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ABSTRACT

A study is made of the scattering of high energy protons by protons. Several types of "cutoffs" are introduced into the singular tensor interaction proposed by Christian and Noyes; the triplet P state radial equations are then solved by essentially exact numerical integration methods. The resulting cross sections show a more pronounced disagreement with experiment than do the Born approximation cross sections of Christian and Noyes. Calculations are carried out in the vicinity of 350 Mev and 120 Mev.

I. INTERPRETATION OF HIGH ENERGY p-p SCATTERING

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INTRODUCTION

Several experiments have been carried out on the scattering of protons by protons at energies greater than 100 Mev[1-4]. The resulting differential cross sections are characterized by spherically symmetric angular distributions (in the center of mass system) and by a lack of dependence on energy. Between scattering angles of 20^{o} and 160^{o} and between energies of 120 Mev and 350 Mev the cross section is about four or five millibarns per steradian. The results have been interpreted by Christian and Noyes[5] (hereafter referred to as "CN"), by Jastrow[6], and by Case and Pais[7]. In the CN analysis (350 Mev) a square well singlet interaction was used which gave almost no scattering at angles greater than 40^{o}. The problem then was to find a triplet interaction yielding an essentially isotropic differential cross section. It was observed that any triplet central potential is undesirable since the cross section due to it would vanish at 90^{o} (the wave function is antisymmetric), accordingly a tensor force model was chosen. (The wave function must of course still be anti-symmetric; however, with a noncentral potential the anti-symmetrization is not expressed in terms of the polar scattering angle, θ, alone, but by the azimuthal angle, ϕ, as well. The antisymmetric spin scattering matrix $S(\theta, \phi) - S(\pi - \theta, \phi + \pi)$ does not necessarily vanish at $\theta = \pi/2$ as it would if there were no ϕ dependence.)

In order to obtain the desired "flat" cross section, Christian and Noyes found it necessary to use a potential with a "highly singular" radial dependence $e^{-r/R}/r^2$. All triplet state calculations were carried out in Born approximation. Jastrow, on the other hand, attempted to obtain agreement with experiment by introducing a hard core into the singlet interaction, thus permitting greater momentum transfers and accordingly a substantial amount of large angle (90°) scattering. The triplet interaction was not then required to yield an isotropic cross section. Neither the CN nor the Jastrow interpretation was entirely successful in fitting the experimental data, the principal difficulty being too large a theoretical peak in the forward direction due mostly to scattering of the singlet D state. However it was not in the spirit of the analyses to indulge in a detailed program of "curve fitting" but rather to illustrate the important features of the various interactions chosen. This philosophy applies as well to the present paper.

It is proposed here to examine more critically the triplet state calculations of Christian and Noyes, and, in particular, to investigate the validity of their use of the Born approximation. Singlet scattering will be ignored. There is reason to suspect that results of the Born approximation applied to a highly singular potential may not be even qualitatively correct. Consider, for example, the radial equation for the $^3P_0$ state (Appendix, Equation A18). In the vicinity of the origin this takes the form

$$\frac{d^2u}{dy^2} + \frac{\lambda_0}{y^2} u = 0 \quad , \quad y = kr \tag{1}$$

(Choosing $\lambda_0 > 0$ implies that the nuclear potential is effectively attractive in this state, and sufficiently deep to dominate the centrifugal term as $r \to 0$.) The solution, for $\lambda_0 > 1/4$, is composed of spherical Bessel functions of imaginary order having an oscillatory singularity at the origin[8]:

$$u \underset{y \to 0}{\longrightarrow} \sqrt{y} \cos \left[\sqrt{\lambda_0 - 1/4} \log y + B \right] .$$

An interaction of this nature can be treated in a physically meaningful way only if the singularity at the origin is in some arbitrary way "cut off". It is evident, however, that the region of the cutoff cannot be arbitrarily small since several oscillations of the wave function within the region would lead to bound states of the di-proton. Consider the integral equation satisfied by the solution to Equation (A18):

$$u = A_1 g_1(y) + 4\lambda g_{-1}(y) \int_0^y \frac{e^{-ay}}{y^2} u g_1 \, dy + 4\lambda g_1(y) \int_y^\infty \frac{e^{-ay}}{y^2} u g_{-1} \, dy . \tag{2}$$

The left hand side of the equation becomes the wave function in Born approximation if the plane wave solution $u = g_1(y)$ is inserted as a trial function in the integrand. The Born approximation is valid if the exact solution does not deviate greatly from the free particle trial function. Near the origin the latter, $g_1(y)$, becomes just $\frac{1}{3} y^2$; the next zero occurs beyond the region in which the nuclear interaction is appreciable, even for energies as high as 350 Mev. It is therefore evident that for sufficiently short cutoffs the Born approximation is invalid since the exact (possibly oscillatory) solution does not resemble the trial function. Examination of the integral Equation (2) shows, moreover, that the presence of a short range cutoff has a negligible influence on the Born calculation itself, simply because the singularity in the potential is masked by the $\frac{1}{3} y^2$ factor from the trial function. (For convenience a square well cutoff may be visualized here; that is, the potential e^{-ay}/y^2 for $y \geq y_o$ is placed equal to the constant e^{-ay_o}/y_o^2 for $y \leq y_o$.) It is evident that the larger the cutoff radius the more nearly valid becomes the first order iteration procedure. On the other hand a long range cutoff cannot be ignored in a Born calculation. It seems, then, that the CN procedure (Born approximation without explicit introduction of cutoff) can be taken seriously only if there exists some kind of cutoff of sufficiently long range to permit first order perturbation methods to have real meaning, yet short enough so that the perturbation calculation itself is not appreciably influenced by its presence. It will be shown here that, strictly speaking, a cutoff fulfilling these two conditions does not exist.

PROCEDURE

The procedure adopted here is to introduce specific cutoffs into the CN interaction and obtain an essentially exact solution to the scattering problem by a numerical integration procedure. The cutoffs considered will be of two types: tensor force "square wells", in which the potential is given by

$$V(r) = \mp 15.2\, S_{12}\, \frac{e^{-r/R}}{(r/R)^2}\ \text{Mev} \quad \text{for} \quad r \geq r_o \qquad S_{12} \equiv 3\vec{\sigma}_1 \cdot \hat{r}\, \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$V(r) = \mp 15.2\, S_{12}\, \frac{e^{-r_o/R}}{(r_o/R)^2}\ \text{Mev} = \text{constant} \quad \text{for} \quad r \leq r_o \tag{3}$$

$$R = 1.6 \times 10^{-13}\ \text{cm.}$$

and "hard cores", where

$$V(r) = \infty \quad \text{for} \quad r \leq r_o \quad \text{and} \quad V(r) = \text{Equation (3) for} \quad r \geq r_o\,. \tag{4}$$

The $\mp$ sign refers to what will be called "attractive" and "repulsive" interactions, respectively. The Born cross section of course is the same for the two signs of the interaction.

In attempting to choose a more or less physically meaningful cutoff radius, r_o, the "nucleon Compton wave length" $\hbar/Mc$ is a convenient guide. Part of the motivation for choosing a radial dependence of the form $e^{-r/R}/r^2$ is its similarity to terms in the phenomenological interactions predicted by meson theories. Such motivation hardly exists at distances as short as $\hbar/Mc$ where, for example, the nucleon structure, as well as relativistic effects, may be expected to play an important role. On the other hand, to introduce a cutoff as large as $3\hbar/Mc$ (about 1/2 the meson Compton wavelength) more or less abandons the similarity to meson potentials. Essentially the same limits on r_o are obtained by a few rough calculations which indicate that a cutoff somewhat smaller than $\hbar/Mc$ would lead to a bound di-proton, and a radius greater than $3\hbar/Mc$ tends to destroy the desired isotropy of the cross section even in Born approximation. (The latter point is illustrated by a plot of the Born tensor amplitude in Figure 1.) The calculations were therefore carried out using a "short range cutoff", $r_o \approx \hbar/Mc$, and a "long range cutoff", $r_o \approx 2\hbar/Mc$, for both the square well and the hard core. The four cases considered will be denoted by the abbreviations:

SRSW: short range square well cutoff; Eq. (3) with $r_o = .24 \times 10^{-13}$ cm.
LRSW: long range square well cutoff; Eq. (3) with $r_o = .48 \times 10^{-13}$ cm.
SRHC: short range hard core cutoff; Eq. (4) with $r_o = .24 \times 10^{-13}$ cm.
LRHC: long range hard core cutoff; Eq. (4) with $r_o = .48 \times 10^{-13}$ cm.

The ${}^3P_o$, ${}^3P_1$ and ${}^3P_2$, ${}^3F_2$ states for the SRSW case were solved by numerical integration and checked by iterating the resulting radial functions (using the integral equations) to produce the same phase shifts and amplitudes to within a few percent. All other states (${}^3F_3$ ${}^3F_4$; ${}^3H_4$ etc.) were included in Born approximation, with cutoffs ignored. Some details of the procedure are given in the Appendix.

The phase shifts for the LRSW cutoff were then obtained by a perturbation method using as trial functions in the integral Equations (A21) the radial functions for the SRSW case, except in the ^3P_o state of the "repulsive" interaction, which was integrated numerically. (The ^3P_o state is effectively attractive in the "repulsive" interaction and repulsive in the "attractive" because of a minus sign appearing in the corresponding matrix element of the tensor operator S_{12}.) Inspection of the differential Equations (A18) and (A19) shows that the effective well depth in the ^3P_o state is twice as great and of the opposite sign as that of the $^3P_1$ state. From the remarks following Equations (A23) in the Appendix, it is apparent that the most important quantity in the coupled system is the P dominant P phase shift. Furthermore, in the P dominant mode the term

$$\frac{e^{-ay}}{y^2}\, 3\sqrt{6}\, w$$

is asymptotically smaller than the term $\frac{e^{-ay}}{y^2}$ u; from the power series expansion it is clear that it also starts out much smaller near the origin. Ignoring for the moment this coupling term, then, and comparing the size of the $^3P_2$ potential to the $^3P_1$ and ^3P_o, it is seen that the latter are, in absolute value, five and ten times as large as the former. Accordingly it is reasonable to think that the ^3P_o phase shift in the "repulsive" case and the $^3P_1$ in the "attractive" will exhibit a great deal more sensitivity to the nature of the cutoff than will the coupled $^3P_2$, $^3F_2$ states. The perturbation calculations for the long range square well cutoffs indeed show just this sort of behaviour. The coupled phase shifts in fact differ negligibly from those of the SRSW cutoff.

The foregoing arguments indicate that a fair approximation to the hard core cutoff cross sections should result from taking the core into consideration only in the ^3P_o and $^3P_1$ states and using the square well cutoff phase shifts in the coupled states. However, the following somewhat more refined procedure was used which still avoids the labor of repeating the coupled numerical integrations. Starting with the unperturbed SRSW solutions, the P-dominant P phase shift, $\delta_{11}^{\;2}$, is added to the "hard sphere" P phase shift,

$$\tan \delta_p^{hs} = + \frac{J_{3/2}(y_o)}{J_{-3/2}(y_o)} \quad .$$

The nature of the approximation can be readily seen by considering a similar procedure for an uncoupled integral equation (see Appendix for notation):

$$\sin \delta_{hc} = \int_0^{y_o} U_a\, u_a\, g_1\, dy + \int_{y_o}^{\infty} U_b\, u_b\, g_1\, dy \approx \sin \delta_p^{hs} + \sin \delta_{sw} \tag{5}$$

$$\delta_{hc} \approx \delta_p^{hs} + \delta_{sw}$$

U_a = strong repulsion (approximates hard core).

U_b = e^{-ay}/y^2 for $y \geq y_o$, = $e^{-ay_o}/y_o^{\,2}$ = constant for $y \leq y_o$.

Trial function u_a = exact solution when $U_b = 0$.

Trial function u_b = exact solution when $U_a = 0$.

δ_{sw} = phase shift for square well cutoff.

δ_{hc} = phase shift for hard core cutoff.

δ_p^{hs} = P phase shift for hard core alone.

$$\int_0^{y_o} U_b u_b g_1 \, dy \quad \text{is neglected.}$$

Analogous treatment of a_{13}^2, a_{31}^2, ${}^{2}_{33}$ shows that the influence of the hard core on these quantities is negligible. The phase shifts δ_{13}^2, δ_{31}^2 (unperturbed SRSW value is $\pi/2$) feel the core somewhat more strongly; however they may deviate as much as 20% from $\pi/2$ without changing the δ^{Jm_s} by more than a few percent. (Equations A8.)

All hard core coupled PF phase shifts were obtained in the manner just indicated; all uncoupled P-state equations were integrated numerically.

RESULTS AND CONCLUSIONS

Phase shifts and differential cross sections at 350 Mev are given in Table II and Figure 2. The "attractive" interaction evidently leads to a greater anisotropy of the triplet cross section than does the "repulsive", regardless of the nature of the cutoff. The near agreement of the exact cross sections at 350 Mev with those calculated in Born approximation is surprising in view of the large discrepancies in the corresponding phase shifts. Similar discrepancies at 129 Mev lead to an exact cross section much larger than that obtained in Born approximation (Figure 3); apparently, then, the close agreement at 350 Mev is accidental.

Figure 2 also indicates that the greater the "volume" of potential removed by the cutoff the greater is the angular variation of the cross section; Figure 1 illustrates the same point in Born approximation.

The SRSW cutoff for the "repulsive" case was calculated in detail at 129 Mev. The results, Figure 3 (curve A) and Table III, show that the predicted scattering is much too great. The trouble comes almost entirely from the large ${}^3P_o$ phase shift. To investigate the effect (at 129 Mev) of modifying the cutoff, attention will be restricted to the ${}^3P_o$ state. (The arguments of the preceding section indicate that the coupled phase shifts are only slightly influenced by the nature of the potential at short range; the ${}^3P_1$ state is repulsive and so obviously insensitive to the cutoff.) A ${}^3P_o$ phase shift of .80 (instead of the 1.8 of Table III) yields roughly the desired cross section (Figure 3, curve B). The required phase shift can be produced, for example, by the combination of a square well cutoff at $.48 \times 10^{-13}$ cm. and a hard core of radius $.24 \times 10^{-13}$ cm., (or, of course, by a hard core alone of radius somewhat larger than $.24 \times 10^{-13}$ cm.). The cross section at 350 Mev will then in any case lie between that of the SRHC and the LRHC cutoffs ~~shown in Figure 2.~~

It is concluded, therefore, that, within the framework of the singlet and triplet models adopted by Christian and Noyes, something similar to the following triplet potential seems to yield the closest approach to the experimental cross sections (Figure 4) at 120 Mev and 350 Mev:

$$V(r) = 15.2\, S_{12} \frac{e^{-r/R}}{(r/R)^2} \text{ Mev} , \qquad r \geq r_1 = .48 \times 10^{-13} \text{ cm.},$$

$$V(r) = 15.2\, S_{12} \frac{e^{-r_1/R}}{(r_1/R)^2} \text{ Mev} = \text{constant}, \qquad r_o \leq r \leq r_1 , \tag{6}$$

$$V(r) = \infty \qquad r \leq r_o = .24 \times 10^{-13} \text{ cm.}$$

It is to be emphasized that significance should be attached only to the necessary degree of "severeness" (i.e., volume of potential affected) of the cutoff and not to its precise nature.

It should be mentioned that a cutoff sufficiently short to increase the ^3P_o phase shift at 129 Mev to 2.2 is not obviously less desirable than (6) (see Figure 3, curve C); the 350 Mev scattering would be changed, but not drastically. The effect of so short a cutoff would be more pronounced at some energy less than 120 Mev where the ^3P_o phase shift will have decreased to $\pi/2$.

Using the potential given by (6), the discrepancy with the experimental forward scattering is considerably greater than originally was indicated by the CN calculations. The disagreement seems sufficiently conclusive to justify ruling out a large class of static potentials for the p-p interaction. The class of inacceptable potentials is by no means exhaustive, however. Whenever a strong short range component (e.g., hard core) is included in the singlet interaction (thus permitting large angle scattering), the triplet potential acquires several more degrees of freedom since the requirement of isotropy may be dropped. In particular, triplet central potentials then merit consideration.

APPENDIX

The nucleon-nucleon scattering problem for a noncentral static potential will be formulated and discussed. The notation and method of treatment adapts conveniently to a description of polarization effects carried out in a concurrent paper[9].

The asymptotic form of the triplet state wave function can be written[10]:

$$\Psi \sim e^{i\,kz}\chi_{\text{inc.}} + \frac{e^{i\,kr}}{r}\,S\,\chi_{\text{inc.}} \tag{A1}$$

$\chi_{\text{inc.}}$ = triplet spin function of initial state where $S(\theta, \phi)$ is the triplet spin scattering operator, the matrix for which is given explicitly in terms of the complex phase shifts, $\delta_\ell^{Jm_s}$, by

$$S = \frac{1}{2ik}\sum_{J,\,m_s,\,\ell}(2\ell+1)(e^{2i\,\delta_\ell^{Jm_s}} - 1)\,\eta_\ell^{Jm_s}\,P_\ell(\cos\theta)$$

$$= \frac{1}{4ik}\begin{array}{c} \begin{array}{ccc} 1 & 0 & -1 \end{array} \;\; m_s' \\ \left|\begin{array}{ccc} A & \sqrt{2}\,B\,e^{-i\,\phi} & C\,e^{-2i\,\phi} \\ -\sqrt{2}\,D\,e^{i\,\phi} & 2\,E & \sqrt{2}\,D\,e^{-i\,\phi} \\ C\,e^{2\,i\,\phi} & -\sqrt{2}\,B\,e^{i\,\phi} & A \end{array}\right| \end{array} \begin{array}{c} m_s \\ 1 \\ 0 \\ -1 \end{array} \tag{A2}$$

$$A = \sum_{\ell} \left[(\ell+2) A_{\ell}^{\ell+1} + (2\ell+1) A_{\ell}^{\ell} + (\ell-1) A_{\ell}^{\ell-1} \right] P_{\ell}$$

$$B = \sum_{\ell} \left[B_{\ell}^{\ell+1} - B_{\ell}^{\ell-1} \right] P_{\ell}'$$

$$C = \sum_{\ell} \left[\frac{1}{\ell+1} A_{\ell}^{\ell+1} - \frac{2\ell+1}{\ell(\ell+1)} A_{\ell}^{\ell} + \frac{1}{\ell} A_{\ell}^{\ell-1} \right] P_{\ell}^{2}$$

$$D = \sum_{\ell} \left[\frac{\ell+2}{\ell+1} A_{\ell}^{\ell+1} - \frac{2\ell+1}{\ell(\ell+1)} A_{\ell}^{\ell} - \frac{\ell-1}{\ell} A_{\ell}^{\ell-1} \right] P_{\ell}'$$

$$E = \sum_{\ell} \left[(\ell+1) B_{\ell}^{\ell+1} + \ell B_{\ell}^{\ell-1} \right] P_{\ell}$$

$$A_{\ell}^{J} = e^{2i\,\delta_{\ell}^{J,\pm 1}} - 1$$

$$B_{\ell}^{J} = e^{2i\,\delta_{\ell}^{J_o}} - 1 \; . \qquad \text{(A3)}$$

$\pi_{\ell}^{Jm_s} P_{\ell}$ is an operator in triplet spin space defined by Eq. (A6). Coulomb scattering is neglected. The boundary conditions of the scattering problem yield also the relationship between the complex phase shifts and the asymptotic form of the radial wave functions. To obtain this relationship, first expand the wave function of the system in eigenfunctions, ψ^{Jm}, of total angular momentum J^2, and J_z. Separate the radial from the spin-angular dependence by means of the expansion

$$\psi^{Jm} = \sum_{\ell} \frac{u_{\ell}^{J}(r)}{r} \psi_{\ell}^{Jm}$$

where the ψ_{ℓ}^{Jm} are eigenfunctions of J^2, J_z and the orbital angular momentum L^2. The Schrodinger equation for the radial functions becomes:

$$\frac{d^2 u_\ell^J}{dr^2} - \frac{\ell(\ell+1)}{r^2} u_\ell^J + k^2 u_\ell^J - \frac{M}{\hbar^2} \sum_{\ell'=J-1}^{J+1} V_{\ell\ell'}^J(r)\, u_{\ell'}^J = 0 \tag{A4}$$

where $V_{\ell\ell'}^J(r) = (\psi_\ell^{Jm}, V(r_1 \sigma_1, \sigma_2)\, \psi_{\ell'}^{Jm})$ is independent of m. The scalar product denotes an integration over the surface of a sphere and summation over spin variables.

For a tensor interaction, the orbital angular momentum is not a constant of the motion and $V(r, \sigma_1, \sigma_2)$ contains off-diagonal elements between states of the same parity[11].

$$S_{12} \equiv 3\, \vec{\sigma}_1 \cdot \hat{r}\; \vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$= \frac{1}{2J+1} \begin{array}{c|ccc|c} & J-1 & J & J+1 & \ell' / \ell \\ \hline & -2(J-1) & 0 & 6\sqrt{J(J+1)} & J-1 \\ & 0 & 2(2J+1) & 0 & J \\ & 6\sqrt{J(J+1)} & 0 & -2(J+2) & J+1 \end{array} \tag{A5}$$

The orthonormal set of spin-angular functions, ψ_ℓ^{Jm}, can be expressed in terms of spherical harmonics and spin functions by means of the Clebsch-Gordon expansion:

$$\psi_{\ell}^{Jm} = \sum_{m_\ell, m_s} s_{Jm_\ell\, m_s}^{\ell s}\; Y_{m_\ell}^{\ell}(\theta, \phi)\, \chi_{m_s}^{s} \qquad \text{and}$$

$$Y_{m_\ell}^{\ell}\, \chi_{m_s}^{s} = \sum_{J} s_{Jm_\ell\, m_s}^{\ell s}\; \psi_{\ell}^{J, m_\ell + m_s} \,.$$

Defining the projection operator

$$\pi_{\ell}^{Jm}\left(Y_{m-m_s}^{\ell}\, \chi_{m_s}^{s}\right) = s_{J,m-m_s,m_s}^{\ell s}\, \psi_{\ell}^{Jm} \qquad \text{so that}$$

(A6)

$$s_{J_o m_s}^{\ell 1}\, \psi_{\ell}^{Jm_s} = \pi_{\ell}^{Jm_s}\left(Y_o^{\ell}\, \chi_{m_s}^{1}\right) ,$$

the general expansion for the wave function of the system takes the form

$$\overline{\psi} = \sum_{Jm} \sum_{\ell=J-1}^{J+1} \sum_{i=1}^{2} \frac{C_i^{Jm}}{s_{J,m-m_s,m_s}^{\ell 1}}\, \frac{u_i^J(r)}{r}\, \pi_{\ell}^{Jm}\left(Y_{m-m_s}^{\ell}\, \chi_{m_s}^{1}\right)$$

(A7)

$$u_{\ell i}^{J} \sim a_{\ell i}^{J} \sin\left(kr - \ell\pi/2 + \delta_{\ell i}^{J}\right)$$

The subscript i is summed over the two regular solutions to the coupled equation in (A4) (see discussion following Equations A20). For uncoupled states, put $C_2^{Jm} = 0$. The asymptotic form of (A7) is the same as (A1) with S defined by (A2) provided that[5]

$$D_J\, e^{2i\delta^{J,\pm 1}_{J-1}} = E_J + 2i\sqrt{\frac{J}{J+1}}\; W = \left(D_J\, e^{2i\delta^{J,0}_{J+1}}\right)^*$$

$$D_J\, e^{2i\delta^{J,0}_{J-1}} = E_J - 2i\sqrt{\frac{J+1}{J}}\; W = \left(D_J\, e^{2i\delta^{J,\pm 1}_{J+1}}\right)^* \qquad \text{(A8)}$$

where:

$$D_J \equiv e^{-i(\delta^J_{J-1,J-1} + \delta^J_{J+1,J+1})} - a^J_{J-1,J+1}\, a^J_{J+1,J-1}\, e^{-i(\delta^J_{J-1,J+1} + \delta^J_{J+1,J-1})}$$

$$E_J \equiv e^{i(\delta^J_{J-1,J-1} - \delta^J_{J+1,J+1})} - a^J_{J-1,J+1}\, a^J_{J+1,J-1}\, e^{i(\delta^J_{J-1,J+1} - \delta^J_{J+1,J-1})}$$

$$W = a^J_{J-1,J+1} \sin(\delta^J_{J-1,J-1} - \delta^J_{J-1,J+1}) = a^J_{J+1,J-1} \sin(\delta^J_{J+1,J+1} - \delta^J_{J+1,J-1})$$

$$a^J_{J-1,J-1} = a^J_{J+1,J+1} = 1 \quad . \qquad \text{(A9)}$$

$\delta^{Jm_s}_{\ell} = \delta^J_{\ell}$ for all uncoupled states.

The subscript i in Equations (A7) here takes on the values $J-1$, $J+1$ instead of 1, 2. The Wronskian conditions (A9) follow immediately from the differential Equations (A4) or (A20):

$$u_1 u_2' - u_2 u_1' + w_1 w_2' - w_2 w_1' = \text{constant} \quad . \qquad \text{(A10)}$$

Boundary conditions at the origin require the constant to be zero; the asymptotic form of (A10) is (A9).

In Born approximation, Equations (A2) and (A8) become:

$$^{B}S = \frac{1}{k} \sum_{J,\ell,m_s} (2\ell+1)\, {}^{B}\delta_{\ell}^{Jm_s}\, \eta_{\ell}^{Jm_s}\, P_{\ell} \tag{A11}$$

$$^{B}\delta_{\ell}^{Jm_s} = {}^{B}\delta_{\ell\ell}^{J} - \varepsilon_{\ell}^{Jm_s}\, {}^{B}a_{\ell L}^{J} \qquad \ell = J-1,\ J+1 \qquad L = 2J - \ell \tag{A12}$$

$$\varepsilon_{J-1}^{J\pm} = -\varepsilon_{J+1}^{J_o} = \frac{1}{\varepsilon_{J+1}^{J\pm}} = -\frac{1}{\varepsilon_{J-1}^{J_o}} = \sqrt{\frac{J}{J+1}} \; .$$

For any linear combination of central and tensor potentials, with arbitrary exchange dependence, $V(r, \sigma_1, \sigma_2) = \left[- J(r)S_{12} - J_c(r)\right]\left[a + bP_x\right]$, Equation (A11) can be written in the closed form[10]:

$$^{B}S = F\,1 + \begin{vmatrix} C_1 & C_2\, e^{-i\phi} & C_3\, e^{-2i\phi} \\ C_2\, e^{i\phi} & -2\,C_1 & -C_2\, e^{-i\phi} \\ C_3\, e^{2i\phi} & -C_2\, e^{i\phi} & C_1 \end{vmatrix} \tag{A13}$$

where

$$F = a\,F(\theta) + b\,F(\pi - \theta) \equiv a\,F_K + b\,F_L \quad ; \quad F(\theta) = \frac{M}{\hbar^2}\int r^2\, J_C(r)\, \frac{\sin Kr}{Kr}\, dr$$

$$C_1 = -\tfrac{1}{2}\, C_+ + \tfrac{3}{2}\cos\theta\, C_- \qquad \text{(A14)}$$

$$C_2 = \frac{3}{\sqrt{2}}\sin\theta\, C_-$$

$$C(\theta) = \frac{M}{\hbar^2}\int r^2\, J(r)\, \frac{g_2(Kr)}{Kr}\, dr$$

$$C_3 = -\frac{3}{2}\left[C_+ + \cos\theta\, C_-\right]$$

$$C_\pm = a\,C_K \pm b\,C_L \equiv a\,C(\theta) \pm b\,C(\pi - \theta) .$$

The procedure for calculating S will be to remove from the Born scattering matrix (A13) the first few terms of its partial wave expansion (A11), and to replace them by the corresponding terms in the exact scattering matrix. The result will then correspond to a scattering matrix containing explicitly the phase shifts of the few lowest angular momentum states and implicitly the Born approximation on all higher states.

$$S = {}^{B}S + S'$$

$$A'^{J}_{\ell} = e^{2i\,\delta^{J,\pm1}_{\ell}} - 1 - 2i\;{}^{B}\delta^{J,\pm1}_{\ell} \quad ; \quad B'^{J}_{\ell} = e^{2i\,\delta^{J,0}_{\ell}} - 1 - 2i\;{}^{B}\delta^{J,0}_{\ell} \qquad \text{(A15)}$$

where S' is defined analogously to equations (A2), (A3), but with A'^{J}_{ℓ}, B'^{J}_{ℓ} replacing A^{J}_{ℓ}, B^{J}_{ℓ}.

For p-p scattering, replace $S(\theta, \phi)$ with $S(\theta, \phi) - S(\pi - \theta, \phi + \pi)$.

The triplet contribution to the differential scattering cross section, reduced to terms containing just Legendre polynomials is:

(A16)

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{triplet}} = \left(\frac{d\sigma}{d\Omega}\right)^{\text{Born}}_{\text{trip.}} + \left(\frac{d\sigma}{d\Omega}\right)_{TI} + \left(\frac{d\sigma}{d\Omega}\right)_{CI} + \left(\frac{d\sigma}{d\Omega}\right)' = \frac{1}{4}\,\mathrm{Tr}(S^{+}S)$$

$$\left(\frac{d\sigma}{d\Omega}\right)^{\text{Born}}_{\text{trip.}} = \frac{1}{4}\,\mathrm{Tr}({}^{B}S^{+}\,{}^{B}S) = \frac{3}{4}\left|F\right|^2 + 6\left[a^2 C_K^{\,2} + b^2 C_L^{\,2} - ab\, C_K C_L\right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{T.I.} = -\frac{1}{4k}\left\{\left[aC_K + bC_L\right]\left[\varepsilon_0(\Delta_0 + \Delta_2 P_2) + \varepsilon_2(\Delta_1 P_1 + \Delta_3 P_3)\right]\right.$$

$$\left. + \left[aC_K - bC_L\right]\left[\varepsilon_2(\nu_0 + \nu_2 P_2) + \varepsilon_0 \nu_1 P_1\right]\right\}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{CI.} = \frac{1}{4\,k}\left[aF_K + bF_L\right]\left[\varepsilon_0(\Delta_0^{C} + \Delta_2^{C} P_2)\right.$$

$$\left. + \varepsilon_2(\Delta_1^{C} P_1 + \Delta_3^{C} P_3)\right].$$

$$\left(\frac{d\sigma}{d\Omega}\right)' = \frac{\varepsilon_4}{4k^2}\left[\Lambda_0 + \Lambda_1 P_1 + \Lambda_2 P_2 + \Lambda_3 P_3 + \Lambda_4 P_4\right] = \frac{1}{4}\,\mathrm{Tr}(S'^{+} S')$$

(For n-p scattering, $\varepsilon_0 \equiv \varepsilon_2 \equiv \varepsilon_4 \equiv 1.$) (For p-p scattering, $\varepsilon_0 \equiv 0$, $\varepsilon_2 \equiv 2$, $\varepsilon_4 \equiv 4$ and $a \equiv 1$, $b \equiv -1.$)

$$\Delta_0 = -\Delta_0^{10} + \Delta_0^{11} + \tfrac{3}{2}\Delta_2^{11} - \tfrac{5}{2}\Delta_2^{21} + \Delta_2^{31}$$

$$\Delta_2 = -2\Delta_2^{10} - \Delta_2^{11} + 5\Delta_2^{21} + \Delta_2^{31} - 3\Delta_2^{30}$$

$$\Delta_1 = -\Delta_1^{00} + \tfrac{3}{2}\Delta_1^{11} + \tfrac{3}{2}\Delta_1^{21} - 2\Delta_1^{20} + 3\Delta_3^{21}$$

$$\Delta_3 = -2\Delta_3^{21} - 3\Delta_3^{20}$$

$$\Delta_0^{c} = \Delta_0^{10} + 2\Delta_0^{11}$$

$$\Delta_2^{c} = 2\Delta_2^{10} + \Delta_2^{11} + 5\Delta_2^{21} + 3\Delta_2^{30} + 4\Delta_2^{31}$$

$$\nu_0 = 3\Delta_1^{00} + \tfrac{3}{2}\Delta_1^{21} + \Delta_3^{21} - \tfrac{9}{2}\Delta_1^{11}$$

$$\nu_2 = 6\Delta_1^{20} - 6\Delta_1^{21} + 9\Delta_3^{20} - 4\Delta_3^{21}$$

$$\nu_1 = 3\Delta_0^{10} - 3\Delta_0^{11} + 6\Delta_2^{10} - \tfrac{3}{2}\Delta_2^{11} - \tfrac{15}{2}\Delta_2^{21} + 3\Delta_2^{31}$$

$$\Delta_1^{c} = \Delta_1^{00} + 3\Delta_1^{11} + 2\Delta_1^{20} + 3\Delta_1^{21}$$

$$\Delta_3^{c} = 3\Delta_3^{20} + 2\Delta_3^{21}.$$

(A17)

$$\Lambda_0 = \Delta_{11}^{000} + 3\Delta_{11}^{111} + 2\Delta_{11}^{220} + 3\Delta_{11}^{221} + 3\Delta_{33}^{220} + 2\Delta_{33}^{221} + \Delta_{00}^{110} + 2\Delta_{00}^{111} + 2\Delta_{22}^{110}$$

$$+ \Delta_{22}^{111} + 5\Delta_{22}^{221} + 3\Delta_{22}^{330} + 4\Delta_{22}^{331} \quad .$$

$$\Lambda_2 = 4\Delta_{11}^{020} + \frac{3}{2}\Delta_{11}^{111} + 9\Delta_{11}^{121} + 2\Delta_{11}^{220} + \frac{3}{2}\Delta_{11}^{221} + 6\Delta_{13}^{020} + 6\Delta_{13}^{121}$$

$$+ \frac{12}{7}\Delta_{13}^{220} - \frac{6}{7}\Delta_{13}^{221} + \frac{24}{7}\Delta_{33}^{220} + \frac{8}{7}\Delta_{33}^{221} + 4\Delta_{02}^{110} + 2\Delta_{02}^{111}$$

$$+ 10\Delta_{02}^{121} + 6\Delta_{02}^{130} + 8\Delta_{02}^{131} + 2\Delta_{22}^{110} - \frac{1}{2}\Delta_{22}^{111} + \frac{25}{14}\Delta_{22}^{221}$$

$$+ \frac{24}{7}\Delta_{22}^{331} + \frac{24}{7}\Delta_{22}^{330} + 5\Delta_{22}^{121} + \frac{40}{7}\Delta_{22}^{231} - \frac{8}{7}\Delta_{22}^{131} + \frac{12}{7}\Delta_{22}^{130} \quad .$$

$$\Lambda_4 = \frac{72}{7}\Delta_{13}^{220} + \frac{48}{7}\Delta_{13}^{221} + \frac{18}{7}\Delta_{33}^{220} - \frac{8}{7}\Delta_{33}^{221} + \frac{40}{7}\Delta_{22}^{221} + \frac{100}{7}\Delta_{22}^{231}$$

$$+ \frac{4}{7}\Delta_{22}^{331} + \frac{36}{7}\Delta_{22}^{131} + \frac{72}{7}\Delta_{22}^{130} + \frac{18}{7}\Delta_{22}^{330} \quad .$$

(Eq. A17 cont.)

(Eq. (A17) cont.)

$$\Lambda_1 = 2\Delta_{10}^{010} + 6\Delta_{01}^{111} + 6\Delta_{01}^{121} + 4\Delta_{01}^{120} + 4\Delta_{12}^{010} + 3\Delta_{12}^{111} + 9\Delta_{12}^{121}$$

$$- \frac{3}{5}\Delta_{21}^{121} + \frac{4}{5}\Delta_{21}^{120} + 3\Delta_{12}^{221} + \frac{48}{5}\Delta_{12}^{231} + \frac{36}{5}\Delta_{12}^{230} \; .$$

$$\Lambda_3 = 6\Delta_{12}^{121} + \frac{18}{5}\Delta_{21}^{121} + \frac{36}{5}\Delta_{21}^{120} + 12\Delta_{12}^{221} + 6\Delta_{12}^{030} + 12\Delta_{12}^{131}$$

$$+ \frac{12}{5}\Delta_{12}^{231} + \frac{24}{5}\Delta_{12}^{230} + 4\Delta_{03}^{121} + 6\Delta_{03}^{120} \; .$$

where

$$\Delta_\ell^{J1} = \Delta_\ell^{J,-1} = \mathrm{Im}\, A_\ell^{,J} \qquad \Delta_{\ell\ell}^{JJ'1} = \frac{1}{4} R\, A_\ell^{,J}(A_{\ell'}^{,J'})^*$$

$$\Delta_\ell^{J0} = \mathrm{Im}\, B_\ell^{,J} \qquad \Delta_{\ell\ell'}^{JJ'0} = \frac{1}{4} R\, B_\ell^{,J}(B_{\ell'}^{,J'})^*$$

(A17b)

The identity $A_{J-1}^{J} - A_{J+1}^{J} = B_{J-1}^{J} - B_{J+1}^{J}$ follows from (A8), (A9).

The summation has been carried out explicitly over the $^3S_1$, $^3P_0$, $^3P_1$, $^3P_2$, $^3D_1$, $^3D_2$, $^3D_3$, and $^3F_2$ states. In $\left(\frac{d\sigma}{d\Omega}\right)'$, DF interference has been omitted. For p-p scattering, terms which contain an even subscript do not appear and (A17) simplifies considerably.

The radial differential equations for which "exact" solutions were obtained in the present paper will be considered now in more detail.

Let $U = \frac{e^{-ay}}{y^2}$ for $y \geq y_o$, $= \frac{e^{-ay_o}}{y_o^{\,2}}$ for $y \leq y_o$ = constant.

$$^3P_0 \qquad \frac{d^2u(0)}{dy^2} - \frac{2}{y^2}u(0) + u(0) = 4\lambda U u(0) \qquad \text{(A18)}$$

$$^3P_1 \qquad \frac{d^2u(1)}{dy^2} - \frac{2}{y^2}u(1) + u(1) = -2\lambda U u(1) \qquad \text{(A19)}$$

$$\begin{array}{l} ^3P_2 \\ \\ ^3F_2 \end{array} \quad \left\{ \begin{array}{l} \dfrac{d^2u}{dy^2} - \dfrac{2}{y^2}u + u = \dfrac{2}{5}\lambda U(u - 3\sqrt{6}\,w) \\ \\ \dfrac{d^2w}{dy^2} - \dfrac{12}{y^2}w + w = \dfrac{2}{5}\lambda U(4w - 3\sqrt{6}\,u) \end{array} \right\}. \qquad \text{(A20)}$$

$$a = \frac{1}{kR} \quad ; \quad \lambda = \frac{MV_0R^2}{\hbar^2} \quad ; \quad k^2 = \frac{ME}{\hbar^2} \quad ; \quad y = kr \quad .$$

The potential $V(r)$ is given in Equation (3).

There are four independent sets of solutions,

$$\begin{Bmatrix} u_1 \\ w_1 \end{Bmatrix}, \begin{Bmatrix} u_2 \\ w_2 \end{Bmatrix}, \begin{Bmatrix} u_3 \\ w_3 \end{Bmatrix}, \begin{Bmatrix} u_4 \\ w_4 \end{Bmatrix}$$

to equations (A20). Examination of the power series representation in the neighborhood of the origin shows that two of the solutions always vanish at the origin, and the other two are irregular and must be discarded because of the usual arguments on quadratic integrability and conservation of current. The two regular solutions, $\begin{Bmatrix} u_1 \\ w_1 \end{Bmatrix}, \begin{Bmatrix} u_2 \\ w_2 \end{Bmatrix}$ will be called a "fundamental set". Any set of solutions arising from a linear transformation of the fundamental set will also satisfy all of the boundary conditions of the scattering problem and hence may be used to calculate the complex phase shifts δ^{Jm_s}. It is not difficult to give a plausibility argument showing that there ought to be one pair of solutions,

$$\begin{Bmatrix} u_1 \longrightarrow u_{11}^2 \\ w_1 \longrightarrow u_{31}^2 \end{Bmatrix}$$

in which the P state is dominant, at least asymptotically, and another pair,

$$\begin{Bmatrix} u_2 \longrightarrow u_{13}^2 \\ w_2 \longrightarrow u_{33}^2 \end{Bmatrix}$$

in which the F state is dominant. Consider the integral equations corresponding to the coupled differential equations (A20) and their boundary conditions:

$$u^{2}_{1\alpha}(y) = A^{2}_{1\alpha}\, g_1(y) + \tfrac{2}{5}\lambda g_{-1}(y)\int_0^y U\left(u^{2}_{1\alpha} - 3\sqrt{6}\, u^{2}_{3\alpha}\right) g_1\, dy$$

$$+ \tfrac{2}{5}\lambda g_1(y)\int_y^\infty U\left(u^{2}_{1\alpha} - 3\sqrt{6}\, u^{2}_{3\alpha}\right) g_{-1}\, dy\,;$$

(A21)

$$u^{2}_{3\alpha}(y) = A^{2}_{3\alpha}\, g_3(y) + \tfrac{2}{5}\lambda g_{-3}(y)\int_0^y U\left(4\, u^{2}_{3\alpha} - 3\sqrt{6}\, u^{2}_{1\alpha}\right) g_3\, dy$$

$$+ \tfrac{2}{5}\lambda g_3(y)\int_y^\infty U\left(4u^{2}_{3\alpha} - 3\sqrt{6}\, u^{2}_{1\alpha}\right) g_{-3}\, dy$$

$$g_{\pm\ell}(y) = \sqrt{\frac{\pi y}{2}}\; J_{\pm(\ell+\frac{1}{2})}(y)\quad ;\quad g_1 \xrightarrow[y\to 0]{} \frac{y^2}{3}\ ;\quad g_3 \xrightarrow[y\to 0]{} \frac{y^4}{105}\ .$$

The constants $A^{2}_{1\alpha}$, $A^{2}_{3\alpha}$ are arbitrary; the subscripts α denote the duplicity of regular solutions and take on the values 1, 3. The asymptotic form of (A21) yields integral expressions for the amplitudes and phase shifts;

$$A^{2}_{1\alpha} = a^{2}_{1\alpha} \cos \delta^{2}_{1}$$

$$a^{2}_{1\alpha} \sin \delta^{2}_{1\alpha} = -\tfrac{2}{5}\lambda\int_0^\infty U\left(u^{2}_{1\alpha} - 3\sqrt{6}\, u^{2}_{3\alpha}\right) g_1\, dy\ ;$$

(A22 cont.)

(A22 cont.)

$$A^2_{3\alpha} = a^2_{3\alpha} \cos \delta^2_{3\alpha}$$

$$a^2_{3\alpha} \sin \delta^2_{3\alpha} = -\frac{2}{5} \int_0^\infty U(4\, u^2_{3\alpha} - 3\sqrt{6}\, u^2_{1\alpha}) g_3 \, dy$$

where

$$u^2_{1\alpha} \sim a^2_{1\alpha} \sin (y - \pi/2 + \delta^2_{1\alpha})$$

$$u^2_{3\alpha} \sim a^2_{3\alpha} \sin (y - 3\pi/2 + \delta^2_{3\alpha}) .$$

The weighting influence of the $g_3(y)$ term (which is small throughtout the region in which the nuclear potential is large) in the various integrands suggests that the "subdominant" amplitudes a^2_{13}, a^2_{31} might best be kept small by placing $A^2_{13} = A^2_{31} = 0$, which amounts to choosing $\delta^2_{13} = \delta^2_{31} = \pi/2$. In Born approximation, for which the free particle trial functions $u^2_{11} = a^2_{11}\, g_1(y)$, $u^2_{13} = u^2_{31} \equiv 0$, $u^2_{33} = a_{33}\, g_3(y)$ are used, Equations (A22) become:

$$^B\delta^2_{11} = -\frac{2}{5}\lambda \int_0^\infty U(g_1)^2 \, dy \qquad {}^B a^2_{13} = {}^B a^2_{31} = \frac{6}{5}\sqrt{6}\,\lambda \int_0^\infty U\, g_1 g_3 \, dy$$

$$^B\delta^2_{33} = -\frac{8}{5}\lambda \int_0^\infty U(g_3)^2 \, dy \qquad \delta^2_{13} = \delta^2_{31} = \pi/2 \qquad \text{(A23)}$$

It is evident from the behaviour of the functions $g_1(y)$ and $g_3(y)$ that, of the four quantities now describing the coupled state scattering δ_{11}^2, a_{13}^2, a_{31}^2, δ_{33}^2), δ_{11}^2 might be large but the other three are small. (a_{11}^2 and a_{33}^2 may be normalized to unity since only the ratios

$$\frac{a_{13}^2}{a_{33}^2} \quad \text{and} \quad \frac{a_{31}^2}{a_{11}^2} \quad \text{are relevant.)}$$

In general, wherever a comparison of the exact solution with the Born approximation could be made, the latter was found to be quite accurate for the three small quantities (a_{13}^2, a_{31}^2, δ_{33}^2), with only δ_{11}^2 showing marked deviations. Table AI gives the comparison at 350 Mev for the SRSW case.

To integrate equations (A20) numerically, it is convenient to start with a power series solution near the origin (where the potential is constant). The roots of the indicial equations are:

$$\begin{aligned} \alpha_1 &= 6, & \beta_1 &= 4 \\ \alpha_2 &= 2, & \beta_2 &= 4 \\ \alpha_3 &= -1, & \beta_3 &= 1 \\ \alpha_4 &= -1, & \beta_4 &= -3 \end{aligned} \qquad \text{where} \qquad \begin{aligned} u_i &= \sum_{n=0}^{\infty} a_n^i \, y^{n+\alpha_i} \\ w_i &= \sum_{n=0}^{\infty} b_n^i \, y^{n+\beta_i} \end{aligned} \tag{A24}$$

The fundamental set of regular solutions can therefore be taken[12] to be:

$$u_1 = \sum_0^\infty a_n y^{n+6} \qquad u_2 = u_1 \log y + \sum_0^\infty C_n y^{n+2}$$

$$w_1 = \sum_0^\infty b_n y^{n+4} \qquad w_2 = w_1 \log y + \sum_0^\infty d_n y^{n+4} \quad . \tag{A25}$$

The recurrences relations are:

$$\left\{ \begin{array}{l} a_n \left[(n+6)(n+5) - 2 \right] + K_1 a_{n-2} + K b_n = 0 \\ b_n \left[(n+4)(n+3) - 12 \right] + K_2 b_{n-2} + K a_{n-4} = 0 \end{array} \right\}$$

$$\left\{ \begin{array}{l} a_{n-4}(2n+3) + C_n \left[(n+2)(n+1) - 2 \right] + K_1 C_{n-2} + K d_{n-4} = 0 \\ b_n(2n+7) + d_n \left[(n+4)(n+3) - 12 \right] + K_2 d_{n-2} + K C_n = 0 \end{array} \right\} \tag{A26}$$

where

$$K_o = \frac{2}{5} \lambda e^{-a y_o} y_o^2 \quad ; \quad K = \pm 3\sqrt{6}\, K_o \quad ; \quad K_1 = 1 \mp K_o \quad ; \quad K_2 = 1 \mp 4K_o \, .$$

upper sign: "attractive" lower sign: "repulsive" .

The quantities a_o and d_o are undetermined; the former merely defines the normalization and the latter represents the arbitrary amount of solution $\alpha = 1$ that may be mixed in solution $\alpha = 2$.

In some cases the coupled equations were integrated on a differential analyzer; in others, a desk calculator was used. To check the phase shifts, the resulting radial functions were used as trial functions in the integral equations. For the uncoupled equations, a method recently described by G. J. Kynch[13] was used. Its advantage lies in the fact that the nuclear phase shift is integrated directly, whereas in an integration of the wave function most of the effort is "wasted" in obtaining the centrifugal phase shift. If the quantity $\tan^{-1}\left((-1)^{\ell} S(y')\right)$ represents the phase shift which would obtain if the potential for $y \geq y'$ were placed equal to zero, then

$$(-1)^{\ell+1} \frac{d S_\ell}{dy} = V(g_\ell + S g_{-\ell})^2 \quad \text{where} \quad \frac{d^2 u}{dy^2} - \frac{\ell(\ell+1)}{y^2} u + u = Vu \tag{A27}$$

$S(y)$ is either monotonically increasing or decreasing depending on whether the potential is repulsive or attractive. For a square well cutoff, the power series expansion for $S(y)$ (P state) is given by:

$$S(y) = S_0 y^5 + S_2 y^7 + S_4 y^9 + S_6 y^{11} \qquad \text{let } V = \varepsilon = \text{constant}$$

$$S_0 = \frac{\varepsilon}{45} \qquad S_4 = \frac{\varepsilon}{9}\left(S_0^2 - \frac{4}{15} S_0 - \frac{2}{3} S_2 + \frac{1}{525}\right)$$

$$S_2 = -\frac{2\varepsilon}{21}\left(S_0 + \frac{1}{30}\right) \qquad S_6 = \frac{\varepsilon}{11}\left(S_0^2 + 2 S_0 S_2 + \frac{4}{35} S_0 - \frac{4}{15} S_2 - \frac{2}{3} S_4 - \frac{4}{42{,}525}\right). \tag{A28}$$

Within a hard core $S(y) = -g_1(y)/g_{-1}(y)$ (P state).

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TABLE AI

Triplet p-p phase shifts and amplitudes at 350 Mev for singular tensor potential with short range square well cutoff. (SRSW case).

	δ^2_{11}	a^2_{13}	a^2_{31}	δ^2_{33}	δ^2_{31}	δ^2_{13}
Repulsive Exact	.218	-.101	-.098	.050	$\pi/2$	$\pi/2$
Attractive Exact	.018	.084	.085	-.077	$\pi/2$	$\pi/2$
Repulsive Born	.088	-.099	-.099	.067	$\pi/2$	$\pi/2$
Attractive Born	-.088	.099	.099	-.067	$\pi/2$	$\pi/2$

TABLE II

Triplet p-p phase shifts at 350 Mev for singular tensor potential with various cutoffs. (Cross sections plotted in Fig. 2.)

		SHORT RANGE SQUARE WELL CUTOFF			LONG RANGE SQUARE WELL CUTOFF		SHORT RANGE HARD CORE		LONG RANGE HARD CORE	
		ATTRACTIVE	REPULSIVE	BORN (REPULSIVE)	ATTRACTIVE	REPULSIVE	ATTRACTIVE	REPULSIVE	ATTRACTIVE	REPULSIVE
$^3P_2, {}^3F_2$	B_1^2	- .002 +i.242	- .048 +i.184	$^B\delta_1^{20}$ = -.033	Same as SRSW		.005 +i.170	- .029 +i.116	- .035 -i.185	- .008 -i.239
	A_1^2	- .022 -i.100	- .152 +i.567	$^B\delta_1^{21}$ = .169	Same as SRSW		- .028 -i.171	- .119 +i.503	- .129 -i.514	- .027 +i.158
	B_3^2	- .017 -i.016	.018 -i.060	$^B\delta_3^{20}$ = -.014	Same as SRSW		- .013 -i.016	.013 -i.062	.012 -i.020	- .016 -i.066
	A_3^2	- .037 -i.357	- .086 +i.323	$^B\delta_3^{21}$ = .188	Same as SRSW		- .045 -i.356	- .078 +i.325	- .082 -i.349	- .035 +i.331
$^3P_0$	δ_1^0	- .626	2.00	.880	-.616	1.24	- .64	.91	- .66	.32
$^3P_1$	δ_1^1	.580	- .360	-.440	.540	- .351	.38	- .37	.04	- .47

TABLE III

Phase shifts for triplet p-p scattering at 129 Mev, using repulsive singular tensor interaction with short range square well (SRSW) cutoff.

	Exact	Born
$^3P_0$	$\delta_1^{\,0} = 1.81$	$^B\delta_1^{\,0} = .590$
$^3P_1$	$\delta_1^{\,1} = -.240$	$^B\delta_1^{\,1} = -.295$
$^3P_2, {}^3F_2$	$B_1^{\,2} = -.008 + i.012$	$^B\delta_1^{\,20} = -.021$
	$A_1^{\,2} = -.037 + i.288$	$^B\delta_1^{\,21} = .112$
	$B_3^{\,2} = .002 - i.082$	$^B\delta_3^{\,20} = -.024$
	$A_3^{\,2} = -.027 + i.195$	$^B\delta_3^{\,21} = .109$

FIGURE CAPTIONS

Figure 1: Born tensor amplitudes, $C(\theta)$ (Equation A14), for singular potentials with various ranges ($x_o = \frac{r_o}{R}$) of square well cutoffs. The radial dependence of the potentials is indicated on the plot. $x = r/R$.

Figure 2: Differential cross sections (center of mass system) for triplet p-p scattering (neglecting coulomb) at 350 Mev using a singular tensor potential $15.2\ S_{12}\ \frac{e^{-r/R}}{(r/R)^2}$ Mev with various cutoffs. Dotted curves show Born cross sections, solid curves are "exact". Phase shifts are in Table II. "Short range" means $r_o = .24 \times 10^{-13}$ cm.; "long range" $= .48 \times 10^{-13}$ cm.

Figure 3: Differential cross sections for triplet p-p scattering (neglecting coulomb) at 129 Mev using cutoff singular tensor potential. Curve A: SRSW cutoff. Curves B, C have cutoffs adjusted to give the $^3P_0$ phase shifts indicated on the plot.

Figure 4: P-p scattering at 350 Mev, 129 Mev using the cutoff singular tensor potential given by Equation (6) and a square well singlet interaction (Christian and Noyes). Coulomb scattering neglected. The experimental points at 350 Mev and those at 120 Mev denoted by • are taken from Chamberlain, Segre, and Wiegand[1]. The points, x, at 105 Mev are from Birge, Kruse, and Ramsey[3].

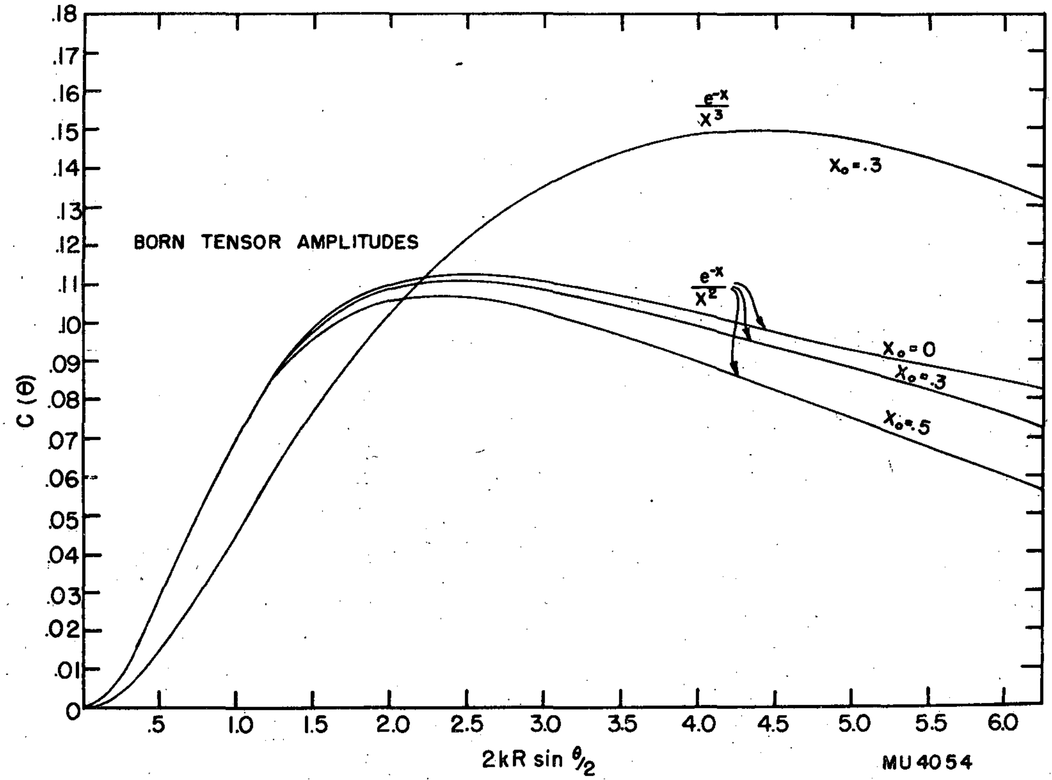
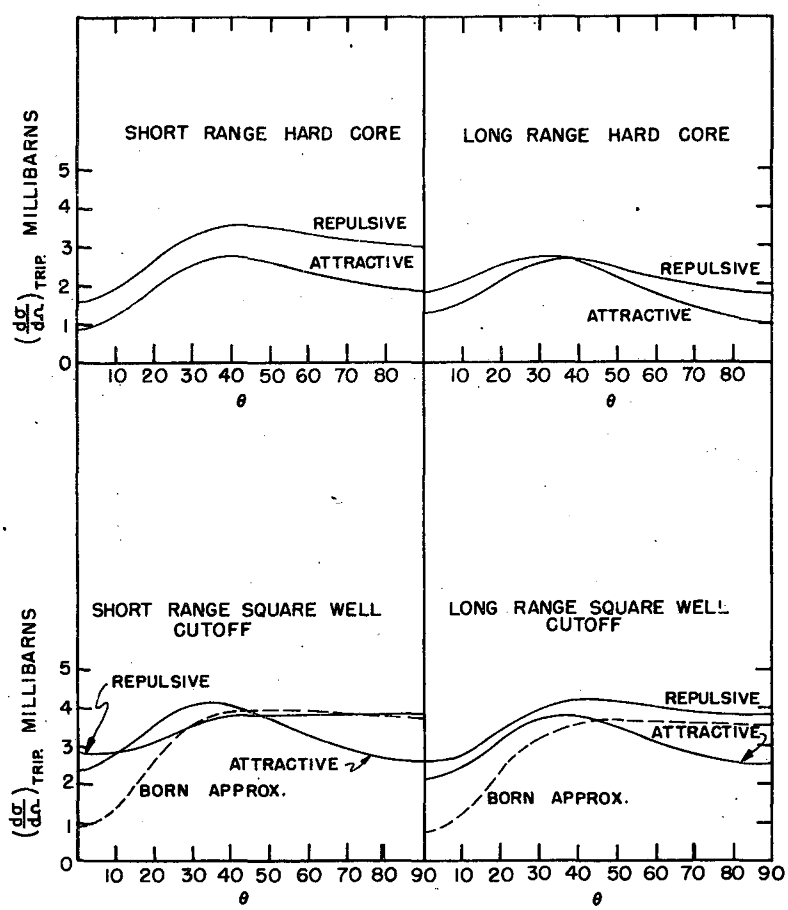

BORN TENSOR AMPLITUDES
C (θ)
2kR sin θ/2
e^-x / X^3
X₀ = .3
e^-x / X^2
X₀ = 0
X₀ = .3
X₀ = .5
MU 4054

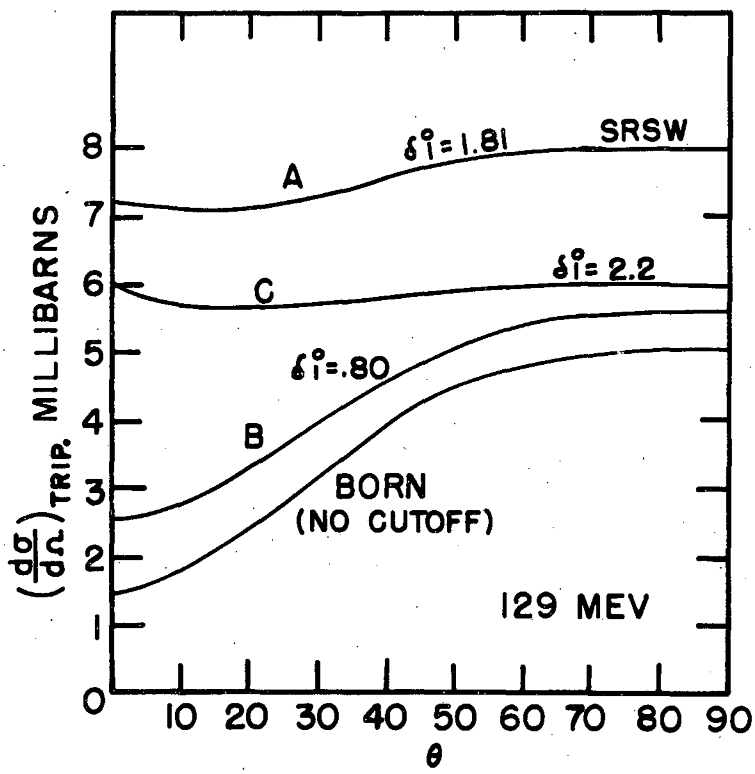
FIG 1



SHORT RANGE HARD CORE
LONG RANGE HARD CORE
REPULSIVE
ATTRACTIVE
REPULSIVE
ATTRACTIVE
$\left(\frac{d\sigma}{d\Omega}\right)_{TRIP.}$ MILLIBARNS
θ
SHORT RANGE SQUARE WELL CUTOFF
LONG RANGE SQUARE WELL CUTOFF
REPULSIVE
ATTRACTIVE
BORN APPROX.
REPULSIVE
ATTRACTIVE
BORN APPROX.
$\left(\frac{d\sigma}{d\Omega}\right)_{TRIP.}$ MILLIBARNS
θ

FIG. 2

MU 4055



8
7
6
5
4
3
2
1
0
$\left(\frac{d\sigma}{d\Omega}\right)_{TRIP.}$ MILLIBARNS
A
$\delta_1^o = 1.81$
SRSW
C
$\delta_1^o = 2.2$
$\delta_1^o = .80$
B
BORN
(NO CUTOFF)
129 MEV
10 20 30 40 50 60 70 80 90
θ

FIG. 3

MU 4056

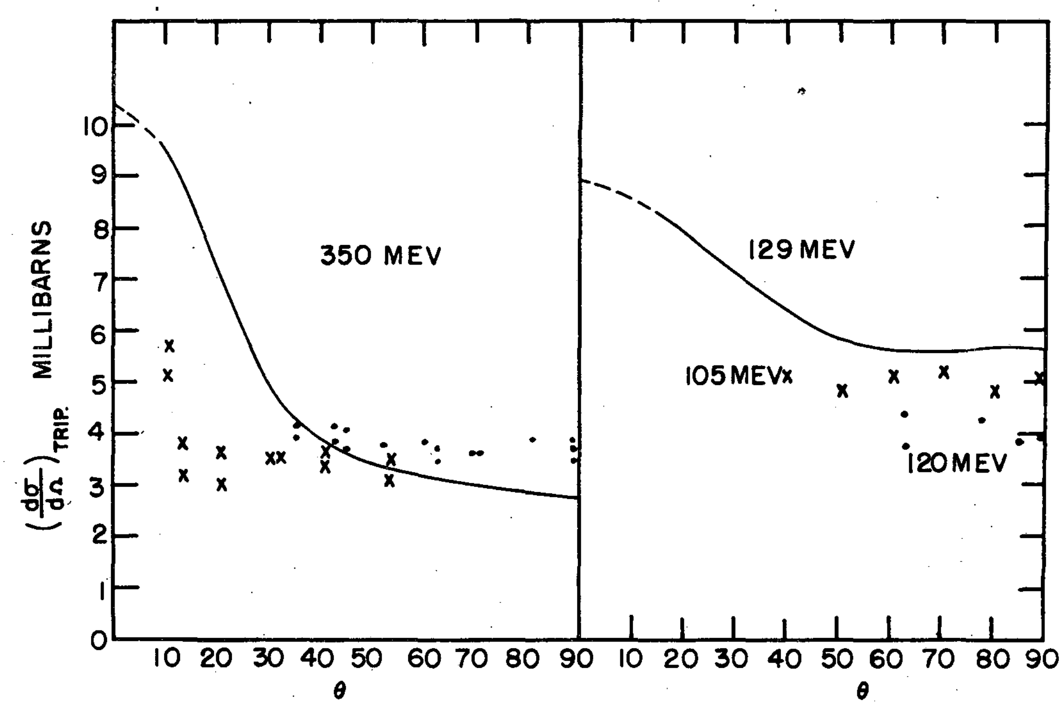

MILLIBARNS
$\left(\frac{d\sigma}{d\Omega}\right)_{TRIP.}$
350 MEV
129 MEV
105 MEV
120 MEV
θ
θ

FIG. 4

MU 4057

POLARIZATION EFFECTS IN NUCLEON-NUCLEON SCATTERING

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ABSTRACT

If a beam of unpolarized nucleons is scattered from a target of unpolarized nucleons, the scattered particles are polarized (in a direction normal to the scattering plane) provided that the interaction contains tensor or spin-orbit forces. The polarization can be detected by means of a second similar scattering since the cross section then contains an azimuthal dependence:

$$I(\theta, \phi) = I_0(\theta)(1 + \mathcal{E} \cos \phi) ,$$

where $\mathcal{E}(\theta)$ is essentially the square of the polarization. Calculations are carried out by the author for a double p-p scattering using the tensor interaction described in the preceding paper, and for a double n-p scattering using the central and tensor potential of Christian and Hart (containing the "half exchange" dependence proposed by Serber). The polarization produced by the first scattering at the optimum angle of $\theta \approx 50^{\circ}$ was found to vary from 6% at 40 Mev to 33% at 285 Mev for n-p scattering and from 10% at 129 Mev to 15% at 350 Mev for p-p scattering. The n-p results (previously published) are consistent with the azimuthal asymmetry detected in a double scattering experiment reported by L. Wouters.

II. POLARIZATION EFFECTS IN NUCLEON-NUCLEON SCATTERING

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May 29, 1952

SCATTERING OF A POLARIZED BEAM

For a single nucleon-nucleon collision in a definite initial spin state χ_i, the intensity of the scattered state is given by $(\bar{S}\chi_i, \bar{S}\chi_i)$, the expectation value of $\bar{S}^+\bar{S}$. S is the 3X3 triplet spin scattering matrix defined in the Appendix of preceding paper[1]; $\bar{S}$ (4X4 dimensions) is the same with singlet states included. The result of a measurement to which many scattering events contribute is necessarily the average expectation value of the measured quantity taken over an ensemble of all possible initial states of the system. The totality of information concerning a system can be expressed in terms of the q-dimensional density matrix $\rho^{(q)}_{ji} = \sum_{\alpha} g_\alpha a_i^{*\alpha} a_j^{\alpha}$ where $\sum_i a_i^{\alpha} u_i$ is the wave function of the system in the state α, g_α is probability of occurrence, and u_i a complete set of expansion functions. Following the method of Wolfenstein and Ashkin[2,3], let $\rho^{(4)}$ refer to the initial spin states of the two-nucleon system; then the differential scattering cross section is given by $\mathrm{Tr}(\rho^{(4)}\bar{S}^+\bar{S})$. Consider for the moment an ensemble of one particle (spin $\frac{1}{2}$) systems; a measurement of spin will yield the result $\langle\vec{\sigma}_1\rangle = \mathrm{Tr}(\rho^{(2)}\vec{\sigma}_1)$, from which it follows that the (two dimensional) density matrix can be written $\rho^{(2)} = \frac{1}{2}\left[1 + \langle\vec{\sigma}_1\rangle \cdot \vec{\sigma}_1\right]$. The four dimensional density matrix describing a spin state ensemble of two-particle systems is given by the "direct product"[4] of the density matrices for the one particle ensembles, provided that the states of one particle are not correlated with those of the other:

$$\rho^{(4)}(1, 2) = \rho^{(2)}(1) \times \rho^{(2)}(2) \qquad \text{or}$$

$$(\rho^{(4)})_{ij;i'j'} = \left[\rho^{(2)}(1)\right]_{ii'} \left[\rho^{(2)}(2)\right]_{jj'} .$$

Hence,

$$\rho^{(4)} = \tfrac{1}{4}(1 + \langle\vec{\sigma}_1\rangle \cdot \vec{\sigma}_1) \times (1 + \langle\vec{\sigma}_2\rangle \cdot \vec{\sigma}_2) \tag{1}$$

The differential cross section for a beam of particles of polarization $\vec{P}_1 = \dfrac{\langle\vec{\sigma}_1\rangle}{I_1}$ scattered from an unpolarized target $\langle\vec{\sigma}_2\rangle = 0$, is therefore given by:

$$\begin{aligned} \mathrm{Tr}(\rho^{(4)}\, \bar{S}^{+} \bar{S}) &= \frac{I_o}{4}\mathrm{Tr}(\bar{S}^{+}\bar{S}) + \tfrac{1}{4}\langle\vec{\sigma}_1\rangle \cdot \mathrm{Tr}(\vec{\sigma}_1 \times 1\, \bar{S}^{+}\bar{S}) \\ &= \frac{I_o}{4}\mathrm{Tr}(\bar{S}^{+}\bar{S}) + \frac{1}{8}\langle\vec{\sigma}_1\rangle \cdot \mathrm{Tr}(\vec{\sigma}\, S^{+} S) \end{aligned} \tag{2}$$

where $\vec{\sigma}$ is the triplet spin operator, and I_o the intensity of the incident beam. The second equality follows from the absence of matrix elements in S between triplet and singlet states, hence the latter do not contribute to the "polarization term" $\frac{1}{8}\langle\vec{\sigma}_1\rangle\cdot Tr(\vec{\sigma}\, S^+ S)$.

For an interaction of the form

$$\left[A(r) + \vec{\sigma}_1\cdot\vec{\sigma}_2\, B(r)\right]\left[a + b\, P_x\right] \quad ,$$

S is proportional to the (triplet) unit matrix and so the polarization term vanishes. In the case of a tensor or spin-orbit force, it follows from Eq. (A2) (reference 1) (or can be proved by symmetry arguments[5]) that the polarization term in Eq. (2) is nonvanishing and proportional to the component of polarization of the incident beam normal to the scattering plane. Detection of an azimuthal dependence of this type in the nucleon-nucleon scattering cross section would therefore constitute direct evidence for the presence of noncentral forces. The problem now to be considered is that of producing the incident polarized beam of high energy (S states alone do not contribute to polarization) nucleons.

If an unpolarized beam strikes an unpolarized target, the polarization of the scattered beam is given by

$$\frac{(\bar{S}\chi_i,\ \vec{\sigma}_i\,\bar{S}\chi_i)}{(\bar{S}\chi_i,\ \bar{S}\chi_i)} = \frac{Tr(\vec{\sigma}\, S\, S^+)}{Tr(\bar{S}\,\bar{S}^+)}\ .$$

where $\rho^{(4)} = \frac{1}{4}\,1$ is the density matrix describing the initial system. A proof, based on the transformation properties of S, that

$$Tr(\vec{\sigma}\, S\, S^+) = Tr(\vec{\sigma}\, S^+\, S)$$

has been given by Wolfenstein and Ashkin[3]. An algebraic tour de force, however, using the form (A2) (reference 1) for S, yields the equality:

$$\mathrm{Tr}(\sigma_y S^{+} S) - \mathrm{Tr}(\sigma_y S S^{+}) = \frac{\tan\theta\cos\phi}{4k^2}\, I_m \sum_J \left[P^2_{J+1} + J(J+1)P_{J+1} \right]$$

$$\times \left[(A^J_{J+1} - A^J_{J-1}) - (B^J_{J+1} - B^J_{J-1}) \right]$$

$$\times \left[A - E - C \right] = 0 \qquad (3)$$

which vanishes immediately for purely central or $S \cdot L$ forces (uncoupled, therefore $\delta_\ell^{Jm_s} = \delta_\ell^J$) and does so for tensor forces as a consequence of the Wronskian conditions (A9), (A17b) reference 1.

If the z_1 direction is taken as that of the incident beam, then $\mathrm{Tr}(\sigma_{z_1} S^{+} S) \equiv 0$ may be readily confirmed. Placing the x_1-axis in the scattering plane, (ϕ_1=0), the polarization is given by:

$$P_{1y_1}(\theta_1, \phi_1=0) = \frac{1}{8I_1}\,\mathrm{Tr}(\sigma_{y_1} S_1^{+} S_1) = \frac{Q_1(\theta_1)}{I_1(\theta_1)} \; ; \quad P_{x_1} = P_{z_1} = 0 \qquad (4)$$

$$I_1 = \tfrac{1}{4}\,\mathrm{Tr}(S_1^{+} S_1)$$

where the subscripts 1 will be used throughout to denote the first scattering.

In the first scattering, introduce the subscript (b) to represent the particles originally in the incident beam, and (t) to denote those from the target. The polarization of the two scattered beams is the same:

$$\langle\vec{\sigma}_b\rangle = \mathrm{Tr}(\vec{\sigma}_b \times 1\ \bar{S}^{+} \bar{S}) = \mathrm{Tr}(1 \times \sigma_t\ \bar{S}^{+} \bar{S}) = \langle\vec{\sigma}_t\rangle$$

$$= \tfrac{1}{2}\,\mathrm{Tr}(\vec{\sigma}\ S^{+}\ S) \quad . \qquad (5)$$

The nucleons emerging at some laboratory angle (Θ, Φ) will be used to form the incident beam for a second scattering. If particles (b) are to be used, the center of mass angles are $\theta = 2\Theta$ and $\phi = \Phi$; for particles (t), however, $\theta = \pi - 2\Theta$ and $\phi = \Phi + \pi$. Consider, for example, the experiment of Wouters[6] in which incident protons produce a neutron beam by means of a (p,n) reaction. The (p,n) collision is described by $S(\theta, \phi)$, and the polarization of neutrons observed at Θ, Φ is $\frac{\langle\sigma_t\rangle(\theta, \phi)}{I(\theta)}$ where $\theta = \pi - 2\Theta$ and $\phi = \Phi + \pi$. The scattering matrix itself carries all information on the exchange nature of the interaction. In the case of two protons the S matrix is antisymmetric so it is of course immaterial whether $\theta = 2\Theta$, $\phi = \Phi$ or $\theta = \pi - 2\Theta$, $\phi = \Phi + \pi$ is used.

The subscript 1 will be used hereafter in place of (b) or (t) to indicate that the operator in question refers to once-scattered particles which form an incident beam for the second scattering.

THE DOUBLE SCATTERING PROBLEM

The coordinate system for the second scattering, $(x_2\ y_2\ z_2)$ is obtained by rotating $(x_1\ y_1\ z_1)$ about the y_1 axis until the z axis lies along the new incident beam (Figure 1). Hence $P_{y_1} = P_{y_2}$ is unchanged, and represents (in the form of $\langle\vec{\sigma}_1\rangle$) just the quantity that must appear in the density matrix for the new initial state

$$\rho^{(4)} = \rho_1^{(2)} \times \rho_2^{(2)} = \tfrac{1}{4}(1 + \langle\vec{\sigma}_1\rangle\cdot\vec{\sigma}_1) \times (1+\langle\vec{\sigma}_2\rangle\cdot\vec{\sigma}_2) . \tag{6}$$

The subscript 2 refers to particles of the second target. The latter is supposed to be unpolarized, so that $\langle\vec{\sigma}_2\rangle = 0$. The differential cross section for the second scattering is obtained from (6), (2), (3), and (4):

$$\left(\frac{d\sigma}{d\Omega}\right)_2 = J(\theta_1, \theta_2, \phi_2) = I_1(\theta_1)I_2(\theta_2) + Q_1(\theta_1)Q_2(\theta_2) \cos \phi_2 . \tag{7}$$

$I_1(\theta_1)$ and $I_2(\theta_2)$ are the differential cross sections with polarization terms omitted.

In the case of p-p scattering, or n-p scattering with exchange dependence $1 \pm P_x$, so that interaction occurs only in orbital angular momentum states of the same parity, then the condition

$$(S^+ S)(\theta, \phi) = (S^+ S)(\pi - \theta, \phi + \pi)$$

implies $Q(\theta) = - Q(\pi - \theta)$ so that $Q(\pi/2) = 0$. The contribution to the polarization at $\theta = \pi/2$ must therefore come exclusively from odd-even interference terms; the possibility of such a measurement suggests a test of the $1 + P_x$ dependence proposed by Serber.

Ignoring for the moment the fact that the second scattering occurs at a somewhat lower energy than the first, and assuming the two involve the same types of particles (i.e. both n-p or both p-p), then the measured ratio at the optimum angles $\theta_1 = \theta_2 = \theta_{max.}$ is:

$$R \equiv \frac{J(\phi_2=0)}{J(\phi_2=\pi)} = \frac{1+(Q/I)^2}{1-(Q/I)^2} \geq 1 . \qquad (8)$$

Barring a somewhat remarkable dependence of $Q(\theta)$ on energy, a ratio greater than 1 should in general be expected as the experimental result whenever $\theta_1 \approx \theta_2$. A relationship which led to Equation (3):

$$\sqrt{2}(B - D) \cot \theta - (A - C - E) = \sum_J \left[P^2_{J+1} + J(J+1)P_{J+1} \right] \left[(A^J_{J+1} - A^J_{J-1}) - (B^J_{J+1} - B^J_{J-1}) \right] = 0 \qquad (9)$$

can be used to simplify appreciably the form of $Q(\theta)$ by eliminating A-C.

$$Q(\theta) = \frac{\mathcal{E}_4}{8k^2} I_m \left[E'(B' + D')^* + D^{*\prime} B' \cot \theta \right] + \frac{\mathcal{E}_2}{4k} (C_+ + F)R(B + D) \equiv Q' + Q_I \qquad (10)$$

$\mathcal{E}_2$, $\mathcal{E}_4$ are defined in (A17) reference 1.

RESULTS AND CONCLUSIONS

For p-p scattering, $Q(\theta)$ is plotted in Figure 2 for all cases considered in reference 1 except the long range hard core model which has been omitted because the coupled phase shifts were found only roughly. The dominant term of Eq. (10), which alone yields a value of $Q(\theta)$ correct to within 50% or so is quite simple; for singular potentials $(C_K - C_L)$ is very small, so that $Q' >> Q_I$; only P states have been kept:

$$Q(\theta) \approx I_m \left\{ B_1^{'2*} \left[B_1^{'0} + \frac{3}{2} (A_1^{'1} - A_1^{'2}) \right] \right\} P_2^1 \ (\cos \theta) . \tag{11}$$

The importance of obtaining accurate values for the coupled $^3P_2$ phase shifts is clear; even rigorously there is no contribution to the polarization from the $^3P_0$ and $^3P_1$ states alone. The polarization

$$P(\theta, \phi{=}0) = \frac{Q(\theta)}{I(\theta)}$$

is plotted in Figure 3; the value of $I(\theta)$ was taken in all cases to be the predicted triplet cross section for the potential model used; that is, the singlet scattering as assumed negligible for $\theta \geq 50^\circ$. If, instead, it is assumed that singlet scattering can be introduced in such a way as to bring the cross section in each case up to the experimental value of 4 millibarns, then Figure 2, rather than Figure 3, shows more clearly the dependence of polarization on choice of cutoff. With the potential given by Equation (6) of reference 1, the polarization (at $\theta \approx 50^\circ$) is 10% ($R \approx 1.02$) at 129 Mev and 15% ($R \approx 1.05$) at 350 Mev.

For n-p scattering, the tensor and central interaction of Christian and Hart (containing the "half exchange" dependence proposed by Serber) is used. In Figure 4, $Q(\theta)$ is plotted for energies of 40, 90, 200, 285 Mev. A similar plot of the polarization $Q(\theta)/I(\theta)$ was given in an earlier report[7]. A comparison of $Q(\theta)$ with Q/I illustrates the point that $I(\theta)$ alone carries almost the entire energy dependence of the polarization.

If odd state forces were introduced into the triplet n-p interaction (by changing the $1 + P_x$ dependence), the polarization could be considerably larger because of the contribution from S-P interference:

$$Q_{SP} = \frac{1}{8k^2} I_m \left\{ B_0^1 \left[B_1^2 - B_1^0 + \frac{3}{2} (A_1^1 - A_1^1) \right]^* \right\} \sin\theta \quad . \tag{12}$$

To obtain some idea of the magnitude of this term, suppose the same amount of triplet odd state interaction is introduced into the n-p Hamiltonian as was used for the p-p interaction in the preceding paper[1]. Interpolating the p-p phase shifts to obtain rough values at 200 Mev, the result is $Q_{SP} \approx .5 \sin\theta$ millibarns leading to $R(\pi/2) \approx 1.03$. Hence, although the asymmetry is appreciably influenced by the presence of odd states, the quoted uncertainty in the experimental results of Wouters[6] is too great to permit any sharp conclusions to be drawn on the question of the exchange dependence of the n-p interaction. The desirability of further experiments on n-p double scattering is, however, indicated.

This work was performed under the auspices of the Atomic Energy Commission.

REFERENCES

1. Don R. Swanson, (preceding paper). The notation of this reference will be used throughout.

2. L. Wolfenstein, Phys. Rev. 76, 541 (1949).

3. L. Wolfenstein and J. Ashkin, Phys. Rev. 85, 947 (1952).

4. H. Weyl, "Theory of Groups and Quantum Mechanics", Ch. II.

5. L. Wolfenstein, Phys. Rev. 75, 1664 (1949).

6. L. Wouters, Phys. Rev. 84, 1069 (1951).

7. Don R. Swanson, Phys. Rev. 84, 1068 (1951).

FIGURE CAPTIONS

Figure 1: Coordinate axes for double scattering problem.

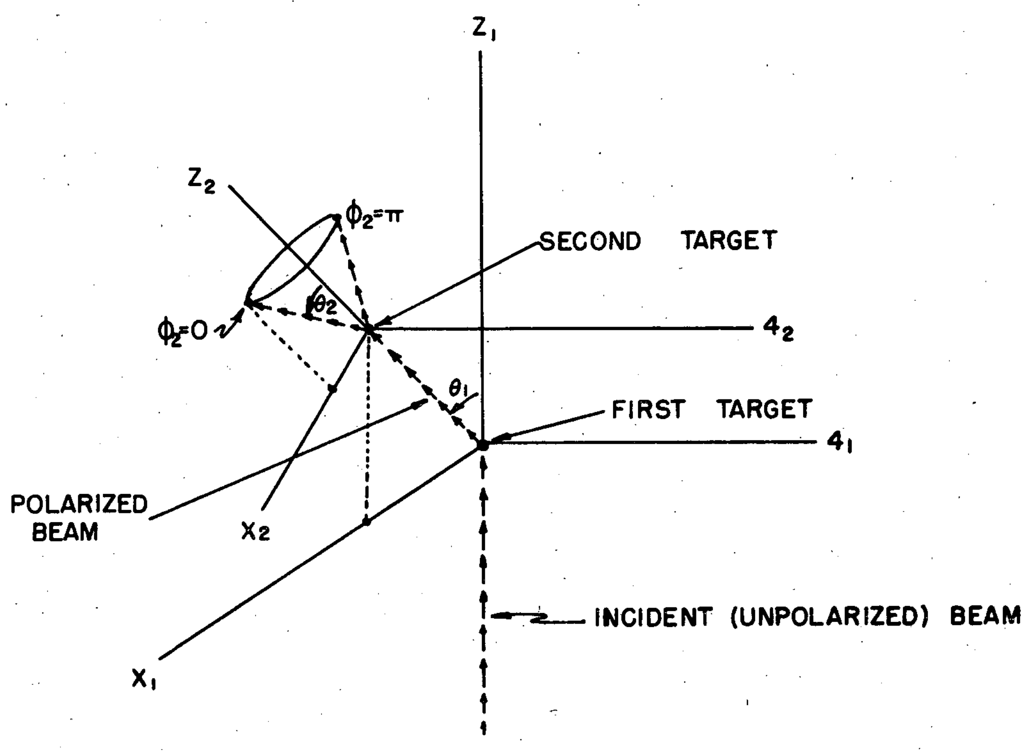
Figure 2: Values of $Q(\theta) = \frac{1}{8}\mathrm{Tr}(\sigma_y S^+ S)$ for p-p scattering at (lab. system) energies of 350 Mev, 129 Mev. θ = scattering angle in center of mass system. The interactions indicated (cutoff singular tensor) are those for which cross sections were computed in reference 1 (Figures 2, 3)[1].

Figure 3: Values of $\frac{Q(\theta)}{I_t(\theta)}$ for p-p scattering at 350 Mev. $I_t(\theta)$ = triplet cross section. Polarization is given by

$$\frac{Q(\theta)}{I_t(\theta) + I_s(\theta)} \quad ;$$

$I_s(\theta)$ = singlet cross section. The function plotted hence represents the polarization at those angles ($\theta > 50^\circ$ for Christian and Noyes model) for which singlet scattering is negligible. The interactions indicated (cutoff singular tensor) are those for which cross sections were computed in reference 1.

Figure 4: Values of $Q(\theta) = \frac{1}{8}\mathrm{Tr}(\sigma_y S^+ S)$ for n-p scattering at the energies indicated. The interaction used is that of Christian and Hart. A similar plot of polarization (Q/I) is given in reference 7.


Z1
Z2
φ2=π
SECOND TARGET
θ2
φ2=0
42
θ1
FIRST TARGET
41
POLARIZED
BEAM
X2
INCIDENT (UNPOLARIZED) BEAM
X1

MU4058

FIG. 1

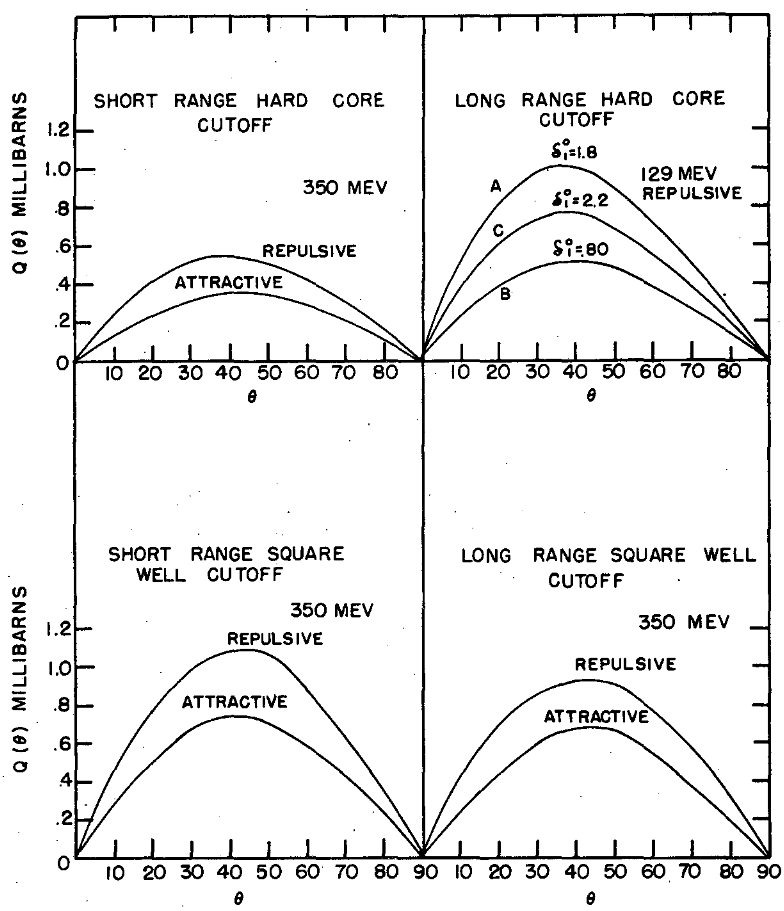

SHORT RANGE HARD CORE CUTOFF
350 MEV
REPULSIVE
ATTRACTIVE
LONG RANGE HARD CORE CUTOFF
129 MEV REPULSIVE
δ_1°=1.8
δ_1°=2.2
δ_1°=.80
A
B
C
SHORT RANGE SQUARE WELL CUTOFF
350 MEV
REPULSIVE
ATTRACTIVE
LONG RANGE SQUARE WELL CUTOFF
350 MEV
REPULSIVE
ATTRACTIVE
Q (θ) MILLIBARNS
θ
0
.2
.4
.6
.8
1.0
1.2
10 20 30 40 50 60 70 80 90

FIG. 2

MU4059

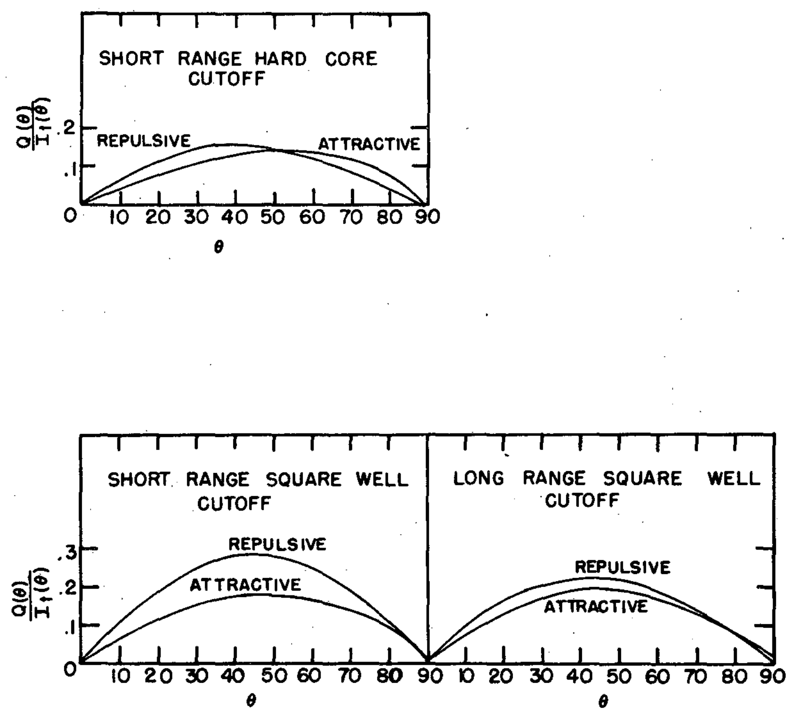

SHORT RANGE HARD CORE
CUTOFF
REPULSIVE
ATTRACTIVE
$\frac{Q(\theta)}{I_t(\theta)}$
.2
.1
0
10 20 30 40 50 60 70 80 90
θ
SHORT RANGE SQUARE WELL
CUTOFF
REPULSIVE
ATTRACTIVE
LONG RANGE SQUARE WELL
CUTOFF
REPULSIVE
ATTRACTIVE
.3
.2
.1
0
10 20 30 40 50 60 70 80 90 10 20 30 40 50 60 70 80 90
θ
θ

FIG. 3

MU 4060

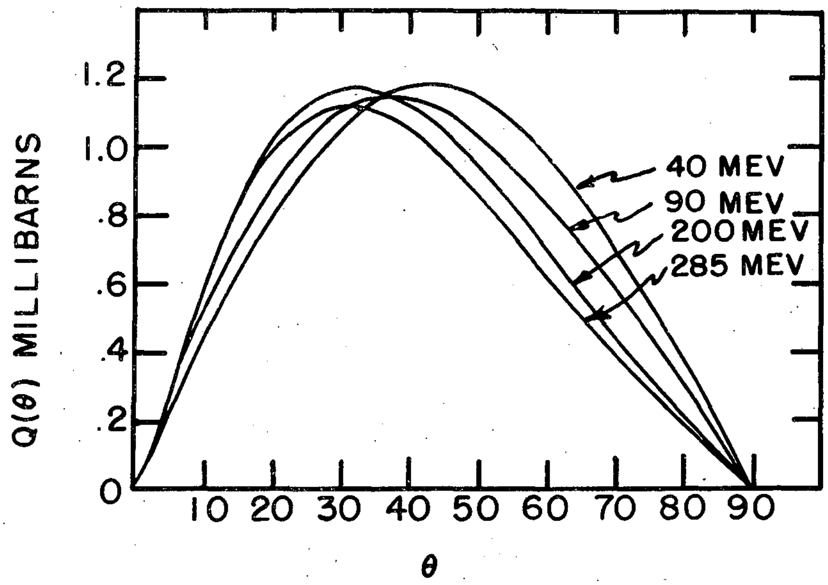

1.2
1.0
.8
.6
.4
.2
0
Q(θ) MILLIBARNS
10 20 30 40 50 60 70 80 90
θ
40 MEV
90 MEV
200 MEV
285 MEV

FIG. 4

MU 4061