considered less liquid than (the riskier) AA corporate bonds.

Chapter Three studies how financial crises affect R&D activities, which is one of the key determinants of productivity. Recent literature documents the slow recovery and the productivity slowdown in the aftermath of financial crises. Several theoretical papers propose models to rationalize this. Some attribute it to a decline in the level of R&D, while others to a decline in the effectiveness of R&D. Which channel is more empirically relevant is an important question in guiding theoretical works. This chapter contributes to the literature by answering this question. Using data on 30 OECD countries over the years 1981–2016, I estimate the responses of R&D upon recessions and financial crises, using local projections. While recessions are bad times for output and investment in general, most of the responses are coming from the non-R&D part of investment. R&D is overall unresponsive to recessions, even to financial crises. This result suggests that a decline in R&D efficiency is a more empirically relevant channel to the productivity slowdown.

Acknowledgments

I am filled with immeasurable gratitude to my advisors who were incredibly supportive throughout graduate school. This dissertation would not have been possible without their guidance. I am deeply indebted to Thanasis Geromichalos, who spent countless hours advising me, guided me in the right direction, and believed in me even when I was struggling. Thanks to his dedication, I was able to successfully finish my degree. I am grateful to Nico Caramp for his guidance since I was writing my second-year literature review and for his insightful comments and suggestions which helped me break through difficult problems whenever I was faced with one. I am grateful to Andrés Carvajal especially for the reading group that he organized. Both the theory and the empirical parts of the first chapter of this dissertation started as crude ideas presented in the reading group. I also thank James Cloyne. All the empirical techniques that I learned are from his second-year class, and he was always generous with his time and willing to discuss with me whenever I asked for help.

I thank my best friends, Camila Saez and Mariam Yousuf, for their company, support, and encouragement throughout this journey. I would have never made it through this without you. Finally, I thank my family for their unceasing love and support.

Contents

Chapter 1

Liquidity Premium, Credit Costs, and Optimal Monetary Policy

1.1 Introduction

Central banks influence firms' investment through controlling the nominal policy rate, which then gets transmitted to the real rates at which firms borrow. I study this transmission mechanism in a general equilibrium macroeconomic model where firms have two options for external financing: they can issue corporate bonds or obtain bank loans. A theoretical novelty of my model is that corporate bonds are not just stores of value but also serve a liquidity role. The model delivers three predictions. First, an increase in the nominal policy rate can lower the borrowing cost in the corporate bond market, while increasing that in the bank loan market. This is in contrast with the common belief that all rates in the economy move in the same direction in response to changes in monetary policy.[1] I provide empirical evidence that supports the result. Second, a higher nominal policy rate induces firms to substitute corporate bonds for bank loans, and this result is supported by the existing empirical evidence. Third, the Friedman rule is suboptimal so that keeping the cost of holding liquidity at a positive level is socially optimal. The optimal policy rate is an increasing function of the degree of corporate bond liquidity.

To provide a concrete concept of liquidity of corporate bonds, I employ a model in the tradition of monetary-search theory, extended to include firms externally financing their production. Consumers and firms trade in a decentralized market, where trade is bilateral, credit is imperfect, and thus a medium of exchange is necessary. Agents allocate their wealth between money and corporate bonds. Both can serve liquidity purposes, but only a fraction of corporate bond holdings can be used towards trades. This assumption is meant to capture the idea that, when in need of extra money, agents liquidate corporate bonds in a secondary market, but due to frictions, trading delays, intermediation fees, etc., only a fraction of these bonds can be sold. Hence, the fraction of bonds the agents can use is meant to capture the degree of liquidity in the secondary market for those assets. Firms need to raise funds to finance production, and they can do so whether by issuing corporate bonds or by obtaining a bank loan. Naturally, the liquidity properties of corporate bonds affect their equilibrium price and, consequently, the issuance decision of firms.

Incorporating liquidity is the key to obtaining the main result of the paper. Thus, it

[1] Consider the following quote from Jones (2017)'s Macroeconomics textbook: "The Federal Reserve sets ... the federal funds rate, ... effectively setting the rate[s] at which [firms] borrow ... in financial *markets* [emphasis added]." This quote implies that the Federal Reserve implements monetary policy changes by targeting a single nominal rate but anticipates that these changes will be transmitted (symmetrically) to the rates in all financial markets.

is important to justify that this choice is empirically relevant. I highlight the fact that the U.S. corporate bond secondary market underwent a structural change during the early 2000s with the introduction of the Transaction Reporting and Compliance Engine (TRACE) that mandated reporting transaction-related information in all over-the-counter (OTC) transactions. Empirical evidence shows that the corporate bond secondary market liquidity has improved substantially as a result of the increased transparency under the new system, and that liquidity has become a significant component of the corporate bond premium since the TRACE was implemented.[2]

In the model, the nominal policy rate affects the cost of issuing corporate bonds and borrowing from a bank as follows. A higher nominal policy rate increases the opportunity cost of holding money, reduces real money balances, increases the liquidity premium of corporate bonds, and makes issuing corporate bonds less expensive. This pass-through becomes stronger when the corporate bond secondary market is more liquid. The real loan rates are determined in the OTC market for loans where firms and banks are matched and bargain over the size and the interest rate of a loan. With an increase in the nominal policy rate, the cost of holding money increases, agents carry less liquidity, and firms borrow less from a bank since agents can afford less. As a result, the real loan rate increases because it depends positively on the marginal benefit of a loan, and the latter decreases in the loan size.

[2]Two observations are in order. First, Bessembinder, Maxwell, and Venkataraman (2006), Edwards, Harris, and Piwowar (2007), and Goldstein, Hotchkiss, and Sirri (2007) find that the secondary market liquidity increased by 50–84% with the mandatory transaction reporting system. Second, Bao, Pan, and Wang (2011) find that liquidity explains 47% to 60% of the time variation of aggregate bond spreads of high-rated bonds, even larger than the variation that can be explained by credit risk. Also, He and Milbradt (2014) find that liquidity accounts for 44% of credit spreads for investment-grade corporate bonds and 31% for speculative-grade corporate bonds.

To empirically identify the channel through which the nominal policy rate affects the liquidity premium of corporate bonds (which I label the liquidity premium channel of monetary policy transmission), I use the fact that the introduction of the TRACE brought a structural change in the liquidity of the corporate bond secondary market. The liquidity premium channel is identified by the difference in the effect of an increase in the nominal policy rate on the corporate bond premium between the pre- and the post-TRACE periods. I employ the structural vector autoregression (SVAR) and the local projection approaches. To ensure the results are not driven by other macro factors and to limit any potential reverse causality issues, I exploit the high-frequency identified surprises from Federal Funds futures around the Federal Open Market Committee policy announcements as an external instrument, following Gertler and Karadi (2015).

The empirical analysis shows that the liquidity premium of corporate bonds indeed responds negatively to an increase in the nominal policy rate as the model predicts. Direct liquidity measures such as bid-ask spreads and trading volume further confirm the result. In the pre-TRACE period when the secondary market liquidity is low, a higher nominal policy rate still increases the bond premium, which is consistent with Gertler and Karadi (2015). However, surprisingly, in the post-TRACE period when the secondary market is highly liquid, the liquidity premium channel turns out to be so strong that a higher nominal policy rate decreases the bond premium. On the contrary, the real loan rates increase in the nominal policy rate.

Another interesting prediction of the model is that an increase in the nominal policy rate induces firms to substitute corporate bonds for bank loans. When firms have the option of financing both through issuing corporate bonds and borrowing from a bank, firms with large corporate bond issuance rely less on bank loans and thus can negotiate for a lower real loan rate. A higher nominal policy rate makes issuing corporate bonds less expensive, allowing firms to issue more corporate bonds for the strategic purpose of lowering their financing costs. Becker and Ivashina (2014) provide direct empirical support for this theoretical finding.

Lastly, I use the model to study optimal monetary policy for the period, such as the post-TRACE period, when the liquidity premium channel of monetary policy transmission is dominant in the response of the bond premium to the nominal policy rate. A common result in monetary theory is that an increase in the nominal policy rate hurts welfare: a higher nominal policy rate increases the opportunity cost of holding liquidity, induces agents to carry less liquidity, and reduces the quantity of goods they can afford. In my model, however, the Friedman rule—implementing zero nominal policy rate—is suboptimal.[3] The intuition behind the suboptimality of the Friedman rule is as follows. Assume that the nominal policy rate is currently low, so that the borrowing cost in the corporate bond market is high, while that in the bank loan market is low. When meeting a firm for trade, agents face risk: they can meet a firm that obtained a loan from a bank and have large production capacity, or a firm that financed only by issuing corporate bonds and have small production capacity. Increasing the nominal policy rate makes issuing corporate bonds cheaper and borrowing from a bank more expensive, thereby reducing the risk agents face and increasing welfare. The optimal nominal policy rate depends on the corporate bond secondary market liquidity and the distribution of firms along their ways of financing. The more liquid the corporate

[3] A negative relationship between the nominal policy rate and welfare characterizes a large class of monetary models, including Lagos and Wright (2005) and the majority of models that build upon their framework. However, there are exceptions to this rule, especially models with search externality. Later, when I review the related literature, I provide a more detailed discussion of exceptions to this result, and I claim that the channel through which my model can deliver a positive relationship between the nominal policy rate and welfare has not been highlighted before.

bond secondary market, or the more firms financing through issuing corporate bonds, the higher the optimal policy rate.

Related literature. A collection of empirical papers uses monetary policy as a source of aggregate variation and studies its effect on the firm-side of the economy. One strand of such literature examines firms' heterogeneous responses in their investment, interpreting the results as an indication of the presence of financial frictions. The heterogeneity depends on the firms' various characteristics such as cash flows (Oliner and Rudebusch (1992)), size (Gertler and Gilchrist (1994), Bernanke, Gertler, and Gilchrist (1996)), liquid asset holdings (Kashyap, Lamont, and Stein (1994), Jeenas (2019)), default risk (Ottonello and Winberry (2020)), and age/dividend payouts (Cloyne, Ferreira, Froemel, and Surico (2019)). This paper contributes to the literature by studying the heterogeneity in the responses of different financial markets for firms' external financing, including the corporate bond and the bank loan markets, which implies that firms will respond differently depending on their access to markets.

Another related empirical literature is the one that examines the relationship between monetary policy and the liquidity premium of liquid assets. Nagel (2016) and Drechsler, Savov, and Schnabl (2018) provide empirical evidence that the liquidity premium of Treasuries is positively associated with the short-term interest rates. The rationale behind it is the exact same as the one considered in this paper: the short-term interest rates imply a higher opportunity cost of holding money and hence a higher premium for the liquidity service benefits of assets that can be substitutes for money. This paper complements the literature by providing evidence that a similar relationship between monetary policy and the liquidity premium holds also for corporate bonds.[4]

This paper also contributes to the empirical literature that investigates the effect of the introduction of the TRACE to the corporate bond secondary market. A series of papers, such as Bessembinder, Maxwell, and Venkataraman (2006), Edwards, Harris, and Piwowar (2007), Goldstein, Hotchkiss, and Sirri (2007), and Asquith, Covert, and Pathak (2013), study the impact of the mandatory transaction reporting through the TRACE on the trading costs and the liquidity of the corporate bond secondary market. This paper contributes to the literature by looking at the impact of the introduction of the TRACE on the response of the corporate bond market to monetary policy.

Also related is the literature that studies firms' financing choices and the composition of credit, which includes for instance Denis and Mihov (2003), Adrian, Colla, and Shin (2012), Becker and Ivashina (2014), and Schwert (2018). This paper is especially relevant to Becker and Ivashina (2014). One of the theoretical findings of this paper is that firms switch from loans to bonds following an increase in the nominal policy rate. Becker and Ivashina (2014) provide direct empirical support for this finding.

The model in this paper builds on the New Monetarist framework, recent advances in monetary economics, as surveyed in Lagos, Rocheteau, and Wright (2017) and Nosal and Rocheteau (2017). The consumer-side of the model is based on Lagos and Wright (2005). In the model, corporate bonds have a liquidity premium due to the liquidity service they provide, and the monetary policy affects the costs of holding money and in turn the price of corporate bonds through their liquidity premium, following Geromichalos, Licari, and Suárez-Lledó (2007), Lagos (2011), Nosal and Rocheteau (2012), Andolfatto, Berentsen, and

[4]Lagos and Zhang (2020) provide an empirical study on the equity market.

Waller (2013), and Hu and Rocheteau (2015).[5]

Another paper that studies how monetary policy affects corporate finance is Rocheteau, Wright, and Zhang (2018). While in their paper firms finance investment by internal financing or bank loans, the firm-side of the model in this paper focuses solely on external financing, in particular corporate bond issues and bank loans.[6] In addition, this paper integrates the consumer- and the firm-sides and studies how they interact with each other. By doing so, the supply of corporate bonds becomes endogenous, instead of being supplied at an exogenous level. Geromichalos and Herrenbrueck (2016b) also endogenize the supply of assets, but in their model asset issuers and sellers who produce consumption goods are different agents. On the other hand, in this paper firms issue bonds to finance their production.

As for the suboptimality of the Friedman rule, there exist generally two classes of models where positive costs of holding money can be welfare improving.[7] One is the models where inflation has distributive effects (see for example Molico (2006) and Rocheteau, Weill, and Wong (2019)). The other is the models with free entry to search (see for example Rocheteau and Wright (2005) and Berentsen, Rocheteau, and Shi (2007)). When there is search externality, the Friedman rule is optimal if and only if the Hosios (1990) condition is satisfied. When the Hosios (1990) condition does not hold, a deviation from the Friedman rule can be optimal since it can adjust the inefficiently large or small number of agents

[5]While in this paper the liquidity property of assets is direct in the sense that they serve as a medium of exchange or collateral and thus help to facilitate trade in frictional decentralized markets for goods, it can be microfounded by introducing secondary markets where agents can liquidate assets for money or by using information theory. See Berentsen, Huber, and Marchesiani (2014, 2016), Han (2015), Geromichalos and Herrenbrueck (2016a, 2016b, 2017), Geromichalos, Herrenbrueck, and Salyer (2016), Mattesini and Nosal (2016), Herrenbrueck and Geromichalos (2017), Herrenbrueck (2019a), and Madison (2019) for the examples for the former, and Rocheteau, Wright, and Xiao (2018) for the latter.

[6]The OTC market for bank loans in this paper follows Rocheteau, Wright, and Zhang (2018).

[7]For an exhaustive list of the papers in which a deviation from the Friedman rule can be optimal, see Section 6.9 of Nosal and Rocheteau (2017).

who are in search. This paper provides a new rationale for why the Friedman rule can be suboptimal, which is due to the heterogeneity in the effect of monetary policy across different financing sources.

Definitions of premiums. This paper defines the liquidity premium and the bond premium as follows. The liquidity premium is a price premium that investors are willing to pay for the liquidity service that assets provide. It is defined as the price of an asset minus the price if the asset did not provide any liquidity service. This definition follows the New Monetarist literature (see Lagos, Rocheteau, and Wright (2017) and Nosal and Rocheteau (2017) for surveys). The liquidity premium defined in this way moves in the same direction as that in Nagel (2016) and Drechsler, Savov, and Schnabl (2018), who define the liquidity premium in yield, as the yield if an asset did not provide any liquidity service minus the yield of the asset. The definition of the bond premium follows Gilchrist and Zakrajšek (2012), who defines the bond premium as the yield of a bond minus the yield associated with a price that equals the net present value of the cash flows, or the fundamental value, of the bond. Under these definitions, the liquidity premium and the bond premium are negatively correlated.

Structure of the paper. Section 1.2 presents the environment of the model. Section 1.3 characterizes the equilibrium of the model and examines the transmission mechanism of monetary policy to credit costs. Section 1.4 provides empirical analysis and evidence that supports such mechanism. Section 1.5 analyzes how monetary policy affects the composition of firms' credit. Section 1.6 studies optimal monetary policy. Section 1.7 concludes. The appendices contain a theory appendix and an appendix that explains data sources and includes additional figures for robustness checks.

1.2 The Model

The model builds on Lagos and Wright (2005) and introduces firms externally financing their production. Time is discrete and continues forever. Each period is divided into two subperiods. In the first subperiod, there is a decentralized market (DM) where a specialized good is traded. In the second subperiod, three markets open in order: a frictionless centralized market (CM) where agents settle liabilities and trade a consumption good and assets; an over-the-counter (OTC) market for bank loans, as in Rocheteau, Wright, and Zhang (2018); and a competitive market for intermediate goods. The consumption good in the CM is taken as the numeraire.

There are four agents: firms, intermediate good suppliers, banks, and consumers. Firms produce special goods (hereafter, DM goods) in the DM and sell them to consumers. To produce the DM goods, they need to purchase intermediate goods from the intermediate good suppliers, and, to do so, they need to externally finance. The intermediate good suppliers (hereafter, suppliers) can produce intermediate goods and provide them to firms. Bank loans are one of the ways of external financing, and banks do loan services for firms. Consumers buy the DM goods from firms in the DM and consume them. The measure of firms and consumers is 1. The measure of banks is the same as that of firms borrowing from a bank. The measure of suppliers is irrelevant due to constant returns to scale (CRS) in their production.

Agents live forever, except for firms that live one period. Firms are born in the second subperiod and die next period in the second subperiod after settlement. Agents discount across periods, but not subperiods, at rate $\beta \in (0, 1)$. All agents have a linear preference over the numeraire, c, where $c > 0$ is interpreted as consumption of the numeraire and $c < 0$ as production. Additionally, consumers, who consume the DM goods in the first subperiod in the DM, derive utility, $u(q)$, where q is the consumption of the DM goods. u is twice continuously differentiable, $u(0) = 0$, $u'(0) = \infty$, $u'(\infty) = 0$, $u' > 0$ and $u'' < 0$. Firms can transform intermediate goods acquired from suppliers into the DM goods with linear technology.[8] Suppliers can produce intermediate goods at unit cost.

Firms, born in the second subperiod, are in need of intermediate goods to produce the DM goods in the following first subperiod. When purchasing intermediate goods from suppliers, firms need to pay in numeraire, and firms that are just born are assumed not to be able to produce numeraire goods. Firms can acquire numeraire goods either by obtaining a loan from a bank or by issuing one-period real corporate bonds that yield a unit of numeraire in the next second subperiod. For the moment, it is assumed that the measure $1 - \lambda \in (0, 1)$ of firms finance by borrowing from a bank, while the measure λ of firms finance by issuing corporate bonds.[9] The issuance decision of firms endogenously determines the supply of corporate bonds. On the demand side of the corporate bond market are consumers. Banks are not allowed to hold corporate bonds.

The other asset traded in the CM, besides corporate bonds, is money. Monetary authority controls the money supply, and the supply evolves according to $M_{t+1} = (1 + \pi)M_t$, where π is the rate of monetary expansion (or contraction if $\pi < 0$) implemented by lump-sum transfers to (or taxes on) consumers at the beginning of the second subperiod. In a station-

[8]This is without loss of generality, and all go through with a concave production function. Assume that, with k amount of intermediate goods, firms can produce $f(k)$ amount of the DM goods, where f is twice continuously differentiable, $f(0) = 0$, $f'(0) = \infty$, $f'(\infty) = 0$, $f' > 0$ and $f'' < 0$. This in turn means that, to produce q amount of the DM goods, a firm needs $f^{-1}(q)$ amount of intermediate goods. f^{-1} is effectively a convex cost function, and all remaining analysis is the same.

[9]Section 1.5 endogenizes the ways of firms' financing.

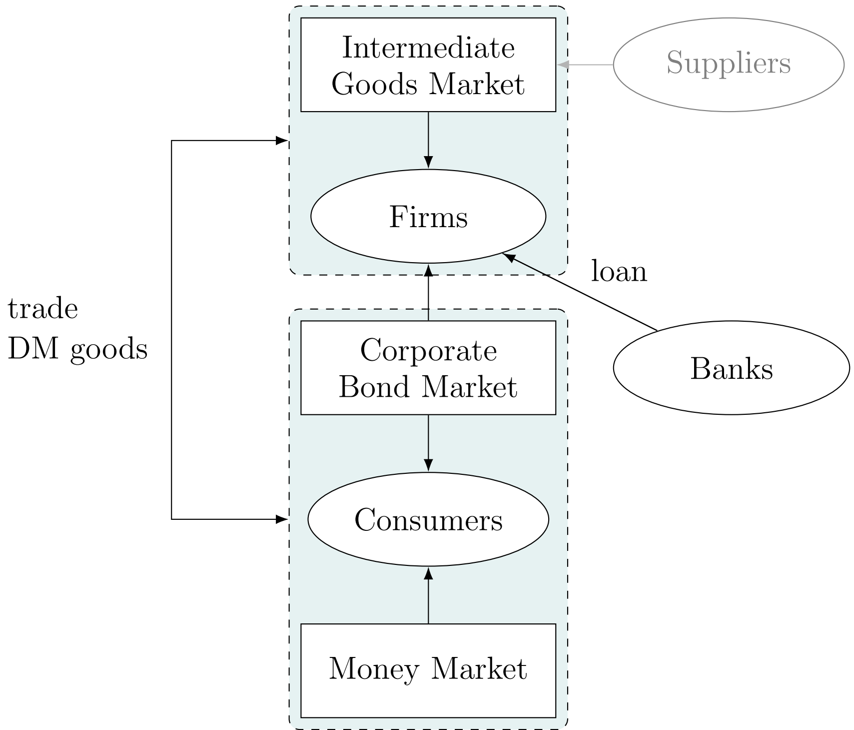

Figure 1.1. The environment of the model

ary equilibrium, π is also the inflation rate. $i \equiv (1+\pi)/\beta - 1$ represents the cost of holding money and, it is the nominal interest rate on an illiquid bond (if such bond were introduced). An equilibrium exists for $i > 0$, or $\pi > \beta - 1$, and the Friedman rule is considered as $i \to 0$, or $\pi \to \beta - 1$.

In the DM, firms produce the DM goods using intermediate goods and sell them to consumers. Trade in the DM is bilateral and agents are anonymous and lack commitment. Thus, trade has to be quid pro quo and necessitates a medium of exchange. Both money and corporate bonds can play this role. But corporate bonds are partially liquid, and only a fraction $\chi \in (0, 1]$ can be used as payment. Consumers meet firms randomly and negotiate over the terms of trade. All consumers match with a firm. The surplus generated within a match is split according to Kalai's proportional bargaining solution, and the consumer's bargaining power is $\theta \in (0,1)$. In the OTC market for loans, all firms match with a bank. Terms of a loan contract are determined through generalized Nash bargaining between a firm and a bank, and the banks' bargaining power is $\eta \in (0,1)$.

The environment of the model is summarized in Figure 1.1.

1.3 Analysis of the Model

1.3.1 Value Functions

Consider a consumer in the second subperiod who carries to the CM financial wealth w denominated in numeraire and chooses a portfolio of real balances (units of money in terms of numeraire) and corporate bonds to bring to the DM. The value function of the consumer in the CM is

$$W^C(w) = \max_{c,\hat{m}\geq 0,\hat{a}\geq 0} c + \beta V^C(\hat{m},\hat{a}) \quad \text{s.t.} \quad c + (1+\pi)\hat{m} + \psi\hat{a} = w + T, \tag{1.1}$$

where W^C and V^C are the value functions of a consumer in the second and the first subperiods, respectively, c is consumption (or production if $c < 0$) of numeraire goods, $\hat{m}$ is real balances (units of money in terms of numeraire), $\hat{a}$ is the amount of corporate bonds purchased, ψ is the price of corporate bonds, and T is the lump-sum transfer in terms of numeraire (or taxes if $T < 0$). Since the rate of return on money is $1/(1+\pi)$, a consumer accumulates $(1+\pi)\hat{m}$ of real balances this period to hold $\hat{m}$ at the start of the next period.

Eliminating c using the constraint, the value function reduces to

$$W^C(w) = w + T + \max_{\hat{m}\geq 0, \hat{a}\geq 0} \{-(1+\pi)\hat{m} - \psi\hat{a} + \beta V^C(\hat{m}, \hat{a})\}, \tag{1.2}$$

which shows that W^C is linear in w and that the choice of $(\hat{m}, \hat{a})$ is independent of w. In the following first subperiod, the consumer randomly matches with a firm and trades the DM goods. In the DM, the consumer bargains with a firm over how many DM goods to purchase from the firm, q, and how much financial wealth to transfer to the firm in return for the DM goods, p. With probability $1-\lambda$, the consumer will match with a firm that finances through borrowing from a bank, and the terms of trade with such firm are denoted by (q_L, p_L). With probability λ, the consumer will match with a firm that finances through issuing corporate bonds, and the terms of trade with such firm are denoted by (q_B, p_B). When purchasing q amount of the DM goods, the consumer derives $u(q)$ of utility from consuming them. After paying p amount of financial wealth to a firm in exchange for the DM goods purchased, the consumer brings $\hat{m}+\hat{a}-p$ amount of leftover financial wealth to the CM. The value function of a consumer who brings $\hat{m}$ amount of real balances and $\hat{a}$ amount of corporate bonds to the DM is

$$V^C(\hat{m}, \hat{a}) = (1-\lambda)\left[u(q_L) + W^C(\hat{m}+\hat{a}-p_L)\right] + \lambda\left[u(q_B) + W^C(\hat{m}+\hat{a}-p_B)\right], \tag{1.3}$$

which, using the linearity of W^C, reduces to

$$V^C(\hat{m}, \hat{a}) = (1-\lambda)\left[u(q_L) - p_L\right] + \lambda\left[u(q_B) - p_B\right] + W^C(\hat{m}+\hat{a}). \tag{1.4}$$

Next consider the value function of an intermediate good supplier in the second subperiod:

$$W^S = \max_{c,k\geq 0} c + \beta W^S \quad \text{s.t.} \quad c + k = p_k k, \tag{1.5}$$

where k is the amount of intermediate goods produced and p_k is their price. Suppliers do not trade in the DM and do not carry any money or corporate bonds due to the cost of holding money and because corporate bonds will be priced at the liquidity premium. In the competitive market for intermediate goods that comes after the CM, suppliers choose the amount of intermediate goods, k, to produce at a linear cost taking its price, p_k, as given. A supplier finds k that maximizes $-k + p_k k$. If the intermediate goods market is active, $p_k = 1$.

In the OTC market for loans, a bank provides a loan to a firm. The terms of a loan contract, denoted by (k, r_ℓ), are determined through bargaining between a firm and a bank: a firm borrows k amount of numeraire from a bank and pays back $(1 + r_\ell)k$ amount of numeraire in the next second subperiod. The value function of a bank in the CM with financial wealth w denominated in numeraire and a loan contract (k, r_ℓ) is

$$W^B(w) = \max_{c} c + \beta W^B((1 + r_\ell)k) \quad \text{s.t.} \quad c + k = w. \tag{1.6}$$

The constraint can be written as $k = w - c$, and this represents the balance sheet of the bank. It indicates that the amount of a loan given to a firm, k, is covered by the financial wealth of the bank, $w - c$, which can be thought of as bank capital. Eliminating c using the constraint, the value function reduces to $W^B(w) = w - k + \beta W^B((1 + r_\ell)k)$.

Now consider a firm in the second subperiod that is just born and finances through borrowing from a bank under the terms of a loan contract (k, r_ℓ). With k amount of numeraire borrowed from a bank, the firm purchases k amount of intermediate goods from suppliers at price $p_k = 1$. In the following first subperiod, the firm matches with a consumer and trade the DM goods, and the terms of trade are denoted by (q_L, p_L). After trading in the DM, the firm brings $k - q_L$ of leftover intermediate goods and p_L of financial wealth to the CM and needs to pay back $(1 + r_\ell)k$ units of numeraire to the bank. The value function in the first subperiod of a firm with a loan contract (k, r_ℓ) is

$$V^F(k, (1 + r_\ell)k) = W^F(k - q_L, p_L, (1 + r_\ell)k), \tag{1.7}$$

where V^F and W^F are the value functions of a firm in the first and the second subperiods, respectively. The value function of a firm in the second subperiod after trading in the DM is

$$W^F(k - q_L, p_L, (1 + r_\ell)k) = \max_c c \quad \text{s.t.} \quad c = k - q_L + p_L - (1 + r_\ell)k, \tag{1.8}$$

which simply reduces to $W^F(k - q_L, p_L, (1 + r_\ell)k) = k - q_L + p_L - (1 + r_\ell)k$.[10]

A firm that finances through issuing corporate bonds first decides the amount of corporate bonds to issue, $\hat{A}$, taking their price ψ in the CM as given. With $\psi\hat{A}$ amount of numeraire acquired by issuing corporate bonds, the firm purchases intermediate goods from suppliers at price $p_k = 1$. In the following first subperiod, the firm matches with a consumer

[10] It is assumed that the firm can use the leftover intermediate goods to produce numeraire goods at unit cost, in case the firm did not use all the intermediate goods it held to produce the DM goods. Allowing a firm to be able to use the leftover intermediate goods enters as an outside option for firms in bargaining over the terms of DM trade, and this technology is not used in equilibrium. This assumption is just for simplifying the exposition.

and trade the DM goods, and the terms of trade are denoted by (q_B, p_B). After trading in the DM, the firm brings $\psi\hat{A} - q_B$ of leftover intermediate goods and p_B of financial wealth to the CM and needs to pay $\hat{A}$ units of numeraire to the consumers who hold the corporate bonds. The value function in the first subperiod of a firm that issued $\hat{A}$ amount of corporate bonds in the previous second subperiod at price ψ is

$$V^F(\psi\hat{A}, \hat{A}) = W^F(\psi\hat{A} - q_B, p_B, \hat{A}). \tag{1.9}$$

The value function of a firm in the second subperiod after trading in the DM is

$$W^F(\psi\hat{A} - q_B, p_B, \hat{A}) = \max_c c \quad \text{s.t.} \quad c = \psi\hat{A} - q_B + p_B - \hat{A}, \tag{1.10}$$

which simply reduces to $W^F(\psi\hat{A} - q_B, p_B, \hat{A}) = \psi\hat{A} - q_B + p_B - \hat{A}$. Using the linearity of W^F, a newborn firm in the second subperiod decides the amount of corporate bonds to issue by solving

$$\max_{\hat{A} \geq 0} \beta V^F(\psi\hat{A}, \hat{A}) = \max_{\hat{A} \geq 0} \beta\{(p_B - q_B) - (1 - \psi)\hat{A}\}. \tag{1.11}$$

1.3.2 Terms of Trade

Consider a meeting in the DM between a consumer who carries m amount of real balances and a amount of corporate bonds and a firm that brings k amount of intermediate goods. The two parties bargain over the quantity of the DM goods to trade, q, and the amount of financial wealth for the consumer to transfer to the firm, p. Corporate bonds are partially liquid, and only a fraction $\chi \in (0,1]$ can be used as a medium of exchange. Thus, the maximum amount of financial wealth that the consumer can use for trade is $m + \chi a$. The firm can produce the DM goods with a linear technology up to k. Trade is as a result subject to both the consumer's liquidity and the firm's capacity constraints: $p \leq m + \chi a$ and $q \leq k$. The total surplus generated within a meeting is split according to Kalai's proportional bargaining solution, and the consumer's bargaining power is $\theta \in (0,1)$. The consumer's continuation value with trade is $u(q) + W^C(m + a - p)$: the consumer derives $u(q)$ of utility from consuming q amount of the DM goods and brings $m + a - p$ amount of leftover financial wealth to the CM after transferring p amount of financial wealth to the firm as a payment. The consumer's continuation value without trade is $W^C(m + a)$. Thus, the consumer's surplus is $u(q) + W^C(m + a - p) - W^C(m + a)$, which, using the linearity of W^C, reduces to $u(q) - p$. The firm's continuation value with trade is $W^F(k - q, p, \cdot)$, where the last argument is the liabilities that the firm needs to pay back in the subsequent second subperiod to either consumers (who hold the corporate bonds if the firm financed through issuing corporate bonds) or a bank (according to a loan contract if the firm financed through a bank loan). The firm brings $k - q$ amount of leftover intermediate goods after producing q amount of DM goods with a linear technology and p amount of financial wealth that it received from the consumer as a payment. The firm's continuation value without trade is $W^F(k, 0, \cdot)$. Thus, the firm's surplus is $W^F(k - q, p, \cdot) - W^F(k, 0, \cdot)$, which, using the linearity of W^F, reduces to $p - q$. The total surplus is the sum of the consumer's surplus and the firm's surplus and equals $u(q) - q$. The bargaining solution is

$$p = v(q) \equiv (1-\theta)u(q) + \theta q, \quad v'(q) > 0, \tag{1.12}$$

$$q = \min\{v^{-1}(m + \chi a), k\}. \tag{1.13}$$

The consumer must transfer $p = v(q)$ amount of financial wealth to the firm to get q amount of the DM goods, and a larger amount of financial wealth needs to be transferred to purchase a larger amount of the DM goods. p solves $u(q) - p = \theta(u(q) - q)$ or $p - q = (1-\theta)(u(q) - q)$ so that the consumer's surplus, $u(q) - p$, becomes θ share of the total surplus, $u(q) - q$, and that the firm's surplus, $p - q$, becomes $1 - \theta$ share of the total surplus. The first best solution to the bargaining problem that maximizes the total surplus is denoted by (p^*, q^*), where $p^* = v(q^*)$ and q^* satisfies $u'(q^*) = 1$. With $m + \chi a$ amount of financial wealth that can be used for trade, the consumer can buy up to $v^{-1}(m + \chi a)$ amount of the DM goods. With k amount of intermediate goods in hand, the firm can produce up to k amount of the DM goods. In equilibrium, $m + \chi a \leq v(q^*)$ and $k \leq q^*$ hold: the consumer will not want to bring more financial wealth than she needs to buy q^* amount of the DM goods, and the firm will not want to bring more intermediate goods than it needs to produce q^* amount of the DM goods. Observing that the total surplus $u(q) - q$ increases in q until $q = q^*$, the shorter side between the consumer's liquidity position and the firm's capacity determines the bargaining solution. Thus, q is given by the minimum between $v^{-1}(m + \chi a)$ and k.

1.3.3 Loan Contract

Consider a meeting in the OTC market for loans in the second subperiod between a bank with w amount of bank capital that can be lent as a loan and a firm that finances through borrowing from a bank. The two parties bargain over the amount of numeraire that the bank lends to the firm, k, and the amount of numeraire that the firm needs to repay to the bank in the next second subperiod, $(1+r_\ell)k$, where r_ℓ is the real lending rate. Terms of a loan contract are determined through generalized Nash bargaining between the firm and the bank, and the bank's bargaining power is $\eta \in (0,1)$. The firm's continuation value with a loan contract (k, r_ℓ) is $\beta V^F(k,(1+r_\ell)k)$, and the firm's continuation value without a loan contract is $\beta V^F(0,0)$. Thus, the firm's surplus is $\beta[V^F(k,(1+r_\ell)k) - V^F(0,0)]$, which, using (1.7) and the linearity of W^F, reduces to $\beta[p_L - q_L - r_\ell k]$. Using (1.12) and (1.13), it further reduces to $\beta[(1-\theta)(u(q_L)-q_L)-r_\ell k]$, where $q_L = \min\{v^{-1}(\tilde{m}+\chi\tilde{a}), k\}$ when the firm believes that a consumer will carry $\tilde{m}$ amount of real balances and $\tilde{a}$ amount of corporate bonds to the DM. The bank's continuation value with a loan contract (k, r_ℓ) is $\beta W^B((1+r_\ell)k+w-k)$, and the bank's continuation value without a loan contract is $\beta W^B(w)$. Thus, the bank's surplus is $\beta[W^B((1+r_\ell)k+w-k) - W^B(w)]$, which, using the linearity of W^B, reduces to $\beta r_\ell k$.[11] The terms of a loan contract specify (k, r_ℓ) that solves

$$\max_{k, r_\ell} \left((1-\theta)(u(\min\{v^{-1}(\tilde{m}+\chi\tilde{a}), k\}) - \min\{v^{-1}(\tilde{m}+\chi\tilde{a}), k\}) - r_\ell k\right)^{1-\eta} \left(r_\ell k\right)^{\eta}. \quad (1.14)$$

[11] It is assumed that a bank can use its bank capital (which is in numeraire) that was not lent in the following way. Assume that a bank lent only $k < w$ amount to a firm and has $w - k$ amount of leftover numeraire in hand. It will then go to the intermediate goods market, exchange the leftover nuemraire with intermediate goods, and, in the next period CM, produce numeraire goods using the intermediate goods at unit cost. I assumed that banks have access to this technology. This technology is not used on the equilibrium path, and it is just for simplifying the exposition.

Since the firm will not want to borrow more than it needs to produce the amount of the DM goods that a consumer can afford, $v^{-1}(\tilde{m}+\chi\tilde{a})$, the bargaining problem simplifies to

$$\max_{k \leq v^{-1}(\tilde{m}+\chi\tilde{a}), r_\ell} \left((1-\theta)(u(k)-k) - r_\ell k\right)^{1-\eta} \left(r_\ell k\right)^{\eta}. \tag{1.15}$$

The solution is such that k maximizes the total surplus, $(1-\theta)(u(k)-k)$, subject to $k \leq v^{-1}(\tilde{m}+\chi\tilde{a})$. Since $u(k)-k$ increases in k until $k = q^*$, $k = \min\{v^{-1}(\tilde{m}+\chi\tilde{a}), q^*\}$. Observing that $v^{-1}(\tilde{m}+\chi\tilde{a}) \leq q^*$ holds in equilibrium, the solution is given by

$$k = v^{-1}(\tilde{m}+\chi\tilde{a}), \tag{1.16}$$

$$r_\ell = \frac{\eta(1-\theta)(u(k)-k)}{k}. \tag{1.17}$$

1.3.4 Equilibrium

First start with the optimal behavior of a firm that finances through issuing corporate bonds. From (1.10), at a given price ψ, the firm chooses the amount of corporate bonds to issue, $A \geq 0$, that maximizes $(p_B - q_B) - (1-\psi)A$, which, using (1.12) and (1.13), reduces to $(1-\theta)(u(q_B) - q_B) - (1-\psi)A$, where $q_B = \min\{v^{-1}(\tilde{m}+\chi\tilde{a}), \psi A\}$ when believing that a consumer will carry $\tilde{m}$ amount of real balances and $\tilde{a}$ amount of corporate bonds to the DM. An equilibrium exists when $1-\psi > 0$, or $\psi < 1$, that is, when borrowing through the corporate bond market is costly. Since the firm will not want to bring more capital to the DM than it needs to produce the amount of the DM goods that a consumer can afford,

$v^{-1}(\tilde{m}+\chi\tilde{a})$, the maximization problem becomes

$$\max_{0\leq A\leq v^{-1}(\tilde{m}+\chi\tilde{a})/\psi}\{(1-\theta)(u(\psi A)-\psi A)-(1-\psi)A\}. \tag{1.18}$$

The solution describes the optimal corporate bond issuance decision of the firm, or the supply of corporate bonds, and is given by

$$A=\min\{v^{-1}(\tilde{m}+\chi\tilde{a})/\psi,\bar{A}\} \tag{1.19}$$

where $\bar{A}$ solves

$$\frac{1}{\psi}-1=(1-\theta)(u'(\psi\bar{A})-1). \tag{1.20}$$

The amount of funds raised through issuing corporate bonds when $\bar{A}\leq v^{-1}(\tilde{m}+\chi\tilde{a})/\psi$, $\psi\bar{A}$, is

$$\psi\bar{A}=(u')^{-1}\left(\frac{1/\psi-1}{1-\theta}+1\right), \tag{1.21}$$

which is an increasing function of ψ, the price of the corporate bonds. The higher price makes financing through issuing corporate bonds less expensive and thus allows firms to raise more funds.

Now consider the optimal behavior of a consumer who chooses a portfolio of real balances and corporate bonds. From (1.2), the consumer chooses the amount of real balances, m, and the amount of corporate bonds, a, that maximize $-(1+\pi)m-\psi a+\beta V^C(m,a)$, which, using

(1.4) and the linearity of W^C, becomes

$$\max_{m\geq 0, a\geq 0}\Big\{-(1+\pi)m-\psi a+\beta m+\beta a+\beta(1-\lambda)[u(q_L)-p_L]+\beta\lambda[u(q_B)-p_B]\Big\}. \quad (1.22)$$

Here I restrict attention to the case where the price of the corporate bonds, ψ, is not so high that firms will not be able to bring enough amount of intermediate goods to the DM to meet the consumers' demand.[12] In this case, $q_B = \psi\bar{A} < v^{-1}(m+\chi a)$, and $q_L = v^{-1}(m+\chi a)$ given (1.16). The consumer's maximization problem becomes

$$\max_{m\geq 0, a\geq 0}\Big\{-(1+\pi)m-\psi a+\beta m+\beta a+\beta(1-\lambda)\big[u(v^{-1}(m+\chi a))-v(v^{-1}(m+\chi a))\big]\Big\}, \quad (1.23)$$

The optimal behavior of the consumer is given by

$$1+\pi = \beta\left\{1+(1-\lambda)\left(\frac{u'(v^{-1}(m+\chi a))}{v'(v^{-1}(m+\chi a))}-1\right)\right\}, \quad (1.24)$$

$$\psi = \beta\left\{1+(1-\lambda)\chi\left(\frac{u'(v^{-1}(m+\chi a))}{v'(v^{-1}(m+\chi a))}-1\right)\right\}, \quad (1.25)$$

where the first is the consumer's money demand and the second is the consumer's bond demand. These expressions simplify to

$$i = (1-\lambda)L(m+\chi a), \quad (1.26)$$

$$\psi = \beta(1+\chi i). \quad (1.27)$$

[12] This essentially means that i is assumed to be not too high (Assumption 1.1 given below). Appendix 1.A.1 provides the characterization of the equilibrium outside this parameter space.

where $i \equiv (1+\pi)/\beta - 1$ and $L(\cdot) \equiv u'(v^{-1}(\cdot))/v'(v^{-1}(\cdot)) - 1$ with $L'(\cdot) < 0$.

The following assumption ensures that the price of the corporate bonds is not too high so that financing through corporate bonds is expensive and that firms financing through corporate bonds are not able to satisfy the consumer's demand in the DM.

Assumption 1.1. $i < \bar{\imath} \equiv \dfrac{(1-\lambda)(1-\beta)\theta}{1-\theta+(1-\lambda)\beta\theta\chi}$.

Notice that this assumption implies that $i < (1-\beta)/(\beta\chi)$ so that $\psi < 1$ and thus also guarantees a well-defined bond supply function.

The equilibrium is defined as below.

Definition 1.1. *A steady state equilibrium of the economy corresponds to a constant sequence $(q_L, q_B, m, a, A, \psi, k, r_\ell)$, where q_L is the DM goods traded between a consumer and a firm that finances through borrowing from a bank, q_B is the DM goods traded between a consumer and a firm that finances through issuing corporate bonds, m is the consumer's real balance holdings, a is the consumer's corporate bond holdings, A is the supply of corporate bonds issued by firms, ψ is the price of corporate bonds, k is the size of a loan that a bank lends to a firm, and r_ℓ is the real lending rate of loans. Under Assumption 1.1, (q_L, q_B) satisfy*

$$q_L = v^{-1}\left(L^{-1}\left(\frac{i}{1-\lambda}\right)\right), \tag{1.28}$$

$$q_B = (u')^{-1}\left(\frac{1-\beta(1+\chi i)}{\beta(1+\chi i)(1-\theta)} + 1\right), \tag{1.29}$$

(m, a, A, ψ) *satisfy*

$$\psi = \beta(1 + \chi i), \tag{1.30}$$

$$A = q_B/\psi, \tag{1.31}$$

$$a = \lambda A, \tag{1.32}$$

$$m = v(q_L) - \chi a, \tag{1.33}$$

and (k, r_ℓ) *satisfy*

$$k = q_L = v^{-1}(m + \chi a), \tag{1.34}$$

$$r_\ell = \frac{\eta(1-\theta)(u(k) - k)}{k}, \tag{1.35}$$

where

$$v(\cdot) = (1-\theta)u(\cdot) + \theta\,\cdot\,, \quad v'(\cdot) > 0, \tag{1.36}$$

$$L(\cdot) = u'(v^{-1}(\cdot))/v'(v^{-1}(\cdot)) - 1, \quad L'(\cdot) < 0. \tag{1.37}$$

1.3.5 Transmission Mechanism of Monetary Policy to Credit Costs

This section focuses on how monetary policy influences the cost of financing that in turn affects economic activity. First start with the price of the corporate bonds. From (1.30), $\partial\psi/\partial i > 0$. The nominal policy rate i affects the price of the corporate bonds through the cost of holding money (which equals i itself) and the liquidity premium of the corporate bonds (which equals $LP \equiv \chi i$ in (1.30)). As i increases, the rate of return on money decreases, and the real balances decrease. Due to less prevalent liquidity in the economy, the role of the corporate bonds as a medium of exchange increases, which in turn leads to an increase in the liquidity premium of the corporate bonds.

I define the excess bond premium following Gilchrist and Zakrajšek (2012), who compute the excess bond premium of a corporate bond as a difference between the yield of the corporate bond and the yield calculated using a price that equals the net present value of the cash flows, or the fundamental value, of the corporate bond.[13] For the one-period real corporate bond, the net present value of its cash flows is β, and the corresponding nominal yield is $i \equiv (1+\pi)/\beta - 1$, while the nominal yield of the corporate bond is $(1+\pi)/\psi - 1$. Hence, the excess bond premium (EBP) is given by

$$EBP = \left(\frac{1+\pi}{\psi} - 1\right) - i, \tag{1.38}$$

and

$$\frac{\partial EBP}{\partial i} = -\frac{\chi(1+i+i(1+\chi i))}{(1+\chi i)^2} < 0. \tag{1.39}$$

The negative impact of the nominal policy rate on the excess bond premium in this model comes through the effect of the nominal policy rate on the liquidity premium: the higher liquidity premium implies the smaller excess bond premium. I label this mechanism the

[13] Gilchrist and Zakrajšek (2012) define this difference as the credit spread, and define the excess bond premium as the credit spread after removing the component due to default risk. Since the corporate bonds in this model do not default, the credit spread equals the excess bond premium.

liquidity premium channel of monetary policy transmission. Furthermore,

$$\frac{\partial \left| \frac{\partial EBP}{\partial i} \right|}{\partial \chi} = \frac{1 + i(2 - \chi)}{(1 + \chi i)^3} > 0, \tag{1.40}$$

which means that the more liquid the corporate bond market, or the higher the degree of corporate bond liquidity, the stronger the liquidity premium channel.

Next consider how the nominal policy rate passes through to the real lending rate for loans. $\partial r_\ell / \partial k < 0$ from (1.35), $\partial k / \partial (m + \chi a) > 0$ from (1.34) and (1.36), and $\partial (m + \chi a)/\partial i < 0$ from (1.26) and (1.37). These together imply $\partial r_\ell / \partial i > 0$. From (1.16), with an increase in the nominal policy rate, the cost of holding money increases, agents carry less liquidity, and firms borrow less from a bank since agents can afford less. As a result, as can be seen from (1.17), the real loan rate increases because it depends positively on the marginal benefit of a loan, and the latter decreases in the loan size. The following proposition summarizes the discussion.

Proposition 1.1. *As the nominal policy rate increases, the liquidity premium of corporate bonds increases, the excess bond premium decreases, and the effect of the nominal policy on the excess bond premium becomes stronger as the corporate bond secondary market becomes more liquid. In addition, a higher nominal policy rate implies a higher price of corporate bonds and a higher real lending rate for loans:*

$$\frac{\partial LP}{\partial i} > 0, \quad \frac{\partial EBP}{\partial i} < 0, \quad \frac{\partial |\partial EBP / \partial i|}{\partial \chi} > 0, \quad \frac{\partial r_\ell}{\partial i} > 0. \tag{1.41}$$

1.3.6 Nominal Policy Rate

In the next section, I turn to the data and provide empirical evidence that supports the monetary policy transmission mechanism of the model summarized in Proposition 1.1. In the analysis of the model, as a nominal policy rate, I have used i, a nominal interest rate on a perfectly illiquid bond. However, in the empirical analysis, following the literature, I am going to use the Treasury rate as the nominal policy rate. The Treasuries are obviously considered highly liquid and thus their rate is a different object than i. In this subsection, before turning to the empirical analysis, I connect i and the Treasury rate, a nominal interest rate on a liquid government bond. To do so, assume that there are government bonds supplied at a fixed amount. Denote the nominal interest rate on the government bonds by i_g. Also, assume that those government bonds are partially liquid, and only a fraction $\chi_g \in (0, 1]$ can be used for liquidity purposes. In this case, in equilibrium, the nominal interest rate on the government bonds is given by

$$i_g = \frac{(1-\chi_g)i}{1+\chi_g i}, \quad \frac{\partial i_g}{\partial i} > 0. \tag{1.42}$$

That is, there is a one-to-one positive relationship between i and i_g. Hence, in the following empirical exercise, I adopt the Treasury rate as the policy rate.[14]

[14]Although theoretically it is possible to generate a negative relationship between i and i_g when one microfounds the asset secondary market in a rigorous way (see Geromichalos and Herrenbrueck (2017) for instance), Herrenbrueck (2019b) empirically shows that, except for Volcker's disinflation period in his first term (1981–1982), the estimated i and the nominal interest rate on the public debt are positively correlated. This gives me another justification for using the Treasury rate as the policy rate for the empirical analysis, given that the sample period of the dataset that I use starts from 1990.

1.4 Empirical Analysis

1.4.1 Structural Change in the Corporate Bond Market

1.4.1.1 Introduction of TRACE and Its Impact on Market Liquidity

Corporate bonds are traded between agents in the secondary market, which is a dealer-oriented over-the-counter (OTC) market. The trading environment of the U.S. corporate bond secondary market used to be highly opaque for decades. Transaction-related information, such as prices and volumes at which corporate bonds were traded, was available only to the parties involved in the transactions. This caused an asymmetric information problem between dealers and traders, and dealers extracted rents from less-informed customers. These rent-seeking behaviors of dealers incurred traders a huge amount of trading costs and made the market illiquid.[15]

However, the scene changed dramatically when the Transaction Reporting and Compliance Engine (TRACE) was introduced to the U.S. corporate bond market, and many of the issues that were hindering the market from being liquid were resolved. With the approval of the Securities and Exchange Commission (SEC), beginning on July 1, 2002, the National Association of Security Dealers (NASD) (which is currently the Financial Industry Regulatory Authority (FINRA)) started to require dealers to report transaction-related information on all over-the-counter trades for publicly issued corporate bonds, such as the identification of traded bonds, the date and the time of execution, trade size, trade price, yields, and whether the dealers bought or sold in the transaction. The TRACE is the platform that the NASD

[15] Biais and Green (2019) provide detailed discussion on how the opaque transaction environment deteriorated the corporate bond secondary market in terms of trading costs and the market liquidity.

developed to facilitate this mandatory reporting.

The amount of the information made public and the timeliness of reporting under the new system were phased in over time from July 1, 2002 to January 9, 2006 based on the size and the credit rating of the bonds. On July 1, 2002, trades in investment-grade corporate bonds with an issuance size of $1 billion or greater, as well as 50 representative non-investment-grade bonds, began to be disseminated to the public. During 2003, trades in 120 selected BBB-rated bonds (on April 14, 2003) and higher-rated bonds (on March 3, 2003) with initial issue sizes over $100 million began to be disseminated to the public. On February 7, 2005, data began to be disseminated for all but newly issued or lightly traded bonds. By January 9, 2006, trades in all publicly issued bonds were disseminated to the public. In addition, the timeliness with which dealers were required to report trades was tightened in stages. Upon the introduction of TRACE, dealers had 75 minutes to report trades. This was reduced on October 1, 2003, to a reporting time of 45 minutes, and on October 1, 2004, to 30 minutes. Since July 1, 2005, dealers have been required to report trades within 15 minutes. Since January 9, 2006, reports have had to be made immediately.[16]

Empirical evidence shows that the post-trade transparency due to the introduction of the TRACE reduced dealers' information advantage relative to traders, led to a significant drop in trading costs, and substantially improved the market liquidity. For example, three papers, Bessembinder, Maxwell, and Venkataraman (2006), Edwards, Harris, and Piwowar (2007), and Goldstein, Hotchkiss, and Sirri (2007) examine how the market liquidity, measured in bid-ask spreads, changed around the period when the TRACE was implemented, and find

[16] For more details on the history of the implementation of the TRACE, see Bessembinder and Maxwell (2008) and Asquith, Covert, and Pathak (2013).

that the secondary market liquidity increased by 50–84% with the mandatory transaction reporting system. The empirical literature documents this substantial improvement in the market liquidity with the introduction of the TRACE as a structural change in the U.S. corporate bond market.

1.4.1.2 Hypotheses

The increased liquidity of the corporate bond secondary market with the introduction of the TRACE can be interpreted as an increase in χ in the model, the fraction of corporate bond holdings that can be used towards trades for liquidity purposes, which essentially is capturing the degree of the secondary market liquidity. This in turn implies that, as Proposition 1.1 states, the liquidity premium channel of monetary policy transmission must be stronger in the period after the TRACE was implemented.

My model was focusing mainly on how monetary policy affects the corporate bond premium through the liquidity premium of the corporate bonds, and, through this channel, a higher nominal policy rate decreases the corporate bond premium, as in Proposition 1.1. However, there is other channel as well through which monetary policy can influence the corporate bond premium. According to the literature on the credit channel of monetary policy transmission, when financial market imperfections are present, a higher nominal policy rate increases the corporate bond premium by tightening credit constraints and subsequently affecting firms' ability to borrow.[17] This means that the end effect of monetary policy on the corporate bond premium depends on the relative strength of the (negative) liquidity premium channel and the (positive) credit channel.

[17]See for instance Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999).

Gertler and Karadi (2015) find that an increase in the nominal policy rate increases the corporate bond premium using the data with the sample period 1979–2012. This suggests that, during that overall period, the credit channel is stronger than the liquidity premium channel. Noting that the liquidity premium channel must be stronger during the post-TRACE period, I make the following hypotheses. When the effect of an increase in the nominal policy rate is positive in the overall period, the magnitude of the effect should be larger during the pre-TRACE period when the negative liquidity premium channel barely exists. On the other hand, the magnitude of the effect should be smaller during the post-TRACE period when the negative liquidity premium channel is active. Or, the effect could potentially be overturned and become negative if the liquidity premium channel is strong enough.

In the empirical analysis in the following sections, I test the hypotheses and show that this is the case. In Section 1.4.2, I compare the effect of monetary policy on the corporate bond premium across the two periods before and after the introduction of the TRACE. In doing so, the liquidity premium channel is identified by the difference in the effects across two periods. In Section 1.4.3, I measure the liquidity premium channel using more direct liquidity measures, such as bid-ask spreads and trading volume. In Section 1.4.4, I examine how bank loan rates respond to monetary policy changes. These altogether provide empirical support to Proposition 1.1.

1.4.2 Corporate Bond Premium and Monetary Policy Shocks

1.4.2.1 Empirical Framework

For the corporate bond premium, I use the excess bond premium measured by Gilchrist and Zakrajšek (2012), which is an extracted component of credit spreads that is not directly attributable to the expected default risk. I estimate the dynamic response of the excess bond premium to a monetary policy shock. I use the structural vector autoregression with external instruments (SVAR-IV) that was introduced by Stock (2008) and Mertens and Ravn (2013), and apply it to monetary policy, following Gertler and Karadi (2015). The SVAR-IV includes four variables: the 1-year Treasury constant maturity rate (as the policy rate), industrial production (100 times log of it), the consumer price index (100 times log of it), and the excess bond premium. To ensure the results are not driven by other macro factors and to limit any potential reverse causality issues, as exogenous variations in the policy rates, I use three-month-ahead financial market surprises from Federal Funds futures in a 30-minute window around the Federal Open Market Committee policy announcements, constructed by Gertler and Karadi (2015). In addition, I use the local projection instrumental variable (LP-IV) approach, following Jordà (2005), Jordà, Schularick, and Taylor (2020). The baseline specification for horizon h is $y_{t+h} - y_{t-1} = \alpha^h + \beta^h r_t + u_{t+h}$, where y are the main variables, r is the policy rate that is instrumented, and β^h refers to the impulse response at horizon h. The same four variables and instruments are used for estimation.

The sample period spans 1990:2–2016:12 with monthly frequency. For both SVAR-IV and LP-IV specifications, 12-month lags of the four main variables and 4-month lags of the instrument are used as control variables, following Gertler and Karadi (2015). I do the unit effect normalization following Stock and Watson (2018) for direct estimation of the dynamic causal effect in the native units relevant to policy analysis. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap for SVAR-IV following Stock and Watson (2018), and using the Newey-West heteroskedastic and autocorrelation consistent standard errors for LP-IV. For each point estimate along the horizons, the 95% confidence interval is given.

To examine potentially different effects of monetary policy shocks on the excess bond premium before and after the introduction of the TRACE, I divide the sample period into two: 1990:2–2003:2 for the pre-TRACE period and 2003:3–2016:12 for the post-TRACE period. Considering the fact that required reporting of corporate bond transactions to the public was phased in over the period 2002:7–2006:1, I choose the midpoint 2003:3 as a benchmark when the mandatory reporting was imposed on a significant portion of corporate bonds. I check that the results are robust to alternative breakpoints.

1.4.2.2 Results

The response of the excess bond premium to a one-percent increase in the nominal policy rate estimated using SVAR-IV is given in Figure 1.2, and the response estimated using LP-IV is given in Figure 1.3. In both Figures 1.2 and 1.3, the left panel is for the entire period, the middle panel is for the pre-TRACE period, and the right panel is for the post-TRACE period. To ensure that the instrument is valid, I check the heteroscedasticity-robust F-statistic from the first-stage regression, and all are safely above the threshold suggested by Stock, Wright, and Yogo (2002) to rule out a reasonable likelihood of a weak instruments problem.

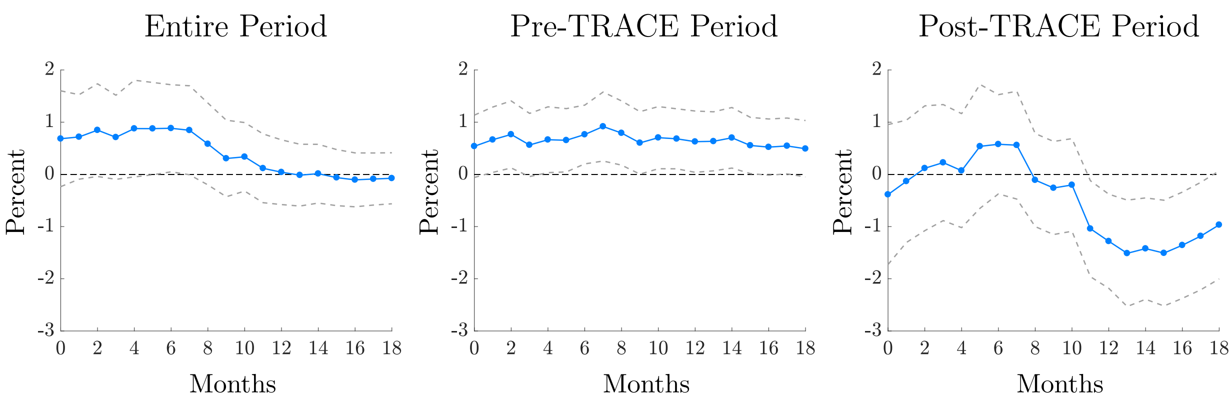

Figure 1.2. Response of the excess bond premium to a one-percent increase in the nominal policy rate for different sample periods, estimated using SVAR-IV with unit effect normalization. Sample period: 1990:2–2016:12; pre-TRACE period: 1990:2–2003:2; post-TRACE period: 2003:3–2016:12. Dashed lines are the 95% confidence interval.

Monetary policy shock considered is a one-percent increase in the 1-year Treasury constant maturity rate, which I consider as the policy rate. When estimated using the entire sample, consistent with the results of Gertler and Karadi (2015), the excess bond premium increases following a one-percent increase in the nominal policy rate. However, the response of the excess bond premium is different when we look at the two different periods: the pre- and the post-TRACE periods. For the pre-TRACE period, the positive response of the excess bond premium to a one-percent increase in the nominal policy rate appears more persistent and significant. On the other hand, for the post-TRACE period, the response of the excess bond premium is not just less strong but it becomes negative. These results are consistent with the hypotheses. The whole period covers both the pre-TRACE period where the negative liquidity premium channel is less effective and the post-TRACE period where the negative liquidity premium channel is more effective. It turns out that during the post-TRACE period the negative liquidity premium channel is strong enough to dominate the positive credit channel.

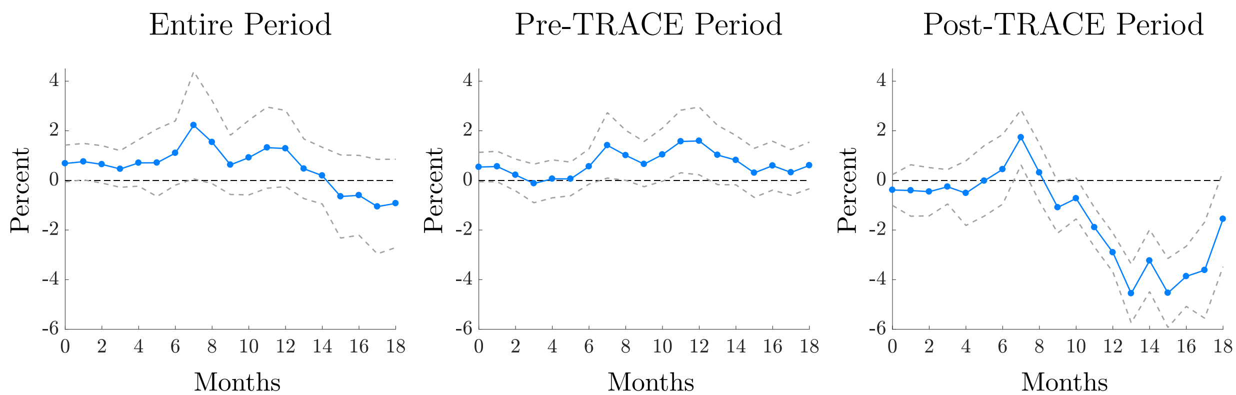

Figure 1.3. Response of the excess bond premium to a one-percent increase in the nominal policy rate for different sample periods, estimated using LP-IV with unit effect normalization. Sample period: 1990:2–2016:12; pre-TRACE period: 1990:2–2003:2; post-TRACE period: 2003:3–2016:12. Dashed lines are the 95% confidence interval.

1.4.2.3 Sensitivity Analysis

This section checks the robustness of the decline in the excess bond premium following an increase in the nominal policy rate during the post-TRACE period.

Factor-augmented LP-IV. As Stock and Watson (2018) point out, if there are more than four shocks that affect the four variables, or if some elements of the four variables are measured with error (such as industrial production, the consumer price index, or the inflation rate), including additional variables that are correlated with the shocks could increase the precision of the estimation. As suggested by Stock and Watson (2018), I add lags of principal components, or factors, computed from the FRED-MD database by McCracken and Ng (2016) to the LP-IV setting. The response of the excess bond premium to a one-percent increase in the nominal policy rate estimated using factor-augmented LP-IV is given in Figure 1.B.1. The additional controls yield results that are consistent with (and stronger than) the results estimated using LP-IV.

Test for a structural break. To test a structural break induced by the introduction of the TRACE, I interact all the regressors in LP-IV and factor-augmented LP-IV with the post-TRACE year dummy. Figure 1.B.2 shows the base and the post-TRACE responses of the excess bond premium to a one-percent increase in the nominal policy rate estimated using LP-IV, and Figure 1.B.3 estimates the responses using factor-augmented LP-IV. As can be seen, an increase in the nominal policy rate has a negative impact for the post-TRACE period. Although the negative impact is not significant in the early horizons, the null hypothesis of no structural break is rejected for all horizons with p-value 0 for both LP-IV and factor-augmented LP-IV.

Zero lower bound. The sample period, especially the post-TRACE period, includes the Great Recession, and, during that period, the short-term interest rate reached the zero lower bound. However, Swanson and Williams (2014) argue that the zero lower bound was not a constraint on the Federal Reserve's ability to manipulate the 2-year rate, which might have been probably less true for the 1-year rate. To address the concern about the zero lower bound, I show the results are robust to using the 2-year Treasury constant maturity rate, instead of the 1-year rate, although, as Gertler and Karadi (2015) point out, the 2-year rate is less relevant with the instrument in the first-stage regression compared to the 1-year rate and thus suffers the weak instruments problem. Figure 1.B.4, 1.B.5 and 1.B.6 show the response of the excess bond premium to a one-percent increase in the nominal policy rate for the entire period, the pre-TRACE period, and the post-TRACE period, respectively using LP-IV, factor-augmented LP-IV, and SVAR-IV, using the 2-year rate. All the results are consistent with those using the 1-year rate.

Estimates during the shorter period around the introduction of the TRACE. Another concern over the fact that the sample period, especially the post-TRACE period, includes the Great Recession is that it was a very different time in terms of monetary policy, for example, in that the central bank used unconventional credit market interventions such as a series of quantitative easing to affect market interest rates. This therefore implies that the pre- and the post-TRACE periods are different not just because of the introduction of the TRACE but because of all that was happening during and after the crisis. To address this concern and to make the pre- and the post-TRACE periods as similar as possible except for the existence of the TRACE, I narrow the sample period to a shorter window around the introduction of the TRACE to exclude the 2008:7–2009:6 crisis period. I consider 1997:11–2003:2 as the pre-TRACE period and 2003:3–2008:6 as the post-TRACE period. Due to the singularity problem with the long lag length, I decrease the lag length of the four main variables to 4 months and that of the instrument to 2 months. Figure 1.B.7 shows the response of the excess bond premium to a one-percent increase in the nominal policy rate for the entire period, the pre-TRACE period, and the post-TRACE period using LP-IV during the shorter sample period. Although the small sample size generates large standard errors and using the short lag length is subject to a weak instruments problem, the results, especially the one for the post-TRACE period, suggest the decline in the excess bond premium following a one-percent increase in the nominal policy rate when the negative liquidity premium channel is active and strong. Figure 1.B.8 and 1.B.9 perform the same exercise using SVAR-IV. Even with the shorter sampler period that does not include the recent crisis, the responses of the excess bond premium to a one-percent increase in the nominal policy rate during the pre- and the post-TRACE periods are extremely contrasting and significant, with the former during the pre-TRACE period being the exact same as in Gertler and Karadi (2015) and the latter during the post-TRACE period being a total opposite.

Different lag lengths. I check the robustness of the results with different lag lengths. When the lag length of the main variables is shorter than 9 months, the first-stage regression suffers a weak instruments problem with both the F-statistic the robust F-statistic being less than the threshold suggested by Stock, Wright, and Yogo (2002). For a lag length longer than or equal to 9 months (I checked up to 24 months), the results remain consistent.

Alternative breakpoints. The obvious alternative breakpoint is 2002:7 when the mandatory reporting was first executed. All the results discussed remain the same. Using other breakpoints such as 2003:4 (when the mandatory reporting was applied to additional 120 selected BBB-rated bonds), 2005:2 (when the mandatory reporting was applied to all but newly issued or lightly traded bonds) or 2006:1 (when transaction information for all publicly issued bonds started to be made public) also does not change the results at all.

1.4.3 Liquidity Premium and Monetary Policy Shocks

In the previous section, the liquidity premium channel of monetary policy transmission is identified indirectly by the difference of the effects of monetary policy on the corporate bond premium across the pre- and the post-TRACE periods. In this section, I measure the liquidity premium channel using direct liquidity measures such as bid-ask spreads and trading volume.

1.4.3.1 Empirical Framework

I add a liquidity measure of corporate bonds to the SVAR-IV and the LP-IV setups described in the previous section. The two most common liquidity measures are bid-ask spreads and trading volume. The measures are based on corporate bond transaction data from the TRACE database. I follow Adrian, Fleming, Shachar, and Vogt (2017) in calculating the measures. The bid-ask spreads compute average daily bid-ask spreads by month across bonds. First, spreads are calculated daily for each bond as the difference between the average (volume-weighted) dealer-to-client buy price (the price at which dealers are willing to buy, or bid) and the average (volume-weighted) dealer-to-client sell price (the price at which dealers are willing to sell, or ask). Then, the spreads are averaged across bonds using equal weighting and across days for each month. The trading volume computes the average daily trading volume by month across bonds. Both liquidity measures enter the specification as 100 times log of them. Because to compute the liquidity measures I use the TRACE database that exists only after the TRACE was introduced, naturally this section focuses solely on the post-TRACE period.

1.4.3.2 Results

The more liquid corporate bonds, the narrower the bid-ask spreads, and the larger the trading volume. In other words, liquidity and the bid-ask spreads are negatively correlated, while liquidity and the trading volume are positively correlated. For all results, I check the heteroscedasticity-robust F-statistic from the first-stage regression to ensure that the results are not subject to a weak instruments problem, and all are safely above the threshold

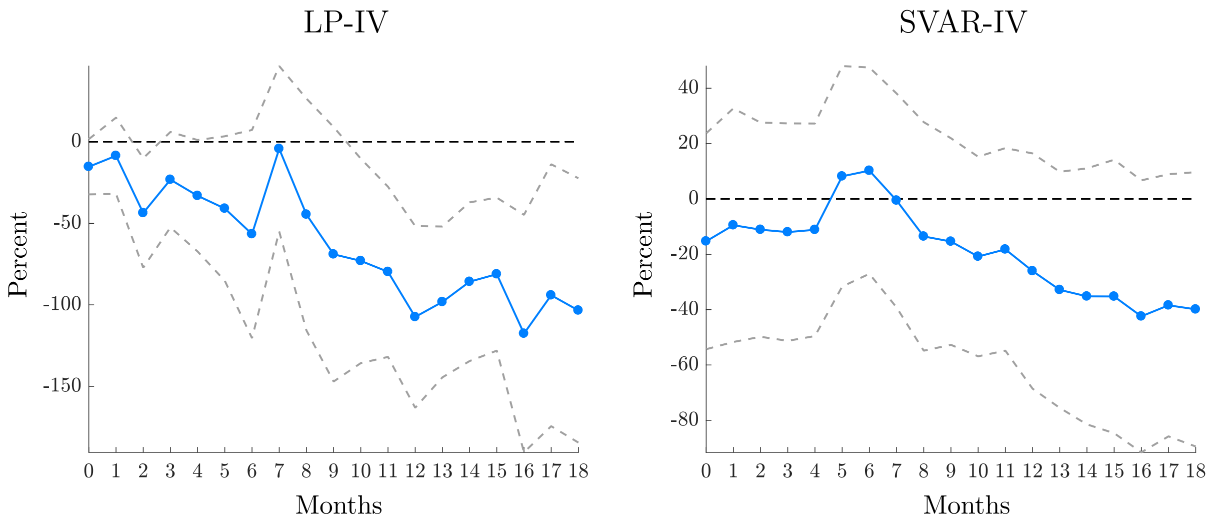

Figure 1.4. Response of the bid-ask spreads of corporate bonds to a one-percent increase in the nominal policy rate during the post-TRACE period, estimated using LP-IV and SVAR-IV with unit effect normalization. Sample period: 2003:3–2016:12. Dashed lines are the 95% confidence interval.

suggested by Stock, Wright, and Yogo (2002). The monetary policy shock considered is a one-percent increase in the 1-year Treasury constant maturity rate. Figure 1.4 estimates the response of the bid-ask spreads to a one-percent increase in the nominal policy rate using SVAR-IV and LP-IV. In both panels, the bid-ask spreads decrease following a one-percent increase in the nominal policy rate. Figure 1.5 estimates the response of the trading volume to a one-percent increase in the nominal policy rate using SVAR-IV and LP-IV. In both panels, the trading volume increases following a one-percent increase in the nominal policy rate. For both the bid-ask spreads and the trading volume, the responses are not significant for SVAR-IV, but the responses are highly significant for LP-IV. The results support the negative liquidity premium channel of monetary policy transmission and are consistent with the theory that suggests that a one-percent increase in the nominal policy rate increases the liquidity premium of corporate bonds.

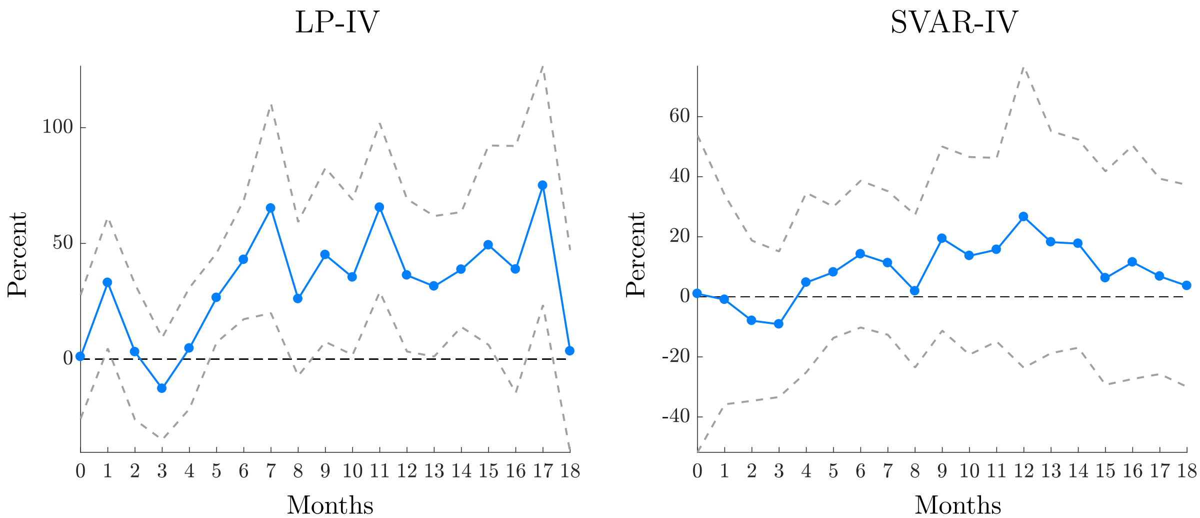

Figure 1.5. Response of the trading volume of corporate bonds to a one-percent increase in the nominal policy rate during the post-TRACE period, estimated using LP-IV and SVAR-IV with unit effect normalization. Sample period: 2003:3–2016:12. Dashed lines are the 95% confidence interval.

1.4.3.3 Sensitivity Analysis

This section checks the robustness of the positive response of the liquidity premium to a one-percent increase in the nominal policy rate, following the checklists from Section 1.4.2.3. I augment the LP-IV setting with the macroeconomic factors from the FRED-MD database by McCracken and Ng (2016). The responses of the liquidity measures to a one-percent increase in the nominal policy rate estimated by factor-augmented LP-IV are given in Figure 1.B.10. While the response of the trading volume is less clear, the response of the bid-ask spreads is consistent with (and stronger than) the results estimated using LP-IV. To address the concern over the 1-year rate hitting the zero lower bound during the Great Recession, I estimate the response of both the liquidity measures to a one-percent increase in the nominal policy rate using LP-IV and SVAR-IV with the 2-year Treasury constant maturity rate instead of the 1-year rate. Figure 1.B.11 and 1.B.12 show the response of the bid-ask spreads and the trading volume, respectively. Although, as Gertler and Karadi (2015) point out, the 2-year rate is less relevant with the instrument in the first-stage regression compared to the 1-year rate and thus suffers the weak instruments problem, all the results are consistent with those using the 1-year rates. The results also remain the same when using different lag lengths for the lag length that does not suffer a weak instruments problem (longer than or equal to 10 months for the bid-ask spreads and 7 months for the trading volume). Using alternative breakpoints does not change the results either.

1.4.4 Bank Loan Rates and Monetary Policy Shocks

This section provides the empirical evidence that an increase in the nominal policy rate raises real bank loan rates, as opposed to the case of corporate bonds.

1.4.4.1 Empirical Framework

I add the business loan rate to the SVAR-IV and the LP-IV setups described in Section 1.4.2. In particular, the real rate is of my interest and I calculate it as the nominal loan rate minus the expected inflation rate. The nominal loan rate is the bank business prime loan rate, and the expected inflation rate is the 5-year forward inflation expectation rate from the Federal Reserve Economic Data (FRED) from the Federal Reserve Bank of St. Louis. The expected inflation rate series exists from 2003, so the sample period considered in this section is the post-TRACE period.

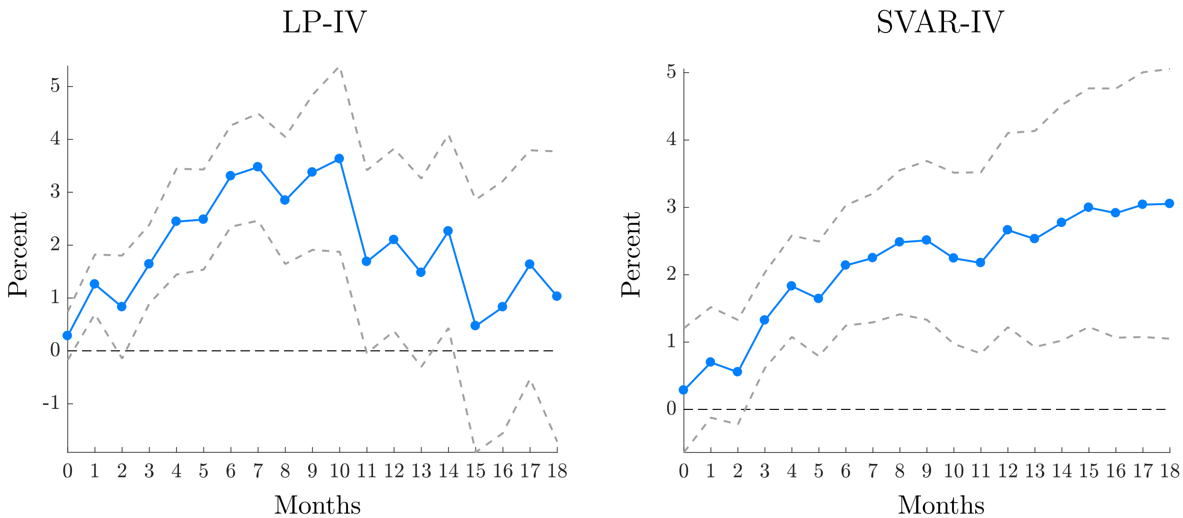

Figure 1.6. Response of the real loan rate to a one-percent increase in the nominal policy rate during the post-TRACE period, estimated using LP-IV and SVAR-IV with unit effect normalization. Sample period: 2003:3–2016:12. Dashed lines are the 95% confidence interval.

1.4.4.2 Results

The heteroscedasticity-robust F-statistics from the first-stage regressions ensure that the results are not subject to a weak instrument problem, and all are safely above the threshold suggested by Stock, Wright, and Yogo (2002). The monetary policy shock considered is a one-percent increase in the 1-year Treasury constant maturity rate. Figure 1.6 estimates the response of the real loan rate to a one-percent increase in the nominal policy rate using SVAR-IV and LP-IV. In both panels, following a one-percent increase in the nominal policy rate, the real bank loan rate increases, and the estimates are significant.

1.4.4.3 Sensitivity Analysis

This section checks the robustness of the positive response of the real bank loan rate to a one-percent increase in the nominal policy rate, following the checklists from Section 1.4.2.3. I augment the LP-IV setting with the macroeconomic factors from the FRED-MD database by McCracken and Ng (2016). The response of the real bank loan rate to a one-percent increase in the nominal policy rate estimated by factor-augmented LP-IV is given in Figure 1.B.13. The results are consistent with those using LP-IV. To address the concern over the 1-year rate hitting the zero lower bound during the Great Recession, I estimate the response of the real bank loan rate to a one-percent increase in the nominal policy rate using LP-IV and SVAR-IV with the 2-year Treasury constant maturity rate instead of the 1-year rate. Although, as Gertler and Karadi (2015) point out, the 2-year rate is less relevant with the instrument in the first-stage regression compared to the 1-year rate and thus suffers the weak instruments problem, the results are in Figure 1.B.14 and all are consistent with those using the 1-year rates. The results also remain the same when using different lag lengths for the lag length that does not suffer a weak instruments problem (longer than or equal to 6 months). Using alternative breakpoints does not change the results either.

1.5 Effect of Monetary Policy on Credit Composition

In this section, I examine how monetary policy changes induce a shift in the composition of credit between corporate bonds and loans at the firm level. While previously the measure $\lambda \in (0, 1)$ of firms were assumed to finance solely through issuing corporate bonds, now those firms that have access to the corporate bond market can also try to obtain a loan from a bank. In addition, a firm meets, or can find, a bank that is willing to give a loan with probability $\alpha \in (0, 1)$, as in Rocheteau, Wright, and Zhang (2018). α can be thought of as a loan application acceptance rate. This means that, among the measure λ of the firms that have access to the corporate bond market, $\alpha\lambda$ will be able to finance through both issuing corporate bonds and obtaining a bank loan. In such case, I assume firms will first decide how many corporate bonds to issue and then go to the OTC market for bank loans.[18] The other $(1-\alpha)\lambda$ will not be able to find a bank that is willing to give a loan and thus will have to finance investment only through issuing corporate bonds. $(1-\alpha)(1-\lambda)$ among the measure $1-\lambda$ of the firms that do not have access to the corporate bond market will not be able to borrow from a bank and thus cannot produce any in the first subperiod. To simplify the presentation, I normalize the measure of consumers to $\alpha+(1-\alpha)\lambda$ so that all the consumers match with a firm in bilateral meetings in the DM.

1.5.1 Value Functions

The value functions of suppliers remain the same as before. A bank in the second subperiod randomly matches with a firm, and there are two types of firms: one that has access to the corporate bond market and the other that does not. Denote the terms of a loan contract between a bank and a firm that cannot issue corporate bonds by (k^L, r_ℓ^L) and the terms of a loan contract between a bank and a firm that can issue corporate bonds by (k^B, r_ℓ^B). The value function of a bank that is willing to give a loan to a firm that cannot issue corporate bonds is

$$W^B(w) = \max_c c + \beta W^B((1+r_\ell^L)k^L) \quad \text{s.t.} \quad c + k^L = w, \tag{1.43}$$

[18]The timing of events is important in getting the desired result that firms use both ways of financing when they have access to both the corporate bond and the bank loan markets. If it is assumed that firms first go to the OTC market for bank loans and then turn to the corporate bond market, then they will not issue any corporate bonds. See Appendix 1.A.2 for details.

and the value function of a bank that is willing to give a loan to a firm that can issue corporate bonds is

$$W^B(w) = \max_c c + \beta W^B((1+r_\ell^B)k^B) \quad \text{s.t.} \quad c + k^B = w. \tag{1.44}$$

The value function of a firm that does not have access to the corporate bond market and thus has to borrow from a bank to finance investment remains the same as before, as in (1.7) and (1.8), but now the terms of a loan contract are denoted by (k^L, r_ℓ^L). The value function of a firm that has access to the corporate bond but could not borrow from a bank is the same as that of a firm that finances investment solely by issuing corporate bonds, as in (1.9) and (1.10).

Consider a firm that has access to the corporate bond market and also finds a bank that is willing to give a loan. The terms of a loan contract are denoted by (k^B, r_ℓ^B). The value function in the first subperiod of the firm that issued $\hat{A}$ amount of corporate bonds at price ψ and that obtained k^B amount of a loan from a bank at a real lending rate r_ℓ^B in the previous second subperiod is

$$V^F(\psi\hat{A} + k^B, \hat{A} + (1+r_\ell^B)k^B) = W^F(\psi\hat{A} + k^B - q, p, \hat{A} + (1+r_\ell^B)k^B), \tag{1.45}$$

where (p, q) are the terms of trade in the following DM. The value function of a firm in the second subperiod after trading in the DM is

$$
\begin{aligned}
W^F(\psi\hat{A} + k^B - q, p, \hat{A} + (1 + r_\ell^B)k^B) &= \max_c \; c \\
\text{s.t.} \quad c = \psi\hat{A} + k^B - q + p - \hat{A} - (1 + r_\ell^B)k^B&,
\end{aligned}
\tag{1.46}
$$

which simply reduces to $W^F(\psi\hat{A}+k^B-q, p, \hat{A}+(1+r_\ell^B)k^B) = \psi\hat{A}+k^B-q+p-\hat{A}-(1+r_\ell^B)k^B$. Using the linearity of W^F, a newborn firm in the second subperiod with a loan contract (k^B, r_ℓ^B) decides the amount of corporate bonds to issue by solving

$$
\max_{\hat{A}\geq 0} \beta V^F(\psi\hat{A} + k^B, \hat{A} + (1 + r_\ell^B)k^B) = \max_{\hat{A}\geq 0} \beta\{(p - q) - (1 - \psi)\hat{A} - r_\ell^B k^B\}. \tag{1.47}
$$

A consumer in the DM matches with a firm that does not have access to the corporate bond market but was able to borrow from a bank with probability $\alpha(1-\lambda)/(\alpha+(1-\alpha)\lambda)$, a firm that has access to the corporate bond market and also was able to borrow from a bank with probability $\alpha\lambda/(\alpha + (1 - \alpha)\lambda)$, and a firm that has access to the corporate bond market but was not able to borrow from a bank with probability $(1 - \alpha)\lambda/(\alpha + (1 - \alpha)\lambda)$. The value function of a consumer who brings $\hat{m}$ amount of real balances and $\hat{a}$ amount of corporate bonds to the DM is

$$
\begin{aligned}
V^C(\hat{m}, \hat{a}) = &\frac{\alpha(1-\lambda)}{\alpha + (1-\alpha)\lambda}\left[u(q_L) - p_L\right] + \frac{\alpha\lambda}{\alpha + (1-\alpha)\lambda}\left[u(q) - p\right] \\
&+ \frac{(1-\alpha)\lambda}{\alpha + (1-\alpha)\lambda}\left[u(q_B) - p_B\right] + W^C(\hat{m} + \hat{a}).
\end{aligned}
\tag{1.48}
$$

1.5.2 Loan Contract

Now there are two types of meetings in the OTC market for loans in the second subperiod: one between a bank and a firm that does not have access to the corporate bond market, and the other between a bank and a firm that has access to the corporate bond market and thus has issued corporate bonds before entering the OTC market for loans. The bargaining problem in the former meeting is the same as in the previous environment, and the solution is given by (1.16) and (1.17).

In the latter meeting, a bank and a firm bargain over the terms of a loan contract, (k^B, r_ℓ^B). Consider a meeting between a bank and a firm that has already raised ψA amount of funds by issuing A amount of corporate bonds at price ψ. I restrict attention as in Section 1.3.4 under Assumption 1.1 to the case where the price of the corporate bonds is not too high so that financing through corporate bonds is expensive and that firms financing through corporate bonds are not able to satisfy the consumer's demand in the DM. The firm's continuation value with a loan contract (k^B, r_ℓ^B) is $\beta V^F(\psi A + k^B, A + (1 + r_\ell^B)k^B)$, and the firm's continuation value without a loan contract is $\beta V^F(\psi A, A)$. Thus, the firm's surplus is $\beta[V^F(\psi A + k^B, A + (1 + r_\ell^B)k^B) - V^F(\psi A, A)]$. Given that firms will not raise funds more than what they need to satisfy the consumer's demand, using (1.12), (1.13) and (1.45), this reduces to $[(1-\theta)(u(\psi A+k^B)-(\psi A+k^B))-(1-\psi)A-r_\ell^B k^B]-[(1-\theta)(u(\psi A)-\psi A)-(1-\psi)A]$ subject to $k^B \leq v^{-1}(\tilde{m} + \chi\tilde{a}) - \psi A$, when the firm and the bank believe that a consumer will carry $\tilde{m}$ amount of real balances and $\tilde{a}$ amount of corporate bonds to the DM. The bank's surplus is $\beta r_\ell k$ as before. The terms of a loan contract specify (k^B, r_ℓ^B) that solves

$$\max_{k^B \leq v^{-1}(\tilde{m}+\chi\tilde{a})-\psi A, r_\ell} \left[r_\ell k\right]^\eta \begin{bmatrix} \big((1-\theta)(u(\psi A + k^B) - (\psi A + k^B)) - (1-\psi)A - r_\ell^B k^B\big) \\ - \big((1-\theta)(u(\psi A) - \psi A) - (1-\psi)A\big) \end{bmatrix}^{1-\eta}. \tag{1.49}$$

The solution is such that k maximizes the total surplus, $(1-\theta)[u(\psi A + k^B) - u(\psi A) - k^B]$, subject to $k^B \leq v^{-1}(\tilde{m} + \chi\tilde{a}) - \psi A$. The solution is given by

$$k^B = v^{-1}(\tilde{m} + \chi\tilde{a}) - \psi A, \tag{1.50}$$

$$r_\ell^B = \eta(1-\theta)\left[\frac{u(\psi A + k^B) - u(\psi A)}{k^B} - 1\right]. \tag{1.51}$$

1.5.3 Bond Supply

From (1.12), (1.13), (1.47), (1.50) and (1.51), at a given price ψ, the firm chooses the amount of corporate bonds to issue, $A \geq 0$, to maximize

$$\max_A (1-\theta)[u(v^{-1}(m + \chi a)) - v^{-1}(m + \chi a)] - (1-\psi)A - r_\ell^B k^B, \tag{1.52}$$

which is equivalent to maximizing

$$\max_A \eta(1-\theta)[u(\psi A) - \psi A] - (1-\psi)A. \tag{1.53}$$

The solution describes the optimal corporate bond issuance decision of the firm, or the supply of corporate bonds, which is given by

$$\frac{1}{\psi} - 1 = \eta(1-\theta)(u'(\psi\bar{A}) - 1), \tag{1.54}$$

and the amount of funds that firms will raise by issuing corporate bonds, ψA, is

$$\psi A = (u')^{-1}\left(\frac{1/\psi - 1}{\eta(1-\theta)} + 1\right). \tag{1.55}$$

1.5.4 Composition of Credit

Now I examine the optimal composition of credit between corporate bonds and bank loans at the firm level. The result in (1.55) shows that a firm wants to issue some amount of corporate bonds before entering the OTC market for bank loans. The intuition is as follows. From (1.51), r_ℓ^B is an increasing function of ψA. This is because firms with large corporate bond issuance rely less on bank loans (as can be seen from (1.50) that k^B is decreasing in ψA) and can negotiate for a lower real loan rate (as can be seen from (1.51) that r_ℓ^B is decreasing in k^B). The benefit of issuing corporate bonds in negotiating for a bank loan is the first term in (1.53), $\eta(1-\theta)[u(\psi A) - \psi A]$, which comes from $-r_\ell^B k^B$ in (1.52). The cost side of issuing corporate bonds is the second term in (1.53), $-(1-\psi)A$, the liabilities that the firm needs to pay to the consumers who are holding the corporate bonds. The concave benefit function and the linear cost function together determine the optimal composition of credit as in (1.50) and (1.55).

Monetary policy changes affect this composition of credit between corporate bonds and bank loans. A higher nominal policy rate, i, decreases the total size of credit as the consumer's demand declines due to the higher cost of holding liquidity, as can be seen from (1.50) that $\psi A + k^B$ equals $v^{-1}(m + \chi a)$ which in turn is a decreasing function of i from (1.26). On the other hand, at the same time, as is explained in Section 1.3.5 and Proposition 1.1 says, a higher nominal policy rate makes issuing corporate bonds less expensive, allowing firms to issue more corporate bonds for the strategic purpose of lowering their financing costs, as can be seen from (1.55) that the left-hand side, ψA, is an increasing function of ψ which in turn is an increasing function of i. As a result, with a higher nominal policy rate, firms borrow less from banks. Therefore, as the nominal policy rate increases, the portion of corporate bonds among the total credit becomes larger, and that of bank loans becomes smaller. The following proposition summarizes the discussion.

Proposition 1.2. *As the nominal policy rate increases, the size of total credit decreases. Among the total credit that becomes smaller, firms increase the portion of credit from issuing corporate bonds and decrease that from bank loans:*

$$\frac{\partial(\psi A + k^B)}{\partial i} < 0, \quad \frac{\partial \psi A}{\partial i} > 0, \quad \frac{\partial k^B}{\partial i} < 0, \quad \frac{\partial\left(\dfrac{\psi A}{\psi A + k^B}\right)}{\partial i} > 0, \quad \frac{\partial\left(\dfrac{k^B}{\psi A + k^B}\right)}{\partial i} < 0 \qquad (1.56)$$

Becker and Ivashina (2014) provide direct empirical support for this theoretical finding by showing that firms switch from bank loans to corporate bonds following an increase in the nominal policy rate.

1.6 Optimal Monetary Policy

In this section, I study optimal monetary policy for the period, such as the post-TRACE period, when the liquidity premium channel of monetary policy transmission is dominant in the response of the bond premium to the nominal policy rate. To simplify the presentation, I consider the environment described in Section 1.2.[19] With the settlement market at the end of each period, maximizing welfare is equivalent to maximizing the per-period welfare that equals the sum of the per-period utility of each agent. The per-period utility of suppliers is 0 due to the CRS technology. The per-period utility of firms that finance investment by borrowing from a bank is $p_L - q_L - r_\ell k$ and their total measure is $1 - \lambda$. The per-period utility of firms that finance investment by issuing corporate bonds is $p_B - q_B - (1-\psi)A$ and their total measure is λ. The per-period utility of banks that lend to a firm is $r_\ell k$ and their total measure is $1 - \lambda$. The per-period utility of consumers is $-(1+\pi)m - \psi a + m + a + T + (1-\lambda)[u(q_L) - p_L] + \lambda[u(q_B) - p_B]$, where $T = \pi m$. The per-period utility of all agents sums up to

$$\mathcal{W} \equiv (1-\lambda)[u(q_L) - q_L] + \lambda[u(q_B) - q_B]. \tag{1.57}$$

A common result in monetary theory is that an increase in the nominal policy rate hurts welfare: a higher nominal policy rate increases the opportunity cost of holding liquidity, induces agents to carry less liquidity, and reduces the quantity of goods they can afford.

[19] Discussing optimal monetary policy in the extended environment described in Section 1.5 requires just a simple relabeling. Notice from (1.16) and (1.50) that when a firm has the option of financing both through issuing corporate bonds and borrowing from a bank, such firm will borrow in total from both the corporate bond and the bank loan markets the same amount as the firm that finances only through bank loans. Relabel the fraction of the firms that are borrowing from a bank with or without issuing corporate bonds as $1 - \bar{\lambda}$, instead of $1 - \lambda$. Then, the welfare analysis becomes the exact same as discussed in this section.

In this economy, however, the Friedman rule—implementing zero nominal policy rate—is suboptimal. The intuition is as follows. When meeting a firm for trade, agents can meet a firm that financed only by issuing corporate bonds, or a firm that obtained a loan from a bank. Increasing the nominal policy rate has the opposite effects across the two types of meetings. On the one hand, increasing the nominal policy rate makes issuing corporate bonds less expensive and thus helps firms raise more funds and bring a larger amount of intermediate goods to trades in the former type of meeting. More precisely, a higher nominal policy rate increases the price of corporate bonds by increasing their liquidity premium from (1.30); the higher price of corporate bonds makes issuing corporate bonds cheaper, allowing firms to raise more funds from (1.21); and firms can produce more goods in trades by bringing a larger amount of intermediate goods from (1.29) or (1.31). On the other hand, increasing the nominal policy rate increases the cost of holding money and makes consumers carry less liquidity, which in turn makes firms borrow less from banks due to the lower demand and hurts the latter type of meeting. More precisely, a higher nominal policy rate reduces the real amount of liquidity that consumers carry with themselves for trades from (1.26); due to the lower demand, firms will borrow less from banks from (1.16); and a smaller amount of goods are produced from (1.34). Consider that the nominal policy rate is currently low so that the borrowing cost in the corporate bond market is high and a relatively small amount of goods are produced in the former type of meeting, while the borrowing cost in the bank loan market is low and already a large amount of goods are produced in the latter type of meeting. In such case, the welfare loss from the latter type of meeting is only second order, while the welfare gain from the former type of meeting becomes first order. More precisely, in $\partial \mathcal{W}/\partial i = (1-\lambda) \cdot \partial(u(q_L) - q_L)/\partial i + \lambda \cdot \partial(u(q_B) - q_B)/\partial i$, the first term represents

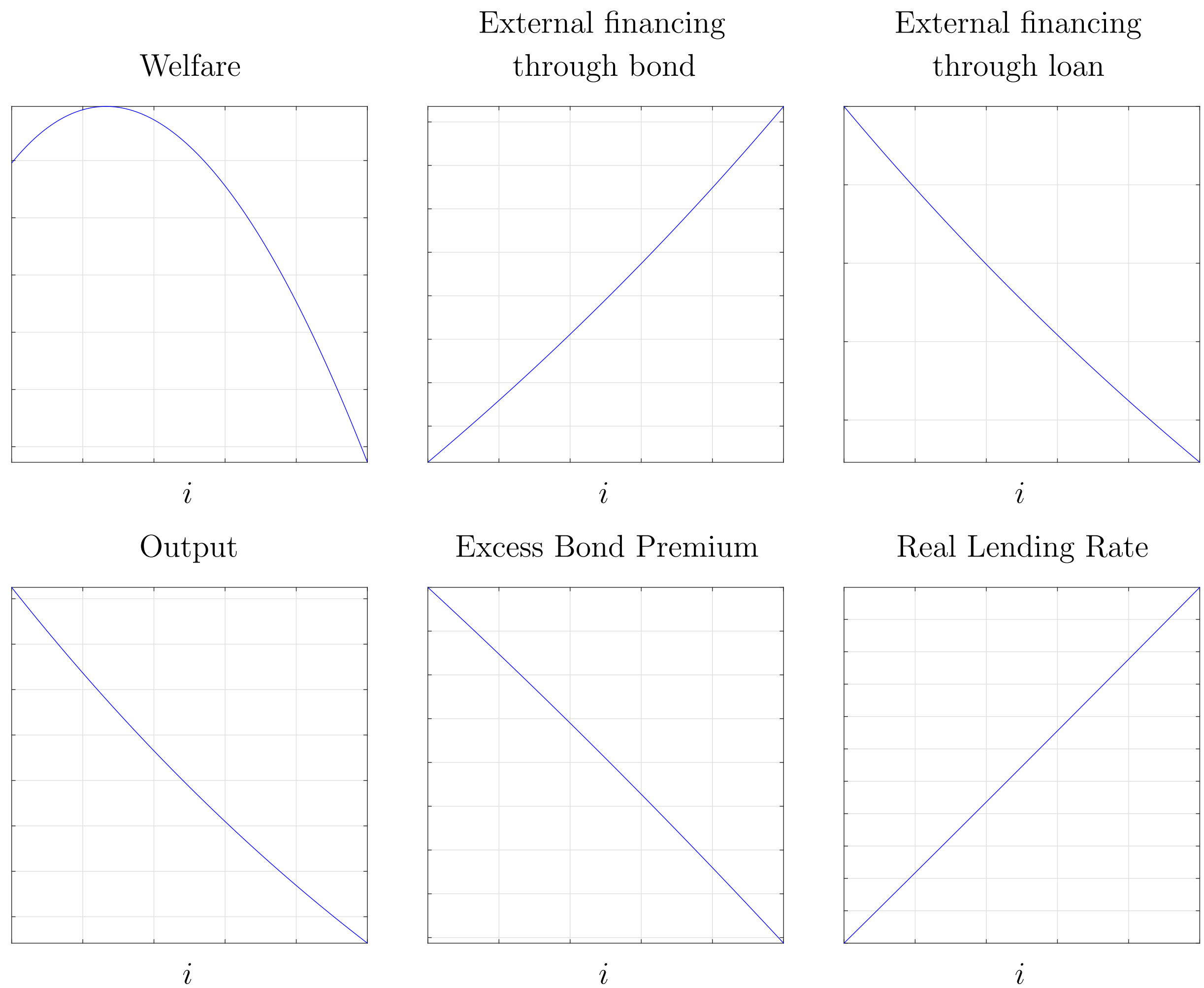

Figure 1.7. Effect of monetary policy on the aggregate outcomes

the welfare loss from the latter type of meeting, and the second term represents the welfare gain from the former type of meeting because $\partial(u(q_L) - q_L)/\partial i < 0$ since $\partial q_L/\partial i < 0$ and because $\partial(u(q_B) - q_B)/\partial i > 0$ since $\partial q_B/\partial i > 0$. However, at the Friedman rule, when $i \to 0$, $\partial(u(q_L) - q_L)/\partial i \to 0$ because $q_L \to q^*$ as $i \to 0$ and $u'(q^*) = 1$. Therefore, when $i \to 0$, $\partial \mathcal{W}/\partial i = \lambda \cdot \partial(u(q_B) - q_B)/\partial i > 0$. That is, at the Friedman rule, increasing the nominal policy rate can be welfare improving. The following proposition summarizes the discussion.

Proposition 1.3. *A deviation from the Friedman rule is optimal, i.e., the optimal monetary policy requires* $i > 0$.

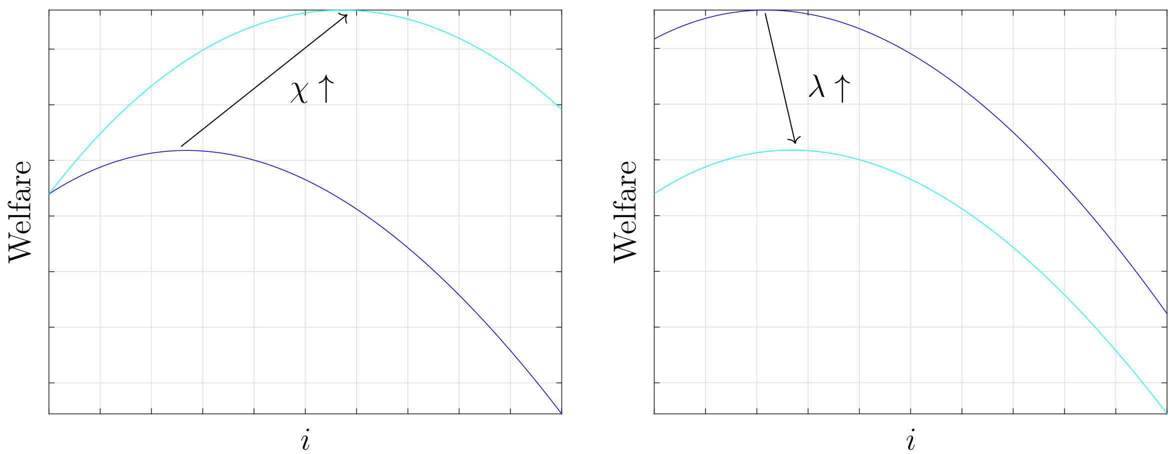

Figure 1.8. The corporate bond secondary market liquidity and the distribution of firms along their ways of financing matters for the optimal policy rate.

Figure 1.7 shows the relationship between the nominal policy rate and the welfare of the economy, along with other aggregate variables. The main force that drives a positive nominal policy rate to be optimal is the liquidity premium channel of monetary policy. Therefore, the stronger the channel, the higher the optimal policy rate. Figure 1.8 provides numerical examples that support this argument. In particular, the optimal nominal policy rate depends on the corporate bond secondary market liquidity and the distribution of firms along their ways of financing. The more liquid the corporate bond secondary market, or the more firms financing through issuing corporate bonds, the higher the optimal policy rate.

1.7 Conclusion

Central banks influence firms' investment through controlling the nominal policy rate, which then gets transmitted to the real rates at which firms borrow. I study this transmission mechanism in a general equilibrium macroeconomic model where firms have two options for external financing: they can issue corporate bonds or obtain bank loans. A theoretical novelty of my model is that corporate bonds are not just stores of value but also serve a liquidity role. The model delivers three predictions. First, an increase in the nominal policy rate can lower the borrowing cost in the corporate bond market, while increasing that in the bank loan market. This is in sharp contrast with the common belief that all rates in the economy move in the same direction in response to changes in monetary policy. I highlight the role of asset liquidity in this result and provide empirical evidence. Second, a higher nominal policy rate induces firms to substitute corporate bonds for bank loans, and this result is supported by the existing empirical evidence. Third, the Friedman rule is suboptimal so that keeping the cost of holding liquidity at a positive level is socially optimal. The optimal policy rate is an increasing function of the degree of corporate bond liquidity.

1.A Theory Appendix

1.A.1 Full Characterization of the Equilibrium

This section characterizes the equilibrium beyond the parameter space in Assumption 1.1. First start with the optimal behavior of a firm that finances investment by issuing corporate bonds. From (1.10), at a given price ψ, the firm chooses the amount of corporate bonds to issue, $A \geq 0$, that maximizes $(p_B - q_B) - (1-\psi)A$, which, using (1.12) and (1.13), reduces to $(1-\theta)(u(q_B) - q_B) - (1-\psi)A$, where $q_B = \min\{v^{-1}(\tilde{m} + \chi\tilde{a}), \psi A\}$ when believing that a consumer will carry $\tilde{m}$ amount of real balances and $\tilde{a}$ amount of corporate bonds to the DM. An equilibrium exists when $1-\psi > 0$, or $\psi < 1$, that is, when borrowing through the corporate bond market is costly. Assumption 1.2, given below, guarantees that this is the case. Since the firm will not want to bring more capital to the DM than it needs to produce the amount of the DM goods that a consumer can afford, $v^{-1}(\tilde{m}+\chi\tilde{a})$, and the maximization problem becomes

$$\max_{0 \leq A \leq v^{-1}(\tilde{m}+\chi\tilde{a})/\psi} \{(1-\theta)(u(\psi A) - \psi A) - (1-\psi)A\}. \tag{1.58}$$

The solution describes the optimal corporate bond issuance decision of the firm, or the supply of corporate bonds, and is given by

$$A = \min\{v^{-1}(\tilde{m} + \chi\tilde{a})/\psi, \bar{A}\} \tag{1.59}$$

where $\bar{A}$ solves

$$\frac{1}{\psi} - 1 = (1-\theta)(u'(\psi\bar{A}) - 1). \tag{1.60}$$

Now consider the optimal behavior of a consumer who chooses a portfolio of real balances and corporate bonds. From (1.2), the consumer chooses the amount of real balances, m, and the amount of corporate bonds, a, that maximize $-(1+\pi)m - \psi a + \beta V^C(m,a)$, which, using (1.4) and the linearity of W^C, becomes

$$\max_{m\geq 0, a\geq 0}\Big\{-(1+\pi)m - \psi a + \beta m + \beta a + \beta(1-\lambda)[u(q_L) - p_L] + \beta\lambda[u(q_B) - p_B]\Big\}. \tag{1.61}$$

When believing that a firm that issues corporate bonds will issue $\tilde{A}$ amount of corporate bonds and bring $\psi\tilde{A}$ amount of capital to the DM and that a firm that borrows from a bank will bring $\tilde{k}$ amount of capital to the DM, $q_L = \min\{v^{-1}(m+\chi a), \tilde{k}\}$ and $q_B = \min\{v^{-1}(m+\chi a), \psi\tilde{A}\}$. Depending on the relative size of $\tilde{k}$, $\psi\tilde{A}$ and $v^{-1}(m+\chi a)$, the maximization problem is:

For $m + \chi a \leq \min\{v(\tilde{k}), v(\psi\tilde{A})\}$,

$$\max_{m\geq 0, a\geq 0}\Big\{-(1+\pi)m - \psi a + \beta m + \beta a + \beta\big[u(v^{-1}(m+\chi a)) - v(v^{-1}(m+\chi a))\big]\Big\}, \tag{1.62}$$

for $\min\{v(\tilde{k}), v(\psi\tilde{A})\} < m + \chi a \leq \max\{v(\tilde{k}), v(\psi\tilde{A})\}$, if $\psi\tilde{A} < \tilde{k}$,

$$\max_{m\geq 0, a\geq 0} \Big\{ -(1+\pi)m - \psi a + \beta m + \beta a + \beta(1-\lambda)\big[u(v^{-1}(m+\chi a)) - v(v^{-1}(m+\chi a))\big]\Big\}, \tag{1.63}$$

and if $\tilde{k} < \psi\tilde{A}$,

$$\max_{m\geq 0, a\geq 0} \Big\{ -(1+\pi)m - \psi a + \beta m + \beta a + \beta\lambda\big[u(v^{-1}(m+\chi a)) - v(v^{-1}(m+\chi a))\big]\Big\}, \tag{1.64}$$

and for $\max\{v(\tilde{k}), v(\psi\tilde{A})\} < m + \chi a$,

$$\max_{m\geq 0, a\geq 0} \Big\{ -(1+\pi)m - \psi a + \beta m + \beta a\Big\}. \tag{1.65}$$

I consider an equilibrium where expectations are rational. (1.64) is not a relevant case with $\psi\tilde{A} < \tilde{k}$ from (1.16) and (1.59), and (1.65) does not have a solution. The solution describes the optimal portfolio choice of real balances and corporate bonds of the consumer, or the demand for real balances and corporate bonds. The solution to (1.63) satisfies

$$m + \chi a = \max\{v(\psi\tilde{A}), \bar{m} + \chi\bar{a}\} \tag{1.66}$$

where $\bar{m} + \chi\bar{a}$ solves

$$1 + \pi = \beta\left\{1 + (1-\lambda)\left(\frac{u'(v^{-1}(\bar{m}+\chi\bar{a}))}{v'(v^{-1}(\bar{m}+\chi\bar{a}))} - 1\right)\right\}, \tag{1.67}$$

which simplify to

$$i = (1-\lambda)L(\bar{m} + \chi\bar{a}), \tag{1.68}$$

where $i \equiv (1+\pi)/\beta - 1$ and $L(\cdot) \equiv u'(v^{-1}(\cdot))/v'(v^{-1}(\cdot)) - 1$ with $L'(\cdot) < 0$. The solution to (1.62) satisfies

$$m + \chi a = \min\{v(\psi\tilde{A}), \bar{\bar{m}} + \chi\bar{\bar{a}}\} \tag{1.69}$$

where $\bar{\bar{m}} + \chi\bar{\bar{a}}$ solves

$$1 + \pi = \beta\left\{1 + \left(\frac{u'(v^{-1}(\bar{\bar{m}} + \chi\bar{\bar{a}}))}{v'(v^{-1}(\bar{\bar{m}} + \chi\bar{\bar{a}}))} - 1\right)\right\}, \tag{1.70}$$

which simplify to

$$i = L(\bar{\bar{m}} + \chi\bar{\bar{a}}). \tag{1.71}$$

For both cases, the price of corporate bonds is given by

$$\psi = \beta(1 + \chi i). \tag{1.72}$$

Note that $\bar{m} + \chi\bar{a}$ is the amount of liquidity consumers would decide to bring to the DM when their liquidity position will be on the shorter side of the bargaining only if they trade with a firm that borrows from a bank, and that $\bar{\bar{m}} + \chi\bar{\bar{a}}$ is the amount of liquidity consumers would decide to bring to the DM when their liquidity position will always be on the shorter side of the bargaining whether a firm they meet finances investment by borrowing from a bank or by issuing corporate bonds. By comparing (1.68) and (1.71), we see that $\bar{m} + \chi\bar{a} < \bar{\bar{m}} + \chi\bar{\bar{a}}$. There are three cases depending on the relative size of $v(\psi\bar{A})$, $\bar{m} + \chi\bar{a}$ and $\bar{\bar{m}} + \chi\bar{\bar{a}}$ given i. Define $\bar{\imath}$ and $\bar{\bar{\imath}}$ as follows:

$$\bar{\imath} \equiv \frac{(1-\lambda)(1-\beta)\theta}{1-\theta+(1-\lambda)\beta\theta\chi}, \tag{1.73}$$

$$\bar{\bar{\imath}} \equiv \frac{(1-\beta)\theta}{1-\theta+\beta\theta\chi}. \tag{1.74}$$

The first is when $i \leq \bar{\imath}$ and $v(\psi\bar{A}) \leq \bar{m} + \chi\bar{a} < \bar{\bar{m}} + \chi\bar{\bar{a}}$. This is when the price of the corporate bonds is not high enough for firms to finance investment enough to fully satisfy the consumer's demand, $\bar{m} + \chi\bar{a}$. Hence, the firms that are borrowing from a bank and thus can satisfy the consumer's demand are at the margin of the consumer's decision on how much liquidity to bring to the DM. The second is when $\bar{\imath} < i \leq \bar{\bar{\imath}}$ and $\bar{m} + \chi\bar{a} < v(\psi\bar{A}) \leq \bar{\bar{m}} + \chi\bar{\bar{a}}$. This is when the price of the corporate bonds is high enough for firms to finance investment enough to satisfy $\bar{m} + \chi\bar{a}$, but not high enough to satisfy $\bar{\bar{m}} + \chi\bar{\bar{a}}$. When this is the case, a consumer will bring liquidity just enough to be able to purchase $\psi\bar{A}$ amount of the DM goods, and such amount of liquidity will make the consumer on the shorter side of the bargaining with both the firms that are borrowing from a bank and the firms that are issuing corporate bonds. The third case is when $\bar{\bar{\imath}} < i$ and $\bar{m} + \chi\bar{a} < \bar{\bar{m}} + \chi\bar{\bar{a}} \leq v(\psi\bar{A})$, that is, when the price of the corporate bonds is high enough to satisfy $\bar{\bar{m}} + \chi\bar{\bar{a}}$. In this case, a consumer will decide the amount of liquidity to bring to the DM with considering both the firms that are borrowing from a bank and the firms that are issuing corporate bonds at the same margin.

The firms with access to the corporate bond market will issue corporate bonds just enough to satisfy $\bar{\bar{m}} + \chi\bar{\bar{a}}$.[20]

Now I specify the assumption that ensures $\psi < 1$ so that borrowing through the corporate bond market is costly.

Assumption 1.2. $i < \dfrac{1-\beta}{\beta\chi}$.

The equilibrium is defined as below.

Definition 1.2. *A steady state equilibrium of the economy corresponds to a constant sequence* $(q_L, q_B, m, a, A, \psi, k, r_\ell)$*, where* q_L *is the DM goods traded between a consumer and a firm that finances investment by borrowing from a bank,* q_B *is the DM goods traded between a consumer and a firm that finances investment by issuing corporate bonds,* m *is the consumer's real balance holdings,* a *is the consumer's corporate bond holdings,* A *is the supply of corporate bonds issued by firms,* ψ *is the price of corporate bonds,* k *is the size of a loan that a bank lends to a firm, and* r_ℓ *is the real lending rate of loans. Under Assumption 1.2,* (q_L, q_B) *satisfy:*

For $i \leq \bar{\imath}$,

$$q_L = v^{-1}\left(L^{-1}\left(\frac{i}{1-\lambda}\right)\right), \tag{1.75}$$

$$q_B = (u')^{-1}\left(\frac{1-\beta(1+\chi i)}{\beta(1+\chi i)(1-\theta)} + 1\right), \tag{1.76}$$

[20] For each given i, there are more equilibria other than those described above. The most trivial one is when no one brings any thinking that everyone else will bring nothing. Although this belief can be consistent in equilibrium, however, such equilibrium is not Pareto efficient. In this paper, I consider the Pareto efficient equilibrium for each i.

for $\bar{\imath} < i \leq \bar{\bar{\imath}}$,

$$q_L = q_B = (u')^{-1}\left(\frac{1-\beta(1+\chi i)}{\beta(1+\chi i)(1-\theta)} + 1\right), \tag{1.77}$$

and for $\bar{\bar{\imath}} < i$,

$$q_L = q_B = v^{-1}(L^{-1}(i)). \tag{1.78}$$

(m, a, A, ψ) *satisfy*

$$\psi = \beta(1+\chi i), \tag{1.79}$$

$$A = q_B/\psi, \tag{1.80}$$

$$a = \lambda A, \tag{1.81}$$

$$m = v(q_L) - \chi a, \tag{1.82}$$

and (k, r_ℓ) *satisfy*

$$k = q_L = v^{-1}(m + \chi a), \tag{1.83}$$

$$r_\ell = \frac{\eta(1-\theta)(u(k)-k)}{k}, \tag{1.84}$$

where

$$v(\cdot) = (1-\theta)u(\cdot) + \theta\,\cdot\,, \quad v'(\cdot) > 0, \tag{1.85}$$

$$L(\cdot) = u'(v^{-1}(\cdot))/v'(v^{-1}(\cdot)) - 1, \quad L'(\cdot) < 0. \tag{1.86}$$

1.A.1.1 Optimal Monetary Policy

Among $i \leq \bar{\imath}$, the welfare-maximizing nominal policy rate depends on the relative size of $(1-\lambda)\cdot\partial(u(q_L)-q_L)/\partial i < 0$ and $\lambda\cdot\partial(u(q_B)-q_B)/\partial i > 0$. When neither λ nor χ is large, the latter force is not so large that the welfare-maximizing policy rate satisfying $\partial\mathcal{W}/\partial i = 0$ exists in the interior. When either λ or χ is large, the latter force becomes so large that the welfare-maximizing policy rate exists on the right boundary at $i = \bar{\imath}$. In addition, note that when $\bar{\imath} < i \leq \bar{\bar{\imath}}$, $\partial\mathcal{W}/\partial i > 0$ as can be seen from (1.77), that when $\bar{\bar{\imath}} \leq i$, $\partial\mathcal{W}/\partial i < 0$ as can be seen from (1.78), and therefore that among $i > \bar{\imath}$, $i = \bar{\bar{\imath}}$ maximizes the welfare. These together imply that when neither λ nor χ is large, there will be a welfare-maximizing policy rate that is less than $\bar{\imath}$, and that when either λ or χ is large, the welfare-maximizing policy rate will be $\bar{\bar{\imath}}$. Figures 1.A.1 and 1.A.2 illustrate these observations. Figures 1.A.3 (for small λ and small χ), 1.A.4 (for large λ and small χ) and 1.A.5 (for small λ and large χ) show the effect of the nominal policy rate on different variables, including the welfare, the amount of external financing through bonds and loan, the real balance, the excess bond premium, the real lending rate, and the average output. In all figures, there are two kinks, and the first and the second correspond to $i = \bar{\imath}$ and $i = \bar{\bar{\imath}}$, respectively. Exceptions are the figures for the amount of external financing through issuing corporate bonds that display one kink, which corresponds to $i = \bar{\bar{\imath}}$ as can be seen from (1.76), (1.77) and (1.78). In all figures, we can see that the Friedman rule when $i \to 0$ is not optimal. Also, notice that the relationship between the welfare and the average output is not monotone, due to the heterogeneity in the effect of the nominal policy rate across the firms using different financing sources. Figure 1.A.6 illustrates this point.

1.A.2 Loan Contract with a Different Timing of Events

There are two types of meetings in the OTC market for loans in the second subperiod: one between a bank and a firm that does not have access to the corporate bond market, and the other between a bank and a firm that has access to the corporate bond market and could issue corporate bonds to finance investment in addition to obtaining a loan from a bank. The bargaining problem in the former meeting is the same as in the previous environment, and the solution is given by (1.16) and (1.17).

In the latter meeting, a bank and a firm bargain over the terms of a loan contract, (k^B, r_ℓ^B). As before, I restrict attention to the case in which the firm's capacity is on the shorter side of the bargaining in the DM if the firm finances investment solely by issuing corporate bonds. Define $\bar{A}$ that solves (1.20) at given ψ:

$$\bar{A} \equiv (u')^{-1}\left(\frac{1-\psi}{\psi(1-\theta)} + 1\right) \Big/ \psi. \tag{1.87}$$

If a firm borrows more than $\psi\bar{A}$ from a bank, the firm will have no incentive to issue corporate bonds to raise more numeraire. On the other hand, if a firm borrows less than $\psi\bar{A}$ from a bank, the firm will issue A amount of corporate bonds so that it raises in total $\psi\bar{A} = \psi A + k^B$

amount of numeraire.

First consider the latter case in which a firm borrows less than $\psi\bar{A}$ from a bank and will issue A amount of corporate bonds so that it raises in total $\psi\bar{A} = \psi A + k^B$ amount of numeraire. The firm's continuation value with a loan contract is $\beta V^F(\psi A + k^B, A + (1 + r_\ell^B)k^B)$, where $A = (\psi\bar{A} - k^B)/\psi$ so that $\psi A + k^B = \psi\bar{A}$. The firm's outside option is to issue corporate bonds. When the firm could not borrow from a bank, it will issue $\bar{A}$ amount of corporate bonds, and the firm's continuation value without a loan contract will be $\beta V^F(\psi\bar{A}, \bar{A})$. Thus, the firm's surplus is $\beta[V^F(\psi\bar{A}, A+(1+r_\ell^B)k^B) - V^F(\psi\bar{A}, \bar{A})]$, which, using (1.47), reduces to $\beta[(1-\psi)(\bar{A}-A) - r_\ell^B k^B]$. As before, the bank's surplus is $\beta r_\ell^B k^B$. The terms of a loan contract specify (k^B, r_ℓ^B) that solve

$$\max_{k^B \leq \psi\bar{A}, r_\ell^B} \left[(1-\psi)(\bar{A}-A) - r_\ell^B k^B\right]^{1-\eta} \left[r_\ell^B k^B\right]^{\eta}. \tag{1.88}$$

Using $A = (\psi\bar{A} - k^B)/\psi$, the bargaining problem becomes

$$\max_{k^B \leq \psi\bar{A}, r_\ell^B} \left[((1-\psi)/\psi - r_\ell^B)k^B\right]^{1-\eta} \left[r_\ell^B k^B\right]^{\eta}. \tag{1.89}$$

The solution k^B maximizes the total surplus, $((1-\psi)/\psi)k^B$, subject to $k^B \leq \psi\bar{A}$. Under Assumption 1.1, $\psi < 1$ and $k^B = \psi\bar{A}$, which means the former case is the relevant one.

Now consider the former case in which a firm borrows more than $\psi\bar{A}$ from a bank and has no further incentive to issue corporate bonds. The firm's continuation value with a loan contract is $\beta V^F(k^B, (1+r_\ell^B)k^B)$. The firm's continuation value of issuing corporate bonds without a loan contract is $\beta V^F(\psi\bar{A}, \bar{A})$. Thus, the firm's surplus is $\beta[V^F(k^B, (1+r_\ell^B)k^B) -$

$V^F(\psi\bar{A},\bar{A})]$, which, using (1.12), (1.13) and (1.47), reduces to $\beta[(1-\theta)(u(k^B)-k^B)-r_\ell^B k^B-(1-\theta)(u(\psi\bar{A})-\psi\bar{A})+(1-\psi)\bar{A}]$. As before, the bank's surplus is $\beta r_\ell^B k^B$. The terms of a loan contract specify (k^B, r_ℓ^B) that solve

$$\max_{\psi\bar{A}\leq k^B\leq v^{-1}(\tilde{m}+\chi\tilde{a}),r_\ell^B} \begin{bmatrix} (1-\theta)(u(k^B)-k^B)-r_\ell^B k^B \\ \\ -(1-\theta)(u(\psi\bar{A})-\psi\bar{A})+(1-\psi)\bar{A} \end{bmatrix}^{1-\eta} \left[r_\ell^B k^B\right]^\eta, \tag{1.90}$$

where $v^{-1}(\tilde{m}+\chi\tilde{a})$ is the amount of the DM goods that a consumer can afford when believing that a consumer will carry $\tilde{m}$ amount of real balances and $\tilde{a}$ amount of corporate bonds to the DM, and a firm will not want to borrow more than it needs to produce $v^{-1}(\tilde{m}+\chi\tilde{a})$ amount of the DM goods. The solution is such that k^B maximizes the total surplus, $(1-\theta)(u(k^B)-k^B)-(1-\theta)(u(\psi\bar{A})-\psi\bar{A})+(1-\psi)\bar{A}$, subject to $\psi\bar{A}\leq k^B\leq v^{-1}(\tilde{m}+\chi\tilde{a})$ and thus the solution is as in (1.16). Therefore, both the firm that has an outside option in bargaining and the firm that does not will borrow the same amount of loan from a bank.

The real lending rate, however, will be different between the firm that has an outside option in bargaining and the firm that does not. The real lending rate for the firm that does not have access to the corporate bond market is given by (1.17). On the other hand, the real lending rate for the firm that has access to the corporate bond market is

$$r_\ell^B = \frac{\eta[(1-\theta)(u(k^B)-k^B)-(1-\theta)(u(\psi\bar{A})-\psi\bar{A})+(1-\psi)\bar{A}]}{k^B}, \tag{1.91}$$

where k^B is given by (1.16).

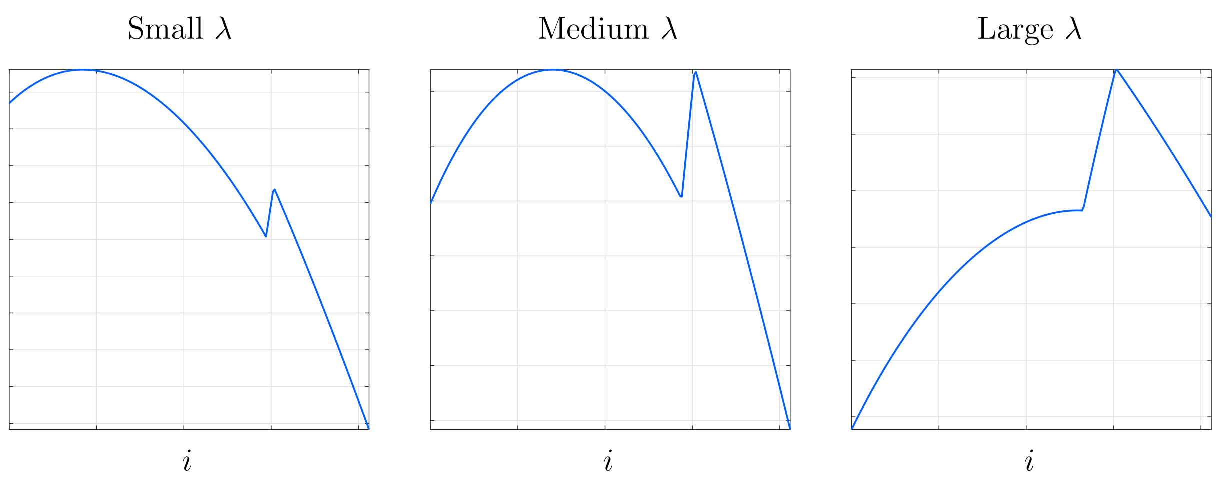

Figure 1.A.1. Effect of monetary policy on the welfare of the economy for different values of λ. Parameter values: Log utility; $\beta = 0.97$; $\lambda = 0.1$ (left), 0.165 (middle), 0.35 (right); $\chi = 0.15$; $\eta = 0.8$; $\theta = 0.95$.

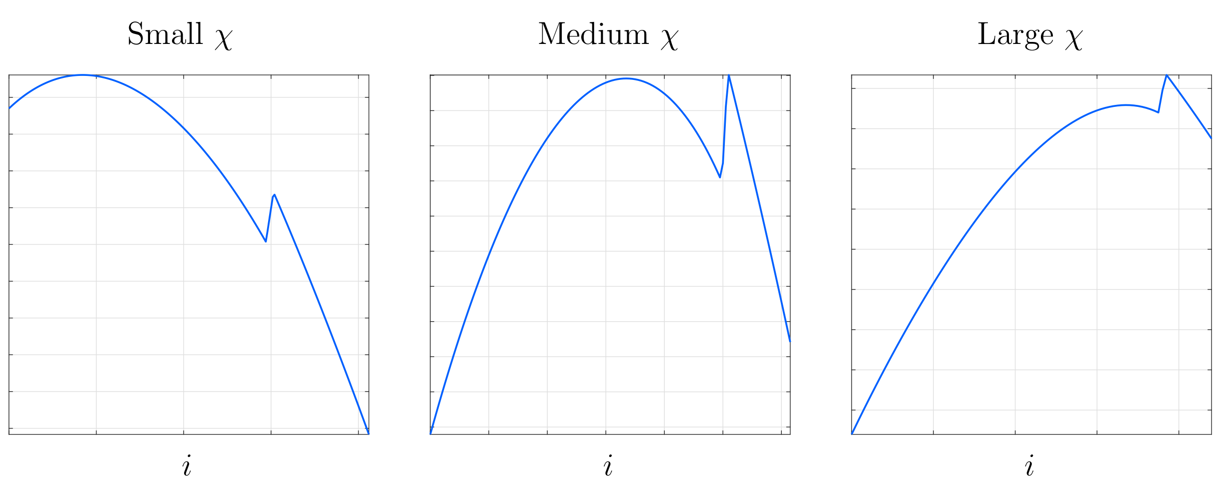

Figure 1.A.2. Effect of monetary policy on the welfare of the economy for different values of χ. Parameter values: Log utility; $\beta = 0.97$; $\lambda = 0.1$; $\chi = 0.15$ (left), 0.25 (middle), 0.35 (right); $\eta = 0.8$; $\theta = 0.95$.

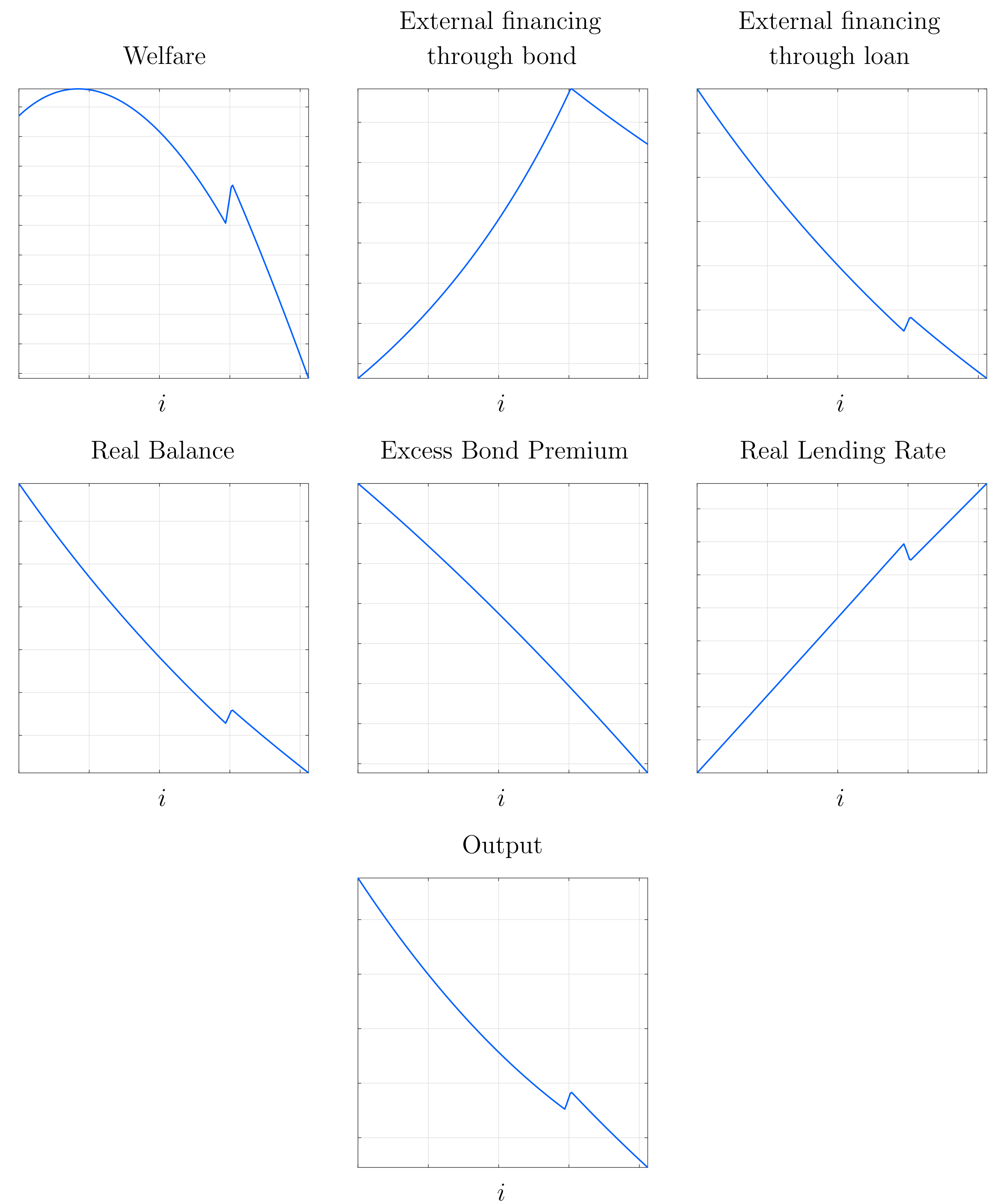

Figure 1.A.3. Effect of monetary policy, when the fraction of the firms with access to the corporate bond market is small (small λ) and the corporate bond secondary market is not so liquid (small χ). Parameter values: Log utility; $\beta = 0.97$; $\lambda = 0.1$; $\chi = 0.15$; $\eta = 0.8$; $\theta = 0.95$.

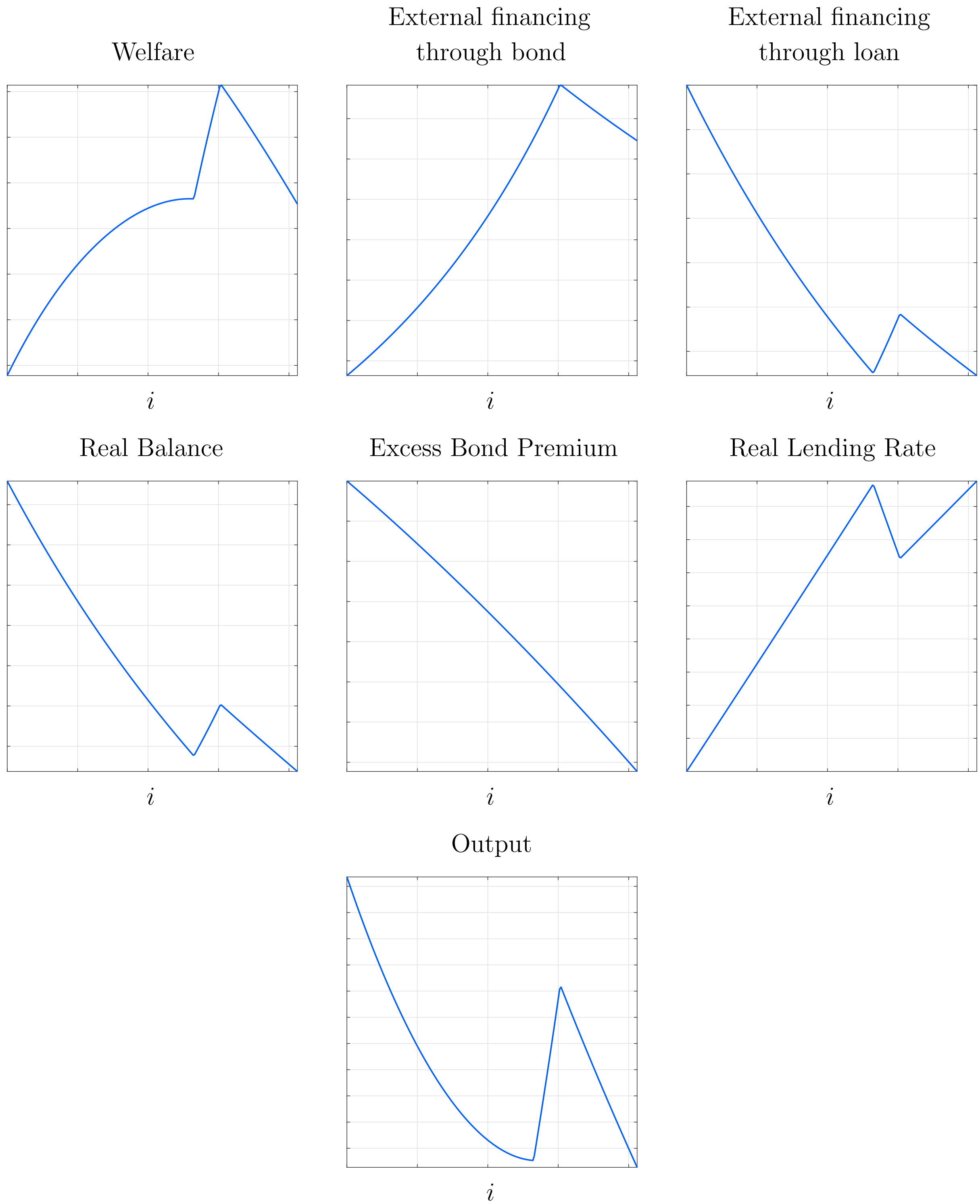

Figure 1.A.4. Effect of monetary policy, when the fraction of the firms with access to the corporate bond market is large (large λ) and the corporate bond secondary market is not so liquid (small χ). Parameter values: Log utility; $\beta = 0.97$; $\lambda = 0.35$; $\chi = 0.15$; $\eta = 0.8$; $\theta = 0.95$.

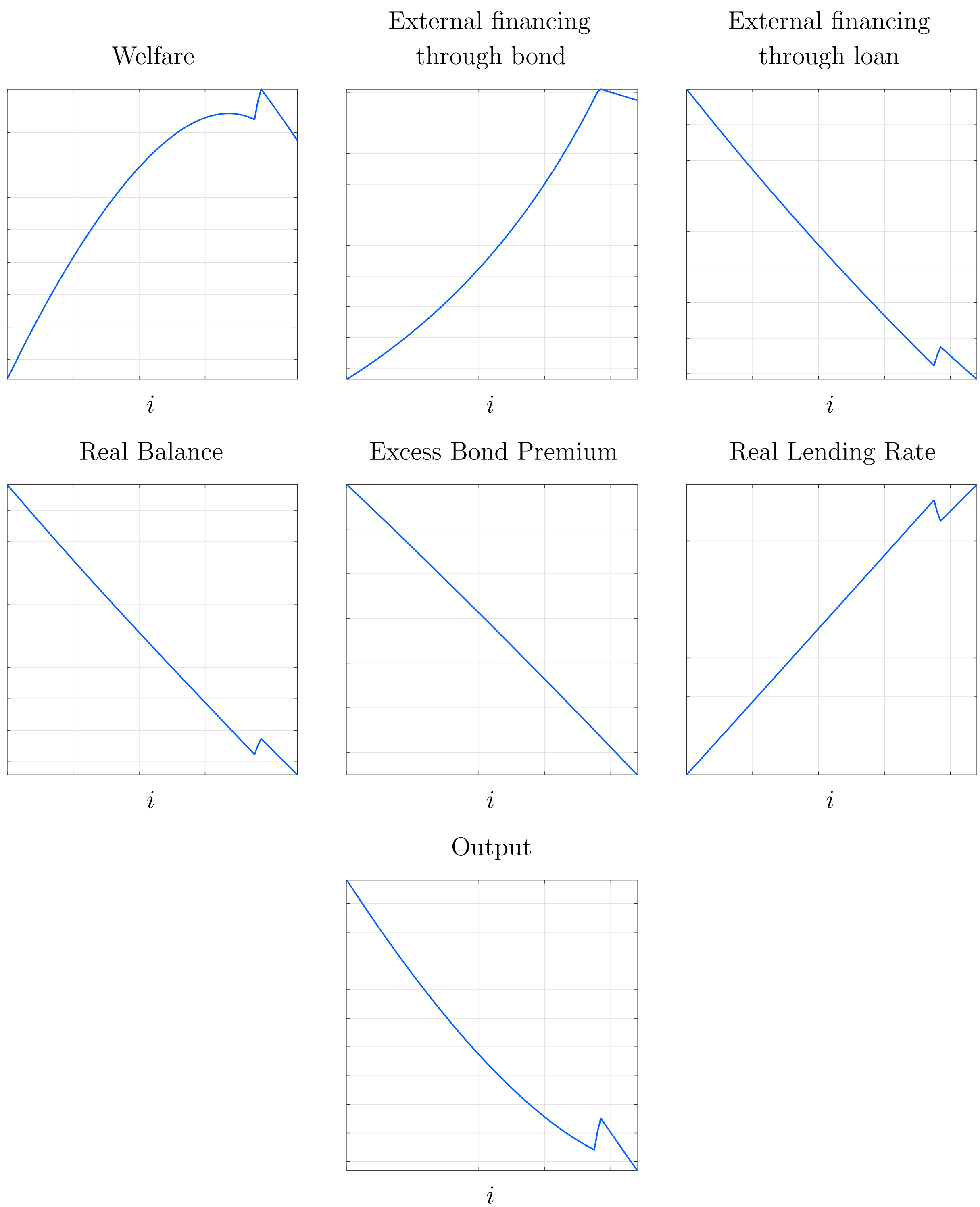

Figure 1.A.5. Effect of monetary policy, when the fraction of the firms with access to the corporate bond market is small (small λ) and the corporate bond secondary market is highly liquid (large χ). Parameter values: Log utility; $\beta = 0.97$; $\lambda = 0.1$; $\chi = 0.35$; $\eta = 0.8$; $\theta = 0.95$.

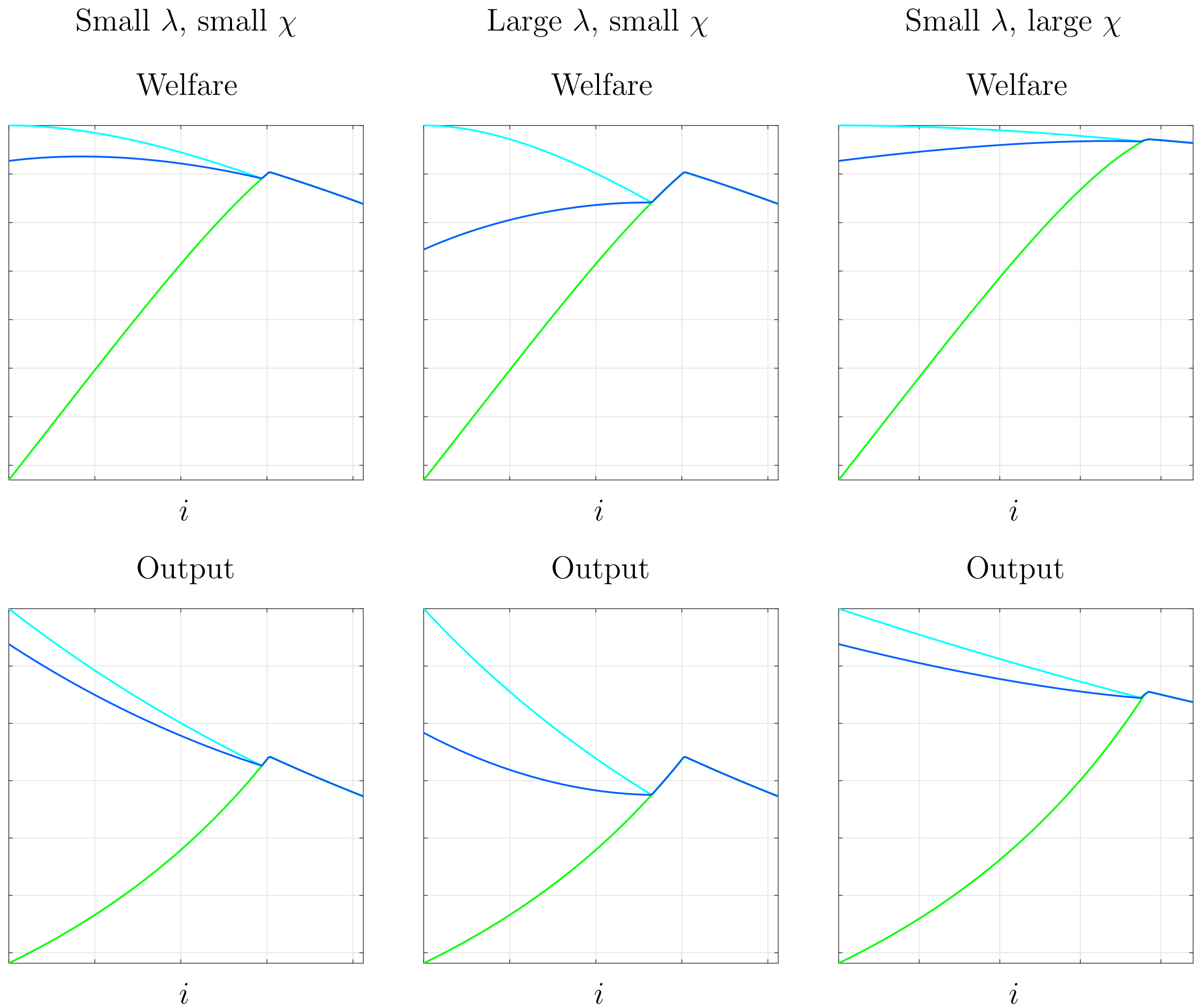

Figure 1.A.6. Effect of monetary policy on the welfare and the output of the economy for small λ (the fraction of the firms with access to the corporate bond market) and small χ (the liquidity of the corporate bond secondary market) (left), large λ and small χ (middle), and small λ and large χ (right). For each figure for the welfare, the top line (in bright blue) plots $u(q_L) - q_L$, the bottom line (in bright green) plots $u(q_B) - q_B$, and the middle line (in blue) plots $(1-\lambda)[u(q_L) - q_L] + \lambda[u(q_B) - q_B]$. For each figure for the output, the top line (in bright blue) plots q_L, the bottom line (in bright green) plots q_B, and the middle line (in blue) plots $(1-\lambda)q_L + \lambda q_B$. For the parameter values used, refer to the notes in Figure 1.A.3 for small λ and small χ, Figure 1.A.4 for large λ and small χ, and Figure 1.A.5 for small λ and large χ.

1.B Appendix for Empirical Analysis

1.B.1 Data

For the macro time-series data, I use data from the Federal Reserve Economic Data (FRED) from the Federal Reserve Bank of St. Louis. The 1-year and the 2-year policy rates are the 1-Year and the 2-year Treasury Constant Maturity Rates (FRED series `GS1` and `GS2`). Industrial production is Industrial Production Index (FRED series `INDPRO`). Consumer Price Index is Consumer Price Index for All Urban Consumers: All Items in U.S. City Average (FRED series `CPIAUCSL`). The expected inflation rate is the 5-Year Forward Inflation Expectation Rate (FRED series `T5YIFRM`). The excess bond premium is constructed by Gilchrist and Zakrajšek (2012) and keeps updated by Favara, Gilchrist, Lewis, and Zakrajšek at https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/files/ebp_csv.csv. Monetary policy shocks are the high-frequency identified surprises from Federal Funds futures around the Federal Open Market Committee policy announcements constructed by Gertler and Karadi (2015), and the series updated until 2016:12 is from Jarocinśki and Karadi (2020).

1.B.2 Additional Figures

Figure 1.A.7 replicates Gertler and Karadi (2015) with unit effect normalization. Figures 1.B.1–1.B.14 are for sensitivity analysis. Figures 1.C.1–1.C.18 show the responses of all variables, not only the variables of main focus (the excess bond premium (EBP), the bid-ask spreads, the trading volume, and the real bank loan rate), to a one-percent increase in the nominal policy rate for all different specifications. For quick references, refer to Table 1.A.1.

Table 1.A.1. Additional figures

Sample Period	Entire Period	Pre-TRACE Period	Post-TRACE Period			
Variable of focus	EBP	EBP	EBP	Bid-Ask Spreads	Trading Volume	Loan Rate
LP-IV	1.C.1	1.C.2	1.C.3	1.C.10	1.C.13	1.C.16
FALP-IV	1.C.4	1.C.5	1.C.6	1.C.11	1.C.14	1.C.17
SVAR-IV	1.C.7	1.C.8	1.C.9	1.C.12	1.C.15	1.C.18

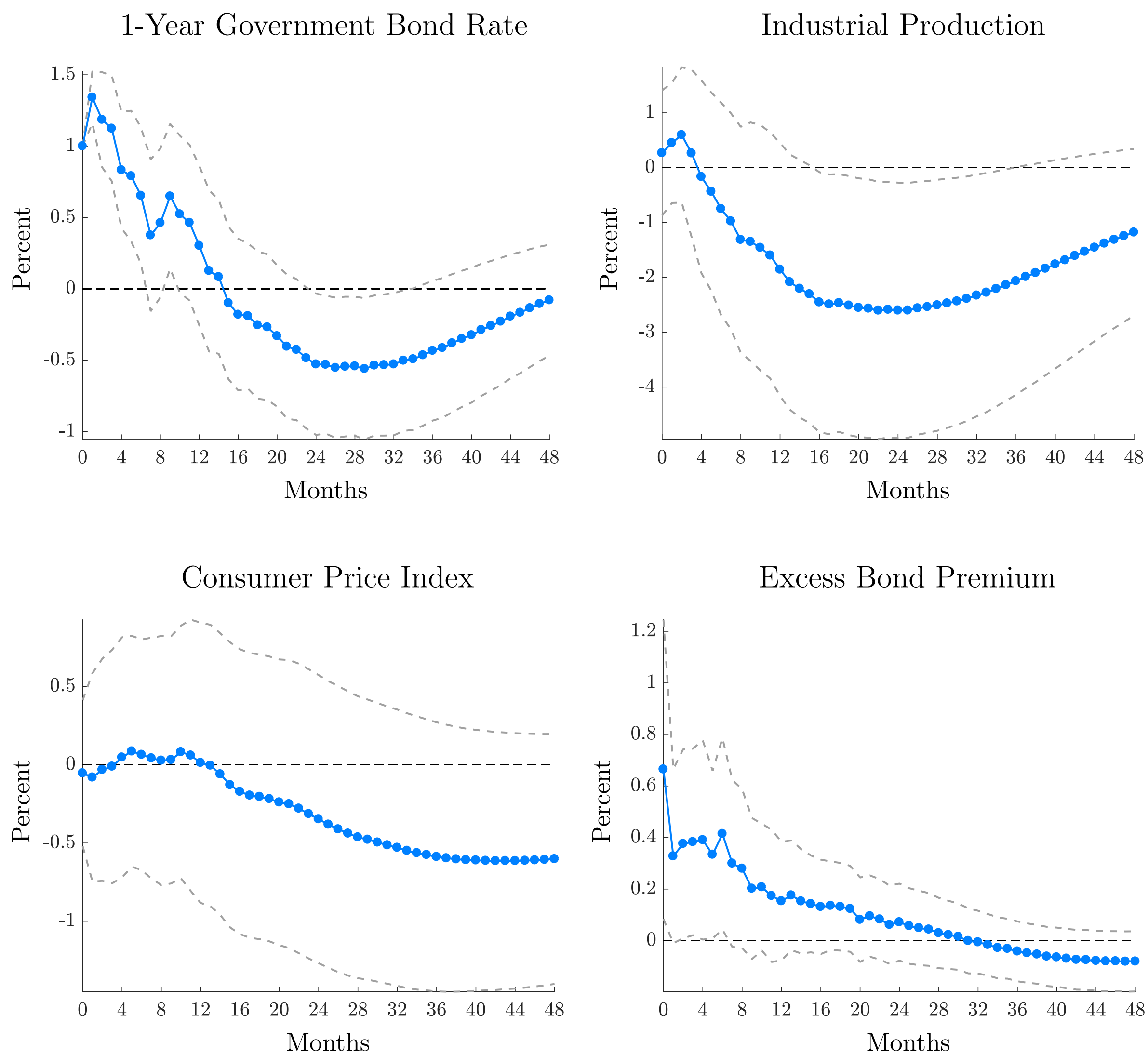

Figure 1.A.7. Replication of Gertler and Karadi (2015). Response of the 1-year government bond rate, industrial production, the consumer price index, and the excess bond premium to a one-percent increase in the nominal policy rate, estimated using SVAR-IV with unit effect normalization, during the entire period. Sample period: 1979:7-2012:6. 12-month lags of the four main variables and 4-month lags of the instrument are included. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 22.4, and the heteroscedasticity-robust first-stage F-statistic is 21.

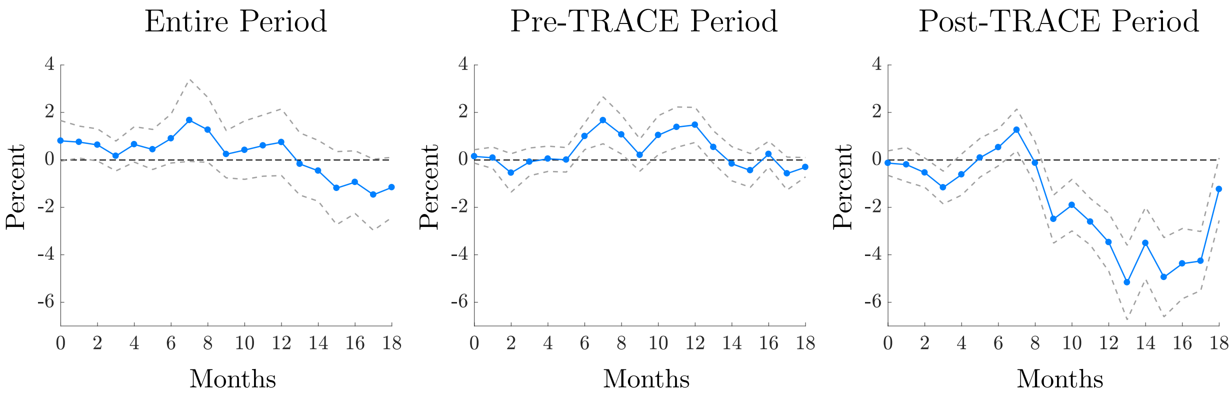

Figure 1.B.1. Response of the excess bond premium to a one-percent increase in the nominal policy rate for different sample periods, estimated using Factor-Augmented LP-IV with unit effect normalization. Sample period: 1990:2–2016:12; pre-TRACE period: 1990:2–2003:2; post-TRACE period: 2003:3–2016:12. Dashed lines are the 95% confidence interval. For details, refer to the notes in Figure 1.C.4 for the entire period, Figure 1.C.5 for the pre-TRACE period, and Figure 1.C.6 for the post-TRACE period.

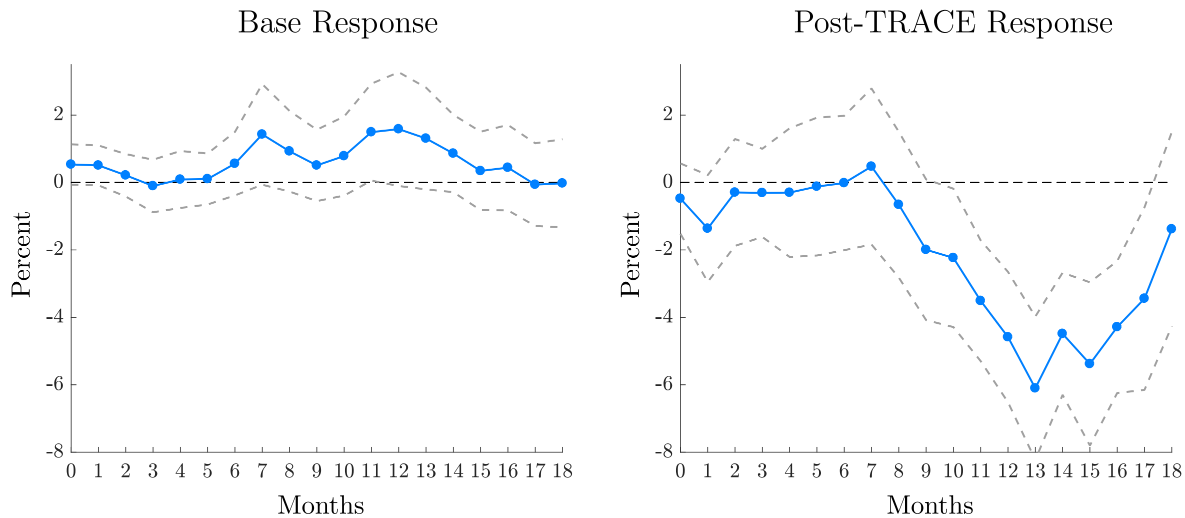

Figure 1.B.2. Base and post-TRACE responses of the excess bond premium to a one-percent increase in the nominal policy rate, estimated using LP-IV with unit effect normalization. Sample period: 1990:2–2016:12; post-TRACE period: 2003:3–2016:12. 12-month lags of the four main variables and 4-month lags of the instrument are included. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 15, and the heteroscedasticity-robust first-stage F-statistic is 10.7.

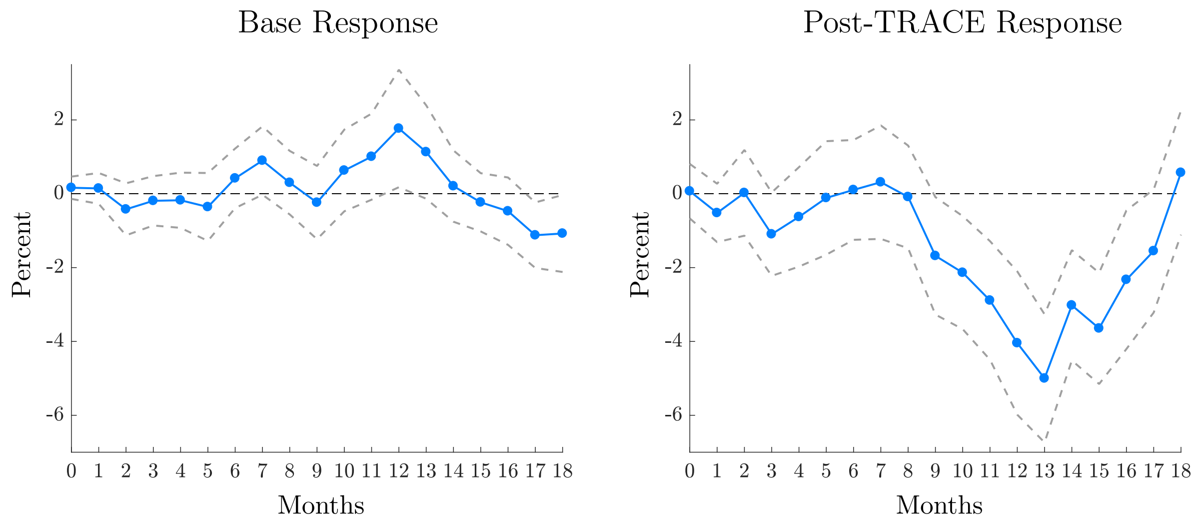

Figure 1.B.3. Base and post-TRACE responses of the excess bond premium to a one-percent increase in the nominal policy rate, estimated using Factor-Augmented LP-IV with unit effect normalization. Sample period: 1990:2–2016:12; post-TRACE period: 2003:3–2016:12. 12-month lags of the four main variables and the FRED-MD factors and 4-month lags of the instrument are included. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 13.7, and the heteroscedasticity-robust first-stage F-statistic is 18.6.

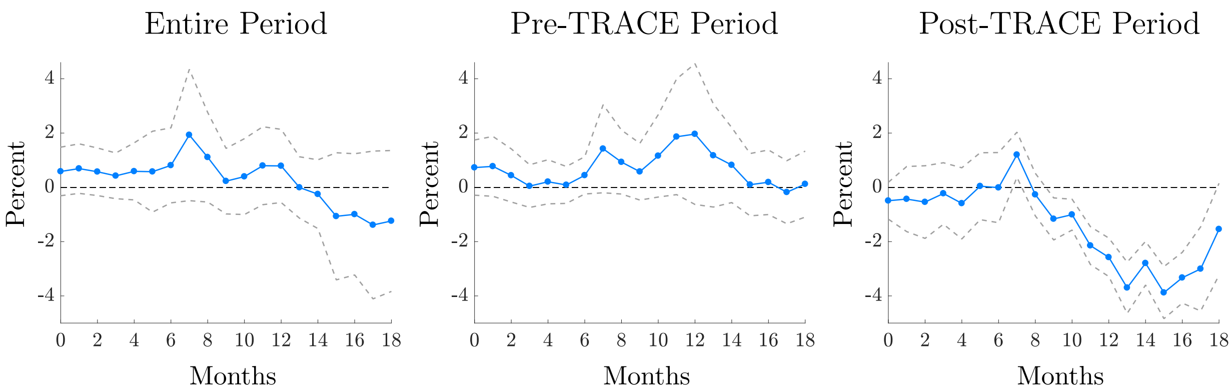

Figure 1.B.4. Response of the excess bond premium to a one-percent increase in the nominal policy rate for different sample periods, estimated using LP-IV with unit effect normalization. Sample period: 1990:2–2016:12; pre-TRACE period: 1990:2–2003:2; post-TRACE period: 2003:3–2016:12. 12-month lags of the four main variables (2-year Treasury constant maturity rate, industrial production, inflation rate, and the excess bond premium) and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic and the heteroscedasticity-robust first-stage F-statistic are 14.2 and 8.3 for the entire period, 7.7 and 9 for the pre-TRACE period, and 10.6 and 12.7 for the post-TRACE period.

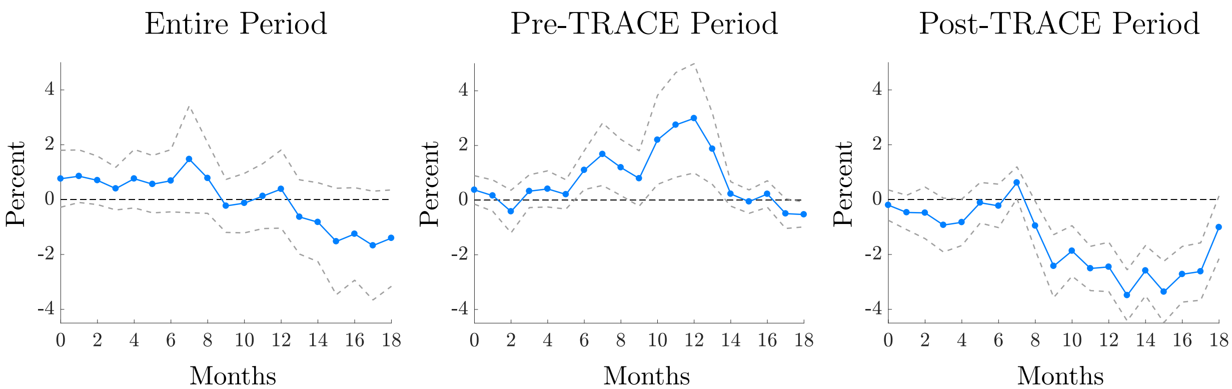

Figure 1.B.5. Response of the excess bond premium to a one-percent increase in the nominal policy rate for different sample periods, estimated using Factor-Augmented LP-IV with unit effect normalization. Sample period: 1990:2–2016:12; pre-TRACE period: 1990:2–2003:2; post-TRACE period: 2003:3–2016:12. 12-month lags of the four main variables (2-year Treasury constant maturity rate, industrial production, inflation rate, and the excess bond premium) and the FRED-MD factors and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic and the heteroscedasticity-robust first-stage F-statistic are 12.5 and 13 for the entire period, 3.2 and 11 for the pre-TRACE period, and 12.2 and 23 for the post-TRACE period.

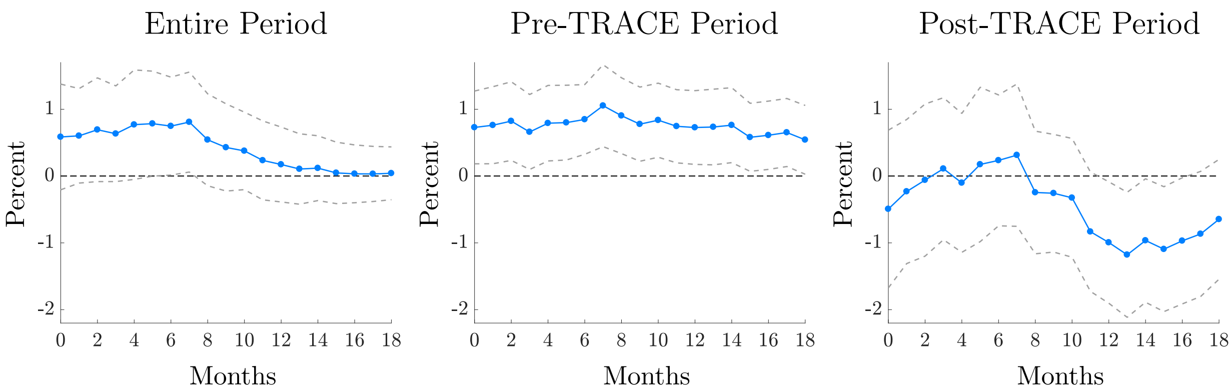

Figure 1.B.6. Response of the excess bond premium to a one-percent increase in the nominal policy rate for different sample periods, estimated using SVAR-IV with unit effect normalization. Sample period: 1990:2–2016:12; pre-TRACE period: 1990:2–2003:2; post-TRACE period: 2003:3–2016:12. 12-month lags of the four main variables (2-year Treasury constant maturity rate, industrial production, inflation rate, and the excess bond premium) and 4-month lags of the instrument are included. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap. Dashed lines are the 95% confidence interval. The first-stage F-statistic and the heteroscedasticity-robust first-stage F-statistic are 14.2 and 8.3 for the entire period, 7.7 and 9 for the pre-TRACE period, and 10.6 and 12.7 for the post-TRACE period.

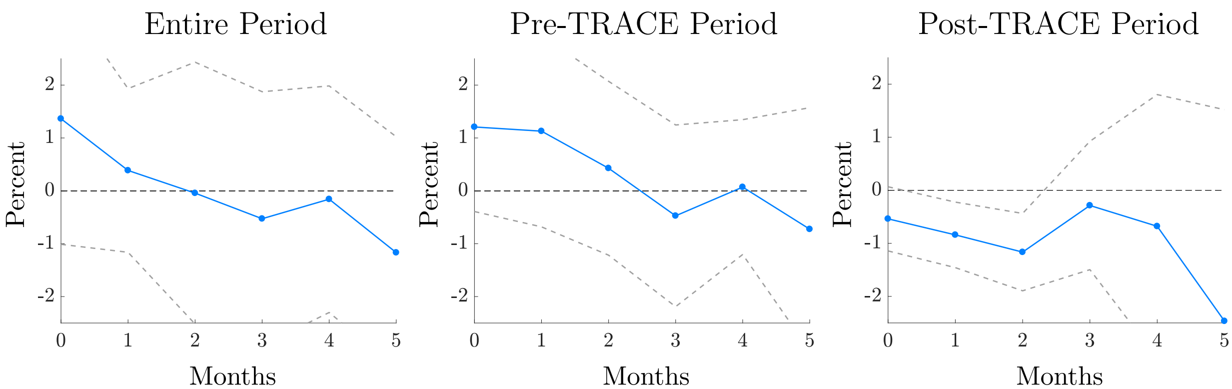

Figure 1.B.7. Response of the excess bond premium to a one-percent increase in the nominal policy rate for different sample periods, estimated using LP-IV with unit effect normalization, during the short period around the introduction of the TRACE. Sample period: 1997:11–2008:6; pre-TRACE period: 1997:11–2003:2; post-TRACE period: 2003:3–2008:6. 4-month lags of the four main variables and 2-month lags of the instrument are included. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap. Dashed lines are the 95% confidence interval. The first-stage F-statistic and the heteroscedasticity-robust first-stage F-statistic are 2.5 and 1.7 for the entire period, 2.9 and 2.7 for the pre-TRACE period, and 4.2 and 5.6 for the post-TRACE period.

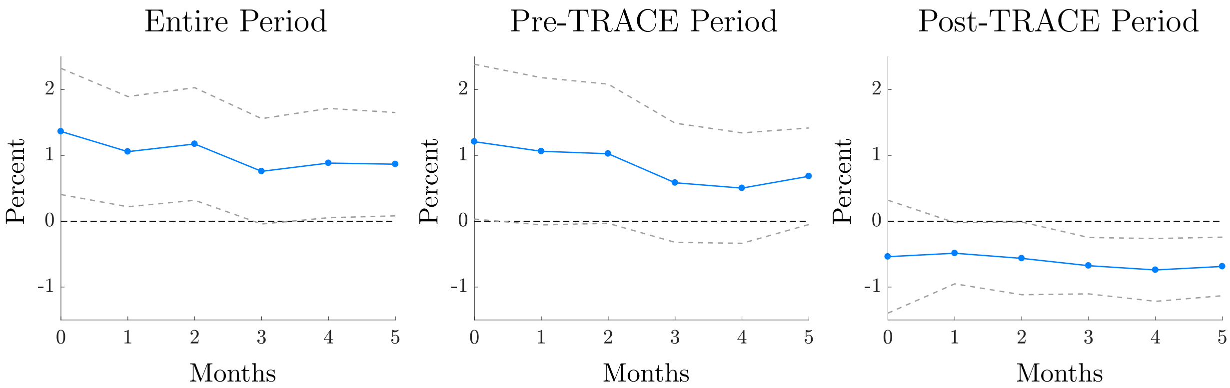

Figure 1.B.8. Response of the excess bond premium to a one-percent increase in the nominal policy rate for different sample periods, estimated using SVAR-IV with unit effect normalization, during the short period around the introduction of the TRACE. Sample period: 1997:11–2008:6; pre-TRACE period: 1997:11–2003:2; post-TRACE period: 2003:3–2008:6. 4-month lags of the four main variables and 2-month lags of the instrument are included. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap. Dashed lines are the 95% confidence interval. The first-stage F-statistic and the heteroscedasticity-robust first-stage F-statistic are 2.5 and 1.7 for the entire period, 2.9 and 2.7 for the pre-TRACE period, and 4.2 and 5.6 for the post-TRACE period.

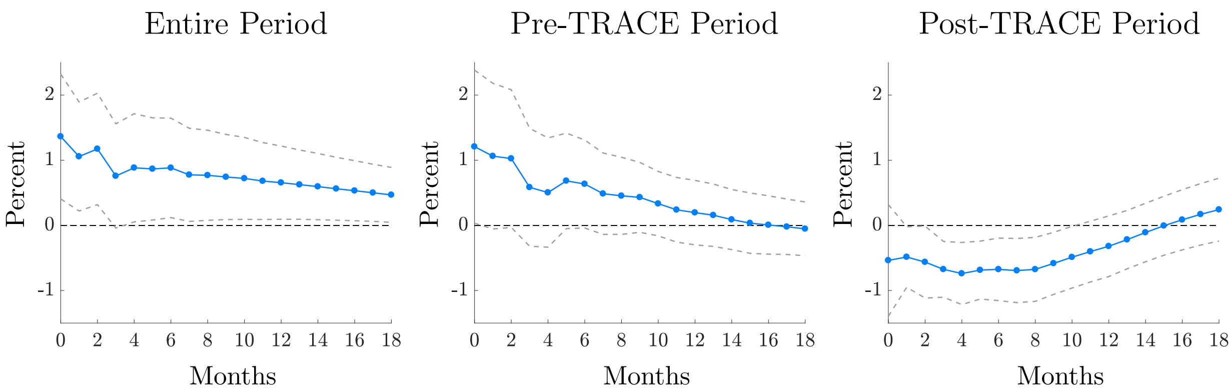

Figure 1.B.9. Response of the excess bond premium to a one-percent increase in the nominal policy rate for different sample periods, estimated using SVAR-IV with unit effect normalization, during the short period around the introduction of the TRACE. Sample period: 1997:11–2008:6; pre-TRACE period: 1997:11–2003:2; post-TRACE period: 2003:3–2008:6. 4-month lags of the four main variables and 2-month lags of the instrument are included. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap. Dashed lines are the 95% confidence interval. The first-stage F-statistic and the heteroscedasticity-robust first-stage F-statistic are 2.5 and 1.7 for the entire period, 2.9 and 2.7 for the pre-TRACE period, and 4.2 and 5.6 for the post-TRACE period.

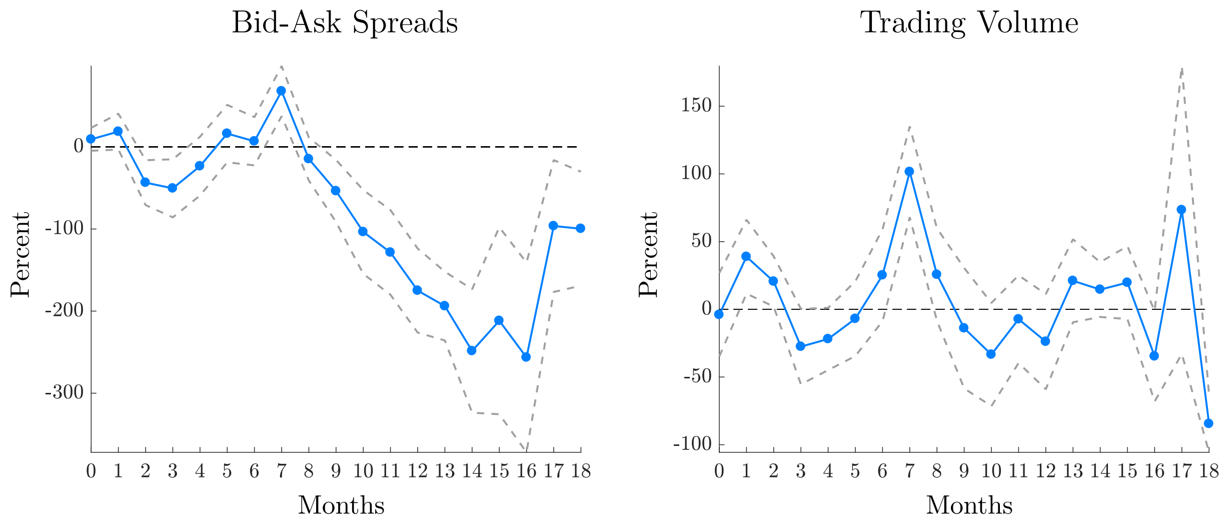

Figure 1.B.10. Response of the bid-ask spreads and the trading volume of corporate bonds to a one-percent increase in the nominal policy rate during the post-TRACE period, estimated using Factor-Augmented LP-IV with unit effect normalization. Sample period: 2003:3–2016:12. Dashed lines are the 95% confidence interval. For details, refer to the notes in Figure 1.C.11 for the bid-ask spreads and Figure 1.C.14 for the trading volume.

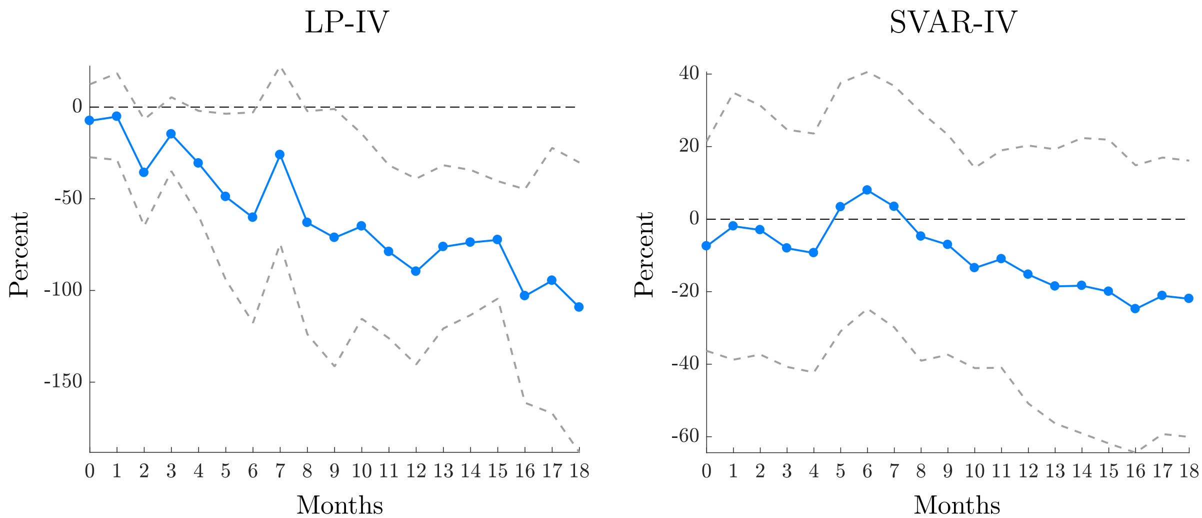

Figure 1.B.11. Response of the bid-ask spreads of corporate bonds to a one-percent increase in the nominal policy rate during the post-TRACE period, estimated using LP-IV (left) and SVAR-IV (right) with unit effect normalization. Sample period: 2003:3–2016:12. 12-month lags of the six main variables (2-year Treasury constant maturity rate, industrial production, inflation rate, the excess bond premium, the bid-ask spreads, and inflation expectation) and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors for LP-IV and the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap for SVAR-IV. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 9, and the heteroscedasticity-robust first-stage F-statistic is 15.6.

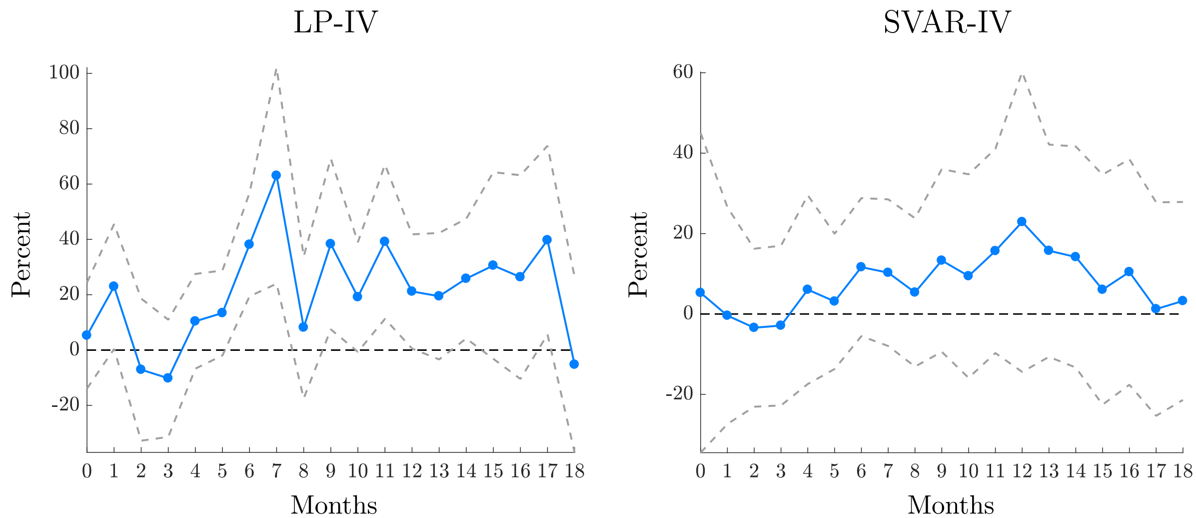

Figure 1.B.12. Response of the trading volume of corporate bonds to a one-percent increase in the nominal policy rate during the post-TRACE period, estimated using LP-IV (left) and SVAR-IV (right) with unit effect normalization. Sample period: 2003:3–2016:12. 12-month lags of the six main variables (2-year Treasury constant maturity rate, industrial production, inflation rate, the excess bond premium, the trading volume, and inflation expectation) and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors for LP-IV and the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap for SVAR-IV. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 11.5, and the heteroscedasticity-robust first-stage F-statistic is 21.

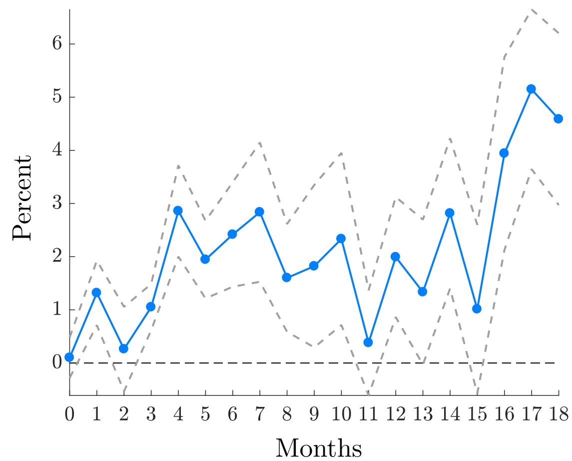

Figure 1.B.13. Response of the real loan rate to a one-percent increase in the nominal policy rate during the post-TRACE period, estimated using Factor-Augmented LP-IV with unit effect normalization. Sample period: 2003:3–2016:12. Dashed lines are the 95% confidence interval. For details, refer to the notes in Figure 1.C.17.

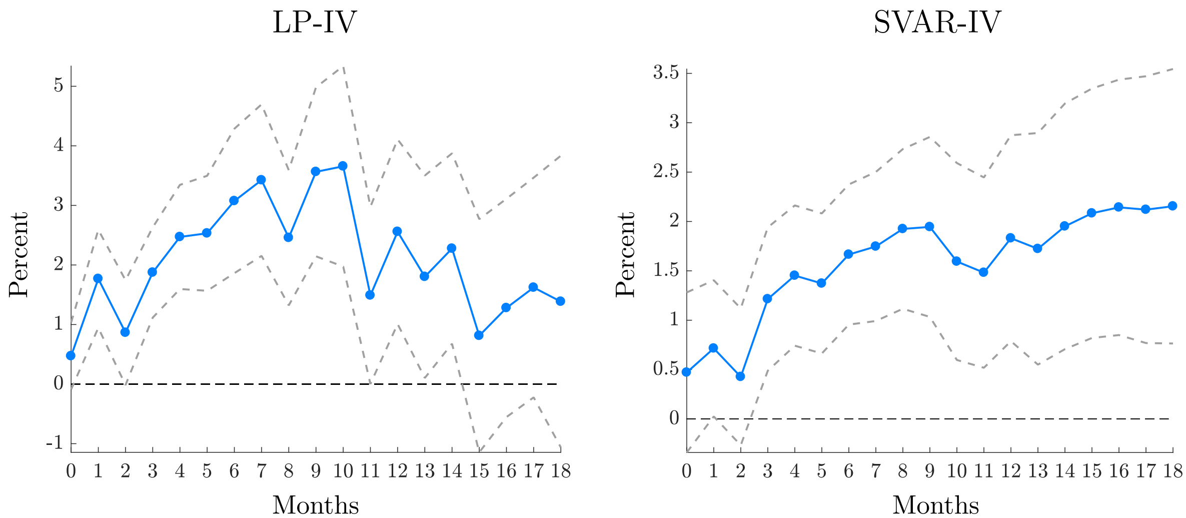

Figure 1.B.14. Response of the real loan rate to a one-percent increase in the nominal policy rate during the post-TRACE period, estimated using LP-IV (left) and SVAR-IV (right) with unit effect normalization. Sample period: 2003:3–2016:12. 12-month lags of the five main variables (2-year Treasury constant maturity rate, industrial production, inflation rate, the excess bond premium, and the real loan rate) and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors for LP-IV and the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap for SVAR-IV. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 8.6, and the heteroscedasticity-robust first-stage F-statistic is 11.7.

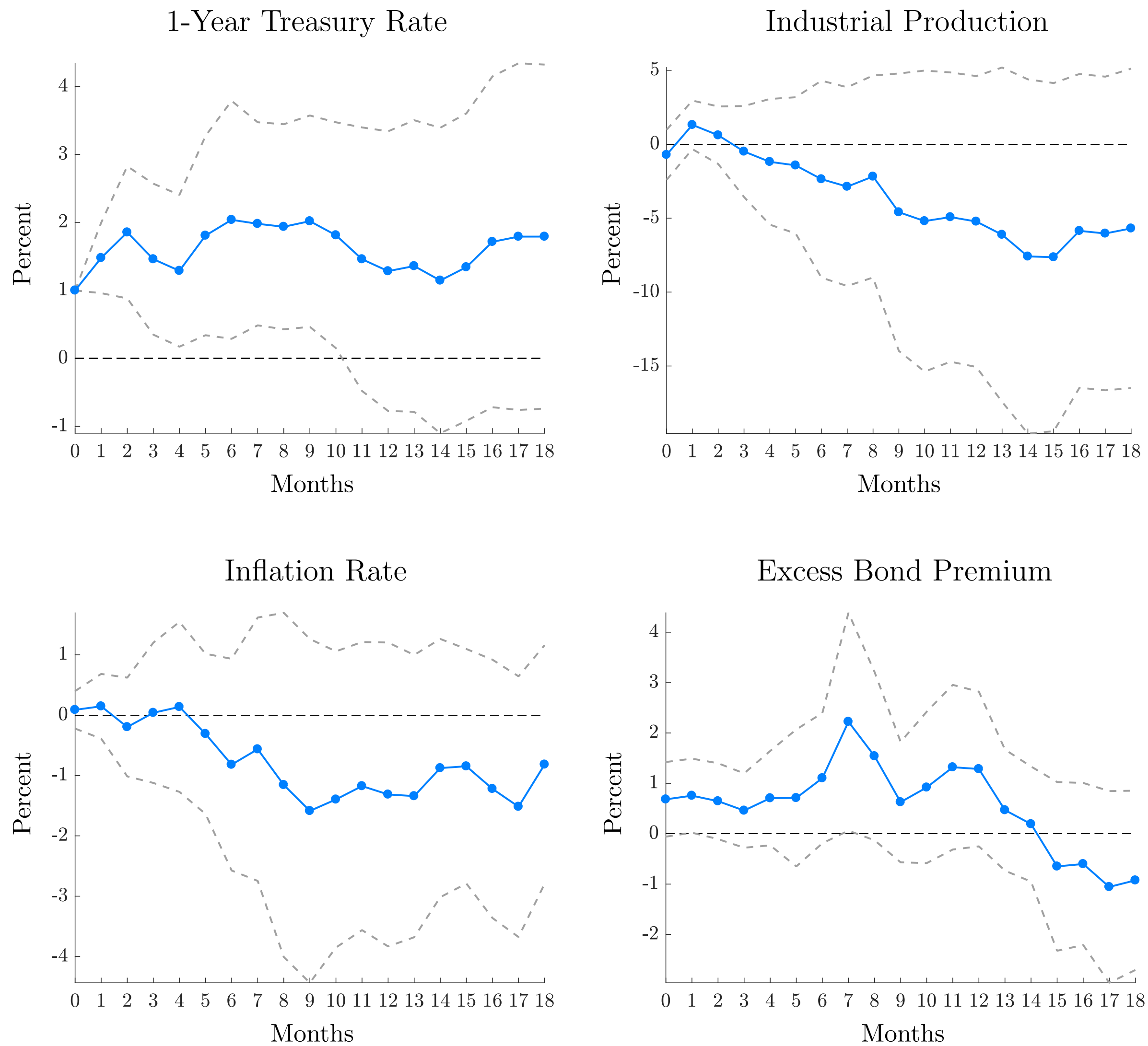

Figure 1.C.1. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, and the excess bond premium to a one-percent increase in the nominal policy rate, estimated using LP-IV with unit effect normalization, during the entire period. Sample period: 1990:2–2016:12. 12-month lags of the four main variables and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 22, and the heteroscedasticity-robust first-stage F-statistic is 14.5.

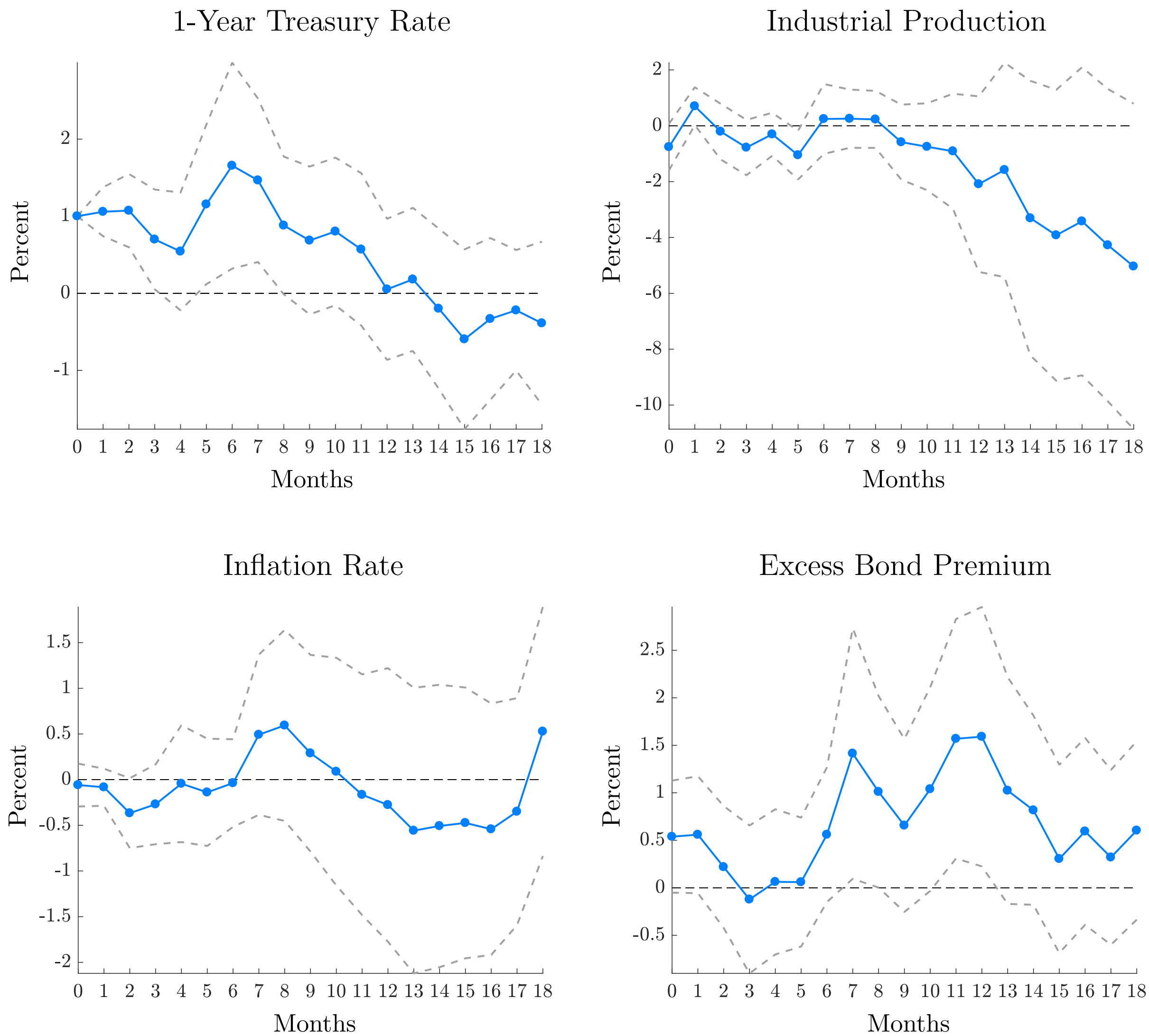

Figure 1.C.2. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, and the excess bond premium to a one-percent increase in the nominal policy rate, estimated using LP-IV with unit effect normalization, during the pre-TRACE period. Sample period: 1990:2–2003:2. 12-month lags of the four main variables and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 15.6, and the heteroscedasticity-robust first-stage F-statistic is 19.4.

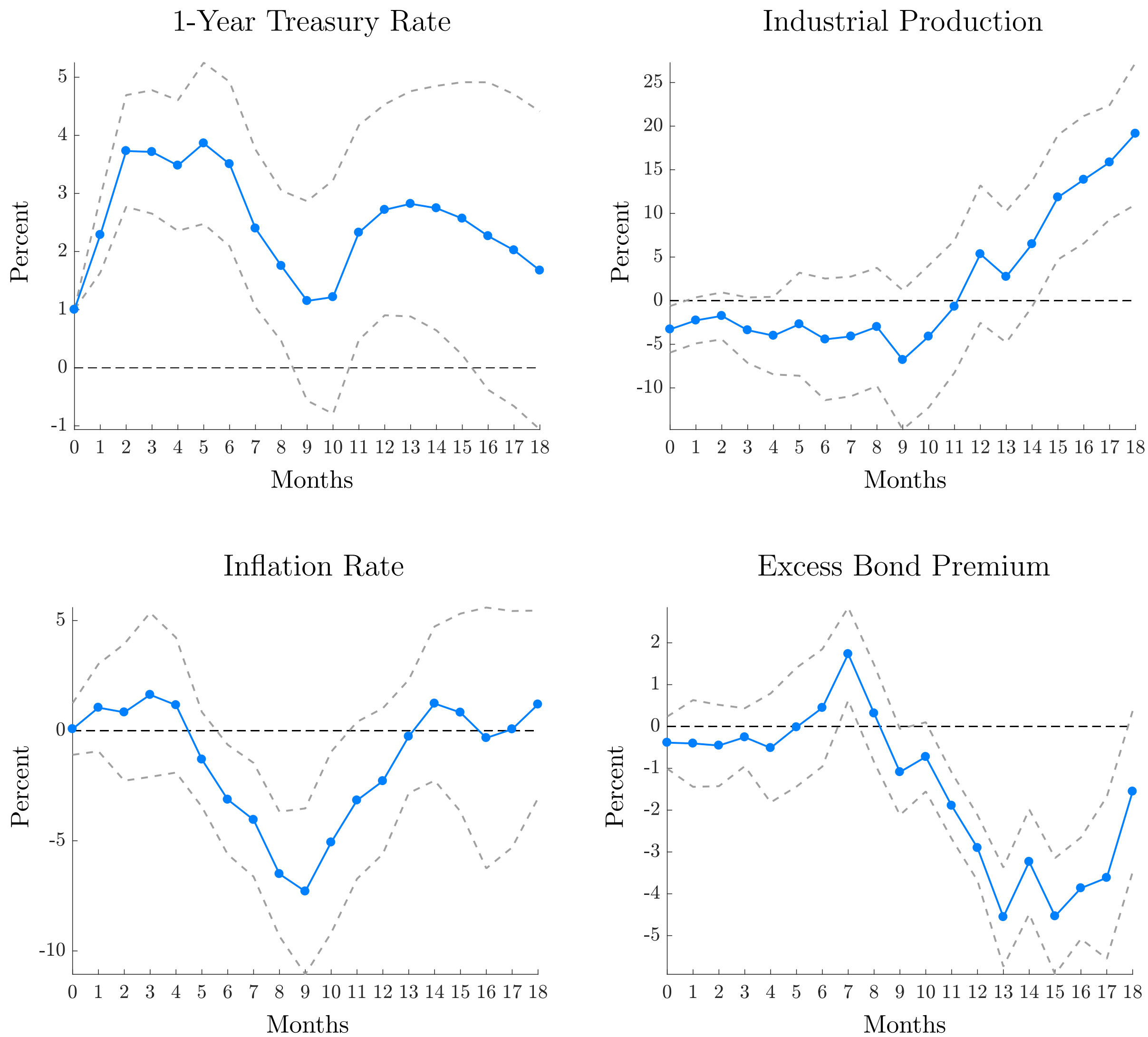

Figure 1.C.3. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, and the excess bond premium to a one-percent increase in the nominal policy rate, estimated using LP-IV with unit effect normalization, during the post-TRACE period. Sample period: 2003:3–2016:12. 12-month lags of the four main variables and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 13.1, and the heteroscedasticity-robust first-stage F-statistic is 14.7.

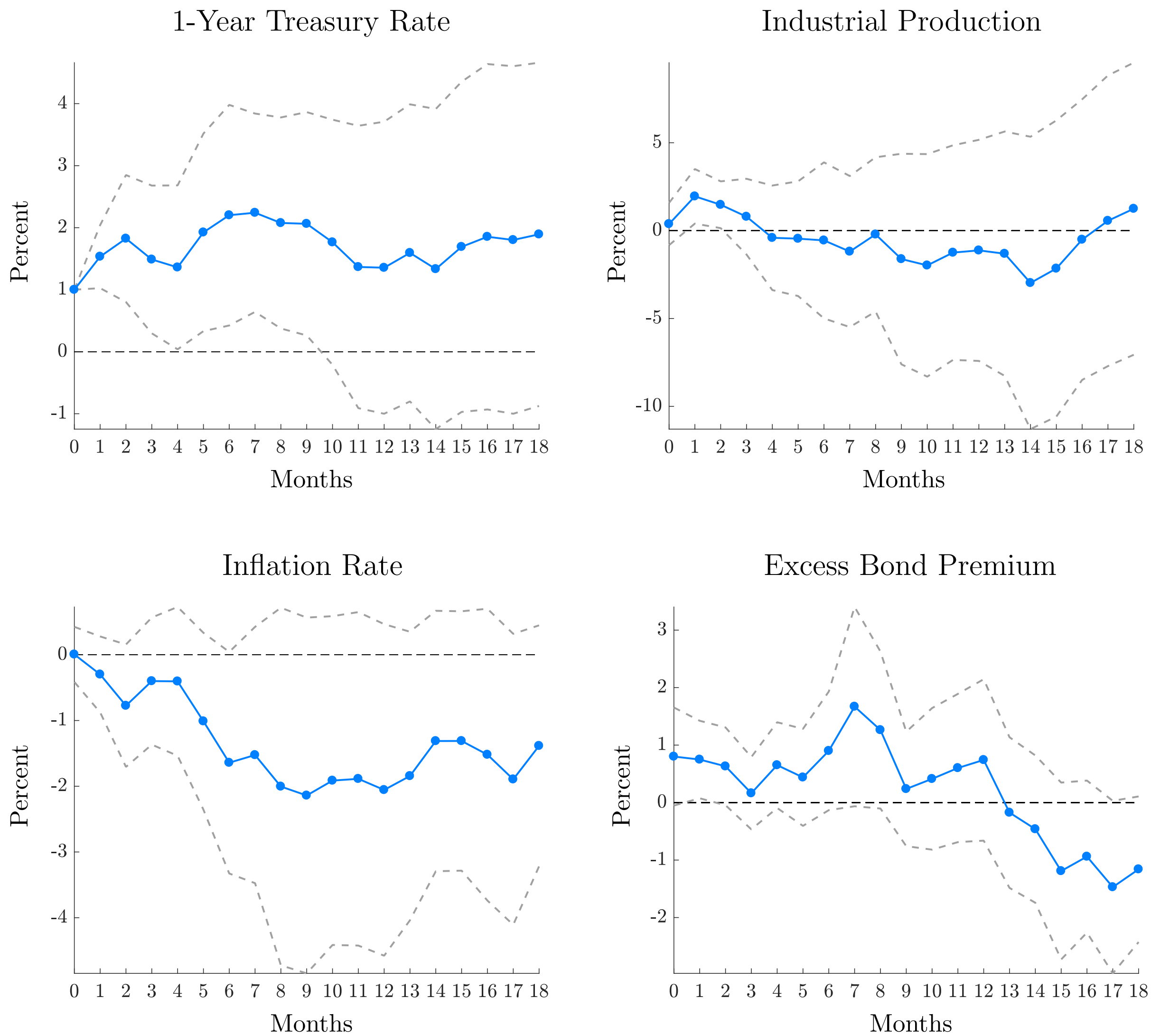

Figure 1.C.4. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, and the excess bond premium to a one-percent increase in the nominal policy rate, estimated using Factor-Augmented LP-IV with unit effect normalization, during the entire period. Sample period: 1990:2–2016:12. 12-month lags of the four main variables and the FRED-MD factors and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 19.1, and the heteroscedasticity-robust first-stage F-statistic is 22.3.

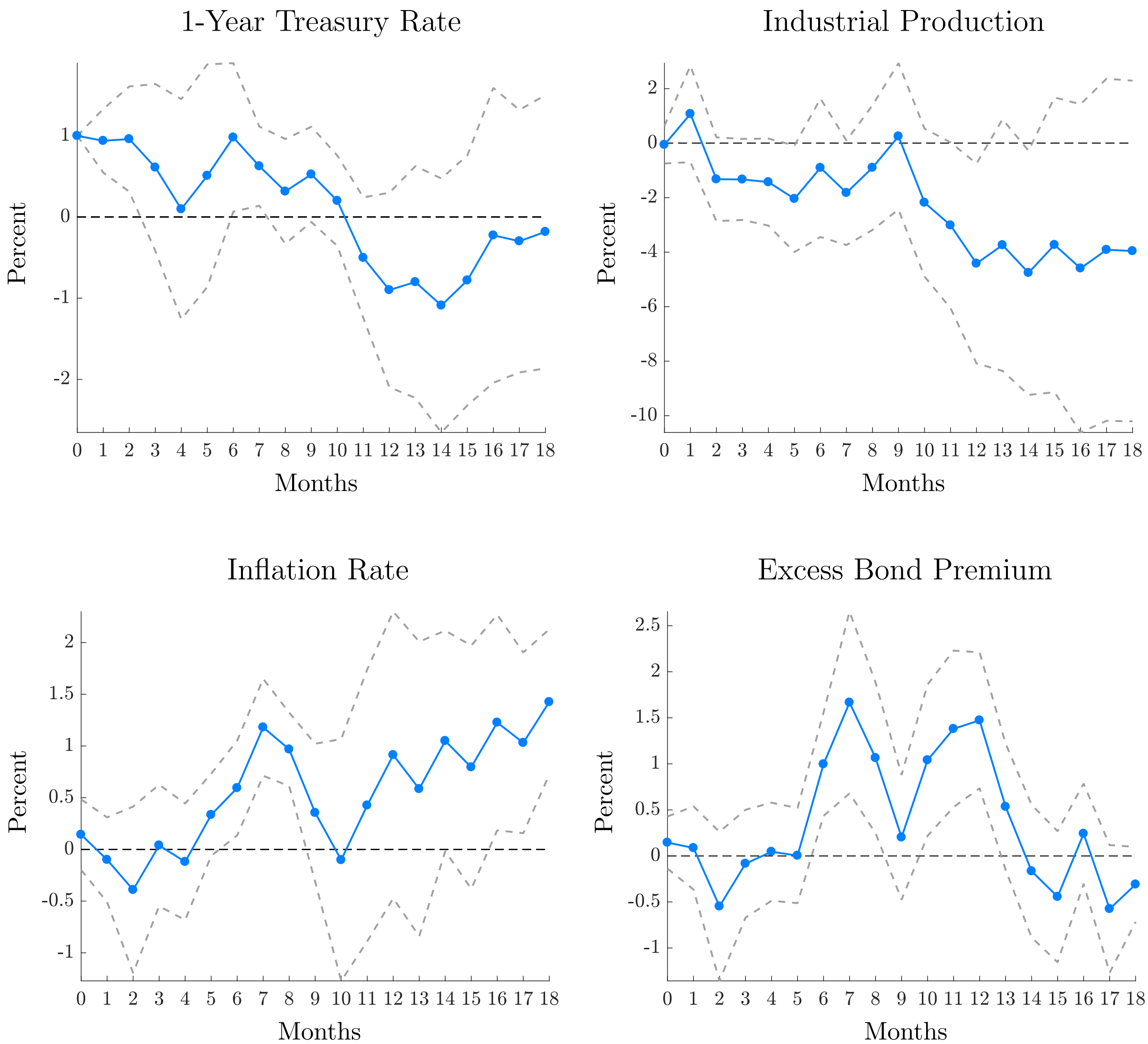

Figure 1.C.5. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, and the excess bond premium to a one-percent increase in the nominal policy rate, estimated using Factor-Augmented LP-IV with unit effect normalization, during the pre-TRACE period. Sample period: 1990:2–2003:2. 12-month lags of the four main variables and the FRED-MD factors and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 6, and the heteroscedasticity-robust first-stage F-statistic is 21.4.

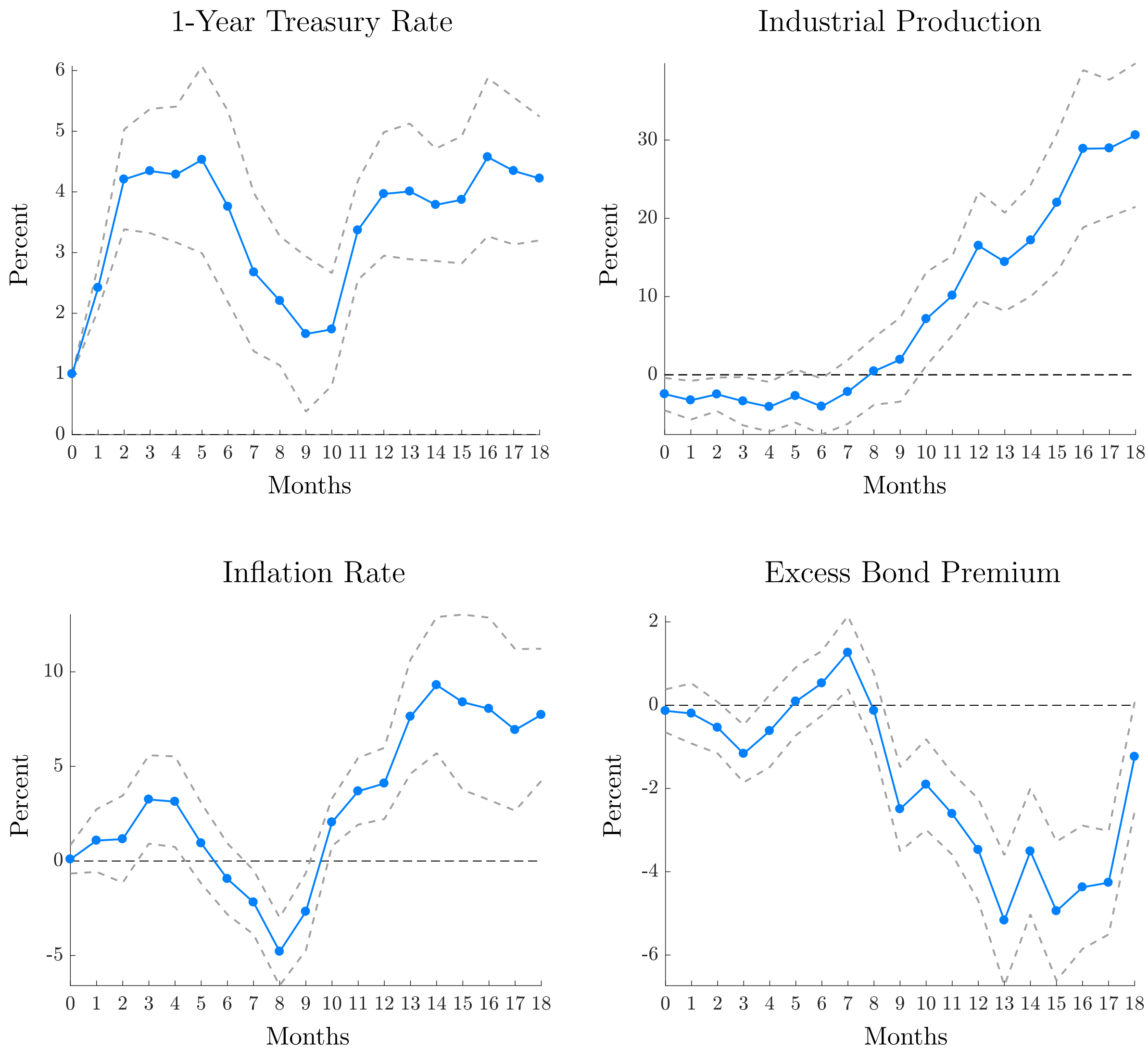

Figure 1.C.6. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, and the excess bond premium to a one-percent increase in the nominal policy rate, estimated using Factor-Augmented LP-IV with unit effect normalization, during the post-TRACE period. Sample period: 2003:3–2016:12. 12-month lags of the four main variables and the FRED-MD factors and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 10.7, and the heteroscedasticity-robust first-stage F-statistic is 20.4.

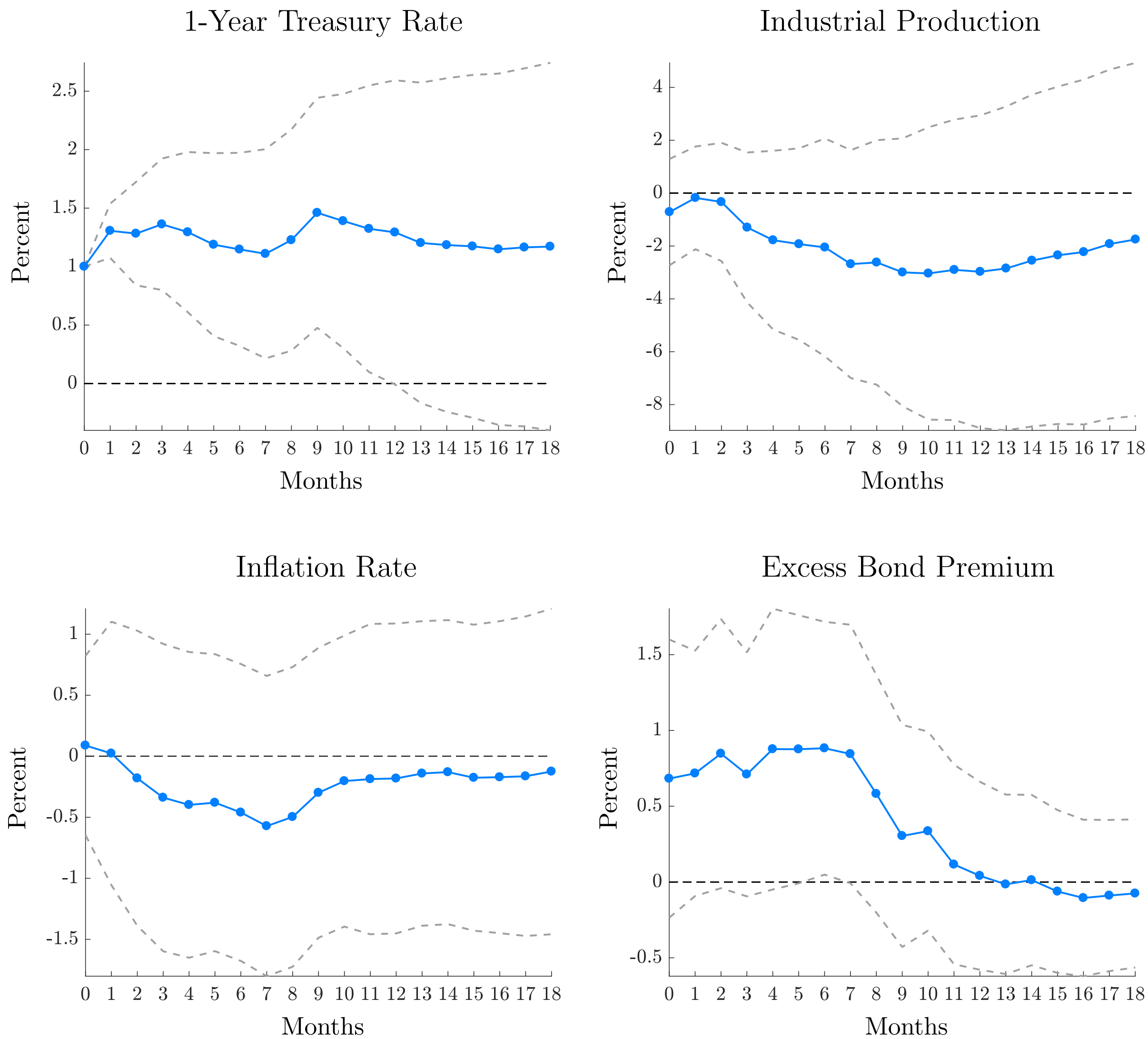

Figure 1.C.7. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, and the excess bond premium to a one-percent increase in the nominal policy rate, estimated using SVAR-IV with unit effect normalization, during the entire period. Sample period: 1990:2–2016:12. 12-month lags of the four main variables and 4-month lags of the instrument are included. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 22, and the heteroscedasticity-robust first-stage F-statistic is 14.5.

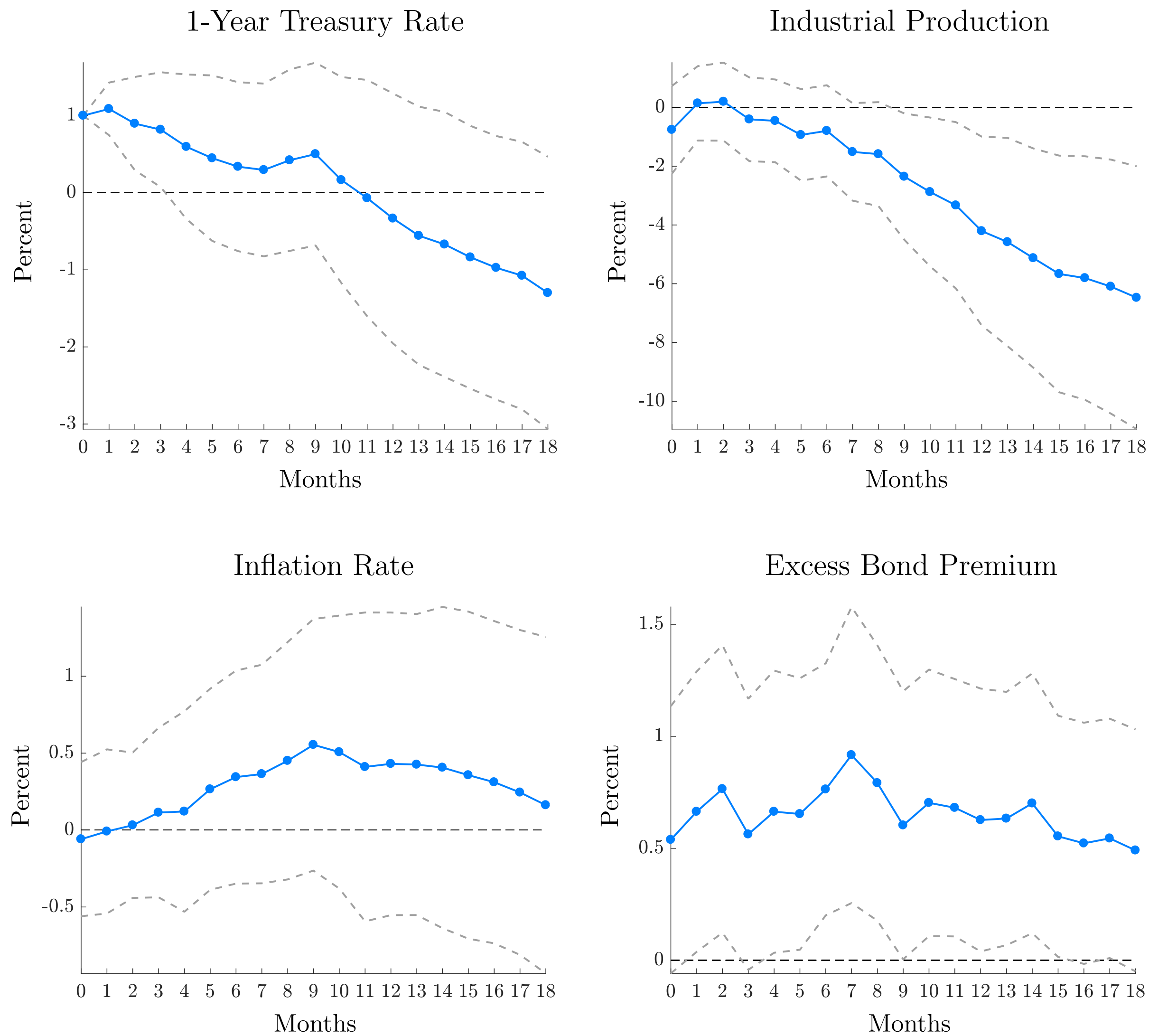

Figure 1.C.8. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, and the excess bond premium to a one-percent increase in the nominal policy rate, estimated using SVAR-IV with unit effect normalization, during the pre-TRACE period. Sample period: 1990:2–2003:2. 12-month lags of the four main variables and 4-month lags of the instrument are included. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 15.6, and the heteroscedasticity-robust first-stage F-statistic is 19.4.

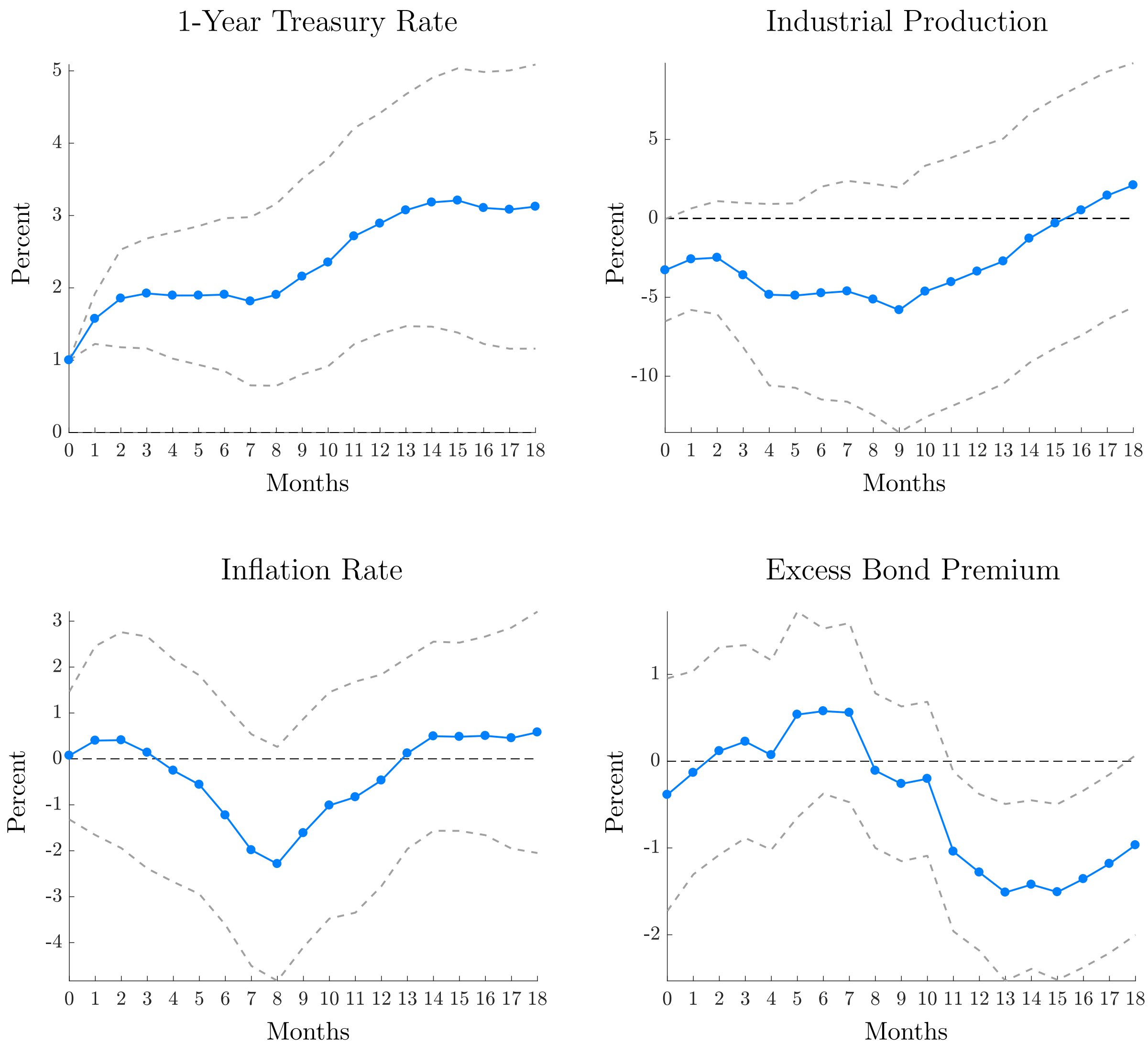

Figure 1.C.9. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, and the excess bond premium to a one-percent increase in the nominal policy rate, estimated using SVAR-IV with unit effect normalization, during the post-TRACE period. Sample period: 2003:3–2016:12. 12-month lags of the four main variables and 4-month lags of the instrument are included. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 13.1, and the heteroscedasticity-robust first-stage F-statistic is 14.7.

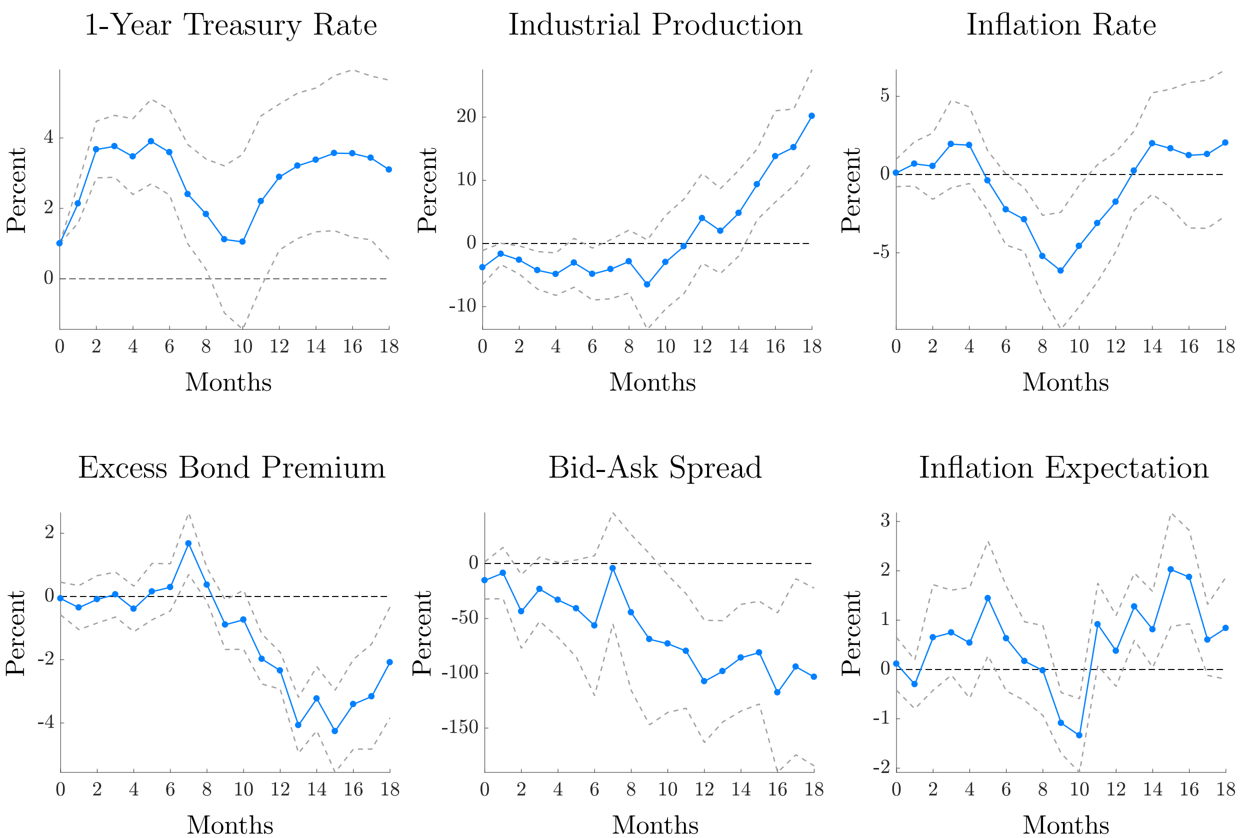

Figure 1.C.10. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, the excess bond premium, the bid-ask spreads, and the inflation expectation, estimated using LP-IV with unit effect normalization, during the post-TRACE period. Sample period: 2003:3–2016:12. 12-month lags of the six main variables and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 11.4, and the heteroscedasticity-robust first-stage F-statistic is 17.5.

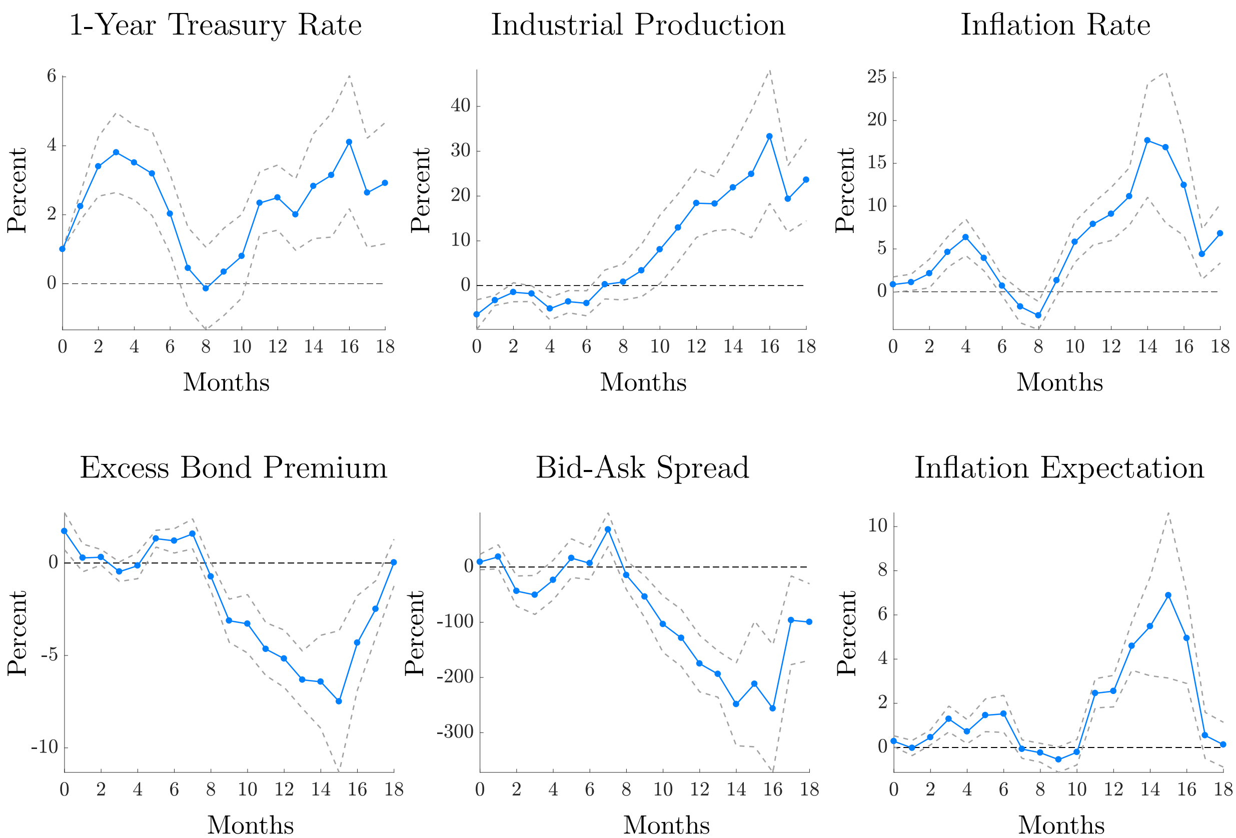

Figure 1.C.11. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, the excess bond premium, the bid-ask spreads, and the inflation expectation, estimated using Factor-Augmented LP-IV with unit effect normalization, during the post-TRACE period. Sample period: 2003:3–2016:12. 12-month lags of the six main variables and the FRED-MD factors and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 5.6, and the heteroscedasticity-robust first-stage F-statistic is 25.9.

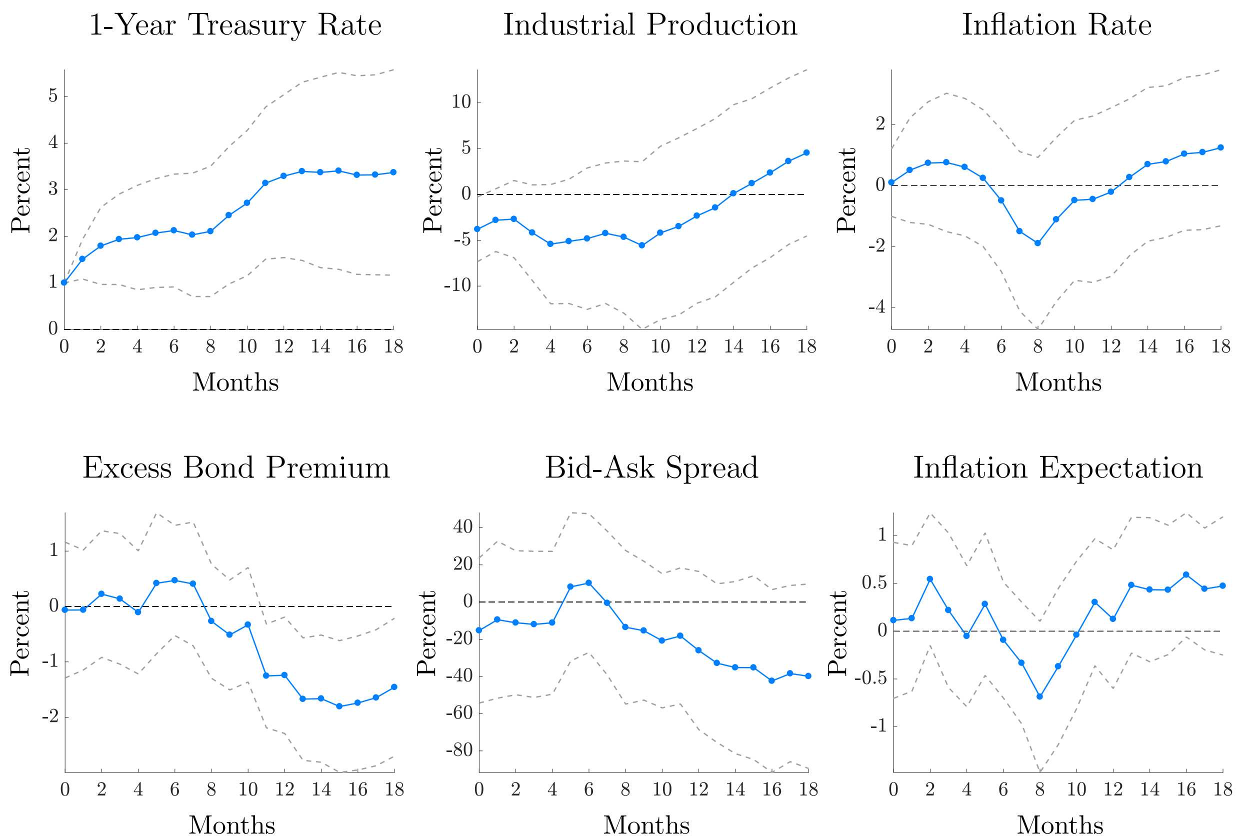

Figure 1.C.12. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, the excess bond premium, the bid-ask spreads, and the inflation expectation, estimated using SVAR-IV with unit effect normalization, during the post-TRACE period. Sample period: 2003:3–2016:12. 12-month lags of the six main variables and 4-month lags of the instrument are included. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 11.4, and the heteroscedasticity-robust first-stage F-statistic is 17.5.

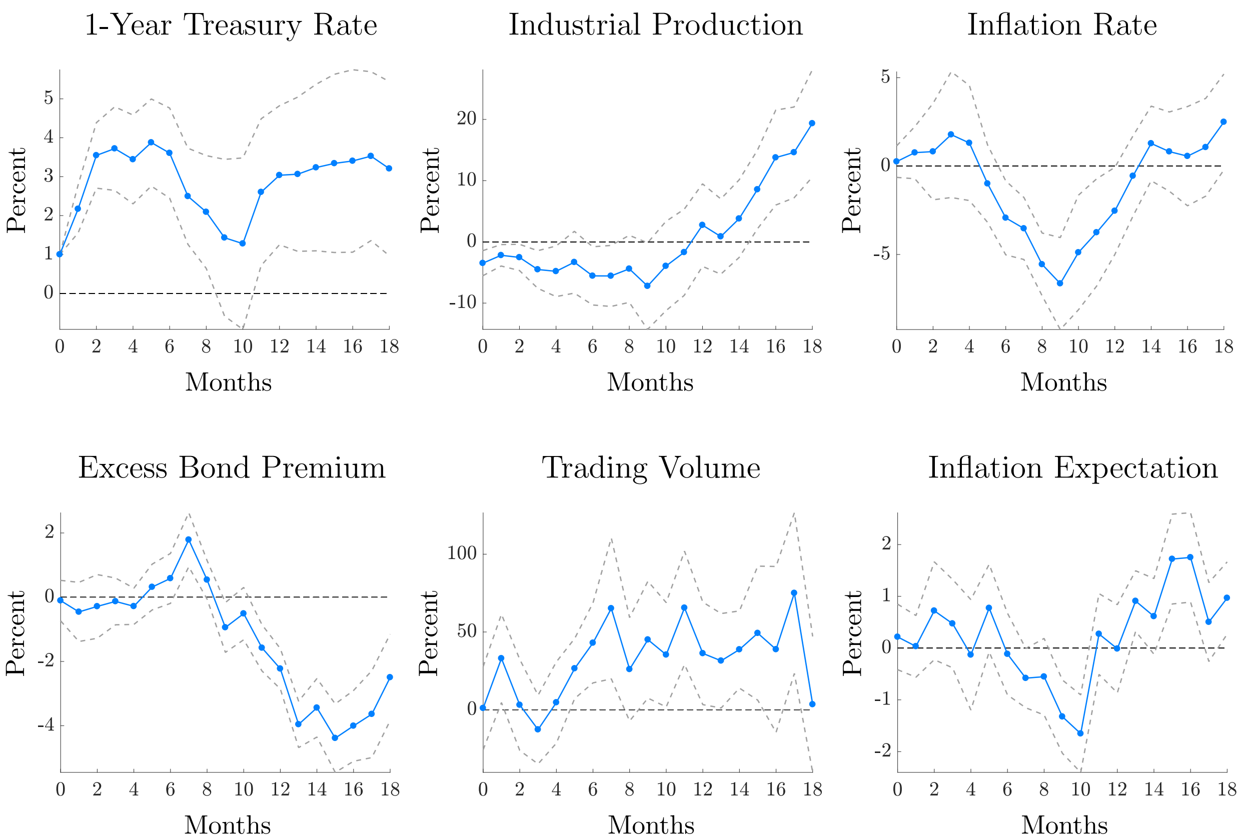

Figure 1.C.13. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, the excess bond premium, the trading volume, and the inflation expectation, estimated using LP-IV with unit effect normalization, during the post-TRACE period. Sample period: 2003:3–2016:12. 12-month lags of the six main variables and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 13.8, and the heteroscedasticity-robust first-stage F-statistic is 25.9.

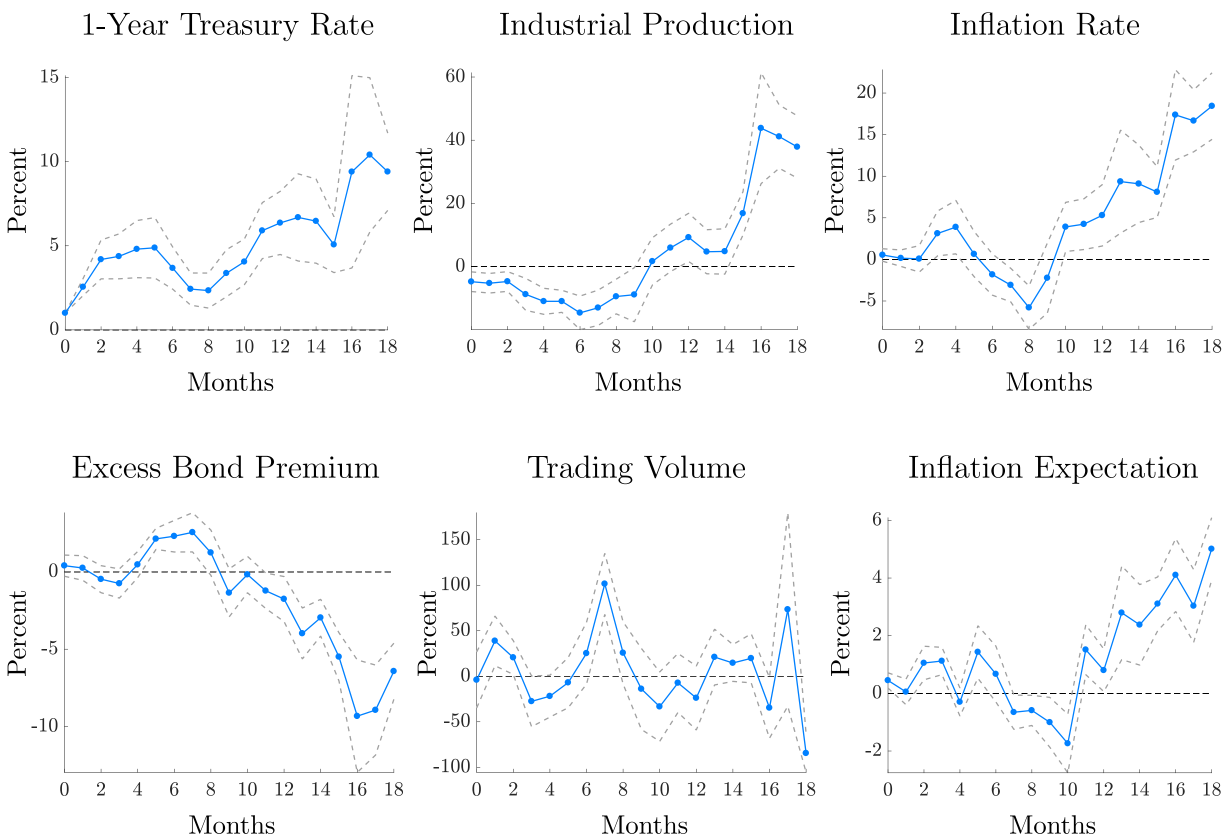

Figure 1.C.14. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, the excess bond premium, the trading volume, and the inflation expectation, estimated using Factor-Augmented LP-IV with unit effect normalization, during the post-TRACE period. Sample period: 2003:3–2016:12. 12-month lags of the six main variables and the FRED-MD factors and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 5.5, and the heteroscedasticity-robust first-stage F-statistic is 27.5.

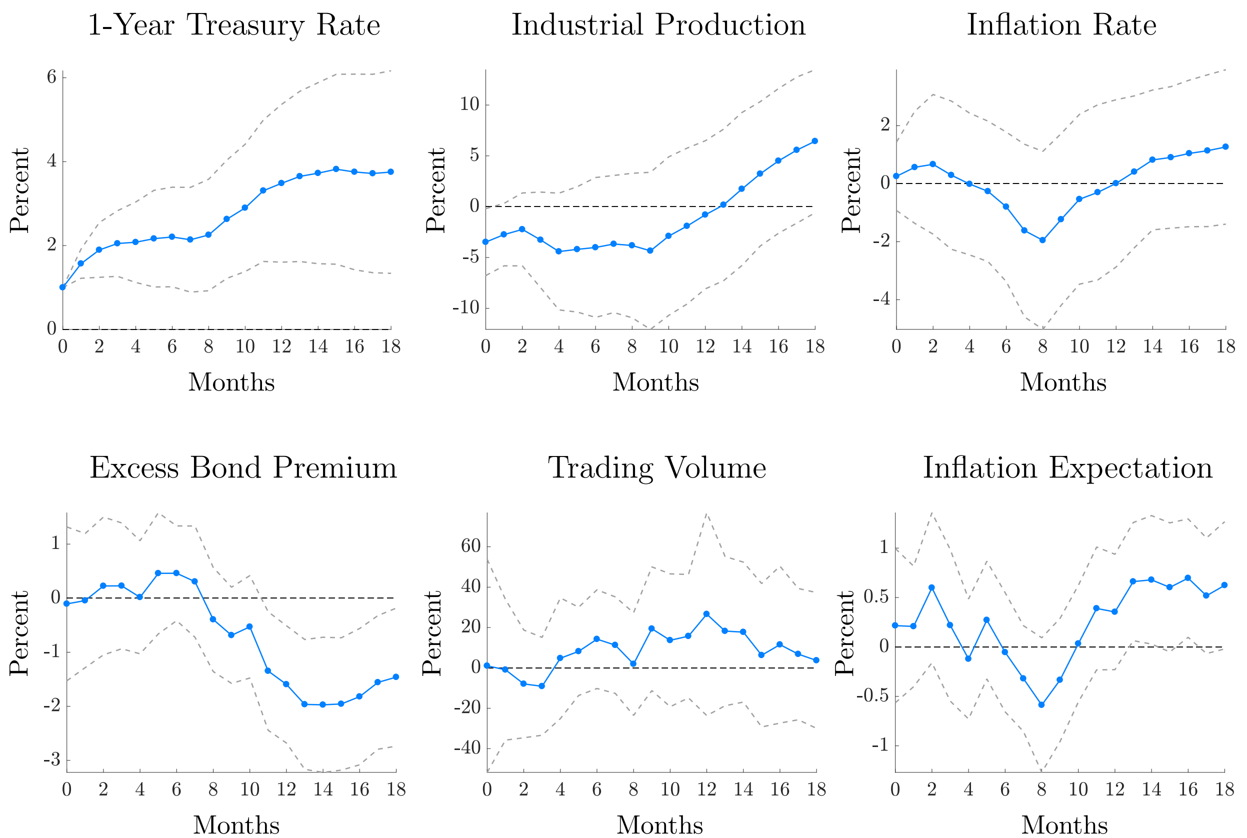

Figure 1.C.15. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, the excess bond premium, the trading volume, and the inflation expectation, estimated using SVAR-IV with unit effect normalization, during the post-TRACE period. Sample period: 2003:3–2016:12. 12-month lags of the six main variables and 4-month lags of the instrument are included. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 13.8, and the heteroscedasticity-robust first-stage F-statistic is 25.9.

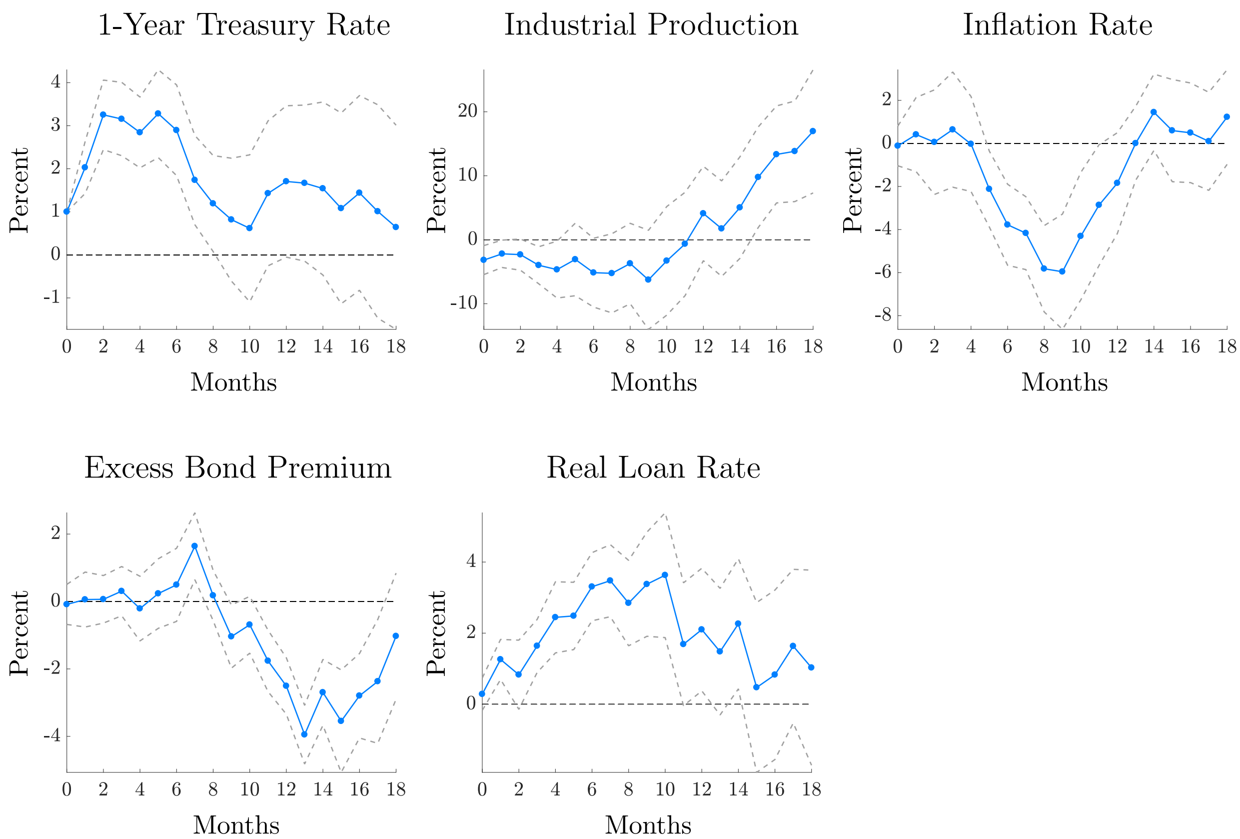

Figure 1.C.16. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, the excess bond premium, the real loan rate, and the inflation expectation, estimated using LP-IV with unit effect normalization, during the post-TRACE period. Sample period: 2003:3–2016:12. 12-month lags of the five main variables and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 13.4, and the heteroscedasticity-robust first-stage F-statistic is 19.4.

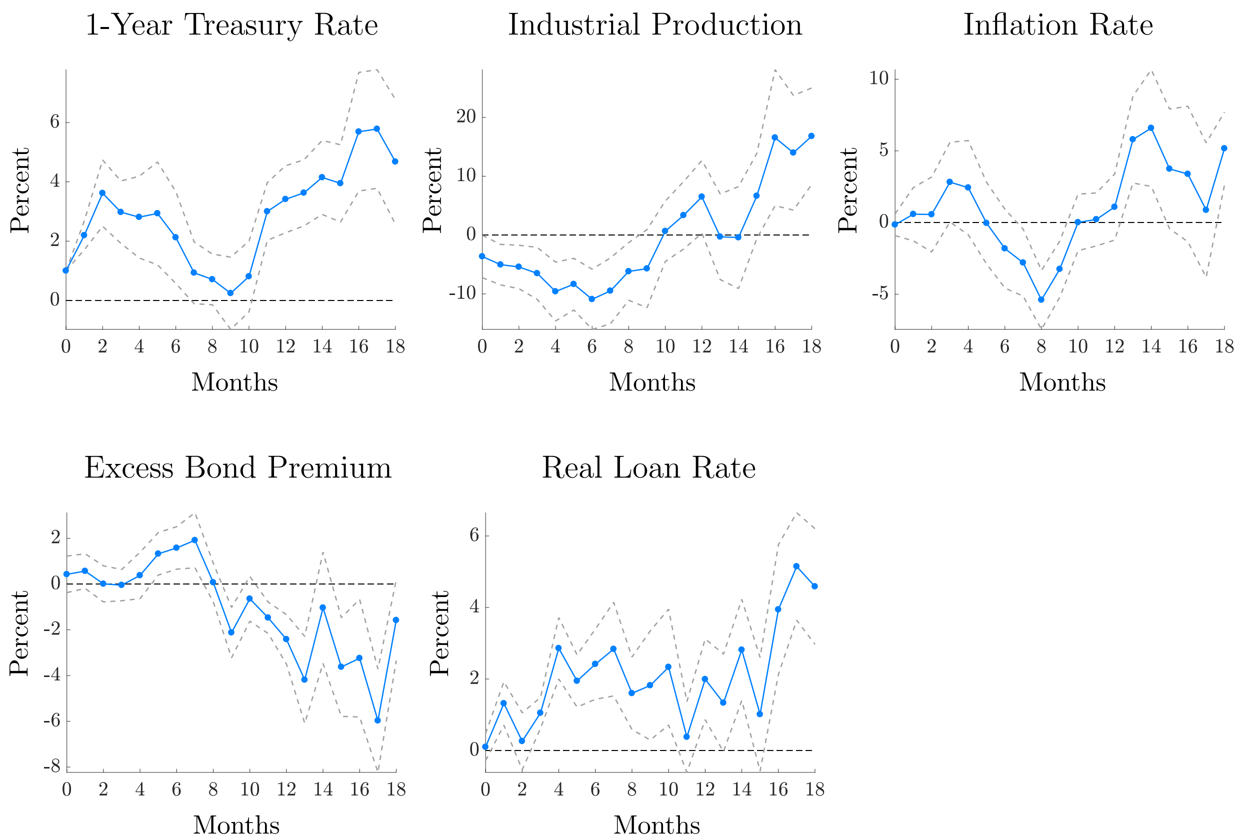

Figure 1.C.17. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, the excess bond premium, the real loan rate, and the inflation expectation, estimated using Factor-Augmented LP-IV with unit effect normalization, during the post-TRACE period. Sample period: 2003:3–2016:12. 12-month lags of the five main variables and the FRED-MD factors and 4-month lags of the instrument are included. Standard errors are calculated using the Newey-West heteroskedastic and autocorrelation consistent standard errors. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 6, and the heteroscedasticity-robust first-stage F-statistic is 14.5.

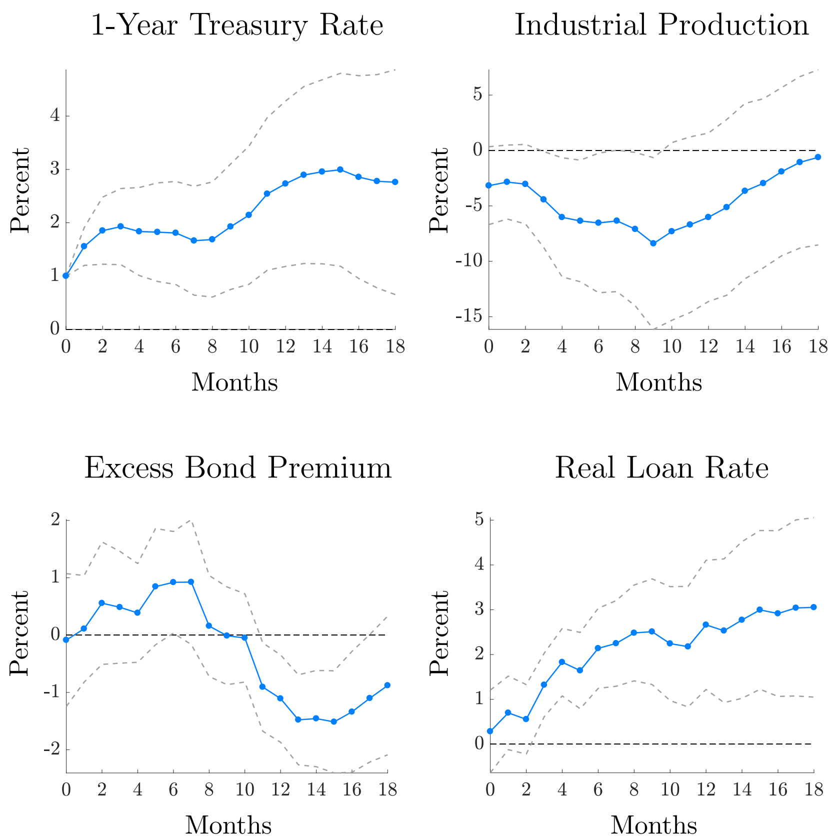

Figure 1.C.18. Response of the 1-year Treasury constant maturity rate, industrial production, inflation rate, the excess bond premium, the real loan rate, and the inflation expectation, estimated using SVAR-IV with unit effect normalization, during the post-TRACE period. Sample period: 2003:3–2016:12. 12-month lags of the five main variables and 4-month lags of the instrument are included. Standard errors are calculated using the sample variances computed from 1,000 draws from a parametric Gaussian bootstrap. Dashed lines are the 95% confidence interval. The first-stage F-statistic is 13.4, and the heteroscedasticity-robust first-stage F-statistic is 19.4.

Chapter 2

Asset Safety versus Asset Liquidity (with Athanasios Geromichalos and Lucas Herrenbrueck)

2.1 Introduction

Recently, there has been a lot of attention on the role of safe assets and liquid assets in the macroeconomy. Many economists, both academics and practitioners, seem to believe that safer assets are also more liquid, and some go a step further by practically using the two terms as synonyms or by merging them into the single term "safe and liquid assets".[1] However, the terms are not synonyms: Safety refers to the probability that the (issuer of

[1] The examples are numerous, so for the sake of brevity we highlight just two. From the IMF's 2012 Global Financial Stability Report: "Safe assets are a desirable part of a portfolio from an investors perspective, as they [...] are highly liquid, permitting investors to liquidate positions easily." And at the 2017 American Economic Association meeting, one session was titled: "How safe and liquid assets impact monetary and financial policy".

the) asset will pay the promised cash flow, at maturity, and liquidity refers to the ease with which an investor can sell the asset if needed, before maturity.[2] Mixing up an asset's safety and liquidity is not just semantics; it can lead to false conclusions and misguided policy recommendations.

For instance, when a credit rating agency characterizes a certain bond as AAA, should investors think of this as an assessment (only) of its safety or also of its liquidity? And, if the answer is affirmative, how can one explain the fact that (the virtually default free) AAA corporate bonds are considered less liquid than their riskier AA counterparts? Moreover, a recent literature in empirical macro-finance measures the so-called safety premium as the spreads between AAA and BAA bonds, assuming that these types of bonds are equally (il)liquid. But if certain assets carry different liquidity premia *because* they have different safety characteristics (as indicated by the conventional wisdom and confirmed by our theory), bonds of "equal liquidity" may be tricky to identify. Finally, policy makers and financial regulators are often concerned about liquidity in certain assets markets. If safety implies liquidity, could we just improve safety and let liquidity follow?

These questions reveal that it is essential to carefully study the relationship between asset safety and asset liquidity, rather than just assume that one implies the other. To do so, we build a multi-asset model in which an asset's safety and liquidity are well-defined and *distinct* from one another. Treating safety as a primitive, we examine the relationship between an asset's safety and its liquidity. We show that the commonly held belief that

[2] Although there are economists who adopt slightly different definitions for both of these terms. For instance, Gorton and Ordonez (2013) emphasize that an important aspect of *safe* assets is that they are "information insensitive". Also, a large number of papers in the New Monetarist literature, assume that an asset's *liquidity* refers to the ease with which that asset can be used to purchase consumption, e.g., by serving as a means of payment; see Lagos, Rocheteau, and Wright (2017). For a careful comparison of the various approaches, see the Literature Review (Section 2.1.1).

"safer assets will be more liquid" is generally justified, but with important exceptions. We then describe the conditions under which a riskier asset can be more liquid than its safe(r) counterparts, and use our model to rationalize several safety-liquidity reversals observed in the data. Finally, we highlight a surprising implication of our model about the effect of an increase in the supply of safe assets on welfare.

To answer the research question at hand we build a dynamic general-equilibrium model with two assets, A and B. The concept of asset safety is straightforward in our framework: asset A is "safe" in the sense that it always pays the promised cash flow, whereas asset B may default with a certain probability, known to everyone.[3] The concept of liquidity is more involved; specifically, we define an asset's liquidity as the ease with which an agent can sell it for cash (if needed). To capture this idea, we employ the monetary model of Lagos and Wright (2005), extended to incorporate asset trade in over-the-counter (OTC) secondary asset markets á la Duffie, Gârleanu, and Pedersen (2005). Another important ingredient we introduce is an entry decision made by the agents: each asset trades in a distinct OTC market, and agents choose to visit the market where they expect to find the best terms. Thus, in our model, an asset's liquidity depends on the *endogenous* choice of agents to visit the secondary market where that asset is traded, not the exogenous characteristics of that market.

More precisely, after agents make their portfolio decisions, two shocks are realized. The first is an idiosyncratic shock that determines whether an agent will have a consumption opportunity in that period, and the second is an aggregate shock that determines whether

[3] Modeling asset A as a default-free asset is not necessary for the main results; all one needs is that asset A is *safer* than asset B, i.e., that it defaults with a lower probability.

asset B will default in that period. Since purchasing the consumption good necessitates the use of a medium of exchange (i.e., money) and carrying money is costly, in equilibrium, agents who receive a consumption opportunity will visit a secondary market to sell assets and boost their cash holdings. Hence, assets have *indirect liquidity* properties (they can be sold for cash, although they do not serve directly as means of payment), and their equilibrium price in the primary market will typically contain a *liquidity premium*, i.e., it will exceed the fundamental value of holding the asset to maturity.

The first result of the paper is that, other things equal, the safer asset carries a higher liquidity premium, and that premium is increasing in the default probability of asset B.[4] The intuition behind this result is as follows. An agent who turns out to be an asset seller can only visit one OTC market at a time; since, typically, assets are costly to own due to the liquidity premium, agents choose to 'specialize' ex-ante in asset A or B. Unlike sellers, who are committed to visit the market of the asset in which they chose to specialize, asset buyers are free to visit any market they wish, since their money is good to buy any asset. As a result, in the event of default, all the asset buyers (even those who had chosen to specialize in asset B) will rush into the market for asset A. Naturally, the *ex-post* possibility of a market flooded with buyers (in the event of default) is a powerful force attracting agents to specialize in asset A *ex-ante*, as they realize that in this market they will have a high expected trade probability, if they turn out to be sellers. This is crucial because it is the sell-probability that affects an asset's issue price: an agent who buys an asset (in the primary market) is willing to pay a higher price if she expects that it will be easy to sell that asset

[4] This statement adopts the liquidity premium as the measure of an asset's liquidity. Later in the analysis, we also consider an alternative measure of liquidity, namely, trade volume, and show that the result is still valid. That is, we show that trade volume is higher in the secondary market for the safer asset, and that the difference in trade volumes between markets A and B is increasing in the default probability of asset B.

down the road. It is mainly through this channel that even a small default probability for asset B can be *magnified* into a big endogenous liquidity advantage for asset A, even with constant returns to scale (CRS) in the OTC matching technology.

So far we have assumed that all parameters other than asset safety are kept equal. Allowing for differences in asset supplies delivers the second important result of the paper.[5] Even with slight increasing returns to scale (IRS) in OTC matching, demand curves can be *upward sloping*, because an asset in large supply is likely to be more liquid. Consequently, asset B can be more liquid than asset A, despite being less safe, as long as the supply of the former is large enough compared to the latter.

The intuition is as follows. As we have seen, our model gives rise to an endogenous channel whereby a safer asset also acquires a liquidity advantage. However, whether this advantage will materialize also depends on the relative supply of the safe asset. If the supply of asset A is limited, as more agents choose to specialize in that asset each one of them will only hold a small amount, and any bilateral meeting in the market for asset A will generate a small surplus. This effect, which we dub the "*dilution effect*", tends to make an asset in large supply more attractive to agents. Now, with the dilution effect in mind, consider an increase in the supply of asset B. As the supply rises, more agents are willing to trade in the secondary market for asset B because of the increase in the expected trading surplus (conditional on no-default). Generally, asset buyers are more sensitive to this increase because their entry choice is more 'elastic' due to the lack of precommitment. As a result,

[5] There are two more parameters held equal in the background: the efficiency of matching in each OTC market and the bargaining power of buyers versus sellers in each OTC market, often put together under the umbrella of "OTC market micro-structure". Since our goal is to develop a theory that links asset safety and asset liquidity in an unbiased way, we assume that these parameters are always equal in both OTC markets. This guarantees that any difference in liquidity between the two assets is driven exclusively by differences in safety and not by exogenous market characteristics.

the trade probability in market B for sellers increases by far more than that for buyers, and, as we have highlighted, it is the sell-probability that matters most for the determination of the issue price. If, to the channel described so far, one adds (even slight) IRS in the matching technology, the agents' incentive to coordinate on the market of the asset in high supply becomes so strong that demand curves can slope upwards, and the less safe asset can carry the higher liquidity premium.

An interesting fact that has recently drawn the attention of economists is that, in the U.S., the virtually default-free AAA bonds are less liquid than (the less safe) AA corporate bonds (see Section 2.4.2.1 for details and empirical evidence). Our model can shed some light on this puzzling empirical observation. In recent years, regulations introduced to improve the stability and transparency of the financial system (most prominently, the Dodd-Frank Act) have made it especially hard for corporations to attain the AAA score. As a result, the supply of such bonds has fallen dramatically. During the same time, the yield on AA corporate bonds has been *lower* than that on AAA bonds, even without controlling for the risk premium associated with the riskier AA bonds. While it is plausible to attribute this differential to a higher liquidity premium enjoyed by AA corporate bonds—and this is precisely what practitioners have claimed—existing models of asset liquidity cannot capture this stylized fact (for details, see Section 2.1.1). Our 'indirect liquidity' approach, coupled with endogenous market entry, is key for explaining why an asset in limited supply tends to be illiquid.

This is not the only case where the commonly held belief, "safety and liquidity go together", is violated. Christensen and Mirkov (2019) highlight yet another class of bonds—Swiss Confederation Bonds—that are considered extremely safe, yet not particularly liquid.

And, vice versa, Beber, Brandt, and Kavajecz (2008) report that Italian government bonds are among the most liquid, but also the most risky of Euro-area sovereign bonds.

The model also delivers a surprising result regarding welfare. A large body of recent literature highlights that the supply of safe assets has been scarce, and that increasing this supply would be beneficial for welfare (see for example Caballero, Farhi, and Gourinchas, 2017). In our model this result is not necessarily true: there exists a region of parameter values for which welfare is decreasing in the supply of the safe asset. The intuition is as follows. In our model agents have the opportunity to acquire additional cash by selling assets in the secondary market. When the safe asset becomes more plentiful, agents expect that it will be easier to acquire extra cash *ex-post* and, thus, choose to hold less money *ex-ante*. This channel depresses money demand, which, in turn, decreases the values of money and of the trade that the existing money supply can support.

2.1.1 Literature Review

Our paper is related to the recent "New Monetarist" literature (reviewed by Lagos, Rocheteau, and Wright, 2017) that has highlighted the importance of asset liquidity for the determination of asset prices. See for example Geromichalos, Licari, and Suárez-Lledó (2007), Lagos (2011), Nosal and Rocheteau (2012), Andolfatto, Berentsen, and Waller (2013), and Hu and Rocheteau (2015). In these papers the liquidity properties of assets are 'direct', in the sense that assets serve as a media of exchange or collateral, thus, helping to facilitate trade in frictional decentralized markets for *goods*. In our paper, on the other hand, asset liquidity is *indirect*, and it stems from the fact that agents can sell assets for money in secondary *asset* markets. This approach to asset liquidity is not only empirically relevant, but also integrates the concepts of liquidity adopted by monetary economics and finance (see Geromichalos and Herrenbrueck, 2016a, for details). The indirect liquidity approach is employed in a number of recent papers, including Berentsen, Huber, and Marchesiani (2014, 2016), Mattesini and Nosal (2016), Han (2015), Herrenbrueck and Geromichalos (2017), Herrenbrueck (2019a), and Madison (2019).

Naturally, our paper is also related to the growing literature that studies the role of safe assets in the macroeconomy. Examples of such papers include Gorton, Lewellen, and Metrick (2012), Piazzesi and Schneider (2016), He, Krishnamurthy, and Milbradt (2019), Caballero, Farhi, and Gourinchas (2017), Gorton (2017). None of these papers study explicitly the relationship between asset safety and asset liquidity. Also, to the best of our knowledge, our paper is the only one that highlights the possibility that welfare can be decreasing in the supply of the safe asset.

Our paper is related to Andolfatto and Martin (2013) who consider a model where a physical asset, whose expected short-run return is subject to a news shock, can serve as medium of exchange. The authors show that the non-disclosure of news can enhance the asset's property as an exchange medium. As we have already highlighted, the concept of (indirect) liquidity adopted here is different, and so is the concept of safety.[6] Here, an asset's safety is simply the (ex-ante) probability with which the assets will not pay the promised cash flow, which, in turn, is a function of the issuer's credit worthiness. This probability is public knowledge and can be thought of (or approximated by) a credit rating agency's

[6] In fact, the authors of that paper never use the term "safety". However, the idea that some assets are more "information sensitive" than others is close to the definition of safety adopted by Gorton and Ordonez (2013); see footnote 2.

score. Rocheteau (2011) studies a model where bonds serve as media of exchange alongside with money. The author shows that if the bond holders (and goods buyers) have private information about the bond's return, then money will endogenously arise as more liquid asset (i.e., a better medium of exchange). Our paper studies the link between asset safety and liquidity, assuming that the various assets' safety characteristics are public knowledge, i.e., we do not have a story of private information.

Our paper is also related to Lagos (2010), who considers a model where bonds, whose return is deterministic, and stocks, whose return is stochastic, compete as media of exchange. The author quantitatively demonstrates that the equity premium puzzle can be explained through a liquidity differential between the safe and the risky asset. Jacquet (2021) employs a similar model, but includes a larger variety of asset classes and ex-ante heterogeneous agents. The author shows that the equilibrium displays a "class structure" in the sense that agents with different liquidity needs will only be willing to hold assets of a certain risk structure. Our paper differs from the aforementioned papers, not only because it employs a different model of liquidity, but also because it predicts that an asset in large(r) supply may carry a higher liquidity premium. This result cannot be obtained in Lagos (2010), or other related papers, as in these papers the asset demand curve is typically decreasing. Thus, our model of indirect liquidity and endogenous market entry has the unique ability to rationalize why assets in limited supply can be highly illiquid, even when they enjoy a high credit rating (e.g., AAA corporate bonds in the U.S.).

He and Milbradt (2014) study a one-asset model where defaultable corporate bonds are traded in an OTC secondary market, and show that the inverse bid-ask spread, which is their proxy for bond liquidity, is positively related with credit ratings. However, in their model the probability of trade between agents is exogenous. We define liquidity as the ease with which an investor can sell her assets, if needed. We build a two-asset (easily extended to an N-asset) model, where the probability of selling an asset depends on the *endogenous* decision of agents to visit the various asset markets, which, in turn, is a function of each asset's safety characteristics. Also, He and Milbradt (2014) employ the model of Duffie, Gârleanu, and Pedersen (2005) where assets are indivisible, i.e., agents can hold either 0 or 1 units of the asset. Our model also incorporates OTC secondary asset trade *à la* Duffie, Gârleanu, and Pedersen (2005), but does so within the monetary model of Lagos and Wright (2005), which allows us to study perfectly divisible asset supplies, and opens up a number of new insights. Such insights include the possibility of upward-sloping demand curves, the possibility that a riskier asset can be more liquid in general equilibrium, and the fact that welfare can be decreasing in the supply of safe assets.

In related empirical work, Krishnamurthy and Vissing-Jorgensen (2012) clearly distinguish between asset safety and liquidity, and extract safety and liquidity premia from the data to explain why Treasury yields have been decreasing. Their model identifies the safety premium through the spreads between AAA and BAA bonds, assuming that both of these types of bonds are equally illiquid. The present paper demonstrates that certain assets may carry different liquidity premia precisely because they are characterized by different default risks. This, in turn, highlights that we need more theory that studies the relationship between asset safety and liquidity, and we view the present paper as a part of this important agenda.

2.2 The Model

We start with the description of the physical environment. Section 2.2.1 contains a discussion of some key modeling choices.

Our model is a hybrid of Lagos and Wright (2005) (henceforth, LW) and Duffie, Gârleanu, and Pedersen (2005). Time is discrete and continues forever. Each period is divided into three subperiods, characterized by different types of trade (for an illustration, see Figures 2.C.1–2.C.2 below). In the first subperiod, agents trade in OTC secondary asset markets. In the second subperiod, they trade in a decentralized goods market (DM). Finally, in the third subperiod, agents trade in a centralized market (CM). The CM is the typical settlement market of LW, where agents settle their old portfolios and choose new ones. The DM is a decentralized market characterized by anonymity and imperfect commitment, where agents meet bilaterally and trade a special good. These frictions make a medium of exchange necessary, and we assume that only money can serve this role. The OTC markets allow agents with different liquidity needs to rebalance their portfolio by selling assets for money.

Agents live forever and discount future between periods, but not subperiods, at rate $\beta \in (0,1)$. There are two types of agents, consumers and producers, distinguished by their roles in the DM. The measure of each type is normalized to the unit. Consumers consume in the DM and the CM and supply labor in the CM; producers produce in the DM and consume and supply labor in the CM. All agents have access to a technology that transforms one unit of labor in the CM into one unit of the CM good, which is also the numeraire. The preferences of consumers and producers within a period are given by $\mathcal{U}(X,H,q) = X - H + u(q)$ and $\mathcal{V}(X,H,q) = X - H - q$, respectively, where X denotes consumption of CM goods, H is labor supply in the CM, and q stands for DM goods produced and consumed. We assume that u is twice continuously differentiable, with $u' > 0$, $u'(0) = \infty$, $u'(\infty) = 0$, and $u'' < 0$. The term q^* denotes the first-best level of trade in the DM, i.e., it satisfies $u'(q^*) = 1$. All goods are perishable between periods.

Notice that in this model the agents dubbed "producers" will never choose to hold any assets, as long as these assets are priced at a premium for their liquidity. The reason is simple; a producer's identity is permanent, so why would she ever pay this premium when she knows that she will never have a liquidity need (in the DM)? As a result, all the interesting portfolio choices in this model are made by the "consumers". Thus, henceforth, we will refer to the "consumers" simply as "agents". When we use the terms "buyer" and "seller", it will be exclusively to characterize the role of these agents in the secondary asset market. We now describe all the assets available in this economy.

There is a perfectly divisible object called fiat money that can be purchased in the CM at the price φ in terms of CM goods. The supply of money is controlled by a monetary authority, and follows the rule $M_{t+1} = (1 + \mu)M_t$, with $\mu > \beta - 1$. New money is introduced if $\mu > 0$, or withdrawn if $\mu < 0$, via lump-sum transfers in the CM. Money has no intrinsic value, but it possesses all the properties that make it an acceptable medium of exchange in the DM, e.g., it is portable, storable, and recognizable by everyone in the economy. Using the Fisher equation, we summarize the money growth rate by $i = (1 + \mu + \beta)/\beta$; the rate i will be a useful benchmark as the yield on a completely illiquid asset. (Thus, i should not be thought of as representing the yield on T-bills; see Geromichalos and Herrenbrueck (2017) for a discussion, and Herrenbrueck (2019b) for evidence.)

There are also two types of assets, asset A and asset B. These are one-period, nominal bonds with a face value of one dollar; their supply is exogenous and denoted by S_A and S_B, respectively. Asset j can be purchased at price p_j, $j = \{A, B\}$, in the CM, which we think of as the primary market. After leaving the CM agents receive an idiosyncratic consumption shock (discussed below) and may trade these assets (before maturity) in a secondary OTC market. Each asset j trades in a distinct secondary market, which we dub OTC_j, $j = \{A, B\}$. To make things tractable, we assume that agents can only hold either asset A or asset B, and can visit only one OTC market per period.[7] Thus, we say they "specialize" in holding asset A or B. However, agents are free to choose any quantity of money and the asset of their choice.

The economy is characterized by two shocks, both of which are revealed after the CM closes and before the OTC round of trade opens. The first is an aggregate shock that determines whether asset B will default or not in that period. More precisely, with probability π each unit of asset B pays the promised dollar, but with probability $1 - \pi$, asset B defaults and pays nothing. Throughout the paper we assume that asset A is a perfectly safe and default-free asset.[8] This aggregate default shock is *iid* across time.

The second shock is an idiosyncratic consumption shock that determines whether an agent will have an opportunity/desire to consume in the forthcoming DM. We assume that a fraction $\ell < 1$ of agents will obtain such an opportunity and the rest will not. Thus, a measure ℓ of agents will be of type C ("Consuming"), and a measure $1 - \ell$ of agents will be of type N ("Not consuming"). This shock is *iid* across agents and time. Since the various

[7] See Section 2.2.1 for a detailed discussion of this modeling choice.

[8] Our results are robust to different model specifications. For instance, modeling asset A as a default-free asset is done for simplicity and because many real-world assets characterized as AAA are virtually default free. However, all one needs is that asset A defaults with a lower probability than asset B. Similarly, when asset B defaults, it defaults completely. Qualitatively, our results would not change if we assumed that the default is partial; i.e., at default, asset B pays only $x < 100$ cents on the dollar.

types are realized after agents have made their portfolio choices in the CM, N-types will typically hold some cash that they do not need in the current period, and C-types may find themselves short of cash, since carrying money is costly. Placing the OTC round of trade after the CM but before the DM allows agents to reallocate money into the hands of the agents who need it most, i.e., the C-types.[9]

As we have discussed, agents can only trade in one OTC market per period, and they will choose to trade in the market where they expect to find the best terms. Suppose that a measure C_j of C-types and a measure N_j of N-types have chosen to trade in the market for asset $j = \{A, B\}$ (of course, these measures will be determined endogenously). Then, the matching technology

$$f(C_j, N_j) = \left(\frac{C_j N_j}{C_j + N_j}\right)^{1-\rho} (C_j N_j)^{\rho}, \quad \rho \in [0, 1], \tag{2.1}$$

determines the measure of successful matches in OTC_j. The suggested matching function satisfies $f(C, N) \leq \min\{C, N\}$, and is useful because it admits both constant and increasing returns to scale (CRS and IRS, respectively) as subcases: when $\rho = 0$, the matching technology features CRS, while $\rho > 0$ implies IRS. Within each successful match the buyer and seller split the available surplus based on proportional bargaining (Kalai, 1977), with $\theta \in (0, 1)$ denoting the seller's (C-type's) bargaining power.[10] Notice that the matching technology and the bargaining protocol are identical in both OTC markets. This guarantees that any

[9] The first paper to incorporate this idea into the LW framework is Berentsen, Camera, and Waller (2007), but there the reallocation of money takes place through a competitive banking system.

[10] The proportional bargaining solution of Kalai (1977) has important advantages over Nash bargaining (Nash Jr, 1950). First, it is significantly more tractable. Second, in recent work, Rocheteau, Hu, Lebeau, and In (2018) solve a sophisticated model of bargaining with strategic foundations, and find that, under fairly general conditions, their solution converges to the proportional one.

differences in liquidity between assets A and B will be driven by differences in safety, and not by exogenous market characteristics (see footnotes 5 and 11).

Since all the action of the model takes place in the CM and, more importantly, the OTC markets, we wish to keep the DM as simple as possible. To that end, we assume that all C-type consumers match with a producer, and they make a take-it-or-leave-it offer (i.e., C-type consumers grasp all the surplus in the DM).

Figures 2.C.1 and 2.C.2 summarize the timing of events and the important economic actions of the model. A few details are worth emphasizing. First notice that agents who turn out to be C-types are *committed* to visit the OTC market of the asset they chose to specialize in. (One cannot sell asset B in OTC_A.) However, this is not true for N-types: an agent who turns out to be an N-type can visit either OTC market, because *her money is good* to buy any type of assets. This has an important consequence. In the default state (see Figure 2.C.2), OTC_B will shut down so *all* N-types will rush into OTC_A. And what about the agents who specialized in asset B and turned out to be C-types? Unfortunately, they must proceed to the DM only with the money that they carried from the CM. But it is important to remember that agents are aware of this possibility and may choose to hold asset B anyway. Part of what makes this choice optimal is that they may pay a low(er) price for asset B and choose to carry more money as a precaution.

2.2.1 Discussion of Modeling Choices

Since this is one of the first monetary models to incorporate *multiple* OTC markets and non-trivial entry decisions into these markets, some modeling choices deserve further expla-

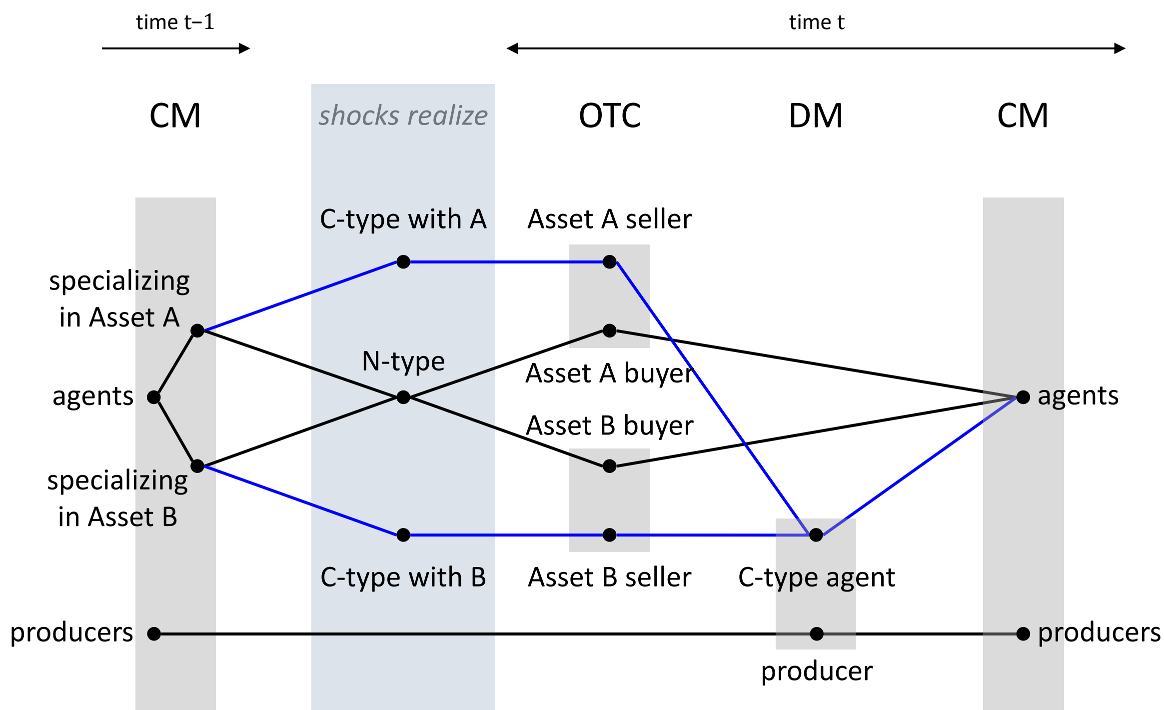

Figure 2.C.1. Timeline when both assets pay out

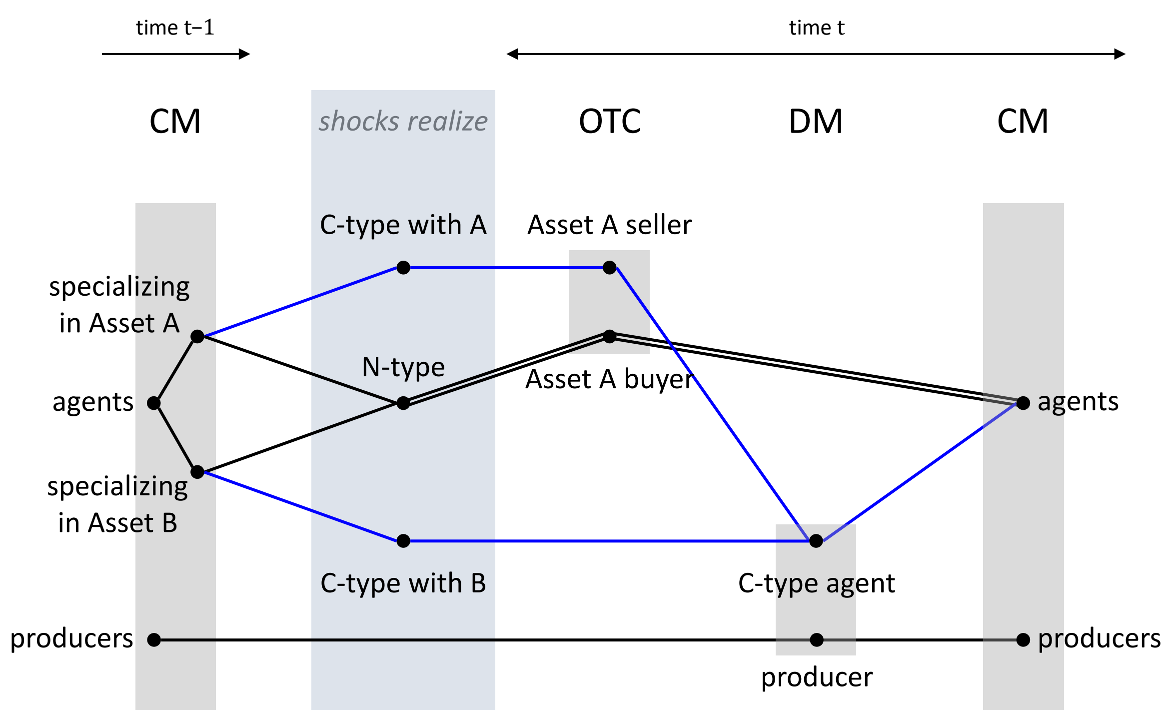

Figure 2.C.2. Timeline when asset B defaults

nation.[11]

First, we assume that the two OTC markets are segmented. This is certainly a realistic assumption: Treasuries and municipal (or corporate) bonds do trade in secondary markets that are completely distinct. A second assumption worth discussing is that agents can only hold either asset A or asset B, and can visit only one OTC market per period. This implies some loss of generality but not too much. As shown in Geromichalos and Herrenbrueck (2016b) for the case where both assets are safe, specialization is actually a *result* which follows from the fact that agents can only visit one OTC market per period. Here, full specialization will still be the endogenous outcome if the supply of asset A is not too large and/or asset B does not default with a very high probability.[12]

Assuming, effectively, that agents can visit only one market per period is not meant to be taken literally. Following the recent literature in finance, we model the secondary asset markets as OTC markets characterized by search and matching. To that end, we employ a standard matching function, described by equation (2.1), which is meant to capture the idea that a seller (buyer) is more likely to trade in a market with many buyers (sellers), and, importantly, she will prefer to visit such a market. If agents could (and did) visit both OTC markets, then both markets would have the same number of participants (all agents would just go to both markets), market entry would become trivial, and so would asset liquidity. Our assumption generates a rich yet tractable environment, where sellers' market entry decisions are driven by their belief about buyers' entry decisions, and vice versa, giving rise to

[11] Geromichalos and Herrenbrueck (2016b) also consider a model with two OTC markets and agents choosing which market to visit, but in that paper all assets have the same risk characteristics. (What *is* different is the matching technology in each market.) Hence, one of the two key ingredients of the present model—asset payout risk—is absent from that paper.

[12] Otherwise, there may be partial specialization where some agents only hold asset A, and other agents hold B but also small amounts of A which they plan to sell in case B defaults.

a number of interesting and empirically relevant results, described in Section 2.3.5. In sum, the assumption under consideration is a stark way to capture the idea that, even if some investors could visit multiple markets, they will visit the market where they expect to find better trading conditions *more frequently*.

2.3 Analysis of the Model

2.3.1 Summary of Value Functions and Bargaining Solutions

In order to streamline the analysis, we relegate the derivations of the value functions and the solutions of the various bargaining problems to Appendix 2.A.1-2.A.2. Here is a summary of the main results. As is standard in models that build on LW, all agents have linear value functions in the CM, a result that follows from the (quasi) linear preferences. This makes the bargaining solution in the DM easy to characterize. Consider a DM meeting between a producer and a C-type agent who carries m units of money, and define $m^* \equiv q^*/\varphi$ as the amount of money that (given the price φ) allows the agent to purchase the first-best quantity, q^*. Then, either $m \geq m^*$, and the buyer can purchase q^*, or $m < m^*$, and she spends all of her money to purchase the amount $q = \varphi m < q^*$.

Next, consider a meeting in OTC_j, $j = \{A, B\}$, where the N-type brings a quantity $\tilde{m}$ of money, and the C-type brings a portfolio (m, d_j) of money and asset j. Since money is costly to carry, in equilibrium we will have $m < m^*$, and the C-type will want to acquire the amount of money that she is missing in order to reach m^*, namely, $m^* - m$. Whether she will be able to acquire that amount of money depends on her asset holdings. If her asset holdings are enough (of course, how much is "enough" depends on the bargaining power θ), then she will acquire exactly $m^* - m$ units of money. If not, then she will give up all her assets to obtain an amount of money $\xi(m, d_j) < m^* - m$, which is increasing in d_j (the more assets she has, the more money she can acquire) and decreasing in m (the more money she carries, the less she needs to acquire through OTC trade). This last case, where assets are scarce, is especially interesting, because it is precisely then that having a few more assets would have allowed the agent to alleviate the binding cash constraint, which is why an asset price will carry a liquidity premium.[13] A take-away point of this discussion is that the OTC terms of trade depend only on the C-type's portfolio.

2.3.2 Matching Probabilities

Next, consider the matching probabilities in each OTC market. Let $e_C \in [0, 1]$ be the fraction of C-type agents who specialize in asset A and are thus committed to trading in OTC_A, no matter the eventual aggregate state. And let $e_N^s \in [0, 1]$ be the fraction of N-type agents who enter OTC_A in state s, where $s = \{n, d\}$ denotes the aggregate state (n for "normal" and d for "default"). Then, in state s, $e_C \ell$ is the measure of C-types and $e_N^s(1 - \ell)$ is the measure of N-types who enter OTC_A. Similarly, $(1 - e_C)\ell$ is the measure of C-types and $(1 - e_N^s)(1 - \ell)$ is the measure of N-types who enter OTC_B. Letting α_{ij}^s denote the matching probability of an i-type who enters OTC_j in state s, we have:

[13] This discussion assumes that $m + \tilde{m} \geq m^*$, i.e., that the money holdings of the C-type and the N-type pulled together is enough to allow the C-type to purchase the first best quantity q^*. Allowing for $m + \tilde{m} < m^*$ adds many complications to the model without offering any valuable insights (see Geromichalos and Herrenbrueck, 2016a). Therefore, in what follows, we will assume that we are always in the region where $m + \tilde{m} \geq m^*$. This condition will be always satisfied as long as inflation is not too large, so that all agents carry at least *half* of the first-best amount of money.

$$\alpha_{CA}^n \equiv \frac{f[e_C\ell, e_N^n(1-\ell)]}{e_C\ell}, \qquad \alpha_{CB}^n \equiv \frac{f[(1-e_C)\ell, (1-e_N^n)(1-\ell)]}{(1-e_C)\ell},$$
$$\alpha_{NA}^n \equiv \frac{f[e_C\ell, e_N^n(1-\ell)]}{e_N^n(1-\ell)}, \qquad \alpha_{NB}^n \equiv \frac{f[(1-e_C)\ell, (1-e_N^n)(1-\ell)]}{(1-e_N^n)(1-\ell)},$$
$$\alpha_{CA}^d \equiv \frac{f[e_C\ell, e_N^d(1-\ell)]}{e_C\ell}, \qquad \alpha_{CB}^d \equiv \frac{f[(1-e_C)\ell, (1-e_N^d)(1-\ell)]}{(1-e_C)\ell},$$
$$\alpha_{NA}^d \equiv \frac{f[e_C\ell, e_N^d(1-\ell)]}{e_N^d(1-\ell)}, \qquad \alpha_{NB}^d \equiv \frac{f[(1-e_C)\ell, (1-e_N^d)(1-\ell)]}{(1-e_N^d)(1-\ell)}.$$

2.3.3 Optimal Portfolio Choice

As is standard in models that build on LW, all agents choose their optimal portfolio in the CM independently of their trading histories in previous markets. In our model, in addition to choosing an optimal portfolio of money and assets, $(\hat{m}, \hat{d}_A, \hat{d}_B)$, agents also choose which OTC market they will enter in order to sell or buy assets, once the shocks have been realized. The agent's choice can be analyzed with an objective function, $J(\hat{m}, \hat{d}_A, \hat{d}_B)$, which we derive in Appendix 2.A.3 and reproduce here for convenience:

$$\begin{aligned} J(\hat{m}, \hat{d}_A, \hat{d}_B) \equiv & -\varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) + \beta\hat{\varphi}(\hat{m} + \hat{d}_A + \pi\hat{d}_B) \\ & + \beta\ell\Big(u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m} + \pi\max\{\alpha_{CA}^n\mathcal{S}_{CA}, \alpha_{CB}^n\mathcal{S}_{CB}\} + (1-\pi)\alpha_{CA}^d\mathcal{S}_{CA}\Big), \end{aligned}$$

where $\mathcal{S}_{Cj}$ is the surplus of an agent who turns out to be a C-type and trades in OTC_j:

$$\mathcal{S}_{Cj} = u(\hat{\varphi}(\hat{m} + \xi_j(\hat{m}, \hat{d}_j))) - u(\hat{\varphi}\hat{m}) - \hat{\varphi}\chi_j(\hat{m}, \hat{d}_j).$$

In the above expression, ξ_j stands for the amount of money that the agent can acquire by selling assets, and χ_j stands for the amount of assets sold in OTC_j, $j = \{A, B\}$.

The interpretation of J is straightforward. The first term is the cost of choosing the portfolio $(\hat{m}, \hat{d}_A, \hat{d}_B)$. This portfolio yields the expected payout $\hat{\varphi}(\hat{m} + \hat{d}_A + \pi\hat{d}_B)$ in next period's CM (the second term of J). The portfolio also offers certain liquidity benefits, but these will only be relevant if the agent turns out to be a C-type; thus, the term in the second line of J is multiplied by ℓ. The C-type can enjoy at least $u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{m}$ just with the money that she brought from the CM. Furthermore, she can enjoy an additional benefit by selling assets for cash in the secondary market. How large this benefit is depends on the market choice of the agent (the term inside the max operator) and on the realization of the aggregate shock: if asset B defaults, an event that happens with probability $1 - \pi$, a C-type who specialized in that asset has no benefit. A default of asset B is not the only reason why the C-type may not trade in the OTC markets; it may just be that she did not match with a trading partner. This is why the various surplus terms $\mathcal{S}_{Cj}$ are multiplied by the α-terms, i.e., the matching probabilities described in the previous section.[14]

2.3.4 Equilibrium

We focus on steady-state equilibria. Before we move on to characterizing possible equilibria, we first need to understand their structure. We have twelve endogenous variables to be determined in equilibrium (not including the terms of trade in the OTCs):

- equilibrium prices: φ, p_A, p_B
- equilibrium real balances: z_A, z_B
- equilibrium entry choices: $e_C (\equiv e_C^n = e_C^d)$, e_N^n, e_N^d

[14] There are two reasons why the objective function does not contain any term that represents the event in which the agent is an N-type. First, and most obviously, N-types are defined as the agents who do not get to consume in the DM. Second, the OTC terms of trade, χ and ξ, depend only on the portfolio of the C-type. An intuitive explanation was presented in Section 2.3.1. For the details, see Appendix 2.A.2.2.

- equilibrium DM production: $q_{0A}(\equiv q_{0A}^n = q_{0A}^d)$, $q_{1A}(\equiv q_{1A}^n = q_{1A}^d)$,

  $q_{0B}(\equiv q_{0B}^n = q_{0B}^d)$, $q_{1B}(\equiv q_{1B}^n)$

In this list of equilibrium variables, the asset prices are obvious, and z_j, $j = \{A, B\}$, is simply the real balances held by the typical agent who chooses to specialize in asset j. The remaining terms deserve some discussion. First, notice that the fraction of C-types who enter OTC$_A$, e_C, does not depend on the aggregate state $s = \{n, d\}$. This is because C-types are committed to visiting the OTC market of the asset they chose to specialize in (and this choice is effectively made before the realization of the shock).

Regarding the DM production terms q_{kj}, $k = \{0, 1\}$ indicates whether the C-type did ($k = 1$) or did not ($k = 0$) trade in the preceding OTC market, and $j = \{A, B\}$ indicates the asset in which she specializes. For example, q_{0A} is the amount of DM good purchased by an agent who specialized in asset A and did not match in OTC$_A$, and so on. These terms do not depend on the aggregate state $s = \{n, d\}$. To see why, notice that q_{0A} depends only on the amount of real balances that the agent carried from the CM (this agent did not trade in the OTC), and that choice was made before s was realized. The same reasoning applies to q_{0B}. How about the term q_{1A}?[15] This term depends on the real balances that the agent carried from the CM (which, we just argued, is independent of the shock realization), and on the amount of assets that this agent carries from the CM (see Section 2.3.1). How many asset does this agent carry? The answer is S_A/e_C: the exogenous asset supply, S_A, divided by the measure of agents who specialize on asset A. Since S_A is a parameter, and e_C is independent of the state s, the same will be true for the term q_{1A}.

To simplify the exposition of the equilibrium analysis, we now establish that the variables

[15] Notice that the term q_{1B}^d is left undefined, since OTC$_B$ shuts down in the default state.

$\{z_A, z_B, \varphi, p_A, p_B\}$ follow immediately from $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, e_C, e_N^n\}$ (and e_N^d is always equal to 1). First, since the C-types have all the bargaining power in the DM, the equilibrium real balances satisfy

$$z_A = q_{0A},\; z_B = q_{0B}. \tag{2.2}$$

Second, the equilibrium price of money solves the money market clearing condition:

$$\varphi M = e_C q_{0A} + (1 - e_C) q_{0B}. \tag{2.3}$$

Third, the equilibrium asset prices solve the following asset demand equations (reproduced from (2.23) in Appendix 2.A.4):

$$p_A = \frac{1}{1+i}\left(1 + \ell \frac{\theta}{\omega_\theta(q_{1A})}\left(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d\right)(u'(q_{1A}) - 1)\right), \tag{2.4}$$

$$p_B = \frac{1}{1+i}\left(\pi + \ell \frac{\theta}{\omega_\theta(q_{1B})}\pi\alpha_{CB}^n(u'(q_{1B}) - 1)\right), \tag{2.5}$$

where:

$$\omega_\theta(q) \;\equiv\; \theta + (1-\theta)u'(q) \;\geq\; 1.$$

Next, we study the determination of $\{q_{0A},\, q_{1A},\, q_{0B},\, q_{1B},\, e_C,\, e_N^n\}$, keeping in mind that all other variables follow easily once these "core" variables have been determined.

2.3.4.1 Core Variable Equilibrium Conditions

To determine the six core variables we have six equilibrium conditions. First, we have two money demand equations for agents who specialize in asset A and B:[16]

$$
\begin{aligned}
i = {} & \ell\Big(1-\theta\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)\Big)(u'(q_{0A})-1) \\
& + \ell\theta\frac{\omega_\theta(q_{0A})}{\omega_\theta(q_{1A})}\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A})-1), \qquad (2.6)
\end{aligned}
$$

$$
i = \ell(1-\theta\pi\alpha^n_{CB})(u'(q_{0B})-1)+\ell\theta\frac{\omega_\theta(q_{0B})}{\omega_\theta(q_{1B})}\pi\alpha^n_{CB}(u'(q_{1B})-1). \qquad (2.7)
$$

Next, we have the trading protocol in OTC$_j$, $j=\{A,B\}$, that links q_{0j} and q_{1j}:

$$
\begin{aligned}
q_{1A} &= \min\Big\{q^*,\, q_{0A}+\varphi\tilde{\xi}_A\Big\}, \quad \varphi d_A = z(\tilde{\xi}_A), \\
q_{1B} &= \min\Big\{q^*,\, q_{0B}+\varphi\tilde{\xi}_B\Big\}, \quad \varphi d_B = z(\tilde{\xi}_B),
\end{aligned}
$$

where:

$$
z(\tilde{\xi}) \equiv (1-\theta)\Big(u(\varphi(m+\tilde{\xi}))-u(\varphi m)\Big)+\theta\varphi\tilde{\xi},
$$

$$
d_A = \begin{cases} S_A/e_C, & \text{if } e_C>0, \\ 0, & \text{otherwise,} \end{cases}
$$

$$
d_B = \begin{cases} S_B/(1-e_C), & \text{if } e_C<1, \\ 0, & \text{otherwise.} \end{cases}
$$

[16] The details of this derivation can be found in Appendix 2.A.5. What is important to remember here is that agents who choose to specialize in different assets will typically carry different amounts of money. Not surprisingly, agents who choose to carry the less safe asset B self-insure against the probability of default (and the shutting down of OTC$_B$) by carrying more money.

The equations for d_A, d_B can be interpreted as asset market clearing with 'free disposal': we require agents to choose either asset A or B to specialize in, so it is possible that everyone chooses the same asset. In that case, demand for the other asset is zero. If demand for an asset is positive, the market for that asset clears with equality.

The other equations also have intuitive interpretations. They state that if the agent's asset holdings are large, then $q_{1j} = q^*$, because the agent will acquire (through selling assets) the money necessary to purchase the first-best quantity, and no more. On the other hand, if the agent's asset holdings are scarce, she will give up all her assets and purchase an amount of DM good equal to q_{0j} (the amount she could have purchased without any OTC trade) plus $\varphi\tilde{\xi}_j$ (the additional amount she can now afford by selling assets for extra cash). The terms d_j represent the amount of assets held by the typical agent who specializes in asset j. With some additional work, we can re-write the OTC bargaining protocols in a form that involves only core equilibrium variables (and parameters):

$$q_{1A} = \min\left\{ q^*,\, q_{0A} + \frac{\frac{S_A}{M}\frac{e_C q_{0A} + (1-e_C)q_{0B}}{e_C} - (1-\theta)\Big(u(q_{1A}) - u(q_{0A})\Big)}{\theta} \right\}, \tag{2.8}$$

$$q_{1B} = \min\left\{ q^*,\, q_{0B} + \frac{\frac{S_B}{M}\frac{e_C q_{0A} + (1-e_C)q_{0B}}{1-e_C} - (1-\theta)\Big(u(q_{1B}) - u(q_{0B})\Big)}{\theta} \right\}. \tag{2.9}$$

Our last two equilibrium conditions come from the optimal OTC market entry decisions of agents. An important remark is that the OTC surplus of N-types does not depend on their portfolios (see Section 2.3.1 or 2.A.2.2), whereas the OTC surplus of C-types does depend on their portfolios. Hence, in making their entry decisions, C-types consider not only the expected surplus of entering in either market, as is the case for N-types, but also the cost associated with each entry decision. Another way of stating this is to say that e_C is determined *ex-ante* and represents the decision to specialize in asset A, while e_N^n is determined *ex-post* and represents the fraction of N-types who enter OTC_A in the normal state. Therefore, the optimal entry of C-types is characterized by:

$$e_C = \begin{cases} 1, & \tilde{\mathcal{S}}_{CA} > \tilde{\mathcal{S}}_{CB} \\ 0, & \tilde{\mathcal{S}}_{CA} < \tilde{\mathcal{S}}_{CB} \\ \in [0,1], & \tilde{\mathcal{S}}_{CA} = \tilde{\mathcal{S}}_{CB} \ , \end{cases} \tag{2.10}$$

where:

$$\begin{aligned} \tilde{\mathcal{S}}_{CA} =& -iq_{0A} - ((1+i)p_A - 1)\Big((1-\theta)(u(q_{1A}) - u(q_{0A})) + \theta(q_{1A} - q_{0A})\Big) \\ & + \ell(u(q_{0A}) - q_{0A}) + \ell\Big(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d\Big)\mathcal{S}_{CA}, \\ \mathcal{S}_{CA} =& \ \theta\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big), \end{aligned}$$

and:

$$\begin{aligned} \tilde{\mathcal{S}}_{CB} =& -iq_{0B} - ((1+i)p_B - \pi)\Big((1-\theta)(u(q_{1B}) - u(q_{0B})) + \theta(q_{1B} - q_{0B})\Big) \\ & + \ell(u(q_{0B}) - q_{0B}) + \ell\pi\alpha_{CB}^n\mathcal{S}_{CB}, \\ \mathcal{S}_{CB} =& \ \theta\Big(u(q_{1B}) - u(q_{0B}) - q_{1B} + q_{0B}\Big). \end{aligned}$$

The optimal entry of N-types is simpler, and characterized by:

$$e_N^n = \begin{cases} 1, & \alpha_{NA}^n \mathcal{S}_{NA} > \alpha_{NB}^n \mathcal{S}_{NB} \\ 0, & \alpha_{NA}^n \mathcal{S}_{NA} < \alpha_{NB}^n \mathcal{S}_{NB} \\ \in [0,1], & \alpha_{NA}^n \mathcal{S}_{NA} = \alpha_{NB}^n \mathcal{S}_{NB} \ , \end{cases} \tag{2.11}$$

in the normal state, where:

$$\mathcal{S}_{NA} = (1-\theta)\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big),$$
$$\mathcal{S}_{NB} = (1-\theta)\Big(u(q_{1B}) - u(q_{0B}) - q_{1B} + q_{0B}\Big),$$

and by:

$$e_N^d = \begin{cases} 1, & e_C > 0 \\ \in [0,1], & e_C = 0 \ , \end{cases} \tag{2.12}$$

in the default state.

We can now define the steady-state equilibrium of the model.

Definition 2.1. *For given asset supplies* $\{A, B\}$*, the steady-state equilibrium for the core variables of the model consists of the equilibrium quantities and entry choices,* $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, e_C, e_N^n\}$*, such that* (2.6)*,* (2.7)*,* (2.8)*,* (2.9)*,* (2.10) *and* (2.11) *hold. The equilibrium real balances,* $\{z_A, z_B\}$*, satisfy* (2.2)*, the equilibrium price of money,* φ*, solves* (2.3)*, and the equilibrium asset prices,* $\{p_A, p_B\}$*, solve* (2.4) *and* (2.5)*.*

2.3.5 Equilibrium Market Entry

In this section we analyze the optimal entry decision of agents, which is a key channel in our model. More precisely, we want to study the best response of the representative C-type, who takes as given e_C (the proportion of other C-types who enter market A), and the associated optimal entry decision of N-types (in the normal state), $e_N^n(e_C)$. This task becomes easier by recognizing that there are three opposing forces at work. We dub them the congestion effect, the coordination effect, and the dilution effect.

The congestion effect means that a high e_C will *discourage* the representative C-type from going to market A because it implies a low matching probability. However, a high e_C also means that many N-types are attracted to market A, i.e., a high e_N^n, and a high e_N^n is a force that *encourages* the representative C-type to visit OTC$_A$. This is the coordination effect, which may completely or more than completely offset congestion. A more subtle force is the dilution effect. When e_C is high, many agents specialize in asset A, and each one of them carries a small fraction of the (fixed) asset supply. As a result, the surplus generated in a meeting in OTC$_A$ will be small. This is yet another force that *discourages* the representative C-type from entering market A when e_C is high, because that agent forecasts that few N-types will be attracted to that market.

Moving to the formal analysis, we construct equilibria as fixed points of e_C. To be specific: first, we fix a level of e_C; then we solve for the optimal portfolio choices through equations (2.6)-(2.9) and (2.11); and finally, we define the C-types' *best response function*:

$$G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB},$$

where the surplus terms have the optimal choices substituted. This function measures the relative benefit to an *individual* C-type from specializing in asset A over asset B, assuming a proportion e_C of all *other* C-type agents specialize in A, and all other decisions (portfolios and entry of N-types) are conditionally optimal. We say that a value of e_C is part of an "interior" equilibrium if $e_C \in (0,1)$ and $G(e_C) = 0$, or a "corner" equilibrium if $e_C = 0$ and $G(0) \leq 0$ or $e_C = 1$ and $G(1) \geq 0$.

Proposition 2.1. *The following types of equilibria exist, and have these properties:*

(a) There exists a corner equilibrium where $e_C = 0$, $e_N^n = e_N^d = 0$.

(b) There exists a corner equilibrium where $e_C = 1$, $e_N^n = e_N^d = 1$.

(c) Assume $\rho = 0$ *(CRS) and asset supplies are low enough so that assets are scarce in OTC trade. Then,* $\lim_{e_C \to 0+} G(e_C) > 0 > G(0)$*; the equilibrium at the B-corner is not robust to small trembles.*

(d) Assume $\rho = 0$ *(CRS) and asset supplies are low enough so that assets are scarce in OTC trade. Then,* $\lim_{e_C \to 1} G(e_C) < G(1)$*. If* $\pi \to 1$*, then the limit is negative, and the equilibrium at the A-corner is not robust, either.*

(e) Assume $\rho = 0$ *(CRS),* $\pi \to 1$*, and asset supplies are low enough so that assets are scarce in OTC trade. Then, there exists at least one interior equilibrium which is robust.*

(f) Given $\rho > 0$ *(IRS),* $\lim_{e_C \to 0+} G(e_C) \neq G(0)$*.*

(g) Given $\rho > 0$ *(IRS),* $\lim_{e_C \to 1-} G(e_C) = G(1) > 0$*; the equilibrium at the A-corner is robust.*

Proof. See Appendix 2.A.7.1. □

Figures 2.C.3 and 2.C.4 illustrate these results; the former for the case of CRS, and

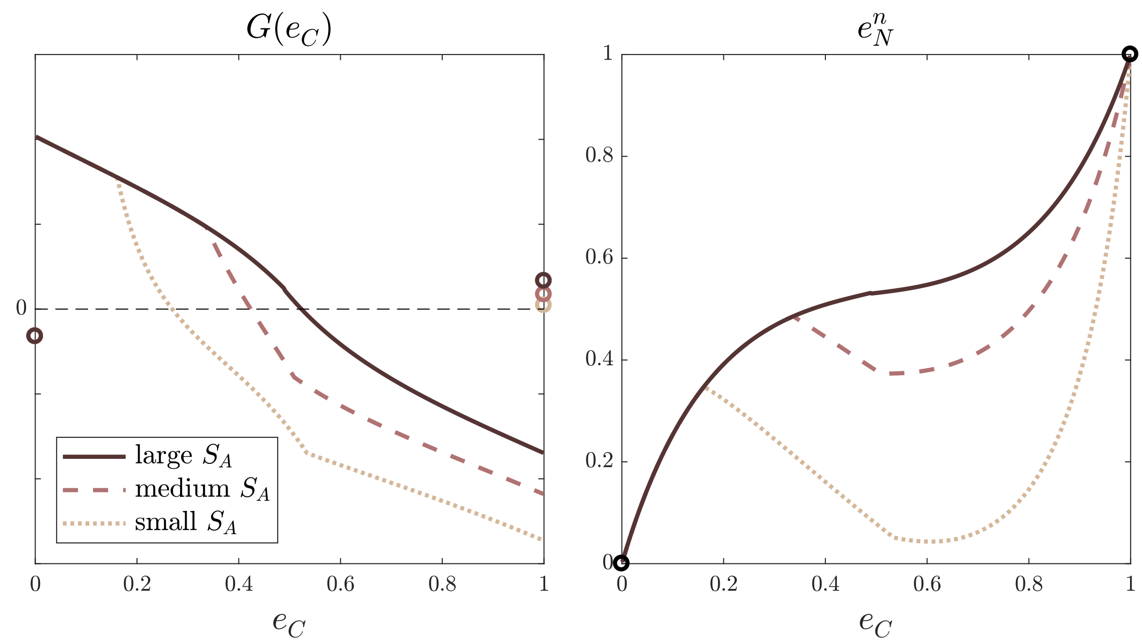

Figure 2.C.3. C-types' incentive to deviate and N-types' optimal entry choice, given e_C, for the case of CRS. The figure depicts the function $G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB}$ (left panel) and the optimal response of N-types, e_N^n (right panel), as functions of aggregate e_C, assuming CRS in matching ($\rho = 0$). Equilibrium entry is illustrated for three levels of asset supply S_A, keeping the supply of asset B constant. Here, $\pi = 0.95$.

the latter for the case of IRS. The left panel of each figure depicts the individual C-type's best response function, $G(e_C)$. Since this function depends not only on the behavior of fellow C-types, but also on that of N-types, on the right panel of each figure we show the optimal entry choice of N-types, $e_N^n(e_C)$, as a function of e_C. The figures also illustrate how equilibrium entry is affected by changes in the supply of asset A, keeping the supply of asset B constant.

As indicated in the right panel of each figure, we have $e_N^n(0) = 0$ and $e_N^n(1) = 1$: when all C-types are concentrated in one market, the N-types will follow. Generally, the higher e_C is, the more N-types would like to go to market A: this is just the coordination effect and it tends to make $e_N^n(e_C)$ increasing. Whether it will be strictly increasing or not, ultimately depends on the strength of the dilution effect relative to the coordination effect. This is why in both figures, $e_N^n(e_C)$ is increasing when A is large: it is a large asset supply that weakens the dilution effect.[17]

Next, we have $G(0) < 0$ and $G(1) > 0$, while $e_N^n(0) = 0$ and $e_N^n(1) = 1$. This illustrates parts (a) and (b) of Proposition 2.1; the corners are always equilibria (marked with circles on the left panel of the figures). However, with CRS these equilibria are not robust (unless π is so small that the entire G-function is positive, in which case the A-corner is the only equilibrium); this illustrates parts (c) and (d) of the proposition.[18] Also, with CRS the congestion effect is so dominant that the G-function is globally decreasing in the interior, as shown in part (e) of the proposition and illustrated in Figure 2.C.3. Therefore, there exists a robust interior equilibrium where the representative C-type is indifferent between entering market A or B; i.e., $G(e_C) = 0$. As the supply of asset A increases, so does the equilibrium value of e_C, because a larger asset supply weakens the dilution effect and increases the incentives of agents to concentrate on market A.

Figure 2.C.4 illustrates equilibrium entry under various values of S_A for the case of IRS. Naturally, the two corner solutions are still equilibria, and since IRS strengthen the coordination effect, the equilibrium where all agents go to OTC$_A$ ($e_N^n = e_C = 1$) is now robust (part (g) of the proposition). This may or may not be true for the equilibrium with $e_N^n = e_C = 0$, depending on the values of π and ρ.[19] Figure 2.C.4 demonstrates the

[17] There is also a difference between the two figures. In Figure 2.C.3 (CRS case), $e_N^n(e_C)$ is strictly increasing in its entire domain. However, in Figure 2.C.4 (IRS case), and for the case of large S_A, $e_N^n(e_C)$ reaches 1 for a rather small value of e_C and becomes flat afterwards. This is because with IRS, the desire of N-types to go to the market with many C-types, i.e., the coordination effect, is supercharged.

[18] More precisely, they are not "trembling hand perfect" Nash equilibria. Consider for example the equilibrium with $e_N^n = e_C = 1$ (a similar argument applies to the one with $e_N^n = e_C = 0$). Since all N-types visit market A, the representative C-type also wishes to visit that market. (Why try to trade in a ghost town, which OTC$_B$ is in this case?) However, if an arbitrarily small measure ε of N-types visited market B by error, the representative C-type would have an incentive to deviate to market B, where her chance of matching is now extremely high (since $e_C = 1$, she would be the only C-type in that market).

[19] Consider first the equilibrium with $e_N^n = e_C = 1$. With IRS the desire to go to OTC$_A$ (where all agents

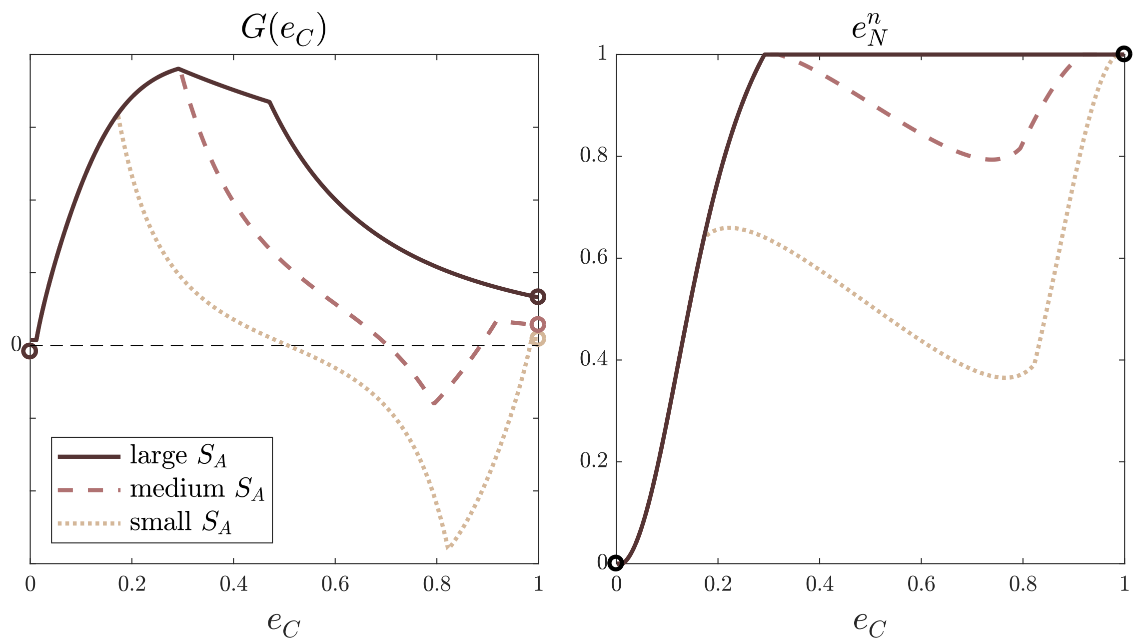

Figure 2.C.4. C-types' incentive to deviate and N-types' optimal entry choice, given e_C, for the case of IRS. The figure depicts the function $G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB}$ (left panel) and the optimal response of N-types, e_N^n (right panel), as functions of aggregate e_C, assuming IRS in matching. Equilibrium entry is illustrated for three levels of asset supply S_A, keeping the supply of asset B constant. Here, $\pi = 0.95$ and $\rho = 0.3$.

case of non-robustness; as shown in part (f) of the proposition, the best response function is discontinuous at the B-corner, though it is continuous at the A-corner. With the coordination effect amplified, multiple interior equilibria are typical (as in the case of "small S_A" and "medium S_A"). However, the only interior equilibrium that is robust is the one where G has a negative slope. A rise in S_A will lead to an increase in the (interior and robust) equilibrium value of e_C. But with IRS, another interesting possibility arises: if S_A is large enough, the desire of agents to coordinate on OTC_A is so strong that interior equilibria cease to exist.

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are concentrated) is so strong that, even if some N-types visit OTC_B by error, the representative C-type no longer has an incentive to deviate to that market (unlike the CRS case; see footnote 18). But the channel described so far is relevant for both corners. So why is the equilibrium where all agents go to OTC_B not always robust as well? Because OTC_B is the market of the asset that may default. When that happens (ex-post), all N-types will rush to market A, i.e., $e_N^d = 1$, and this creates an additional incentive for the representative C-type to deviate to market A (a decision made ex-ante). This additional incentive will be relatively large, when π is low (high default probability) and ρ is low (weak coordination effect). Therefore, the equilibrium with $e_N^n = e_C = 0$ is likely to be non-robust for relatively low values of π and ρ.

This is depicted in the "large S_A" case in the figure, where one can see that the A-corner (with $e_N^n = e_C = 1$) is the unique robust equilibrium entry outcome.

2.3.6 Liquidity Premia

Most of our main results will be about the *liquidity premia* assets A and B may carry in equilibrium. As we have seen in the asset pricing equations (2.4) and (2.5), asset prices consist of the fundamental value multiplied by a premium that reflects the possibility of selling the asset in the OTC market. We define the fundamental value of an asset as the equilibrium price that would emerge if this possibility was eliminated. In that case, agents would value the assets only for their payouts at maturity, and the equilibrium prices would be given by $1/(1+i)$, for asset A, and $\pi/(1+i)$, for asset B.

The liquidity premium of asset j, denoted by L_j, is therefore defined as the percentage difference between an asset's price and its fundamental value:

$$p_A = \frac{1}{1+i}(1+L_A), \qquad p_B = \frac{\pi}{1+i}(1+L_B), \tag{2.13}$$

where:

$$\begin{aligned} L_A &= \ell \cdot \left(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d\right) \cdot \frac{\theta}{\omega_\theta(q_{1A})} \cdot (u'(q_{1A}) - 1), \\ L_B &= \ell \cdot \alpha_{CB}^n \cdot \frac{\theta}{\omega_\theta(q_{1B})} \cdot (u'(q_{1B}) - 1). \end{aligned} \tag{2.14}$$

Each liquidity premium is the product of four terms. First, the probability that an agent turns out to be a C-type and thus needs liquidity at all (ℓ). Second, given that the agent is a C-type, the expected probability of matching in the respective OTC market (conditional on the market being open). Third, the share of the marginal surplus captured by the C-type (θ/ω_θ), which is endogenous but constrained to the interval $(0, \theta]$. And fourth, the marginal surplus of the match: the utility gained by a consumer who brings one more unit of real balances into the DM, net of the production cost ($u'(q_{1j}) - 1$).

Thus, there are two ways a liquidity premium can be zero: either the relevant OTC market is closed ($\alpha_{Cj} = 0$), or assets are so plentiful that selling an extra asset in the OTC does not create additional surplus in the DM ($q_{1j} = q^*$, thus $u'(q_{1j}) = 1$). In the latter case, the asset is still "liquid", but its liquidity is *inframarginal* so it does not affect the price.

2.4 Main Results

2.4.1 Result 1: Safe and Liquid

The first result of the paper is that, other things equal, the safer asset (A) tends to be more liquid. We demonstrate this result employing two measures of liquidity: the liquidity premium and the volume of trade in each OTC market. Throughout Section 2.4.1, we assume that the supplies of the two assets are equal ($S_A = S_B$), in order to focus on liquidity differences purely due to the assets' safety differential. Because of the complexity of our model, a full analytical characterization is impossible and we break our analysis into two stages.[20] First, we take a local approximation of our model around $\pi = 1$, assuming CRS

[20] Our model has six 'core' equilibrium variables, most of which show up in multiple equations; these equations are non-linear and include kinks, due to the various branches of the bargaining solutions and the agents' market entry decisions. Simply put, every time a parameter value changes, all six endogenous variables are affected by simultaneous and, typically, opposing forces. For more detail, one can inspect matrix equation (2.33) in the Appendix, which describes the effect of changes in π on the core variables in general equilibrium, keeping in mind that this matrix is evaluated at the limit as $\pi \to 1$.

($\rho = 0$). In this case, a symmetric interior equilibrium exists where the two assets are perfect substitutes and their equilibrium prices are equal, and the perturbation of this equilibrium with small changes in π can be solved in closed form (see Appendix 2.A.7). Second, in order to obtain global results away from $\pi \to 1$, and to conduct comparative statics with respect to ρ, we solve the model numerically.

Proposition 2.2. *Assume that asset supplies S_A and S_B are equal and are low enough so that assets are scarce in OTC trade. Then:*

(a) At $\pi = 1$, there exists a symmetric equilibrium where $e_C = e_N = 0.5$, $q_{0A} = q_{0B}$, $q_{1A} = q_{1B}$, and $L_A = L_B$.

(b) Assume $\rho = 0$ (CRS) and $(1-\ell)\theta$ is sufficiently large. Then, locally, $\pi < 1$ implies $L_A > L_B$: the safer asset is more liquid.

Proof. See Appendix 2.A.7.2. $\square$

Naturally, when $\pi = 1$, the two assets are perfect substitutes and their equilibrium prices (and liquidity premia) will be equal. However, as π falls below 1, the liquidity premium of asset A generally exceeds that of asset B. Near the symmetric equilibrium, the derivative of the difference between the liquidity premia with respect to π is:

$$\left.\frac{d(L_A - L_B)}{d\pi}\right|_{\pi\to 1} = \ell\theta \frac{u'(q_1) - 1}{w_\theta(q_1)}(\alpha^n_{CA} - \alpha^d_{CA}) + \ell\theta \frac{u''(q_1)}{w_\theta(q_1)^2} \times \frac{d(q_{1A} - q_{1B})}{d\pi} + \ell\theta \frac{u'(q_1) - 1}{w_\theta(q_1)} \times \frac{d(\alpha^n_{CA} - \alpha^n_{CB})}{d\pi}$$

The first term on the right-hand side represents the negative *direct effect*: the probability of meeting a buyer for asset A is always lower in the normal state than in the state where B

defaults ($\alpha_{CA}^n < \alpha_{CA}^d$), therefore the liquidity advantage of asset A increases as B becomes less safe ($\pi \downarrow$). But this liquidity advantage is magnified by the endogenous responses of agents to *perceived* default risk, which affect what happens even in the normal state. Consider the second term in the equation. An agent who specializes in asset B despite the default risk will self-insure by carrying more money, which translates (after OTC trade) to a higher q_{1B}, resulting in a lower trading surplus (indicated by multiplication with $u''(q_1) < 0$) and thus a lower liquidity premium for asset B; thus, this *intensive margin effect* is also negative and always reinforces the direct effect.

Finally, there is the the third term in the equation, representing an *extensive margin effect*: generally, when $\pi < 1$, N-types respond more strongly to the lower trading surplus in the B-market, thus the matching probability for C-types is higher in OTC_A. If so, then all three effects point in the same direction and thus the overall sign of the equation is negative, as per part (b) of Proposition 2.2. Analytically, we can show that this is indeed the case when $(1 - \ell)\theta$ is sufficiently large; numerically, we can find counterexamples, but the overall negative sign is still the predominant result.[21]

Figure 2.C.5 illustrates our result for a range of π, and for both CRS and an intermediate degree of IRS. In each of these cases, the difference between L_A and L_B is positive and strictly decreasing in π. It is important to remind the reader that this differential is purely due to liquidity; it is not a risk premium. Indeed, decreasing π makes agents less willing to hold asset B because that asset is now at higher risk of default, but that effect is already included

[21] To be precise, we checked the sign for all combinations of θ and ℓ in $\{0.1, 0.5, 0.9\}$, and asset supplies of $S_A = S_B \in \{.02, .05, .10, .15\}$, with $\rho = 0$, $i = .1$, and $M = 1$ maintained. Out of these 36 parameter combinations, in four of them the assets are so plentiful that both liquidity premia are zero for any π; in three of them, all with maximal ℓ and minimal asset supplies, the sign is reversed so that $L_A < L_B$ when $\pi < 1$, i.e., the safer asset is less liquid; in the remaining 29 cases, we have the 'normal' result where the safer asset is more liquid. For more details, see Appendix 2.A.7.2.

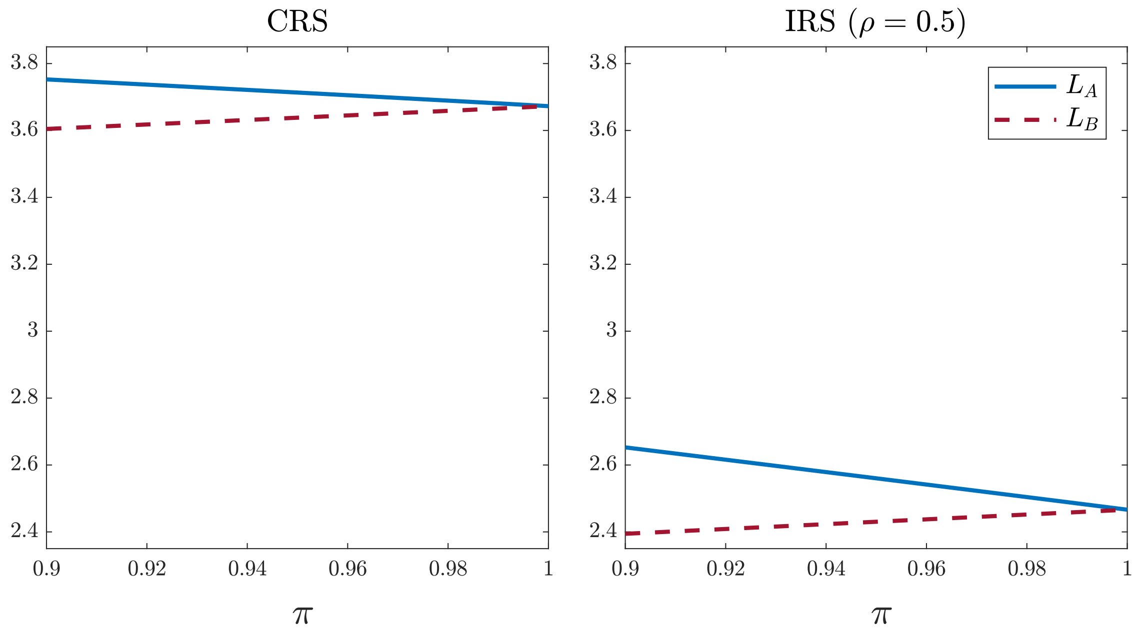


Figure 2.C.5. Liquidity premia as functions of π. The figure depicts the liquidity premia of assets A, B as functions of π, assuming symmetric asset supplies. The left panel illustrates the case of a CRS matching technology, and the right panel represents the the case of IRS ($\rho = 0.5$).

in the fundamental value of the assets (see equations 2.13). The new result here is that as asset B becomes less safe it also enjoys a smaller liquidity premium on top of the smaller fundamental value.

The intuition behind Result 1 is as follows. Unlike C-types, who are committed to visit the market of the asset in which they chose to specialize, N-types are free to visit any market they wish, since their money is good to buy any asset. Consequently, in the event of default, all the N-types (even those who had chosen to specialize in asset B) will rush into OTC_A. Of course, agents who are currently making their portfolio and market entry decisions in the CM correctly anticipate this possibility. Thus, the chance of a market flooded with buyers *ex-post* (i.e., OTC_A in the event of default) serves as a powerful incentive attracting agents to specialize in asset A *ex-ante*, as they forecast that this market will offer a high matching probability, if they turn out to be C-types.

The discussion following equations (2.14) reveals why this is important for liquidity: an agent who buys an asset today (in the primary market) is willing to pay a higher price if she expects that it will be easy to sell that asset 'down the road', and, importantly, it is the C-types who sell assets down the road. Through this channel, any positive default probability for asset B translates into a matching advantage for C-types in OTC_A. This, in turn, translates into a higher liquidity premium for asset A, because that premium depends on the (anticipated) ease with which the agent can sell the asset if she turns out to be a C-type. Naturally, this channel, and the liquidity differential between the two assets, will be magnified if matching is characterized by IRS.

This last point can be seen more clearly in Figure 2.C.6. Instead of liquidity premia, we plot the percentage difference between the two asset prices, $(p_A - p_B)/p_A$, for various values of ρ (and as functions of π), and we contrast them to the difference between the fundamental values.[22] Thus, any difference between the curve labeled "fundamental" and the curves representing the various ρ's is a *pure* liquidity difference. The bottom panel of this figure performs the same exercise, but for high values of ρ (including $\rho = 1$, i.e., the congestion-free matching function adopted by Duffie, Gârleanu, and Pedersen (2005) and most of the papers that build on their framework). This figure highlights that with strong IRS ($\rho \to 1$), even a tiny default probability for asset B can be magnified into an enormous liquidity advantage for asset A. This is visualized by the function $(p_A - p_B)/p_A$, which jumps discontinuously at $\pi \to 1$ as long as $\rho \geq .87$ in the example.

The description of the mechanism behind Result 1 also highlights that as π decreases,

[22] Clearly, the percentage difference between the fundamental values of assets A and B is $1 - \pi$.

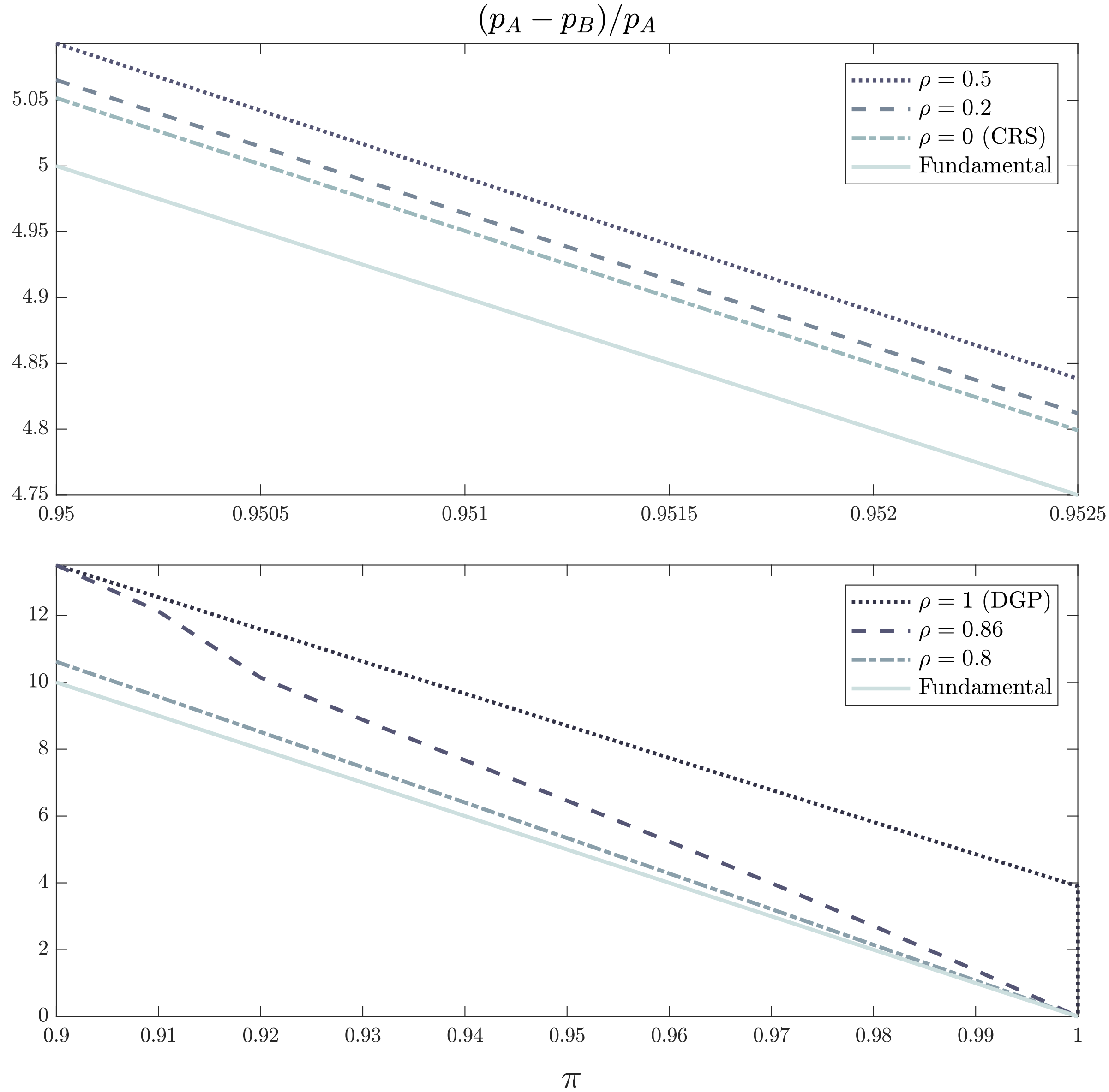


Figure 2.C.6. Price differentials as functions of π. The top panel depicts the price differential $(p_A - p_B)/p_A$ as a function of π, for various values of ρ, assuming symmetric asset supplies. The curve dubbed "Fundamental" represents the percentage difference between the price of assets A and B, if the liquidity channel was shut down, namely, the term $1 - \pi$. The difference between the "Fundamental" curve and the curves corresponding to the various ρ's represent a pure liquidity difference between the two assets. The bottom panel repeats the same exercise for high values of ρ, including the "congestion-free" case where $\rho = 1$. For high enough ρ and for values of π arbitrarily close to 1, the price differential $(p_A - p_B)/p_A$ jumps discontinuously, representing a large liquidity advantage of asset A versus asset B.

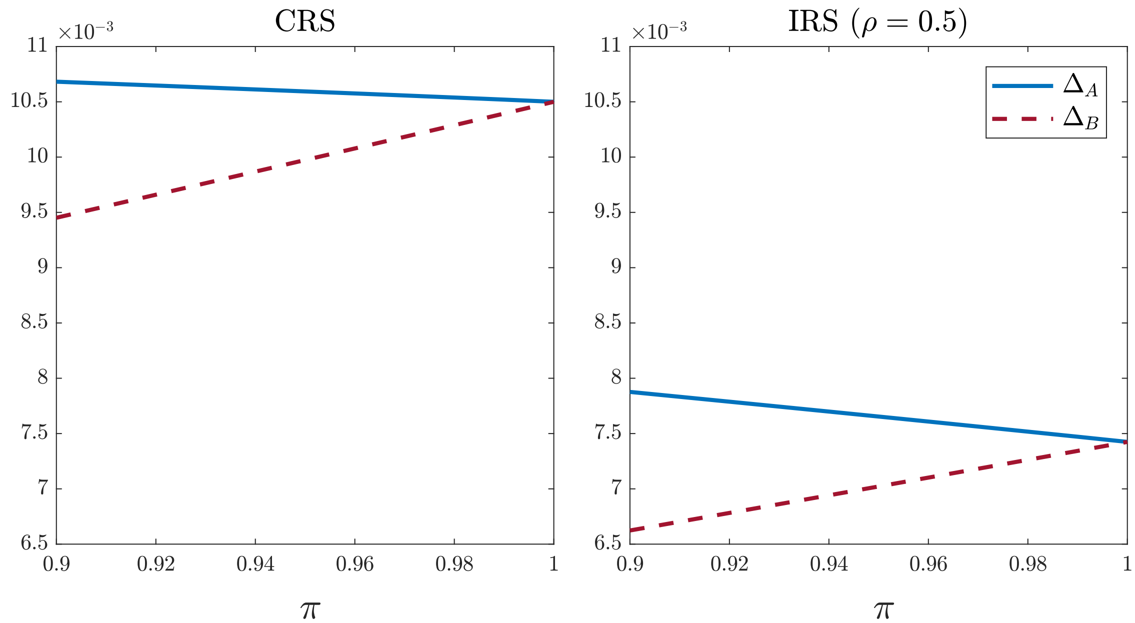


Figure 2.C.7. OTC trade volumes as functions of π. The figure depicts the trade volumes in OTC_A and OTC_B, Δ_A and Δ_B as defined in Appendix 2.A.6, as functions of π, for $S_A = S_B$. In the left panel we assume the matching technology is CRS ($\rho = 0$), and in the right panel we have $\rho = 0.5$.

more agents will choose to coordinate in the market for asset A. Thus, it is not only the liquidity premium of asset A that increases in the default probability, but also the volume of trade in that market. This is illustrated in Figure 2.C.7, which graphs the trade volumes in the two OTC markets as functions of π for the cases of CRS and IRS. (The details of the derivation of OTC trade volume are relegated to Appendix 2.A.6.) As seen in the figure, the trade volume is higher in the secondary market for the safer asset, and the difference in trade volumes between OTC_A and OTC_B is decreasing in π.[23] Since secondary market trade volume is often adopted in the finance literature as a measure of an asset's liquidity, we view this result as an alternative way of establishing that a safer asset will also be more liquid—*other things being equal*.

[23] Within the context of a different model, Velioglu and Üslü (2019) obtain a result with similar flavor. They develop a multi-asset version of Duffie, Gârleanu, and Pedersen (2005) and find that safer assets trade in larger quantities.

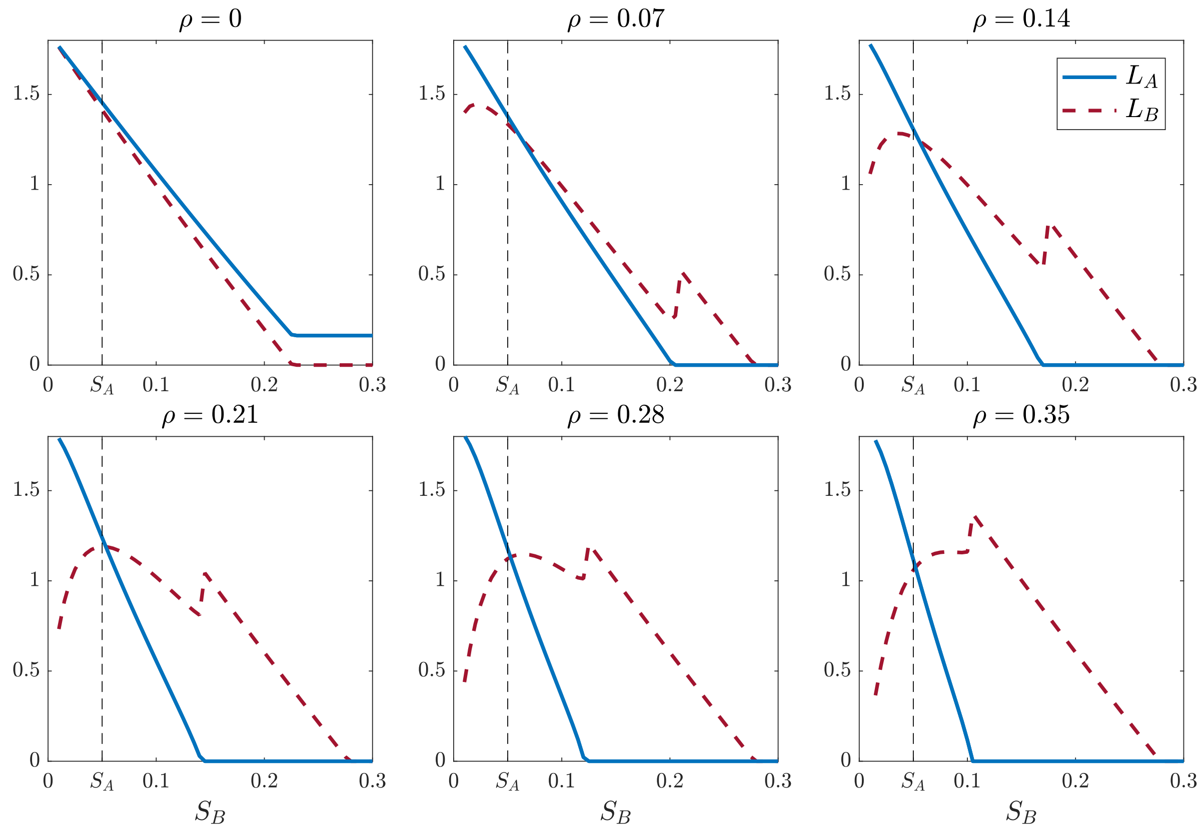


Figure 2.C.8. Liquidity premia with varying degrees of IRS. The figure depicts the liquidity premia of assets A and B as functions of S_B, for a constant S_A, and for varying degrees of IRS. The dashed vertical line indicates the (fixed) supply of asset A; π is set to 0.95.

2.4.2 Result 2: Safer yet Less Liquid

Of course, other things are not always equal, and we are particularly interested in asset supplies.[24] Allowing for differences in asset supplies delivers the second important result of the paper: even with slight IRS in OTC matching, the coordination channel becomes so strong that asset demand curves can be upward sloping. Consequently, asset B can carry a higher liquidity premium than the safe asset A, as long as the supply of the former is sufficiently larger than that of the latter.

Figure 2.C.8 depicts the liquidity premia for assets A and B as functions of the supply

[24] Recall that the matching efficiency and the bargaining protocol are assumed to be identical in both markets, because we do not want to give one of the assets an exogenous liquidity advantage.

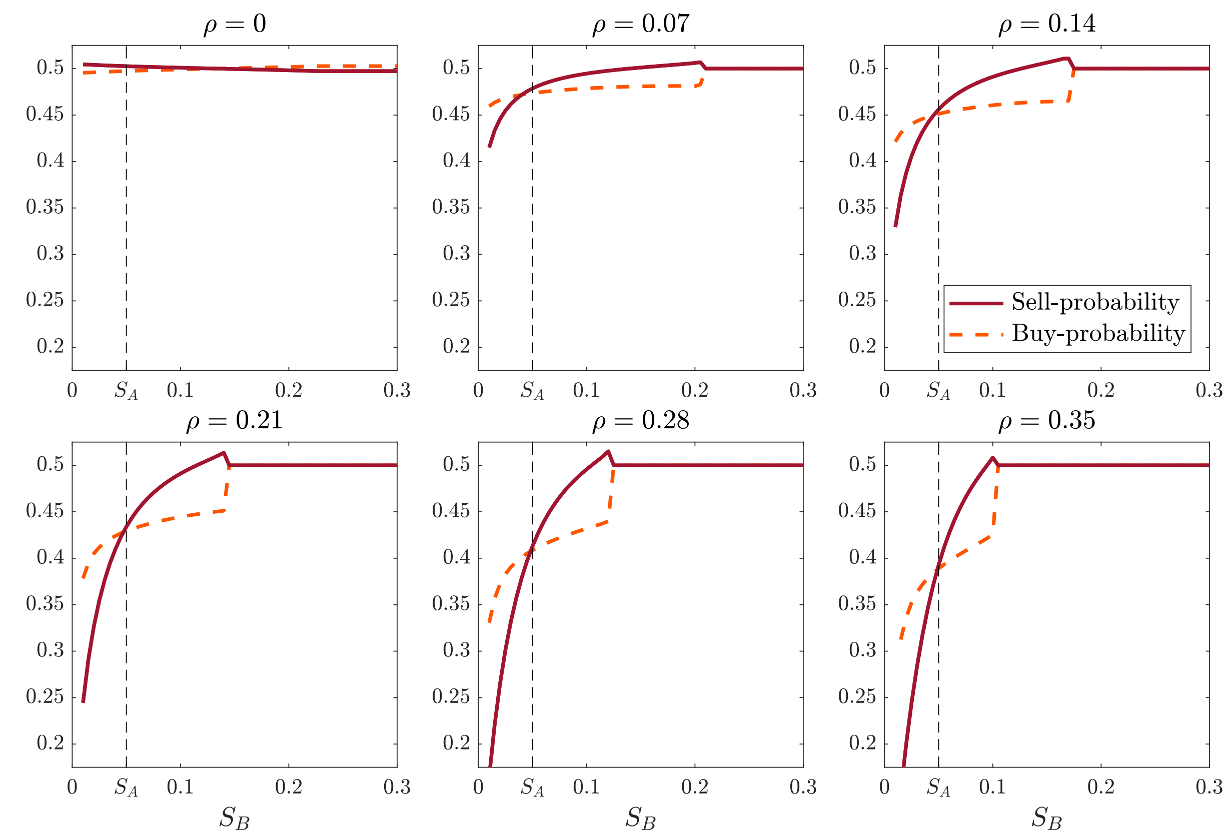


Figure 2.C.9. Sell- and buy-probabilities in the OTC_B. The figure depicts the sell-probability, α^n_{CB}, and the buy-probability, α^n_{NB}, in the secondary market for asset B, in the normal state, as a function of S_B (and for varying degrees of IRS). The dashed vertical line indicates the (fixed) supply of asset A; for the default probability, we set $\pi = 0.95$.

S_B, keeping S_A fixed, and for various degrees of IRS in matching. First, notice that the liquidity premium on asset A is always decreasing in S_B. With CRS (top-left panel), this is also true for the liquidity premium on asset B, as is standard in existing models of asset liquidity. However, with even a small degree of IRS, the coordination channel becomes so strong that asset demands can slope upwards. And if S_B is significantly larger than S_A, we observe $L_B > L_A$, i.e., the less safe asset emerges as more liquid.

The mechanism of this result is as follows. As we have seen, our model has a channel whereby a safer asset also enjoys an endogenous liquidity advantage. However, whether this advantage will materialize depends on the relative strength of the dilution effect: If the supply of asset A is limited, as more agents choose to specialize in that asset, each one of them will only hold a small amount, and any bilateral meeting in OTC_A will generate a small surplus. Keeping this effect in mind, consider an increase in the supply of asset B. As a result, more agents are willing to trade in OTC_B because of the increase in the expected trading surplus in that market (conditional on no-default). Crucially, N-types respond more elastically to this increase because their market entry choice is not governed by their asset specialization choice (which is already sunk). Consequently, the trade probability in market B for C-types increases by far more than that for N-types, as illustrated in Figure 2.C.9. Why this is important for liquidity should now be transparent: the agent will be willing to pay a high liquidity premium for an asset if she expects a high probability of selling that asset (conditional on needing to sell, i.e., being a C-type). And with some IRS in matching, the aforementioned channel becomes so strong that the premium an agent is willing to pay for an asset is increasing in that asset's supply.

Figure 2.C.10 summarizes Results 1 and 2. It depicts the liquidity premia of assets A and B as functions of S_B, keeping S_A fixed, with a slight degree of IRS, $\rho = 0.2$. When the supplies of the two assets are equal ($S_B = S_A$), asset A carries a higher liquidity premium (Result 1). However, as S_B increases further, we enter the region where the demand for asset B becomes upward sloping, until eventually L_B surpasses L_A (Result 2).

2.4.2.1 Rationalizing the Illiquidity of AAA Corporate Bonds

An interesting fact that has recently drawn the attention of practitioners (but not so much that of academic researchers yet) is that, in the U.S., the virtually default-free AAA bonds are less liquid than (the riskier) AA corporate bonds. Figure 2.C.11 plots the time-series

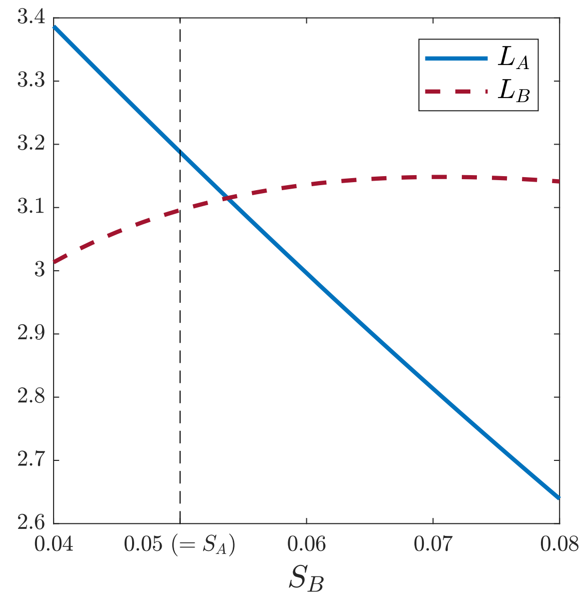


Figure 2.C.10. Liquidity premia. The figure depicts the liquidity premia of assets A and B as functions of S_B, for a constant S_A, and for $\rho = 0.2$. The dashed vertical line indicates the (fixed) supply of asset A; π is set to 0.95.

yields of AAA versus AA corporate bonds (as well as their difference) on the top panel, and, as a reference point, it does the same for the AAA versus AA municipal bond yields in the bottom panel.[25] The bottom panel is consistent with what one would expect to see: the riskier AA municipal bonds command a higher yield than the one on AAA municipal bonds, because investors who choose to hold the former want to be compensated for their higher default probability.

Interestingly, this logical pattern is reversed in the case of corporate bonds. Indeed, on the top panel of the figure, we see that in the past 5 years, the yield on AA corporate bonds has been consistently lower than that on AAA bonds. Why do investors command a higher yield in order to hold (the virtually default-free) AAA corporate bonds? Many practitioners have claimed that this is so because the secondary market for AAA corporate

[25] The data on municipal bonds comes from Standard & Poor's, and the data on corporate bonds comes from Federal Reserve Economic Data (FRED). The original data is on a daily base, but, to make the graphs more legible, it is converted to a monthly base. The graphs show the historical yields for the past 10 years.

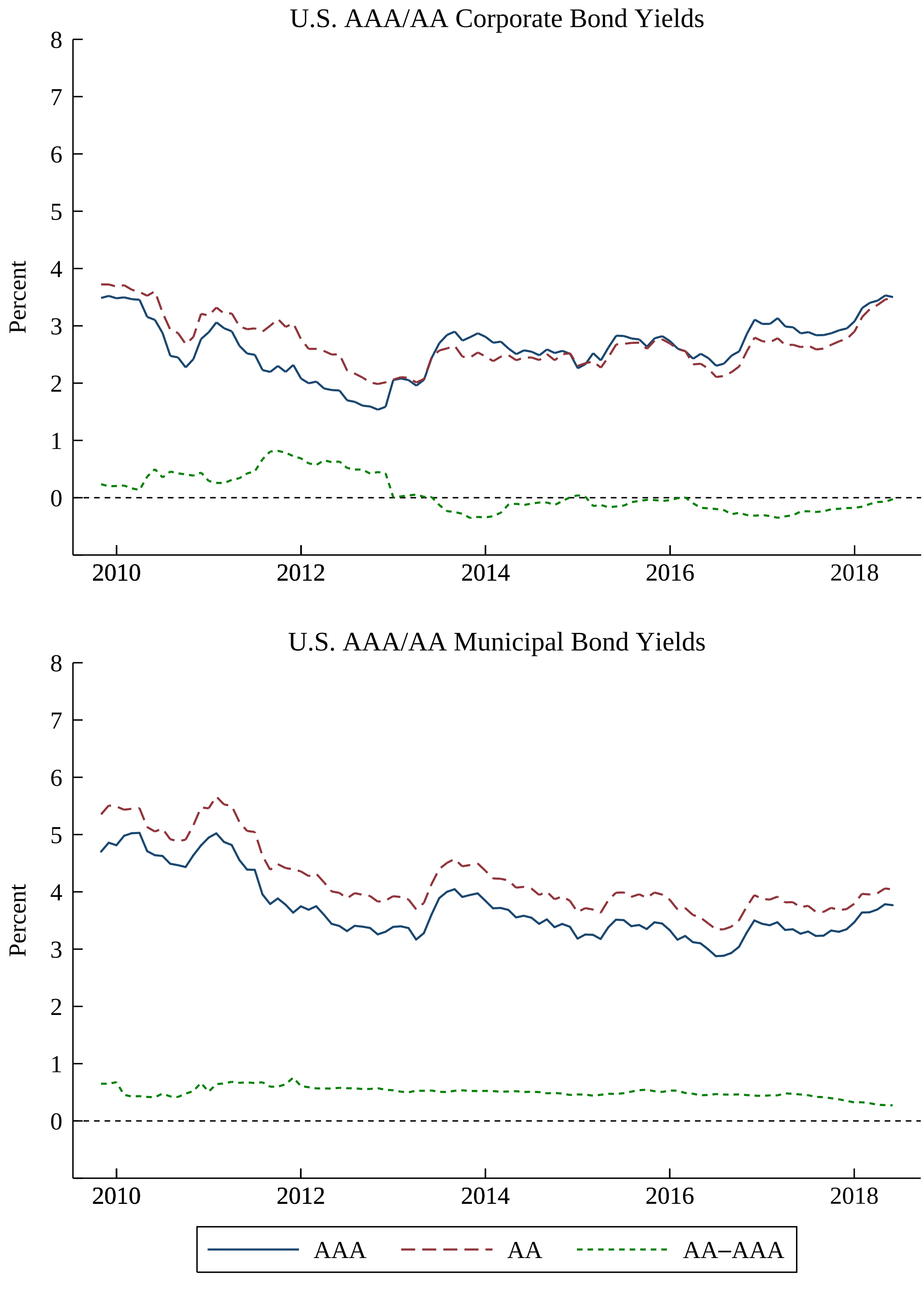


Figure 2.C.11. Historical yields of AAA/AA corporate/municipal bonds. Sources: FRED, Standard & Poor's.

bonds is extremely illiquid. This narrative is consistent with the observations depicted in Figure 2.C.11, and it is supported by further evidence. For instance, He and Milbradt (2014) document that the bid-ask spread in the market for AAA corporate bonds is higher than the one in the market for AA corporate bonds. Additionally, in recent years Bloomberg has ceased constructing its price index for AAA-rated corporate bonds, due to the dearth of outstanding bonds and the lack of secondary market trading. Of course, a high bid-ask spread and a low trade volume are both strong indicators of an illiquid market.

Our model could shed some light on this empirical observation, if it was the case that AAA corporate bonds have a scarce supply relative to AA corporate bonds. This turns out to be overwhelmingly true. In the years following the financial crisis, regulations introduced to improve the stability and transparency of the financial system (such as the Dodd-Frank Act) have made it especially hard for corporations to attain the AAA score. This resulted in a large decrease in the outstanding supply of this class of bonds.[26] As a benchmark of comparison, in June 2018, the outstanding supply of AAA over AA corporate bonds was 1/10, while the same statistic for municipal bonds was 1/3.

While it is plausible to attribute the irregularity observed on the top panel of Figure 2.C.11 to 'some liquidity story', existing models of liquidity cannot help us understand this puzzling observation (see a review of the literature in Section 2.1.1). In these papers, the asset demand curves are decreasing, hence, an asset in large (small) supply will tend to have a low (high) liquidity premium. Our model formalizes the idea that an asset in very scarce supply will be illiquid, even if it maintains an excellent credit rating. And our

[26] The number of AAA-rated corporations in the U.S., never high, decreased to four—Automatic Data Processing, Exxon Mobil, Johnson & Johnson, and Microsoft—in 2011. Automatic Data Processing got downgraded in 2014, and Exxon Mobil in 2016. Today, there are only two AAA-rated companies.

'indirect liquidity' approach, coupled with endogenous market entry, is key for delivering this empirically relevant result.

It should be pointed out that the case of AAA versus AA US corporate bonds is not the only one where the commonly held belief that "safety and liquidity go together" is violated. Christensen and Mirkov (2019) highlight yet another class of bonds—Swiss Confederation Bonds—that are considered extremely safe, yet not particularly liquid. Furthermore, Beber, Brandt, and Kavajecz (2008) report that Italian government bonds are among the most liquid, despite also being among the riskiest Euro-area sovereign bonds. The authors justify this observation by pointing to the large supply of Italian debt, which is consistent with our model's prediction.

2.4.3 Result 3: Safe Asset Supply and Welfare

In our final result, we highlight an important implication of our model about the effect of an increase in the supply of safe assets on welfare. A large body of recent literature highlights that the supply of safe assets has been scarce, and that increasing this supply would be beneficial for welfare (see, for example, Caballero, Farhi, and Gourinchas, 2017). In our model this result is not necessarily true. In particular, welfare may not be monotonic in S_A.

First, let us define the welfare function of this economy, which is the C-type agent's surplus in the DM, averaged between agents who had the opportunity to rebalance their portfolios in the OTC round of trade, and those who did not.[27] Clearly, one also needs to remember that here we have agents who chose to specialize in different assets, and two

[27] In models that build on LW, steady-state welfare depends only on the volume of DM trade. Hence, a sufficient statistic for welfare is how close the average DM production is to the first-best quantity, q^*.

possible aggregate states (default and no-default). In the normal state, welfare is:

$$\begin{aligned}
\mathcal{W}^n &= \big(e_C\ell - f(e_C\ell, e_N^n(1-\ell))\big)\cdot\big(u(q_{0A}) - q_{0A}\big) + f(e_C\ell, e_N^n(1-\ell))\cdot\big(u(q_{1A}) - q_{1A}\big) \\
&\quad + \big((1-e_C)\ell - f((1-e_C)\ell, (1-e_N^n)(1-\ell))\big)\cdot\big(u(q_{0B}) - q_{0B}\big) \\
&\quad + f((1-e_C)\ell, (1-e_N^n)(1-\ell))\cdot\big(u(q_{1B}) - q_{1B}\big) \\
&= e_C\ell\cdot\big[(1-\alpha_{CA}^n)\big(u(q_{0A}) - q_{0A}\big) + \alpha_{CA}^n\big(u(q_{1A}) - q_{1A}\big)\big] \\
&\quad + (1-e_C)\ell\cdot\big[(1-\alpha_{CB}^n)\big(u(q_{0B}) - q_{0B}\big) + \alpha_{CB}^n\big(u(q_{1B}) - q_{1B}\big)\big],
\end{aligned}$$

and in the default state, it is:

$$\begin{aligned}
\mathcal{W}^d &= \big(e_C\ell - f(e_C\ell, 1-\ell)\big)\cdot\big(u(q_{0A}) - q_{0A}\big) + f(e_C\ell, 1-\ell)\cdot\big(u(q_{1A}) - q_{1A}\big) \\
&\quad + (1-e_C)\ell\cdot\big(u(q_{0B}) - q_{0B}\big) \\
&= e_C\ell\cdot\big[(1-\alpha_{CA}^d)\big(u(q_{0A}) - q_{0A}\big) + \alpha_{CA}^d\big(u(q_{1A}) - q_{1A}\big)\big] + (1-e_C)\ell\cdot\big(u(q_{0B}) - q_{0B}\big).
\end{aligned}$$

We define aggregate welfare as:

$$\mathcal{W} = \pi\mathcal{W}^n + (1-\pi)\mathcal{W}^d. \tag{2.15}$$

Figure 2.C.12 plots equilibrium welfare as a function of the supply of the safe asset, and highlights the case in which welfare is non-monotonic in S_A. This result may seem surprising at first. A higher supply of asset A enhances the liquidity role of that asset (or, equivalently, allows for more secondary market asset trade), which, in turn, should allow agents to purchase more goods in the DM. While not wrong, this intuition is incomplete.

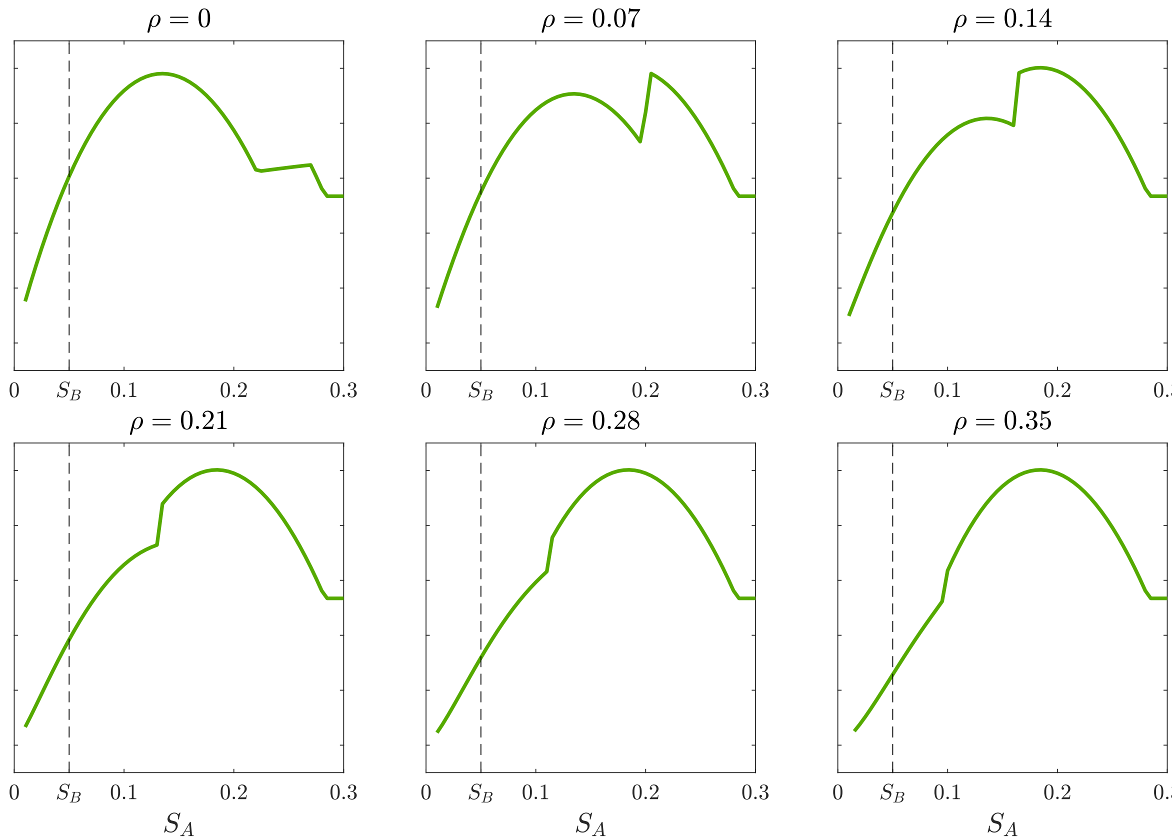


Figure 2.C.12. Safe asset supply and welfare. The figure depicts equilibrium welfare as a function of S_A, for various values of ρ, including $\rho = 0$ (CRS). The dashed vertical line indicates the (fixed) supply of asset B; π is set to 0.95.

What is missing is that when the safe asset becomes more plentiful, agents expect that it will be easier to acquire extra cash ex-post and, thus, they choose to hold less of it ex-ante. In other words, our model is characterized by an externality: agents prefer to carry assets rather than money, and they wish to acquire money in the secondary market(s) only after they have learned that they really need it (i.e., only if they have turned out to be a C-type). But someone has to bring the money, and that someone will not be adequately compensated. This channel depresses the demand for money, which, in turn, decreases the value of money and the volume of trade that the existing money supply can support.

An interesting detail seen in Figure 2.C.12 is that welfare always decreases when S_A

is large enough. This feature of equilibrium can be explained a follows. As S_A increases, the amount of DM goods purchased by an agent who traded in OTC$_A$, q_{1A}, also increases, because that agent was able to sell more assets and boost her money holdings. On the other hand, as S_A increases, the amount of DM goods purchased by an agent who did not trade in OTC$_A$, q_{0A}, decreases, because the higher asset supply induced that agent to carry fewer money balances *ex-ante* (see previous paragraph). Hence, an increase in S_A generates two opposing effects on welfare: the surplus term $u(q_{1A}) - q_{1A}$ (involving agents who traded in OTC$_A$) increases, but the surplus term $u(q_{0A}) - q_{0A}$ (involving agents who did not trade in OTC$_A$) decreases.[28] While it is hard to know which effect prevails for any value of S_A, what is certain is that if S_A keeps rising, there will come a point where the *marginal* liquidity benefit of more A-assets will be zero (because $q_{1A} \to q^*$ implies $u'(q_{1A}) \to 1$). Near that point, an increase in S_A still hurts welfare by depressing $u(q_{0A}) - q_{0A}$ (because $u'(q_{0A}) \gg 1$), but now it generates no countervailing benefit.

2.5 Conclusion

We argue that understanding the link between an asset's safety and its liquidity is crucial. To this end, we present a general equilibrium model where asset safety and asset liquidity are well-defined and *distinct* from one another. Treating safety as a primitive, we examine the relationship between an asset's safety and liquidity. We show that the commonly held belief that "safety implies liquidity" is generally justified, but there may be exceptions. In

[28] Of course, this is a general equilibrium model where any change in S_A affects not only the terms q_{0A}, q_{1A}, but also the terms q_{0B}, q_{1B}. However, the latter is a secondary effect which turns out to be quantitatively not too important.

particular, we highlight that a safe asset in scarce supply may be less liquid than a less-safe asset in large supply. Thus, our model can rationalize the puzzling observation that AAA corporate bonds in the U.S. are less liquid than (the riskier) AA corporate bonds. Contrary to a recent literature on the role of safe assets, we show that in our model increasing the supply of the safe asset is not always beneficial for welfare.

2.A Appendix

2.A.1 Value Functions

2.A.1.1 Value Functions in the CM

Consider first an agent who enters the CM with m units of money and d_j units of asset j, $j = \{A, B\}$ in state $s = \{n, d\}$. The value function of the agent is given by

$$W_s(m, d_A, d_B) = \max_{\substack{X,H,\\ \hat{m},\hat{d}_A,\hat{d}_B}} \left\{ X - H + \beta\, \mathbb{E}_{s,i}\Big[\max\Big\{\Omega^s_{iA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^s_{iB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\}\Big] \right\}$$

$$\begin{aligned} \text{s.t. } & X + \varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) = H + \varphi(m + \mu M + d_A + d_B), && \text{if } s = n \quad \text{(normal)}, \\ & X + \varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) = H + \varphi(m + \mu M + d_A), && \text{if } s = d \quad \text{(default)}, \end{aligned}$$

where variables with hats denote portfolio choices for the next period, and $\mathbb{E}$ is the expectation operator over states and types of consumers. Ω^s_{ij} denotes a value function of an i-type agent, $i = \{C, N\}$, who enters the OTC market for asset j in state s, and it is described in the next section. Replacing $X - H$ from the budget constraint yields

$$
\begin{aligned}
W_s(m, d_A, d_B) = {} & \varphi(m + \mu M + d_A + d_B \cdot \mathbb{I}\{s = n\}) \\
& + \max_{\hat{m}, \hat{d}_A, \hat{d}_B} \Bigg\{ - \varphi(\hat{m} + p_A \hat{d}_A + p_B \hat{d}_B) \\
& \qquad + \beta\pi\ell \max\Big\{\Omega^n_{CA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^n_{CB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\} \\
& \qquad + \beta\pi(1-\ell) \max\Big\{\Omega^n_{NA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^n_{NB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\} \\
& \qquad + \beta(1-\pi)\ell \max\Big\{\Omega^d_{CA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^d_{CB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\} \\
& \qquad + \beta(1-\pi)(1-\ell) \max\Big\{\Omega^d_{NA}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big), \Omega^d_{NB}\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\Big\} \Bigg\},
\end{aligned}
\tag{2.16}
$$

where $\mathbb{I}$ is the indicator function, and we have used the fact that asset B defaults with probability $1 - \pi$ and that an agent becomes C-type with probability ℓ. This can be simply written as follows:

$$
\begin{aligned}
W_n(m, d_A, d_B) &= \varphi(m + d_A + d_B) + \Lambda, \\
W_d(m, d_A, d_B) &= \varphi(m + d_A) + \Lambda,
\end{aligned}
\tag{2.17}
$$

where Λ collects the remaining terms that do not depend on the current states.

The value function for a producer is much simpler. Note that producers will not want to leave the CM with a positive amount of money or assets, as long as the assets are priced at a liquidity premium. The reason is that a producer's identity is permanent; so, there is no reason for her to bring money or buy assets with paying a liquidity premium when she knows that she will never have a liquidity need in the DM. Therefore, when entering the CM, a producer will only hold money that she received as payment in the preceding DM.

Thus, the value function for a producer is given by

$$W^P(m) = \max_{X,H} \left\{ X - H + \beta\, \mathbb{E}_s \left[V_s^P \right] \right\}$$

$$\text{s.t. } X = H + \varphi m,$$

where V_s^S denotes a value function of a producer in the DM in state s that will be described later. Notice that the value function does not depend on states of the economy. Using the budget constraint, we can re-write the value function as follows:

$$W^P(m) = \varphi m + \beta \left(\pi V_n^P + (1-\pi) V_d^P \right) \equiv \varphi m + \Lambda^P.$$

Note that all agents have linear value functions in the CM. This is standard in models that build on LW, a result that follows from the (quasi) linear preferences, and it makes the bargaining solution in the DM easy to characterize.

2.A.1.2 Value Functions in the OTC markets

In the OTC markets, C-type agents are selling assets, and N-type agents are buying assets. Let $\Omega_{ij}^s(m, d_A, d_B)$ denote a value function of an agent of type i who decides to enter OTC_j in state s. ξ_j is the amount of money that gets transferred to a C-type, and χ_j the amount of asset j that gets transferred to an N-type in a typical match in OTC_j. These terms of trade are described in the next section. The value functions are given by

$$
\begin{aligned}
\Omega^n_{CA}(m, d_A, d_B) &= \alpha^n_{CA} V_n(m + \xi_A, d_A - \chi_A, d_B) + (1 - \alpha^n_{CA}) V_n(m, d_A, d_B),\\
\Omega^d_{CA}(m, d_A, d_B) &= \alpha^d_{CA} V_d(m + \xi_A, d_A - \chi_A, d_B) + (1 - \alpha^d_{CA}) V_d(m, d_A, d_B),\\
\Omega^n_{CB}(m, d_A, d_B) &= \alpha^n_{CB} V_n(m + \xi_B, d_A, d_B - \chi_B) + (1 - \alpha^n_{CB}) V_n(m, d_A, d_B),\\
\Omega^d_{CB}(m, d_A, d_B) &= V_d(m, d_A, d_B),\\
\Omega^n_{NA}(m, d_A, d_B) &= \alpha^n_{NA} W_n(m - \xi_A, d_A + \chi_A, d_B) + (1 - \alpha^n_{NA}) W_n(m, d_A, d_B),\\
\Omega^d_{NA}(m, d_A, d_B) &= \alpha^d_{NA} W_d(m - \xi_A, d_A + \chi_A, d_B) + (1 - \alpha^d_{NA}) W_d(m, d_A, d_B),\\
\Omega^n_{NB}(m, d_A, d_B) &= \alpha^n_{NB} W_n(m - \xi_B, d_A, d_B + \chi_B) + (1 - \alpha^n_{NB}) W_n(m, d_A, d_B),\\
\Omega^d_{NB}(m, d_A, d_B) &= W_d(m, d_A, d_B),
\end{aligned} \tag{2.18}
$$

where V_s denotes a C-type agent's value function in the DM in state s. Note that OTC_B shuts down when asset B defaults, and thus N-type agents proceed directly to the CM, whereas C-type agents move on to the DM.

2.A.1.3 Value Functions in the DM

In the DM, C-type agents meet producers. Let q denote the quantity of DM goods traded and τ the total payment in units of money. These terms of trade are described in the next section. The value function of an agent who enters the DM with a portfolio (m, d_A, d_B) in state s is given by

$$V_s(m, d_A, d_B) = u(q) + W_s(m - \tau, d_A, d_B), \tag{2.19}$$

The value function of a producer, who enters with no money or assets, is given by

$$V^P = -q + W^P(\tau),$$

which does not depend on states of the economy.

2.A.2 Terms of Trade

2.A.2.1 Terms of Trade in the DM

Consider a meeting between a producer and a C-type agent with a portfolio (m, d_A, d_B). The two parties bargain over a quantity q to be produced by the producer and a cash payment τ to be made by the agent. The agent makes a take-it-or-leave-it offer maximizing her surplus subject to the producer's participation condition and the cash constraint:

$$\max_{\tau,\, q} \left\{ u(q) + W_s(m - \tau, d_A, d_B) - W_s(m, d_A, d_B) \right\}$$
$$\text{s.t.} \quad -q + W^P(\tau) - W^P(0) = 0, \quad \tau \leq m.$$

Using the linearity of the CM value functions, the C-type agent's surplus becomes $u(q) - \varphi\tau$ and the producer's surplus $-q + \varphi\tau$. This implies that the bargaining solution must satisfy $q(m) = \varphi\tau(m)$—that is, the producer will require $\tau(m)$ units of money for producing $q(m)$ of goods. When the agent has enough money to have the optimal level produced, that is, when $\varphi m \geq q^*$, q^* will be produced. Otherwise, φm will be produced. Define $m^* \equiv q^*/\varphi$ as the amount of money that allows an agent to purchase the first-best quantity, q^*. Then, the solution can be expressed in a concise way:

$$\begin{aligned} q(m) &= \min\{q^*, \varphi m\} \, \big(= \varphi\tau(m)\big), \\ \tau(m) &= \min\{m^*, m\}, \quad m^* \equiv q^*/\varphi. \end{aligned} \tag{2.20}$$

Since an agent will never choose to hold $m > m^*$ due to the cost of carrying money, we will focus on the binding branch of the bargaining solution, $q(m) = \varphi m$ and $\tau(m) = m$.

2.A.2.2 Terms of Trade in the OTC Markets

Consider a meeting in OTC$_j$ between a C-type agent with a portfolio (m, d_A, d_B) who wants to sell assets and an N-type agent with $(\tilde{m}, \tilde{d}_A, \tilde{d}_B)$ who wants to buy assets. Let χ_j be the amount of asset j will be traded for ξ_j amount of money as a result of bargaining. The Kalai bargaining applies with the asset seller's bargaining power denoted by θ. Then, the bargaining surplus of an i-type consumer from an OTC$_j$ trading in state s, $\mathcal{S}_{ij}^s$, are given by

$$\begin{aligned}
\mathcal{S}_{CA}^n &= V_n(m + \xi_A, d_A - \chi_A, d_B) - V_n(m, d_A, d_B) = u(\varphi(m + \xi_A)) - u(\varphi m) - \varphi\chi_A, \\
\mathcal{S}_{NA}^n &= W_n(\tilde{m} - \xi_A, \tilde{d}_A + \chi_A, \tilde{d}_B) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi\xi_A + \varphi\chi_A, \\
\mathcal{S}_{CA}^d &= V_d(m + \xi_A, d_A - \chi_A, d_B) - V_d(m, d_A, d_B) = u(\varphi(m + \xi_A)) - u(\varphi m) - \varphi\chi_A, \\
\mathcal{S}_{NA}^d &= W_d(\tilde{m} - \xi_A, \tilde{d}_A + \chi_A, \tilde{d}_B) - W_d(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi\xi_A + \varphi\chi_A, \\
\mathcal{S}_{CB}^n &= V_n(m + \xi_B, d_A, d_B - \chi_B) - V_n(m, d_A, d_B) = u(\varphi(m + \xi_B)) - u(\varphi m) - \varphi\chi_B, \\
\mathcal{S}_{NB}^n &= W_n(\tilde{m} - \xi_B, \tilde{d}_A, \tilde{d}_B + \chi_B) - W_n(\tilde{m}, \tilde{d}_A, \tilde{d}_B) = -\varphi\xi_B + \varphi\chi_B.
\end{aligned}$$

Notice that $\mathcal{S}_{CA}^n = \mathcal{S}_{CA}^d$ and $\mathcal{S}_{NA}^n = \mathcal{S}_{NA}^d$; thus, the solutions will not depend on states of the economy. $\mathcal{S}_{CB}^d$ and $\mathcal{S}_{CB}^d$ are not defined since OTC$_B$ shuts down when asset B defaults. Thus, we will simply write as follows: $\mathcal{S}_{CA}(\equiv \mathcal{S}^n_{CA} = \mathcal{S}^d_{CA})$, $\mathcal{S}_{NA}(\equiv \mathcal{S}^n_{NA} = \mathcal{S}^d_{NA})$, $\mathcal{S}_{CB}(\equiv \mathcal{S}^n_{CB})$, and $\mathcal{S}_{NB}(\equiv \mathcal{S}^n_{NB})$. The expressions for the surpluses can be simplified as follows:

$$\mathcal{S}_{Cj} = u(\varphi(m+\xi_j)) - u(\varphi m) - \varphi\chi_j,$$

$$\mathcal{S}_{Nj} = -\varphi\xi_j + \varphi\chi_j.$$

Since money is costly to carry, in equilibrium, C-type agents will bring $m < m^*$ and want to acquire the amount of money that she is missing in order to reach m^*, namely, $m^* - m$. Whether she will be able to acquire that amount of money depends on her asset holdings. If her asset holdings are enough, then she will be able to acquire $m^* - m$ units of money. If not, she will give up all her assets to obtain as much money as possible.

An assumption behind this discussion is that N-type's money holdings never limit the trade. That is, we assume that $m + \tilde{m} \geq m^*$, i.e., that the money holdings of the C-type and the N-type pulled together is enough to allow the C-type to purchase the first best quantity q^*, hence ignoring the constraint $\xi_j \leq \tilde{m}$ in the bargaining problem. This will be true in equilibrium as long as inflation is not too large so that all agents carry at least *half* of the first-best amount of money (see also footnote 13).

Thus, the bargaining problem is described by

$$\max_{\xi_j,\chi_j} \mathcal{S}_{Cj} \quad \text{s.t.} \quad \mathcal{S}_{Cj} = \frac{\theta}{1-\theta}\mathcal{S}_{Nj}, \quad \chi_j \leq d_j.$$

From the Kalai constraint, we get

$$\varphi\chi_j = z(\xi_j) \equiv (1-\theta)\Big(u(\varphi(m+\xi_j)) - u(\varphi m)\Big) + \theta\varphi\xi_j,$$

which says that the asset seller has to give up $z(\xi_j)/\varphi$ amount of asset j to acquire ξ_j amount of money. Note that $z'(\xi_j) > 0$, and recall that the optimal amount of money that the asset seller wants to achieve is $m^* - m$. When the asset seller has enough assets to compensate $m^* - m$, that is, when $\varphi d_j \geq z(m^* - m)$, $m^* - m$ will be traded. Otherwise, d_j will be traded. The solution can be expressed in a concise way:

$$\begin{aligned} &\chi_j(m, d_j) = \min\{d_j^*, d_j\} \big(= z(\xi_j(m, d_j))/\varphi\big), \;\; d_j^* \equiv \frac{z(m^* - m)}{\varphi}, \\ &\xi_j(m, d_j) = \min\{m^* - m, \tilde{\xi}_j(m, d_j)\}, \;\; \varphi d_j = z(\tilde{\xi}_j). \end{aligned} \tag{2.21}$$

With the discussion above in mind, note that the solution does not depend on the N-type consumer's portfolio, but only on the C-type's. Also, note that $\xi_j(m, d_j)$ is increasing in d_j (the more assets a C-type has, the more money she can acquire) and decreasing in m (the more money a C-type carries, the less she needs to acquire through OTC trade).

2.A.3 Objective Function

As is standard in models that build on LW, all agents choose their optimal portfolio in the CM independently of their trading histories in previous markets. In our model, in addition to choosing an optimal portfolio of money and assets, $(\hat{m}, \hat{d}_A, \hat{d}_B)$, agents also choose which OTC market they will enter in order to sell or buy assets, once the shocks have been realized. To analyze the agent's choice, we substitute the agent's value functions in the OTC markets and the DM (equations (2.18) and (2.19)) into the maximization operator of the CM value function (2.16) and use the linearity of the CM value functions (equation (2.17)), dropping the terms that do not depend on the choice variables, to obtain

$$
\begin{aligned}
-\varphi(\hat{m}+p_A\hat{d}_A+p_B\hat{d}_B)+\beta\pi\ell\Big(\hat{\varphi}(\hat{d}_A+\hat{d}_B)+u(\hat{\varphi}\hat{m})+\max\{\alpha^n_{CA}\mathcal{S}_{CA},\alpha^n_{CB}\mathcal{S}_{CB}\}\Big)& \\
+\beta\pi(1-\ell)\hat{\varphi}(\hat{m}+\hat{d}_A+\hat{d}_B)& \\
+\beta(1-\pi)\ell\Big(\hat{\varphi}\hat{d}_A+u(\hat{\varphi}\hat{m})+\alpha^d_{CA}\mathcal{S}_{CA}\Big)& \\
+\beta(1-\pi)(1-\ell)\hat{\varphi}(\hat{m}+\hat{d}_A),&
\end{aligned}
$$

from which we finally get the objective function:

$$
\begin{aligned}
J(\hat{m},\hat{d}_A,\hat{d}_B)\equiv&-\varphi(\hat{m}+p_A\hat{d}_A+p_B\hat{d}_B)+\beta\hat{\varphi}(\hat{m}+\hat{d}_A+\pi\hat{d}_B)\\
&+\beta\ell\Big(u(\hat{\varphi}\hat{m})-\hat{\varphi}\hat{m}+\pi\max\{\alpha^n_{CA}\mathcal{S}_{CA},\alpha^n_{CB}\mathcal{S}_{CB}\}+(1-\pi)\alpha^d_{CA}\mathcal{S}_{CA}\Big)\\
=&-\beta\hat{\varphi}i\hat{m}-\beta\hat{\varphi}(1+i)\left(p_A-\frac{1}{1+i}\right)\hat{d}_A-\beta\hat{\varphi}(1+i)\left(p_B-\frac{\pi}{1+i}\right)\hat{d}_B\\
&+\beta\ell\Big(u(\hat{\varphi}\hat{m})-\hat{\varphi}\hat{m}+\pi\max\{\alpha^n_{CA}\mathcal{S}_{CA},\alpha^n_{CB}\mathcal{S}_{CB}\}+(1-\pi)\alpha^d_{CA}\mathcal{S}_{CA}\Big),
\end{aligned}
\tag{2.22}
$$

with $i\equiv(1+\mu)/\beta-1$, where

$$
\mathcal{S}_{Cj}=\theta\left[u(\hat{\varphi}(\hat{m}+\xi_j(\hat{m},\hat{d}_j)))-u(\hat{\varphi}\hat{m})-\hat{\varphi}\xi_j(\hat{m},\hat{d}_j)\right].
$$

2.A.4 Asset Demand

Asset demand equations are derived from the first-order conditions of the objective function (2.22) with respect to $\hat{d}_A$ and $\hat{d}_B$:

$$
\begin{aligned}
&\{\hat{d}_A\}\quad (1+i)p_A-1=\ell\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)\frac{1}{\hat{\varphi}}\frac{\partial\mathcal{S}_{CA}}{\partial\hat{d}_A},\\
&\{\hat{d}_B\}\quad (1+i)p_B-\pi=\ell\pi\alpha^n_{CB}\frac{1}{\hat{\varphi}}\frac{\partial\mathcal{S}_{CB}}{\partial\hat{d}_B},
\end{aligned}
$$

where

$$\frac{\partial \mathcal{S}_{Cj}}{\partial \hat{d}_j} = \frac{\partial \mathcal{S}_{Cj}}{\partial \tilde{\xi}_j}\frac{\partial \tilde{\xi}_j}{\partial \hat{d}_j} = \theta\left(u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j))-1\right)\hat{\varphi}\frac{\partial \tilde{\xi}_j}{\partial \hat{d}_j},$$
$$\hat{\varphi} = z'(\tilde{\xi}_j)\frac{\partial \tilde{\xi}_j}{\partial \hat{d}_j} = \hat{\varphi}\left(\theta+(1-\theta)u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j))\right)\frac{\partial \tilde{\xi}_j}{\partial \hat{d}_j},$$

where the second equation is from total differentiation of $\hat{\varphi}\hat{d}_j = z(\tilde{\xi}_j(\hat{m},\hat{d}_j))$.

From above, we can get the asset demand equations (2.4) and (2.5) by expressing in terms of the equilibrium quantities:

$$\begin{aligned}(1+i)p_A - 1 &= \ell\theta\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A})-1)\frac{1}{\theta+(1-\theta)u'(q_{1A})},\\ (1+i)p_B - \pi &= \ell\theta\pi\alpha^n_{CB}(u'(q_{1B})-1)\frac{1}{\theta+(1-\theta)u'(q_{1B})}.\end{aligned} \tag{2.23}$$

2.A.5 Money Demand

Money demand equations are derived from the first-order conditions of the objective function (2.22) with respect to $\hat{m}$:

$$\begin{aligned}\{\hat{m}\}\quad i &= \ell\left((u'(\hat{\varphi}\hat{m})-1)+\left(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\right)\frac{1}{\hat{\varphi}}\frac{\partial \mathcal{S}_{CA}}{\partial \hat{m}}\right),\\ i &= \ell\left((u'(\hat{\varphi}\hat{m})-1)+\pi\alpha^n_{CB}\frac{1}{\hat{\varphi}}\frac{\partial \mathcal{S}_{CB}}{\partial \hat{m}}\right),\end{aligned}$$

where

$$\frac{\partial \mathcal{S}_{Cj}}{\partial \hat{m}} = \theta\left(u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j))-u'(\hat{\varphi}\hat{m})\right)+\theta\left(u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j))-1\right)\hat{\varphi}\frac{\partial \tilde{\xi}_j}{\partial \hat{m}},$$
$$0 = (1-\theta)\left(u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j))-u'(\hat{\varphi}\hat{m})\right)+(1-\theta)u'(\hat{\varphi}(\hat{m}+\tilde{\xi}_j))\frac{\partial \tilde{\xi}_j}{\partial \hat{m}}+\theta\frac{\partial \tilde{\xi}_j}{\partial \hat{m}},$$

where the second equation is from total differentiation of $\hat{\varphi}\hat{d}_j = z(\tilde{\xi}_j(\hat{m}, \hat{d}_j))$.

From above, we can get the money demand equations (2.6) and (2.7) by expressing in terms of the equilibrium quantities:

$$\begin{aligned}
i &= \ell\Big(1-\theta\Big(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d\Big)\Big)(u'(q_{0A})-1) \\
&\quad + \ell\theta\Big(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d\Big)(u'(q_{1A})-1)\left(1-\frac{(1-\theta)(u'(q_{1A})-u'(q_{0A}))}{\theta+(1-\theta)u'(q_{1A})}\right), \qquad (2.24) \\
i &= \ell(1-\theta\pi\alpha_{CB}^n)(u'(q_{0B})-1) + \ell\theta\pi\alpha_{CB}^n(u'(q_{1B})-1)\left(1-\frac{(1-\theta)(u'(q_{1B})-u'(q_{0B}))}{\theta+(1-\theta)u'(q_{1B})}\right).
\end{aligned}$$

2.A.6 OTC Trade Volume

The OTC trade volumes in the normal state are defined by

$$\Delta_A^n \equiv f(e_C\ell, e_N^n(1-\ell))\cdot\chi_A(m, d_A),$$

$$\Delta_B^n \equiv f((1-e_C)\ell, (1-e_N^n)(1-\ell))\cdot\chi_B(m, d_B),$$

where

$$\begin{aligned}
\chi_j(m, d_j) &= \min\{d_j^*, d_j\}, \quad d_j^* \equiv \frac{z(m^*-m)}{\varphi}, \\
z(\xi) &\equiv (1-\theta)\Big(u(\varphi(m+\xi)) - u(\varphi m)\Big) + \theta\varphi\xi, \\
d_A &= \frac{S_A}{e_C}, \quad d_B = \frac{S_B}{1-e_C}.
\end{aligned}$$

These reduce as below:

$$\Delta_A^n = e_C \ell\, \alpha_{CA}^n \cdot \min\left\{ \frac{M\Big((1-\theta)(u(q^*)-u(q_{0A}))+\theta(q^*-q_{0A})\Big)}{e_C q_{0A}+(1-e_C)q_{0B}}, \frac{S_A}{e_C} \right\},$$

$$\Delta_B^n = (1-e_C)\ell\, \alpha_{CB}^n \cdot \min\left\{ \frac{M\Big((1-\theta)(u(q^*)-u(q_{0B}))+\theta(q^*-q_{0B})\Big)}{e_C q_{0A}+(1-e_C)q_{0B}}, \frac{S_B}{1-e_C} \right\}.$$

The OTC trade volume of market A in the default state is defined by[29]

$$\begin{aligned}\Delta_A^d &\equiv f(e_C\ell, 1-\ell)\cdot \chi_A(m, d_A)\\ &= e_C \ell\, \alpha_{CA}^d \cdot \min\left\{ \frac{M\Big((1-\theta)(u(q^*)-u(q_{0A}))+\theta(q^*-q_{0A})\Big)}{e_C q_{0A}+(1-e_C)q_{0B}}, \frac{S_A}{e_C} \right\}.\end{aligned}$$

Then, the OTC trade volumes, averaged across the normal and default states, are

$$\begin{aligned}\Delta_A &\equiv \pi \Delta_A^n + (1-\pi)\Delta_A^d,\\ \Delta_B &\equiv \pi \Delta_B^n.\end{aligned} \tag{2.25}$$

2.A.7 Proofs

2.A.7.1 Classification of Equilibria

Proof of Proposition 2.1. (a) Assume $e_C = 0$. Then, $e_N^n = e_N^d = 0$ is clearly the best response of N-types. We claim that there is no profitable deviation of a C-type, i.e., $G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB} < 0$ when $e_C = 0$, $e_N^n = e_N^d = 0$. First, notice that when $e_C = 0$, that is, when nobody is holding asset A, $q_{0A} = q_{1A}\,(\equiv \bar{q})$ and $\tilde{\mathcal{S}}_{CA} = -\,i\bar{q} + \ell(u(\bar{q}) - \bar{q})$. If it were the case that nobody was purchasing asset B either, then $q_{0B} = q_{1B}\,(= \bar{q})$, $\tilde{\mathcal{S}}_{CB} = -\,i\bar{q} + \ell(u(\bar{q}) - \bar{q})$

[29]The OTC trade volume of market B in the default state is 0, since OTC_B shuts down.

and $G(e_C) = 0$. Since q_{1B} increases and q_{0B} decreases as agents hold asset B, it remains to show that $d\tilde{\mathcal{S}}_{CB}/dq_{1B} > 0$, keeping in mind that q_{0B} also changes as q_{1B} changes.

When $e_C = 0$ and $e_N^n = e_N^d = 0$, $\alpha_{CB}^n = 1 - \ell$:

$$\tilde{\mathcal{S}}_{CB} = -iq_{0B} - \Big(\ell\frac{\theta}{\omega_\theta(q_{1B})}\pi(1-\ell)(u'(q_{1B})-1)\Big)\Big((1-\theta)(u(q_{1B})-u(q_{0B}))+\theta(q_{1B}-q_{0B})\Big)$$
$$+\ell(u(q_{0B})-q_{0B})+\ell\pi(1-\ell)\theta\Big(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\Big).$$

Then,

$$\frac{d\tilde{\mathcal{S}}_{CB}}{dq_{1B}} = -\frac{dq_{0B}}{dq_{1B}}\begin{pmatrix} i - \dfrac{\ell((1-(1-\ell)\pi)\theta+(1-\theta)u'(q_{1B}))u'(q_{0B})}{\theta+(1-\theta)u'(q_{1B})} \\ +\dfrac{\ell(\theta+(1-(1+(1-\ell)\pi)\theta)u'(q_{1B}))}{\theta+(1-\theta)u'(q_{1B})}\end{pmatrix}$$
$$-\frac{1}{(\theta+(1-\theta)u'(q_{1B}))^2}(1-\ell)\ell\pi\theta\begin{pmatrix}(1-\theta)(u(q_{1B})-u(q_{0B})) \\ +\theta(q_{1B}-q_{0B})\end{pmatrix}u''(q_{1B}).$$

The coefficient of dq_{0B}/dq_{1B} in the first term is

$$i-\frac{\ell((1-(1-\ell)\pi)\theta+(1-\theta)u'(q_{1B}))u'(q_{0B})}{\theta+(1-\theta)u'(q_{1B})}+\frac{\ell(\theta+(1-(1+(1-\ell)\pi)\theta)u'(q_{1B}))}{\theta+(1-\theta)u'(q_{1B})}$$
$$=i-\Big(\frac{\ell((1-(1-\ell)\pi)\theta+(1-\theta)u'(q_{1B}))u'(q_{0B})}{\theta+(1-\theta)u'(q_{1B})}-\frac{\ell((1-(1-\ell)\pi)\theta+(1-\theta)u'(q_{1B}))}{\theta+(1-\theta)u'(q_{1B})}\Big)$$
$$-\Big(\frac{\ell((1-(1-\ell)\pi)\theta+(1-\theta)u'(q_{1B}))}{\theta+(1-\theta)u'(q_{1B})}-\frac{\ell(\theta+(1-(1+(1-\ell)\pi)\theta)u'(q_{1B}))}{\theta+(1-\theta)u'(q_{1B})}\Big)$$
$$=i-\ell\left(1-\frac{\theta}{\theta+(1-\theta)u'(q_{1B})}\pi(1-\ell)\right)(u'(q_{0B})-1)$$
$$-\ell\frac{\theta}{\theta+(1-\theta)u'(q_{1B})}\pi(1-\ell)(u'(q_{1B})-1),$$

which is 0 since it is equivalent to the money demand equation (2.7). Thus,

$$\frac{d\tilde{\mathcal{S}}_{CB}}{dq_{1B}} = -\frac{1}{(\theta + (1-\theta)u'(q_{1B}))^2}(1-\ell)\ell\pi\theta \begin{pmatrix} (1-\theta)(u(q_{1B}) - u(q_{0B})) \\ \\ +\,\theta(q_{1B} - q_{0B}) \end{pmatrix} u''(q_{1B}) > 0. \tag{2.26}$$

Therefore, $G(e_C) < 0$ when $e_C = 0$, $e_N^n = e_N^d = 0$. $\square$

(b) Assume $e_C = 1$. Then, $e_N^n = e_N^d = 1$ is clearly the best response of N-types. We claim that there is no profitable deviation of a C-type, i.e., $G(e_C) \equiv \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB} > 0$ when $e_C = 1$, $e_N^n = e_N^d = 1$. First, notice that when $e_C = 1$, that is, when nobody is holding asset B, $q_{0B} = q_{1B}\,(\equiv \bar{q})$ and $\tilde{\mathcal{S}}_{CB} = -i\bar{q} + \ell(u(\bar{q}) - \bar{q})$. If it were the case that nobody was purchasing asset A either, then $q_{0A} = q_{1A}\,(= \bar{q})$, $\tilde{\mathcal{S}}_{CA} = -\,i\bar{q} + \ell(u(\bar{q}) - \bar{q})$ and $G(e_C) = 0$. Since q_{1A} increases and q_{0A} decreases as agents hold asset A, it remains to show that $d\tilde{\mathcal{S}}_{CA}/dq_{1A} > 0$, keeping in mind that q_{0A} also changes as q_{1A} changes.

When $e_C = 1$ and $e_N^n = e_N^d = 1$, $\alpha_{CA}^n = \alpha_{CA}^d = 1 - \ell$:

$$\begin{aligned}\tilde{\mathcal{S}}_{CA} = &-iq_{0A} - \left(\ell\frac{\theta}{\omega_\theta(q_{1A})}(1-\ell)(u'(q_{1A}) - 1)\right)\Big((1-\theta)(u(q_{1A}) - u(q_{0A})) + \theta(q_{1A} - q_{0A})\Big) \\ &+ \ell(u(q_{0A}) - q_{0A}) + \ell(1-\ell)\theta\Big(u(q_{1A}) - u(q_{0A}) - q_{1A} + q_{0A}\Big).\end{aligned}$$

Then,

$$\begin{aligned}\frac{d\tilde{\mathcal{S}}_{CA}}{dq_{1A}} = &-\frac{dq_{0A}}{dq_{1A}}\left(i - \frac{\ell(\ell\theta + (1-\theta)u'(q_{1A}))u'(q_{0A})}{\theta + (1-\theta)u'(q_{1A})} + \frac{\ell(\theta + (1-(2-\ell)\theta)u'(q_{1A}))}{\theta + (1-\theta)u'(q_{1A})}\right) \\ &- \frac{1}{(\theta + (1-\theta)u'(q_{1A}))^2}(1-\ell)\ell\theta\Big((1-\theta)(u(q_{1A}) - u(q_{0A})) + \theta(q_{1A} - q_{0A})\Big)u''(q_{1A}).\end{aligned}$$

The coefficient of dq_{0A}/dq_{1A} in the first term is

$$\begin{aligned}
& i - \frac{\ell(\ell\theta + (1-\theta)u'(q_{1A}))u'(q_{0A})}{\theta + (1-\theta)u'(q_{1A})} + \frac{\ell(\theta + (1-(2-\ell)\theta)u'(q_{1A}))}{\theta + (1-\theta)u'(q_{1A})} \\
&= i - \left(\frac{\ell(\ell\theta + (1-\theta)u'(q_{1A}))u'(q_{0A})}{\theta + (1-\theta)u'(q_{1A})} - \frac{\ell(\ell\theta + (1-\theta)u'(q_{1A}))}{\theta + (1-\theta)u'(q_{1A})} \right) \\
&\quad - \left(\frac{\ell(\ell\theta + (1-\theta)u'(q_{1A}))}{\theta + (1-\theta)u'(q_{1A})} - \frac{\ell(\theta + (1-(2-\ell)\theta)u'(q_{1A}))}{\theta + (1-\theta)u'(q_{1A})} \right) \\
&= i - \ell \left(1 - \frac{\theta}{\theta + (1-\theta)u'(q_{1A})}(1-\ell) \right) (u'(q_{0A}) - 1) \\
&\quad - \ell \frac{\theta}{\theta + (1-\theta)u'(q_{1A})}(1-\ell)(u'(q_{1A}) - 1),
\end{aligned}$$

which is 0 since it is equivalent to the money demand equation (2.6). Thus,

$$\frac{d\tilde{\mathcal{S}}_{CA}}{dq_{1A}} = -\frac{1}{(\theta + (1-\theta)u'(q_{1A}))^2}(1-\ell)\ell\theta \begin{pmatrix} (1-\theta)(u(q_{1A}) - u(q_{0A})) \\ \\ +\theta(q_{1A} - q_{0A}) \end{pmatrix} u''(q_{1A}) > 0. \quad (2.27)$$

Therefore, $G(e_C) > 0$ when $e_C = 1$, $e_N^n = e_N^d = 1$. $\square$

(c) First observe the value of α_{CA}^n, α_{CA}^d and α_{CB}^n as $e_C \to 0+$. While $\alpha_{CA}^n = \alpha_{CA}^d = 0$ at $e_C = 0$, this is not the case when $e_C \to 0+$. From the optimal entry decision by N-types (2.11),

$$e_N^n = \frac{e_C(1 - e_C\ell - \ell S_{NB}/S_{NA} + e_C\ell S_{NB}/S_{NA}}{-(1-\ell)(-e_C - S_{NB}/S_{NA} + e_C S_{NB}/S_{NA})}.$$

Using this, as $e_C \to 0+$,

$$\begin{aligned}
\alpha_{CA}^n &\to \lim_{e_C \to 0+} \frac{(1-\ell)e_N^n}{\ell e_C + (1-\ell)e_N^n} = \lim_{e_C \to 0+} \frac{(1-\ell)\frac{e_C(1-e_C\ell-\ell S_{NB}/S_{NA}+e_C\ell S_{NB}/S_{NA}}{-(1-\ell)(-e_C-S_{NB}/S_{NA}+e_C S_{NB}/S_{NA})}}{\ell e_C + (1-\ell)\frac{e_C(1-e_C\ell-\ell S_{NB}/S_{NA}+e_C\ell S_{NB}/S_{NA}}{-(1-\ell)(-e_C-S_{NB}/S_{NA}+e_C S_{NB}/S_{NA})}} \\
&= \lim_{e_C \to 0+} 1 + e_C\ell(-1+\frac{S_{NB}}{S_{NA}}) - \ell\frac{S_{NB}}{S_{NA}} = 1 - \ell\frac{S_{NB}}{S_{NA}} \, (> 1-\ell), \\
\alpha_{CA}^d &\to \lim_{e_C \to 0+} \frac{1-\ell}{\ell e_C + (1-\ell)} = 1.
\end{aligned}$$

On the other hand, α_{CB}^n continuously converges to its value, $1-\ell$, at $e_C = 0$ as $e_C \to 0+$.

Hence, as $e_C \to 0+$, $\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d > \pi\alpha_{CB}^n$. Therefore,

$$\begin{aligned}
\tilde{\mathcal{S}}_{CA} =& - iq_{0A} - \left(\ell\frac{\theta}{\omega_\theta(q_{1A})}\left(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d\right)(u'(q_{1A})-1)\right)\begin{pmatrix}(1-\theta)(u(q_{1A})-u(q_{0A})) \\ \\ +\theta(q_{1A}-q_{0A})\end{pmatrix} \\
&+ \ell(u(q_{0A})-q_{0A}) + \ell\left(\pi\alpha_{CA}^n + (1-\pi)\alpha_{CA}^d\right)\theta\left(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\right) \\
>& - iq_{0A} - \left(\ell\frac{\theta}{\omega_\theta(q_{1A})}\pi\alpha_{CB}^n(u'(q_{1A})-1)\right)\left((1-\theta)(u(q_{1A})-u(q_{0A}))+\theta(q_{1A}-q_{0A})\right) \\
&+ \ell(u(q_{0A})-q_{0A}) + \ell\pi\alpha_{CB}^n\theta\left(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\right) \\
\geq& - iq_{0B} - \left(\ell\frac{\theta}{\omega_\theta(q_{1B})}\pi\alpha_{CB}^n(u'(q_{1B})-1)\right)\left((1-\theta)(u(q_{1B})-u(q_{0B}))+\theta(q_{1B}-q_{0B})\right) \\
&+ \ell(u(q_{0B})-q_{0B}) + \ell\pi\alpha_{CB}^n\theta\left(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\right) = \tilde{\mathcal{S}}_{CB},
\end{aligned}$$

where the first inequality comes from

$$
\Big[-iq_{0A}-\Big(\ell\frac{\theta}{\omega_\theta(q_{1A})}\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A})-1)\Big)\begin{pmatrix}(1-\theta)(u(q_{1A})-u(q_{0A}))\\ \\ +\theta(q_{1A}-q_{0A})\end{pmatrix}
$$

$$
+\ell(u(q_{0A})-q_{0A})+\ell\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big)\Big]
$$

$$
-\Big[-iq_{0A}-\Big(\ell\frac{\theta}{\omega_\theta(q_{1A})}\pi\alpha^n_{CB}(u'(q_{1A})-1)\Big)\Big((1-\theta)(u(q_{1A})-u(q_{0A}))+\theta(q_{1A}-q_{0A})\Big)
$$

$$
+\ell(u(q_{0A})-q_{0A})+\ell\pi\alpha^n_{CB}\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big)\Big]
$$

$$
=\frac{\ell[(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA})-\pi\alpha^n_{CB}]\theta(q_{1A}-q_{0A})}{\theta+(1-\theta)u'(q_{1A})}\left(\frac{u(q_{1A})-u(q_{0A})}{q_{1A}-q_{0A}}-u'(q_{1A})\right)>0,
$$

in which we used $\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}>\pi\alpha^n_{CB}$ and the strict concavity of u; and the second inequality comes from that $\lim_{e_C\to 0+}q_{1B}\leq\lim_{e_C\to 0+}q_{1A}=q^*$ and (2.26). Therefore, $G(e_C)>0$ as $e_C\to 0+$. □

(d) All we need to show is that one of α^n_{CA}, α^d_{CA} and α^n_{CB} is discontinuous at $e_C=1$. Here, the discontinuity arises in α^n_{CB}. From the optimal entry decision by N-types (2.11),

$$
e^n_N=\frac{e_C(1-e_C\ell-\ell S_{NB}/S_{NA}+e_C\ell S_{NB}/S_{NA}}{-(1-\ell)(-e_C-S_{NB}/S_{NA}+e_CS_{NB}/S_{NA})}.
$$

Using this, as $e_C\to 1$,

$$
\begin{aligned}
\alpha^n_{CB}&\to\lim_{e_C\to 1}\frac{(1-\ell)(1-e^n_N)}{\ell(1-e_C)+(1-\ell)(1-e^n_N)}\\
&=\lim_{e_C\to 1}\frac{(1-\ell)(1-\frac{e_C(1-e_C\ell-\ell S_{NB}/S_{NA}+e_C\ell S_{NB}/S_{NA}}{-(1-\ell)(-e_C-S_{NB}/S_{NA}+e_CS_{NB}/S_{NA})})}{\ell(1-e_C)+(1-\ell)(1-\frac{e_C(1-e_C\ell-\ell S_{NB}/S_{NA}+e_C\ell S_{NB}/S_{NA}}{-(1-\ell)(-e_C-S_{NB}/S_{NA}+e_CS_{NB}/S_{NA})})}\\
&=\lim_{e_C\to 1}1+\ell(-1+e_C-e_C\frac{S_{NA}}{S_{NB}})=1-\ell\frac{S_{NA}}{S_{NB}}\,(>1-\ell).
\end{aligned}
$$

On the other hand, $\alpha^n_{CB} = 0$ at $e_C = 1$. Now assume $\pi \to 1$. Unlike α^n_{CB}, α^n_{CA} and α^d_{CA} continuously converge to their values at $e_C = 1$, which are both $1-\ell$. Hence, as $e_C \to 1$ and $\pi \to 1$, $\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA} < \pi\alpha^n_{CB}$. Therefore,

$$
\begin{aligned}
\tilde{\mathcal{S}}_{CB} =& -iq_{0B} - \Big(\ell\frac{\theta}{\omega_\theta(q_{1B})}\pi\alpha^n_{CB}(u'(q_{1B})-1)\Big)\Big((1-\theta)(u(q_{1B})-u(q_{0B}))+\theta(q_{1B}-q_{0B})\Big) \\
&+\ell(u(q_{0B})-q_{0B}) + \ell\pi\alpha^n_{CB}\theta\Big(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\Big) \\
>& -iq_{0B} - \Big(\ell\frac{\theta}{\omega_\theta(q_{1B})}\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)(u'(q_{1B})-1)\Big)\begin{pmatrix}(1-\theta)(u(q_{1B})-u(q_{0B})) \\ +\theta(q_{1B}-q_{0B})\end{pmatrix} \\
&+\ell(u(q_{0B})-q_{0B}) + \ell\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\Big) \\
\geq& -iq_{0A} - \Big(\ell\frac{\theta}{\omega_\theta(q_{1A})}\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)(u'(q_{1A})-1)\Big)\begin{pmatrix}(1-\theta)(u(q_{1A})-u(q_{0A})) \\ +\theta(q_{1A}-q_{0A})\end{pmatrix} \\
&+\ell(u(q_{0A})-q_{0A}) + \ell\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1A})-u(q_{0A})-q_{1A}+q_{0A}\Big) = \tilde{\mathcal{S}}_{CA},
\end{aligned}
$$

where the first inequality comes from

$$
\begin{aligned}
&\Big[-iq_{0B} - \Big(\ell\frac{\theta}{\omega_\theta(q_{1B})}\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)(u'(q_{1B})-1)\Big)\begin{pmatrix}(1-\theta)(u(q_{1B})-u(q_{0B})) \\ +\theta(q_{1B}-q_{0B})\end{pmatrix} \\
&\quad+\ell(u(q_{0B})-q_{0B}) + \ell\Big(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA}\Big)\theta\Big(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\Big)\Big] \\
&-\Big[-iq_{0B} - \Big(\ell\frac{\theta}{\omega_\theta(q_{1B})}\pi\alpha^n_{CB}(u'(q_{1B})-1)\Big)\Big((1-\theta)(u(q_{1B})-u(q_{0B}))+\theta(q_{1B}-q_{0B})\Big) \\
&\quad+\ell(u(q_{0B})-q_{0B}) + \ell\pi\alpha^n_{CB}\theta\Big(u(q_{1B})-u(q_{0B})-q_{1B}+q_{0B}\Big)\Big] \\
=&\ \frac{\ell[(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA})-\pi\alpha^n_{CB}]\theta(q_{1B}-q_{0B})}{\theta+(1-\theta)u'(q_{1B})}\Big(\frac{u(q_{1B})-u(q_{0B})}{q_{1B}-q_{0B}}-u'(q_{1B})\Big) < 0,
\end{aligned}
$$

in which we used $\pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA} < \pi\alpha^n_{CB}$ and the strict concavity of u; and the second inequality comes from that $\lim_{e_C\to 1} q_{1A} \le \lim_{e_C\to 1} q_{1B} = q^*$ and (2.27). Therefore, $G(e_C) < 0$ as $e_C \to 1$. □

(e) From (c) and (d), we have $\lim_{e_C\to 0+} G(e_C) > 0 > \lim_{e_C\to 1} G(e_C)$ when $\pi \to 1$. The continuity of G immediately implies that there exists at least one robust interior equilibrium.

□

(f) All we need to show is that one of α^n_{CA}, α^d_{CA} and α^n_{CB} is discontinuous at $e_C = 1$. Here, the discontinuity arises in α^d_{CA}. As $e_C \to 0+$,

$$\alpha^d_{CA} \to \lim_{e_C\to 0+} \frac{1-\ell}{(\ell e_C + (1-\ell))^{1-\rho}} = (1-\ell)^\rho.$$

On the other hand, $\alpha^d_{CA} = 0$ at $e_C = 0$. □

(g) All we need to show is that all α^n_{CA}, α^d_{CA} and α^n_{CB} continuously converge to their values at $e_C = 1$ as $e_C \to 1$. From the optimal entry decision by N-types (2.11),

$$e^n_N = \frac{-1 + e_C\ell + e_C\ell\left(\frac{(1-e_C)S_{NB}/S_{NA}}{e_C}\right)^{\frac{1}{1-\rho}}}{-(1-\ell)\left(1 + \left(\frac{(1-e_C)S_{NB}/S_{NA}}{e_C}\right)^{\frac{1}{1-\rho}}\right)}.$$

Then, as $e_C \to 1$,

$$\begin{aligned}\alpha^n_{CA} &= \frac{(1-\ell)e^n_N}{(\ell e_C + (1-\ell)e^n_N)^{1-\rho}} \\ &= \left(\frac{1}{1 + \left(\left(-1 + \frac{1}{e_C}\right)\frac{S_{NB}}{S_{NA}}\right)^{\frac{1}{1-\rho}}}\right)^\rho \left(1 - e_C\ell\left(1 + \left(\left(-1 + \frac{1}{e_C}\right)\frac{S_{NB}}{S_{NA}}\right)^{\frac{1}{1-\rho}}\right)\right) \to 1-\ell\end{aligned}$$

$$
\begin{aligned}
\alpha^d_{CA} &= \frac{1-\ell}{(\ell e_C + (1-\ell))^{1-\rho}} \to 1-\ell \\
\alpha^n_{CB} &= \frac{(1-\ell)(1-e^n_N)}{(\ell(1-e_C) + (1-\ell)(1-e^n_N))^{1-\rho}} \\
&= \left(\frac{1}{1+\left(\left(-1+\frac{1}{e_C}\right)\frac{S_{NB}}{S_{NA}}\right)^{-\frac{1}{1-\rho}}}\right)^{\rho}\left(1-(1-e_C)\ell\left(1+\left(\left(-1+\frac{1}{e_C}\right)\frac{S_{NB}}{S_{NA}}\right)^{-\frac{1}{1-\rho}}\right)\right) \to 0,
\end{aligned}
$$

and, at $e_C = e^n_N = 1$,

$$
\begin{aligned}
\alpha^n_{CA} &= \frac{(1-\ell)e^n_N}{(\ell e_C + (1-\ell)e^n_N)^{1-\rho}} = 1-\ell \\
\alpha^d_{CA} &= \frac{1-\ell}{(\ell e_C + (1-\ell))^{1-\rho}} = 1-\ell \\
\alpha^n_{CB} &= \frac{(1-\ell)(1-e^n_N)}{(\ell(1-e_C) + (1-\ell)(1-e^n_N))^{1-\rho}} = 0.
\end{aligned}
$$

Therefore, α^n_{CA}, α^d_{CA} and α^n_{CB} continuously converge to their values at $e_C = 1$ as $e_C \to 1$, and $G(e_C)$ also continuously converges to its value at $e_C = 1$, which is greater than 0, as $e_C \to 1$. □

2.A.7.2 When Safety Implies Liquidity

Proof of Proposition 2.2.

(a) Guess-and-verify: at $\pi = 1$, all the equilibrium equations are symmetric between the A and B markets, so $q_{0A} = q_{0B}$, $q_{1A} = q_{1B}$, and $L_A = L_B$ are satisfied. And $e_C = e_N = 0.5$ implies $\alpha_{CA} = \alpha_{CB}$ as well as $\alpha_{NA} = \alpha_{NB}$, so symmetry is complete.

(b) In any interior equilibrium where $e_C \in (0,1)$ and both assets are scarce and valued for liquidity so that $q_{1A} < q^*$, $q_{1B} < q^*$, we can totally differentiate the equilibrium equations around the scarce-interior equilibrium:

Post-trade quantities (equations 7–8):

$$q_{1A} = \min\left\{ q^*,\, q_{0A} + \frac{\frac{S_A}{M}\frac{e_C q_{0A} + (1-e_C)q_{0B}}{e_C} - (1-\theta)\Big(u(q_{1A}) - u(q_{0A})\Big)}{\theta} \right\}$$

$$q_{1B} = \min\left\{ q^*,\, q_{0B} + \frac{\frac{S_B}{M}\frac{e_C q_{0A} + (1-e_C)q_{0B}}{1-e_C} - (1-\theta)\Big(u(q_{1B}) - u(q_{0B})\Big)}{\theta} \right\}$$

Focusing on the scarce branch, total differentiate yields

$$\frac{w_\theta(q_{1A})}{\theta}\, dq_{1A} = \frac{S_A/M + w_\theta(q_{0A})}{\theta}\, dq_{0A} + \frac{S_A}{M}\frac{1-e_C}{e_C\theta}\, dq_{0B} - \frac{S_A}{M}\frac{q_{0B}}{e_C^2\theta}\, de_C \tag{2.28}$$

$$\frac{w_\theta(q_{1B})}{\theta}\, dq_{1B} = \frac{S_B/M + w_\theta(q_{0B})}{\theta}\, dq_{0B} + \frac{S_B}{M}\frac{e_C}{(1-e_C)\theta}\, dq_{0A} + \frac{S_B}{M}\frac{q_{0A}}{(1-e_C)^2\theta}\, de_C \tag{2.29}$$

Money demand (equations 5–6):

$$i = \ell(1-\theta\bar{\alpha}_{Cj})(u'(q_{0j})-1) + \ell\theta\frac{w_\theta(q_{0j})}{w_\theta(q_{1j})}\bar{\alpha}_{Cj}(u'(q_{1j})-1),$$

which is equivalent to

$$i = \ell\left(1 - \frac{\theta}{w_\theta(q_{1j})}\bar{\alpha}_{Cj}\right)(u'(q_{0j})-1) + \ell\frac{\theta}{w_\theta(q_{1j})}\bar{\alpha}_{Cj}(u'(q_{1j})-1), \quad j = A, B$$

where

$$w_\theta(q) \equiv \theta + (1-\theta)u'(q)$$

$$\bar{\alpha}_{CA} \equiv \pi\alpha^n_{CA} + (1-\pi)\alpha^d_{CA}$$

$$\bar{\alpha}_{CB} \equiv \pi\alpha^n_{CB}.$$

Total differentiation yields

$$\begin{aligned} 0 &= \ell\left(1 - \frac{\theta}{w_\theta(q_{1j})}\bar{\alpha}_{Cj}\right)u''(q_{0j})\,dq_{0j} \\ &+ \ell\frac{\theta}{w_\theta(q_{1j})^2}\bar{\alpha}_{Cj}\Big(u''(q_{1j})w_\theta(q_{1j}) - (u'(q_{1j}) - u'(q_{0j}))w'_\theta(q_{1j})\Big)dq_{1j} \\ &+ \ell\frac{\theta}{w_\theta(q_{1j})}(u'(q_{1j}) - u'(q_{0j}))\,d\bar{\alpha}_{Cj} \end{aligned} \tag{2.30}$$

Liquidity premium:

Define a new variable:

$$\bar{L}_j \equiv \ell\frac{\theta}{w_\theta(q_{1j})}\bar{\alpha}_{Cj}(u'(q_{1j}) - 1), \quad j = A, B$$

where $\bar{L}_A = L_A = (1+i)p_A - 1$ and $\bar{L}_B = \pi L_B = (1+i)p_B - \pi$. Total differentiation yields

$$d\bar{L}_j = \ell\frac{\theta}{w_\theta(q_{1j})^2}\bar{\alpha}_{Cj}\begin{pmatrix} u''(q_{1j})w_\theta(q_{1j}) \\ \\ -\,(u'(q_{1j}) - 1)w'_\theta(q_{1j}) \end{pmatrix}dq_{1j} + \ell\frac{\theta}{w_\theta(q_{1j})}(u'(q_{1j}) - 1)\,d\bar{\alpha}_{Cj}, \quad j = A, B$$

C's entry choice (equations following equation 9):

$$\mathcal{S}_{Cj} = \theta\Big(u(q_{1j}) - u(q_{0j}) - q_{1j} + q_{0j}\Big)$$

$$\tilde{\mathcal{S}}_{Cj} = -\,iq_{0j} - \bar{L}_j\Big((1-\theta)(u(q_{1j}) - u(q_{0j})) + \theta(q_{1j} - q_{0j})\Big) + \ell(u(q_{0j}) - q_{0j}) + \ell\bar{\alpha}_{Cj}\mathcal{S}_{Cj}$$

Total differentiation yields

$$d\mathcal{S}_{Cj} = \theta(u'(q_{1j}) - 1)\,dq_{1j} - \theta(u'(q_{0j}) - 1)\,dq_{0j}$$

$$\begin{aligned} d\tilde{\mathcal{S}}_{Cj} = &-\Big((1-\theta)(u(q_{1j}) - u(q_{0j})) + \theta(q_{1j} - q_{0j})\Big)d\bar{L}_j + \ell\mathcal{S}_{Cj}\,d\bar{\alpha}_{Cj} \\ &+ \Big(-\,i + \bar{L}_j w_\theta(q_{0j}) + \ell(u'(q_{0j}) - 1)\Big)dq_{0j} - \bar{L}_j w_\theta(q_{1j})\,dq_{1j} + \ell\bar{\alpha}_{Cj}\,d\mathcal{S}_{Cj} \end{aligned}$$

where

$$\begin{aligned} &\Big(-\,i + \bar{L}_j w_\theta(q_{0j}) + \ell(u'(q_{0j}) - 1)\Big)dq_{0j} - \bar{L}_j w_\theta(q_{1j})\,dq_{1j} + \ell\bar{\alpha}_{Cj}\,d\mathcal{S}_{Cj} \\ = &\Big(-\,i + \bar{L}_j w_\theta(q_{0j}) + \ell(u'(q_{0j}) - 1) - \ell\bar{\alpha}_{Cj}\theta(u'(q_{0j}) - 1)\Big)dq_{0j} \\ &+ \Big(-\,\bar{L}_j w_\theta(q_{1j}) + \ell\bar{\alpha}_{Cj}\theta(u'(q_{1j}) - 1)\Big)dq_{1j} \\ = &\,0 \end{aligned}$$

since the coefficient of dq_{0j} is equivalent to the first-version money demand. Thus,

$$d\tilde{\mathcal{S}}_{Cj} = -\Big((1-\theta)(u(q_{1j}) - u(q_{0j})) + \theta(q_{1j} - q_{0j})\Big)d\bar{L}_j + \ell\mathcal{S}_{Cj}\,d\bar{\alpha}_{Cj}$$

Therefore, we have

$$G(e_C) = \tilde{\mathcal{S}}_{CA} - \tilde{\mathcal{S}}_{CB}$$

and total differentiation yields

$$\begin{aligned} dG &= d\tilde{\mathcal{S}}_{CA} - d\tilde{\mathcal{S}}_{CB} \\ &= -\Big((1-\theta)(u(q_{1A}) - u(q_{0A})) + \theta(q_{1A} - q_{0A})\Big) d\bar{L}_A + \ell\mathcal{S}_{CA}\, d\bar{\alpha}_{CA} \\ &\quad + \Big((1-\theta)(u(q_{1B}) - u(q_{0B})) + \theta(q_{1B} - q_{0B})\Big) d\bar{L}_B - \ell\mathcal{S}_{CB}\, d\bar{\alpha}_{CB} \end{aligned} \tag{2.31}$$

N's entry choice (equations following equation 10):

$$\alpha^n_{NA}\mathcal{S}_{NA} = \alpha^n_{NB}\mathcal{S}_{NB}$$

$$\mathcal{S}_{Nj} = (1-\theta)\Big(u(q_{1j}) - u(q_{0j}) - q_{1j} + q_{0j}\Big)$$

Total differentiation yields

$$\mathcal{S}_{NA}\, d\alpha^n_{NA} + \alpha^n_{NA}\, d\mathcal{S}_{NA} = \mathcal{S}_{NB}\, d\alpha^n_{NB} + \alpha^n_{NB}\, d\mathcal{S}_{NB}$$

$$d\mathcal{S}_{Nj} = (1-\theta)(u'(q_{1j}) - 1)\, dq_{1j} - (1-\theta)(u'(q_{0j}) - 1)\, dq_{0j}$$

Thus,

$$\mathcal{S}_{NA}\, d\alpha^n_{NA} + \alpha^n_{NA}(1-\theta)(u'(q_{1A}) - 1)\, dq_{1A} - \alpha^n_{NA}(1-\theta)(u'(q_{0A}) - 1)\, dq_{0A} \tag{2.32}$$

$$= \mathcal{S}_{NB}\, d\alpha^n_{NB} + \alpha^n_{NB}(1-\theta)(u'(q_{1B}) - 1)\, dq_{1B} - \alpha^n_{NB}(1-\theta)(u'(q_{0B}) - 1)\, dq_{0B}$$

Matching probabilities (Section 3.2):

$$\alpha_{CA}^n = e_N^n(1-\ell)\left[e_N^n(1-\ell)+e_C\ell\right]^{\rho-1}$$

$$\alpha_{CB}^n = (1-e_N^n)(1-\ell)\left[(1-e_N^n)(1-\ell)+(1-e_C)\ell\right]^{\rho-1}$$

$$\alpha_{NA}^n = e_C\ell\left[e_N^n(1-\ell)+e_C\ell\right]^{\rho-1}$$

$$\alpha_{NB}^n = (1-e_C)\ell\left[(1-e_N^n)(1-\ell)+(1-e_C)\ell\right]^{\rho-1}$$

$$\alpha_{CA}^d = (1-\ell)\left[(1-\ell)+e_C\ell\right]^{\rho-1}$$

$$\alpha_{NA}^d = e_C\ell\left[(1-\ell)+e_C\ell\right]^{\rho-1}$$

$$\alpha_{CB}^d = \alpha_{NB}^d = 0$$

Total differentiation yields

$$\begin{aligned}
d\alpha_{CA}^n =& -(1-\rho)\ell(1-\ell)e_N^n\left[e_N^n(1-\ell)+e_C\ell\right]^{\rho-2}de_C \\
&+\left[\frac{\alpha_{CA}^n}{e_N}-(1-\rho)(1-\ell)^2e_N^n\left[e_N^n(1-\ell)+e_C\ell\right]^{\rho-2}\right]de_N^n \\
d\alpha_{CB}^n =& (1-\rho)\ell(1-\ell)(1-e_N^n)\left[(1-e_N^n)(1-\ell)+(1-e_C)\ell\right]^{\rho-2}de_C \\
&-\left[\frac{\alpha_{CB}^n}{1-e_N^n}-(1-\rho)(1-\ell)^2(1-e_N^n)\left[(1-e_N^n)(1-\ell)+(1-e_C)\ell\right]^{\rho-2}\right]de_N^n \\
d\alpha_{NA}^n =& \left[\frac{\alpha_{NA}^n}{e_C}-(1-\rho)\ell^2e_C\left[e_N^n(1-\ell)+e_C\ell\right]^{\rho-2}\right]de_C \\
&-(1-\rho)(1-\ell)\ell e_C\left[e_N^n(1-\ell)+e_C\ell\right]^{\rho-2}de_N^n \\
d\alpha_{NB}^n =& -\left[\frac{\alpha_{NB}^n}{1-e_C}-(1-\rho)\ell^2(1-e_C)\left[(1-e_N^n)(1-\ell)+(1-e_C)\ell\right]^{\rho-2}\right]de_C \\
&+(1-\rho)(1-\ell)\ell(1-e_C)\left[(1-e_N^n)(1-\ell)+(1-e_C)\ell\right]^{\rho-2}de_N^n
\end{aligned}$$

$$d\alpha^d_{CA} = -(1-\rho)\ell(1-\ell)\left[(1-\ell)+e_C\ell\right]^{\rho-2} de_C$$

$$d\alpha^d_{NA} = \left[\frac{\alpha^d_{NA}}{e_C} - (1-\rho)\ell^2 e_C\left[(1-\ell)+e_C\ell\right]^{\rho-2}\right] de_C$$

$$d\alpha^d_{CB} = d\alpha^d_{NB} = 0$$

Therefore, we have

$$d\bar{\alpha}_{CA} = \pi\, d\alpha^n_{CA} + (1-\pi)\, d\alpha^d_{CA} + (\alpha^n_{CA} - \alpha^d_{CA})\, d\pi$$

$$d\bar{\alpha}_{CB} = \pi\, d\alpha^n_{CB} + \alpha^n_{CB}\, d\pi$$

Now restrict attention to the symmetric equilibrium with CRS matching. If $S_A = S_B \equiv S$ and $\pi \to 1$, then a symmetric equilibrium exists where $e_C = e^n_N = 1/2$. When $\rho = 0$, the matching probabilities becomes

$$\bar{\alpha}_{CA} = \bar{\alpha}_{CB} = \alpha^n_{CA} = \alpha^n_{CB} = 1-\ell$$

$$\alpha^n_{NA} = \alpha^n_{NB} = \ell$$

$$\alpha^d_{CA} = \frac{2(1-\ell)}{2-\ell}$$

$$\alpha^d_{NA} = \frac{\ell}{2-\ell}$$

$$\alpha^d_{CB} = \alpha^d_{NB} = 0\,,$$

which in turn implies $q_{0A} = q_{0B} \equiv q_0$ and $q_{1A} = q_{1B} \equiv q_1$. Total differentiation yields

$$
\begin{aligned}
&d\alpha^n_{CA} = -d\alpha^n_{CB} = -d\alpha^n_{NA} = d\alpha^n_{NB} = -\,2\ell(1-\ell)\,de_C + 2\ell(1-\ell)\,de^n_N \\
&d\alpha^d_{CA} = -d\alpha^d_{NA} = -(1-\ell)\ell\left(\frac{2-\ell}{2}\right)^{-2} de_C \\
&d\alpha^d_{CB} = d\alpha^d_{NB} = 0
\end{aligned}
$$

Assuming CRS matching ($\rho = 0$), put together (2.28), (2.29), (2.30), (2.31), (2.32) in matrix form:

$$
\mathbf{A}\boldsymbol{u} = \boldsymbol{b}\,d\pi, \tag{2.33}
$$

where:

$$
\mathbf{A} = \begin{bmatrix}
\mathfrak{a} & -\mathfrak{a} & -\mathfrak{b} & 0 & -\mathfrak{c} & 0 \\
-\mathfrak{a} & \mathfrak{a} & 0 & -\mathfrak{b} & 0 & -\mathfrak{c} \\
-\mathfrak{d} & 0 & -\mathfrak{e} & 0 & \mathfrak{f} & \mathfrak{g} \\
\mathfrak{d} & 0 & 0 & -\mathfrak{e} & \mathfrak{g} & \mathfrak{f} \\
\mathfrak{h} & -\mathfrak{h} & \mathfrak{j} & -\mathfrak{j} & -\mathfrak{k} & \mathfrak{k} \\
-\mathfrak{m} & \mathfrak{m} & \mathfrak{n} & -\mathfrak{n} & 0 & 0
\end{bmatrix}, \quad
\boldsymbol{u} = \begin{bmatrix} de_C \\ de^n_N \\ dq_{1A} \\ dq_{1B} \\ dq_{0A} \\ dq_{0B} \end{bmatrix}, \quad
\boldsymbol{b} = \begin{bmatrix} -\dfrac{\mathfrak{a}}{2(2-\ell)} \\ \dfrac{\mathfrak{a}}{2\ell} \\ 0 \\ 0 \\ 0 \\ \dfrac{\mathfrak{m}}{2\ell(2-\ell)} \end{bmatrix},
$$

and:

$$
\begin{aligned}
\mathfrak{a} &= \frac{2(1-\ell)\ell\theta[u'(q_0)-u'(q_1)]}{w_\theta(q_1)}, \\
\mathfrak{b} &= -\frac{(1-\ell)\theta w_\theta(q_0)}{w_\theta(q_1)^2}u''(q_1), \\
\mathfrak{c} &= -\frac{\ell\theta+(1-\ell)u'(q_1)}{w_\theta(q_1)}u''(q_0), \\
\mathfrak{d} &= \frac{4q_0 S/M}{\theta}, \\
\mathfrak{e} &= \frac{w_\theta(q_1)}{\theta}, \\
\mathfrak{f} &= \frac{S/M+w_\theta(q_0)}{\theta}, \\
\mathfrak{g} &= \frac{S/M}{\theta}, \\
\mathfrak{h} &= 4(1-\ell), \\
\mathfrak{j} &= \frac{u'(q_1)-1}{u(q_1)-u(q_0)-q_1+q_0}, \\
\mathfrak{k} &= \frac{u'(q_0)-1}{u(q_1)-u(q_0)-q_1+q_0}, \\
\mathfrak{m} &= \frac{4(1-\ell)\ell^2\theta[u(q_1)-u(q_0)-(q_1-q_0)u'(q_1)]}{w_\theta(q_1)}, \\
\mathfrak{n} &= -\frac{(1-\ell)\ell\theta[(1-\theta)(u(q_1)-u(q_0))+\theta(q_1-q_0)]}{w_\theta(q_1)^2}u''(q_1).
\end{aligned}
$$

Note that $\mathfrak{a}$ to $\mathfrak{n}$ are all positive. With a symbolic software package, it is easy to check that the solution is given by:

$$
\boldsymbol{u} = \begin{bmatrix}
-\dfrac{-\mathfrak{cehm}-\mathfrak{bfhm}+\mathfrak{bghm}+2\mathfrak{afhn}-2\mathfrak{aghn}}{4\mathfrak{d}(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex]
\dfrac{\mathfrak{cehm}+\mathfrak{bfhm}-\mathfrak{bghm}-2\mathfrak{cdjm}-2\mathfrak{bdkm}-2\mathfrak{afhn}+2\mathfrak{aghn}+4\mathfrak{adkn}}{4\mathfrak{d}(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex]
-\dfrac{\left(\begin{array}{c}\mathfrak{c}^2\mathfrak{ehm}+\mathfrak{bcfhm}+\mathfrak{bcghm}-2\mathfrak{acfjm}-2\mathfrak{acgjm}-2\mathfrak{abfkm}-2\mathfrak{abgkm} \\ +2\mathfrak{acfj}\ell\mathfrak{m}+2\mathfrak{acgj}\ell\mathfrak{m}+2\mathfrak{abfk}\ell\mathfrak{m}+2\mathfrak{abgk}\ell\mathfrak{m}-2\mathfrak{acfhn}-2\mathfrak{acghn}+4\mathfrak{a}^2\mathfrak{fkn} \\ +4\mathfrak{a}^2\mathfrak{gkn}+2\mathfrak{acfh}\ell\mathfrak{n}+2\mathfrak{acgh}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{fk}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{gk}\ell\mathfrak{n}\end{array}\right)}{4(\mathfrak{ce}+\mathfrak{bf}+\mathfrak{bg})(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex]
-\dfrac{\left(\begin{array}{c}-\mathfrak{c}^2\mathfrak{ehm}-\mathfrak{bcfhm}-\mathfrak{bcghm}-2\mathfrak{acfjm}-2\mathfrak{acgjm}-2\mathfrak{abfkm}-2\mathfrak{abgkm} \\ +2\mathfrak{acfj}\ell\mathfrak{m}+2\mathfrak{acgj}\ell\mathfrak{m}+2\mathfrak{abfk}\ell\mathfrak{m}+2\mathfrak{abgk}\ell\mathfrak{m}-2\mathfrak{acfhn}-2\mathfrak{acghn}+4\mathfrak{a}^2\mathfrak{fkn} \\ +4\mathfrak{a}^2\mathfrak{gkn}+2\mathfrak{acfh}\ell\mathfrak{n}+2\mathfrak{acgh}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{fk}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{gk}\ell\mathfrak{n}\end{array}\right)}{4(\mathfrak{ce}+\mathfrak{bf}+\mathfrak{bg})(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex]
-\dfrac{\left(\begin{array}{c}-\mathfrak{bcehm}-\mathfrak{b}^2\mathfrak{fhm}-\mathfrak{b}^2\mathfrak{ghm}-2\mathfrak{acejm}-2\mathfrak{abekm}+2\mathfrak{acej}\ell\mathfrak{m} \\ +2\mathfrak{abek}\ell\mathfrak{m}+2\mathfrak{abfhn}+2\mathfrak{abghn}+4\mathfrak{a}^2\mathfrak{ekn}+2\mathfrak{aceh}\ell\mathfrak{n}-4\mathfrak{a}^2\mathfrak{ek}\ell\mathfrak{n}\end{array}\right)}{4(\mathfrak{ce}+\mathfrak{bf}+\mathfrak{bg})(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})} \\[2ex]
\dfrac{\left(\begin{array}{c}-\mathfrak{bcehm}-\mathfrak{b}^2\mathfrak{fhm}-\mathfrak{b}^2\mathfrak{ghm}+2\mathfrak{acejm}+2\mathfrak{abekm}-2\mathfrak{acej}\ell\mathfrak{m}-2\mathfrak{abek}\ell\mathfrak{m} \\ +4\mathfrak{acehn}+2\mathfrak{abfhn}+2\mathfrak{abghn}-4\mathfrak{a}^2\mathfrak{ekn}-2\mathfrak{aceh}\ell\mathfrak{n}+4\mathfrak{a}^2\mathfrak{ek}\ell\mathfrak{n}\end{array}\right)}{4(\mathfrak{ce}+\mathfrak{bf}+\mathfrak{bg})(-2+\ell)\ell(\mathfrak{cjm}+\mathfrak{bkm}+\mathfrak{chn}-2\mathfrak{akn})}
\end{bmatrix} d\pi \quad (2.34)
$$

Now, look at the liquidity premium:

$$
\begin{aligned}
L_A &= \ell\frac{\theta}{w_\theta(q_{1A})}(\pi\alpha^n_{CA}+(1-\pi)\alpha^d_{CA})(u'(q_{1A})-1) \\
L_B &= \ell\frac{\theta}{w_\theta(q_{1B})}\alpha^n_{CB}(u'(q_{1B})-1).
\end{aligned}
$$

Total differentiation, when $\pi \to 1$ in the symmetric equilibrium, yields

$$
\begin{aligned}
dL_A &= \ell\theta \frac{u''(q_1)}{w_\theta(q_1)^2}\alpha^n_{CA} dq_{1A} + \ell\theta \frac{u'(q_1)-1}{w_\theta(q_1)}(\alpha^n_{CA} - \alpha^d_{CA})d\pi + \ell\theta \frac{u'(q_1)-1}{w_\theta(q_1)} d\alpha^n_{CA} \\
dL_B &= \ell\theta \frac{u''(q_1)}{w_\theta(q_1)^2}\alpha^n_{CB} dq_{1B} + \ell\theta \frac{u'(q_1)-1}{w_\theta(q_1)} d\alpha^n_{CB}.
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
&dL_A - dL_B \\
&= \ell\theta \frac{u''(q_1)}{w_\theta(q_1)^2}(dq_{1A} - dq_{1B}) + \ell\theta \frac{u'(q_1)-1}{w_\theta(q_1)}(\alpha^n_{CA} - \alpha^d_{CA})d\pi + \ell\theta \frac{u'(q_1)-1}{w_\theta(q_1)}(d\alpha^n_{CA} - d\alpha^n_{CB}).
\end{aligned}
$$

Since:

$$
\alpha^n_{CA} = 1-\ell, \quad \alpha^d_{CA} = \frac{2(1-\ell)}{2-\ell}, \quad \text{and} \quad d\alpha^n_{CA} = -d\alpha^n_{CB} = -\,2\ell(1-\ell)(de_C - de^n_N),
$$

we get:

$$
\begin{aligned}
&dL_A - dL_B \\
&= \ell\theta \frac{u''(q_1)}{w_\theta(q_1)^2}(dq_{1A} - dq_{1B}) - \ell\theta \frac{u'(q_1)-1}{w_\theta(q_1)} \frac{\ell(1-\ell)}{2-\ell} d\pi - 4\ell\theta \frac{u'(q_1)-1}{w_\theta(q_1)} \ell(1-\ell)(de_C - de^n_N).
\end{aligned}
$$

In order to have $dL_A - dL_B < 0$, we want each term in $dL_A - dL_B$ to be negative. The second term is obviously negative. To determine the sign of the first term, look at $dq_{1A} - dq_{1B}$. From (2.34),

$$dq_{1A} - dq_{1B} = \frac{\mathfrak{chm}}{2(2-\ell)\ell(\mathfrak{cjm} + \mathfrak{bkm} + \mathfrak{chn} - 2\mathfrak{akn})}.$$

The sign of $dq_{1A} - dq_{1B}$ depends on that of $\mathfrak{cjm} + \mathfrak{bkm} + \mathfrak{chn} - 2\mathfrak{akn}$ in the denominator. We define:

$$\begin{aligned}
\mathfrak{D} &\equiv \mathfrak{cjm} + \mathfrak{bkm} + \mathfrak{chn} - 2\mathfrak{akn} \\
&= \left[4\ell(1-\ell)\frac{\theta}{w_\theta(q_1)}\right]\left[-u''(q_0)\left(1 - \frac{(1-\ell)\theta}{w_\theta(q_1)}\right)\ell(u'(q_1)-1)\frac{S^1}{S}\cdots\right. \\
&\qquad \left. -u''(q_0)\left(1 - \frac{(1-\ell)\theta}{w_\theta(q_1)}\right)(1-\ell)\frac{-u''(q_1)}{w_\theta(q_1)}S^\theta - u''(q_1)\frac{(1-\ell)\theta}{w_\theta(q_1)}\ell(u'(q_0)-1)\frac{S^0}{S}\right],
\end{aligned}$$

where:

$$\begin{aligned}
S &\equiv u(q_1) - u(q_0) - q_1 + q_0 > 0 \\
S^\theta &\equiv (1-\theta)(u(q_1) - u(q_0)) + \theta(q_1 - q_0) > 0 \\
S^1 &\equiv u(q_1) - u(q_0) - u'(q_1)(q_1 - q_0) > 0 \\
S^0 &\equiv u(q_1) - u(q_0) - u'(q_0)(q_1 - q_0) < 0.
\end{aligned}$$

$S^1 > 0$ and $S^0 < 0$ due to the strict concavity of u. For the first term in $dL_A - dL_B$ to be negative, we want $\mathfrak{D} > 0$ so that $dq_{1A} - dq_{1B} > 0$. The first and the second terms in the second bracket in $\mathfrak{D}$ are positive, whereas the third term is negative. If $\theta \to 0$ or $\ell(1-\ell) \to 0$, then $\mathfrak{D} > 0$. In case of quadratic utility, $u(q) \equiv (1+\gamma)q - q^2/2$ with $q^* = \gamma$, we can show that $\mathfrak{D} > 0$ is always the case for all (ℓ, θ). First, observe the following from the sum of the second and the third terms in the second bracket in $\mathfrak{D}$:

$$
\begin{aligned}
& - u''(q_0)\left(1-\frac{(1-\ell)\theta}{w_\theta(q_1)}\right)(1-\ell)\frac{-u''(q_1)}{w_\theta(q_1)}S^\theta - u''(q_1)\frac{(1-\ell)\theta}{w_\theta(q_1)}\ell(u'(q_0)-1)\frac{S^0}{S} \\
> & - u''(q_0)\ell(1-\ell)\frac{-u''(q_1)}{w_\theta(q_1)}S^\theta - u''(q_1)\frac{(1-\ell)\theta}{w_\theta(q_1)}\ell(u'(q_0)-1)\frac{S^0}{S} \\
= & \frac{-u''(q_1)}{w_\theta(q_1)}\ell(1-\ell)\frac{1}{S}\left[-u''(q_0)S^\theta S + (u'(q_0)-1)\theta S^0\right],
\end{aligned}
$$

where the first inequality comes from $u'(q_1) = 1 + q^* - q_1 \geq 1 > \ell$. Denote $\Upsilon(\theta) \equiv -u''(q_0)S^\theta S + (u'(q_0)-1)\theta S^0$. Observe that $\Upsilon(\theta = 0) = -u''(q_0)(u(q_1) - u(q_0))S > 0$; $\Upsilon(\theta = 1) = (q_1 - q_0)^2(q^* - q_1)/2 > 0$; and $d\Upsilon/d\theta = (u'(q_0)-1)S^0 + S^2u''(q_0) < 0$. Therefore, $\Upsilon > 0$ and $\mathfrak{D} > 0$. For other cases, including log utility, we verified numerically and could not find any case where $\mathfrak{D} > 0$ is not satisfied. $\mathfrak{D} > 0$ implies that $dq_{1A} - dq_{1B} > 0$ and that the first term in $dL_A - dL_B$ is negative.

To determine the sign of the last term in $L_A - dL_B$, look at $de_C - de_N^n$. From (2.34),

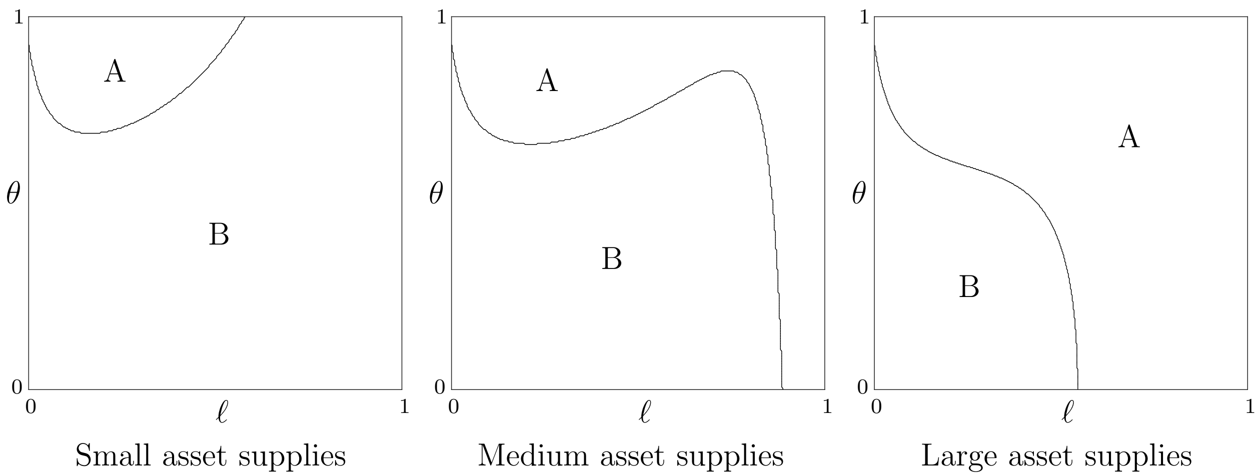
$$
de_C - de_N^n = -\frac{\mathfrak{cjm} + \mathfrak{bkm} - 2\mathfrak{akn}}{2(2-\ell)\ell\,\mathfrak{D}}.
$$

Since $\mathfrak{D} > 0$, the sign of $de_C - de_N^n$ depends on that of $\mathfrak{cjm} + \mathfrak{bkm} - 2\mathfrak{akn}$ in the numerator:

$$
\begin{aligned}
\mathfrak{cjm} + \mathfrak{bkm} - 2\mathfrak{akn} = & \left[4\ell^2(1-\ell)\frac{\theta}{w_\theta(q_1)}\right]\dots \\
& \times\left[-u''(q_0)\left(1-\frac{(1-\ell)\theta}{w_\theta(q_1)}\right)(u'(q_1)-1)\frac{S^1}{S} - u''(q_1)\frac{(1-\ell)\theta}{w_\theta(q_1)}(u'(q_0)-1)\frac{S^0}{S}\right].
\end{aligned}
$$

For the third term in $dL_A - dL_B$ to be negative, we want $\mathfrak{cjm} + \mathfrak{bkm} - 2\mathfrak{akn} < 0$ so that $de_C - de_N^n > 0$. The first term in the second bracket is positive, whereas the second term is negative. From the equation, we can see that if $(1-\ell)\theta$ is sufficiently large, $\mathfrak{cjm} + \mathfrak{bkm} - 2\mathfrak{akn}$

becomes negative, $de_C - de_N^n$ becomes positive, and the third term in $dL_A - dL_B$ becomes negative. Below is the figure that numerically shows in the (ℓ, θ) plane the parameter space where the third term in $dL_A - dL_B$ is negative (A) and where it is not (B):



In region A, the third term in $dL_A - dL_B$ is negative, so all the components of $dL_A - dL_B$ are negative, while in region B the third term is positive. Under the sufficient condition that $(1 - \ell)\theta$ is large enough, we will always be in region A so that all the components of $dL_A - dL_B$ become negative. Finally, $dL_A - dL_B < 0$ in turn implies that near $\pi = 1$ we have $L_A > L_B$. $\square$

Chapter 3

Financial Crises and R&D Dynamics

3.1 Introduction

The slow recovery has been documented as one of the stylized facts that characterize financial crises (e.g., Reinhart and Rogoff (2009), Jordà, Schularick, and Taylor (2013)). Many attribute the slow recovery to persistent demand shortfalls. A sustained decline in spending by borrowers in the process of deleveraging is one of the popular demand-side stories.

Meanwhile, recently, a number of authors started to consider supply-side factors for the slow recovery after crises. For example, Reifschneider, Wascher, and Wilcox (2015) observes from the recent financial crisis a huge loss in productivity that reflects a steep decline in capital accumulation and slower growth in multifactor productivity. Queralto (2016) argues that a decline in productivity was also the case for emerging countries. Using the sample of East Asia financial crises in 1990s, he shows that there was a sustained drop in labor productivity in those episodes.

The natural question that follows is what led to the productivity slowdown after crises.

The literature has been developed to the direction toward impacts of crises on productivity-enhancing investment, such as research and development (R&D). There could be two hypotheses. One is that a reduction in R&D, and the other is a reduction in R&D effectiveness, which could be affected by who is doing R&D or driven by an exogenous shock on the R&D effectiveness. In other words, the former is related to the level of R&D, while the latter is related to the efficiency of R&D. Although, when it comes to general capital accumulating investment, there are a series of papers that consider the efficiency effects through misallocation (e.g., Midrigan and Xu (2014), Buera and Moll (2015), Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez (2017)), the literature that tries to relate R&D to business cycle and productivity fluctuations has been focusing on the level effects. The literature on R&D and business cycles combines the growth theories with the business cycle frameworks. For example, Anzoategui, Comin, Gertler, and Martinez (2017) use the expanding variety model of Romer (1990), and Garga and Singh (2017) use the quality ladder model of Aghion and Howitt (1992). Both focus on the level effects of crises on R&D. That is, their stories are that after crises, the level of R&D decreases, and this leads to the productivity slowdown.

Which channel between the level and the efficiency effects is more relevant should be an empirical question. However, so far there is no research that tried to identify this. Anzoategui, Comin, Gertler, and Martinez (2017) and Garga and Singh (2017) are trying to justify their approaches that focus on the level effects by plotting R&D over the years for the case of the United States. However, it is hard to say, based on a single time-series plot, what they provide is decisive evidence. Thus, we need to examine the effects of crises on R&D in a more rigorous way and with a broader set of data observations.

This paper contributes to the literature by trying to answer this question. Using the panel dataset on the 30 OECD countries over the years 1981–2016, I examine the effects of business cycle fluctuations, focusing on normal recessions and financial crises, on R&D investment, using local projections. The responses of R&D to recessions are in a sharp contrast with the responses of other variables, such as output and non-R&D investment. It is well established that recessions are bad times for output and investment, with financial crises being more painful and followed by slower recovery, and indeed we can see this also from the analysis here. However, most of the responses of investment are coming from its non-R&D portion. I could not find statistically significant evidence that the level of R&D activities decrease after recessions or crises. R&D investment is overall unresponsive to recessions, even to financial crises. This result suggests that, when one wishes to incorporate R&D activities into the business cycle models, it should be modelled in the way that the productivity slowdown that results from crises should be driven not through a decline in the level of R&D but rather through a decline in the efficiency of R&D.

Although this paper suggests some evidence that changes in the R&D effectiveness is more relevant channel, whether a decline in the R&D efficiency is an endogenous feature from misallocation of R&D resources across heterogenous firms or it is coming from an exogenous shock on R&D productivity is another question, and this paper does not answer to this question. To answer this question, more micro-level data is required to observe what is happening across heterogenous firms. While more focusing on the decline in the level of R&D, actually in their model Anzoategui, Comin, Gertler, and Martinez (2017) also incorporates an exogenous shock to R&D efficiency, and they show some independent evidence that there is a decline in R&D productivity after crises based on the measure of R&D productivity as the number of patent applications relative to the number of R&D researchers. However, this evidence does not necessarily support their view that the decline in the R&D efficiency is a result of an exogenous shock to it. Without further firm-level evidence, it could be equally plausible that the decline in R&D efficiency was resulted from an endogenous misallocation of R&D resources across firms. We need to see firm-level data to see whether the decline in R&D productivity is coming from an exogenous shock to the economy or it is an endogenous phenomenon from misasllocation of R&D resources across heterogeneous firms. I turn this to future research.

The rest of the paper is organized as follows. Section 2 explains about the constructed dataset. Section 3 analyzes the responses of macroeconomic variables, to recessions and financial crises, with a main focus on R&D variables. Section 4 concludes.

3.2 Business Cycle

3.2.1 Data

In order to examine the dynamics of R&D along business cycles, especially the responses of R&D investment to normal and financial recessions, R&D related data was collected for 30 OECD countries over the years 1981–2016, among which 23 are advanced and 7 are emerging countries. The twenty-three advanced countries included are Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, the United Kingdom, Greece, Ireland, Iceland, Italy, Japan, Luxembourg, Netherlands, Norway, New Zealand, Portugal, Sweden and the United States of America. The seven emerging countries included are Czech Republic, Estonia, Hungary, Israel, Latvia, Mexico and Slovenia.[1]

The R&D related data are aggregate R&D investment, R&D investment by firms, R&D investment by the government, and the number of researchers per 1,000 people employed. The main focus is R&D investment by firms, but all the results are going through all the R&D related variables in a robust way.

Other variables used for analysis include national accounts data on real GDP per capita, investment, price levels and inflation, short- and long-term interest rates on government securities (usually 3 months tenor at the short end, and 5 years at the long end). For most indicators, I relied on the Jordà–Schularick–Taylor (JST) Macrohistory Database, which covers the period 1870–2013. For the countries not included in the JST Database and for the recent period 2014–2016 not covered by the JST Database, I collected data from the same sources documented in the accompanied documentation.[2]

The other main variable is the financial crisis years, defined by the JST Database. Financial recessions are referred to as recessions that coincided with financial crisis. The other recessions that did not coincide with financial crisis will be referred to as normal recessions. For the countries not included in the JST Database, financial recession dates were collected from the systemic banking crisis years in Laeven and Valencia (2012).

The sources of the variables used are listed in Data Appendix in detail.

[1] I separated the countries into advanced and emerging countries according to their membership starting dates. Among 23 countries classified as advanced countries, the last to join was New Zealand in 1973. Among 7 countries classified as emerging countries, the first to join was Mexico in 1994. The rest of 6 countries joined subsequently in 1995 (Czech Republic), 1996 (Hungary), 2010 (Estonia, Israel, Slovenia) and 2016 (Latvia). Because the seven countries were not the members of the OECD for the significant period out of the whole period 1981–2016 when the R&D related data is available, I classified those seven countries as emerging countries. But all the results presented in this paper remain almost same whether focusing on advanced countries or emerging countries, or both. Hence, the classification of advanced and emerging countries, in the end, is immaterial.

[2] The 13 countries not covered by the JST Databse are Austria, Czech Republic, Estonia, Greece, Hungary, Ireland, Iceland, Israel, Latvia, Luxembourg, Mexico, New Zealand and Slovenia.

3.2.2 Chronology of Turning Points in Economic Activity

As is indicated in Jordà, Schularick, and Taylor (2013), while the NBER keeps records of the US business cycles, most countries do not have agencies that determine turning points in economic activity. For the countries not included in the JST Database, following Jordà, Schularick, and Taylor (2011), I use the Bry and Boschan (1971) algorithm to identify peaks and troughs. Using the OECD quarterly GDP data, the algorithm identifies potential turning points as the local minima and maxima in the series. Following Harding and Pagan (2002), candidate points must then satisfy two conditions: phases are at least 2 quarters long, and complete cycles are at least 5 quarters long. The number of observations on both sides over which local minima and maxima are computed is set to 2.[3]

After peaks are identified as local maxima, recessions were sorted into two types: those that were associated with financial crises and those that were not. The resulting chronology of business cycle peaks is shown in Table 3.A.1, where "N" denotes normal peaks, and "F" denotes peaks associated with a systemic financial crises. Total 150 peaks are identified over the years 1981–2016 in the 30-country sample in Table 3.A.1. Summary statistics are given in Table 3.A.2. Among all, 36 are financial recessions and 114 are normal recessions. The number of recessions in 23 advanced countries are total 125, among which 26 are financial recessions and 99 are normal recessions. The number of recessions in 7 emerging countries are 25, among which 10 are financial recessions and 15 are normal recessions.[4]

[3]Quarterly GDP data in 1980s is not available for a number of European countries: Austria, Belgium, Czech Republic, Denmark, Estonia, Finland, Germany, Greece, Hungary, Iceland, Ireland, Israel, Italy, Japan, Latvia, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Portugal, Slovenia, Spain, Sweden, Switzerland and the United Kingdom. For these countries, business cycle peaks could not be identified in that early period. This is why there are no peaks in 1980s listed for these countries in Table 3.A.1.

[4]In Jordà, Schularick, and Taylor (2013), which covered 14 advanced countries, the mean of financial recession indicator was 0.22, while here I have 0.24 for all 30 countries and 0.20 for 23 advanced countries.

3.3 Dynamics of R&D

3.3.1 Methodology

The main message of Jordà, Schularick, and Taylor (2013) is that relative to typical recessions, financial crisis recessions are costlier and more painful to all aspects of economy, such as output and investment, followed by deeper recessions and slower recoveries. The key question of this paper is whether this is also the case for R&D investment. Overall movements of growth rates of R&D investment in the sample period 1981–2016 are shown in Figure 3.1, along with growth rates of real GDP per capita. Overall movements of changes in the number of researchers per 1,000 employed are shown in Figure 3.2.

To answer the question rigorously, making use of the dataset consisting of 150 business cycles peaks in 30 OECD countries in 1981–2016, I examine responses of R&D investment to the onset of the recession in the subsequent recession and recovery phases that follow the peak. The methodology follows Jordà, Schularick, and Taylor (2013). For completeness, the empirical strategy is presented here again.

The local projection method by Jordà (2005) is used in estimation. The macroeconomic variables included in the economic system that is analyzed are as follows: real GDP per capita; aggregate real investment per capita and real R&D investment per capita; real investment per capita and real R&D investment per capita by firms; real investment per capita and real R&D investment per capita by the government; the number of researchers per 1,000 people employed; real short- and long-term interest rates.

We are interested in the changes in the levels in postpeak years of the variables of interest relative to their levels in the peak year. Hence, the peak year reference levels are set to 0, and deviations from that references in each of the postpeak years are measured in terms of percentage point changes. the change measure is then given by the difference in the logarithm of variables times 100 for GDP, investment and R&D variables, and by the simple time difference in raw variables for the number of researchers per 1,000 people employed and interest rates.

Now the notations are formally introduced. Let i be the country index, $t(r)$ the time index when recessions occurred, and k the variable index for the macroeconomic variables. We want to characterize the changes in the macroeconomic variables from the start of the recession to some distant horizon H, that is, from time $t(r)$ to $t(r)+H$. Let y^k, $k = 1, \ldots, K$, denote each of the macroeconomic variables. Here we have 10 main variables:

$$\begin{bmatrix} y^1 \\ y^2 \\ y^3 \\ y^4 \\ y^5 \\ y^6 \\ y^7 \\ y^8 \\ y^9 \\ y^{10} \end{bmatrix} = \begin{bmatrix} \text{log(real GDP per capita)} * 100 \\ \text{log(real investment per capita)} * 100 \\ \text{log(real R\&D investment per capita)} * 100 \\ \text{log(real firm investment per capita)} * 100 \\ \text{log(real firm R\&D investment per capita)} * 100 \\ \text{log(real government investment per capita)} * 100 \\ \text{log(real government R\&D investment per capita)} * 100 \\ \text{the number of researchers per 1,000 people employed} \\ \text{real short-term interest rates} \\ \text{real long-term interest rates} \end{bmatrix}.$$

The dependent variables are $[\Delta_h y^1_{it(r)+h} \dots \Delta_h y^{10}_{it(r)+h}]$, where Δ_h indicates the difference between the reference peak year level at time $t(r)$ and the level in the hth, $h = 1, \dots, H$, horizon at time $t(r)+h$. The control variables are $Y_{it(r)}, \dots, Y_{it(r)-p}$, where $Y_{it(r)} = [\Delta y^1_{t(r)}\ \Delta y^2_{t(r)}\ \Delta y^3_{t(r)}\ \Delta y^4_{t(r)}\ \Delta y^5_{t(r)}\ \Delta y^6_{t(r)}\ \Delta y^7_{t(r)}\ y^8_{t(r)}\ y^9_{t(r)}\ y^{10}_{t(r)}]$ and Δ is the first difference. The responses of the macroeconomic variables are estimated by

$$\Delta_h y^k_{it(r)+h} = \alpha^k_i + \theta^k_{h,N} N + \theta^k_{h,F} F + \sum_{j=0}^{p} \Gamma^k_j Y_{it(r)-j} + u^k_{it(r)}, \quad k = 1, \dots, 10, \quad h = 1, \dots, H,$$

where α^k_i are country fixed effects. The key treatment variables are the indicators for whether the peak comes before a normal recession or a financial recession. θ^k_N is the common constant associated with normal recession treatment ($N = 1$), and θ^k_F is the constant associated with financial recession treatment ($F = 1$). A history of p lags of the control variables Y at time $t(r)$ are included with coefficients Γ, and u is the error term. The cumulated responses of each variable to normal/financial recessions are estimated by $\{\theta^k_{h,N}\}^H_{h=1}$ and $\{\theta^k_{h,F}\}^H_{h=1}$. For each impulse response, 1.96 standard deviation confidence intervals are presented. $H = 8$ and $p = 2$ are used.

3.3.2 Results

The estimated cumulated responses of the macroeconomic variables are depicted in Figures 3.3 and 3.4. Figure 3.3 uses all countries, and Figure 3.4 focuses on advanced countries. The first half of the results are the cumulated responses of 10 level variables, which are shown in the following order: real GDP per capita, real investment per capita, real non-R&D investment per capita, real R&D investment per capita, real investment per capita by firms, real non-R&D investment per capita by firms, real R&D investment per capita by firms, real investment per capita by the government, real non-R&D investment per capita by the government, and real R&D investment per capita by the government. The second half of the results are the cumulated responses of 9 ratio variables, which focus on R&D investment and are shown in the following order: R&D investment-to-GDP ratio, R&D investment-to-total investment ratio, firm R&D investment-to-GDP ratio, firm R&D investment-to-total investment ratio, firm R&D investment-to-firm investment ratio, government R&D investment-to-GDP ratio, government R&D investment-to-total investment ratio, government R&D investment-to-government investment ratio, and the number of researchers per 1,000 employed.

The results on real GDP per capita and real investment per capita are consistent with Jordà, Schularick, and Taylor (2013). Along the path upon recessions, financial recessions are more painful and followed by slower recovery. The path after normal recessions sits well above the path after financial recessions. This is true also for real investment per capita by firms and the government.

However, things get different when it comes to R&D investment. The aggregate real R&D investment per capita and real R&D investment per capita by firms and the government are in general unresponsive. The estimates along the path are not statistically significantly different from zero. The path after normal recessions does not necessarily sit above the path after financial recessions. In the horizons close to the reference peak year, the path after normal recessions even sits below the path after financial recessions.

The unresponsiveness of R&D investment can be seen more dramatically when the results are shown in terms of ratios. For R&D investment by the aggregate economy, firms and the government relative to GDP, aggregate investment and their own total investment, the paths after normal and financial recessions are now flipped to what we would expect. The path after financial recessions sits above the path after normal recessions. Moreover, the paths are not any more sloping downward. Rather, they look like slightly sloping upward. Indeed, the point estimates along the path of the ratio variables are positive, although they are not statistically significant. It is because R&D variables, which are in the numerator of the ratios, are unresponsive while GDP and aggregate investment, which are in the denominator of the ratios, are decreasing after recessions. So, the ratios increase. These results hold both for all countries and advanced countries only.

We can observe similar responses to those of the ratio variables also from the number of researchers per 1,000 employed. The point estimates of the cumulated responses over horizons of the number of researchers per 1,000 employed are positive. Some of them in the early horizons after normal recessions are statistically significant, and almost all after financial recessions are statistically significant.

Thus, we learned from the analysis that the responses of R&D investment to recessions are in a sharp contrast with the responses of non-R&D investment. Recessions are bad times for investment in general, with financial recessions being more painful and followed by slower recovery. While this is true, most of the responses are coming from the responses of the non-R&D part of investment. R&D investment is overall unresponsive to recessions, even to financial recessions. This result suggests that the recently documented productivity slowdown after financial crises is not from a decline in the level of R&D, but rather from a decline in the R&D effectiveness.

3.3.3 Discussion

[5] So, what could be the rationale behind the persistence of R&D activities? Thinking in the framework of a type of Bloom (2009), we could conjecture that R&D adjustment costs would play a significant role. A possible story is that there exist huge R&D adjustment costs in hiring researchers. It should be relatively easy to adjust capital stock in that selling capital stock in bad times and buying in good times would not be difficult even in a large scale adjustment. On the contrary, it is hard to imagine easily being able to hire a bunch of quality researchers at one time. Unlike low-quality labor force, there are not many replacements in the hiring market for researchers. This might be the reason why firms get reluctant in firing researchers and cutting down their R&D investment during bad times since, if they do so, it would be hard to go back to the optimal level or to the original trend once the economy recovers.[6] How high the adjustment costs would be is another question to be explored. As Bloom (2009) structurally measures capital and labor adjustment costs using a micro-level data, we would need a firm-level R&D related data in order to measure R&D adjustment costs.

3.4 Conclusion

The slow recovery after financial crises is one of the stylized facts that characterize the aftermath of financial crises. Recent literature documents that the slow recovery is largely from

[5] I use this subsection for very preliminary discussion on possible theoretical explanations for the result of the unresponsiveness of R&D to recessions.

[6] An extreme example that we could imagine would be that, if we assume that firms can hire at most one researcher at one time, firing one researchers at one point of time means having one less researchers forever in the future.

the slowdown in the productivity. Two hypotheses that could lead to the productivity slowdown are that the level of R&D has decreased after financial crises and that the effectiveness of R&D has decreased after financial crises. Recent literature tries to rationalize the productivity slowdown after financial crises by incorporating R&D and endogenous productivity changes into the business cycle frameworks. In so doing, which channel is more empirically relevant between the level effects and the effectiveness effects is an important question. This paper contributes to the literature by answering this question. Using the 30-OECD-country panel dataset over the years 1981–2016 and estimating by local projections, it is hard to find evidence of the level effects to R&D. While recessions are bad times for output and investment in general, most of the responses are coming from the non-R&D part of investment. R&D is overall unresponsive to recessions, even to financial crises. This result suggests that the productivity slowdown after financial crises is not from a decline in the level of R&D, but rather from a decline in R&D efficiency. Whether this decline in R&D efficiency is coming an exogenous shock to the economy or it is an endogenous phenomenon that results from misallocation of R&D resources across heterogenous firms is another question that should be explored. To answer this question, we need micro firm-level data, and I turn this to future research.

3.A Data Appendix

This appendix documents the sources of the R&D variables used to build the dataset.

Real R&D investment per capita. OECD.Stat; Science, Technology and Patents; Research and Development Statistics; Historical Series; Gross domestic expenditure on R-D by sector of performance and source of funds; Measure: 2010 Dollars—Constant prices and PPPs; Sector of performance: Total intramural, Business enterprise, Government.

http://stats.oecd.org/Index.aspx?DataSetCode=GERD_FUNDS

Number of researchers per 1,000 employed. OECD Data; Innovation and Technology; Research and development (R&D); Researchers. https://data.oecd.org/rd/researchers.htm

3.B Tables and Figures

Table 3.A.1. Business cycles peaks. “N” denotes a normal business cycle peak, and “F” denotes a peak associated with a financial crisis. Business cycle peaks identified by the Bry and Boschan (1971) algorithm are classified into N or F, using crises dates in the Jordà–Schularick–Taylor Macrohistory Database and Laeven and Valencia (2012).

AUS	N	1981	2000	2005	2008						
	F	1989									
AUT	N	2000	2012	2013							
	F	2008									
BEL	N	2000	2011								
	F	2008									
CAN	N	1981	1986	1989	1995	2001	2006	2008	2011	2014	2017
	F	——									
CZE	N	2008	2011								
	F	1996									
CHE	N	1981	1990	1994	1998	2000	2011	2014	2016		
	F	2008									
DEU	N	1992	1995	2001	2012	2014					
	F	2008									
DNK	N	1992	1997	2001	2006	2011	2015	2017			
	F	1987	2007								
ESP	N	1992	2010								
	F	2008									
EST	N	1998	2007								
	F	1992									
FIN	N	2007	2011	2013	2017						
	F	1991									
FRA	N	1983	1990	1992	2002	2011					
	F	2008									
GBR	N	——									
	F	1990	2007								
GRC	N	2004	2015								
	F	2007									
HUN	N	2006	2011								
	F	1991	2008								

Table 3.1 (Continued). Business cycles peaks. "N" denotes a normal business cycle peak, and "F" denotes a peak associated with a financial crisis. Business cycle peaks identified by the Bry and Boschan (1971) algorithm are classified into N or F, using crises dates in the Jordà–Schularick–Taylor Macrohistory Database and Laeven and Valencia (2012).

IRL	N	2012					
	F	2007					
ISL	N	1997	2000	2005	2011	2016	
	F	2007					
ISR	N	1998	2000	2008	2011	2015	
	F	——					
ITA	N	1992	1997	2001	2002	2004	2011
	F	2007					
JPN	N	2001	2008	2010	2015		
	F	1997					
LVA	N	1998					
	F	1995	2007				
LUX	N	2002	2003	2011	2015	2016	
	F	2007					
MEX	N	2006	2008				
	F	1981	1994				
NLD	N	2001	2002	2011			
	F	2008					
NOR	N	1981	2000	2007	2012	2015	
	F	1987	1991				
NZL	N	1992	1997	2000	2004	2007	2010
	F	——					
PRT	N	2002	2010				
	F	2007					
SVN	N	2011					
	F	1992	2008				
SWE	N	1996	2011				
	F	1990	2007				
USA	N	1990	2000	2012	2015		
	F	1981	2007				

Table 3.A.2. Summary statistics for the treatment variables. The annual sample runs from 1981 to 2016 for 30 countries, among which 23 are advanced countries and 7 are emerging countries.

All countries	All recessions	Financial recessions	Normal recessions
Financial recession indicator (F), mean	0.24	1	0
Observations	150	36	114
Normal recession indicator (F), mean	0.76	0	1
Observations	150	36	114
Advanced countries	**All recessions**	**Financial recessions**	**Normal recessions**
Financial recession indicator (F), mean	0.20	1	0
Observations	125	26	99
Normal recession indicator (F), mean	0.80	0	1
Observations	125	26	99
Emerging countries	**All recessions**	**Financial recessions**	**Normal recessions**
Financial recession indicator (F), mean	0.4	1	0
Observations	25	10	15
Normal recession indicator (F), mean	0.6	0	1
Observations	25	10	15

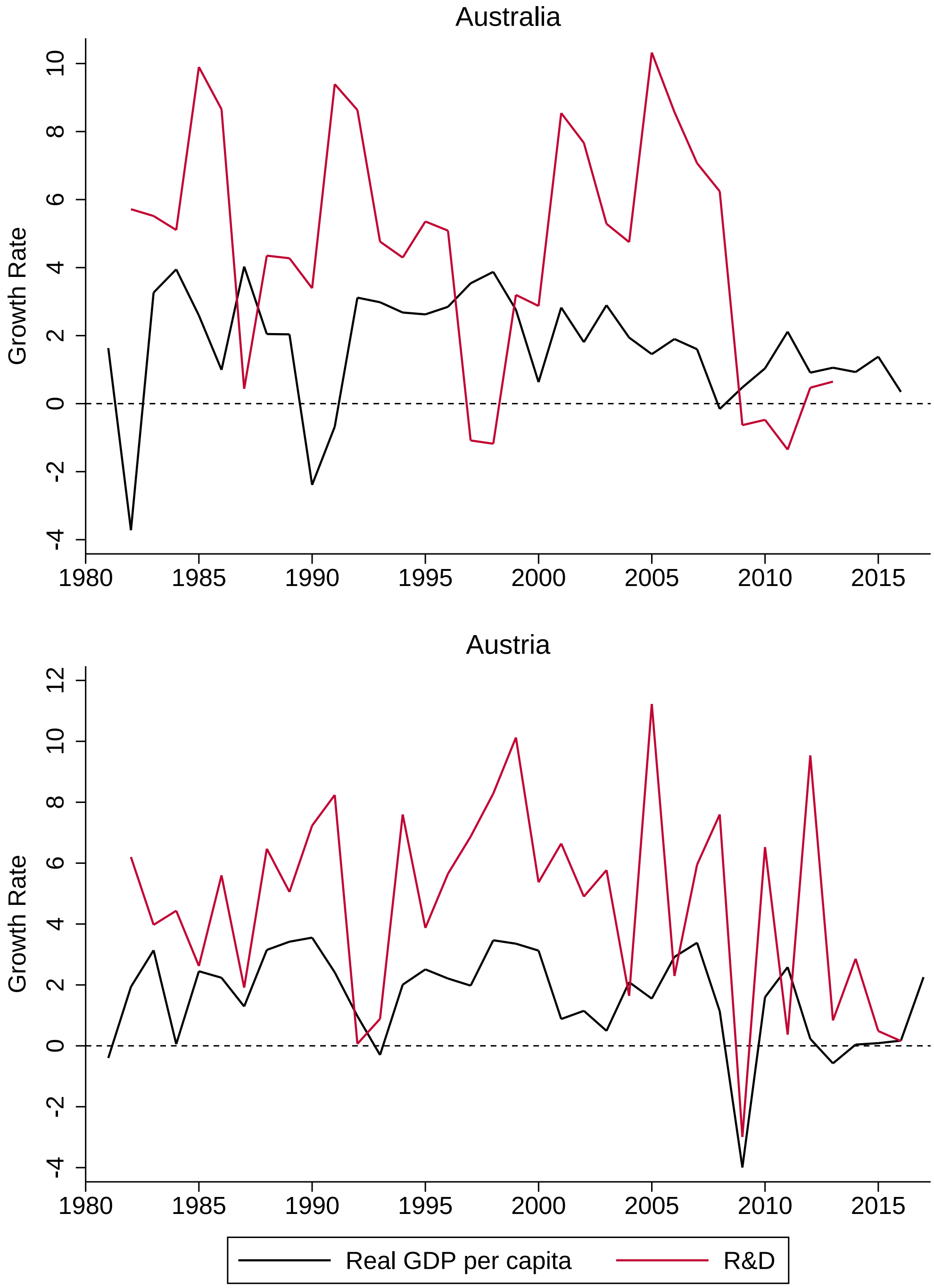


Figure 3.1. Growth rates of real GDP per capita and R&D investment of 23 advanced countries in 1981–2016. Real GDP per capita data is from the Jordà–Schularick–Taylor Macrohistory Database. R&D data is from the OECD. Countries are in the following order: Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, UK, Greece, Ireland, Iceland, Italy, Japan, Luxembourg, Netherlands, Norway, New Zealand, Portugal, Sweden, USA.

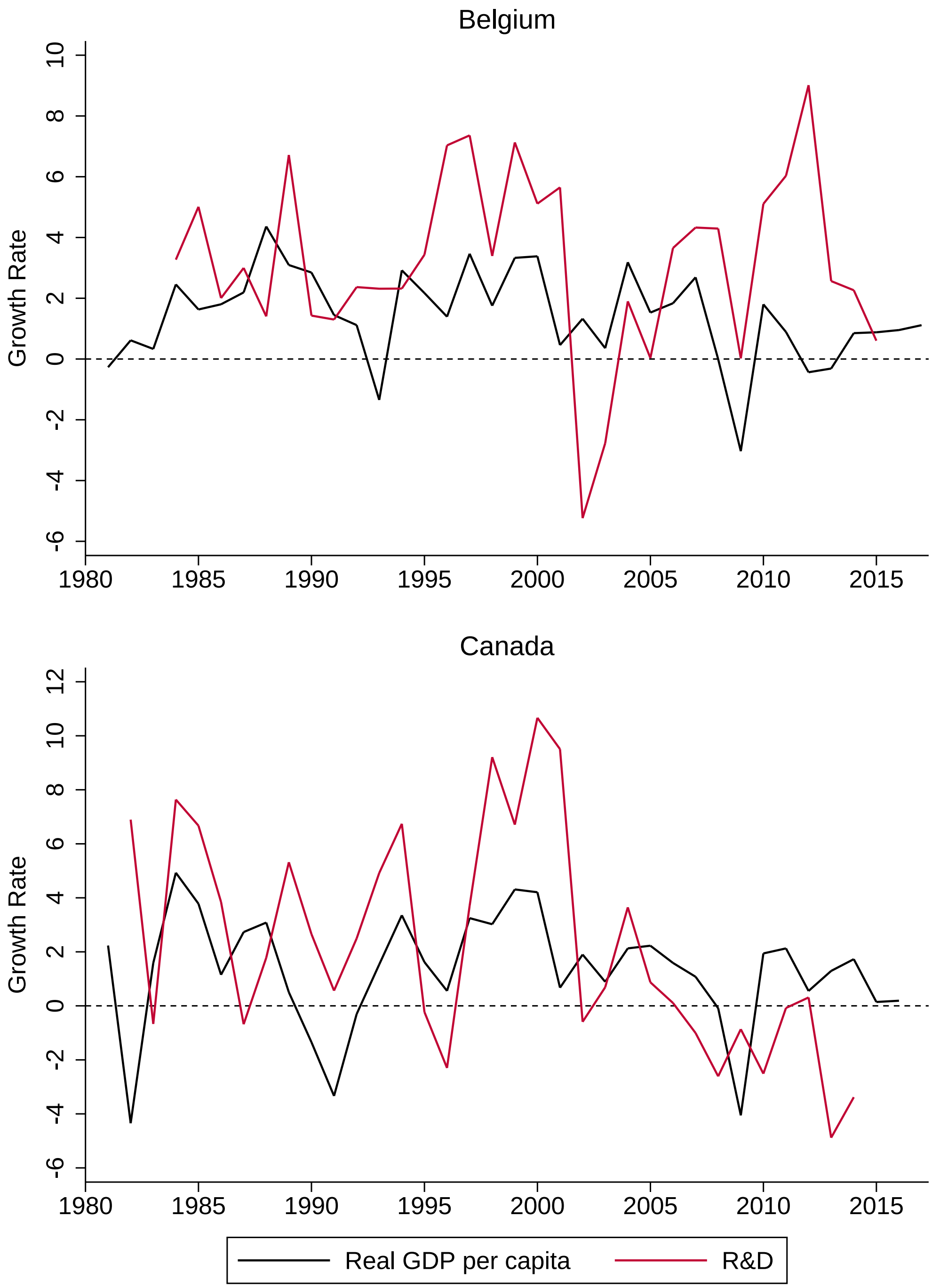
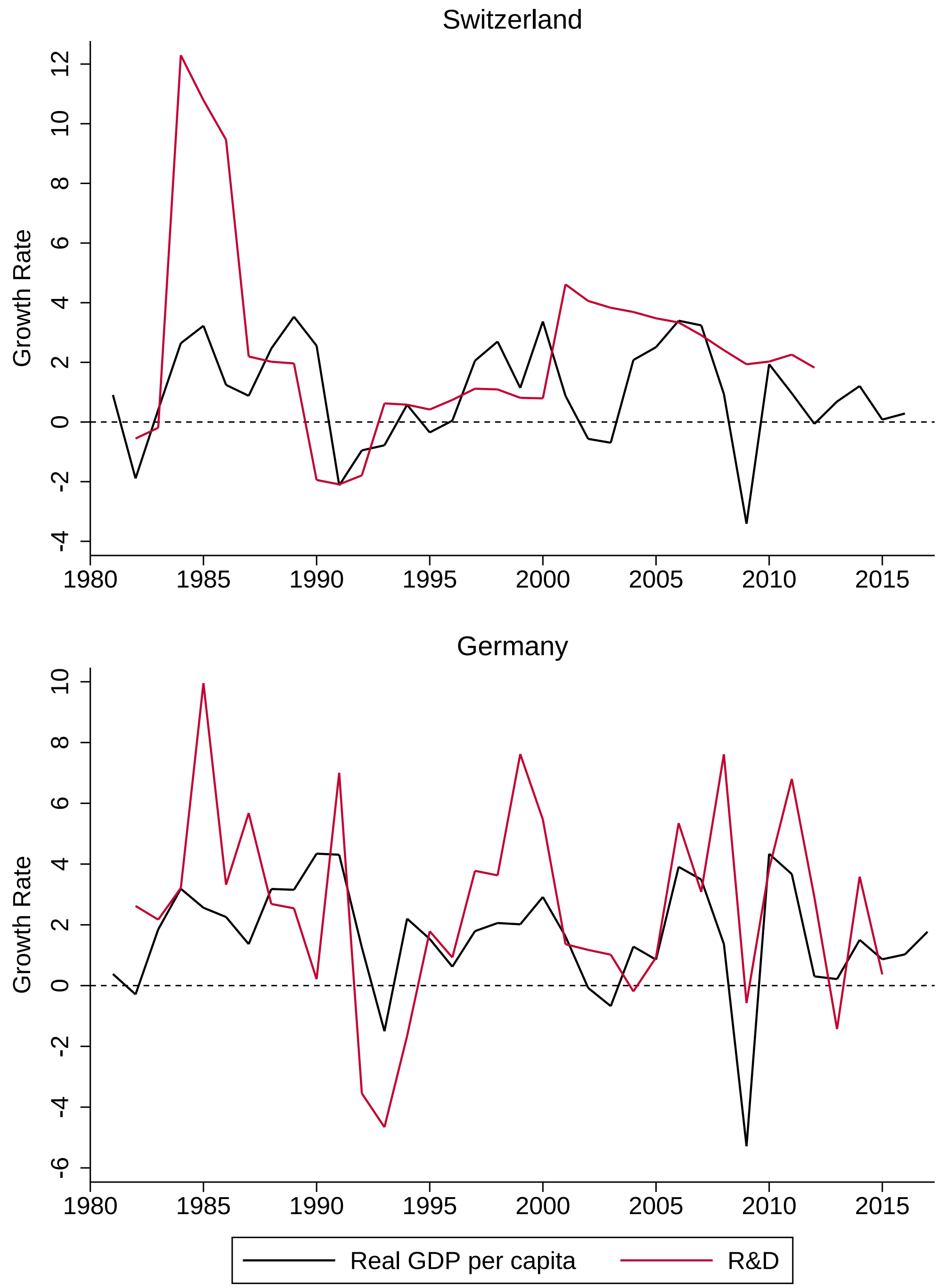


Figure 3.1 (Continued). Growth rates of real GDP per capita and R&D investment of 23 advanced countries in 1981–2016. Real GDP per capita data is from the Jordà–Schularick–Taylor Macrohistory Database. R&D data is from the OECD. Countries are in the following order: Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, UK, Greece, Ireland, Iceland, Italy, Japan, Luxembourg, Netherlands, Norway, New Zealand, Portugal, Sweden, USA.



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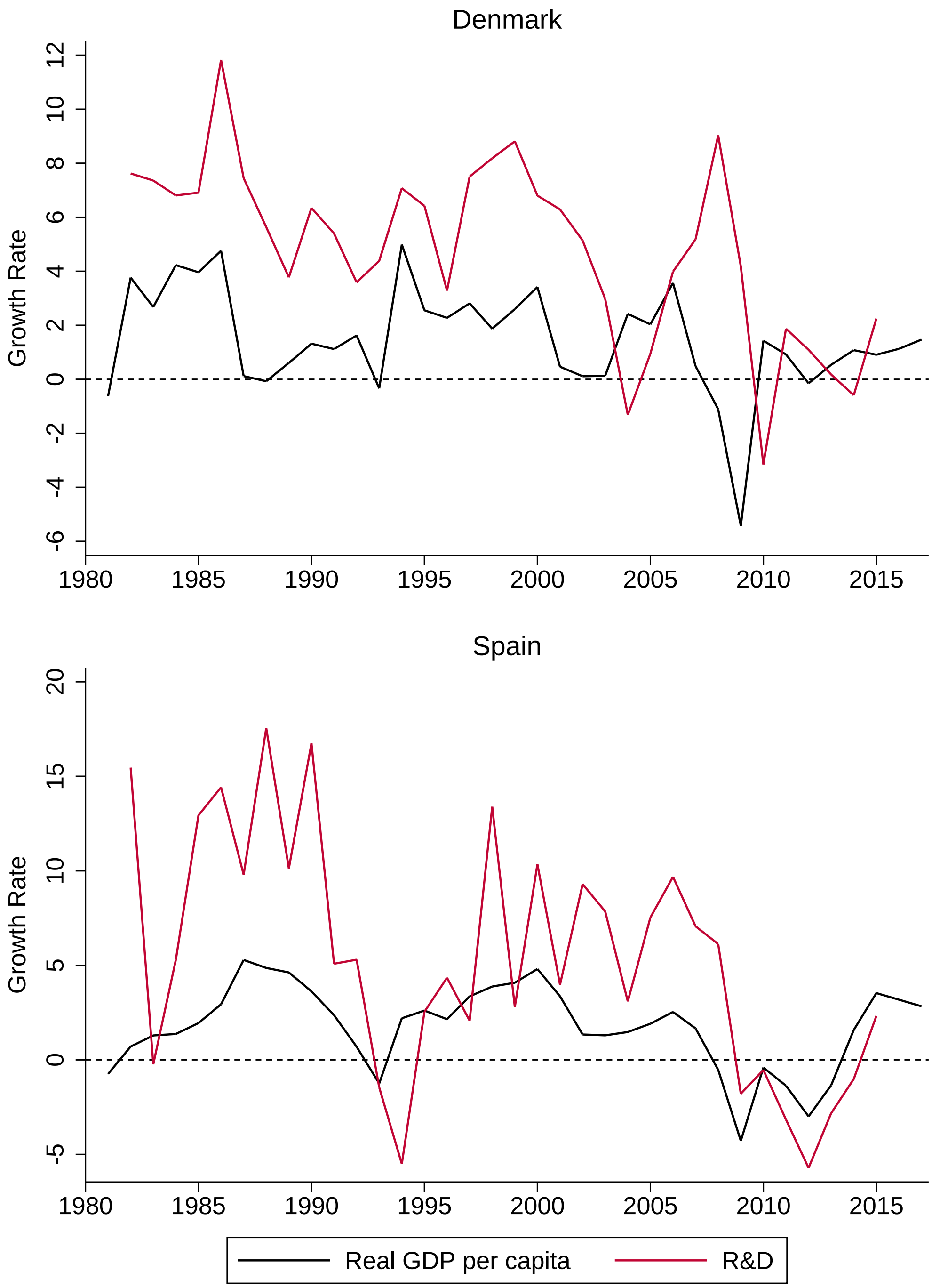

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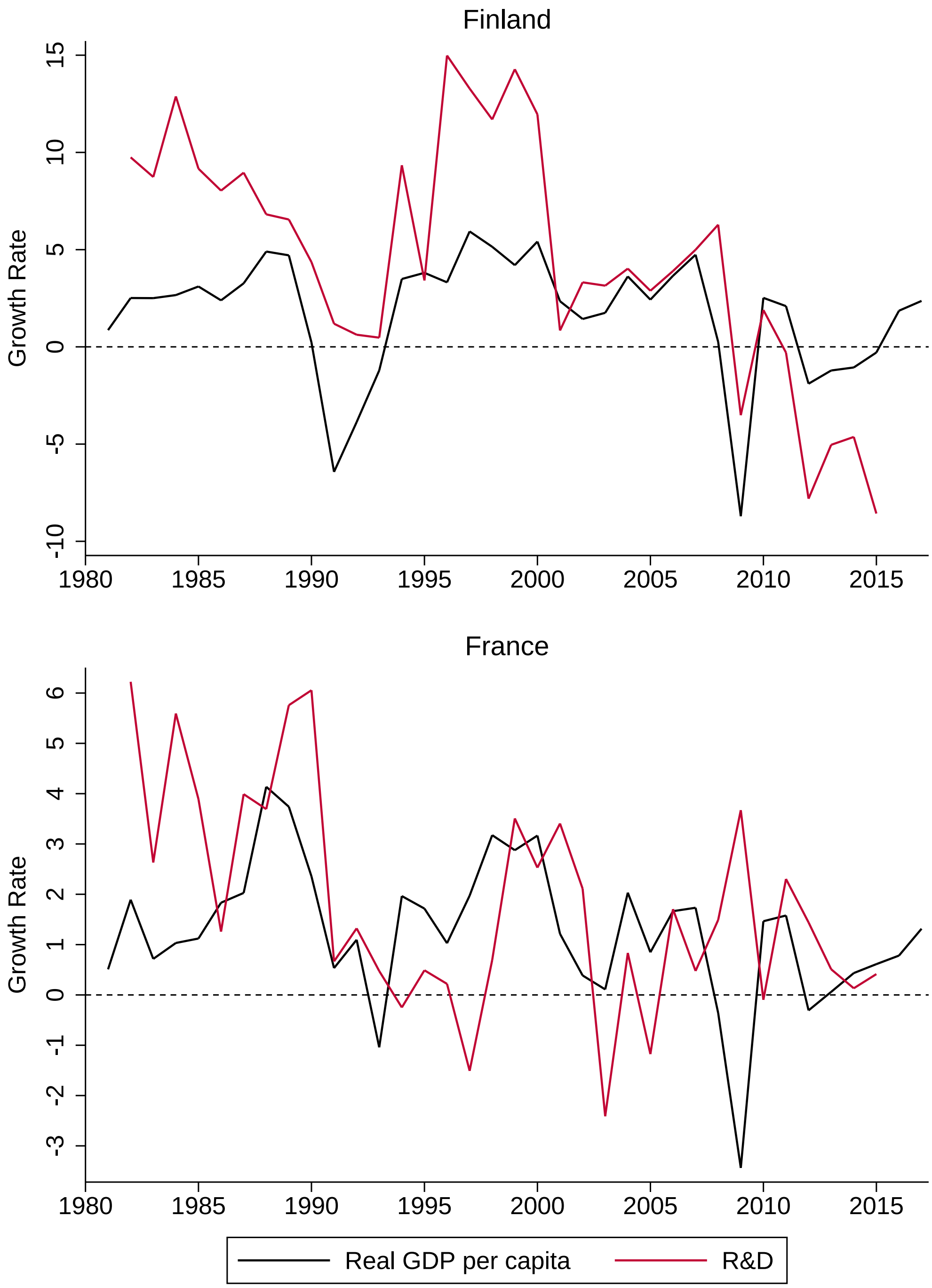

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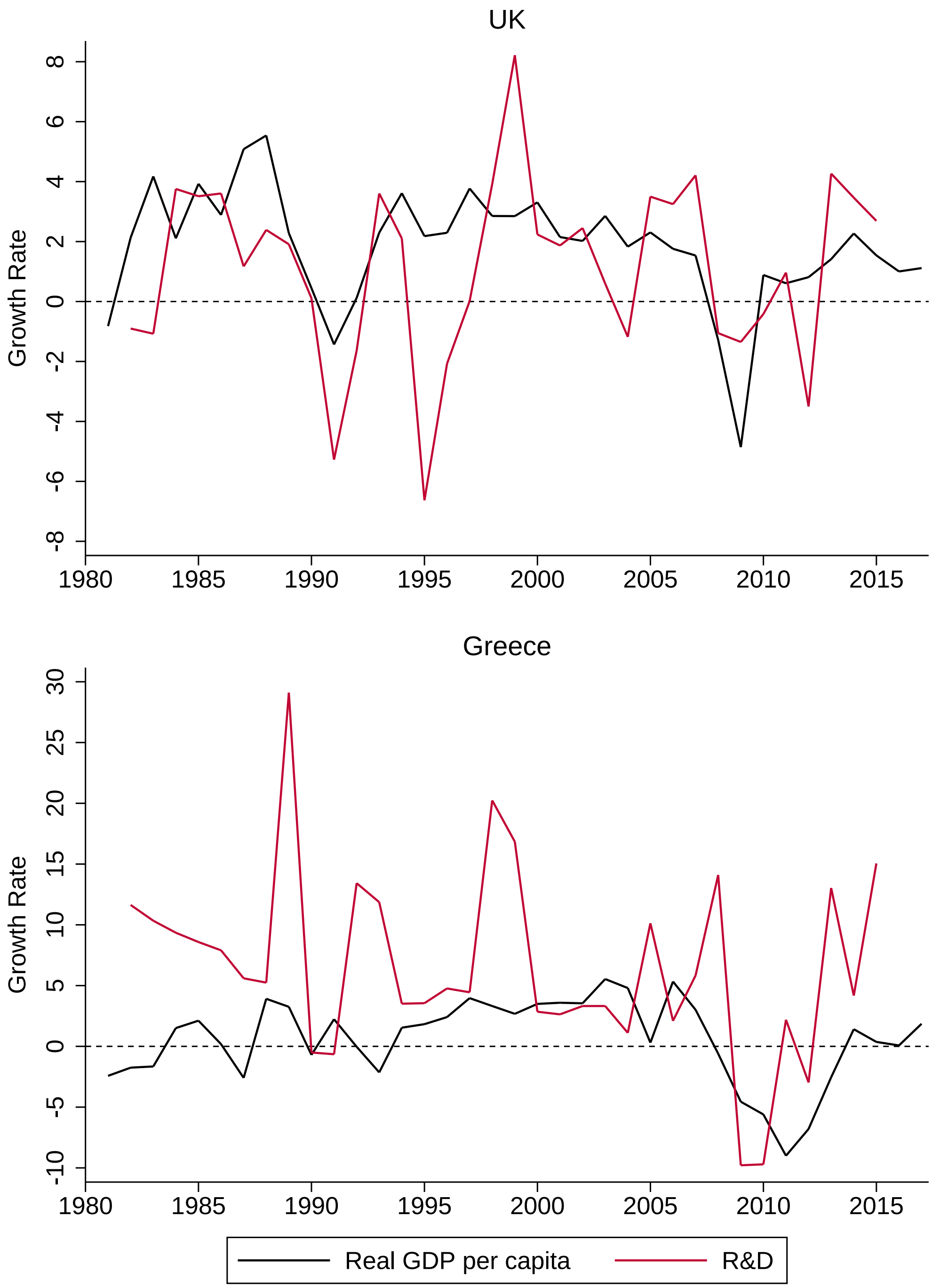

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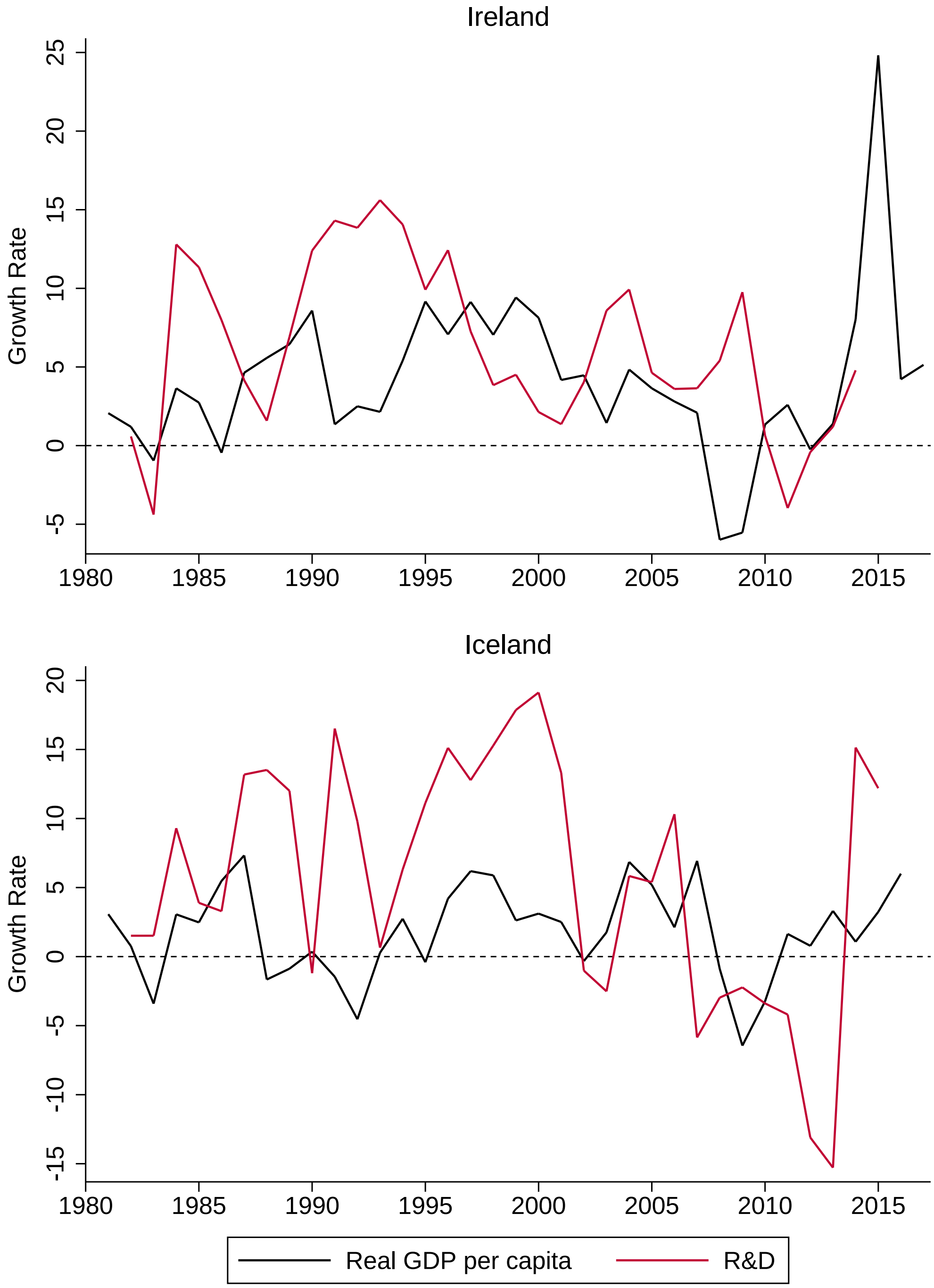

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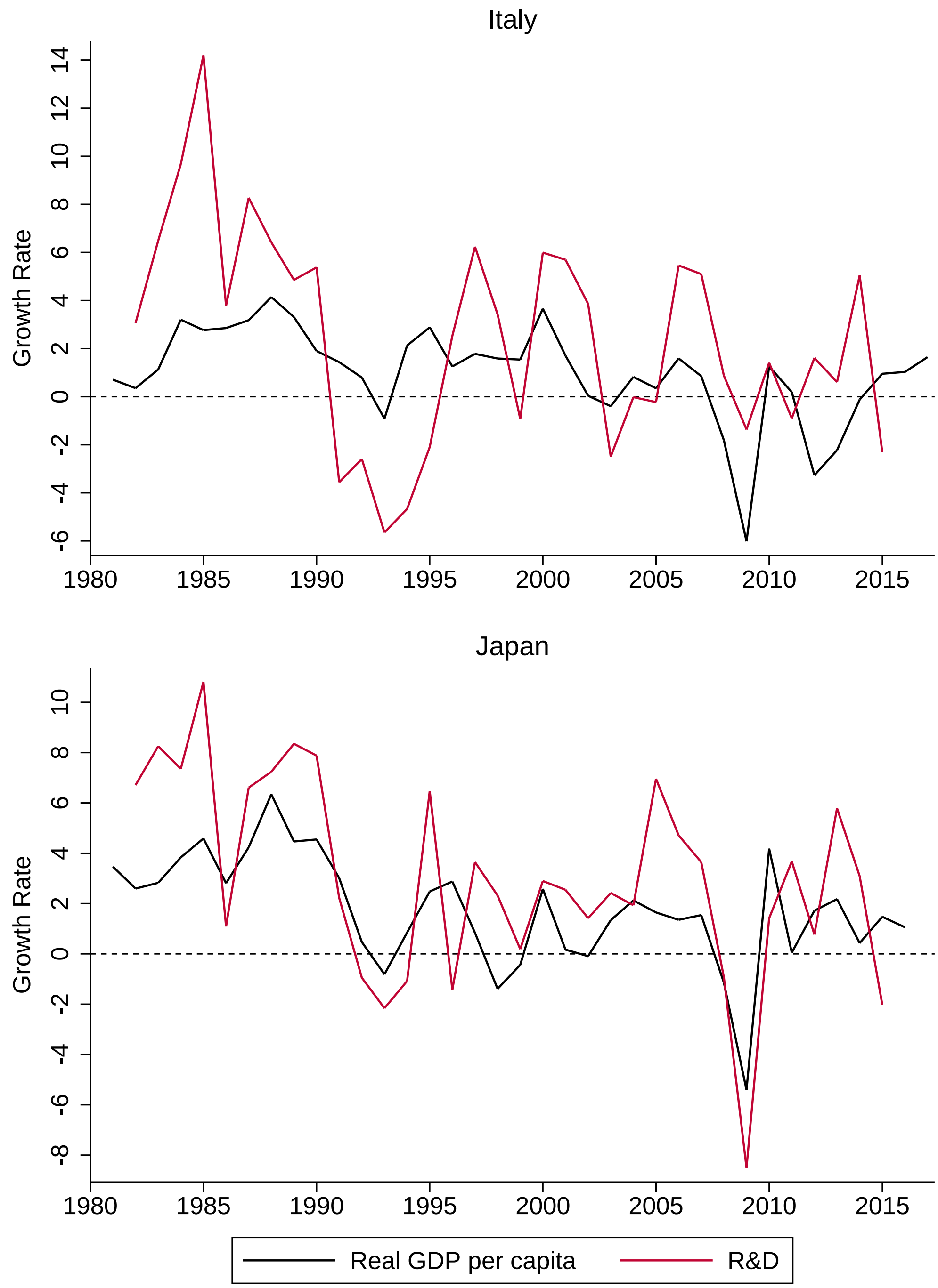

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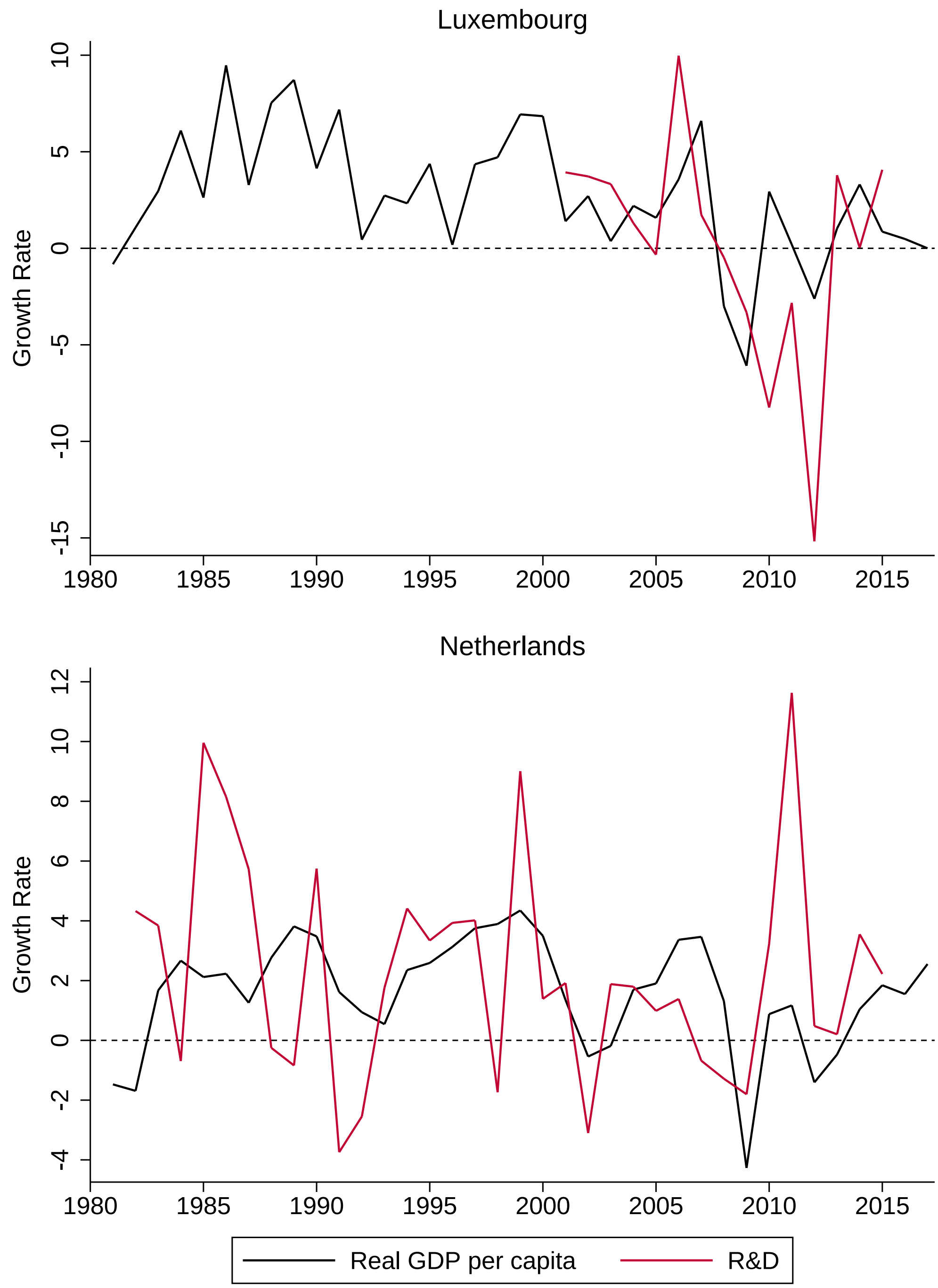

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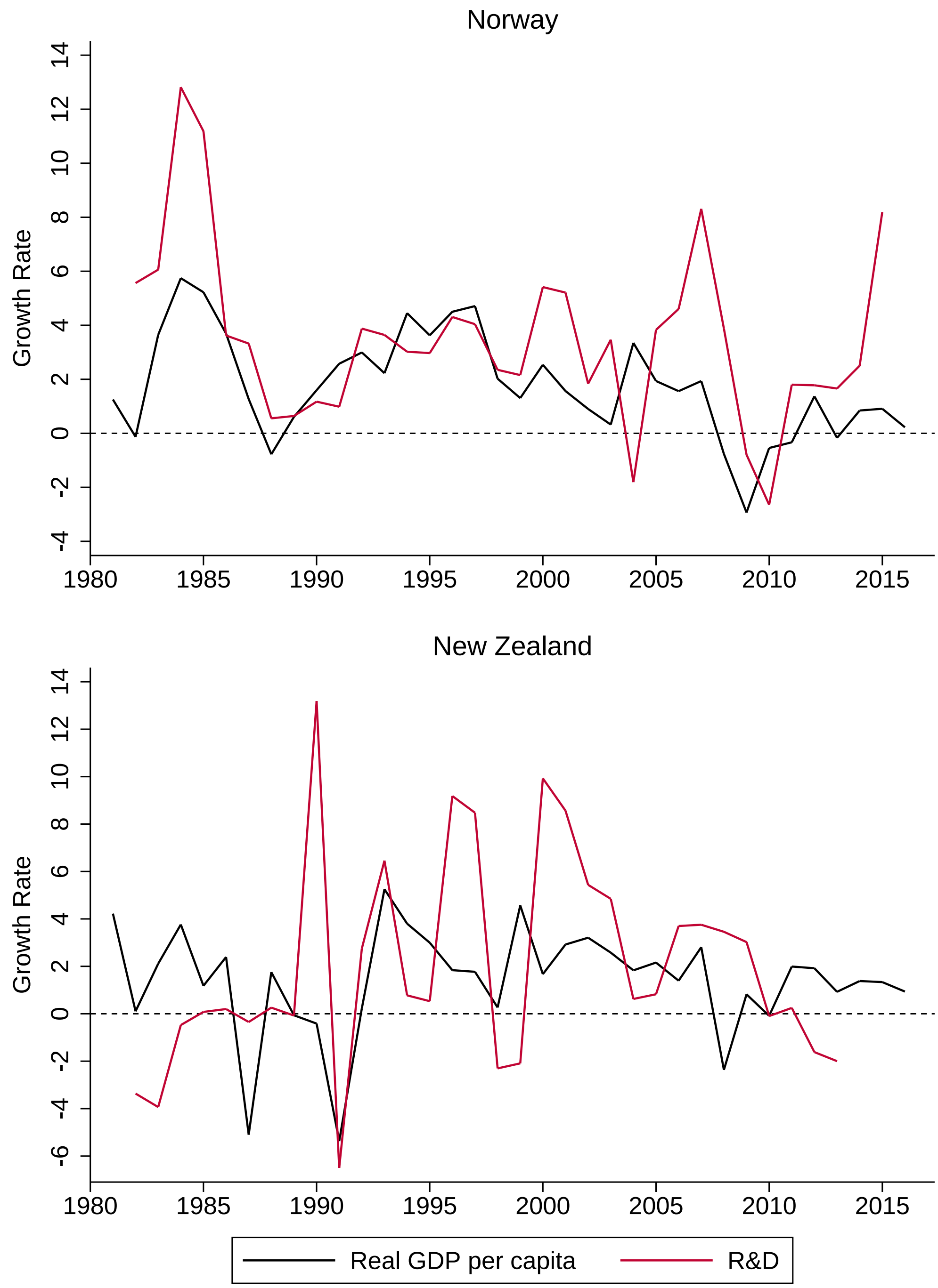

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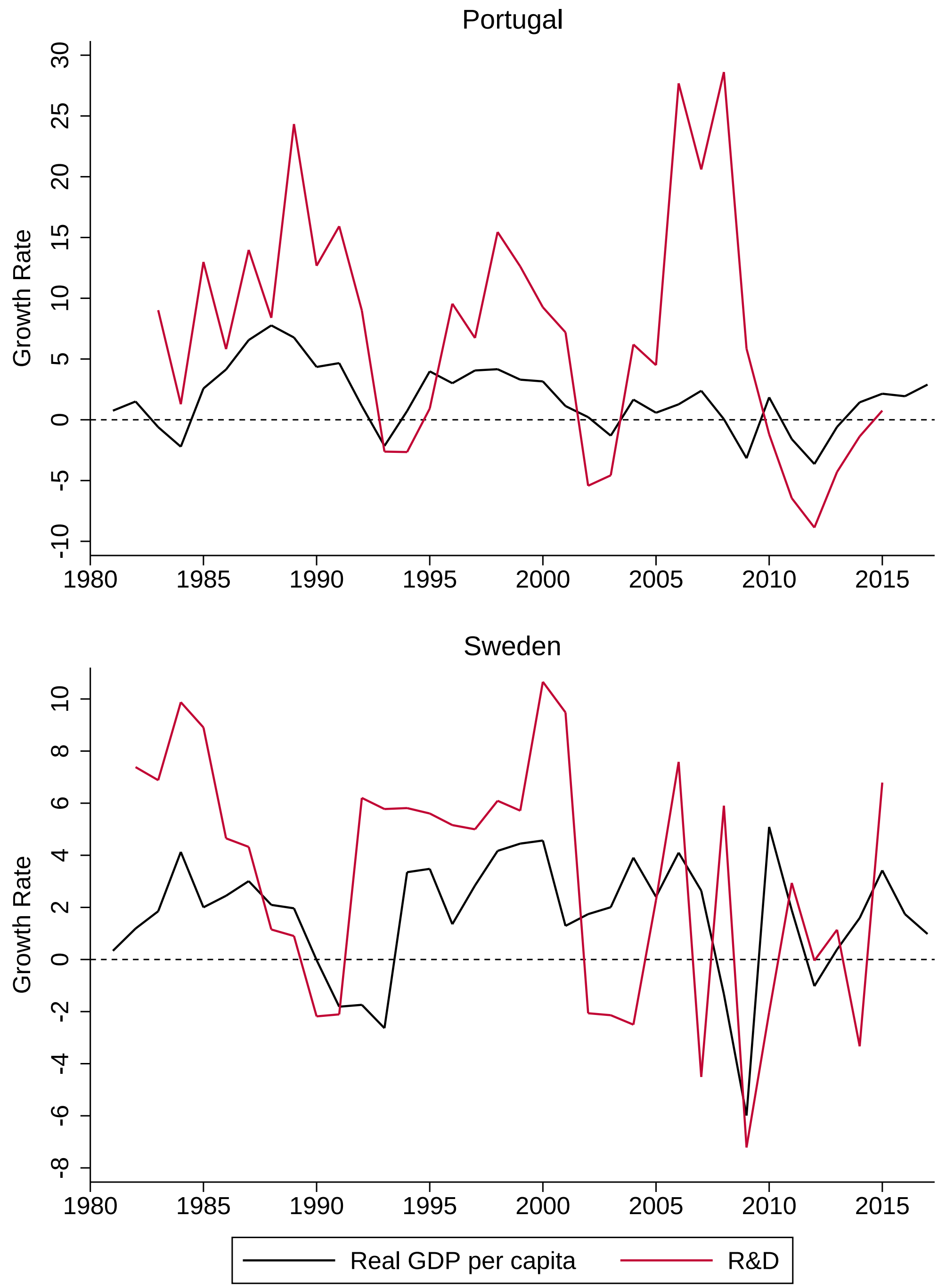

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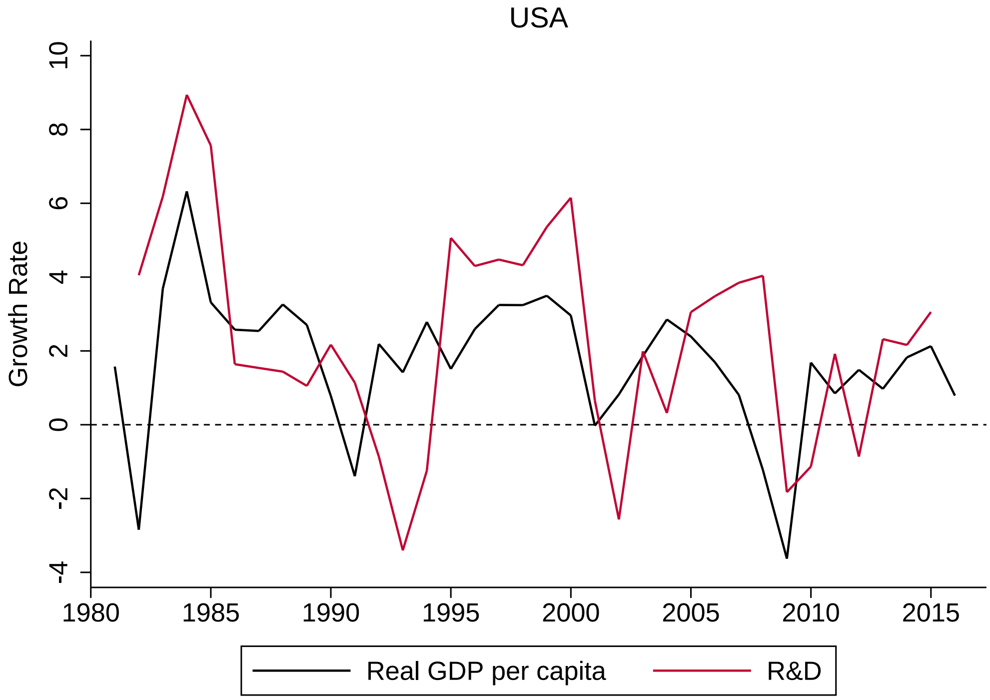

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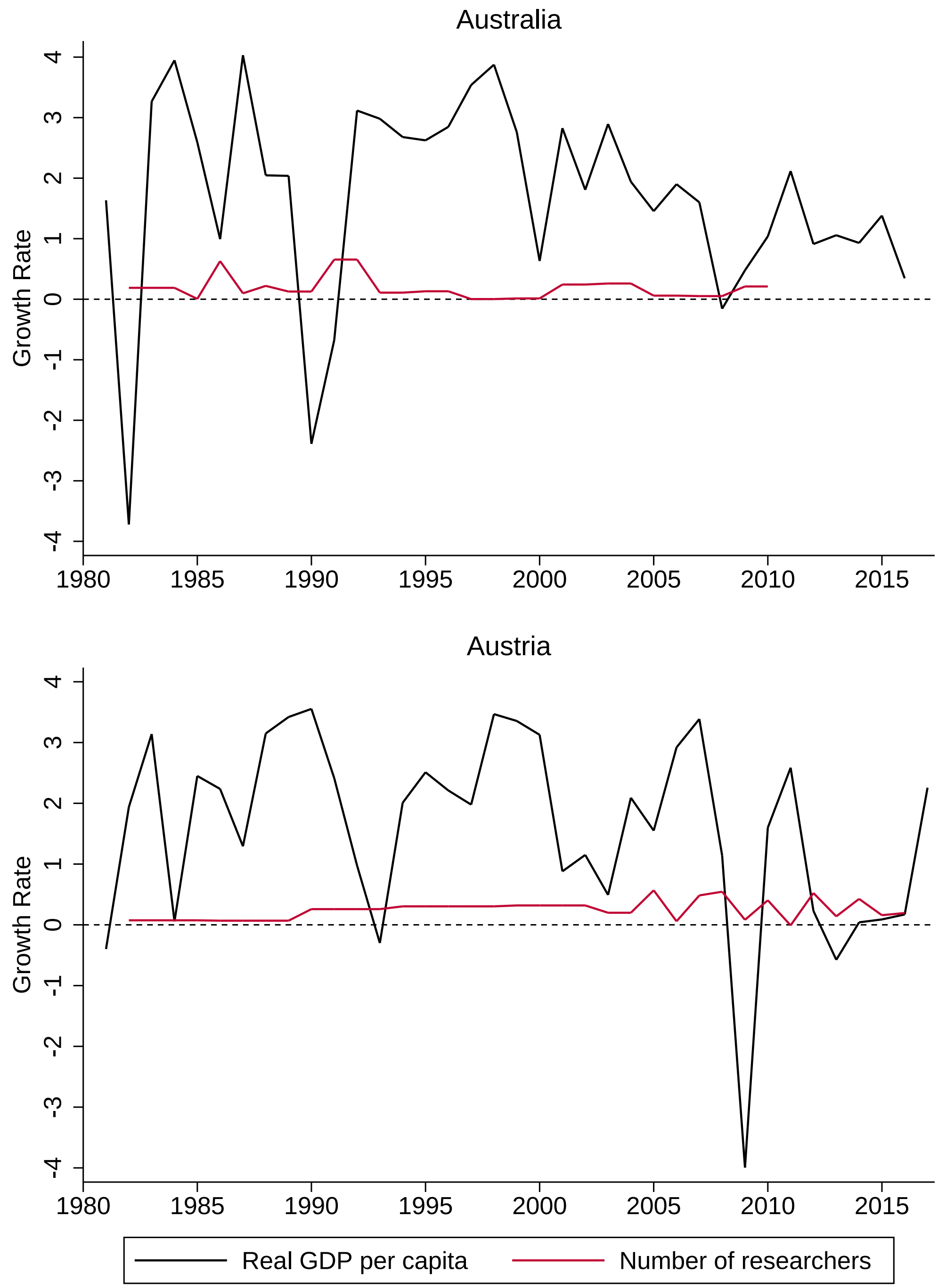
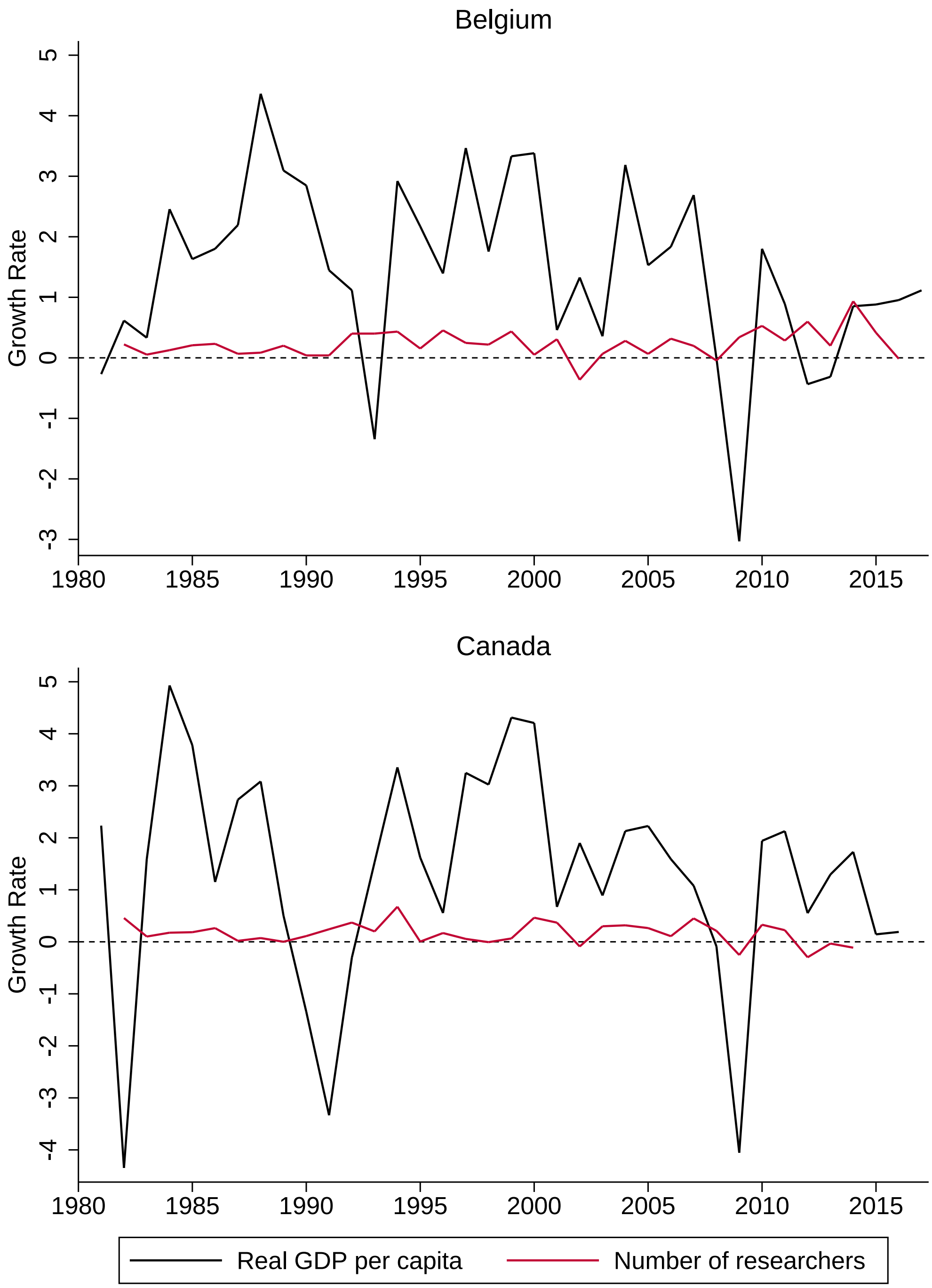


Figure 3.2. Growth rates of real GDP per capita and the number of researchers per 1,000 employed of 23 advanced countries in 1981–2016. Researcher data is from the OECD. Countries are in the following order: Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, UK, Greece, Ireland, Iceland, Italy, Japan, Luxembourg, Netherlands, Norway, New Zealand, Portugal, Sweden, USA.



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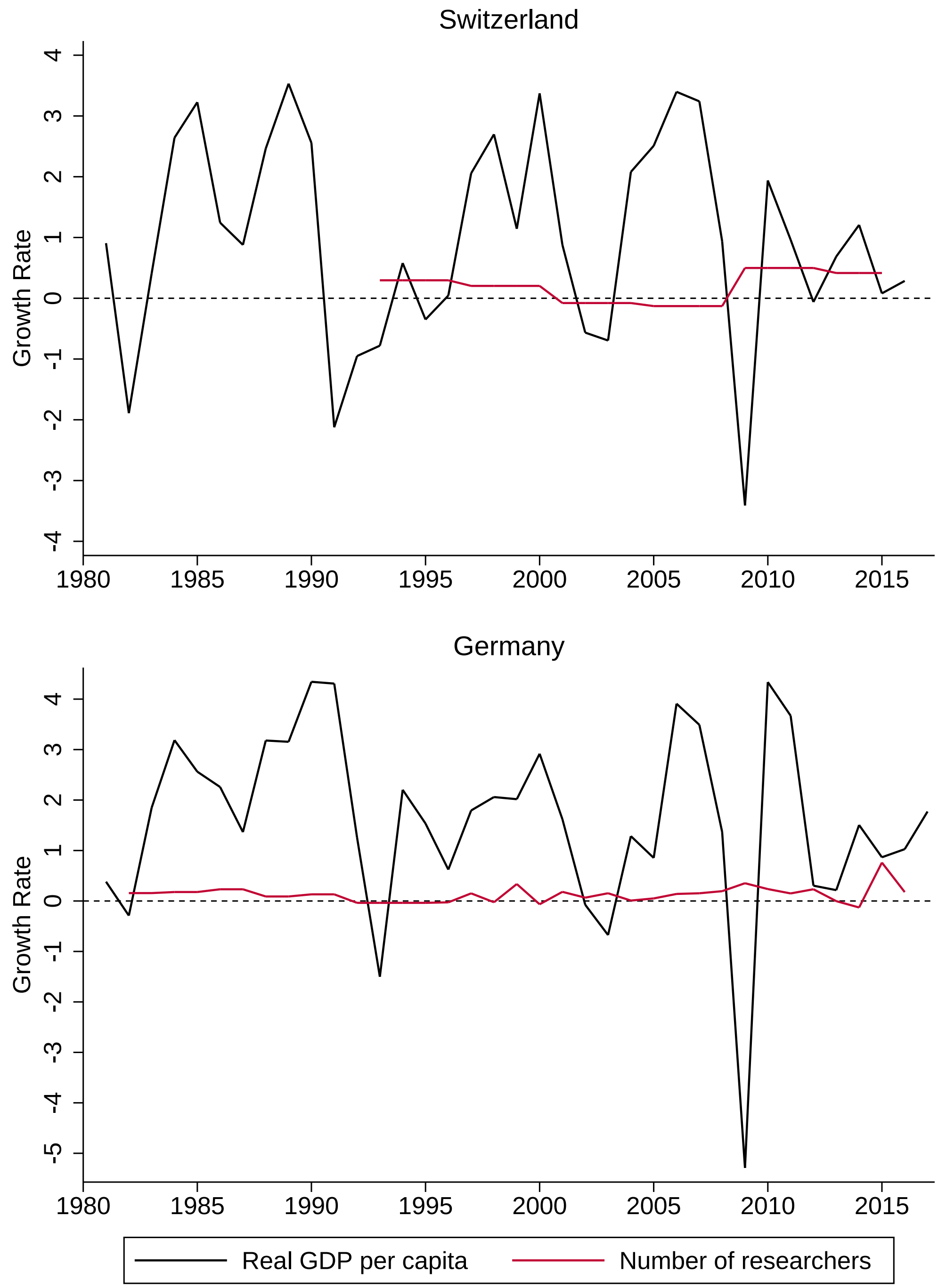

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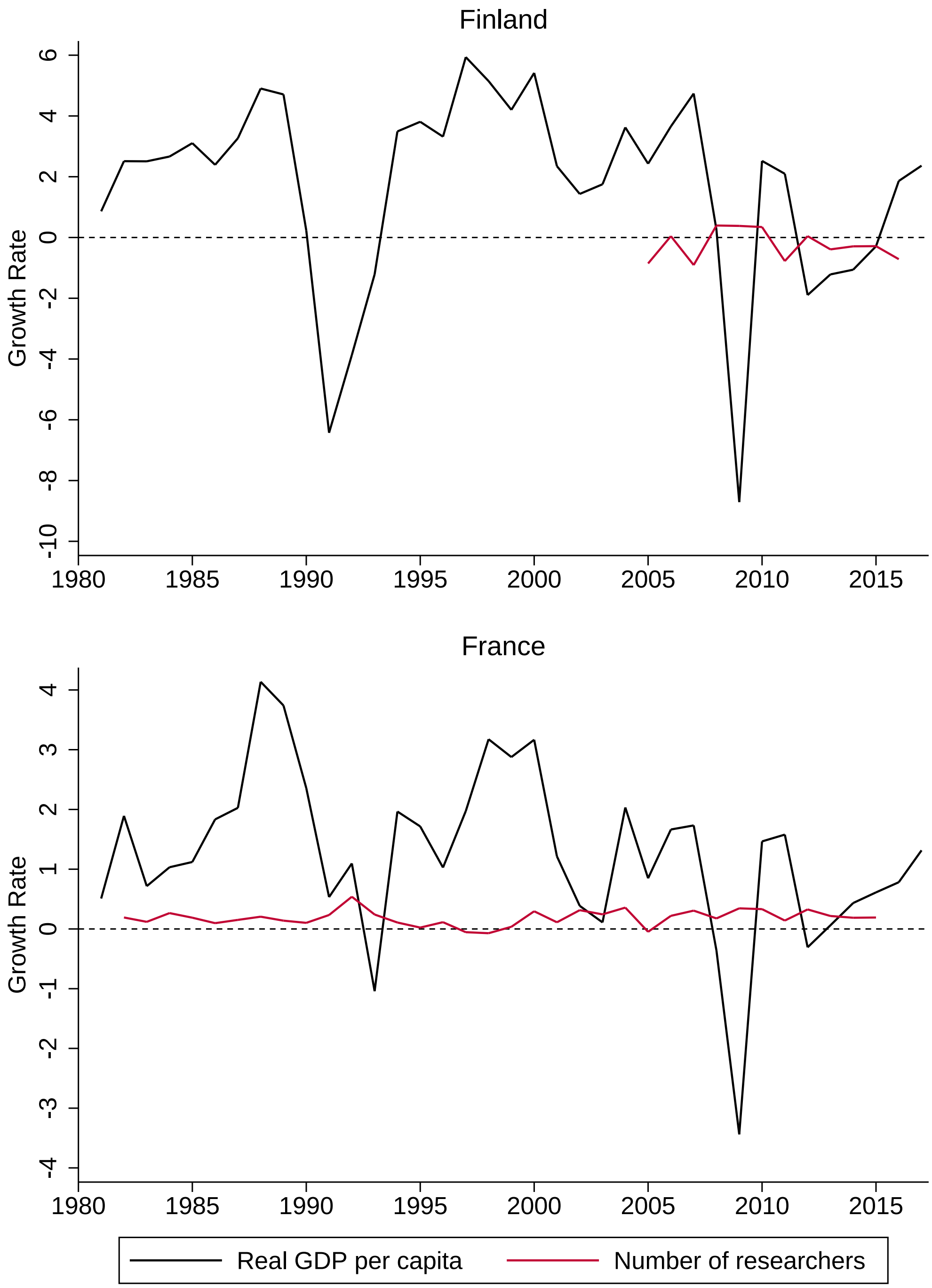

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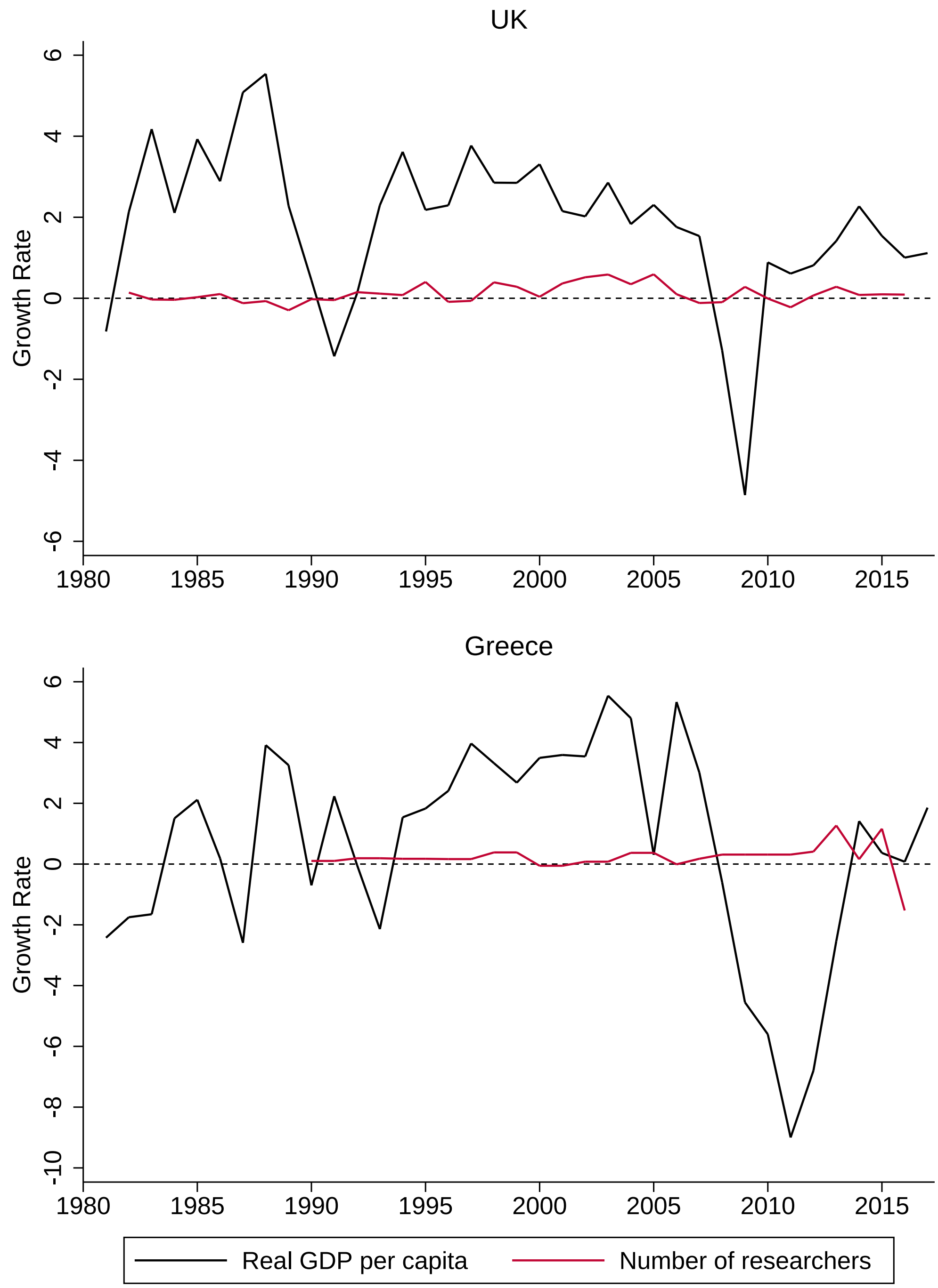

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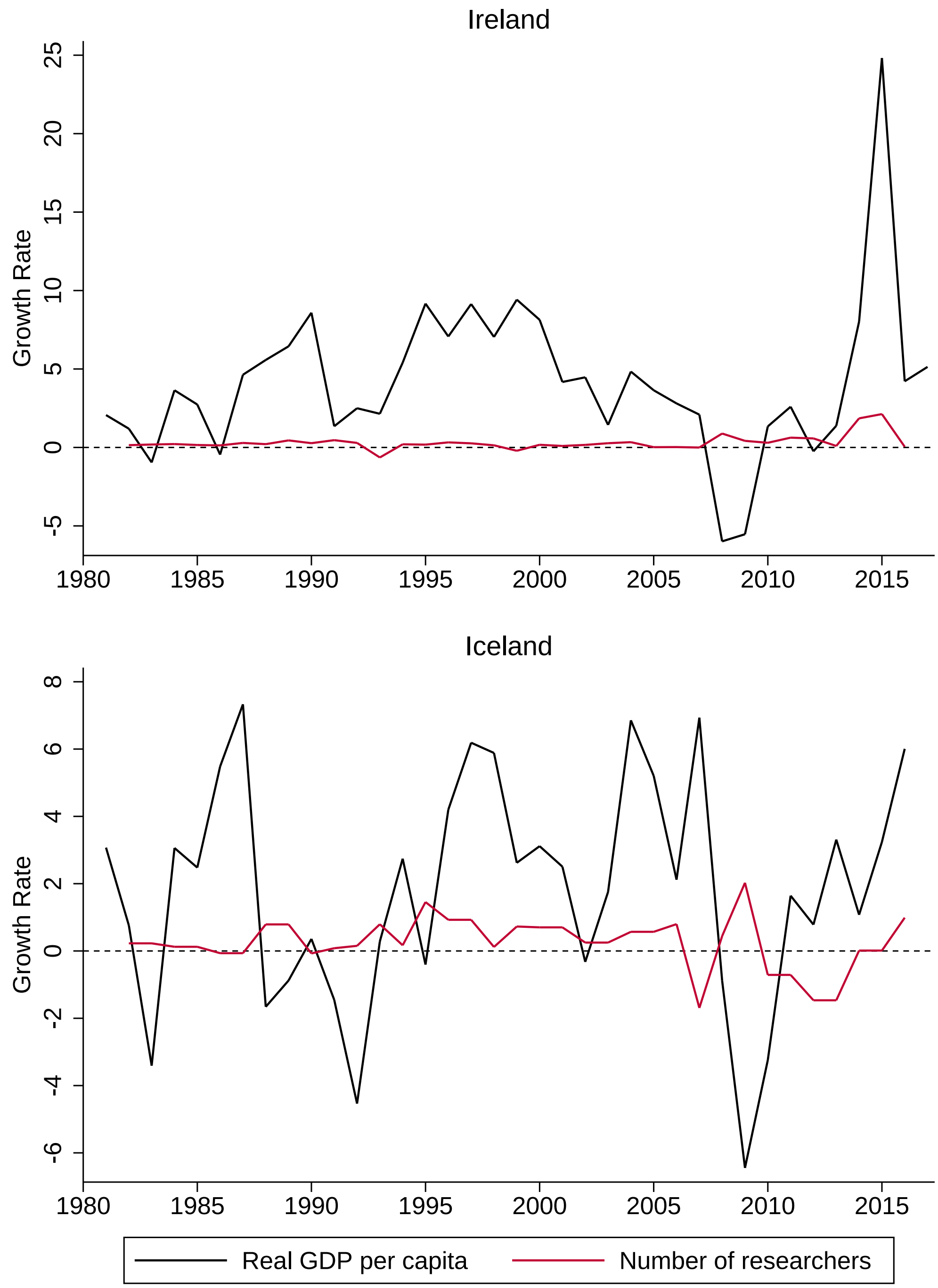

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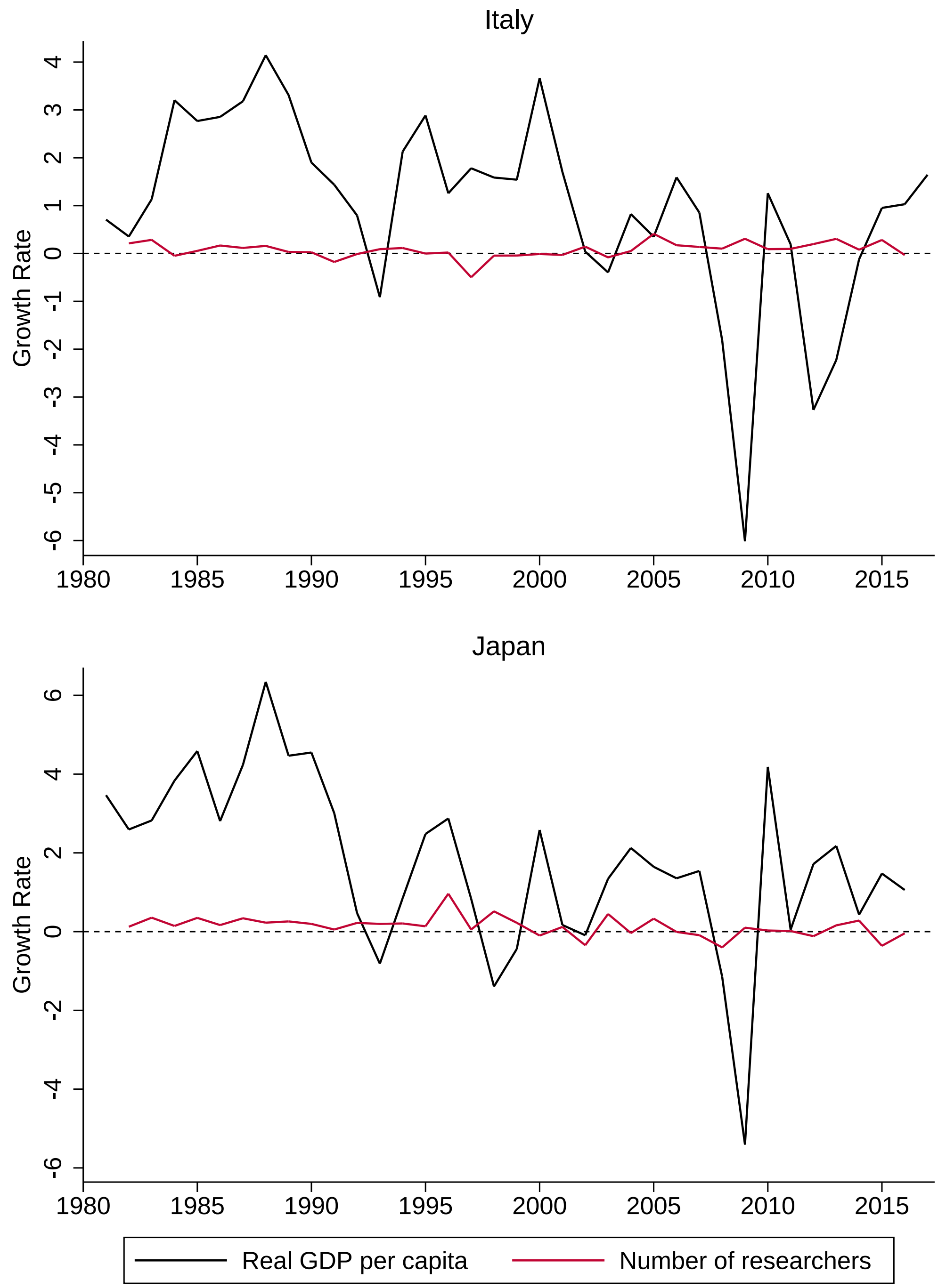

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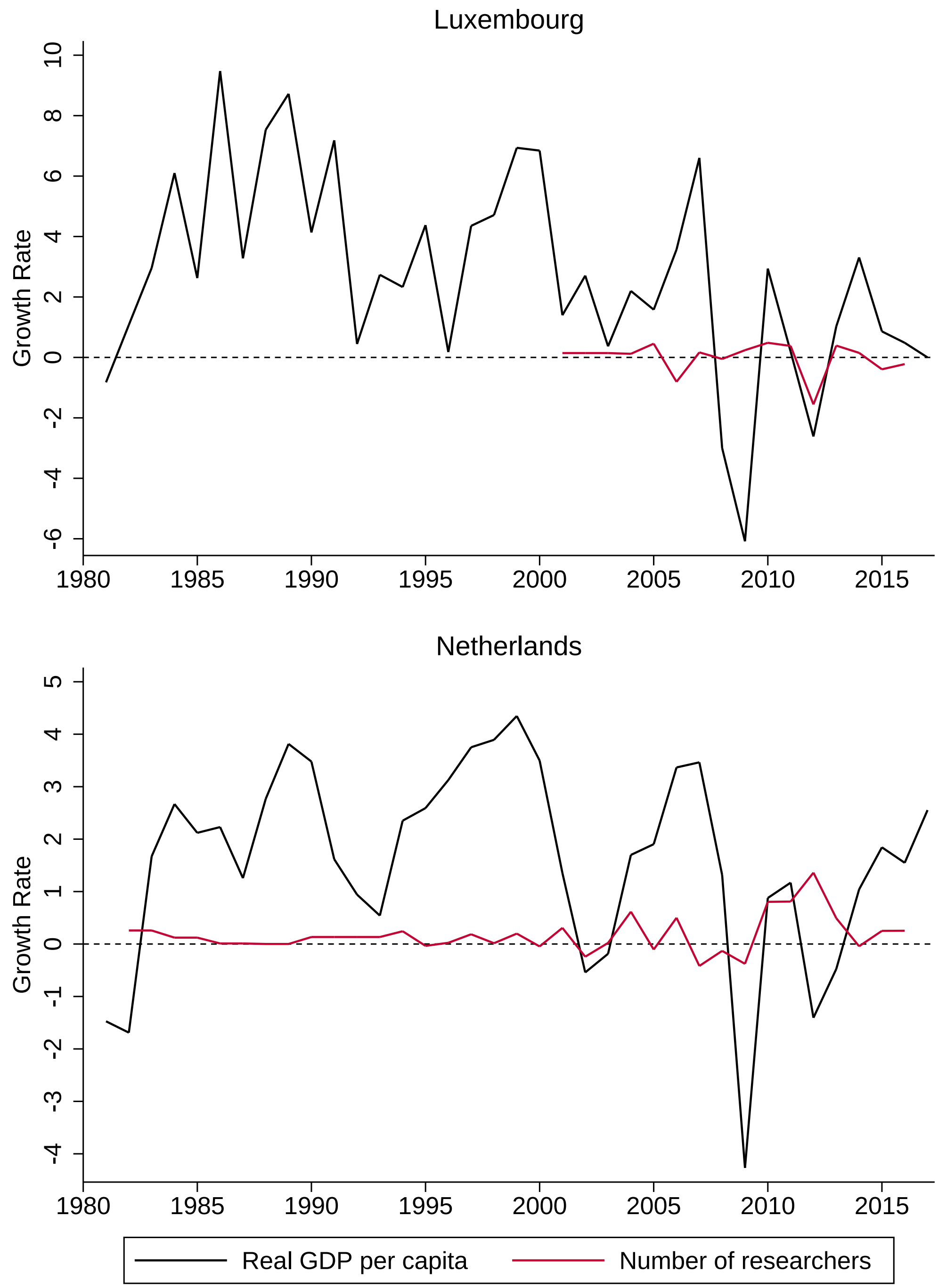

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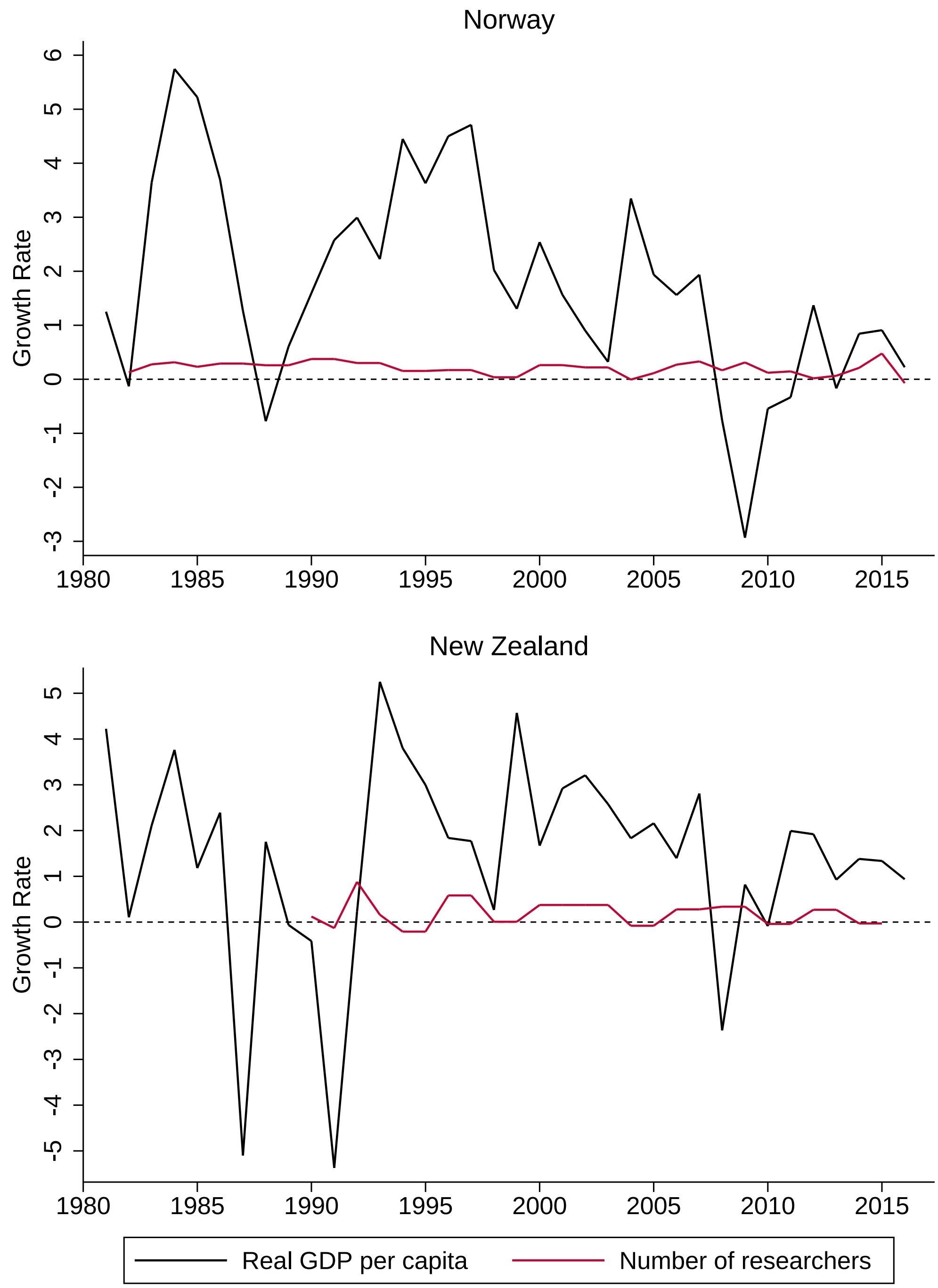

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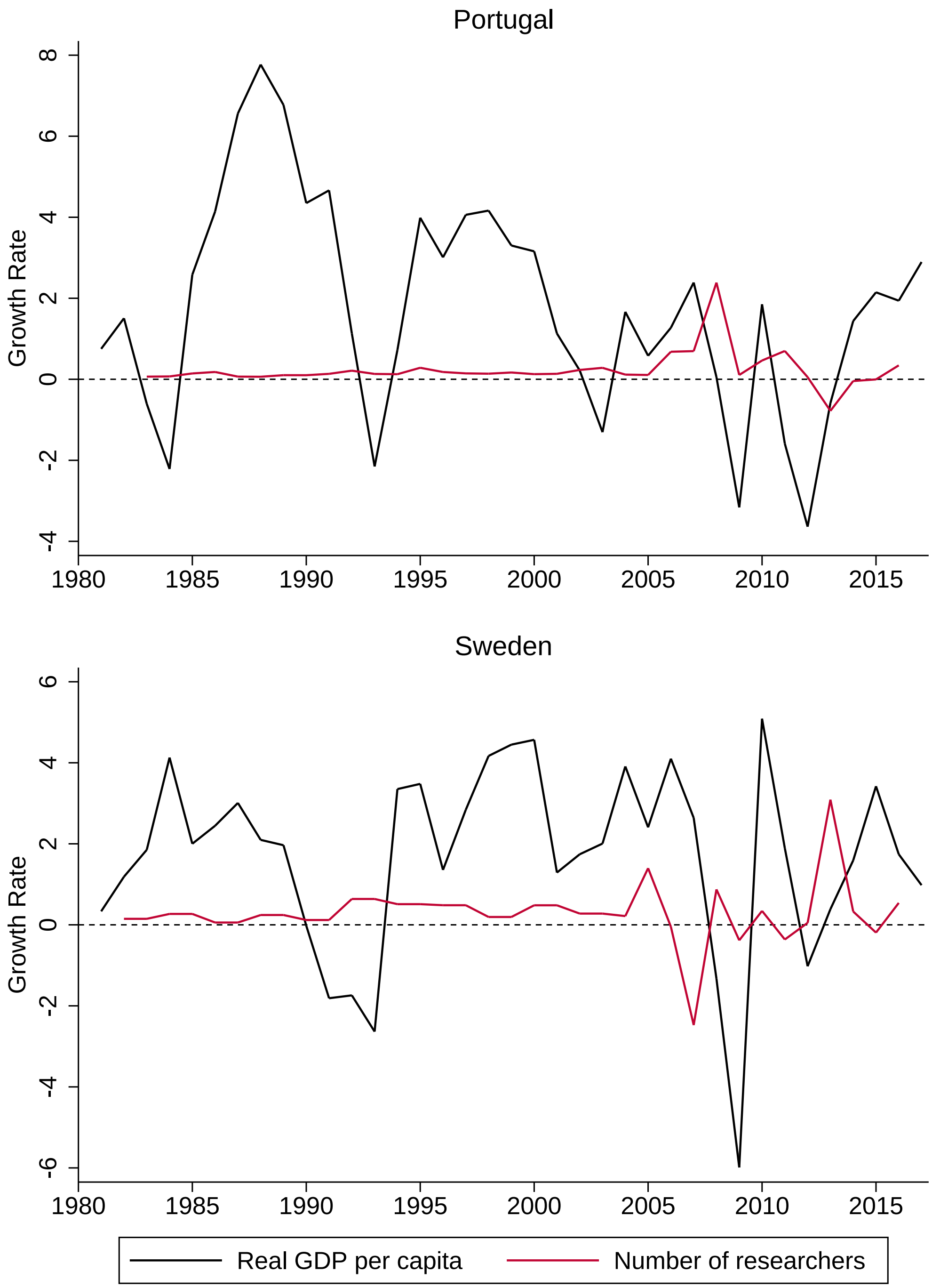

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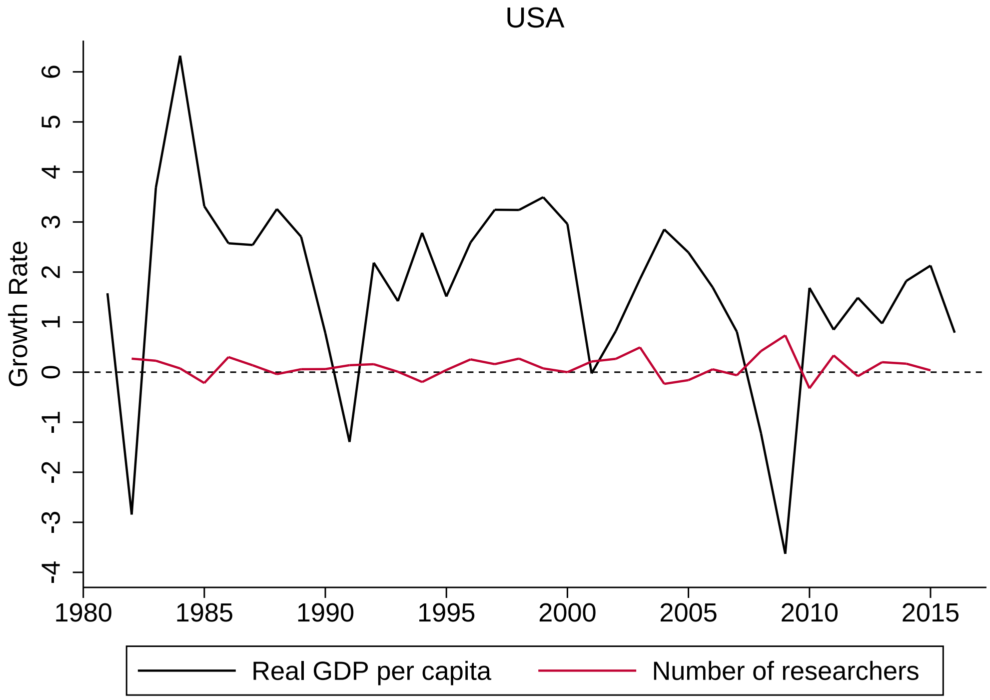

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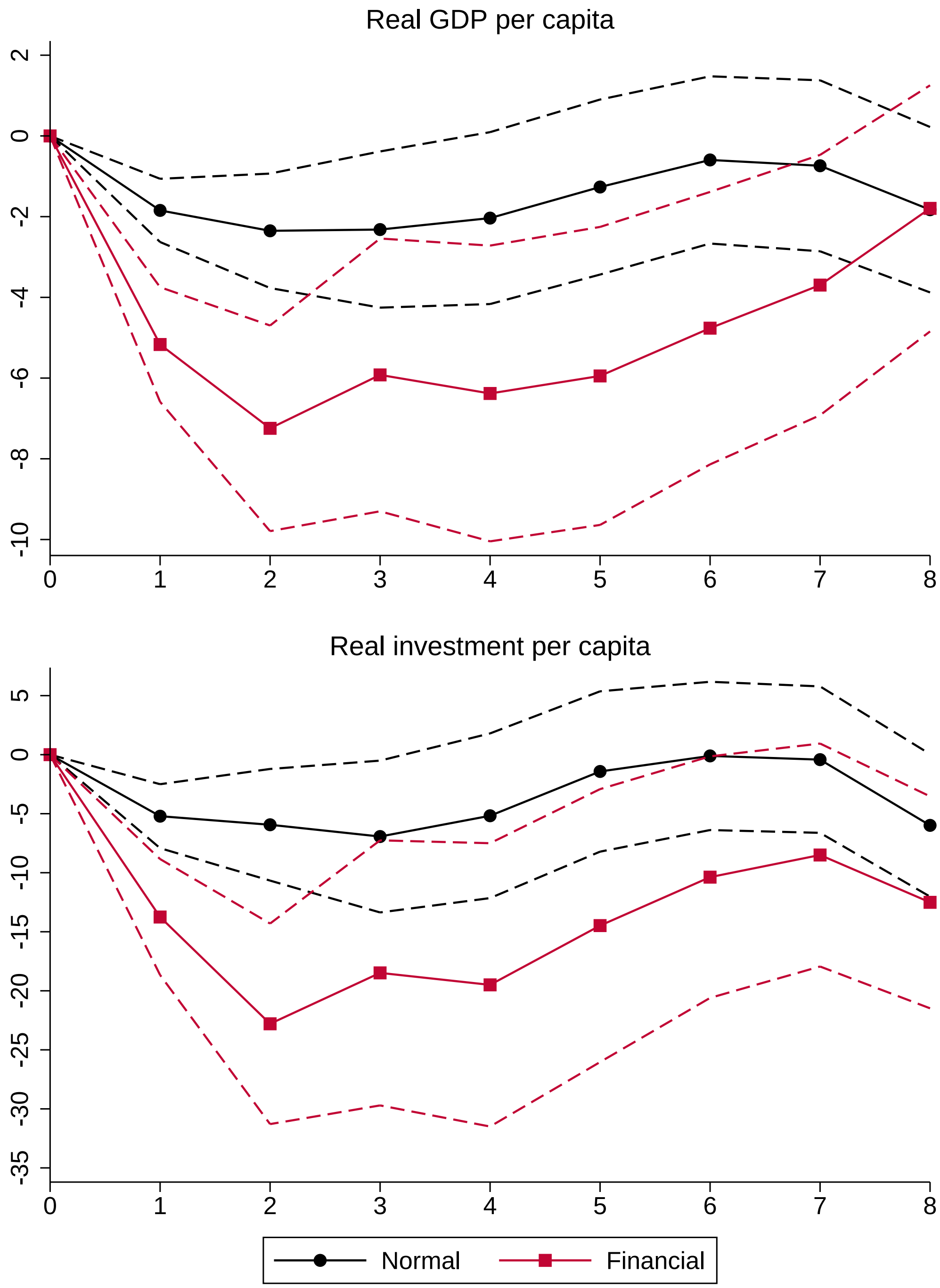
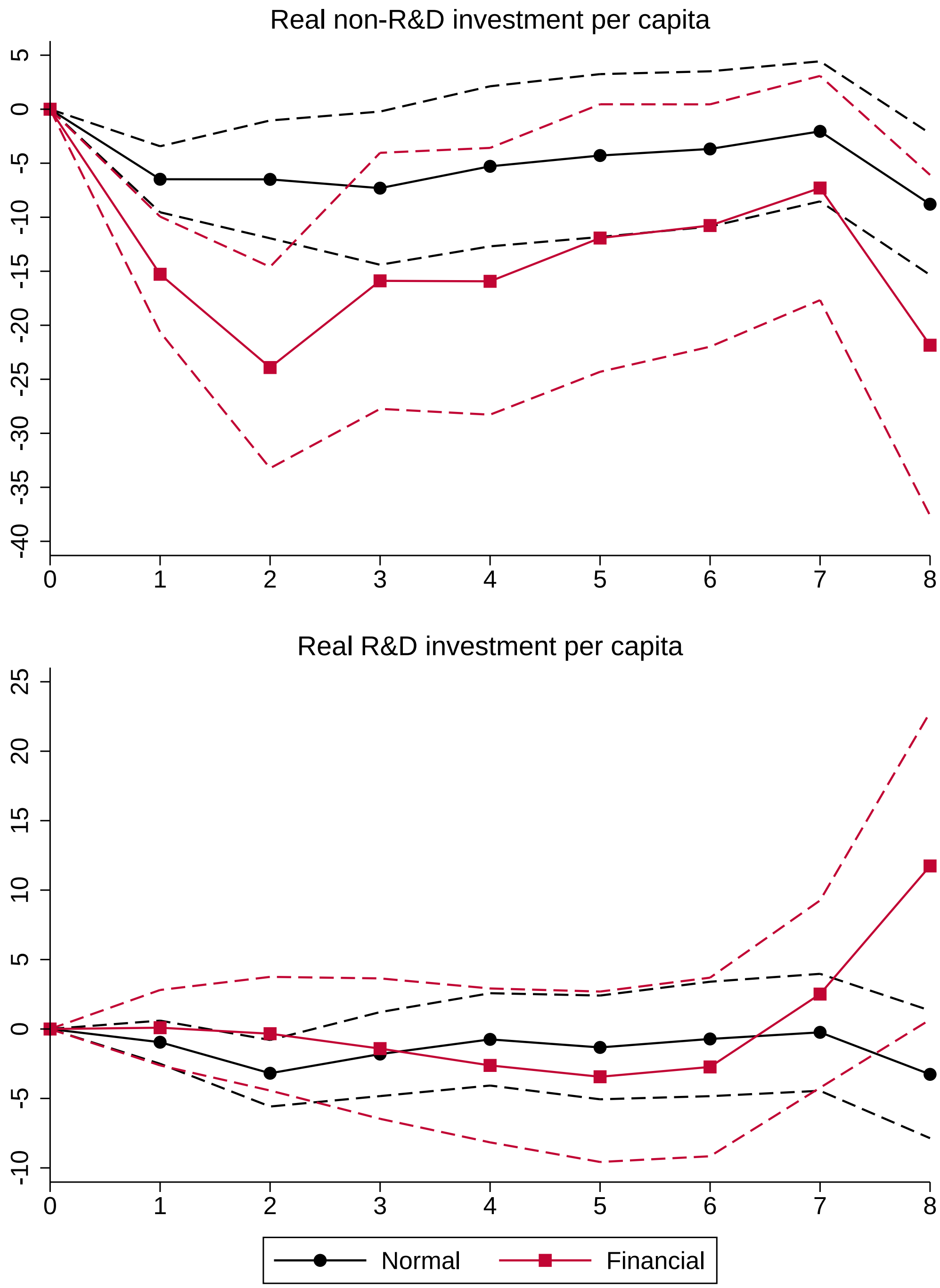


Figure 3.3. Cumulated responses of all countries that correspond to $\{\theta^k_{h,N}\}^8_{h=1}$ and $\{\theta^k_{h,F}\}^8_{h=1}$ in $\Delta_h y^k_{it(r)+h} = \alpha^k_i + \theta^k_{h,N} N + \theta^k_{h,F} F + \sum_{j=0}^{2} \Gamma^k_j Y_{it(r)-j} + u^k_{it(r)}$, estimated by local projections. The vertical axis shows the percentage point changes from the reference peak year level. Circled black markers show the path upon normal recessions, and squared red markers show the path upon financial recessions. For each impulse response, 1.96 standard deviation confidence intervals are presented.



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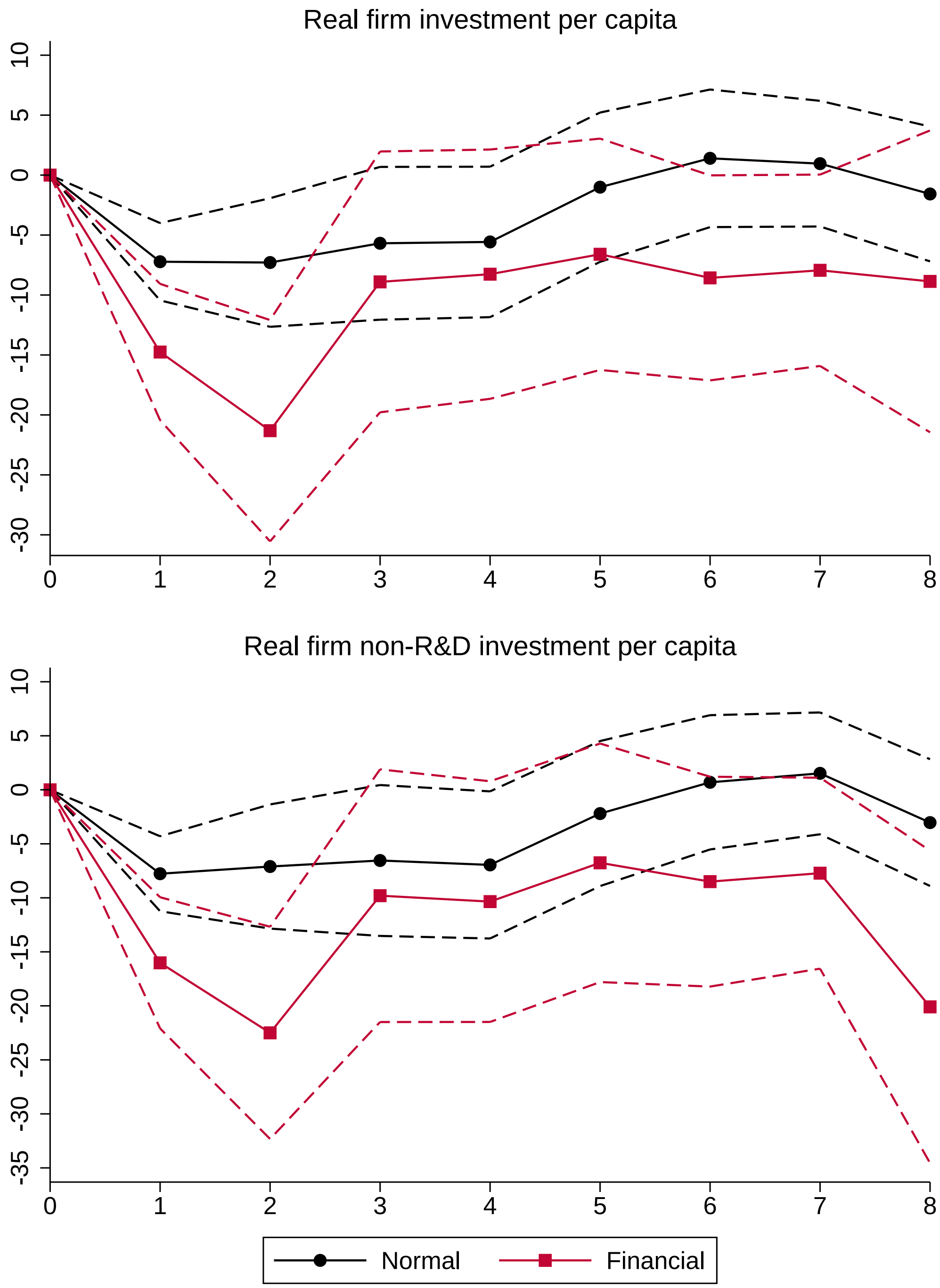

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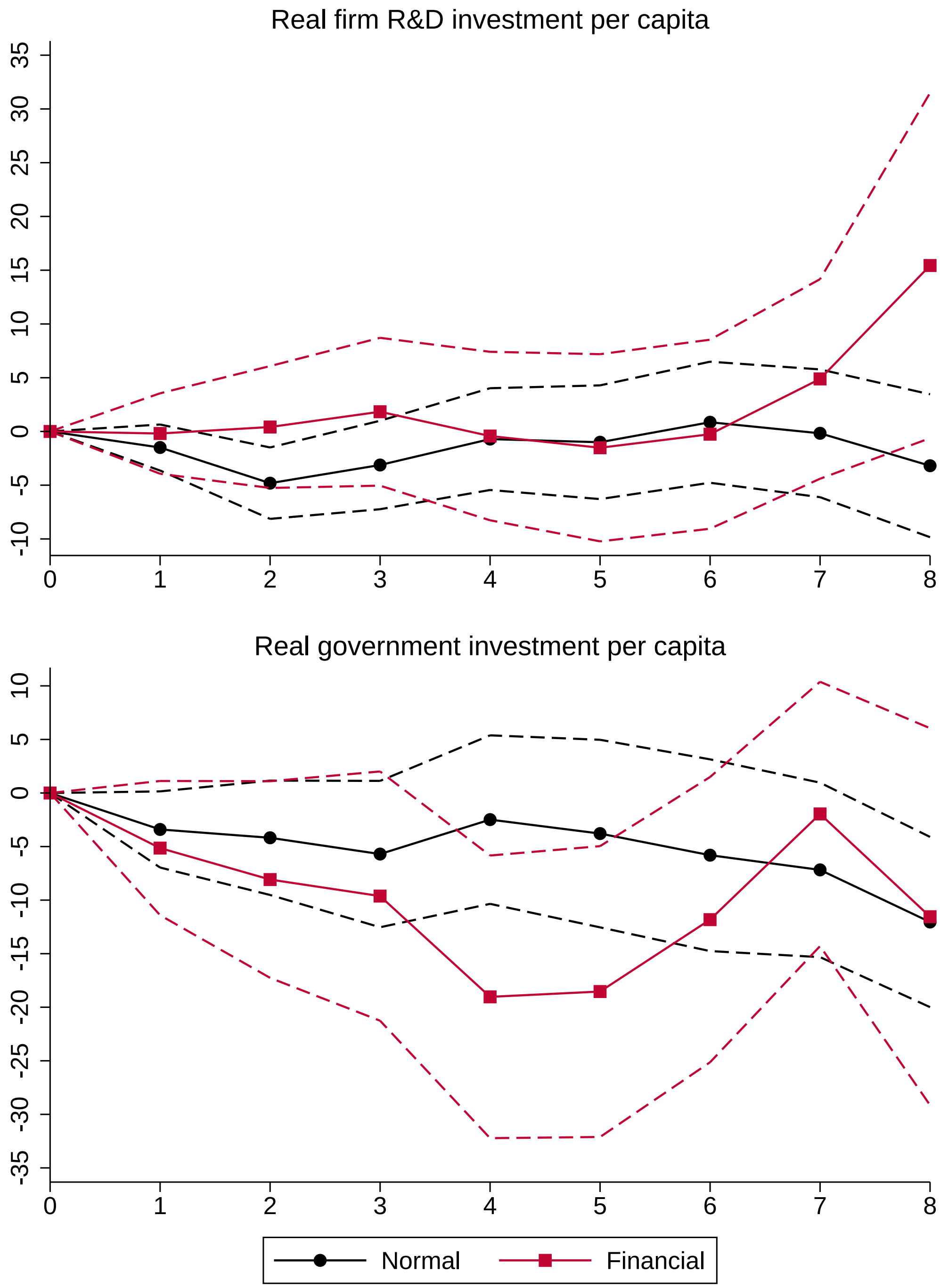

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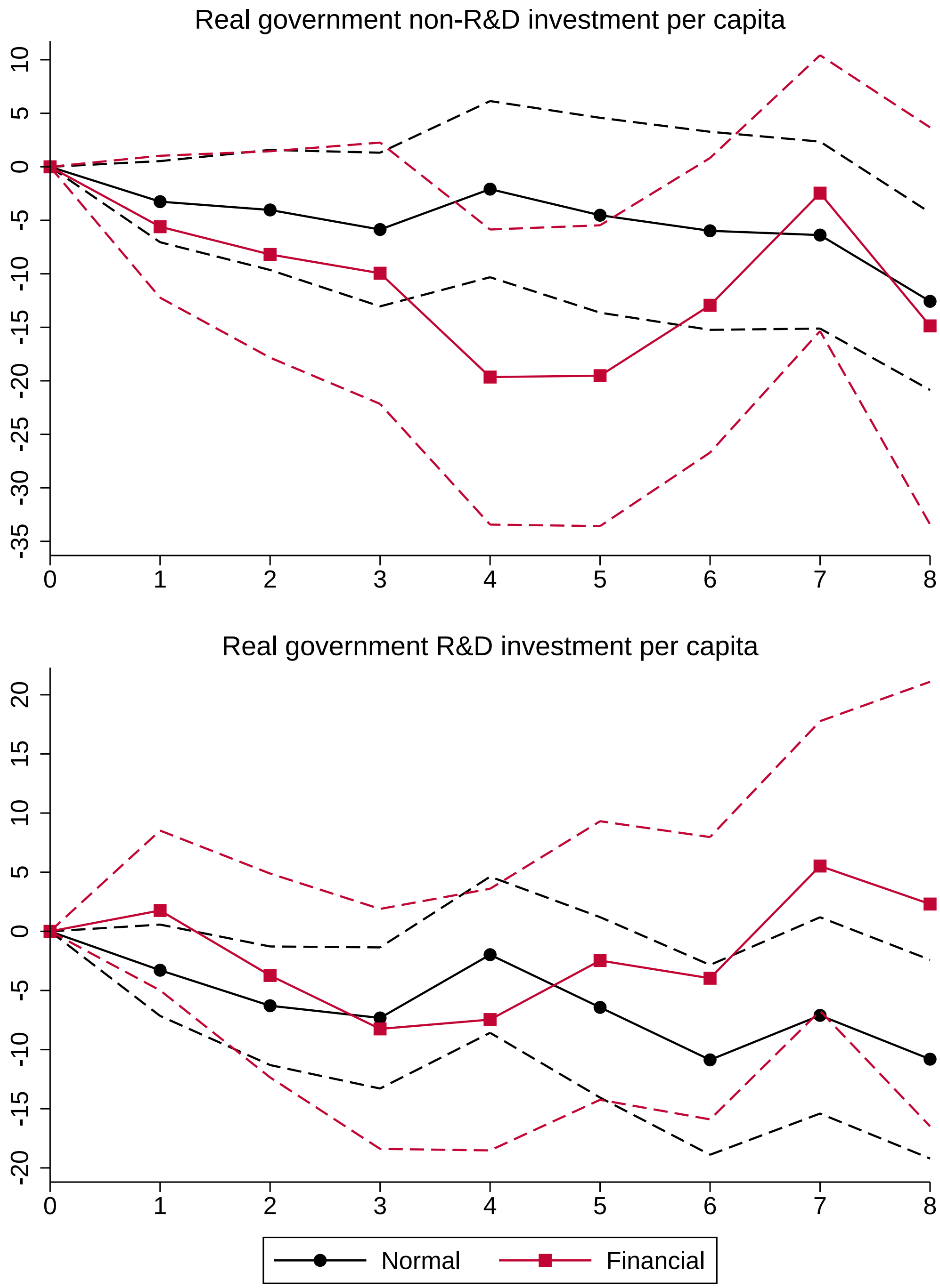

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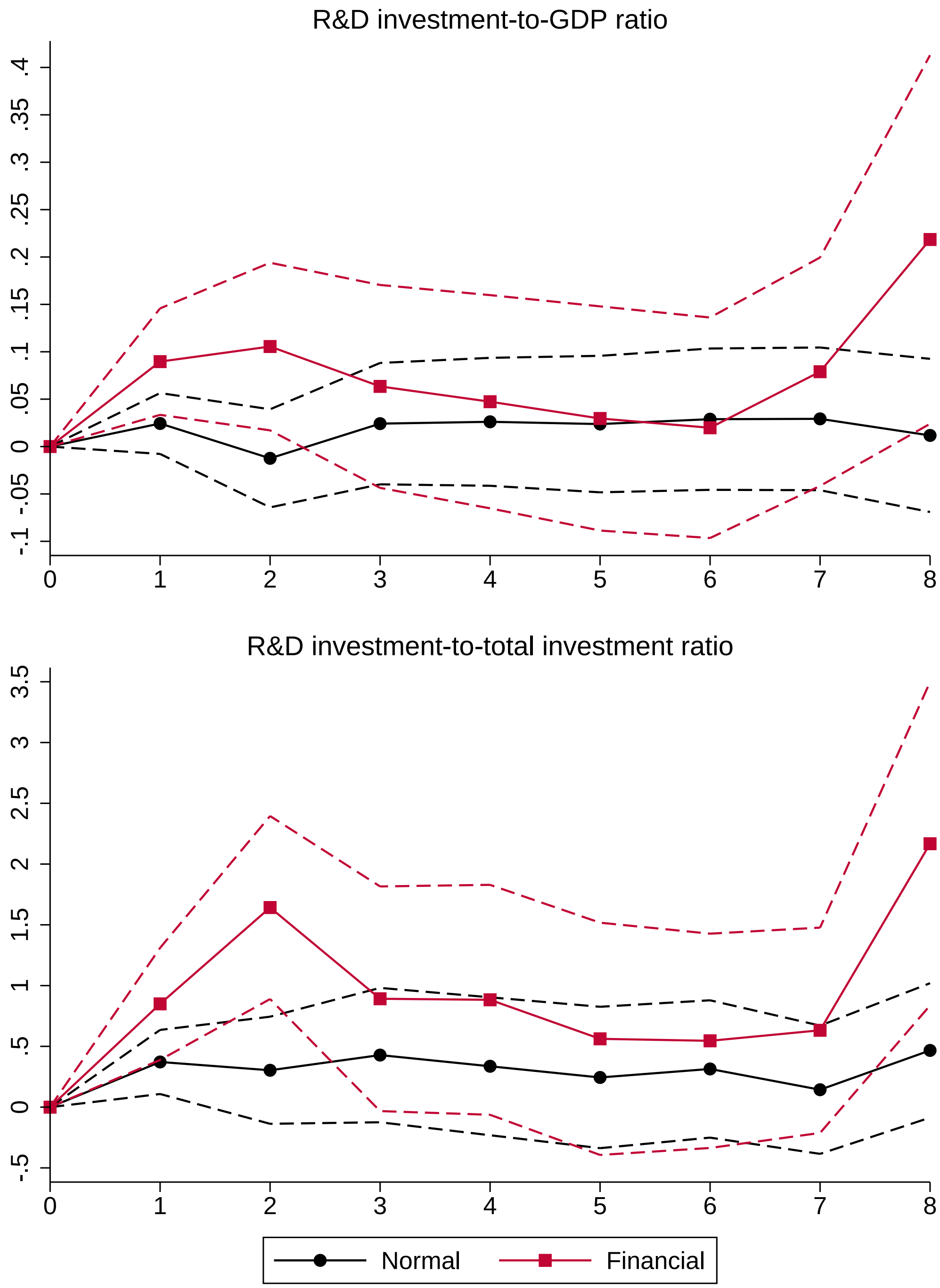

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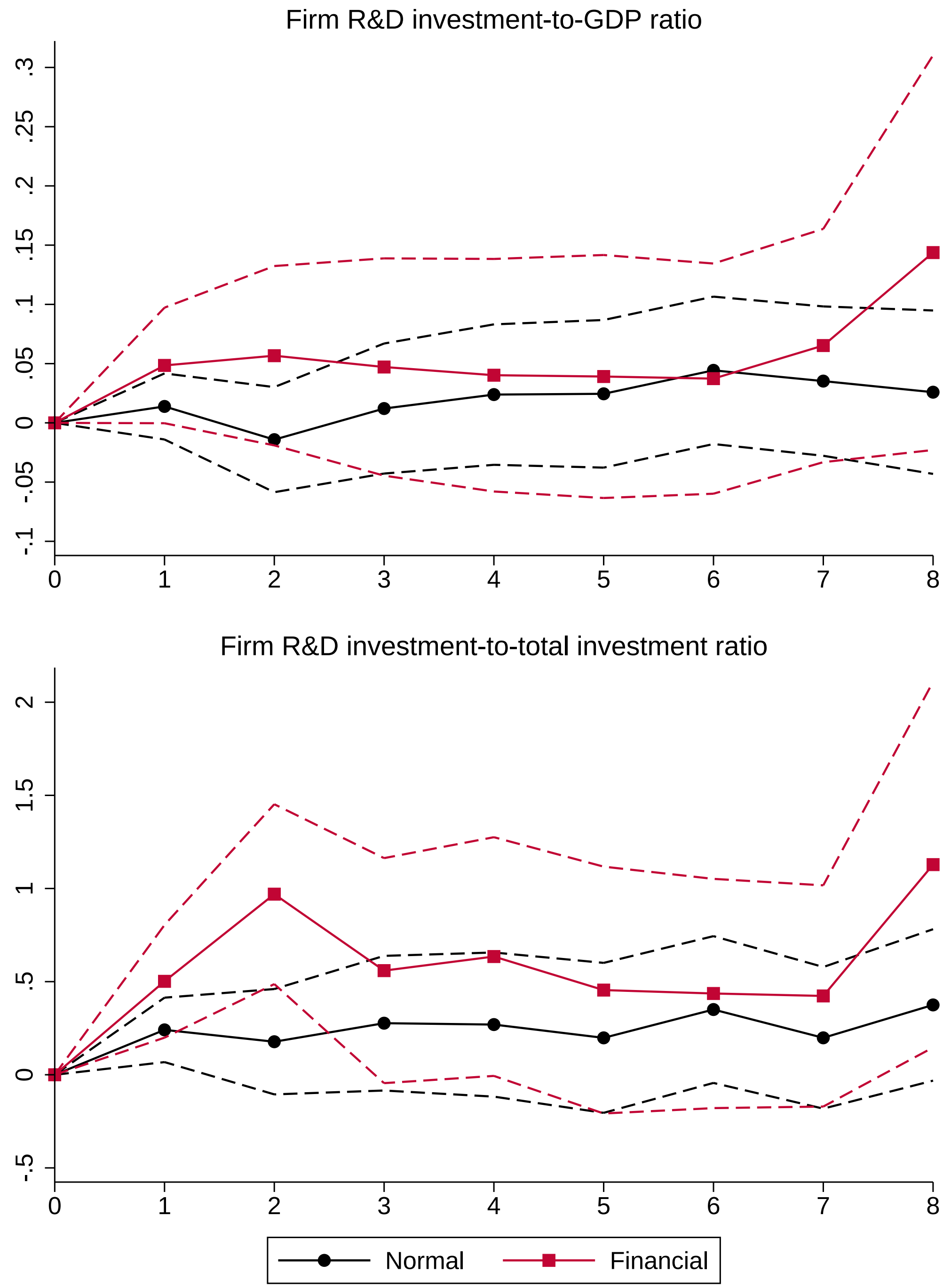

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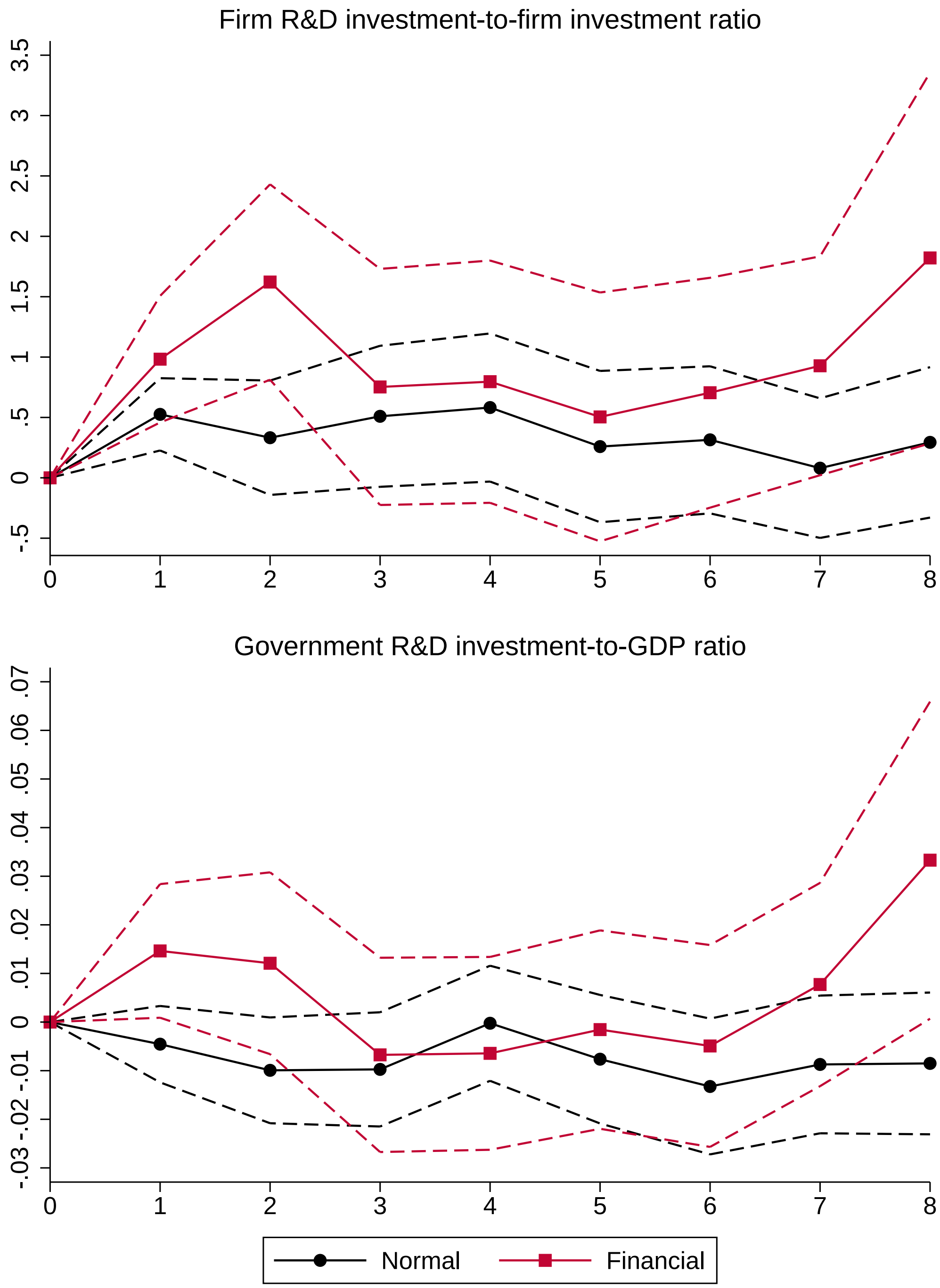
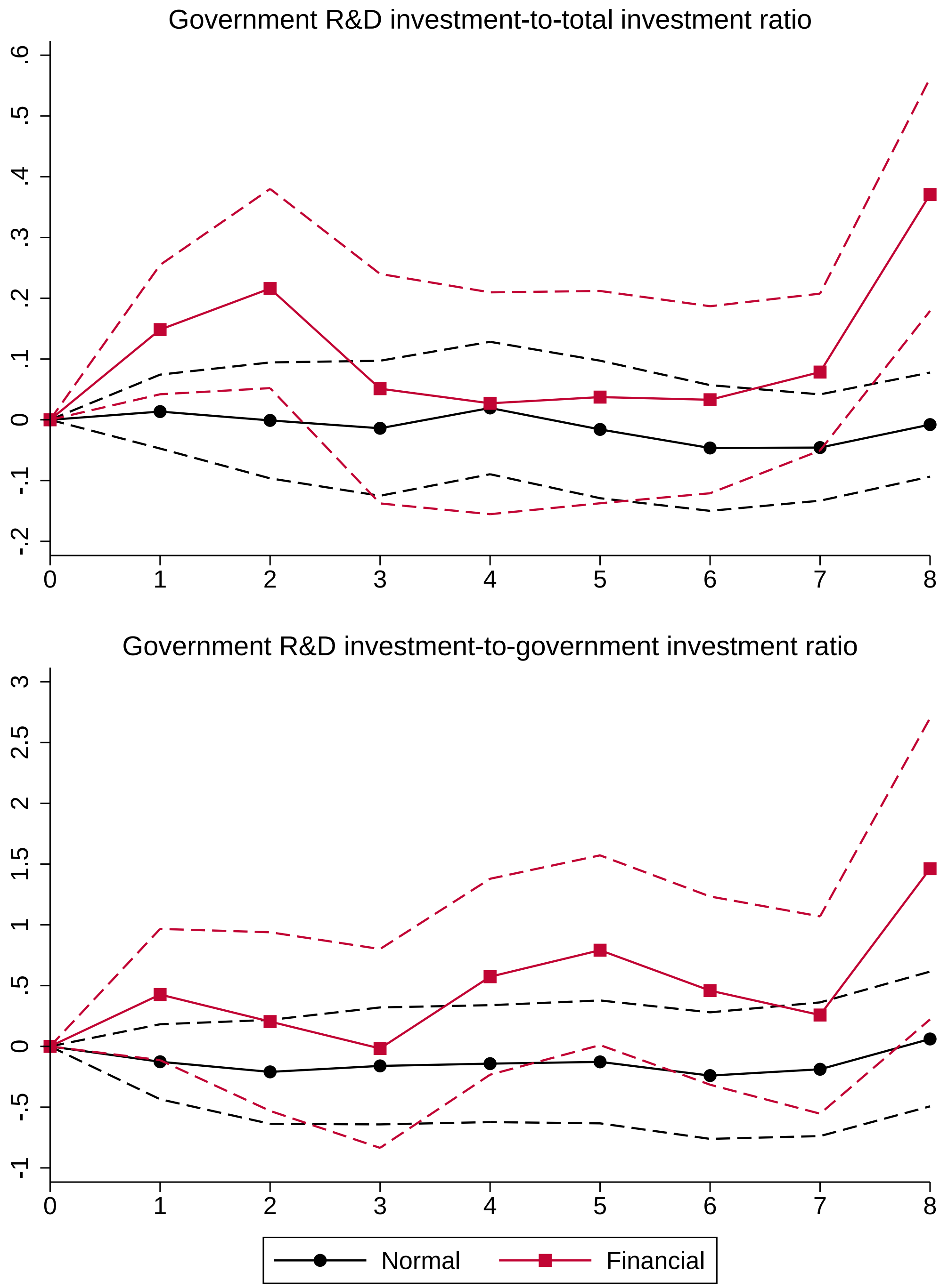


Figure 3.3 (Continued). Cumulated responses of all countries that correspond to $\{\theta^k_{h,N}\}^8_{h=1}$ and $\{\theta^k_{h,F}\}^8_{h=1}$ in $\Delta_h y^k_{it(r)+h} = \alpha^k_i + \theta^k_{h,N} N + \theta^k_{h,F} F + \sum^2_{j=0} \Gamma^k_j Y_{it(r)-j} + u^k_{it(r)}$, estimated by local projections. The vertical axis shows the percentage point changes from the reference peak year level. Circled black markers show the path upon normal recessions, and squared red markers show the path upon financial recessions. For each impulse response, 1.96 standard deviation confidence intervals are presented.



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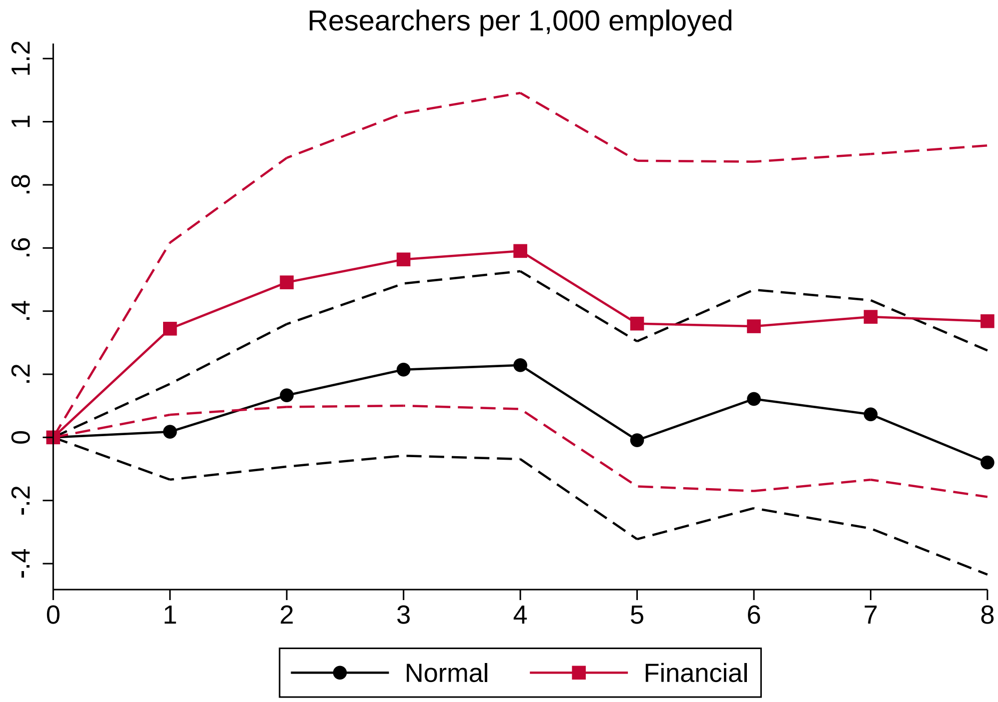

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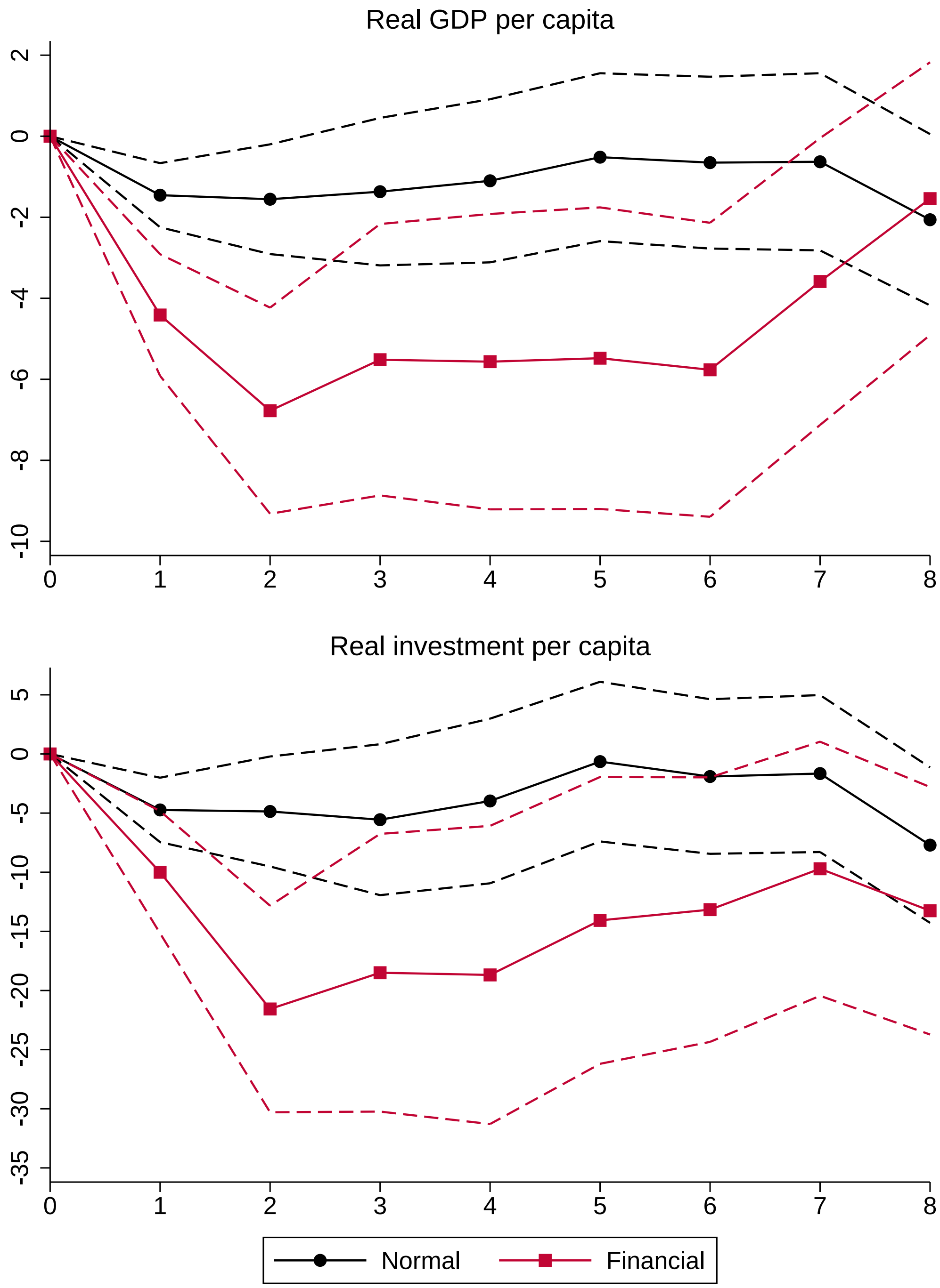

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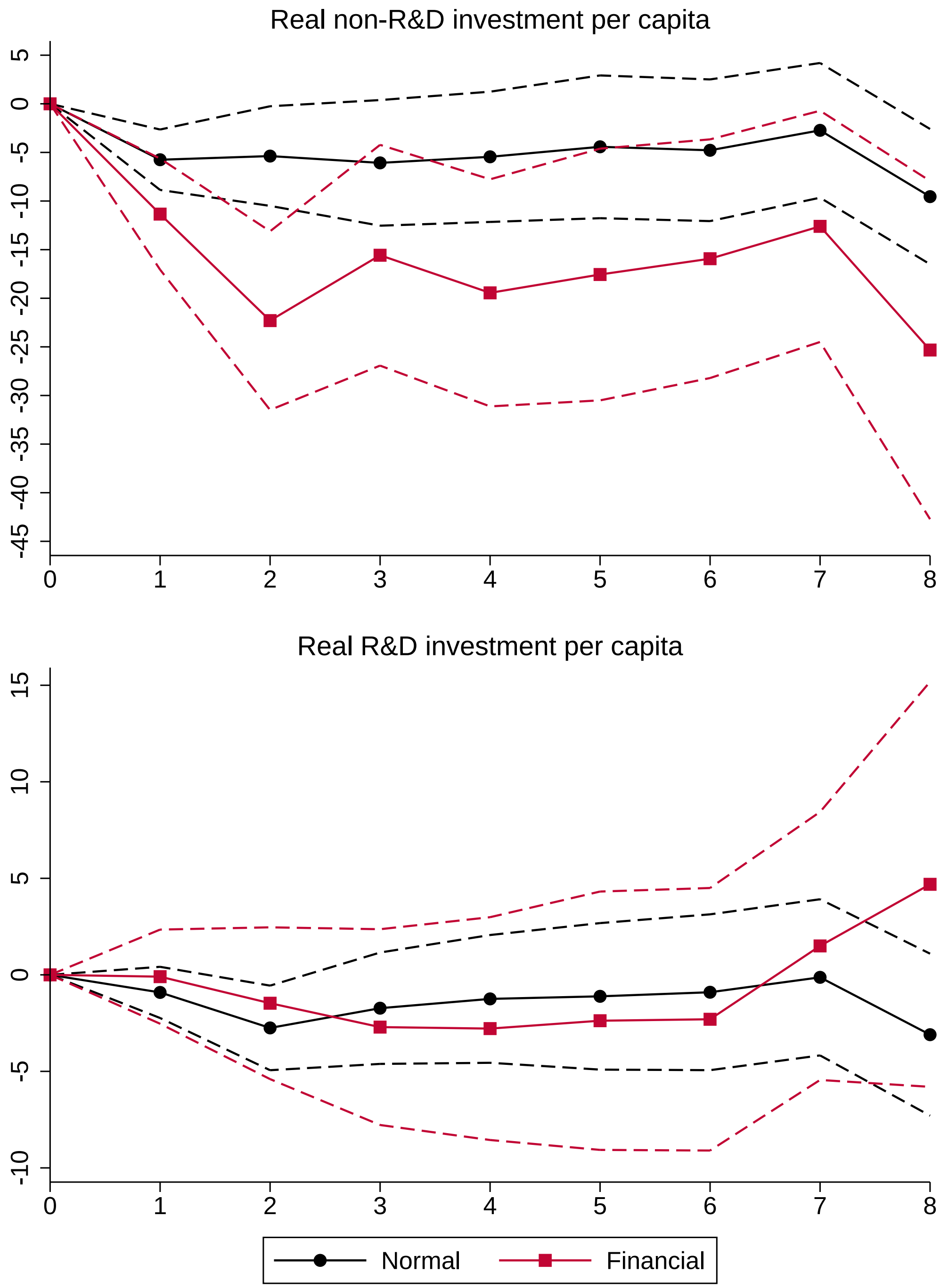

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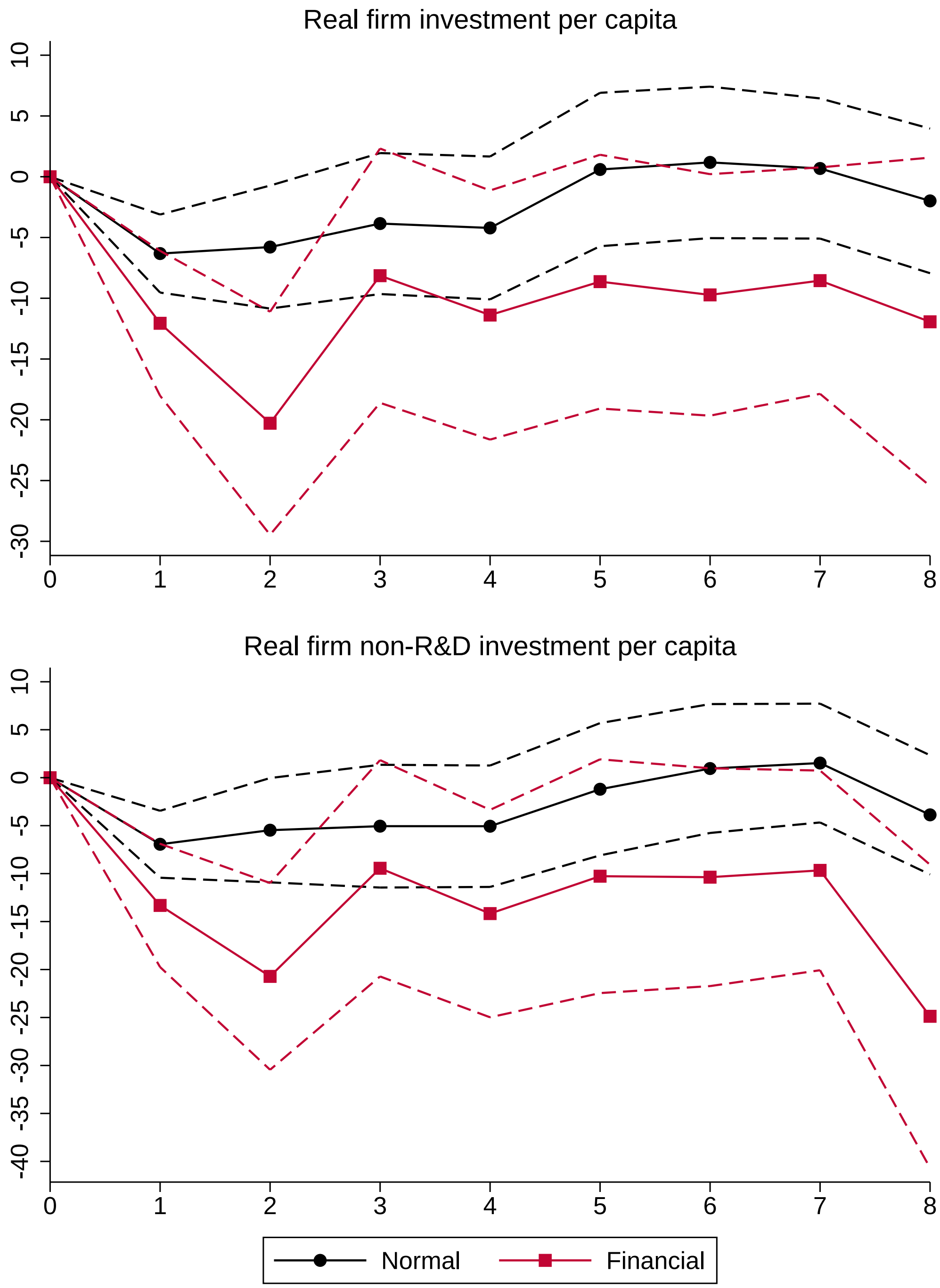

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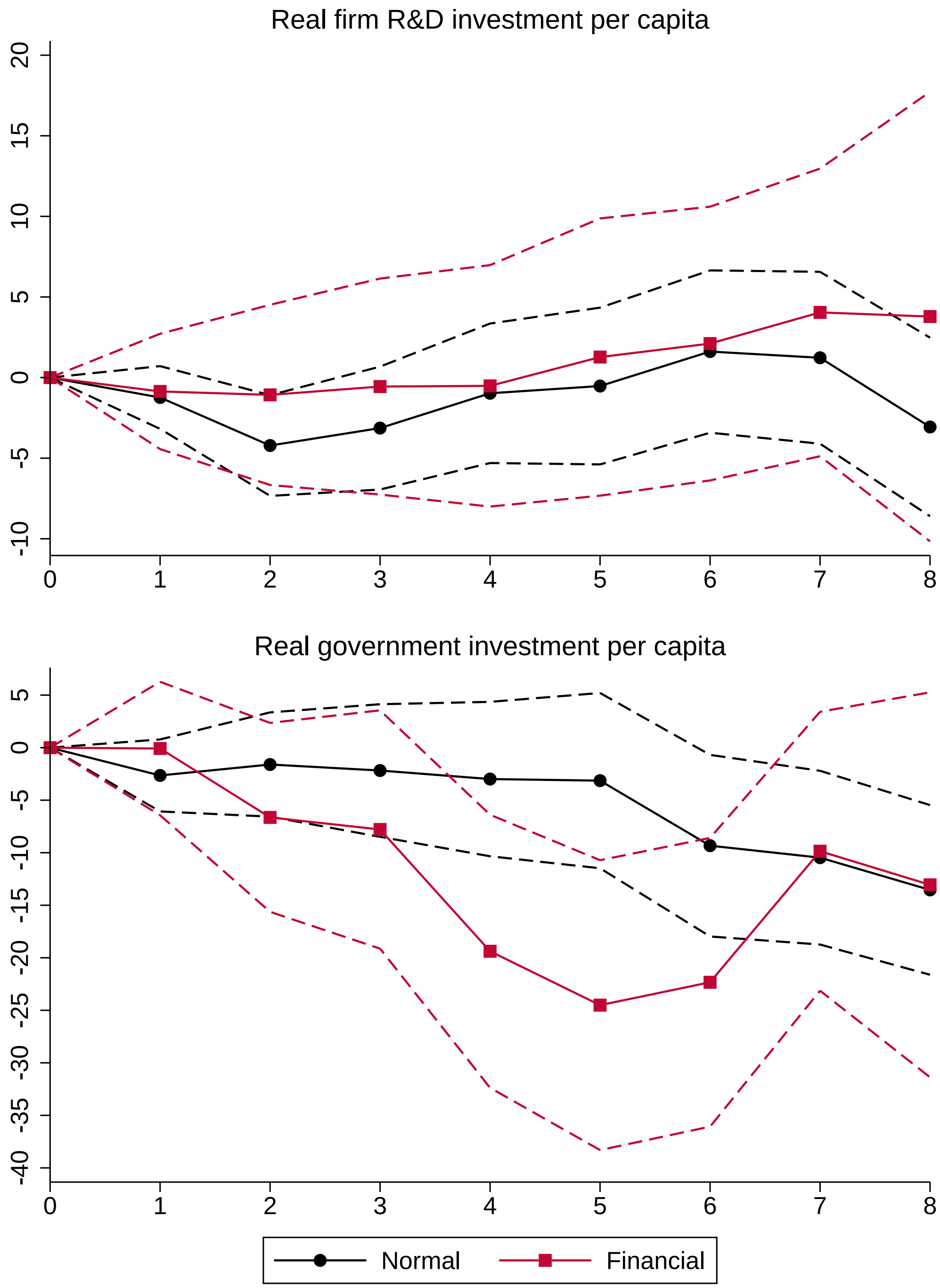

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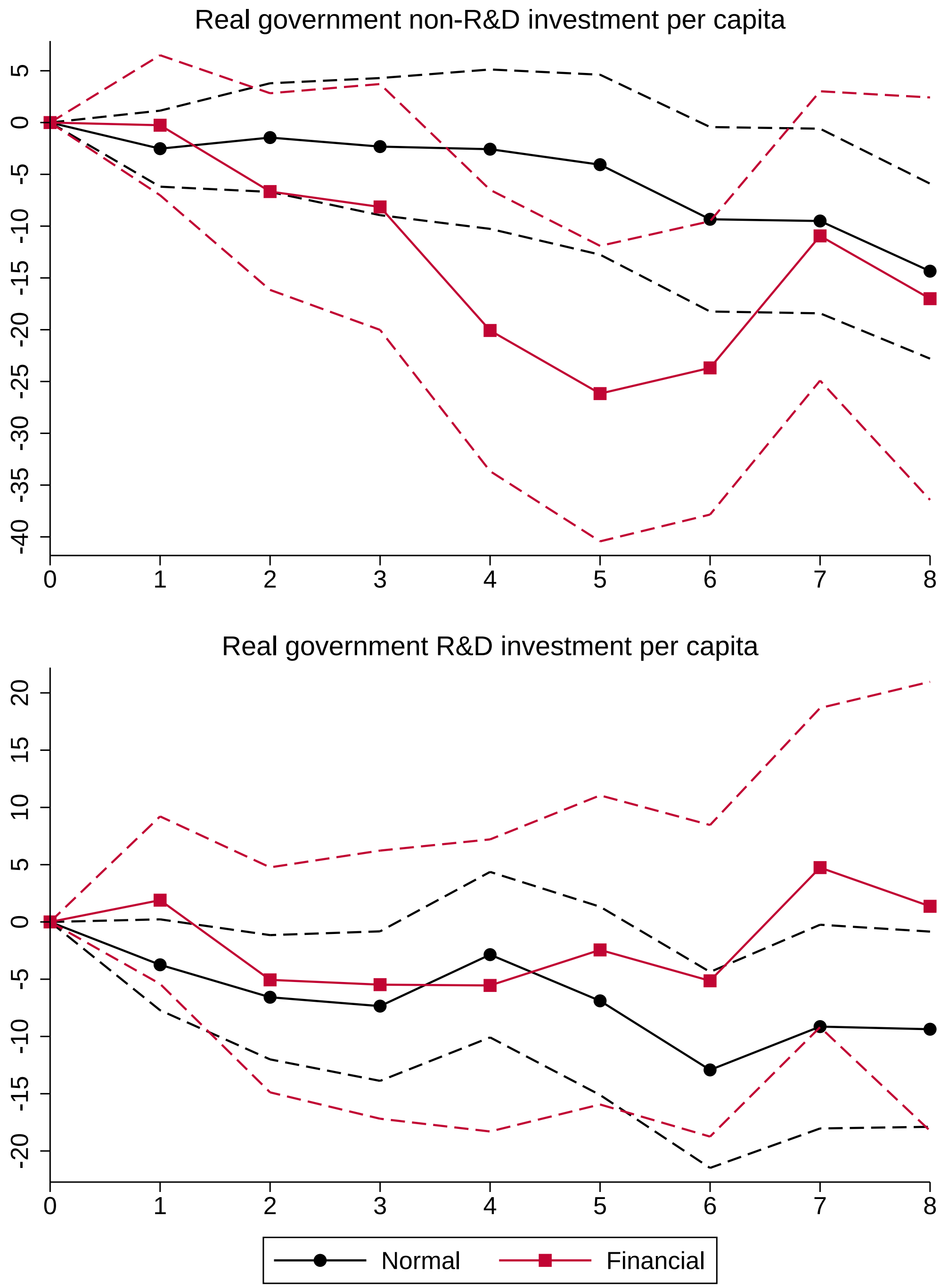

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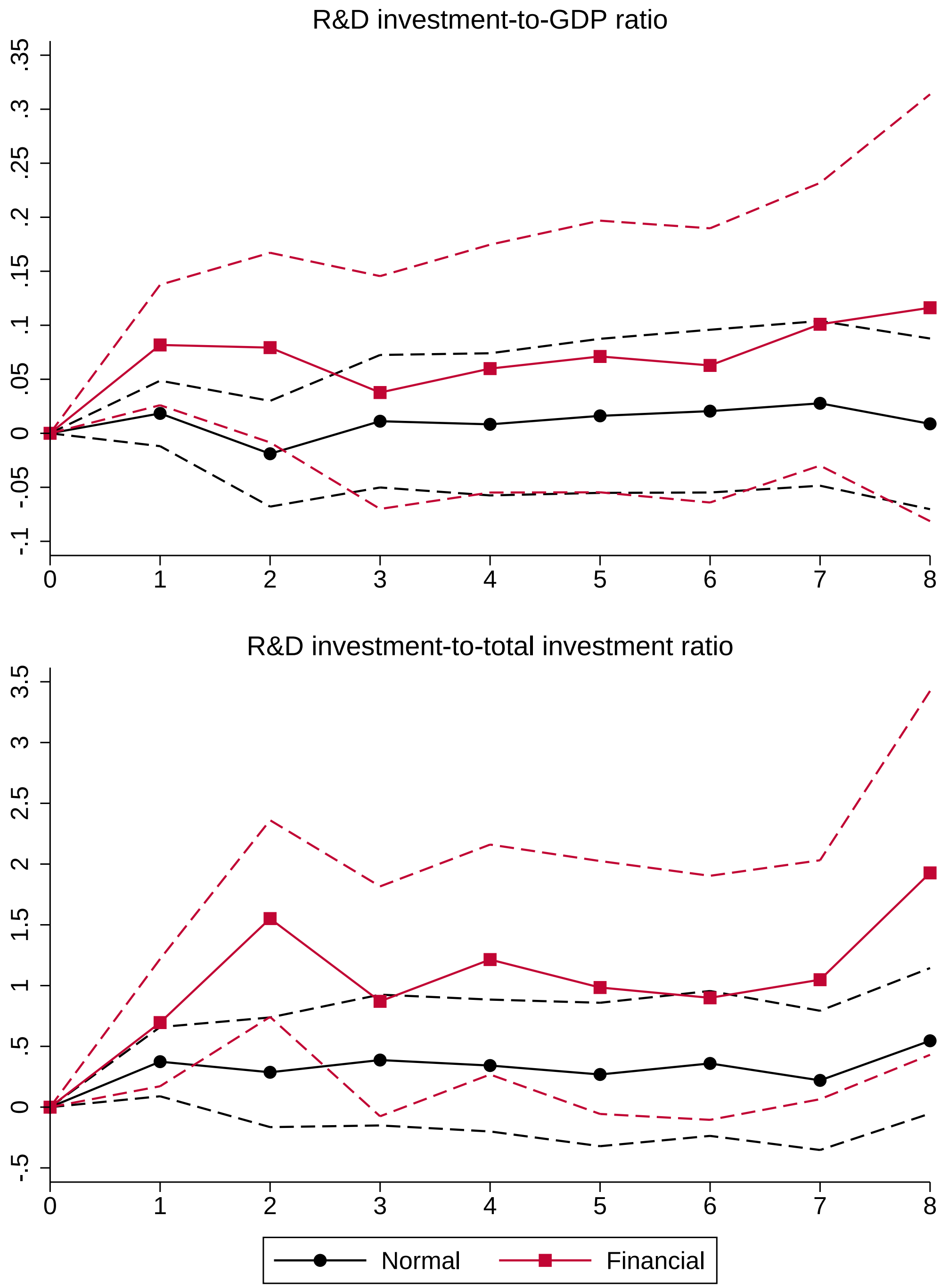

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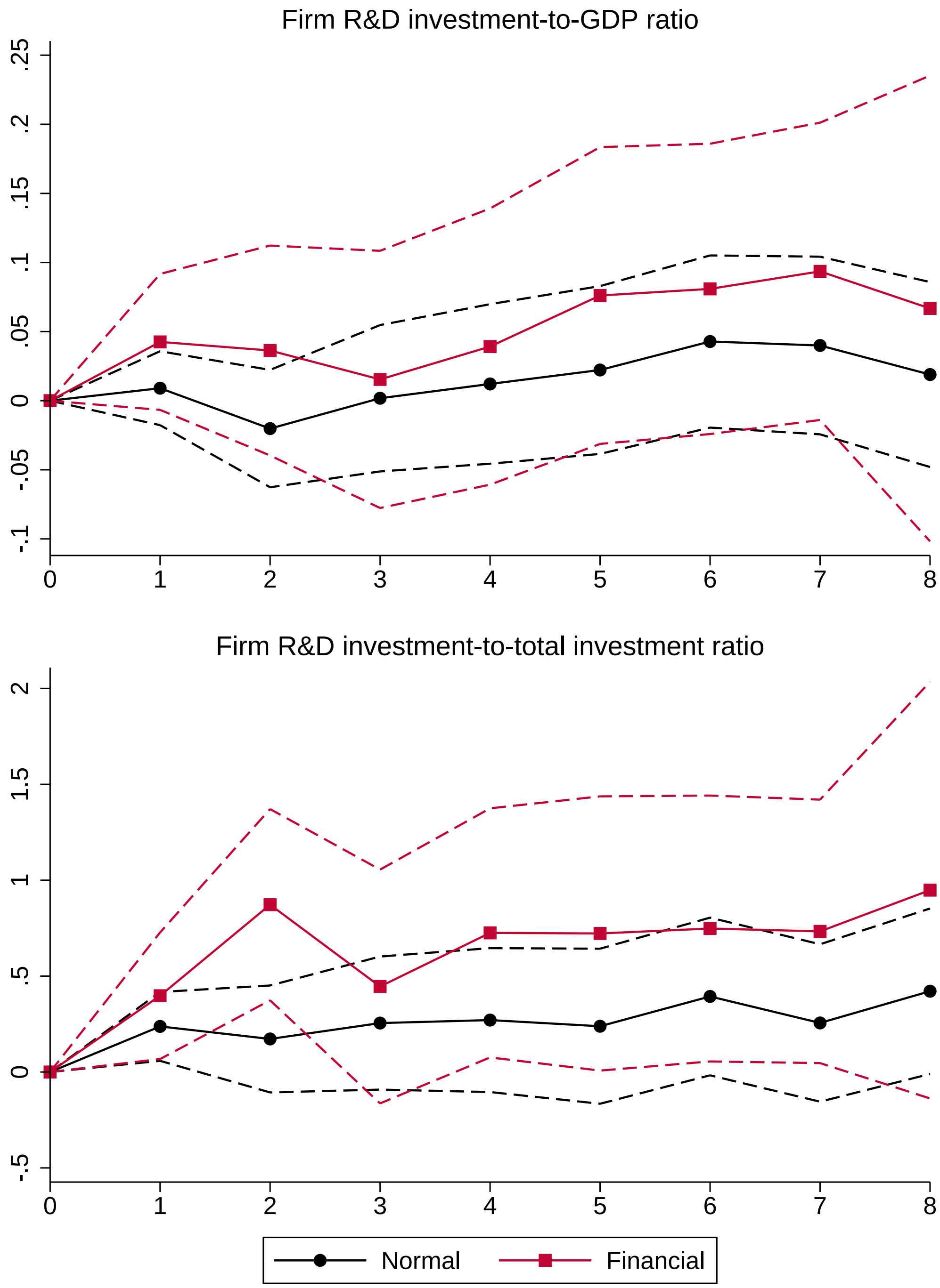

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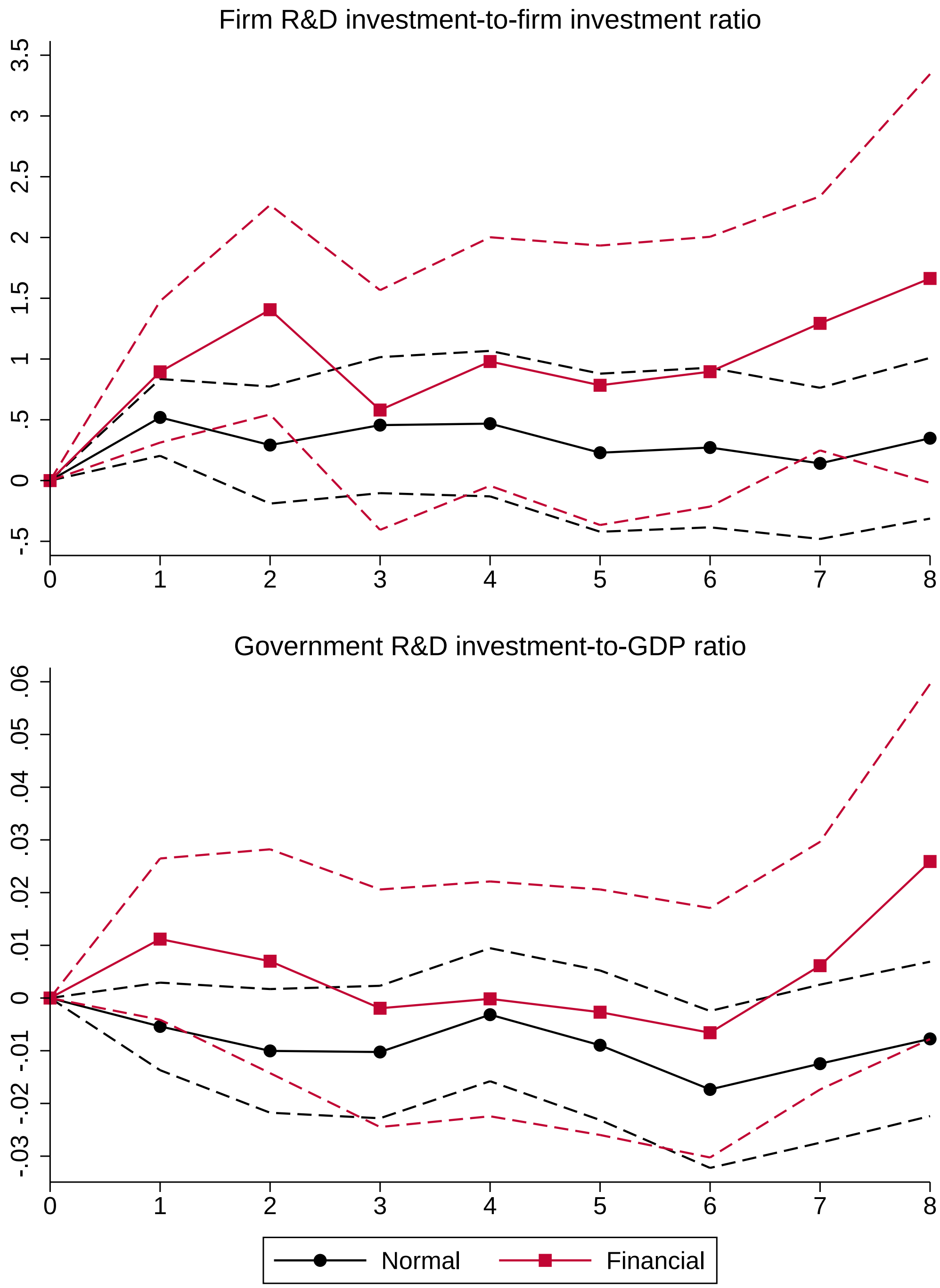

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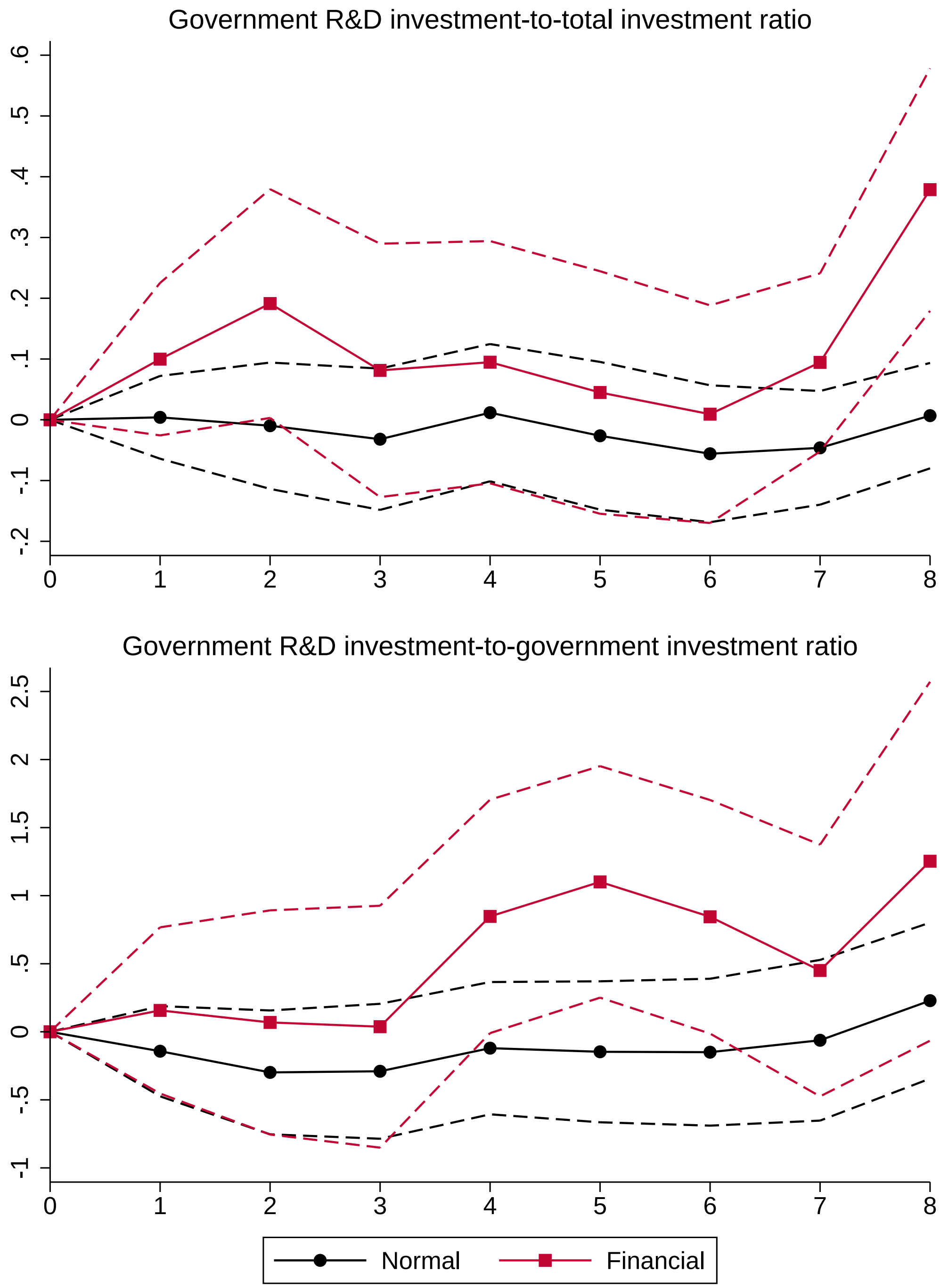

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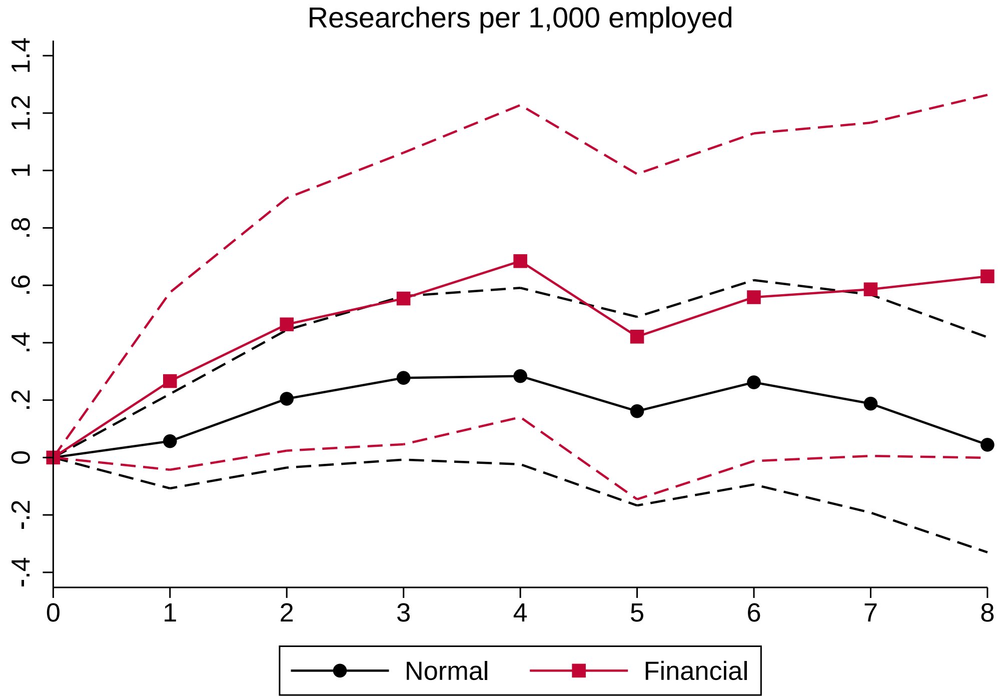

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