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Author
Chattopadhyay, S.

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MASTER

S. Chattopadhyay, A. Faltens, and L. Smith

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Accelerator and Fusion Research Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

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STUDY OF THE BEAM BREAKUP MODE IN LINEAR INDUCTION ACCELERATORS FOR HEAVY IONS*

S. Chattopadhyay, A. Faltens, and L. Smith
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

Abstract

A simple theoretical study and numerical estimate is presented for the transverse amplitude growth of a nonrelativistic heavy ion beam in an induction linac, as envisaged for use in commercial power plants, due to the nonregenerative coherent beam breakup mode. An equivalent electrical circuit has been used to represent the accelerating induction modules. Our calculation shows that for the parameters of interest, the beam breakup amplitude for a heavy ion beam grows extremely slowly in the time scales of interest, to magnitudes insignificant for transport purposes. It is concluded that the coherent beam breakup mode does not pose any serious threat to the stability of a high current (kA) heavy ion beam in an induction linac.

I. Introduction

High current heavy ion beams are being actively studied as potential drivers for inertial confinement fusion. Such high current nonneutral beams are subject to coherent and incoherent, transverse and longitudinal, collective instabilities arising from the beam space charge (self-force) and its interaction with the environment (external impedances, cavities etc.). In this paper, we study the growing coherent transverse motion of a high current (~kA) heavy ion beam due to an oscillatory transverse mode (analogous to TM_{110} mode of a pill-box cavity) excited by the beam in the accelerating modules. The subject has been studied extensively in connection with electron linacs by several authors[1-5], who computed the upper limit of transportable total charge set by the growth of beam breakup amplitude. However no such study has been reported for heavy ion beams transported by induction linacs.

II. Model For Transport

Our theoretical model of transport is a semi-infinite series of identical accelerating induction modules with identical focussing elements between them (see Fig. 1). If the beam centroid is off center (or if the beam is centered in an azimuthally asymmetric structure), it will excite a transversely deflecting mode in the modules. The induced electromagnetic fields act on later parts of the beam, causing a transverse motion of the beam centroid. The amplitude of the coherent beam oscillation increases from head to tail in a bunch, the cavity excitation increases in time at any location and increases in distance along the accelerator.

Our analysis is based on the following assumptions:

(a) Only one effective resonant mode of frequency Ω and quality factor Q is of significance.

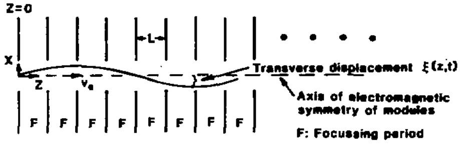

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Fig. 1

(b) Focussing can be treated in the smooth approximation i.e. focussing fields of quadrupoles or interrupted solenoids can be replaced by their average values.

(c) There is no acceleration.

(d) The process is 'non-regenerative', i.e. there is no propagation of electromagnetic fields from one induction module to the next and information is carried only by perturbations on the beam.

(e) The rate of amplitude growth is small compared to Ω.

III. Induction Module Response

An induction linac module differs drastically from an r.f. cavity in its response to excitation by a particle beam. There is no accelerating mode as such[6]; the longitudinal interaction of beam and module is best represented by an equivalent circuit involving the external drive, corresponding typically to a frequency of a few megacycles and strongly overdamped by the low drive-impedance. For the asymmetric modes of interest to the beam breakup phenomenon, the module looks like a pill-box with conducting end walls and a lossy outer wall traversed longitudinally by one or more conducting straps. Accordingly, we take as a model the excitation of the TM_{110} mode of a pill-box cavity with a radius of about half a meter and a Q of about 10.

The vector potential can be written as:

$$A_z = A(t)\, J_1\left(\frac{\Omega}{c} r\right) \cos\theta$$

and A satisfies the differential equation:

$$\ddot{A} + \omega\dot{A} + \Omega^2 A = \frac{\mu_0 \Omega^3 I}{\pi c j_1^2 J_0(j_1) J_2(j_1)}\, \xi(\tau) \qquad (1)$$

where I is the beam current, $\xi(\tau)$ is the transverse beam displacement at time τ following beam arrival at the module and the other symbols have their conventional meanings. In traversing the cavity, the beam experiences a change in slope (see Fig. 2) given by:

$$\Delta\xi' = -\frac{(Ze)\ell}{m v_0} B_y = \frac{(Ze)}{2 m v_0} \cdot \frac{\ell\Omega}{c} A$$

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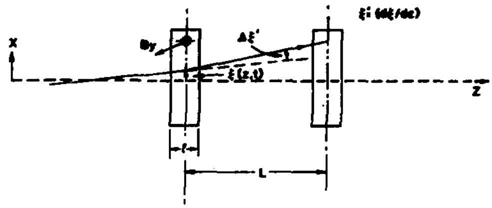

Fig. 2

where Z is the charge state of the ions, ℓ the effective cavity length (including a transit time factor) and mv_0 the particle momentum. Using the solution of eqn. (1), we then have:

$$\frac{\Delta\xi'}{L} = -iG\int_0^\tau dt\, e^{-\alpha(\tau-t)}\,\xi(t)\left[e^{i\Omega(\tau-t)} - e^{-i\Omega(\tau-t)}\right]$$

where $$G = \frac{Z^2 r_p \ell}{ALv_0}\left[\frac{\dot{N}\Omega^3}{c^2 j_1{}^2 J_0(j_1)J_2(j_1)}\right]$$

with $r_p = (e^2/4\pi\epsilon_0 m_p c^2)$(classical proton radius), L the distance between modules, $\dot{N}$ the current in particles per second, and A the atomic number.

IV. Equation of Motion and Solution in Closed Form

In the approximation that both focussing and the impulses from the modules can be replaced by their average values (smooth approximation), the transverse displacement is then determined by an integro-differential equation:

$$\left[\frac{\partial^2}{\partial z^2} + \omega_\beta{}^2\right]\xi(z,\tau) = -iG\int_0^\tau dt\, e^{-\alpha(\tau-t)} \times \xi(z,t)\left[e^{i\Omega(\tau-t)} - e^{-i\Omega(\tau-t)}\right]$$

where ω_β is the coherent spatial betatron frequency. Then, with a change of variable,

$$\xi(z,\tau) = e^{-\alpha\tau}\left[X(z,\tau)e^{i\Omega\tau} + X^*(z,\tau)e^{-i\Omega\tau}\right] \qquad (2)$$

where $X(z,\tau)$ is a slowly varying function of τ, we arrive at the equation[7]:

$$\left[\frac{\partial^2}{\partial z^2} + \omega_\beta{}^2\right]X(z,\tau) = -iG\int_0^\tau dt\, X(z,t) \qquad (3)$$

We have neglected a rapidly varying term in $e^{2i\Omega\tau}$ in arriving at eqn. (3). We now take a Laplace transform of eqn. (3) in τ obtaining:

$$\frac{\partial^2\tilde{x}(z,s)}{\partial z^2} + \left[\omega_\beta{}^2 + \frac{iG}{s}\right]\tilde{x}(z,s) = 0$$

with the immediate solution:

$$\tilde{x}(z,s) = \tilde{x}(0,s)\,\mathrm{Cos}\left[\left(\omega_\beta{}^2 + \frac{iG}{s}\right)^{1/2} z\right]$$

For an initial displacement

$$\xi(0,\tau) = d\, e^{-\alpha\tau}\,\mathrm{Cos}\,\Omega\tau$$

we have $x(0,\tau) = \frac{d}{2}$ and $\tilde{x}(0,s) = \frac{d}{2s}$

Thus: $$\tilde{x}(z,s) = \frac{d}{2s}\,\mathrm{Cos}\left[\left(\omega_\beta{}^2 + \frac{iG}{s}\right)^{1/2} z\right]$$

Using infinite and binomial series expansions for the cosine and $(\omega_\beta{}^2 + iG/s)^n$ respectively and making use of the Laplace inversion formula

$$L^{-1}\left(\frac{1}{s^{n+1}}\right) = \frac{s^n}{n!}$$

we get an expression for $X(z,\tau)$ involving a double sum over integers, one of which can be summed in closed form to give spherical Bessel functions. We finally get:

$$X(z,\tau) = \frac{d}{2}\sum_{\ell=0}^{\infty}(-i)^\ell\,\frac{1}{(\ell!)^2}\left(\frac{Gz\tau}{2\omega_\beta}\right)^\ell(\omega_\beta z)j_\ell(\omega_\beta z) \qquad (4)$$

After a few betatron wavelengths down the accelerator, $\omega_\beta z \gg 1$ and we use:

$$j_{\ell-1}(\omega_\beta z) \xrightarrow[(\omega_\beta z)\to\infty]{} \frac{1}{(\omega_\beta z)}\,\mathrm{Cos}\left[\omega_\beta z - \tfrac{1}{2}\ell\pi\right] \qquad (5)$$

Using (4), (5) and (2), we finally arrive at the expression for the transverse beam displacement $\xi(z,\tau)$ at location z and time τ following the arrival of the front of the beam, in closed form, as follows:

$$\xi(z,\tau) = \frac{d}{2}e^{-\alpha\tau}\left[\mathrm{Cos}(\omega_\beta z - \Omega\tau)I_0\left(\sqrt{\frac{2Gz\tau}{\omega_\beta}}\right) + \mathrm{Cos}(\omega_\beta z + \Omega\tau)\,J_0\left(\sqrt{\frac{2Gz\tau}{\omega_\beta}}\right)\right] \qquad (6)$$
$$(\omega_\beta z \gg 1)$$

where J_0 and I_0 are zero-order Bessel and modified Bessel functions respectively.

We note that in the limit of no focussing at all ($\omega_\beta = 0$), we have:

$$x(z,\tau) = \frac{d}{2}\sum_{\ell=0}^{\infty}(-i)^\ell\,\frac{(Gz^2\tau)^\ell}{\ell!(2\ell)!}$$

so that the absolute square of the slowly varying amplitude grows as:

$$|X(z,\tau)|^2 = \frac{d^2}{4}\sum_{n=0}^{\infty}(Gz^2\tau)^{2n}\sum_{m=0}^{2n}\frac{(-1)^{n-m}}{m!(2m)!(2n-m)!(4n-2m)!}$$

in agreement with Panofsky and Bander[2] and hence is expected to scale similarly as

$$|X(z,\tau)|^2 \sim e^s \quad \text{with} \quad s = (Gz^2\tau)^{1/3}$$

V. Numercial Estimates:

We observe from expression (6) that the beam displacement is damped on the whole if $\alpha > (Gz/2\omega_\beta)$; if $\alpha \ll (Gz/2\omega_\beta)$, the maximum in τ of the amplitude of displacement comes at $\tau = (Gz/2\omega_\beta\alpha^2)$ and has a magnitude:

$$x = \frac{d}{2}\sqrt{\frac{8\omega_\beta}{2\pi Gz}}\; e^{Gz/2\omega_\beta\alpha}$$

As a numerical example, we consider an induction linac that accelerates singly charged Uranium ions, with a 30° phase advance between modules. Example beam parameters[6] for two significant cases and parameters of equivalent induction module cavities are listed in Table I below.

Table 1

Square Bucket – Transverse – BF = 0.5

	Δ	s
g_4 = .0022	.1	.0036
g_3 = .0022	.01	.0012
g_4 = .0022	.01	.0015
$g_3 = g_4$ = .0022	.01	.0020
$g_1 = g_2 = g_3 = g_4$ = .0022	.1	.0090
$g_1 = g_2 = g_3 = g_4$ = .0022	.01	.0021

Sinusoidal Bucket – Transverse

	Δf_s	BF	s
g_4 = .0022	.015	.7	.0022
g_4 = .0022	.025	.9	.0029
g_4 = .0022	.0015	.7	.00051
$g_1 = g_2 = g_3 = g_4$ = .0022	.015	.7	.003
$g_1 = g_2 = g_3 = g_4$ = .0022	.025	.9	.0045
$g_1 = g_2 = g_3 = g_4$ = .0022	.0015	.7	.00065

Sinusoidal-Like Bucket – Longitudinal – BF = .7

	Δf_s	s_{200}	s_{1000}
$g_1 = g_2 = g_3 = g_4$ = .0022	.15	.0096	.0065
$g_1 = g_2 = g_3 = g_4$ = .0022	.015	.0054	.0043

Table 2

Coasting Beam Theory – Transverse

	Δ	s
g_3 = .0022	.1	.0038
	.05	.0033
	.01	.0013
	.005	.00062
g_4 = .0022	.1	.004
	.05	.0035
	.025	.0028
	.015	.0022
	.01	.0017
	.005	.00088
$g_1 = g_2 = g_3 = g_4$	.1	.014
	.05	.011
= .0022	.025	.0084
	.015	.0059
	.01	.0042
	.005	.002
$g_3 = g_4$ = .002 effective	.01	.0022
$g_1 = g_2 = g_3 = g_4$ effective	.01	.0028

For the square bucket, single harmonic (ℓ = 3,4) rates compare remarkably well with coasting beam theory. However, when both harmonics are present, the cooling rate is significantly different from that of coasting beam theory, where rates for each harmonic are simply added. For a square bucket, Schottky signals ℓ and m are coupled with a weighting

$$\left[\frac{\sin\,(\ell-m)\theta_0}{(\ell-m)\theta_0}\right]^2 \tag{8}$$

where θ_0 is the half length of the bunch. The last entries in Table 2 give rates calculated with coasting beam theory, using an effective gain

$$g_{\ell\,\mathrm{eff}} = \sum_m g_m \left[\frac{\sin\,(\ell-m)\theta_0}{(\ell-m)\theta_0}\right]^2 \tag{9}$$

to evaluate the ϵ_ℓ. Agreement is good. These results clearly demonstrate the interference of neighboring harmonics; but for a long bunch, this interference does not totally cancel the effects of using neighboring harmonics in a cooling system.

For transverse cooling in a sinusoidal RF bucket, cooling rates for a synchrotron oscillation spread Δf_s are comparable to coasting beam rates with $\Delta = \Delta f_s$. Again, with several harmonics simultaneously acting, there is a degradation of coasting beam rates by a factor of 2. It should be noted that the longitudinal random load provides a uniform distribution in phase space.

Finally, the last entries in Table 1 are for longitudinal runs. Effective cooling rates/step are given after 200 and 1000 correction steps are given. The phase space orbits are elliptical with amplitude variation of synchrotron frequency. Cooling rates degraded as mixing lessens with higher phase space density.

Acknowledgement

We thank L.J. Laslett for providing his coasting beam cooling code and for giving his invaluable advice in modifying it.

Conclusions

Synchrotron frequency spread provides the necessary mixing mechanism for bunched beam cooling. In addition, it appears that the natural nonlinearities of a single, long full RF bucket can provide mixing comparable to a coasting beam for harmonics of higher frequency than those associated with the gross bunch structure. However, as the bunch length decreases degradation of cooling occurs as the mixing mechanism couples neighboring Schottky bands.[6]

References

1. Tevatron Phase 1 Project, FNAL, 1980.
2. J. Gareyte, CERN p-p Complex, 11th International Conference on High Energy Accelerators, 79.
3. J. Bisognano, Vertical Transverse Stochastic Cooling, LBL Internal Report, BECON 9.
4. J. Bisognano, Kinetic Equations for Longitudinal Stochastic Cooligm, LBL Report, LBID 119.
5. J.L. Laclare, Bunched Beam Instabilities, 11th International Conference on High Energy Accelerators, 526.
6. H. Herr, D. Mohl, Bunched Beam Stochastic Cooling, CERN-EP- Note/79-34.