Title

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Neutron-Neutron Scattering Length from a Comparison of the $^2H(p,n)2p$ and $^2H(n,p)2n$ Reactions*

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Abstract: A study of the reaction $^2H(p,n)2p$ is reported which re-resolves the discrepancy among the values of the $^1S_0$ n-n scattering length extracted by three different groups from the mirror reaction $^2H(n,p)2n$. The value $a_n = -16.7^{+2.6}_{-3.0}$ fm obtained is now consistent with results from the $^2H(\pi^-,2n)\gamma$ and $^3H(d,{}^3He)2n$ reactions.

Studies of the differential energy spectra of the reaction $^2H(n,p)2n$ near 14 MeV (laboratory energy) have yielded values for the $^1S_0$ neutron-neutron scattering length which display a considerable spread:

$$a_n = -21.7 \pm 1 \text{ fm},^1 \; -23.6^{+2.0}_{-1.6} \text{ fm},^2 \text{ and } -14 \pm 3 \text{ fm},^3$$

where the assigned error in Ref. 1 was due only to statistics. References 1 and 3 used similar analyses appropriate to a long-range process that results from the large spatial extension of the deuteron.[4] Thus, the disagreement between those two results is unexplained, and its clarification is important to the validity of the assumption of charge symmetry in the nucleon-nucleon interaction.[5] Voitovetskii et al.[2] used a formalism based on Feynman diagrams. The result near the high energy end of the proton spectrum is similar to that of the Watson-Migdal[6,7] treatment which is, however, more appropriate for a short-range process. It has been pointed out that an analysis based on either the long-range assumption[4] or on a more rigorous three-body theory[8] would give a smaller value of a_n. Figure 1a shows that the three sets of data are self-consistent within the experimental errors, statistical uncertainties, and differences in resolution. Thus, the different values obtained for a_n result from differences among the analyses employed. An experimental test of the theoretical formulations is clearly desirable.

In contrast, a study[9] of the reaction $^3H(d,^3He)2n$ near 32 and 40 MeV has provided a value

$$a_n = -16.1 \pm 1.0 \text{ fm} \quad .$$

The validity of the Watson-Migdal theory employed in this work was verified through analysis of data from the mirror reaction $^3He(d,t)2p$. The deduced proton-proton scattering length agreed with the value known from low-energy p-p scattering.

Similarly, study of the reaction $^2H(p,n)2p$ can provide a test of the theory used in the analysis of data from the reaction $^2H(n,p)2n$. Spectra at 30 and 50 MeV[10] from $^2H(p,n)2p$ obtained with 1.4 and 2.0 MeV resolution respectively have been fitted quite well with the impulse-approximation of R. J. N. Phillips,[4] although the relatively poor resolution reduced the sensitivity of the fits to variations of the scattering length. Data have also been obtained at 14.1 MeV[11] and at 8.9 MeV,[12] and they are in qualitative agreement with the impulse approximation prediction.

We have studied the $^2H(p,n)2p$ reaction near 20 MeV laboratory energy, using protons from the Berkeley 88-inch variable-energy cyclotron. The target was gaseous deuterium, 99.9% pure, at one atmosphere pressure, enclosed in a cell with 13 mg/cm^2 aluminum entrance and exit windows. The neutron detection was accomplished with a proton-recoil spectrometer.[13] The overall resolution of the system, as determined empirically with the reaction $^{14}N(p,n)^{14}O$, was 600 keV in the relevant high energy region of the spectrum. Spectra were measured between 5° and 12° in the laboratory. They showed a small anisotropy of the neutron peak, in agreement with other experiments.[14] Figure 2a and 2b show the high energy region of the spectra obtained at 5° and 8° in the laboratory.

The differential energy spectrum can be written as

$$\frac{d^2\sigma}{dE d\Omega} = \frac{2\pi}{\hbar} \frac{1}{v_i} \sum_{\text{spins}} \left| T_{if} \right|^2 \rho \qquad (1)$$

where T_{if} is the transition matrix element, ρ is the density of final states, and v_i is the velocity of the projectile.

In general, $T_{if} = \int \Psi_f^\dagger V \Psi_i d\tau$, where V is the interaction causing the transition. In a reaction of the form

$$A + B \rightarrow X + 2N$$

the final-state interaction of the two-nucleon pair (2N) in a ^1S state of low relative energy results in a peak at the high energy end of the spectrum of particle X. Restricting our discussion to that region of the spectrum, and assuming that there the effect of the interaction between X and N is negligible, one can factor the wave function $\Psi_f = \Psi_{2N} \phi_R \Psi_X$, where ϕ_R describes the relative motion between X and the 2N system. For $r \geq b$, where b is the radius at which the internal and external wave functions are matched,

$$\Psi_{2n} = e^{i\delta}(\sin kr + \delta)/kr$$

and

$$\Psi_{2p} = e^{i\delta}[F_o(kr) \cos \delta + G_o(kr) \sin \delta]/kr \quad ,$$

where k is the relative N-N momentum in units of $\hbar$, δ is the ${}^1S_0$ phase shift, and $F_o(kr)$ and $G_o(kr)$ are the regular and irregular Coulomb S-wave functions. Thus, we have

$$\begin{aligned} T_{if} &= \frac{e^{-i\delta}\sin \delta}{k} \int (f_n \phi_R \Psi_X)^\dagger \, V\Psi_i d\tau \\ &= \frac{e^{-i\delta}\sin \delta}{k} \, g_n(\theta,k) \end{aligned} \qquad \text{for nn} \qquad (2a)$$

$$\begin{aligned} T_{if} &= \frac{e^{-i\delta}\sin \delta}{kC} \int (f_p \phi_R \Psi_X)^\dagger \, V\Psi_i d\tau \\ &= \frac{e^{-i\delta}\sin \delta}{kC} \, g_p(\theta,k) \end{aligned} \qquad \text{for pp} \qquad (2b)$$

with

$$\begin{aligned} f_n(k,r) &= (\sin kr \cot \delta + \cos kr)/r \quad , \quad r \geq b \\ &= f_n^o(k,r) \quad , \quad r \leq b \end{aligned}$$

$$\begin{aligned} f_p(k,r) &= C[F_o(kr) \cot \delta + G_o(kr)]/r \quad , \quad r \geq b \\ &= f_p^o(k,r) \quad , \quad r \leq b \end{aligned}$$

where $C^2 = 2\pi\eta/(\exp 2\pi\eta - 1)$, $\eta = e^2/(\hbar v)$, v is the p-p relative velocity, and θ is the c.m. angle of particle X. Equations (2) reduce to the Watson-Migdal short-range approximation and to the long-range impulse approximation under the appropriate assumptions.

For $r \leq b$ and for the small values of $\underline{k}$ which are important here, Ψ_{2n} and Ψ_{2p} are equal to within a few percent[15] and, in any case, contribute little to g_n and g_p because of the small overlap with the deuteron wave function contained in the initial state, Ψ_i. Also, it can be seen[16] that f_n and f_p have a remarkably similar energy dependence in the important range of values of $\underline{kr}$. Consequently, for mirror reactions at equivalent center of mass energies, we assume that

$$g_n(\theta,k) = \text{Const.}\ g_p(\theta,k) \qquad (3)$$

An experimental determination of $|g_p|^2$ thus provides a $|g_n|^2$ which can be used in the analysis of nn final-state spectra. The important point is that this obviates the need for a direct calculation of g_n via (2a). Therefore, the necessary approximations and uncertainties of such a calculation are eliminated. As an example, Fig. 1c shows a verification of (3) in the context of the calculation of Phillips.[4]

We have compared our $^2H(p,n)2p$ spectra with those calculated from (1) and (2b), using the ^1S effective range expansion for δ, with the known scattering length, $a_p = -7.7$ fm, and effective range, $r_e = 2.63$ fm.[17] This provides an experimental determination of $|g_p|^2$. With $|g_n|^2$ given by (3),[18] a comparable analysis was made of the statistically best nn final-state data.[2] Figure 1b shows the resulting best fit calculation, with

$$a_n = -16.7\ ^{+2.6}_{-3.0}\ \text{fm} \quad .$$

The probable errors were determined from a χ^2 criterion with a fixed (optimum) value for the spectrum endpoint energy.

The value of a_n deduced in our analysis of these $^2H(p,n)2p$ and $^2H(n,p)2n$ data is consistent with the values $a_n = -16.4 \pm 1.3$ fm and $a_n = -16.1 \pm 1$ fm determined respectively from the $^2H(\pi^-,2n)\gamma$[19] and $^3H(d,{}^3He)2n$[9] reactions.

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16. This point will be demonstrated in more detail in a later paper; but, for example, as $k \rightarrow 0$, $f_n = -(a_n)^{-1} + r^{-1}$ and $f_p = -(a_p)^{-1} + r^{-1}$ for $r \geq b$.

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18. Since our data were taken at 20 MeV, we have determined from the $^2H(p,n)2p$ data of Ref. 11 at 14 MeV that $|g_p|^2$ has the same dependence on k at these two incident proton energies.

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Figure Captions

Fig. 1a. Experimental spectra of the reaction $^2H(n,p)2n$ near 14 MeV. Triangles correspond to Ref. 1, circles to Ref. 2, and squares to Ref. 3. The data of Refs. 1 and 3 were energy shifted to superimpose them properly on the data of Ref. 2, for comparison purposes.

Fig. 1b. The dots are the $^2H(n,p)2n$ data of Ref. 2 corrected with the form factor $|g_n(\theta,k)|^2$ obtained from the mirror reaction $^2H(p,n)2p$. The solid line is a plot of the best fit with $a_n = -16.7$ fm. The dashed line is a plot of the form factor $|g_n(\theta,k)|^2$ as a function of the energy of the third particle (proton).

Fig. 1c. Form factors from the calculation of Ref. 4 are indicated with solid lines, the dashed line is the ratio $|g_p|^2/|g_n|^2$. The dash-dot line is the phenomenological form factor deduced from the $^2H(p,n)2p$ reaction.

Fig. 2. Data from the reaction $^2H(p,n)2p$ at 19.7 MeV obtained in the present experiment, and at 14.1 MeV taken from Ref. 11. Dashed lines are Watson-Migdal curves calculated with $a_p = -7.7$ fm. Solid lines are Watson-Migdal curves with $a_p = -13.3$ fm which show that the data can be simulated with a large value of a_p. Curves are normalized to the same area

Fig. 2a. Data at 19.7 MeV and 5° lab.

Fig. 2b. Data at 19.7 MeV and 8° lab.

Fig. 2c. Data at 14.1 MeV from Ref. 11, at 3° lab. The resolution at the high energy region of the spectrum is approximately 1 MeV.

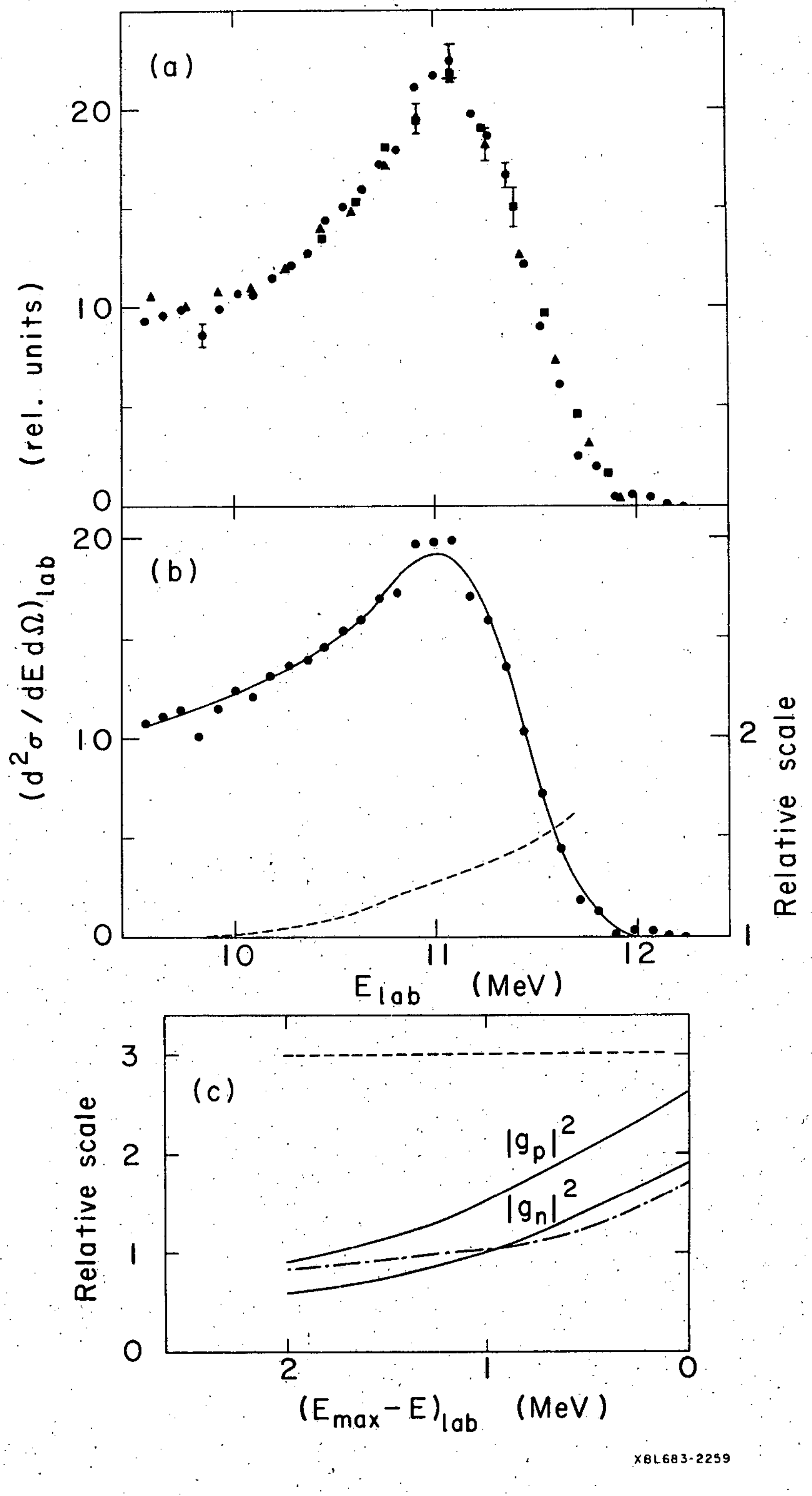
(a)
20
10
0
(rel. units)
(b)
$(d^2\sigma/dE\,d\Omega)_{lab}$
Relative scale
10
11
12
E_{lab} (MeV)
(c)
3
2
1
0
Relative scale
$|g_p|^2$
$|g_n|^2$
$(E_{max}-E)_{lab}$ (MeV)
XBL683-2259

Fig. 1

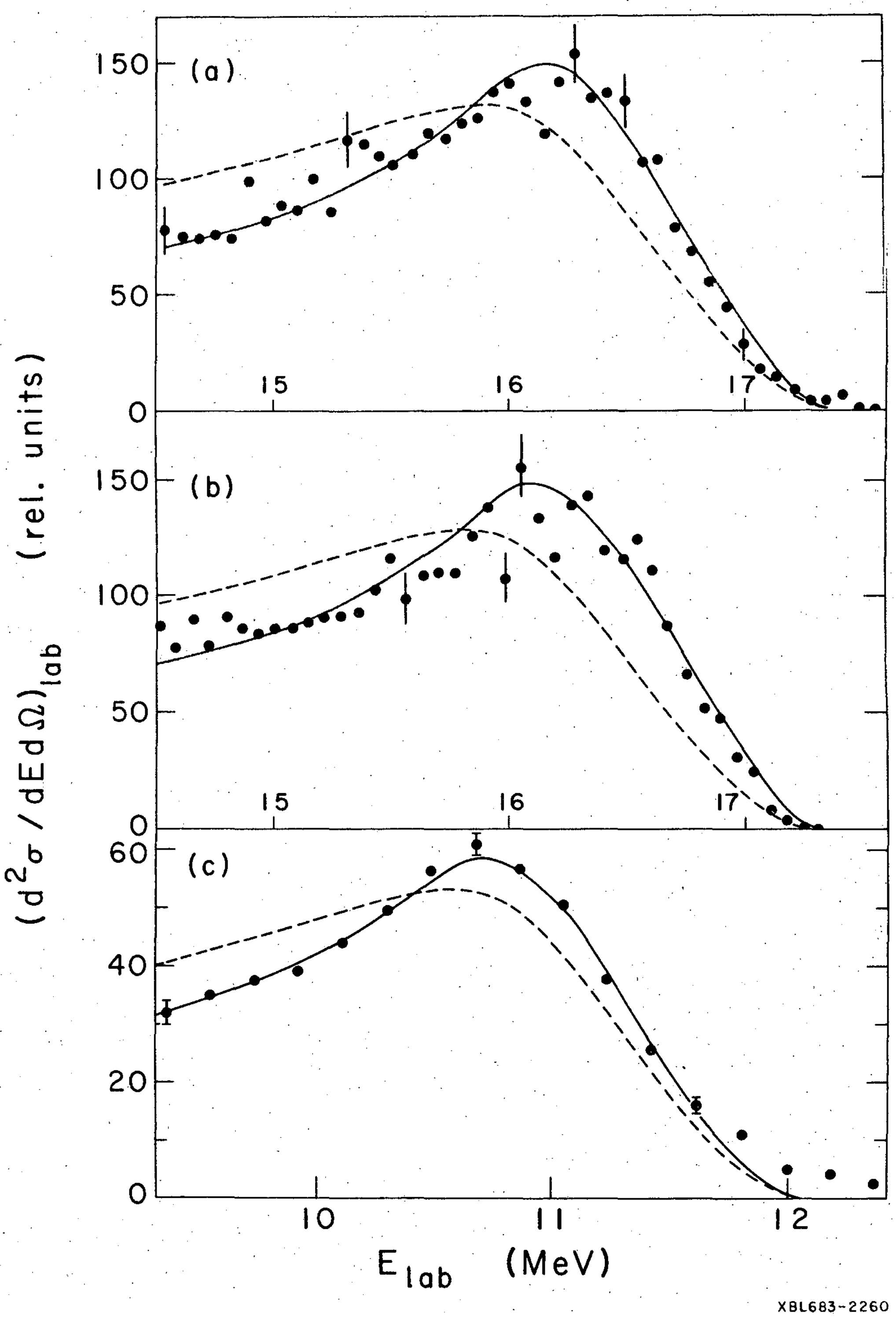

Fig. 2