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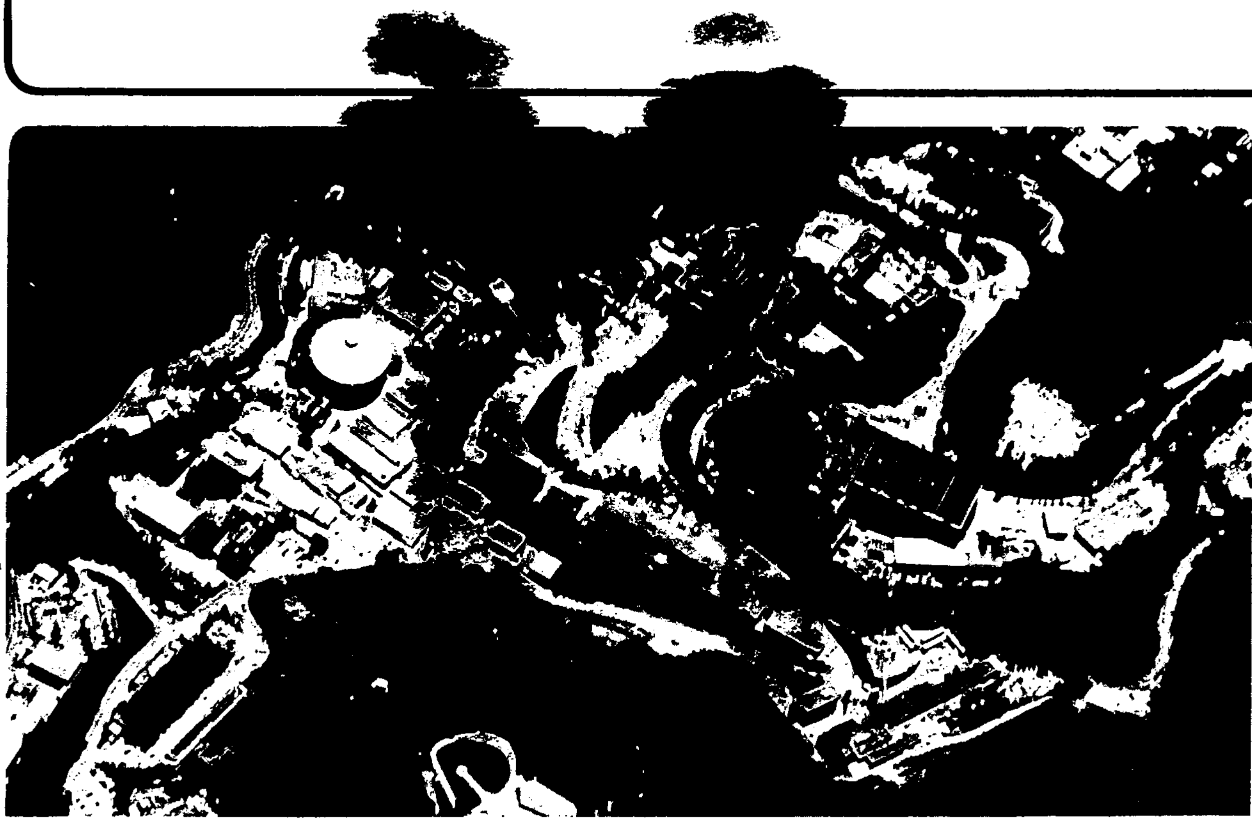

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N-ALITY DEPENDENCE OF STRING TENSION*

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ABSTRACT

We show that in SU(N) gauge models that confine in 2 + 1 dimensions, for $N = 2k$ or $2k + 1$ there are k scales for the area law of the Wilson loop, depending on the quark representation. All representations Λ with the same N-ality or the dual N-ality, $N-n(\Lambda)$, have the same string tension. In the lowest approximation, the ratio of string tensions is the ratio of Casimirs $C_2(\tilde{\Lambda})/C_2(\tilde{\Lambda}')$, where $\tilde{\Lambda}$ is the totally antisymmetric representation of the same N-ality as Λ. The screening of the adjoint representation and higher dimensional representations of a given N-ality is dynamical.

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I. INTRODUCTION

We will study the dependence of the string tension on the quark representation for SU(N) gauge theories. Since in the world of strong interactions the gauge group is SU(3) and there are only fundamental representation quarks, this would be an academic exercise if we did not believe the most likely candidates for more fundamental theories of the electroweak interactions are also theories of nonabelian gauge fields coupled only to (perhaps massless) fermions. There has been interesting speculation about the dynamics of such theories.[1,2,3] One important reasonable feature of these speculations is the generation of a sequence of mass scales - a phenomena refered to by Raby, Dimopoulos, and Susskind[2] as tumbling. A sequence of scales arises from the representation dependence of the relative strength of perturbative gauge interactions between fermions. The perturbative analysis is applied to the spontaneous gauge symmetry breaking picture of tumbling; many of these models also have a complimentary confining picture.[4] Our analysis could have possible relevance for this confining picture. To make contact with the spontaneous gauge symmetry breaking picture of tumbling scale generation we would have to understand the (probably more important) representation dependence of spontaneous chiral symmetry breaking - a problem we can not yet analyse in a reasonable way. Nevertheless we expect that for gauge theories with fermions in more than one representation of the gauge group, the sets of string tensions could be reflected in sets of energy scales for the resulting particle spectrum.

Even as the problem we wish to address with dynamical fermions is too complicated to analyse at present, the simplified problem of representation dependence of string tension in SU(N) gauge theories is also too complicated to address in the real 4-dimensional theory. Our first simplification is to consider the SU(N) gauge theory in 3-dimensions where 't Hooft's[5] topological analysis of phases of SU(N) gauge theories is accompanied by a physical picture of confinement.

't Hooft's analysis follows from commutation relations between a spacial Wilson loop operator, $W_\Lambda(C,t)$, in the representation Λ of SU(N), and an operator $M_{n^*}(\vec{x}^*,t)$ that acts on fields by gauge transforming them by $\Omega^{[\vec{x}^*]}(\vec{x},t)$; this gauge transformation has the property that as $\vec{x}$ encirles $\vec{x}^*$, Ω does not return to its original value but acquires a Z_N phase,

$$\Omega^{[\vec{x}^*]}(\theta=2\pi) = e^{i\frac{2\pi}{N}n^*}\,\Omega^{[\vec{x}^*]}(\theta=0) \tag{1.1}$$

The 't Hooft commutation relations in 2 + 1 dimensions are[5,6]

$$M_{n^*}(\vec{x}^*,t)\,W_\Lambda(C,t) = W_\Lambda(C,t)\,M_{n^*}(\vec{x}^*,t)\,e^{i\frac{2\pi}{N}n^*n(\Lambda)\,n(\vec{x}^*,C)} \tag{1.2}$$

where $n(\vec{x}^*,C)$ is the number of times the curve C loops around $\vec{x}^*$, and $n(\Lambda)$ is the N-ality of the representation Λ, the number of fundamental minus the number of antifundamental representations from which the representation Λ is built by tensor products. $n(\Lambda)$ is defined mod N, so the dual N-ality, $N-n(\Lambda)$ is equivalent

to $-n(\Lambda)$. We denote by $|n(\Lambda)|$ either N-ality. These commutation relations imply that in the confining phase, which is characterized by $\langle M \rangle \neq 0$, which spontaneously breaks a global (dual) Z_N^* symmetry, the Wilson loop operator creates a magnetic domain. Along the curve C a Bloch wall is created separating two domains that differ in orientation of magnetization by $\exp(i\,\frac{2\pi}{N}\,n(\Lambda))$. This Bloch wall carries a finite energy per unit length and corresponds to an electric vortex. A kink like soliton connects two neighboring degenerate vacuua that differ in orientation by $e^{i\frac{2\pi}{N}}$, and carries the finite energy per unit length of the wall. One might expect that Bloch walls separating vacuua that differ in orientation by $(e^{i\frac{2\pi}{N}})^{n(\Lambda)}$ carry $n(\Lambda)$ units of flux and have different energy per unit length, perhaps like the energy of $n(\Lambda)$ separate solitions.

As a second simplification to the analysis of the quark representation of dependence of string tension, we will address these questions in specific models[7,8] that explicitly realize 't Hooft's confinement picture.[8] In these models additional adjoint representation scalar fields are added to the SU(N) gauge theories in order to explicitly select the set of degrees of freedom 't Hooft[9] argues would arise from a unitary like gauge fixing condition of the pure gauge theory, appropriate for studying long distance confinement physics. These models of the confining phase have been shown by one of us[8] to be dual, as implied by the 't Hooft commutation relations, to the Mandelstam[10]-Nielson-Olesen[11] non-abelian vortex models. They are SU(N) generalizations of Polyakov's[12] compact QED confinement model, which have also been studied by Wadia and Das,[7] in which the charged fields of SU(N) are screened by adjoint representation scalar field condensates; that is, SU(N) is spontaneously broken to $U(1)^{N-1}$. These scalar fields also introduce an explicit scale so that a well defined semiclassical analysis exists. The magnetic monopoles of these models in 3 + 1 dimensions become instantons in the 2 + 1 dimensional models.

In our explicit models we will see that Block walls separating vacuua by $n(\Lambda)$ will carry $n(\Lambda)$ units of flux and have different energies, although the above guess for the magnitude of their energies is too naive. Quark sources in all representations with N-ality $|n(\Lambda)|$ are confined by electric vortices with the same string tension, but those with different $|n(\Lambda)|$ have different string tensions. The charges of the higher dimensional representations of a given N-ality are dynamically screened down to the charges of the totally antisymmetric (lowest dimesional) representation of the given N-ality. For SU(N) with N = 2k or 2k + 1, there are k different string tensions. In the lowest approximation the ratio of string tensions is the ratio of Casimirs, $C_2(\tilde{\Lambda})/C_2(\tilde{\Lambda}')$, where $\tilde{\Lambda}$ is the totally antisymmetric representation with the same N-ality as Λ.

II. REPRESENTATION DEPENDENT STRING TENSION.

The Wilson loop in these models[12,7,8] is

$$\langle W_\Lambda(C)\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\Theta\, e^{-S(A,\Theta)}\,\frac{\mathrm{tr}_\Lambda P\, e^{i\oint_C A_k dx_k}}{\dim\Lambda}, \tag{2.1}$$

where A_k is a $\dim\Lambda \times \dim\Lambda$ matrix in the representation Λ of the Lie algebra of SU(N). The Euclidean action is

$$S(A,\Theta) = \int d^3x \Big\{ \tfrac{1}{2}\,\mathrm{tr}\, F_{ik}^2 + \sum_{a=1}^{N(N-1)/2} \mathrm{tr}\big(D_k\Theta_a\big)^2 + V\big(\Theta_1, \Theta_2, \cdots, \Theta_{N(N-1)/2}\big)\Big\}, \tag{2.2}$$

where $F_{ik} = \partial_i A_k - \partial_k A_i + ie\,[A_i, A_k]$, $A_k = \vec{A}_k \cdot \frac{\vec{\lambda}}{2}$, $D_k\Theta = \partial_k\Theta + ie[A_k,\Theta]$. When this field theory is approximated semiclassically it leads to the effective field theory

$$\frac{Z^{-1}}{\dim\Lambda}\sum_{\vec{\mu}}\int \mathcal{D}\vec{\phi}\, \exp\Big(-\int d^3R \Big\{\tfrac{1}{2}(\nabla\vec{\phi})^2 + \frac{e^2}{16\pi^2}M^2 \sum_{|\vec{m}|}\Big(1-\cos\Big[\frac{4\pi}{e}\vec{m}\cdot(\vec{\phi}+\vec{\mu}\,\Phi_c)\Big]\Big)\Big\}\Big). \tag{2.3}$$

$\vec{\phi}$ is an N-1 component vector of scalar fields which have the interpretation of magnetostatic potentials. $\vec{m}$ is an N-1 component root vector of SU(N) and the sum of $|\vec{m}|$ is over all N(N-1)/2 positive roots. The magnetic monopoles in $SU(N) \to U(1)^{N-1}$ have adjoint representation charges $\frac{4\pi}{e}\vec{m}$, and a density proportional to M^2, where

$$M^2 = \mathrm{const.}\,\frac{M_W^{7/2}}{e^3}\exp\Big(-\frac{4\pi}{e}M_W\,\epsilon\Big), \tag{2.4}$$

where ϵ is a function of $\frac{\lambda}{e^2}$ of order unity.

The external potential

$$\Phi_C(\vec{R}) = \frac{e}{4\pi} \int_{\partial S=C} d\vec{S}_x \cdot \nabla \frac{1}{|\vec{x}-\vec{R}|} \tag{2.5}$$

comes from the Wilson loop and is interpreted as the potential from a sheet S bounded by C of magnetic dipoles that is immersed in the monopole plasma. The monopoles of this dipole sheet have charge $e\vec{\mu}$ where $\mu = \vec{\mu}(\Lambda)$ is an N-1 component weight vector of the representation Λ; the sum over $\vec{\mu}$ is over all dim (Λ) weights.

In the lowest approximation, the effective field theory is dominated by the solution to the classical field equation,

$$-\nabla^2\phi + \frac{e}{4\pi} M^2 \sum_{|\vec{m}|} \vec{m} \sin \frac{4\pi}{e} \vec{m}\cdot\left(\vec{\phi} + \vec{\mu}\Phi_C\right) = 0 , \tag{2.6}$$

or equivalently, for the total magnetic potential of both the monopole plasma and dipole sheet,

$$\frac{e}{4\pi}\vec{\chi} = \vec{\phi} + \vec{\mu}\Phi_C , \tag{2.7}$$

$$-\nabla^2\vec{\chi}(\vec{R}) + M^2 \sum_{|\vec{m}|} \vec{m} \sin \vec{m}\cdot\vec{\chi}(\vec{R}) = 4\pi\vec{\mu} \int_{\partial S=C} d\vec{S}_x \cdot \nabla \delta^3(\vec{x}-\vec{R}) . \tag{2.8}$$

With the Wadia-Das[7] like ansatz, $\chi = \vec{\mu}\chi$, this equation can be re-written,

$$-\nabla^2\chi + M^2 \sum_{|\vec{m}|} \frac{\vec{\mu}\cdot\vec{m}}{\vec{\mu}^2} \sin \vec{m}\cdot\vec{\mu}\chi = 4\pi \int_{\partial S=C} d\vec{S}\cdot\nabla\delta^3 . \tag{2.9}$$

For a given representation Λ, such an equation exists for each $\mu(\Lambda)$. Much of our following analysis concerns the group theory of nontrivial soliton solutions to this equation.

A. Fundamental Representations

For all totally antisymmetric representations $\tilde{\Lambda}$, all weights have the same length,

$$\vec{\mu}^2(\tilde{\Lambda}) = \frac{C_2(\tilde{\Lambda})}{N+1} \tag{2.10}$$

where C_2 is the quadratic Casamir. Equation (2.9) will be seen to have a domain wall solution whenever the only values for the projection $\vec{m}\cdot\vec{\mu}$ ($\vec{\mu}$ is considered to be fixed) are

$$\vec{m}\cdot\vec{\mu} = 0, \pm\frac{1}{2} . \tag{2.11}$$

All roots $\vec{m}$ such that $\vec{m}\cdot\vec{\mu} = \pm 1/2$ give the same contribution in the sum over $\vec{m}$ in Eq. (2.9). The number of such roots is

$$\Gamma(\tilde{\Lambda}) = \frac{2N}{N+1} C_2(\tilde{\Lambda}) . \tag{2.12}$$

From Eqs. (2.10) and (2.12) it follows that for all totally anti-symmetric representations the same equation results,

$$-\nabla^2\frac{\chi}{2} + \frac{N}{2} M^2 \sin\frac{\chi}{2} = 2\pi \int_{\partial S=C} d\vec{S}\cdot\nabla\delta^3 . \tag{2.13}$$

The same equation is derived for each $\vec{\mu}(\tilde{\Lambda})$ of the representation $\tilde{\Lambda}$.

For a Wilson loop created in the Euclidean t-x plane, well inside the loop χ is a function only of y, the coordinate perpendicular to the sheet of the loop. The solution to Eq. (2.13) is a solution to the homogeneous equation with a 2π discontinuity implied by right hand side. The solution with the required discontinuity is[12]

$$\frac{\chi}{2}(y) = 4\,\epsilon(y)\tan^{-1} e^{-\sqrt{\frac{N}{2}}M|y|}\,, \tag{2.14}$$

where $\epsilon(y) = \begin{cases} +1 & y > 0 \\ -1 & y < 0 \end{cases}$.

The string tension can be computed from this approximation to the Wilson loop,[13]

$$\langle W_\Lambda(C)\rangle \simeq \exp\left(-\frac{e^2}{16\pi^2}\int d^3R\left\{\frac{1}{2}(4\vec{\mu}^{\,2})\left(\nabla\frac{\chi}{2}\right)^2 + M^2\sum_{|\vec{m}|}\left[1-\cos\vec{m}\cdot\vec{\mu}\chi\right]\right\}\right)$$

$$= \exp\left(-\frac{e^2}{4\pi^2}\frac{C_2(\tilde{\Lambda})}{N+1}\int d^3R\left\{\frac{1}{2}\left(\nabla\frac{\chi}{2}\right)^2 + \frac{N}{2}M^2\left(1-\cos\frac{\chi}{2}\right)\right\}\right)$$

$$= \exp\left(-\left[\frac{e^2}{4\pi^2}\frac{C_2(\tilde{\Lambda})}{N+1}\,8\sqrt{\frac{N}{2}}M\right]A\right), \tag{2.15}$$

where A is the area of the Wilson loop, and we have used Eqs. (2.10) and (2.12) to get to the next to last line. Therefore, the string tension, σ, is representation dependent,

$$\sigma = \frac{2e^2}{\pi^2}\frac{C_2(\tilde{\Lambda})}{N+1}\sqrt{\frac{N}{2}}\,M\,. \tag{2.16}$$

For nondynamical quarks, M^2 is independent of quark representation; it is proportional to the density of vacuum monopoles and is

nonanalytic in the coupling. (See Eq. 2.4).

The representation $\tilde{\Lambda}$ and its dual have the same Casimir. The representations with different Casimirs differ in N-ality $|n(\tilde{\Lambda})|$. Therefore, the ratio of string tensions for representations with different $|n(\tilde{\Lambda})|$ are ratios of Casimirs. In contrast to single gluon exchange which gives the same ratio of Casimirs, this is a non-perturbative confinement result (although certainly not an exact result).

B. Adjoint Representation

For the adjoint representation, all roots have unit length. In the sum over all m in Eq. (2.9) there is one m equal to the particular root, $\vec{\alpha}$. The other roots have projections $m\cdot\vec{\alpha} = 0, \pm\frac{1}{2}$. The number of zero projections is $\frac{(N-2)!}{2(N-4)!}$, and the number of m's with projections $\pm\frac{1}{2}$ is

$$\Gamma = \frac{N(N-1)}{2} - 1 - \frac{(N-2)!}{2(N-4)!} = 2(N-2)\,. \tag{2.17}$$

We therefore have the equation

$$-\nabla^2\chi + M^2\sin\chi + (N-2)M^2\sin\frac{\chi}{2} = 4\pi\int_{\partial S=C} d\vec{S}\cdot\nabla\delta^3\,. \tag{2.18}$$

For the adjoint loop for SU(2) we have the same equation as for the fundamental representation except for a 4π discontinuity. This discontinuity could be satisfied by a two soliton configuration. Although there is no static solution for two solitons, there is a time dependent solution where the solitons scatter. This would correspond to two fundamental representation sheets floping through each other. For the adjoint loop, then, the physics of Eq. (2.18) can not be approximated by a one dimensional version of the equation, as was the case for the fundamental representations.

The existance of solutions to complicated equations like Eq. (2.18) can be simply understood.[16] First, for a finite energy solution, far from the Wilson loop a solution $\vec{\phi}(x)$ to Eqs. (2.7) and (2.8) must approach a minimum of the potential,

$$V(\vec{\phi}) = \sum_{|\vec{m}|} \left[1 - \cos\left(\frac{4\pi}{e}\, \vec{m}\cdot\vec{\phi}\right)\right] . \tag{2.19}$$

This potential has minima for

$$\vec{\phi} = e\vec{\mu} \times \text{integer} , \tag{2.20}$$

since $2\vec{m} \cdot \vec{\mu}$ is an integer. A solution to these equations with a discontinuity must connect different minimia of the potential; the discontinuity in $\vec{\phi}$, different for different representations $\vec{\mu}(\Lambda)$, represents the distance between these minima. The original sheet of discontinuities across the surface S, bounded by the curve C of the Wilson loop, can be moved around by a gauge transformation without affecting the field strength. This kinematic discontinuity compensates

the dynamical discontinuity of the flux tube sheet so that the field $\vec{\phi}$ can be continuous - that is, as $\vec{x}$ tranverses a path around or through the Wilson loop, $\vec{\phi}(\vec{x})$ returns to the same value. The solitons connect different minima as y passes through the sheet of the Wilson loop, and the discontinuity brings the solution back to the original minimum.

For SU(2), the fundamental representation soliton, with kinematic discontinuity different from the center of the flux tube, is shown in Fig. (1a). A one dimensional slice of the corresponding soliton for the adjoint representation is shown in Fig. (1b). These solitons connect different minima of V with the required discontinuities, as shown in Figs. (2a) and (2b). For SU(3) we plot the minima of $V(\vec{\phi})$ in Fig. (3), and in Fig. (4) we show how the minima are connected by a double soliton appropriate for the adjoint loop. The two solitons are associated with a fundamental and an antifundamental weight; the adjoint loop therefore has a double sheet, one sheet of which has fundamental representation flux and the other antifundamental flux.

The physics of the adjoint representation loop we have just described is incomplete because we have neglected important dynamics. In our models, $SU(N) \to U(1)^{N-1}$, there are $N(N-1)/2$ charged massive W's which have adjoint representation electric charge. These W's contribute to vacuum fluctuations. In the presence of an adjoint representation Wilson loop, flux tubes form, as we have just discussed, but there are fluctuations of the charged W's in this background electric field. The fluctuations of these heavy adjoint fields can then tunnel out of the vacuum, becoming real charged pairs to screen the adjoint loop.

To analyze this physics of the adjoint loop we must more carefully examine the usual Polyakov derivation of the effective field theory;

all the necessary physics is there in the 1-loop semiclassical approximation. The original functional integral is expanded about a set of classical background fields which are approximate saddle points of the action, and then 1-loop quantum fluctuations are included. Consider the 1-loop quantum fluctuations about the contribution of a single monopole (a local minimum of $S(A, \Phi)$) to the functional integral for the expectation value of the Wilson loop. This computation without the Wilson loop present was examined in detail by Polyakov; here we will be somewhat symbolic, indicating only the kinds of connections that arise from the presence of the Wilson loop,

$$\int \mathcal{D}\delta A\, \mathcal{D}\delta\Phi \, \exp\Big(-\Big\{ S(A,\Phi) + \int \mathrm{tr}\, \delta A_\mu \mathcal{D}^{-1}_{\mu\nu}(A,\Phi)\,\delta A_\nu + \int \mathrm{tr}\, \delta\Phi\, \mathcal{D}^{-1}(A,\Phi)\,\delta\Phi + \cdots \Big\}\Big)\, \mathrm{tr}\, P e^{i\oint_C (A_\mu + \delta A_\mu)\, dx_\mu}$$

$$\simeq \int S^{3/2} d^3R \, \frac{e^{-S}}{\sqrt{\det' \mathcal{D}^{-1}_{\mu\nu}(A,\Phi)}\,\sqrt{\det' \mathcal{D}^{-1}(A,\Phi)}} \times\Big(\mathrm{tr}\, P e^{i\oint_C A_\mu dx_\mu} - \frac{1}{2}\int_C \mathrm{tr}\, P e^{i\int_y^x A_\lambda dz_\lambda} \times \mathcal{D}^{A,\Phi}_{\mu\nu}(x,y)\, e^{i\int_x^y A_\lambda dz_\lambda} dx_\mu\, dy_\nu + \cdots\Big).$$

(2.21)

In this expression $D_{\mu\nu}(A,\Phi)$ is the matrix of massless and massive gluon propagators in the background monopole field, and D is the corresponding scalar propagator. (As we are being symbolic, we neglect the ghost determinant and the fact that there are many scalar fields). The determinants are over nonzero modes, and the collective coordinates for the translations zero modes are shown. The second term in round brackets is the first radiative correction term to the Wilson loop in a background monopole field. The exchanged gluons propagating in the background monopole field will renormalize the Coulomb interaction and perimeter law term of ordinary perturbation theory; fluctuations about the trivial vacuum give both Coulomb interactions from the massless gauge fields exchanged, and a perimeter term from the massive fields exchanged,

$$\int_C dx_\mu \int_C dy_\nu\, \mathcal{D}_{\mu\nu}(x,y\,;M_W) \qquad (2.22)$$

where

$$\mathcal{D}_{\mu\nu}(x,y;M_W) = \frac{\delta_{\mu\nu}}{4\pi|\vec{x}-\vec{y}|}\, e^{-M_W|\vec{x}-\vec{y}|} \; . \qquad (2.23)$$

Another source of additional physics is that in the presence of the Wilson loop, the determinants of inverse propogators in the background monopole field depend on the position $\vec{R}$ of the monopole relative to the curve C.

In the background field of a configuration of monopoles and anti-monopoles, the determinants factorize in the dilute gas approximation, (surpressing Θ, and indices),

$$\det \mathcal{D}^{-1}(A_1 + \cdots + A_N) \simeq \det \mathcal{D}^{-1}(A_1)\, \det \mathcal{D}^{-1}(A_2) \times \cdots \times \det \mathcal{D}^{-1}(A_N) \,. \tag{2.24}$$

Usually the factor of $\det^{-1/2} D^{-1}_{(A_i)}$ just renormalizes the term $e^{-S(A_i)}$ for each monopole. Now, however, there is an additional effect due to the presence of the Wilson loop. This is seen by expressing the determinants for the charged W's in terms of loops. This representation follows from the Schwinger proper time expression

$$\ln \det\left(-D_\mu^2(A) + m^2\right) = -\mathrm{tr} \int_0^\infty \frac{ds}{s}\, e^{-m^2 s}\, e^{-s(-D^2)} \,. \tag{2.25}$$

The Tr $e^{-S(-D^2)}$ term can be re-written as a "non-relativistic" Feynman path integral in d + 1 dimensions,

$$\mathrm{tr}\, e^{-s(-D^2)} = \int d^d x \int_{C_x} \mathcal{D}x_\mu(t)\, e^{-\frac{1}{4}\int_0^s \dot{x}_\mu^2 dt} \times \mathrm{tr}\, P\, e^{i \oint_{C_x} A_\mu dx_\mu} \,, \tag{2.26}$$

which sums over all paths C_x that return to the point x in proper time S, of a non-relativistic particle of "mass" 1/2, interacting with the vector potential A^a_μ through the current $e \frac{\lambda^a}{2} \frac{dx_\mu(t)}{dt}$. The determinants can therefore be represented by an arbitrary number of virtual loops of charged W's propagating in the background monopole fields.[17]

The expectation value of the Wilson loop then becomes

$$\begin{aligned}
\langle W(C)\rangle \simeq\ & \frac{1}{Z}\int \mathcal{D}\vec{\phi}\, \exp\left[-\int d^3R\, \tfrac{1}{2}(\nabla\vec{\phi})^2\right] \\
& \sum_{N=0}^{\infty} \frac{1}{N!} \Bigg\{ \prod_{i=1}^{N} \left[\int d^3R_i \left(\mathrm{const.}\, e^2 \frac{M_W^{7/2}}{e^3}\right) e^{-M_{\mathrm{monopole}}} \right. \\
& \left. \sum_{\vec{m}} e^{i\frac{4\pi}{e}\vec{m}\cdot\vec{\phi}(\vec{R}_i)}\, W_{A_i}(C) \right] \\
& \exp\left(3\,\frac{N(N-1)}{2}\int d^3x \int_0^\infty \frac{ds}{s}\, e^{-M_W^2 s} \int_{C_x} \mathcal{D}x_\mu\, e^{-\frac{1}{4}\int_0^s \dot{x}_\mu^2 dt} \right. \\
& \left. \times \left[W_{A_1}(C_x) \times \cdots \times W_{A_N}(C_x) \right] \right) \Bigg\} \\
\simeq\ & \langle W(C)\rangle_0 + 3\,\frac{N(N-1)}{2}\int d^3x \int_0^\infty \frac{ds}{s}\, e^{-M_W^2 s} \\
& \times \int_{C_x} \mathcal{D}x_\mu\, e^{-\frac{1}{4}\int_0^s \dot{x}_\mu^2 dt} \langle W(C)\, W(C_x)\rangle_0 + \cdots \,,
\end{aligned} \tag{2.27}$$

where W_{A_i} is a Wilson loop in the background monopole field A_i, and the subscript zero on the expectation values means the quantum fluctuations of the charged fields are neglected. We have used the unfactorized form of the determinant in Eq.(2.24), but the factorization of W_A, $A = A_1 + \ldots + A_N$, into $W_{A_1} \times W_{A_2} \times \ldots \times W_{A_N}$, valid for the abelian long range fields of the monopoles. We have also neglected the contribution of the spin term in the inverse charged W propagators, which is valid for heavy W's. The unspecified terms involve the correlation functions of W(C) with an arbitrary number of $W(C_{x_i})$; these higher order terms are also supressed for large M_W. (The string tension is the scale to which M_W is compared.) The first Wilson loop term will have an area law even for the adjoint loop. The second term will have a real part that will renormalize the string tension of the first term; for large enough loops, however, for the adjoint loop (and also for higher dimensional representations of a given $n(\Lambda)$) it will also have an imaginary part associated with tunneling of adjoint charge W's, which will lead to the decay of the area law term. In Minkowski space-time the form of the area law term in the amplitude will be,

$$\langle W_{adj}(C)\rangle \simeq \exp\left\{-\frac{T}{2\tau}\,\theta(\sigma L - 2M_W)\right\} \times \exp(i\sigma L T)\,. \tag{2.28}$$

The θ function says that the loops must be large enough to have enough energy in the field to create a real charged pair.

We will calculate the Wilson loop correlation function in Euclidean space-time and then continue to Minkowski space-time. Expressed in terms of the effective field theory, the correlation function of Wilson loops is

$$\langle W(C)W(C_x)\rangle_0 \simeq \sum_{\vec m', \vec\mu} \frac{1}{Z}\int \mathcal{D}\vec\phi\, e^{-\int d^3R \left\{\frac{1}{2}(\nabla\vec\phi)^2 + \frac{e^2}{16\pi^2} M_0^2 \sum_{|\vec m|}\left(1-\cos\left[\frac{4\pi}{e}\vec m\cdot\left(\vec\phi + \vec\mu\,\Phi_C + \vec m'\,\Phi_{C_x}\right)\right]\right)\right\}}. \tag{2.29}$$

The classical equation which minimizes the action for the potential χ, where

$$\vec\chi = \vec\mu\chi = \frac{e}{4\pi}\left(\vec\phi + \vec\mu\,\Phi_C + \vec m'\,\Phi_{C_x}\right), \tag{2.30}$$

is

$$-\nabla^2\chi + M_0^2 \sum_{|\vec m|} \frac{\vec\mu\cdot\vec m}{\vec\mu^2} \sin \vec m\cdot\vec\mu\chi = 4\pi \int_{\partial S = C} d\vec S\cdot\nabla\,\delta^3(\vec y - \vec R) + 4\pi\frac{\vec m'\cdot\vec\mu}{\vec\mu^2}\int_{\partial S' = C_x} d\vec S'\cdot\nabla'\,\delta^3(\vec y' - \vec R)\,. \tag{2.31}$$

For the adjoint representation, with $\vec{\mu}(\mathrm{adj}) = \vec{\alpha}$, this equation becomes

$$-\nabla^2\chi + M_0^2\left(\sin\chi + (N-2)\sin\frac{\chi}{2}\right)$$
$$= 4\pi\int_{\partial S=C} d\vec{S}\cdot\nabla\delta^3(\vec{y}-\vec{R}) + 4\pi\,\vec{m}'\cdot\vec{\alpha}\int_{\partial S'=C_x} d\vec{S}'\cdot\nabla'\delta^3(\vec{y}'-\vec{R}) \quad (2.32)$$

The possible values of $\vec{m}'\cdot\vec{\alpha}$ are 0, $\pm\frac{1}{2}$, ± 1. The largest correlation between loops arises from the term with $\vec{m}' = -\vec{\alpha}$, and for C_x within C, as depicted in Fig. (5). Between the loops there are flux sheets. Inside the loop C_x there is no flux; the field within C_x is cancelled by a field of equal magnitude but opposite direction. Note that for a fundamental (or totally antisymmetric representation) Wilson loop, there can be no complete cancellation of the field within C_x. The area law of the Wilson loop C is therefore reduced by the area of the fluctuation loop C_x,

$$\langle W(C)W(C_x)\rangle_0 \simeq e^{-\sigma(A(C)-A(C_x))} \quad (2.33)$$

Notice that this area law can be reproduced by a constant external field with e times the euclidean electric field strength equal to the string tension,

$$e^{-\sigma(A(C)-A(C_x))} = e^{i\frac{e}{2}\int_{\partial S=C-C_x} F_{\mu\nu}x_\nu dx_\mu} , \quad (2.34)$$

where

$$F_{tx} = -i\,\sigma/e\,. \quad (2.35)$$

This expression for the Wilson loop correlation function, Eqs. (2.33) with (2.34) and (2.35), is to be inserted into the second term or the right hand side of Eq. (2.27). As the loop C becomes arbitrarily large, this approximation to the calculation approaches the usual one of vacuum instability in a constant external background electric field. This propertime path integral representation for $\ell n\,\det(-D_\mu^2(A))$ with $A_\mu = \frac{1}{2}F_{\mu\nu}x_\nu$, $F_{\mu\nu}$ constant, has been analyzed by Affleck, Alvarez and Manton.[18] This famous determinant is known to have both real and imaginary parts; the vacuum decays due to charged particle pair creation. The adjoint loop therefore decays with a decay rate of roughly[19]

$$\frac{1}{T\,Vol} \simeq 3\,\frac{N(N-1)}{2}\,\frac{\sigma^{3/2}}{(2\pi)^{5/2}}\,e^{-\frac{\pi M_w^2}{\sigma}}\,. \quad (2.36)$$

C. Other Representations

We next consider the symmetric representation of SU(N) built out of two fundamental representations, Λ_2. These are now two different lengths for the weight vectors $\vec{\mu}^2(\Lambda_2)$ with correspondingly different projections $\vec{m}\cdot\vec{\mu}$, and numbers of non zero projection Γ,

$$\vec{\mu}^2(\Lambda_2) = \begin{cases} \dfrac{C_2(\tilde{\Lambda}_2)}{N+1}\,, \quad \vec{m}\cdot\vec{\mu} = 0, \pm\frac{1}{2}\,, \quad \Gamma = \dfrac{2N}{N+1}C_2(\tilde{\Lambda}_2) & (2.37a) \\ 2\left(\dfrac{N-1}{N}\right), \quad \vec{m}\cdot\vec{\mu} = 0, \pm 1\,, \quad \Gamma = N-1 & (2.37b) \end{cases}$$

there are therefore two different equations, from Eq. (2.9), for these weights. For the values (2.37a) we have Eq. (2.13) for the $\frac{N(N-1)}{2} = \frac{N!}{2(N-2)!}$ (sum of all <u>different</u> pairs of fundamental weights) weights of the symmetric representation which are the same as those of the antisymmetric representation, $\tilde{\Lambda}_2$. For the N weights Eq. (2.37b), which are two times the fundamental representation weights, we have

$$-\nabla^2\chi + \frac{N}{2}M^2 \sin\chi = 4\pi \int_{\partial S=C} d\vec{S}\cdot\nabla\delta^3 \quad . \qquad (2.38)$$

This is the same equation as the one we obtained for the adjoint representation for SU(2), Eq. (2.18). Therefore, only those weights of Λ_2 which are the same as those of $\tilde{\Lambda}_2$ are confined; the other N weights of Λ_2 are dynamically screened.

For representations with arbitrary $|n(\Lambda)|$, any representation Λ contains all the weights of the totally antisymmetric representation, $\tilde{\Lambda}$, of the same $n(\Lambda)$; these weights will be confined in the same way as $\tilde{\Lambda}$. The other weights of Λ will have different lengths. We have not analysed these weights in general, however, we expect that all other weights of Λ not in $\tilde{\Lambda}$ are dymanically screened, just as for Λ_2. All representations of a given $|n(\Lambda)|$ are screened down to the lowest dimensional representation of that $|n(\Lambda)|$ (the totally antisymmetric representation). In general, then, the ratio of string tensions for representations with different $|n(\Lambda)|$ is the ratio of Casmirs of the corresponding totally antisymmetric representations with the same $|n(\Lambda)|$'s. This screening of representations down to the lowest dimensional representation of a given N-ality is of course not realized by single gluon exchange.

III. AN EXAMPLE

We consider the simplest nontrivial example of SU(4) to illustrate some of our main equations. The weights of the low dimensional representations are as follows: for the fundamental representation (4), the weights are determined from the mutually commuting generators of the Lie algebra,

$$\left(\frac{\lambda^3}{2}, \frac{\lambda^8}{2}, \frac{\lambda^{15}}{2}\right) = \begin{pmatrix} \vec{\mu}_1 & & & 0 \\ & \vec{\mu}_2 & & \\ & & \vec{\mu}_3 & \\ 0 & & & \vec{\mu}_4 \end{pmatrix}$$

$$= \begin{pmatrix} \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{6}}\right) & & & \\ & \left(-\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{6}}\right) & & \\ & & \left(0, -\frac{1}{\sqrt{3}}, \frac{1}{2\sqrt{6}}\right) & \\ & & & \left(0, 0, -\frac{1}{\sqrt{6}}\right) \end{pmatrix} ; \qquad (3.1)$$

for the adjoint representation (15), the weights are determined from the sum of fundamental and antifundamental representation weights (the antifundamental weights are minus the fundamental ones),

$$\vec{m}_{ij} = \vec{\mu}_i - \vec{\mu}_j$$

$$= \begin{pmatrix} 0 & (1,0,0) & \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right) & \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{4}{2\sqrt{6}}\right) \\ & 0 & \left(-\frac{1}{2}, \frac{3}{2\sqrt{3}}, 0\right) & \left(-\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{4}{2\sqrt{6}}\right) \\ & & 0 & \left(0, -\frac{1}{\sqrt{3}}, \frac{4}{2\sqrt{6}}\right) \\ - & & & 0 \end{pmatrix} , \qquad (3.2)$$

where the entries on the lower triangular side of the diagonal are just minus the entries on the upper diagonal side; for the antisymmetric tensor representation (6), the weights are the sums of all unequal pairs of fundamental represention weights,

$$\vec{\mu}_{ij}(6) = \vec{\mu}_i + \vec{\mu}_j$$

$$= \begin{pmatrix} 0 & \left(0, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}\right) & \left(\frac{1}{2}, -\frac{1}{2\sqrt{3}}, \frac{1}{\sqrt{6}}\right) & \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}, -\frac{1}{\sqrt{6}}\right) \\ & 0 & \left(-\frac{1}{2}, -\frac{1}{2\sqrt{3}}, \frac{1}{\sqrt{6}}\right) & \left(-\frac{1}{2}, \frac{1}{2\sqrt{3}}, -\frac{1}{\sqrt{6}}\right) \\ & & 0 & \left(0, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{6}}\right) \\ & & & 0 \end{pmatrix} \qquad (3.3)$$

where the lower triangular entries are the same as the upper ones. Because this representation is self-dual (the 6 and $\bar{6}$ are equal), the weights come in $\pm$ pairs. The symmetric tensor represention (10) has the same weights as the antisymmetric tensor plus the addition of diagonal elements which are twice the fundamental weights. In Table 1 we list the Casamirs, $C_2(\Lambda)$; the lengths of weights, $\vec{\mu}^2$; the different values for the projections $\vec{m}\cdot\vec{\mu}$; and the number of $|\vec{m}|$'s with equal projections. One can then explicitly check our general formulas, Eqs. (2.10), (2.12), (2.17), and (2.19).

IV. CONCLUSIONS

In the introduction we speculated on the energy per unit length of a Bloch wall vortex between domains that differ in orientation by $e^{i\frac{2\pi}{N}n(\Lambda)}$. Since such a vortex carries $|n(\Lambda)|$ times the flux of a fundamental vortex (in a way we will show explicitly below), we speculated it may have $|n(\Lambda)|$ times the energy per unit length. In fact, though, it carries less than $|n(\Lambda)|$ times the energy per unit length. This is because (c.f. Eq. (2.16))

$$C_2([\,|n(\Lambda)|\,]_N) < |n(\Lambda)|\, C_2([1]_N)\,, \tag{5.1}$$

where $[n]_N$ is the totally antisymmetric representation of n fundamentals in SU(N). There is therefore a binding effect; $|n(\Lambda)|$ fundamental vortices placed next to each other would prefer to bind into a single vortex of the representation $[\,|n(\Lambda)|\,]_N$. This more general result follows from

$$C_2([\,n(\Lambda)]_N) < \sum_{\substack{n_1+n_2+\cdots \\ =n}} \left\{ C_2([n_1]_N) + C_2([n_2]_N) + \cdots \right\}$$
$$< n\, C_2([1]_N)\,. \tag{5.2}$$

This is illustrated in Fig. (1).

To compute the flux of the Bloch wall electric vortex, we first consider how the flux is measured in the superconducting phase. There the magnetic flux of a magnetic vortex is defined relative to fundamental representation electric charge by the fundamental

representation Wilson loop,

$$W(C) = \exp\left(ie\frac{\vec{\lambda}}{2}\cdot \oint_C A_\kappa dx_\kappa\right)$$
$$= \exp\left(ie\frac{\vec{\lambda}}{2}\cdot\vec{g}\right) = e^{-i\frac{2\pi}{N}n^*}\,. \tag{5.3}$$

This is satisfied by magnetic charge

$$\vec{g} = \frac{4\pi}{e}\vec{\mu}(\tilde{\lambda})\,, \tag{5.4}$$

where $n(\tilde{\Lambda}) = n^*$; that is, larger flux (associated with n^*) is associated with larger N-ality representation magnetic charge. For the confining phase we can define an electric flux of an electric vortex in a dual way, with respect to fundamental representation magnetic charge,

$$\exp\left(i\frac{4\pi}{e}\frac{\vec{\lambda}}{2}\cdot\int_{-\infty}^{\infty}\vec{E}_x\, dy\right)\,, \tag{5.5}$$

where we have assumed the quarks are on the x axis, and we integrate the x-component of the electric field in the transverse y direction. The relation between the Minkowski and Euclidean space-time fields is

$$(B_x, B_y, B_t) = (F_{yt}, F_{tx}, F_{xy}) \to (-E_y, E_x, B)\,, \tag{5.6}$$

so with Eq. (2.7),

$$\vec{E}_x = -\frac{e}{4\pi}\frac{\partial\vec{\chi}}{\partial y} = -\frac{e}{4\pi}\vec{\mu}(\Lambda)\frac{\partial\chi}{\partial y}\,; \tag{5.7}$$

Using Eq. (2.14),

$$\int_{-\infty}^{\infty} \vec{E}_x \, dy = -\frac{e}{4\pi} \vec{\mu}(\Lambda) \left[\left(\chi(\infty) - \chi(0^+) \right) - \left(\chi(-\infty) - \chi(0^-) \right) \right] \tag{5.8}$$

$$= e \vec{\mu}(\Lambda) ,$$

so we have the flux

$$\exp\left(i \frac{4\pi}{e} \frac{\vec{\lambda}}{2} \cdot e \vec{\mu}(\Lambda) \right) = \exp\left(i \, 4\pi \vec{\mu}(\text{fund}) \cdot \vec{\mu}(\Lambda) \right) \tag{5.9}$$

$$= e^{i \frac{2\pi}{N} n(\Lambda)} ,$$

since $2\vec{\mu}(\text{fund}) \cdot \vec{\mu}(\Lambda) = \frac{n(\Lambda)}{N}$ where $n(\Lambda)$ is defined mod N. Therefore the electric flux of the Block wall is proportional to $|n(\Lambda)|$.

Our analysis of the adjoint loop suggests possible glueball excitations similar in nature to heavy quarkonium systems. (The dymanical screening of the adjoint loop is, of course, essentially the same as that of a fundamental representation Wilson loop when these are heavy dynamical quarks.) As well as the closed loop electric vortex glueball, we have in these models massive charged spin 1 degrees of freedom which feel confining forces. Such states would be very massive since these massive gauge fields are very heavy on the scale of the string tension. We expect our model to give a good effective description of the long distance behavior of the pure gauge theory; heavy bound states, though, are at short distance, unless they have large angular momentum. It is thus unclear how to extrapolate this result of our model to the real world.

Our results indicate that in four dimensions, for theories with quarks in more than one representation of the gauge group, for SU(N) gauge theories with $N \geqslant 4$, there can be more than one string tension. In the most relavent case of the pure gauge theory with dynamical quarks we must first restrict attention to only those sets of representations which maintain asymptotic freedom. To 1-loop, the β function is[14]

$$\beta = -\frac{g^3}{48\pi^2} \left(11 N - 4 \sum_i T(\Lambda_i) N_i \right) , \tag{5.10}$$

where N_i is the number of "flavors" in the representation Λ_i, and $T(\Lambda) = C_2(\Lambda) \frac{\dim(\Lambda)}{\dim(\text{adj})}$. This constraint of asymptotic freedom of course restricts one to relatively low dimensional representations; since $C_2(\Lambda) \geqslant C_2(\tilde{\Lambda})$, it emphasizes the relative importance of totally antisymmetric representations.

Not only do dynamical quarks affect the β-junction, but they also affect the energy scales of the theory. In our models, since for static quarks M^2 sets the scale for string tension, it will set the mass scale for the resulting "hadronic" degrees of freedom when there are dynamical quarks. In these models M^2 plays the role of an instanton density and the group theory for the renormalization of M^2, associated with determinants of inverse propagators in background monopole fields, is the same as for the usual instanton density. To compute the renormalization of M^2 due to dynamical quarks of a given representation Λ, with respect to the SU(2) associated with the embedding of the 't Hooft-Polyakov monopole in SU(N), we first need to know the number of SU(2), multiplets of each "isospin" contained in the representation Λ. For ordinary instantons 't Hooft[15] has computed the 1-loop renormalization of the instanton density: for fermions the instanton density is <u>increased</u> by a factor

$$\exp\left(2\sum_t N_f(t)\,\alpha(t)\right) , \tag{5.11}$$

where $N_f(t)$ is the number of SU(2) multiplets in the isospin representation t, and $\alpha(t)$ is a representation dependent positive number, increasing for higher dimensional representations; for comparison, bosons decrease the instanton density by the factor

$$\exp\left(-\sum_t N_b(t)\,\alpha(t)\right) . \tag{5.12}$$

We expect the same qualitative behavior for the models we are considering. Quarks with mass that is large (compared to the square root

of the string tension) decouple from the renormalization of M^2. If we consider theories with quarks in two representations of the gauge group, and one representation Λ' is heavy and the other, Λ, is light, then these will be a ratio of energy scales for the different quark representations,

$$\frac{C_2(\tilde{\lambda})}{C_2(\tilde{\lambda}')}\ \exp\left[\sum_{t(\Lambda)} N_f(t(\Lambda))\,\alpha(t)\right] \tag{5.13}$$

This formula strikingly suggests that gauge theories with quarks in more than one representation of the gauge group should be able to generate dramatically different energy scales for the resulting hadron spectrum.

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Table 1: Some weight diagram geometry.

SU(4) rep. Λ	Casamir $C_2(\Lambda)$	length of roots, $\vec{\mu}^2(\Lambda)$	projection $\vec{m}\cdot\vec{\mu}$	Number of $\vec{m}$'s with equal projections, $\Gamma(\vec{\mu})$
4	$\frac{15}{8}$	$\frac{3}{8} = \frac{C_2(\tilde{\Lambda}_1)}{N+1}$	$\pm\frac{1}{2}$	$3 = \frac{2N}{N+1}\ C_2(\tilde{\Lambda}_1)$
			0	3
15	$4 = N$	1	1	1
			$\pm\frac{1}{2}$	$4 = 2(N-2)$
			0	1
6	$\frac{5}{2}$	$\frac{1}{2} = \frac{C_2(\tilde{\Lambda}_2)}{N+1}$	$\pm\frac{1}{2}$	$4 = \frac{2N}{N+1}\ C_2(\tilde{\Lambda}_2)$
			0	2
10	$\frac{9}{2} = \frac{(N-1)(N-2)}{N}$	$\frac{1}{2} = \frac{C_2(\tilde{\Lambda}_2)}{N+1}$	$\pm\frac{1}{2}$	$4 = \frac{2N}{N+1}\ C_2(\tilde{\Lambda}_2)$
			0	2
		$\frac{3}{2} = 2\ \frac{N-1}{N}$	1	$3 = N-1$
			0	3

FIGURE CAPTIONS

Fig. 1. SU(2) soliton solutions. Kinematic discontinuities can be displaced with respect to center of soliton.

a) fundamental representation,

b) adjoint representation.

Fig. 2. Potential for SU(2). Solitons of Fig. 1 connect minima for

a) fundamental representation,

b) adjoint representation.

Fig. 3. Extrema of potential for SU(3). The symbols reprsent: •, minima, V = 0; x, maxima, V = 4; ⊙ minima, V = 4; ⊗, maxima, V = 4 ½.

Fig. 4. Potential for SU(3). Solitons connect minima (0, 0) and (1, 0) via $(\frac{1}{2}, -\frac{1}{2\sqrt{3}})$ for adjoint representation.

Fig. 5. Binding of fundamental vortices.

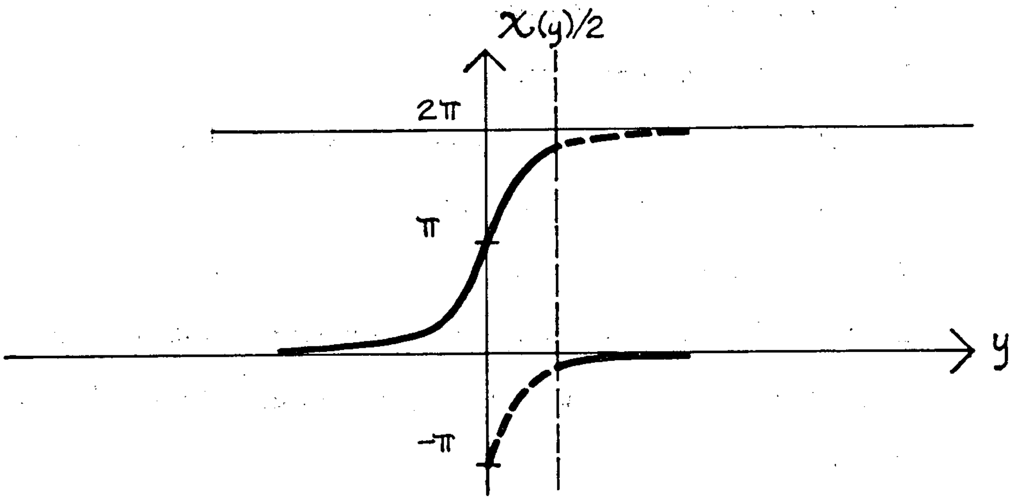

Fig. 1a

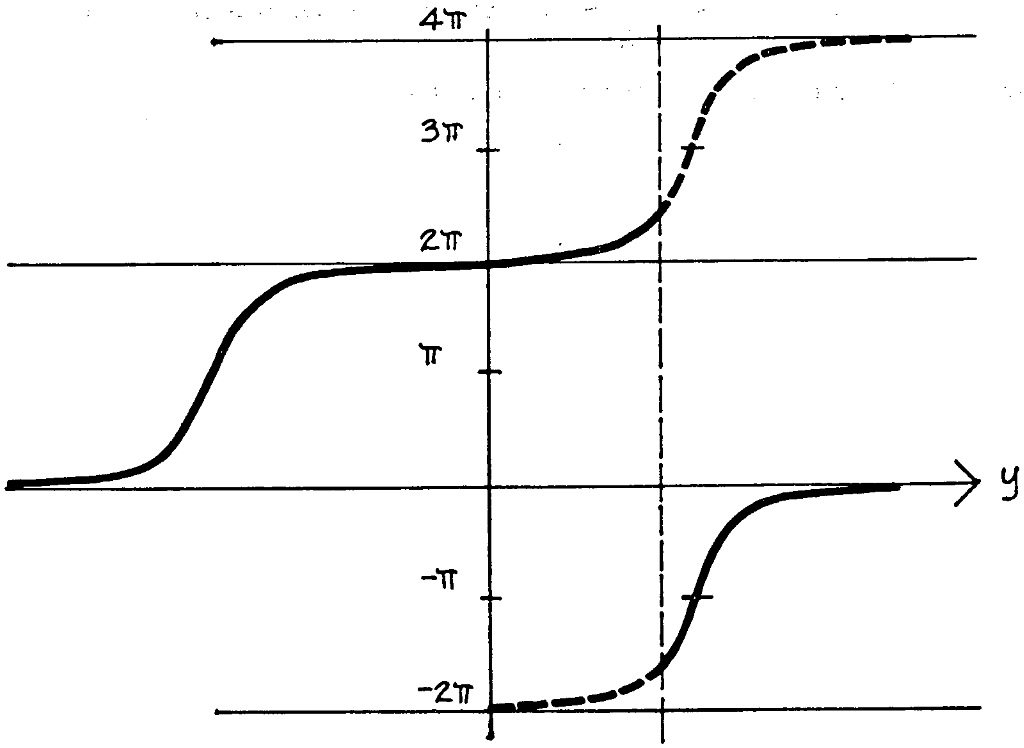

Fig. 1b

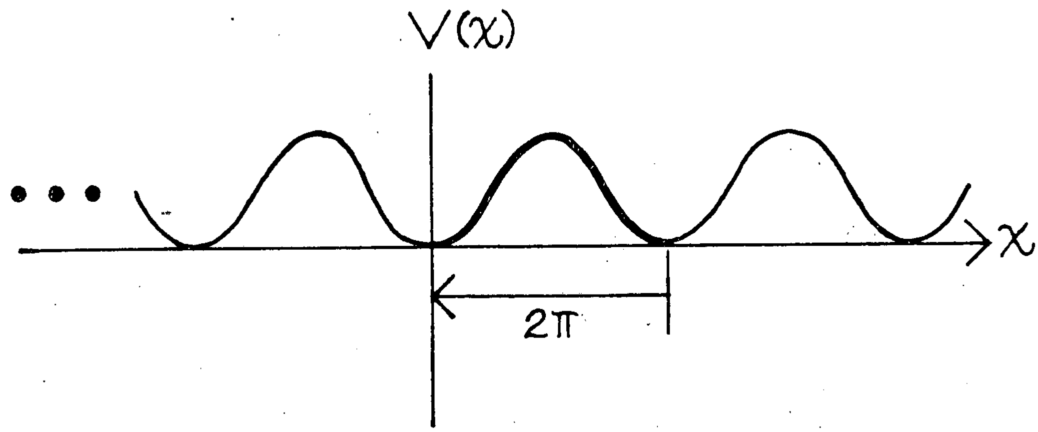

Fig. 2a

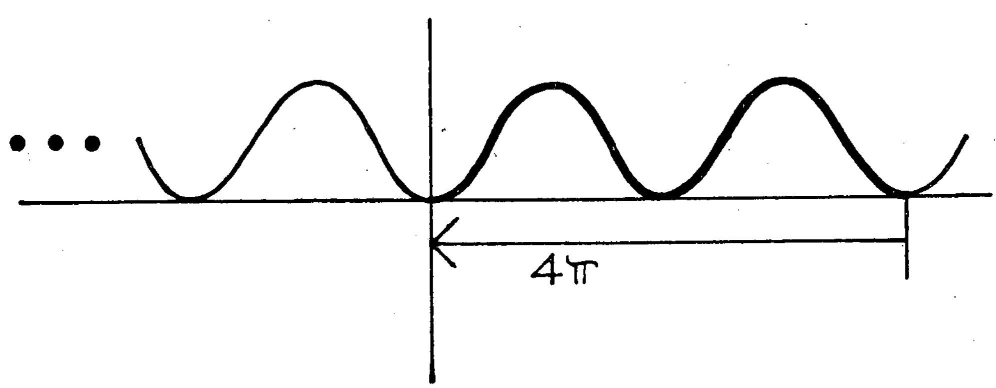

Fig. 2b

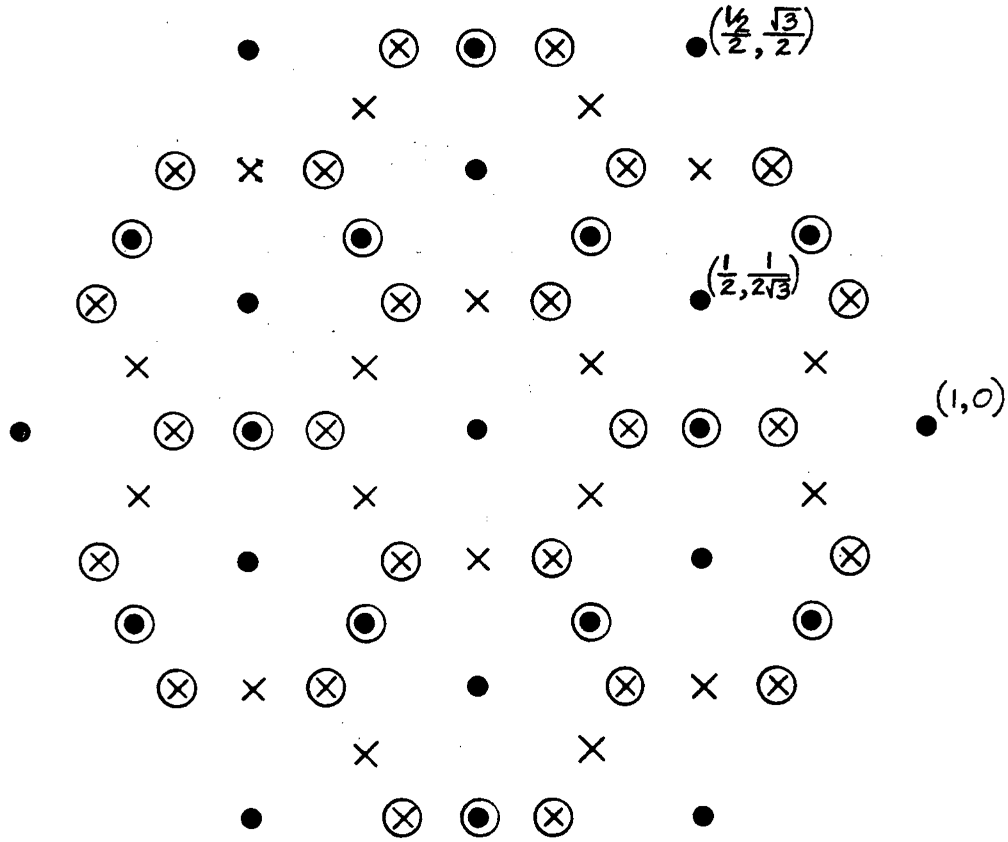
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$
$(1,0)$

Fig. 3

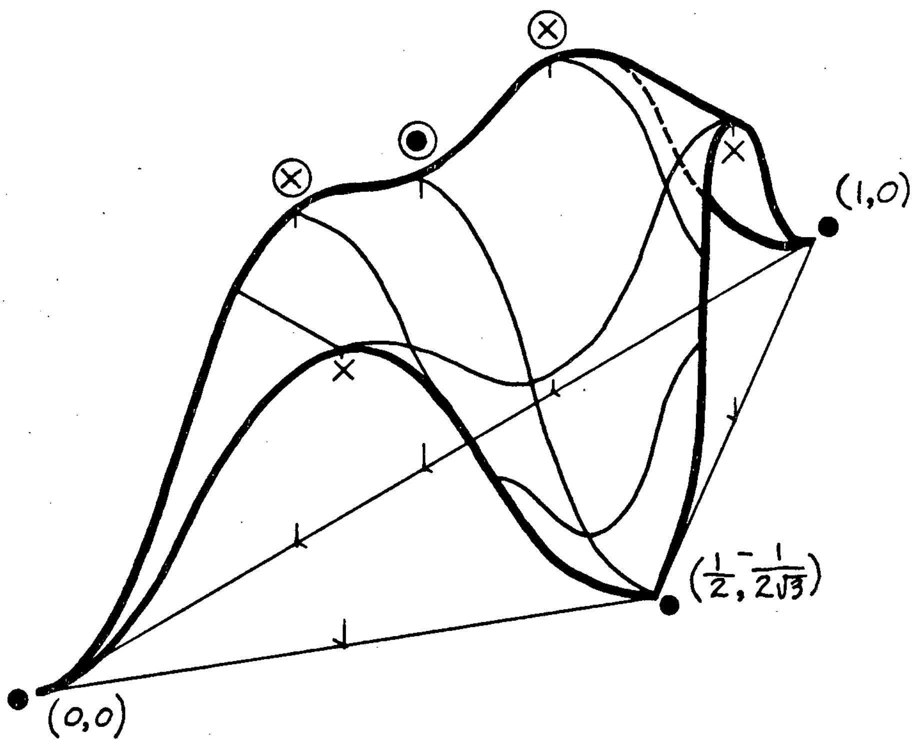

Fig. 4

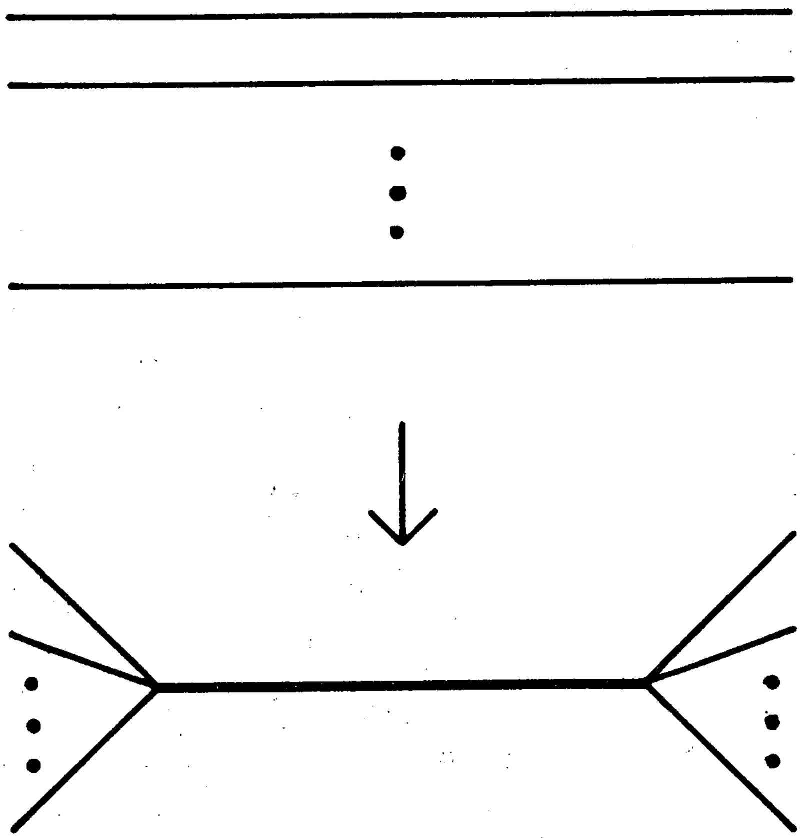

Figure 5

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