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Labor Market Frictions, Interest Rates, and Macroeconomic Policies

A dissertation submitted in partial satisfaction of the
requirements for the degree
Doctor of Philosophy

in

Economics

by

Ji Zhang

Committee in charge:

Professor James D. Hamilton, Chair
Professor Davide Debortoli
Professor Takeo Hoshi
Professor Irina A. Telyukova
Professor Rossen Valkanov

2013

The dissertation of Ji Zhang is approved, and it is acceptable in quality and form for publication on microfilm and electronically:

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Chair

University of California, San Diego

2013

DEDICATION

To my beloved family and esteemed mentor

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I am forever indebted to Professor James D. Hamilton for his endless support as the chair of my committee. Without his patience, encouragement, and invaluable guidance, none of this work would have been possible.

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Chapter 1, in full, is prepared for submission for publication. The dissertation author was the primary author of this material.

Chapter 2, in full, is prepared for submission for publication. The dissertation author was the primary author of this material.

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ABSTRACT OF THE DISSERTATION

Labor Market Frictions, Interest Rates, and Macroeconomic Policies

by

Ji Zhang

Doctor of Philosophy in Economics

University of California, San Diego, 2013

Professor James D. Hamilton, Chair

My dissertation studies the effect of macroeconomic policies both theoretically and empirically.

In Chapter 1, I empirically estimate a DSGE model with search and matching frictions, endogenous job separation, and real wage rigidities to examine the main driving forces behind unemployment fluctuations. I find that shocks to unemployment benefits have historically been important for unemployment fluctuations, and the extension of unemployment benefits during the recent recession contributed to the higher unemployment rate.

In Chapter 2, I study the impact of liquidity shocks on the economy, the effectiveness of alternative government policies, and the role played by the zero lower bound on the nominal interest rate. I find that extended unemployment benefits could

slightly alleviate the big decline in output caused by the liquidity shock through mitigating current consumption decline, but raise unemployment and slow the recovery of the labor market. Unconventional monetary policy and fiscal expansion are very effective in stimulating the economy. The importance length of staying at the zero lower bound depend on type of labor market rigidities.

In Chapter 3, I verify policy implications of New Keynesian models at the zero lower bound empirically. Through analyzing the responses of various yields to macroeconomic announcements, I find that the predictions of New Keynesian models for the behavior of interest rates when the zero lower bound is binding are reliable: nominal rates are less sensitive to news, and real rates respond to shocks in opposite directions from their behavior away from the zero lower bound. This suggests that at least in the short run, fiscal policy is more effective at the zero lower bound. I also find using an identification strategy based on heterogeneity that at the zero lower bound, monetary policy shocks account for less variation of both nominal and real rates, monetary policy is less effective in affecting short- and medium-term real rates, and the effect dies off faster.

Chapter 1

Unemployment Benefits and Matching Efficiency in An Estimated DSGE Model with Labor Market Search Frictions

Abstract. I develop and estimate a DSGE model with search and matching frictions, endogenous separation, and real wage rigidities to examine the main driving forces behind unemployment fluctuations. In contrast to most existing models, shocks to unemployment benefits and matching efficiency are included. I find that a shock to unemployment benefits is important for the cyclical movement of unemployment. This finding is robust to different setups of unemployment benefits policy and different observables used in estimation. On the other hand, matching efficiency changes have little effect on cyclical movement of unemployment. The data favor a formulation in which the separation rate is endogenous and counter-cyclical, playing a very important role in raising unemployment during recessions. During the Great Recession, extended unemployment benefits increased the unemployment rate by 1 percentage point, while a decrease in matching efficiency was of little importance.

1.1 Introduction

After 2007, the unemployment rate kept increasing until it reached 10% at the end of 2009. With the subsequent recovery of the economy, the unemployment rate began to decrease, but at a very slow pace which did not keep up with the increase in output and vacancies. As of the beginning of 2013, the unemployment rate was still almost 8%, far above pre-recession levels (below 5%), despite the fact that the recession has been over for more than 3 years. This problem has drawn close consideration from both policy makers and economists. In addition to the big decline in output and investment, what other factors contributed to the extraordinarily high unemployment rate and the slow recovery of the labor market?

This paper studies the role of mismatches in the labor market and unemployment benefits in accounting for historical variation in unemployment. Recently, there have been many studies of matching efficiency. Some papers focus on studying the mismatch problem during the Great Recession. Dickenson (2010) studies the labor market tightness data of different industries, which suggests that it would be hard to make a case for structural mismatch being a major problem today. Barlevy (2011) and Veracierto (2011) find a big decline in matching efficiency during the Great Recession. Sahin, Song, Topa and Violante (2012) measure the contribution of mismatch to the recent rise in U.S. unemployment and find that mismatch across industries and occupations explains at most 1/3 of the increase in unemployment. Instead of specializing in studying the Great Recession, some studies consider the mismatch problem throughout history. Barnichon and Figura (2011) construct matching efficiency time series from CPS micro data back to 1976, and study the determinants of matching efficiency fluctuations over the last four decades. Michaillat (2011) finds that in bad times, frictional unemployment is only a very small part of total unemployment using a calibrated model of the labor market. All these papers focus on the labor market only, but none of them study it in a complete economic framework. Instead, they use either different versions of calibrated Mortensen-Pissarides model or a matching function as convenient devices to capture the job seeking process without considering its connections to other part of the economy. Furlanetto and Groshenny (2012) make an important advance on these earlier studies, introducing matching effi-

ciency shocks into an estimated DSGE model with labor market search and matching frictions. They find that matching efficiency shocks are irrelevant to unemployment fluctuations historically, but did increase unemployment during the Great Recession. However, their model assumes employment separations are exogenous, and it may be important to see whether alternative formulations are more successful empirically. So in contrast to the above papers, my paper uses a complete general equilibrium structural model with labor market search frictions and endogenous separation to study the importance of matching efficiency shocks over the full period 1976-2011.

All of the above papers focus solely on matching efficiency, but ignore the role of another possible cause of the high unemployment rate – extended unemployment benefits. Empirical works focus on studying the role of extended unemployment benefits in the US labor market during the Great Recession. Valletta and Kuang (2010) measure the increase in involuntary job loses and the average duration of unemployment, and conclude that extended unemployment benefits contributed only a 0.4 percentage point increase in the unemployment rate when it reached 10% during the Great Recession. Fujita (2011) uses monthly CPS data to quantify the effects of extended unemployment benefits in recent years, and suggests that extended benefits have raised male workers' unemployment rate by 1.2 percentage points. Calibrated models are also used in some papers to assess the effects of a countercyclical unemployment benefits policy. Nakajima (2012) measures the effect of extensions of unemployment insurance benefits on the unemployment rate using a calibrated structural model, and finds that the extensions of UI benefits contributed to an 1.2 percentage points increase in the unemployment rate. Moyen and Stahler (2012) make further contribution on the optimal duration of unemployment benefit entitlement across the business cycle, and find a countercyclical policy does harm labor market adjustment in bad times but may be welfare-enhancing because of people's desire to smooth consumption. Landais, Michaillat, and Saez (2013) study the optimal unemployment insurance over the business cycle. Instead of only relying on either labor market data or calibrated model, I introduce unemployment benefits shocks into an estimated medium scale DSGE model and use data and model together to study how these shocks affect labor market dynamics.

Besides the two main tasks above, I also examine whether the data are better

characterized using a model in which the separation rate is exogenous or endogenous. Numerous previous works have evaluated the role of endogenous separation, such as Den Haan, Ramey and Watson (2000) and Fujita and Ramey (2011). These papers focus on how endogenous separation increases the calibrated model's ability in matching key statistics such as labor market volatility in the economy. Instead of focusing on important moments of key variables, I try to study the overall performance of the model in matching data. So I estimate models with different types of separation, and let the marginal likelihood tell which is favored by the data.

In this paper, I build a New Keynesian DSGE model with labor market search and matching frictions, endogenous separation as in Den Haan, Ramey and Watson (2000) and real wage rigidity as in Hall (2005a,b). The rest of the model is similar to that of Smets and Wouters (2007). This kind of models allow us to study the unemployment dynamics under different shocks and policies, while in standard New Keynesian models, only movements in employment or hours of work could be generated. A number of other papers have introduced labor market search and matching frictions into a New Keynesian DSGE model. Walsh (2003, 2005), Krause and Lubik (2007), Blanchard and Gali (2010), Kuester (2010), and Groshenny (2012) focus on the effects of monetary policy and inflation responsiveness to shocks when search and matching frictions exist in the labor market. Gertler, Sala and Trigari (2008), Christiano, Trabandt and Walentin (2010), Gali, Smets and Wouters (2011) try to construct a complete medium scale DSGE model to fit the data better. However, none of these paper allow for shocks to matching efficiency or unemployment benefits, which are the primary interest in my paper.

The main findings of this paper areas follows. First, the shock to unemployment benefits plays a very important role in unemployment fluctuations historically. It accounts for over 15% of unemployment variation. In the Great Recession, extended unemployment benefits increased the unemployment rate by more than one percentage point, while mismatch has very little effect on it, which is consistent with the result in Nakajima (2012) and Dickens (2010). Second, matching efficiency shocks account for less than 5% of unemployment fluctuations historically, which is consistent with the result in Furlanetto and Groshenny (2012), and Michaillat (2011). The importance of matching efficiency shocks depends on the type of job separation. Un-

der exogenous separation, matching efficiency shocks become much more important. However, the third result is, the model with endogenous separation fits the data better, which means the separation rate comoves with the unemployment rate and that counter-cyclical movement of separation is an important cause of high unemployment. This is consistent with the result in den Haan, Ramey and Watson (2000) and Fujita and Ramey (2009, 2010). The second and third result together tell us that in a typical recession, the high unemployment rate is caused more by a higher separation rate instead of by a lower job finding rate resulting from a decrease in matching efficiency. These results are found to be stable under several robustness checks.

The remainder of the paper is structured as follows: Section 2 sets up the model. Section 3 is the estimation of the model parameters. Section 4 is the results from the baseline model. Section 5 gives results of 5 robustness checks. Finally, in section 6 I conclude the paper.

1.2   The Model

The main framework of the model follows Smets and Wouters (2007). There are three types of agents: households, intermediate good firms, and final good firms. And like Smets and Wouters (2007), I introduce a number of exogenous shocks in the model.

1.2.1   Household

There is a representative household in the economy, and there are a continuum of members, indexed by i, measured on $[0,1]$ in the household. Every member has the same period utility function: $\frac{(c_t - h\mathcal{C}_{t-1})^{1-\sigma}}{1-\sigma}$, where the utility depends not only on their own consumption of final goods c_t, but also on the past aggregate consumption in the economy, $\mathcal{C}_{t-1}$. I define h as the habit formation parameter. Unlike Smets and Wouters (2007), I don't include the intensive margin of employment because Gertler, Sala and Trigari (2008) find that most of the cyclical variation in employment in the United States is on the extensive margin and including the intensive margin does not affect the model very much. Leisure is not considered in the utility function here.

Instead, it appears in the budget constraint. That is, the value of being unemployed is measured in consumption goods and taken as a part of the household's income. People in a household pool their income together for consumption. The household does not make the labor supply decision. All unemployed members search on the job market and the frictional search and matching process determines who is employed. The representative household maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - h\mathcal{C}_{t-1})^{1-\sigma}}{1-\sigma} \tag{1.1}$$

s.t.

$$C_t + I_t + \frac{B_t}{\epsilon_t^b r_t P_t} = \int_0^1 \chi_{it} Y_{it}^L di + \frac{B_{t-1}}{P_t} + r_t^k d_t K_{t-1}^H - D(d_t) K_{t-1}^H$$
$$+ D_t + \int_0^1 (1 - \chi_{it})(A_t + G_t^u) di - T_t \tag{1.2}$$

The inter-temporal discount factor is β, and the consumption of the family members at period t is C_t. The consumption C_t is a CES function over a continuum of goods with elasticity of substitution ϵ_t^p,

$$C_t = [\int_0^1 (C_{\widetilde{j}t})^{\frac{\epsilon_t^p - 1}{\epsilon_t^p}} d\widetilde{j}]^{\frac{\epsilon_t^p}{\epsilon_t^p - 1}}, \epsilon_t^p > 1$$

where $\widetilde{j}$ is the index of the differentiated final consumption goods, and ϵ_t^p follows $\log \epsilon_t^p = (1 - \rho^p) \log \epsilon^p + \rho^p \log \epsilon_{t-1}^p - \mu^p \nu_{t-1}^p + \nu_t^p$. All innovations in this paper, including ν_t^p, are i.i.d. random variables with mean 0.

The price for the consumption good is P_t. The investment is represented by I_t. The bond holding is B_t, and the gross nominal interest rate controlled by the central bank is r_t. The risk premium shock is ϵ_t^b, which follows $\log \epsilon_t^b = \rho^b \log \epsilon_{t-1}^b + \nu_t^b$.

Household's disposable real labor income earned by member i is represented by Y_{it}^L. The indicator for employment status, χ_t, equals 1 when the person is employed in period t, and 0 otherwise. The flow value from unemployment includes unemployment benefits paid by the government G_t^u, as well as other factors (such as leisure) that can be measured in units of consumption goods $A_t = \iota^t A$, where ι is the deterministic

growth rate of output. I assume A_t grows at the same rate as output, so that leisure wouldn't become less and less valuable as the economy grows.

The stock of capital at the end of period $t-1$ held by the household is K_{t-1}. The net return to capital is expressed as the return on the capital used minus the cost associated with variations in the degree of capital utilization: $(r_t^k d_t K_{t-1}^H - D(d_t) K_{t-1}^H)$. The income from renting out capital services depends not only on the level of capital stock, but also on its utilization rate d_t. The cost of capital utilization is assumed to be zero when capital is fully used (*i.e.* $D(1) = 0$).

The profit from the final good sector is D_t, and the lump-sum tax is T_t.

The accumulation of capital obeys the following rule:

$$K_t^H = (1 - \delta)K_{t-1}^H + \epsilon_t^I [1 - \psi(I_t/I_{t-1})]I_t \tag{1.3}$$

where $\psi(\cdot)$ is the investment adjustment costs, which equals zero when the investment grows at the deterministic growth trend ι ($\psi(\iota) = 0$). The adjustment cost function also satisfies $\psi'(\iota) = 0$ and $\psi''(\iota) > 0$. ϵ_t^I is the shock to installation cost, which follows $\log \epsilon_t^I = \rho^I \log \epsilon_{t-1}^I + \nu_t^I$.

The representative household maximizes its utility by choosing consumption, bond holdings, investment, capital stock, and capital utilization rate. The first order conditions for the household's problem are:

$$C_t : (C_t - h\mathcal{C}_{t-1})^{-\sigma} = \widetilde{\lambda}_{1t} \tag{1.4}$$

$$B_t : \widetilde{\lambda}_{1t} = \beta \mathbb{E}_t (\widetilde{\lambda}_{1t+1} \epsilon_t^b r_t \frac{P_t}{P_{t+1}}) \tag{1.5}$$

$$I_t : Q_t \psi'(\frac{I_t}{I_{t-1}}) \frac{\epsilon_t^I I_t}{I_{t-1}} - \beta \mathbb{E}_t [Q_{t+1} \frac{\widetilde{\lambda}_{1t+1}}{\widetilde{\lambda}_{1t}} \psi'(\frac{I_{t+1}}{I_t}) \frac{\epsilon_{t+1}^I I_{t+1}}{I_t} \frac{I_{t+1}}{I_t}] + 1$$
$$= Q_t \epsilon_t^I (1 - \psi(\frac{I_t}{I_{t-1}})) \tag{1.6}$$

$$K_t^H : Q_t = \beta \mathbb{E}_t \{ \frac{\widetilde{\lambda}_{1t+1}}{\widetilde{\lambda}_{1t}} [Q_{t+1}(1 - \delta) + d_{t+1} r_{t+1}^k - D(d_{t+1})] \} \tag{1.7}$$

$$d_t : r_t^k = D'(d_t) \tag{1.8}$$

where

$$Q_t = \frac{\widetilde{\lambda}_{2t}}{\widetilde{\lambda}_{1t}} \tag{1.9}$$

represents Tobin's q, and $\widetilde{\lambda}_{1t}$ and $\widetilde{\lambda}_{2t}$ represents the Lagrangian multiplier for the budget constraint and capital accumulation constraint respectively.

1.2.2 Intermediate Good Sector

The intermediate good sector is perfectly competitive, and each firm hires one worker and rents capital to produce identical intermediate goods.

Matching

At the beginning of period t, there are N_t matched workers and firms; $U_t = 1 - N_t$ workers are unmatched. The matched workers at the start of period t travel to their places of employment. At that point, with an exogenous probability $0 \leq \rho^x < 1$ the match is terminated. The remaining $(1-\rho^x)N_t$ pairs of matched workers and firms, indexed by j, jointly observe the realization of social common productivity z_t, and match-specific productivity a_{jt}, which follows a Lognormal distribution with mean 0 and standard deviation σ_a, and then decide whether to continue the match. If a_{jt} is larger than some threshold $\widetilde{a}_{jt}$, the match continues and production occurs. Since all the intermediate good firms are identical ex ante, we can eliminate the subscript j. All the matches with match specific productivity lower than $\widetilde{a}_t$ are endogenously terminated. So the endogenous separation rate is given by

$$\rho^n_t = F(\widetilde{a}_t) = \int_{-\infty}^{\widetilde{a}_t} f(a_t)da_t \tag{1.10}$$

The total separation rate is $\rho_t = \rho^x + (1-\rho^x)\rho^n_t$ and the survival rate is $\rho^s_t = 1 - \rho_t$.

The number of new matches in period t is M_t. These new matches don't produce any good in the current period, and they could only enter production in the next period after surviving from both exogenous and endogenous separations. The total number of matches evolves according to:

$$N_{t+1} = (1 - \rho_{t+1})(N_t + M_t). \tag{1.11}$$

The number of new matches in period t depends on the amount of vacancies posted by the firms, V_t, and the number of unemployed workers, U_t. The matching function $M_t(U_t, V_t)$ takes the form $\epsilon_t^M \mathcal{M} U_t^\zeta V_t^{1-\zeta}$, where $\mathcal{M}$ is the scale parameter standing for the aggregate matching efficiency, and the matching efficiency shock ϵ_t^M follows $\log \epsilon_t^M = \rho^M \log \epsilon_{t-1}^M + \nu_t^M$. In the literature, many papers try to estimate the matching efficiency, and they find that the matching efficiency does change pro-cyclically. A shock to the scale parameter of the matching function allows fluctuations in matching efficiency in the model. An increase in the degree of mismatch, such as skill mismatch and geographic mismatch, worsens the efficiency of the labor market, and could be taken as a negative matching efficiency shock.

The probability of a worker finding a job (the job-finding rate) is given by

$$\rho_t^w = \frac{M_t(U_t, V_t)}{U_t} = \epsilon_t^M \mathcal{M} \theta_t^{1-\zeta}, \tag{1.12}$$

and the probability of a vacancy being filled (the vacancy-filling rate) is

$$\rho_t^f = \frac{M_t(U_t, V_t)}{V_t} = \epsilon_t^M \mathcal{M} \theta_t^{-\zeta}, \tag{1.13}$$

where $\theta_t = V_t/U_t$ is the labor market tightness.

Firm's Decision

The production function of the matched firms follows

$$Y(a_{jt}) = z_t a_{jt} \iota^{t(1-\alpha)} K_{jt}^\alpha. \tag{1.14}$$

The common technology shock z_t follows an AR(1) process: $\log z_t = \rho^z \log z_{t-1} + \nu_t^z$. And ι is the deterministic labor-augmenting growth rate. Intermediate goods are sold in a competitive market at the given price P_t^I.

Firms survived from separations choose capital optimally by maximizing

$$\frac{z_t a_{jt} K_{jt}^\alpha \iota^{t(1-\alpha)}}{\mu_t} - r_t^k K_{jt},$$

where $\mu_t = \frac{P_t}{P_t^I}$ is the price markup. The optimal capital is

$$K^*(a_{jt}) = \iota^t \left(\frac{\alpha z_t a_{jt}}{\mu_t r_t^k} \right)^{\frac{1}{1-\alpha}}. \tag{1.15}$$

Unmatched firms seeking workers have to pay a cost, $\gamma \iota^t$, to post a vacancy. The vacancy posting cost grows at the same deterministic rate as output. The vacancy could be filled with probability ρ_t^f, and the filled vacancy could be separated with probability $1 - \rho_{t+1}$. The unmatched firm will post a vacancy only when the discounted expected future value of doing so is bigger than or equal the cost. Free entry ensures that unmatched firms post vacancies until

$$\gamma \iota^t = \beta \rho_t^f \mathbb{E}_t \left[\frac{\widetilde{\lambda}_{1t+1}}{\widetilde{\lambda}_{1t}} (1 - \rho_{t+1}) J_{t+1} \right] \tag{1.16}$$

where J_{t+1} is the expected future value of a matched firm, which is identical for all firms.

The value of a matched firm with match-specific productivity a_{jt} could be expressed as the net profit obtained from this period's production plus the expected future value of the firm:

$$J(a_{jt}) = \frac{Y(a_{jt})}{\mu_t} - W(a_{jt}) - r_t^k K^*(a_{jt}) + \beta \mathbb{E}_t \left[\frac{\widetilde{\lambda}_{1t+1}}{\widetilde{\lambda}_{1t}} (1 - \rho_{t+1}) J_{t+1} \right] \tag{1.17}$$

where $Y(a_{jt})/\mu_t$ is the firm's revenue from selling the intermediate goods evaluated in terms of final goods, and $W(a_{jt})$ is the real wage of a worker with match-specific productivity a_{jt}.

A matched worker's value, $H^w(a_{jt})$, is equal to the real wage he can get from the work this period, and plus the discounted future value of the work:

$$H^w(a_{jt}) = W(a_{jt}) + \beta \mathbb{E}_t \left\{ \frac{\widetilde{\lambda}_{1t+1}}{\widetilde{\lambda}_{1t}} [(1 - \rho_{t+1}) H_{t+1}^w + \rho_{t+1} H_{t+1}^u] \right\} \tag{1.18}$$

where H_t^u is the value of the unemployed person:

$$H_t^u = A_t + G_t^u + \beta \mathbb{E}_t \left\{ \frac{\widetilde{\lambda}_{1t+1}}{\widetilde{\lambda}_{1t}} [(1 - \rho_{t+1}) \rho_t^w H_{t+1}^w + (1 - (1 - \rho_{t+1}) \rho_t^w) H_{t+1}^u] \right\} \tag{1.19}$$

The economic surplus of a match is $S(a_{jt}) = J(a_{jt}) + H^w(a_{jt}) - H^u_t$. When there is no real wage rigidity, the surplus is divided between the firm and worker through Nash bargaining, and the bargaining power of the worker is η. There is a shock to the bargaining power, which is indicated as ϵ^η_t and follows the AR(1) process: $\log \epsilon^\eta_t = \rho^\eta \log \epsilon^\eta_{t-1} + \nu^\eta_t$. The notional real wage resulting from the Nash bargaining is:

$$W^N(a_{jt}) = \epsilon^\eta_t \eta[\frac{Y(a_{jt})}{\mu_t} - r^k_t K^*(a_{jt}) + \gamma \iota^t \theta_t] + (1 - \epsilon^\eta_t \eta)(A_t + G^u_t)$$

However, when there exists a wage norm, and the real wage is rigid in the sense that the real wage depends on the wage norm, and is the weighted average of the notional wage and the steady state value of the real wage:

$$W(a_{jt}) = \omega^W[\epsilon^\eta_t \eta(\frac{Y(a_{jt})}{\mu_t} - r^k_t K^*(a_{jt}) + \gamma \iota^t \theta_t) + (1 - \epsilon^\eta_t \eta)(A_t + G^u_t)] + (1 - \omega^W)Y^L \iota^t$$

The real wage rigidity index is ω^W. If $\omega^W = 0$, the real wage is solely determined by the steady state surplus, and if $\omega^W = 1$, the real wage is perfectly flexible.

Firms survived from the exogenous separation should make the decision on endogenous separation, that is, decide the threshold of match specific productivity, $\widetilde{a}_t$. Following Krause and Lubik (2007), since under small shocks, real wages are always above workers' reservation wage, the critical value of a_t below which separation takes place is given by $J(\widetilde{a}_t) = 0$. Substituting the real wage and capital use at $\widetilde{a}_t$, the separation threshold is determined by the following equation:

$$\frac{Y(\widetilde{a}_t)}{\mu_t} - W(\widetilde{a}_t) - r^k_t K^*(\widetilde{a}_t) + \frac{\gamma \iota^t}{\rho^f_t} = 0 \tag{1.20}$$

Define the average capital used in production as follows:

$$K^*_t = \int_{\widetilde{a}_t}^{a_{max}} K^*(a_{jt}) \frac{f(a_t)}{1 - F(\widetilde{a}_t)} da_t. \tag{1.21}$$

The aggregate output net of the vacancy posting costs of the intermediate good sector is:

$$Y_t = N_t \iota^t \frac{\mu_t r^k_t}{\alpha} K^*_t - \iota^t \gamma V_t. \tag{1.22}$$

The average real wage is defined as:

$$W = \int_{\tilde{a}_t}^{a_{max}} W(a_{jt}) \frac{f(a_t)}{1 - F(\tilde{a}_t)} da_t$$

$$= \omega^W [\epsilon_t^{\eta} \eta (\frac{1 - \alpha}{\alpha} r_t^k K_t^* + \gamma \iota^t \theta_t) + (1 - \epsilon_t^{\eta} \eta)(A_t + G_t^u)] + (1 - \omega^W) W \iota^t. \tag{1.23}$$

1.2.3 Final Good Sector

The final good sector is monopolistically competitive. Each final good firm, indexed by $\tilde{j}$, buys the output of the intermediate good firms at the price P_t^I, converts this output into a differentiated final good, $Y_{\tilde{j}t}$, with no cost and sells the final good in the market at the price $P_{\tilde{j}t}$. The demand for each variety is

$$Y_{\tilde{j}t} = (\frac{P_{\tilde{j}t}}{P_t})^{-\epsilon_t^p} Y_t \tag{1.24}$$

and the aggregate price is

$$P_t = [\int_0^1 (P_{\tilde{j}t})^{1-\epsilon_t^p} d\tilde{j}]^{\frac{1}{1-\epsilon_t^p}}. \tag{1.25}$$

Prices are sticky in the final good sector. In the following analysis, the index $\tilde{j}$ is eliminated because every firm faces an identical problem. Following Calvo(1983), in each period only a fraction of $(1 - \omega)$ firms can choose their prices optimally. For the firms which could not re-optimize their prices at period t, they can adjust their prices according to the past inflation rate: $P_t = P_{t-1} \Pi_{t-1}^{\xi}$. Let P_t^* be the optimal price set by firms that can reoptimize prices in period t, and the optimization problem for the final good firm is:

$$\max_{P_t^*} \sum_{s=0}^{\infty} \omega^s \mathbb{E}_t \{ \Lambda_{t,t+s} [P_t^* \Pi_{t+s-1,t-1}^{\xi} Y_{t,t+s} - P_{t+s}^I Y_{t,t+s}] \}$$

where

$$Y_{t,t+s} = (\frac{P_t^* \Pi_{t+s-1,t-1}^{\xi}}{P_{t+s}})^{-\epsilon_{t+s}^p} C_{t+s}$$

The result of the optimization problem is:

$$P_t^* = \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \omega^s \Lambda_{t,t+s} C_{t,t+s} \epsilon_{t+s}^p \mu_{t+s}^{-1} P_{t+s}^{1+\epsilon_{t+s}^p} \Pi_{t+s-1,t-1}^{-\xi \epsilon_{t+s}^p}}{\mathbb{E}_t \sum_{s=0}^{\infty} \omega^s \Lambda_{t,t+s} C_{t,t+s} (\epsilon_{t+s}^p - 1) P_{t+s}^{\epsilon_{t+s}^p} \Pi_{t+s-1,t-1}^{\xi(1-\epsilon_{t+s}^p)}} \tag{1.26}$$

where $\mathbb{E}_t \Lambda_{t,t+s} \equiv \beta^s \mathbb{E}_t[(\widetilde{\lambda}_{1t+s}/\widetilde{\lambda}_{1t})(P_t/P_{t+s})]$ is the stochastic discount factor for nominal payoffs, and $\Pi_{t+s,t} = P_{t+s}/P_t$. So the aggregate price is given by

$$P_t = [\omega(P_{t-1}(\frac{P_{t-1}}{P_{t-2}})^{\xi})^{1-\epsilon_t^p} + (1-\omega)(P_t^*)^{1-\epsilon_t^p}]^{\frac{1}{1-\epsilon_t^p}}. \tag{1.27}$$

1.2.4 Government

In order to close the model, we need to specify the monetary policy and fiscal policy. Monetary policy obeys simple Taylor rule:

$$\widehat{r}_t = (1-\phi_r)(\phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t) + \phi_r \widehat{r}_{t-1} + \widehat{\epsilon}_t^r, \tag{1.28}$$

where $\widehat{x}_t$ is the log-deviation from steady state value, and the temporary interest rate shock is given by $\log \epsilon_t^r = \rho^r \log \epsilon_{t-1}^r + \nu_t^r$.

Government budget constraint is of the form:

$$G_t + G_t^{utotal} + \frac{B_{t-1}}{P_t} = T_t + \frac{B_t}{r_t P_t} \tag{1.29}$$

where $G_t^{utotal} = G_t^u U_t$ is the total unemployment benefits.

The unemployment benefits obtained by each unemployed person are $G_t^u = \epsilon_t^{g^u} \overline{rr} Y_t^L$, where $\overline{rr}$ is the replacement rate – the steady state ratio between unemployment benefits and real wage. The unemployment benefits shock $\epsilon_t^{g^u}$ follows $\log \epsilon_t^{g^u} = \rho^{g^u} \log \epsilon_{t-1}^{g^u} + \nu_t^{g^u}$. I include an unemployment benefits shock because I find in the data the weekly unemployment benefits received by an unemployed person do fluctuate throughout time and move counter-cyclically comparing with average real wages. Figure 1.1 plots the real Average Weekly Benefits Amount from 1976Q1 to 2011Q2. This figure tells us the unemployment benefits do fluctuate over time.

Government spending expressed relative to steady state output $g_t^y = \frac{G_t}{Y_{\iota^t}}$ follows the process: $\log g_t^y = (1-\rho^g) \log g^y + \rho^g \log g_{t-1}^y + \nu_t^g + \mu^{gz} \nu_t^z$.

1.2.5 Market Equilibrium

To obtain the goods market equilibrium, the production should equal the household's demand for consumption and investment and the government spending:

$$Y_t = C_t + I_t + G_t + \psi(d_t)K_{t-1}^H \tag{1.30}$$

The equilibrium condition for the capital market is obtained by equalizing the capital used in the intermediate good sector and the capital stock times the utilization rate:

$$n_t K_t^* = d_t K_{t-1}^H. \tag{1.31}$$

1.3 Parameter Estimation

1.3.1 Estimation Equations

The model above is detrended and estimated with Bayesian method using nine key macroeconomic quarterly US time series as observable variables: log difference of real GDP ($dGDP_t$), log difference of real consumption ($dCONS_t$), log difference of real investment ($dINV_t$), log difference of the real wage ($dWAG_t$), log difference of the GDP deflator (INF_t), the federal funds rate (FFR_t), log deviation of the unemployment rate from its mean ($UNEM_t - \overline{UMEM}$), log deviation of vacancies from its mean ($VAC_t - \overline{VAC}$), and log difference of the total government unemployment insurance ($dINS_t$). Every observable is in percentage points, and population growth is abstracted since the variables in the model are all in per capita terms. The time period of the data is from 1976Q1 to 2011Q2.[1]

The data details are described in Table 1.1 to 1.3 in Appendix D. The first 6 observed variables are the same as those in Smets and Wouters (2007) and Gertler,

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[1]I choose 1976 as the first year firstly because I use the dataset constructed by Fujita and Ramey (2009) to form the priors of labor market parameters and use their data on the job-finding rate to do the robustness check, and their data was constructed using CPS micro data back to 1976. The second reason is the unemployment insurance data used in robustness checks also only goes back to 1970's. I also tried to use data back to 1966 in the baseline estimation, which is the same as Smets and Wouters (2007), and the results are not affected. So in order to keep consistent with the data used in robustness checks, I restrict the dataset to the period starting from 1976.

Sala and Trigari (2008). The 7th variable I use is the unemployment rate, which correspond with the unemployment in my model directly. I add 2 new observed variables: vacancies and unemployment insurance. I also add 2 new structural shocks, a matching technology shock and an unemployment benefits shock, to equalize the number of observables and the number of shocks.

The comparison of observed variables and shocks used in Smets and Wouters (2007), Gertler, Sala and Trigari (2008), and this paper is summarized in Table 1.4. And Table 1.5 shows the mapping between each observable and shock. Equation (1.32) are the measurement equations, where d means first difference, $\overline{X}$ is the mean of X, $\bar{\iota} = 100 * (\iota - 1)$ is the quarterly trend growth rate to the real GDP, $\bar{r} = 100 * (r - 1)$ is the quarterly average steady state net nominal interest rate, and $\bar{\pi}_c = 100 * (\pi - 1)$ is the quarterly steady state inflation rate.

$$
\begin{bmatrix}
dGDP_t \\
dCONS_t \\
dINV_t \\
dWAG_t \\
INF_t \\
FFR_t \\
UNEM_t - \overline{UNEM} \\
VAC_t - \overline{VAC} \\
dINS_t
\end{bmatrix}
=
\begin{bmatrix}
\bar{\iota} \\
\bar{\iota} \\
\bar{\iota} \\
\bar{\iota} \\
\bar{\pi}_c \\
\bar{r} \\
0 \\
0 \\
\bar{\iota}
\end{bmatrix}
+
\begin{bmatrix}
\widehat{y}_t - \widehat{y}_{t-1} \\
\widehat{c}_t - \widehat{c}_{t-1} \\
\widehat{i}_t - \widehat{i}_{t-1} \\
\widehat{y}^L_t - \widehat{y}^L_{t-1} \\
\widehat{\pi}_t \\
\widehat{r}_t \\
\widehat{u}_t \\
\widehat{v}_t \\
\widehat{g}^{utotal}_t - \widehat{g}^{utotal}_{t-1}
\end{bmatrix}
\tag{1.32}
$$

1.3.2 Prior and Posterior of the Parameters

Several parameters are calibrated as shown in Table 1.6. The quarterly depreciation rate δ is fixed at 0.025. The elasticity of the production function α is set to be 0.33, and the discount factor β is assumed to be 0.99. Government spending as a proportion of output is fixed at 0.18. The elasticity of substitution among the differentiated final goods, ϵ^p, is set at 11. These parameters are conventionally fixed in the literature. The reason to fix these parameters is that we cannot get information

about them from the data used, and they would be difficult to estimate unless they were used directly in the measurement equations.

The priors of the stochastic processes are set based on the setup in Smets and Wouters (2007): the standard errors of the exogenous innovations are drawn from an Inverse-Gamma distribution with a mean of 0.10 and standard deviation 0.15. The persistence of the $AR(1)$ processes is Beta distributed with mean 0.5 and standard deviation 0.2. The top panel of Table 1.7 shows the prior and posterior distribution of shock processes.

The priors of the conventional structural parameters are also consistent with papers in the literature. For the new parameters related to labor market, I set the mean to be consistent with the data and calibration results in the literature, and choose priors that are reasonably loose. The bottom panel of Table 1.7 shows the prior and posterior distribution of structural parameters.

The estimated real wage rigidity ω^W is 0.39, that is, the real wage is not as rigid as in conventional New Keynesian models. The steady state job finding rate $\overline{\rho}^w$ is 0.72, the same as Fujita and Ramey (2009). This means the job finding rate is 0.35 in monthly basis. The steady state labor market tightness θ is 0.72. The exogenous separation rate is 0.07, and the threshold of the matching-specific productivity for the endogenous separation is 0.75, which indicates the endogenous separation rate is .02. This means the total quarterly separation rate is 0.10.

The estimates of conventional parameters are very close to the results in Smets and Wouters (2007) and Gertler, Sala and Trigari (2008).

1.4   Sources of Fluctuations

In this section, I examine the sources of labor market fluctuations by studying the variance decomposition and impulse responses of variables with respect to the shocks in the model.

1.4.1 Variance Decomposition

Table 1.9 Column 2 shows the variance decomposition of unemployment both right after and 40 quarters after the shocks.

As in most DSGE models and recent literature on unemployment fluctuations, the technology shock is still the main driving force of the economy, and it accounts for almost half of the fluctuations in unemployment. However, that leaves more than half of the variation in unemployment still unexplained. Considering only the technology shock is far from enough for analyzing the labor market.

The effect of the investment specific technology shock on unemployment is not considered important in many papers, such as Smets and Wouters (2003, 2007) and Gali, Smets and Wouters (2011). But here it is the second most important shock and explains 20% of the variation in unemployment in the long run. This is because in addition to the regular channel, endogenous separation and real wage rigidity amplify the response of unemployment to the investment specific technology shock, as I will discuss further below.

The unemployment benefits shock is ignored in other papers, but it appears important empirically. 16% of the unemployment variation is caused by this shock in the short run. Intuitively, the changes in unemployment benefits change the wage required and the surplus created by a matched worker with given match-specific productivity. In this case, firms will change their threshold of endogenous separation, and unemployment is affected as a result.

The matching efficiency shock doesn't account for much of the fluctuations in unemployment. Although we hear claims of high structural unemployment every time when unemployment is high, they have yet to receive much econometric confirmation.

1.4.2 Impulse Response

Figure 1.2 to Figure 1.4 are the impulse responses to the 4 structural shocks that are most important for unemployment dynamics. These impulse responses are calculated with parameter values at the posterior means.

As shown in Figure 1.2, a positive technology shock benefits the economy as a whole. Consumption, investment and output increase and unemployment decreases.

In Figure 1.3, a positive matching efficiency shock increases the efficiency of the matching process and hence increases the job finding rate, so that unemployment decreases. However, unemployment and vacancies move in the same direction instead of in opposite directions under a matching efficiency shock. Although the higher matching efficiency encourages the firms to post more vacancies, the increased labor market tightness due to the decrease in unemployment has the dominant effect and causes the vacancies to decrease.

Figure 1.4 shows the impulse responses to a positive unemployment benefits shock. With higher unemployment benefits, the workers' reservation wage increases and firms increase the separation threshold. Unemployment increases as a result. The main difference between the impulse responses to an unemployment benefits shock and those to a matching efficiency shock is how vacancies and real wage react. Both a decrease in unemployment benefits and an increase in the matching efficiency cause consumption, output and employment to increase. However, in the first case, vacancy would increase and real wage would decrease, whereas in the second case vacancy would decrease and real wage would increase. This is because after a negative unemployment benefits shock although the increased labor market tightness due to the decrease in unemployment discourages the firms to post vacancies, the increased economic surplus has a counter effect on the vacancy posting and leads to an increase in vacancies. In addition, with a negative unemployment benefits shock, the worker's reservation wage also decreases, which leads to a fall in the real wage.

1.4.3 Application: Unemployment over 2008-2011

Figure 1.5 summarizes the historical contribution of each of 9 types of shocks to unemployment fluctuations during the recent recession starting from 2008Q3. The solid line is the log deviation of the unemployment rate from its average level. The bars in different colors represent how much the corresponding shocks affect unemployment. This decomposition is based on the estimation of the baseline model.

During the Great Recession, the decrease in matching efficiency did increase the unemployment rate, however, by less than 0.5% even at the peak. Many people use the following words to describe the current US labor market: "A lot of firms need

workers, but the vacancies cannot be filled. A lot of unemployed workers want to work, but cannot find proper jobs." Although mismatch seems to be a natural reason for this labor market situation, it is a one-sided view. When considering the matching efficiency issues, it is not sufficient to look at the labor market in a static view. Too many people are still unemployed, but many people are finding new jobs and many vacancies are being filled at the same time. So we need to take the large flows both into and out of unemployment into consideration. Diamond (2011) shows that although the average flow rate from unemployment to employment from November 2009 to October 2010 falls to 20% from its average 37% during the last 2 decades, the large increase in unemployment roughly offsets this fall. And the hires per month during November 2009 and October 2010 is 5.7million, which is not far from the 6 million average over the last 20 years. Moreover, there is no evidence showing that we have a widespread difficulty in hiring in some industries or location. Replicating the excercise in Dickens (2010), we do not see any industries with high vacancy-unemployment ratio after the Great Recession (Figure 1.6), and the ratio increased considerably in the early phase of the recent recession, but it has dropped off significantly since then and has already returned to the pre-recession level. This initial rise in mismatch may be taken for structural unemployment. However, Abraham and Kats (1986) tell us that different industries are affected in different phases in business cycles. The appearance of structural mismatch disappear as the recession becoming widespread. Besides the initial rise in mismatch during the recession, the slow recovery of the unemployment after the recession doesn't mean a lower matching efficiency neither. Historically, recovery is always slow after financial crisis. Unemployment will be depleted after a sufficient rise in aggregate demand, but it takes time even though vacancies could recover more quickly.

Then, what caused the increase in unemployment? From the figure, we see that extended unemployment benefits increased the unemployment rate during the Great Recession, and the increase in the unemployment rate caused by extended compensation is more than 1% in 2010. Since 2008, Federal-State Extended Benefit Program (EB) and Emergency Unemployment Compensation 2008 (EUC2008) have largely increased the unemployment benefit. This increase in unemployment benefits increases the value of being unemployed, and hence increases the workers' reservation

wage and makes it harder for them to get new jobs.

In a model without a financial sector the negative shock to the financial market should be mainly captured by the increase in investment adjustment cost. So during the current credit crisis, it is not surprising that the negative shock to investment causes the labor market condition to keep deteriorating. Risk premium shocks also contribute to the increase in unemployment. This is in line with the evidence that the risk premium rose significantly during the Great Recession.

The federal funds rate dropped at the end of 2008, and has stayed at zero since then. Although the central bank can no longer stimulate the economy further by decreasing the federal funds rate at the zero lower bound, the low interest rate still helps decrease the unemployment rate.

1.5 Robustness Checks

This section reports the results of five types of robustness checks. The first two refer to changes in the frictions in the model, the third one is related to a different setup of unemployment benefits policy, the fourth one is the estimation using different observables which help to get a realistic matching efficiency series, and the last one maps the unemployment benefits in the model to the average weekly benefits amount instead of the total benefits paid by the government in the data.

1.5.1 Sensitivity to Model Frictions

Since I introduced two major frictions into the search and matching model to describe the labor market (real wage rigidity and endogenous separation), a natural question raised is whether those frictions are really necessary to capture the dynamics of the data, especially the dynamics of labor market variables.

I reestimate the model when each friction is reduced one at a time, and Table 1.8 presents the marginal likelihood and the estimates of structural parameters. This table gives us an idea of the model performance with respect to the various frictions. For easy comparison, the first column is the results for the baseline model. The flexible real wage model is obtained by setting ω^W, the parameter which represents

the degree of wage rigidity, at 1. And the estimation results for this model is presented in the second column of Table 1.8. The third column is the results for the exogenous separation model. In this model the endogenous separation rate is fixed at zero. Overall, as we can see in Table 1.8, the estimated parameters are relatively robust to changes in frictions, one by one.

1.5.1.1 Endogenous Separation Vs. Exogenous Separation

Comparing with the model with exogenous separation, we can find that the baseline model fits the data better. The data favor a counter-cyclical endogenous separation rate instead of a constant exogenous one. In other words, the high unemployment rate during a recession is at least partly caused by the high separation rate, contrary to the claim that it is solely caused by an extraordinarily low job finding rate, which mainly results from a big fall in the matching efficiency. Actually, in the model with an endogenous separation mechanism, even without the deterioration in the matching efficiency, a higher separation rate could increase the unemployment rate by first causing more people to be fired and second making the labor market more tight so that the job finding rate becomes lower.

Since endogenous separation plays a very essential role in generating the dynamics of unemployment, I now report some further comparisons between the baseline model and the exogenous separation model.

First, I compare the variance decomposition of unemployment in these two models. In Table 1.9, we can find that without endogenous separation, matching efficiency shocks become much more important for labor market fluctuations. In the model with endogenous separation, a lower matching efficiency will make the existing matches more valuable since it becomes harder for an unmatched firm and worker to be matched. This will decrease the threshold for endogenous separation. That is, fewer matches are endogenously terminated. This will reduce the increase in unemployment caused by the less efficient matching process. However, under the setup with exogenous and fixed separation, firms cannot offset the effect of changes in matching efficiency by adjusting their separation decision, hence matching efficiency shocks have bigger influence on labor market dynamics. For the same logic, unemployment benefits shocks are less important under exogenous separation as the separation rate

won't increase even the real wage increases with the unemployment benefits.

Then I compare some impulse responses in the two models. Figure 1.7 to Figure 1.9 are the impulse responses to a technology shock, unemployment benefits shock and matching efficiency shock separately.

Under the impact of one unit of each shock, the variables, especially the unemployment rate, in the baseline model have larger responses than those in the exogenous separation model do. This is true for all the shocks included in the model, except the matching efficiency shock. Unemployment decreases less in the baseline model under a matching efficiency shock. This implies that endogenous separation could amplify all the shocks other than the matching efficiency shock in the model and generate more unemployment fluctuations. From Figure 1.9, endogenous separation dampens the matching efficiency shock. This is because in the baseline model with a higher matching efficiency, both firms and workers could find new matches much easier than before, the surplus of a match decreases and as a result the endogenous separation rate increases, that is the flow into unemployment becomes large as well. This offsets the large flow out of unemployment, and prevents the big decrease in unemployment. However, in the exogenous separation model, there is no such mechanism to counterbalance the positive effect, so that unemployment decreases more. Although endogenous separation dampens the matching efficiency shock, which is not very important for the unemployment dynamics, it amplifies all other shocks. We can still affirm that endogenous separation could help the model generate more volatile unemployment.

1.5.1.2   Rigid Real Wage Vs. Flexible Real Wage

Comparing the marginal likelihood, we also know that the model with a flexible real wage cannot fit the data as well as the baseline model.

Since real wage rigidity is understood to be an important explanation of unemployment volatility in the search and matching model, I also examine the role it plays in this aspect by comparing the impulse responses of the flexible wage model and baseline model.

From Figure 1.7, we find that unemployment has a bigger response to a technology shock in the baseline model, which means technology shocks are amplified by

the real wage rigidity. This result is very standard in the literature.

However, from Figure 1.8 and Figure 1.9, we can get opposite results. That is, real wage rigidity dampens these shocks instead of amplifying them. For example, when there is a positive unemployment benefits shock, jobs become less valuable for the workers and the economic surplus of a match decreases because of the increase in the value of unemployed. In this case, the workers will require higher wages, so that the flexible notional wage by Nash bargaining increases and both firm's and the worker's shares of the surplus remain constant when real wage rigidity is absent. With higher wage and lower surplus, the firms are willing to hire less workers under the positive unemployment benefits shock. However, when the real wage is rigid, it could not increase as much as the flexible notional wage does, so that the worker's share of the surplus falls and the firm's share rises correspondently. An increase in the firm's share of surplus encourages the firms to hire more workers, thereby dampening the effect of the shock. This logic works for the matching efficiency shock, government spending shock, monetary policy shock, and bargaining power shock as well. The more rigid is the wage, the bigger is its dampening effect on these shocks.

Since the real wage rigidity dampens several shocks, some of which are important for unemployment fluctuations, we could not ensure that the real wage rigidity could help the model generate more volatile labor market dynamics.

1.5.2 Estimation with an Alternative Specification of Unemployment Benefits Policy

The states set weekly benefit amounts as a fraction of the individual's average weekly wage up to some state-determined maximum. The total maximum duration available under permanent law is 39 weeks. The regular state programs usually provide up to 26 weeks. The permanent Federal-State Extended Benefits program provides up to 13 additional weeks. The permanent Extended Benefits (EB) program is triggered when the unemployment situation has worsened dramatically. During recessions and while unemployment remains high during recoveries, the federal government has historically created Temporary Emergency Unemployment Compensation (EUC) Program. Thus extended unemployment benefits programs are triggered empirically

by high unemployment rates, causing unemployment benefits per worker to depend on the unemployment rate in the data. However, in the baseline model, changes in unemployment benefits depend on changes in real wages and unemployment benefits shocks. This specification assumes that the unemployment benefits shocks are orthogonal to real wage changes, and assumes no feedback from the unemployment rate to unemployment benefits.

In order to investigate the potential importance of allowing for such feedback, I re-estimate the model with an alternative policy rule. In this case, unemployment benefits received by each unemployed worker depend on lagged unemployment rate in addition to a contemporaneous unemployment shock:

$$\widehat{g}_t^u = \widehat{\epsilon}_t^{g^u} + \widehat{y}_t^L + \phi_u \widehat{u}_{t-1} \tag{1.33}$$

Under this setup, unemployment benefits respond to both wages and the unemployment rate.

By comparing the second and fifth column of Table 1.8, we can find that the posterior means of structural parameters don't change much from the baseline case. The marginal likelihood in this case is slightly larger than that in the baseline case. The fourth column of Table 1.9 gives us the short run and long run variance decomposition of unemployment under this specification of unemployment benefits policy. The importance of unemployment benefits is also robust under different setups in unemployment policy. In the long run, unemployment benefits shocks account for 21% of unemployment fluctuations historically, very similar to the contribution in the baseline case (20%). Matching efficiency shocks explain less than 10% of unemployment changes, comparable to the 9% in the baseline case. Figure 1.10 plots the unemployment benefits shock series implied by the baseline model and the model with alternative unemployment benefits policy, and we can find the two series are almost the same as each other. Allowing for endogeneity of unemployment benefits does not change the main results.

1.5.3   Alternative Measures of Unemployment Benefits

In the estimation of the baseline model, I use data on total unemployment insurance paid by the government. This measurement corresponds to the total unemployment benefits G_t^{utotal} in my model, where $G_t^{utotal} = G_t^u U_t$. However, in the data total unemployment insurance depends on not only unemployment benefits per unemployed person and the number of the unemployed people but also the unemployment duration. Some may worry that the measure in the data is inconsistent with that in the model. In order to test for this, I repeat the analysis with an alternative measure of unemployment benefits which only includes the weekly benefits received by each unemployed worker. I use the log difference of the Average Weekly Benefits for all programs (including the regular program, extended program, and emergency unemployment compensation program) as one observable to substitute the total unemployment insurance paid by the government. This measurement directly corresponds to the unemployment benefits per person defined in my model. The corresponding measurement equation becomes $d \log AWB_t = \bar{\imath} + \widehat{g}_t^u - \widehat{g}_{t-1}^u$.

The last column of Table 1.8 shows the estimation results of structural parameters when average weekly benefit is used as the observable. And the last column of Table 1.9 gives the variance decomposition of unemployment both in the short run (on impact) and in the long run (40 quarters). Unemployment benefits shocks still contributes more than 10% of unemployment fluctuations, and this number is much larger than the contribution of matching efficiency shocks (less than 0.2%). This means under different measures of unemployment benefits, results get from the baseline model are robust.

1.5.4   Estimation with Job-finding Rate and Labor Market Tightness as Observables

Furlanetto and Groshenny (2012) find that when using the unemployment rate and vacancies as observables, it is hard to see much decline in matching efficiency during the Great Recession, while using the job finding rate and labor market tightness as observables results in matching efficiency series that better matching the data. In my baseline model, implied matching efficiency does not decline much. Does that cause

an underestimation on the role played by matching efficiency on unemployment? In order to check whether the importance of unemployment benefits and the irrelevance of matching efficiency I get in previous sections depend on which observables I use, in this part I follow Furlanetto and Groshenny (2012) first using the data on job finding rate constructed by Fujita and Ramey (2009) and labor market tightness to back out matching efficiency series and use that series as one observable instead of vacancies during the estimation.[2] Figure 1.11 is the matching efficiency series implied by the estimated model, and this series has very similar pattern to that derived in Furlanetto and Groshenny (2012), and Barnichon and Figura (2011).

Although different data is used, we can find that estimation results from this estimation are very similar to what were obtained before by comparing the second and sixth column of Table 1.8. Matching efficiency shocks are still unimportant for unemployment as shown in the second to last column of Table 1.9. Matching efficiency shocks explain less than 0.3% of unemployment fluctuations, while unemployment benefits shocks could explain 15% of them.

1.6   Conclusion

Unemployment benefits shocks are responsible for more unemployment fluctuations than the matching efficiency shock. The former accounts for over 15% of unemployment variation, while the latter accounts for only less than 5% of it. In the current recession, extended unemployment benefits contribute to the unemployment rate, while the effect of deteriorating matching efficiency is very small. During the Great Recession, extended unemployment benefits contributed to more than 1% increase in the unemployment rate.

Since the model with endogenous separation fits the data better, we can know that the separation rate should comove with unemployment instead of being constant. Meanwhile, no evidence shows the job finding rate is significantly lowered because of the severely deteriorating matching efficiency during the recession. This means that the high unemployment rate in a recession is not caused by an abnormally low job

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[2] I also use the job-finding rate data in Furlanetto and Groshenny (2012), which differs from Fujita-Ramey dataset in dealing with margin error. Similar results were found with this specification.

finding rate only, but by a higher separation rate together with a low job finding rate.

The endogenous separation magnifies labor market fluctuations significantly, because it amplifies all the shocks in the model except the matching efficiency shock, which is not important for unemployment fluctuations. However, the effect of real wage rigidity is ambiguous, because although it could amplify the technology shock, it also dampens the effects of several other shocks, such as the unemployment benefits shock, which is very important for unemployment fluctuations.

Technology shocks and investment specific technology shocks alone are not enough to explain labor market fluctuations. In addition to technology shocks, changes in unemployment benefits also play a very important role in labor market dynamics.

1.7 Appendix

1.7.1 Equations

1.7.1.1 Stationary Model

$$u_t = 1 - n_t \tag{1.34}$$

$$n_{t+1} = (1 - \rho_{t+1})(n_t + m(u_t, v_t)) = (1 - \rho_{t+1})(n_t + \epsilon_t^M E u_t^\zeta v_t^{1-\zeta}) \tag{1.35}$$

$$\rho_t^w = m(u_t, v_t)/u_t = \epsilon_t^M \mathcal{M} u_t^\zeta v_t^{1-\zeta}/u_t = \epsilon_t^M \mathcal{M} \theta_t^{1-\zeta} \tag{1.36}$$

$$\rho_t^f = m(u_t, v_t)/v_t = \epsilon_t^M \mathcal{M} u_t^\zeta v_t^{1-\zeta}/v_t = \epsilon_t^M \mathcal{M} \theta_t^{-\zeta} \tag{1.37}$$

$$\rho_t = \rho^x + (1 - \rho^x)\rho_t^n = \rho^x + (1 - \rho^x) \int_{-\infty}^{\widetilde{a}_t} f(a_t) da_t = \rho^x + (1 - \rho^x) F(\widetilde{a}_t) \tag{1.38}$$

$$\frac{y(\widetilde{a}_t)}{\mu_t} - w(\widetilde{a}_t) - r_t^k k^*(\widetilde{a}_t) + \frac{\gamma}{\rho_t^f} = 0 \tag{1.39}$$

$$\overline{\beta}\mathbb{E}_t\{\frac{\lambda_{1t+1}}{\lambda_{1t}}(1 - \rho_{t+1})\rho_t^f[\frac{1-\alpha}{\alpha}r_{t+1}^k k_{t+1}^* - w_{t+1} + \frac{\gamma}{\rho_{t+1}^f}]\} = \gamma/\iota \tag{1.40}$$

$$w_t = \eta[\epsilon_t^\eta \eta(\frac{1-\alpha}{\alpha}r_t^k k_t^* + \gamma\theta_t) + (1 - \epsilon_t^\eta \eta)(A + g_t^u)] + (1 - \omega^W)w \tag{1.41}$$

$$1 = \overline{\beta}r_t\mathbb{E}_t[\frac{\lambda_{1t+1}}{\lambda_{1t}}\frac{P_t}{P_{t+1}}] \text{ where } \overline{\beta} = \beta\iota^{-\sigma} \text{ and } \lambda_{1t} = \widetilde{\lambda}_{1t}\iota^{\sigma t} \tag{1.42}$$

$$Q_t = \overline{\beta}\mathbb{E}_t\{\frac{\lambda_{1t+1}}{\lambda_{1t}}[Q_{t+1}(1 - \delta) + d_{t+1}r_{t+1}^k - D(d_{t+1})]\} \tag{1.43}$$

$$Q_t\psi'(\frac{\iota i_t}{i_{t-1}})\frac{\epsilon_t^I \iota i_t}{i_{t-1}} - \overline{\beta}\mathbb{E}_t[Q_{t+1}\frac{\lambda_{t+1}}{\lambda_t}\psi'(\frac{\iota i_{t+1}}{i_t})\frac{\epsilon_{t+1}^I \iota i_{t+1}}{i_t}\frac{\iota i_{t+1}}{i_t}] + 1$$
$$= Q_t(1 - \psi(\frac{\iota i_t}{i_{t-1}})) \tag{1.44}$$

$$r_t^k = D'(d_t) \tag{1.45}$$

$$k_t^H = \frac{1-\delta}{\iota}k_{t-1}^H + \epsilon_t^I(1 - \psi(\frac{\iota i_t}{i_{t-1}}))i_t \tag{1.46}$$

$$k_t^* = \int_{\widetilde{a}_t}^{a_{max}} k(a_{jt})^* \frac{f(a_t)}{1 - F(\widetilde{a}_t)} da_t = (\frac{\alpha z_t}{\mu_t r_t^k})^{\frac{1}{1-\alpha}} \int_{\widetilde{a}_t}^{a_{max}} a_t^{\frac{1}{1-\alpha}} \frac{f(a_t)}{1 - F(\widetilde{a}_t)} da_t$$
$$= (\frac{\alpha z_t}{\mu_t r_t^k})^{\frac{1}{1-\alpha}} \frac{X(\widetilde{a}_t)}{1 - F(\widetilde{a}_t)} \tag{1.47}$$

$$\text{where } X(\widetilde{a}_t) = \int_{\widetilde{a}_t}^{a_{max}} a_t^{\frac{1}{1-\alpha}} f(a_t) da_t$$
$$= e^{\frac{\mu_a}{1-\alpha} + \frac{\sigma_a^2}{2(1-\alpha)^2}} \Phi(\frac{\mu_a + \sigma_a^2/(1-\alpha) - \log \widetilde{a}_t}{\sigma_a})$$

$$\widetilde{k}_t^* = k^*(\widetilde{a}_t) = (\frac{\alpha z_t \widetilde{a}_t}{\mu_t r_t^k})^{\frac{1}{1-\alpha}} \tag{1.48}$$

$$n_t \iota k_t^* = d_t k_{t-1}^H \tag{1.49}$$

$$\lambda_{1t} = (c_t - h/\iota c_{t-1})^{-\sigma} \tag{1.50}$$

$$y_t = n_t \frac{\mu_t r_t^k}{\alpha} k_t^* - \gamma v_t \tag{1.51}$$

$$y_t = c_t + i_t + g_t + D(d_t) k_{t-1}^H / \iota \tag{1.52}$$

$$P_t^{1-\epsilon_t^P} = \omega(P_{t-1}\Pi_{t-1}^\xi)^{1-\epsilon_t^P} + (1-\omega)(P_t^*)^{1-\epsilon_t^P} \text{ where } \Pi_t = \frac{P_t}{P_{t-1}} \tag{1.53}$$

$$g_t^y = \frac{g_t}{y} \tag{1.54}$$

$$g_t^{utotal} = g_t^u u_t = \epsilon_t^{g^u} \overline{rr} y_t^L \tag{1.55}$$

1.7.1.2 Steady State

$$u = 1 - n \tag{1.56}$$

$$\rho n = m(u,v) = (1-\rho)\mathcal{M}u^\zeta v^{1-\zeta} \tag{1.57}$$

$$\rho^w = \frac{m(u,v)}{u} = \mathcal{M}\theta^{1-\zeta} \tag{1.58}$$

$$\rho^f = \frac{m(u,v)}{v} = \mathcal{M}\theta^{-\zeta} \tag{1.59}$$

$$\rho = \rho^x + (1-\rho^x)F(\widetilde{a}) \tag{1.60}$$

$$(1-\omega^W \eta)\frac{1-\alpha}{\alpha} r^k \widetilde{k}^* - \eta\gamma\theta - (1-\eta)(A+g^u) - (1-\omega^W)\eta\frac{1-\alpha}{\alpha} r^k k^* + \frac{\gamma}{\rho^f} = 0 \tag{1.61}$$

$$\overline{\beta}\rho^f(1-\rho)(\frac{1-\alpha}{\alpha}r^k k^* - w + \frac{\gamma}{\rho^f}) = \gamma/\iota \tag{1.62}$$

$$w = \eta(\frac{1-\alpha}{\alpha}r^k k^* + \gamma\theta) + (1-\eta)(A + g^u) \tag{1.63}$$

$$\overline{\beta} = \frac{\pi}{r} \tag{1.64}$$

$$q = 1 \text{ where } \Psi'(\frac{I}{K}) = 1 \tag{1.65}$$

$$1 = \overline{\beta}(1 - \delta + r^k) \tag{1.66}$$

$$r^k = D'(1) \text{ where } d = 1 \tag{1.67}$$

$$\frac{i}{k^H} = 1 - \frac{1-\delta}{\iota} \tag{1.68}$$

$$k^* = \frac{1}{1 - F(\widetilde{a})}(\frac{\alpha}{\mu r^k})^{\frac{1}{1-\alpha}} \int_{\widetilde{a}}^{a_{max}} a^{\frac{1}{1-\alpha}} f(a)da \tag{1.69}$$

$$\widetilde{k}^* = (\frac{\alpha\widetilde{a}}{\mu r^k})^{\frac{1}{1-\alpha}} \tag{1.70}$$

$$nk^*\iota = k^H \tag{1.71}$$

$$y = \frac{n\mu r^k k^*}{\alpha} - \gamma v \tag{1.72}$$

$$y = c + i + g \tag{1.73}$$

$$\lambda_1 = c^{-\sigma}(1 - h/\iota)^{-\sigma} \tag{1.74}$$

$$\mu = \frac{\epsilon^P}{\epsilon^P - 1} \tag{1.75}$$

$$g = g^y y \tag{1.76}$$

$$g^{utotal} = g^u u = \overline{rr}y^L u \tag{1.77}$$

1.7.1.3 Log-linear Model

$$\widehat{u}_t = -\frac{n}{u}\widehat{n}_t \tag{1.78}$$

$$\widehat{n}_{t+1} = (1-\rho)\widehat{n}_t - \frac{\rho}{1-\rho}\widehat{\rho}_{t+1} + \rho[\widehat{\epsilon}_t^M + \zeta\widehat{u}_t + (1-\zeta)\widehat{v}_t] \tag{1.79}$$

$$\widehat{\rho}_t^w = \widehat{\epsilon}_t^M + (\zeta - 1)\widehat{u}_t + (1-\zeta)\widehat{v}_t \tag{1.80}$$

$$\widehat{\rho}_t^f = \widehat{\epsilon}_t^M + \zeta\widehat{u}_t - \zeta\widehat{v}_t \tag{1.81}$$

$$\widehat{\rho}_t = [\frac{(1-\rho^x)\rho^n}{\rho}]\widehat{\rho}_t^n = [\frac{(1-\rho^x)\rho^n}{\rho}]\frac{f(\widetilde{a})\widetilde{a}}{F(\widetilde{a})}\widehat{\widetilde{a}}_t \tag{1.82}$$

$$(1-\omega^W\eta)\frac{1-\alpha}{\alpha}r^k\widetilde{k}^*(\widehat{r}_t^k + \widehat{\widetilde{k}}_t^*) = \omega^W(1-\eta)g^u\widehat{g}_t^u + \omega^W\eta\gamma\theta(\widehat{v}_t - \widehat{u}_t) + \frac{\gamma}{\rho^f}\widehat{\rho}_t^f$$
$$+ \omega^W\eta(\frac{1-\alpha}{\alpha}r^k\widetilde{k}^* + \gamma\theta - A - g^u)\epsilon_t^\eta \tag{1.83}$$

$$-\widehat{\rho}_t^f = \widehat{\lambda}_{1t+1} - \widehat{\lambda}_{1t} - \frac{\rho}{1-\rho}\widehat{\rho}_{t+1} + \frac{\frac{1-\alpha}{\alpha}r^k k^*(\widehat{r}_{t+1}^k + \widehat{k}_{t+1}^*) - w\widehat{w}_{t+1} - \frac{\gamma}{\rho^f}\widehat{\rho}_{t+1}^f}{\frac{1-\alpha}{\alpha}r^k k^* - w + \frac{\gamma}{\rho^f}} \tag{1.84}$$

$$w\widehat{w}_t = \omega^W\eta[\frac{1-\alpha}{\alpha}r^k k^*(\widehat{r}_t^k + \widehat{k}_t^*) + \gamma\theta(\widehat{v}_t - \widehat{u}_t)] + \omega^W(1-\eta)g^u\widehat{g}_t^u$$
$$+ \omega^W\eta(\frac{1-\alpha}{\alpha}r^k k^* + \gamma\theta - A - g^u)\epsilon_t^\eta \tag{1.85}$$

$$\widehat{\lambda}_{1t} = \widehat{r}_t + \mathbb{E}_t(\widehat{\lambda}_{1t+1} - \widehat{\pi}_{t+1}) + \widehat{\epsilon}_t^b \tag{1.86}$$

$$\widehat{q}_t = -(\widehat{r}_t - \mathbb{E}_t\widehat{\pi}_{t+1}) + \overline{\beta}(1-\delta)\mathbb{E}_t\widehat{q}_{t+1} + (1 - \overline{\beta}(1-\delta))\mathbb{E}_t\widehat{r}_{t+1}^k - \widehat{\epsilon}_t^b \tag{1.87}$$

$$\widehat{i}_t = \frac{1}{1+\overline{\beta}\iota}\widehat{i}_{t-1} + \frac{\overline{\beta}\iota}{1+\overline{\beta}\iota}\widehat{i}_{t+1} + \frac{\phi}{\iota^2(1+\overline{\beta}\iota)}\widehat{q}_t - \frac{1}{1+\overline{\beta}\iota}\widehat{\epsilon}_t^I \text{ where } \phi = \frac{1}{\psi''(\iota)} \tag{1.88}$$

$$\widehat{r}_t^k = \sigma_d\widehat{d}_t \tag{1.89}$$

$$\widehat{k}_t^H = \frac{1-\delta}{\iota}\widehat{k}_{t-1}^H + \delta\widehat{i}_t \tag{1.90}$$

$$\widehat{k}_t^* = \frac{\rho^n}{1-\rho^n}\widehat{\rho}_t^n + \frac{1}{1-\alpha}(\widehat{z}_t - \widehat{\mu}_t - \widehat{r}_t^k) + \frac{X'(\widetilde{a})}{X(\widetilde{a})}\widetilde{a}\widehat{\widetilde{a}}_t \tag{1.91}$$

$$\widehat{\widetilde{k}}_t^* = \frac{1}{1-\alpha}(\widehat{z}_t + \widehat{\widetilde{a}}_t - \widehat{\mu}_t - \widehat{r}_t^k) \tag{1.92}$$

$$\widehat{k}_{t-1}^H = \widehat{n}_t + \widehat{k}_t^* - \widehat{d}_t \tag{1.93}$$

$$\widehat{\lambda}_{1t} = \frac{-\sigma}{1-h/\iota}\widehat{c}_t + \frac{\sigma h}{\iota - h}\widehat{c}_{t-1} \tag{1.94}$$

$$\widehat{y}_t = (1 + \gamma\frac{v}{y})(\widehat{n}_t + \widehat{\mu}_t + \widehat{r}_t^k + \widehat{k}_t^*) - \gamma\frac{v}{y}\widehat{v}_t \tag{1.95}$$

$$\widehat{y}_t = \frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t + \widehat{g}_t + \frac{r^k k^H}{y\iota}\widehat{k}_{t-1}^H \tag{1.96}$$

$$\widehat{\pi}_t = \frac{\overline{\beta}}{1+\overline{\beta}\xi}\mathbb{E}_t\widehat{\pi}_{t+1} + \frac{\xi}{1+\overline{\beta}\xi}\widehat{\pi}_{t-1} - \frac{(1-\overline{\beta}\omega)(1-\omega)}{\omega(1+\overline{\beta}\xi)}\widehat{\mu}_t + \widehat{\epsilon}_t^P \tag{1.97}$$

$$\widehat{r}_t = (1 - \phi_r)(\phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t) + \phi_r \widehat{r}_{t-1} + \widehat{\epsilon}_t^r \tag{1.98}$$

$$\widehat{g}_t = \widehat{g}_t^y \tag{1.99}$$

$$\widehat{g}_t^{utotal} = \widehat{\epsilon}_t^{g^u} + \widehat{y}_t^L + \widehat{u}_t \text{ and } \widehat{g}_t^u = \widehat{\epsilon}_t^{g^u} + \widehat{y}_t^L \tag{1.100}$$

1.7.2 Data

Data sources and description are listed in Table 1, 2 and 3.

Table 1.1: Data Description and Sources

Data Title	Data Description	Data Sources
GDPC96	Real Gross Domestic Product Billions of Chained 1996 Dollars Seasonally Adjusted Annual Rate	U.S. Department of Commerce: Bureau of Economic Analysis
GDPDEF	Gross Domestic Product Implicit Price Deflator, 1996=100 Seasonally Adjusted	U.S. Department of Commerce: Bureau of Economic Analysis
PCEC	Personal Consumption Expenditure Billions of Dollars Seasonally Adjusted Annual Rate	U.S. Department of Commerce: Bureau of Economic Analysis
CE16OV	Civilian Employment Sixteen Years & Over, Thousands Seasonally Adjusted, 1996=100	U.S. Department of Labor: Bureau of Labor Statistics
FEDR	Federal Funds Rate Averages of Daily Figures Percent	Board of Governors of the Federal Reserve System
LNS10000000	Labor Force Status Civilian noninstitutional population Seasonally Adjusted	U.S. Department of Labor: Bureau of Labor Statistics
LNSindex	LNS10000000(1992:3)=1	
FPI	Fixed Private Investment Billions of Dollars Seasonally Adjusted Annual Rate	U.S. Department of Commerce: Bureau of Economic Analysis

Table 1.2: Data Description and Sources (continued)

Data Title	Data Description	Data Sources
RWAGE	Nonfarm Business, All Persons Hourly Compensation Duration index, 1992=100	U.S. Department of Labor: Bureau of Economic Analysis
UNRATE	Unemployment Rate Civilian Unemployment Rate Seasonally Adjusted	U.S. Department of Labor: Bureau of Economic Analysis
HELPWANT	Index of Help-Wanted Advertising 1987=100 Seasonally Adjusted	Composite Help-Wanted Index by Regis Barnichon
UNINS	Unemployment Insurance Billions of Dollars Seasonally Adjusted	U.S. Department of Commerce: Bureau of Economic Analysis
AWB	Average Weekly Benefit Amount Dollars Seasonally Adjusted	U.S. Department of Labor: Bureau of Labor Statistics

Table 1.3: Definition of Data Variables

Data Variable	Formula
Output(GDP)	$= \log\left(GDPC96/LNSindex\right) * 100$
Consumption(CONS)	$= \log\left(PCED/(GDPDEF * LNSindex)\right) * 100$
Investment(INV)	$= \log\left(FPI/(GDPDEF * LNSindex)\right) * 100$
Real wage(WAG)	$= \log\left(RWAGE/GDPDEF\right) * 100$
Unemployment insurance(INS)	$= \log\left(UNINS/(GDPDEF * LNSindex)\right) * 100$
Unemployment(UNEM)	$= \log\left(UNRATE\right) * 100$
Inflation(INF)	$= \log\left(GDPDEF/GDPDEF(-1)\right) * 100$
Federal funds rate(FFR)	$= FEDR/4$
Vacancy(VAC)	$= \log\left(HELPWANT/LNSindex\right) * 100$
Average Weekly Benefits(AWB)	$= \log\left(AWB/GDPDEF\right) * 100$

1.7.3   Tables

Table 1.4: Observed Variables and Shocks Comparison

SW (2007)[1]		GST (2008)[2]		Zhang (2011)[3]	
Obs	Shocks	Obs	Shocks	Obs	Shocks
GDP	Gov. Spending	GDP	Gov. Spending	GDP	Gov. Spending
CONS	Risk Prem.	CONS	Risk Prem.	CONS	Risk Prem.
INV	Invest. Tech.	INV	Invest. Tech.	INV	Invest. Tech.
WAG	Wage Markup	WAG	Bargain Power	WAG	Bargain Power
INF	Price Markup	INF	Price Markup	INF	Price Markup
FFR	Monetary	FFR	Monetary	FFR	Monetary
Emp	Technology	Emp	Technology	UNEM	Technology
-	-	-	-	VAC	Matching
-	-	-	-	INS	Unemp. Ben.

[1] SW (2007): Smets and Wouters (2007, AER)
[2] GST (2008): Gertler, Sala, and Trigari (2008, JMCB)
[3] Zhang (2011): this paper

Table 1.5: Mapping Between Observables and Shocks

Variables		Shocks
dGDP	$\Leftarrow$	Government Spending
dCONS	$\Leftarrow$	Risk Premium
dINV	$\Leftarrow$	Investment Specific Technology
dWAG	$\Leftarrow$	Bargaining Power
dINS & dAWB	$\Leftarrow$	Unemployment Benefit
INF	$\Leftarrow$	Price Markup
FFR	$\Leftarrow$	Monetary Policy
UEMP	$\Leftarrow$	Technology
VAC	$\Leftarrow$	Matching Efficiency

Table 1.6: Calibrated Parameters

β	δ	α	g^y	$\bar{\epsilon}^P$
0.99	0.025	0.33	0.18	11

Table 1.7: Prior and Posterior Distribution of Shocks and Structural Parameters

	Prior Distribution			Posterior Distribution			
	Distribution	Mean	St. Dev.	Mode	Mean	5 percent	95 percent
σ_b	InvGamma	0.10	0.15	1.81	2.03	1.34	2.72
σ_η	InvGamma	0.10	0.15	1.87	2.00	1.72	2.25
σ_I	InvGamma	0.10	0.15	0.90	0.90	0.74	1.07
σ_p	InvGamma	0.10	0.15	0.21	0.22	0.20	0.25
σ_z	InvGamma	0.10	0.15	0.59	0.61	0.54	0.67
σ_m	InvGamma	0.10	0.15	8.25	9.45	8.14	10.65
σ_g	InvGamma	0.10	0.15	0.66	0.66	0.59	0.73
σ_{g^u}	InvGamma	0.10	0.15	1.61	1.28	0.65	1.81
σ_r	InvGamma	0.10	0.15	0.25	0.26	0.23	0.28
ρ_b	Beta	0.50	0.20	0.40	0.39	0.27	0.50
ρ_η	Beta	0.50	0.20	0.72	0.70	0.65	0.76
ρ_I	Beta	0.50	0.20	0.93	0.92	0.89	0.96
ρ_p	Beta	0.50	0.20	0.86	0.85	0.81	0.89
ρ_z	Beta	0.50	0.20	0.98	0.98	0.97	0.99
ρ_m	Beta	0.50	0.20	0.94	0.94	0.91	0.97
ρ_g	Beta	0.50	0.20	0.98	0.97	0.97	0.98
ρ_{g^u}	Beta	0.50	0.20	0.97	0.97	0.95	0.98
ρ_r	Beta	0.50	0.20	0.23	0.23	0.12	0.35
μ_p	Beta	0.50	0.20	0.99	0.99	0.98	0.99
μ_{gz}	Beta	0.50	0.20	0.59	0.64	0.49	0.81
ϕ_r	Beta	0.75	0.10	0.77	0.77	0.73	0.81
ϕ_π	Normal	2.20	0.10	2.37	2.34	2.20	2.48
ϕ_y	Beta	0.50	0.20	0.12	0.11	0.08	0.14
h	Beta	0.70	0.10	0.78	0.78	0.74	0.83
$\bar{\iota}$	Normal	0.40	0.10	0.38	0.38	0.37	0.39
ϕ	Normal	0.70	0.05	0.57	0.56	0.46	0.65
ξ	Beta	0.50	0.15	0.90	0.83	0.73	0.94
η	Beta	0.50	0.10	0.35	0.36	0.29	0.42
ω	Beta	0.50	0.10	0.82	0.81	0.80	0.82
σ_d	Normal	1.50	0.10	1.36	1.35	1.28	1.43
$\bar{\pi}_c$	Beta	0.50	0.20	0.93	0.91	0.85	0.98
ω^W	Normal	0.70	0.05	0.38	0.39	0.38	0.40
ρ^w	Normal	0.71	0.05	0.71	0.72	0.63	0.81
θ	Normal	0.63	0.05	0.72	0.75	0.66	0.83
$\bar{rr}$	Normal	0.25	0.05	0.26	0.20	0.11	0.30
ρ^x	Normal	0.07	0.05	0.09	0.07	0.04	0.10
$\widetilde{a}$	Unif(0.7,0.9)[1]	-	-	0.73	0.75	0.70	0.79
σ_a	Normal	0.1	0.05	0.16	0.15	0.11	0.19
ζ	Normal	0.5	0.05	0.58	0.57	0.51	0.64

[1] $\widetilde{a} = 0.7$ corresponds to $\rho^n = 0.01$, and $\widetilde{a} = 0.9$ corresponds to $\rho^n = 0.24$.

Table 1.8: Model Sensitivity – Estimation Results for Structural Parameters in Robustness Checks

	Base-line	No Wage Rigidity	Exo Sep	Benefits Policy with Unemp. feedback	Job-finding Rate as Observable	AWB as Observable
Marginal Likelihood						
	-1246	-1324	-1373	-1268	–	–
Posterior Mean of Structural Parameters						
ϕ_r	0.77	0.77	0.62	0.73	0.82	0.74
ϕ_π	2.34	2.31	2.40	2.33	2.43	2.39
ϕ_y	0.11	0.10	0.10	0.11	0.13	0.02
h	0.78	0.74	0.61	0.75	0.59	0.63
$\bar{\iota}$	0.38	0.40	0.41	0.38	0.40	0.34
ϕ	0.56	0.67	0.84	0.66	0.61	0.64
ξ	0.83	0.74	0.88	0.88	0.71	0.45
η	0.36	0.67	0.32	0.22	0.58	0.72
ω	0.81	0.81	0.72	0.81	0.81	0.76
σ_d	1.35	1.30	1.30	1.38	1.40	1.37
$\overline{\pi}_c$	0.91	0.95	0.93	0.91	0.94	0.97
ω^W	0.39	1	0.39	0.39	0.39	0.39
ρ^w	0.72	0.72	0.71	0.72	0.71	0.71
θ	0.75	0.79	0.77	0.78	0.74	0.76
$\overline{rr}$	0.20	0.21	0.21	0.24	0.26	0.24
ρ^x	0.07	0.06	0.07	0.08	0.06	0.06
$\widetilde{a}$	0.75	0.89	-	0.73	0.75	0.75
σ_a	0.15	0.04	-	0.14	0.19	0.19
ζ	0.57	0.50	0.48	0.60	0.63	0.69
ϕ_u	-	-	-	0.02	-	-

Table 1.9: Variance Decomposition of Unemployment (on impact / 40 quarters)
in the Baseline Model and Models for Robustness Checking (in %)

	Baseline	Exogenous Separation	Benefits Policy with Unemploy. feedback	Job-finding Rate as Observable	AWB as Observable
ν^z	47.12/40.19	66.93/44.79	44.89/39.03	50.76/52.58	52.89/49.06
ν^Θ	0.52/0.99	0.35/0.73	0.43/0.80	2.51/3.87	2.17/3.73
ν^I	23.29/19.79	4.66/6.01	25.69/19.51	25.19/18.93	24.71/17.55
ν^p	4.14/7.32	1.73/3.64	4.30/7.67	3.00/4.59	9.87/15.93
ν^r	0.15/0.32	0.04/0.09	0.11/0.22	0.51/0.84	0.32/0.59
ν^m	*6.30/8.90*	*18.42/30.65*	*6.54/9.63*	*0.24/0.31*	*0.11/0.15*
ν^g	2.17/1.27	1.29/1.70	2.04/1.30	1.59/0.88	0.81/0.64
ν^{g^u}	*15.80/20.16*	*6.35/11.89*	*15.62/21.09*	*14.36/14.97*	*8.41/11.09*
ν^b	0.49/1.06	0.24/0.50	0.38/0.75	1.853.03	0.69/1.26

[1] ν^z: technology shock
[2] ν^Θ: bargaining power shock
[3] ν^I: investment specific technology shock
[4] ν^p: price markup shock
[5] ν^r: monetary policy shock
[6] ν^m: matching efficiency shock
[7] ν^g: government spending shock
[8] ν^{g^u}: unemployment benefits shock
[9] ν^b: risk premium shock

1.7.4 Figures
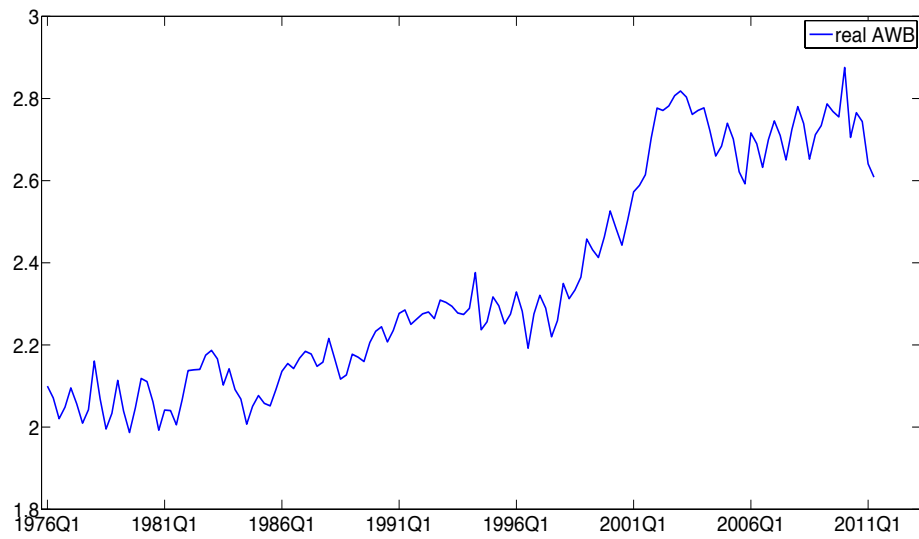
Figure 1.1: Fluctuations in Average Weekly Benefits
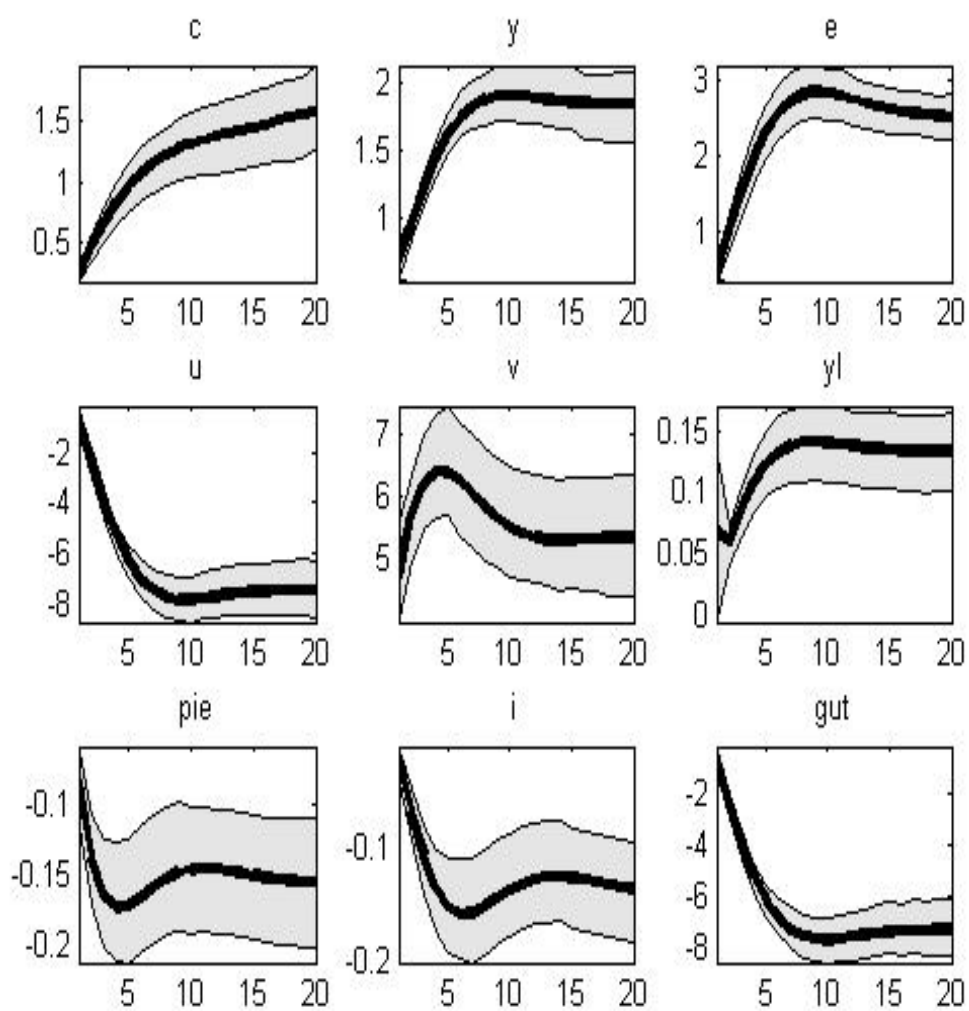
Figure 1.2: Technology Shock
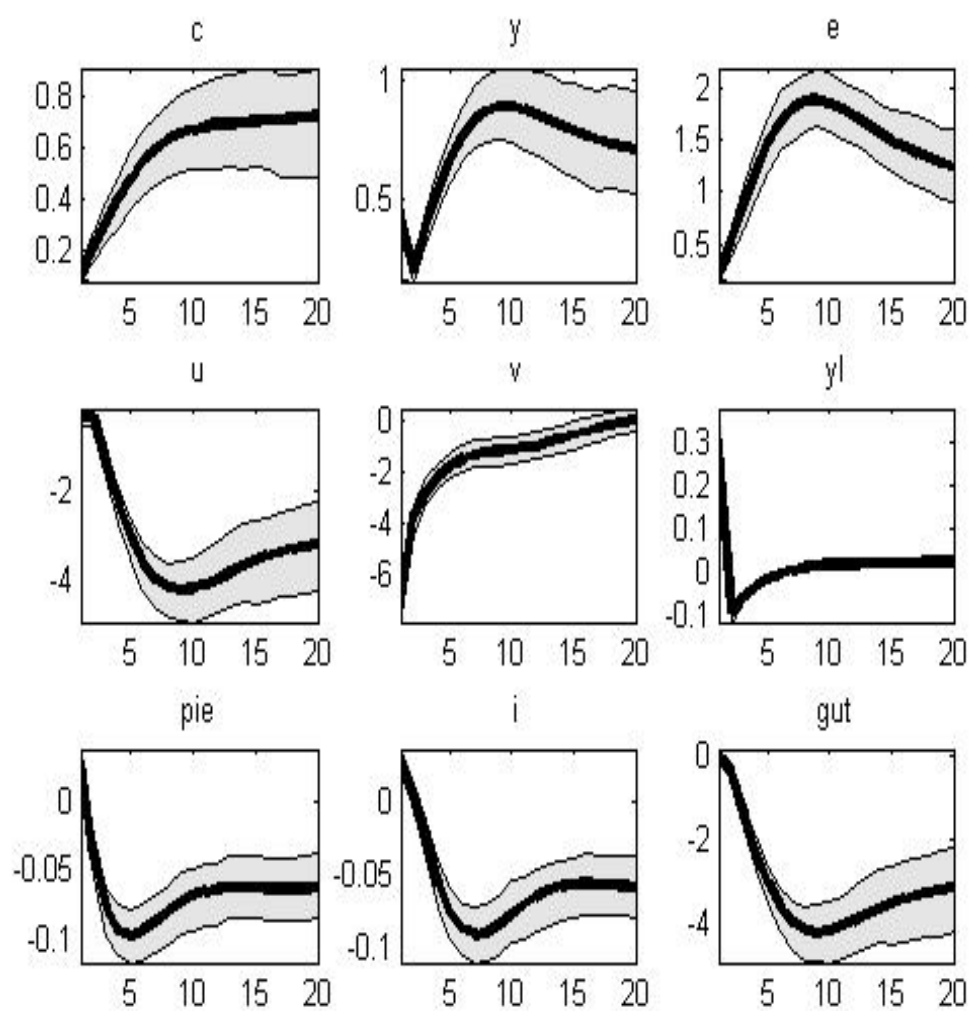
Figure 1.3: Matching Efficiency Shock
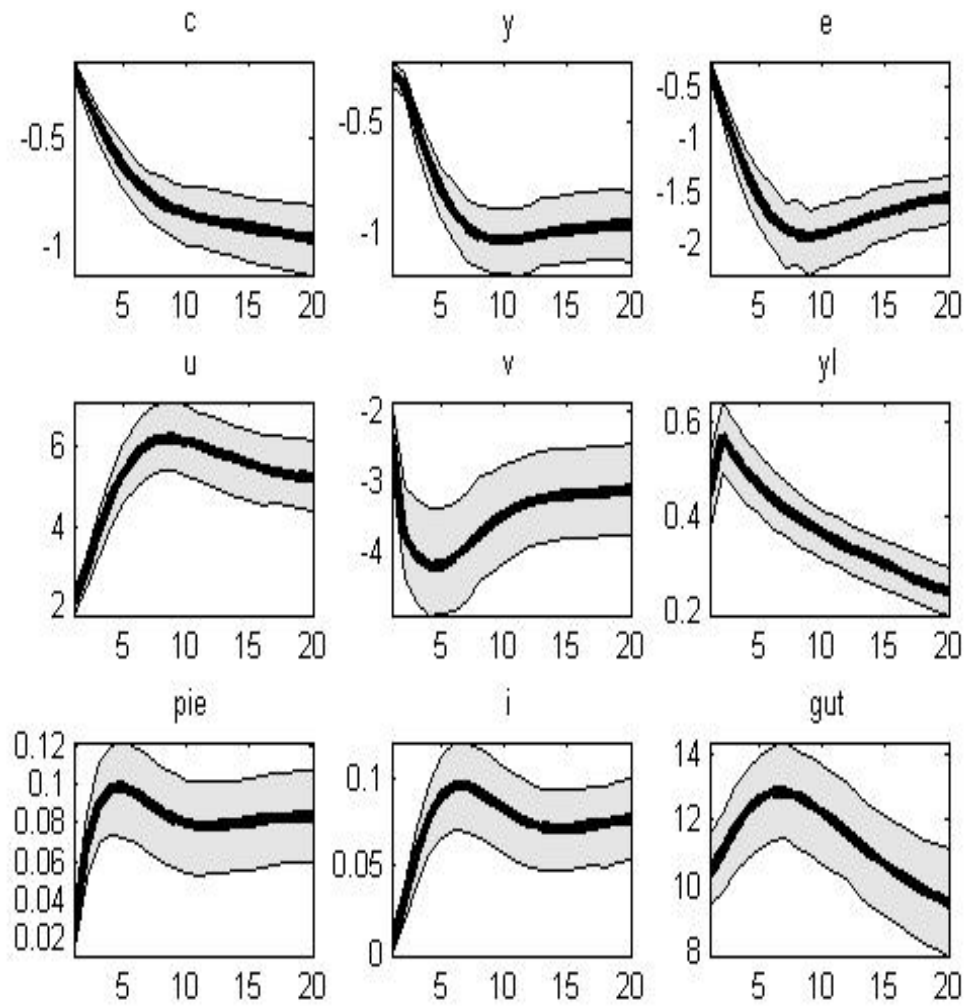
Figure 1.4: Unemployment Benefit Shock
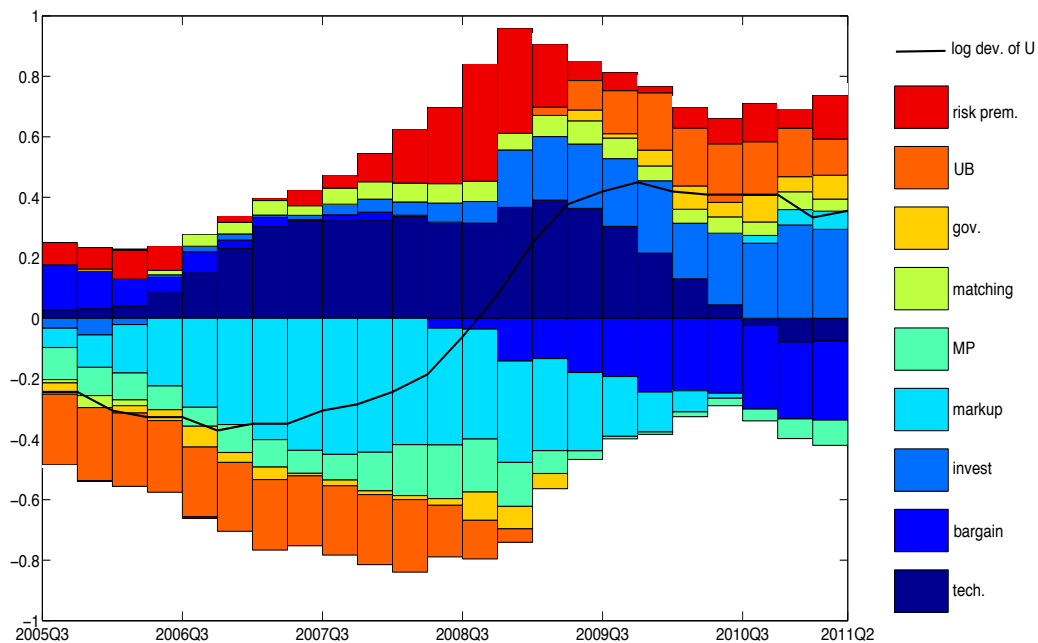
Figure 1.5: Historical Decomposition for Unemployment
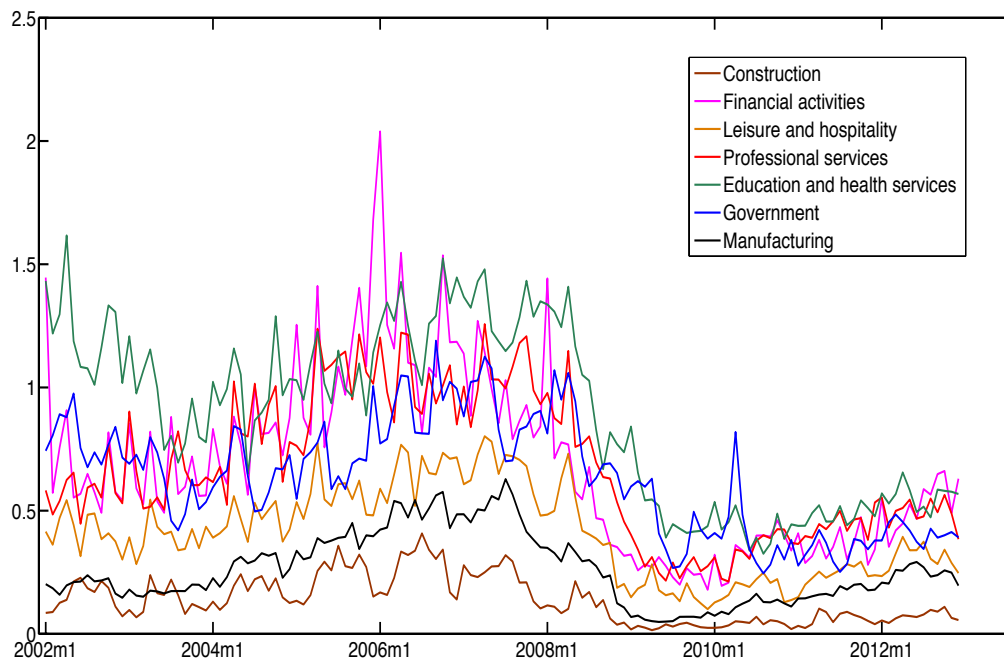
Figure 1.6: Vacancy / Unemployment by Industry
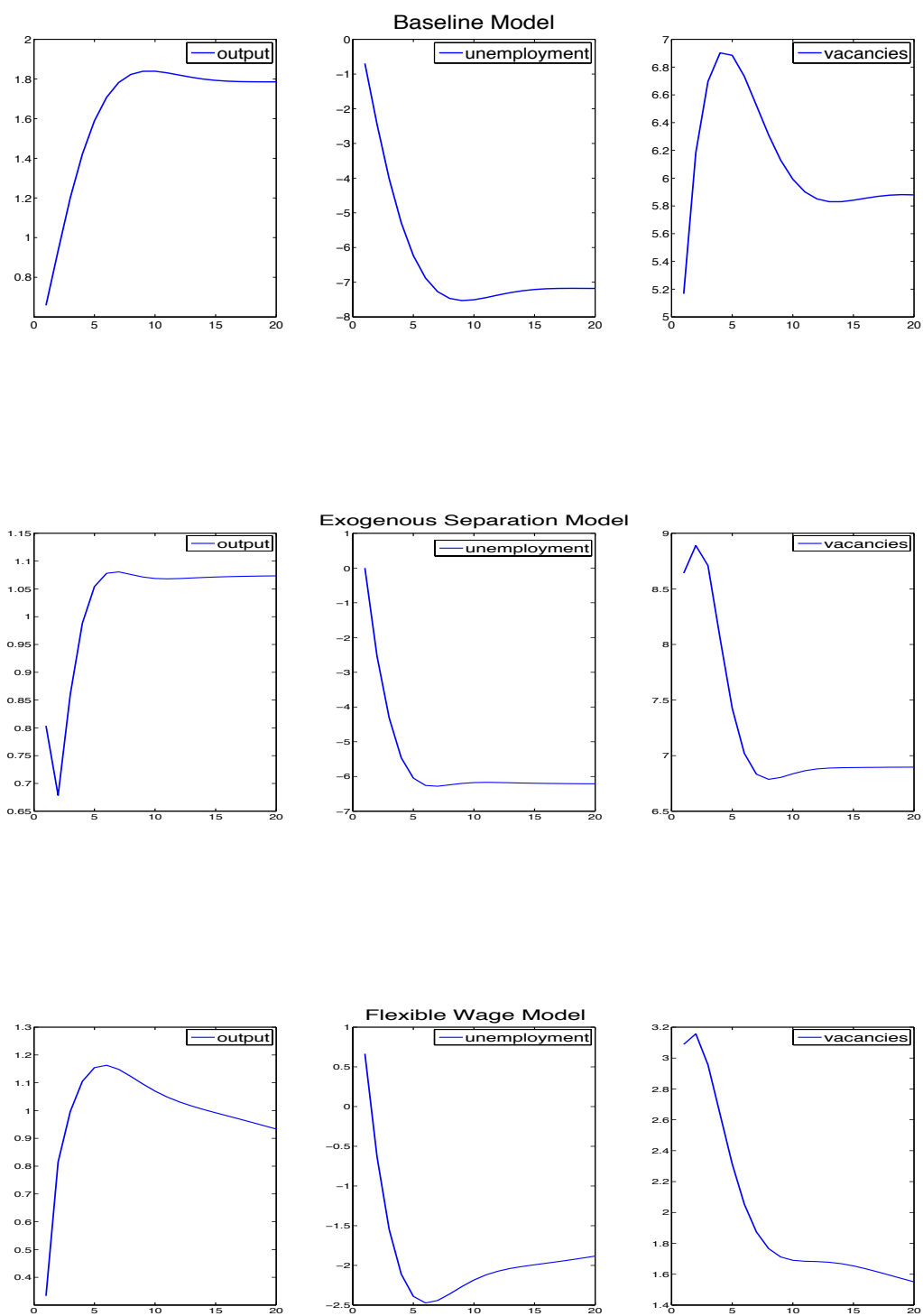
Figure 1.7: Comparison for Impulse Responses to a Technology Shock
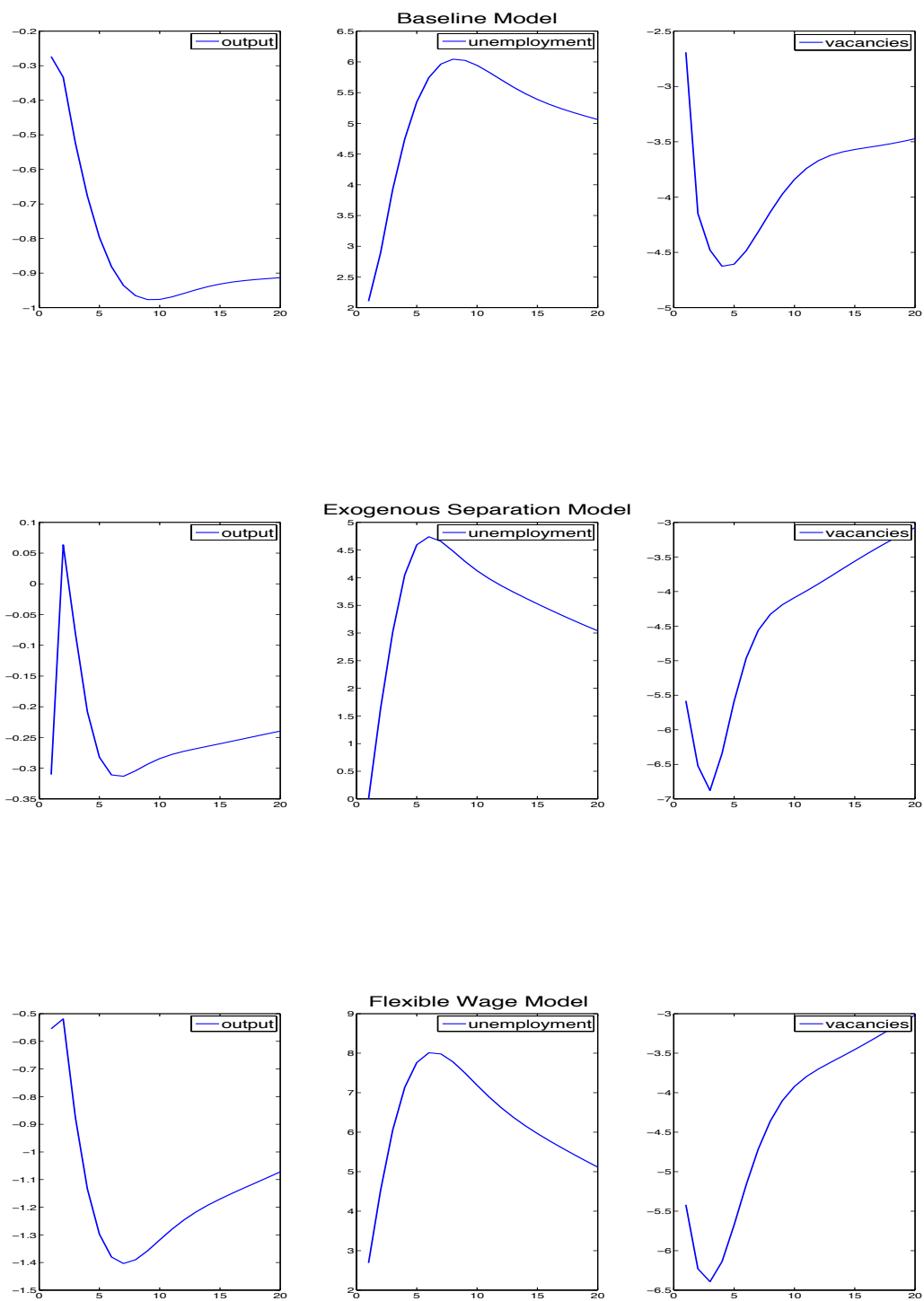
Figure 1.8: Comparison for Impulse Responses to an Unemp. Benefit Shock
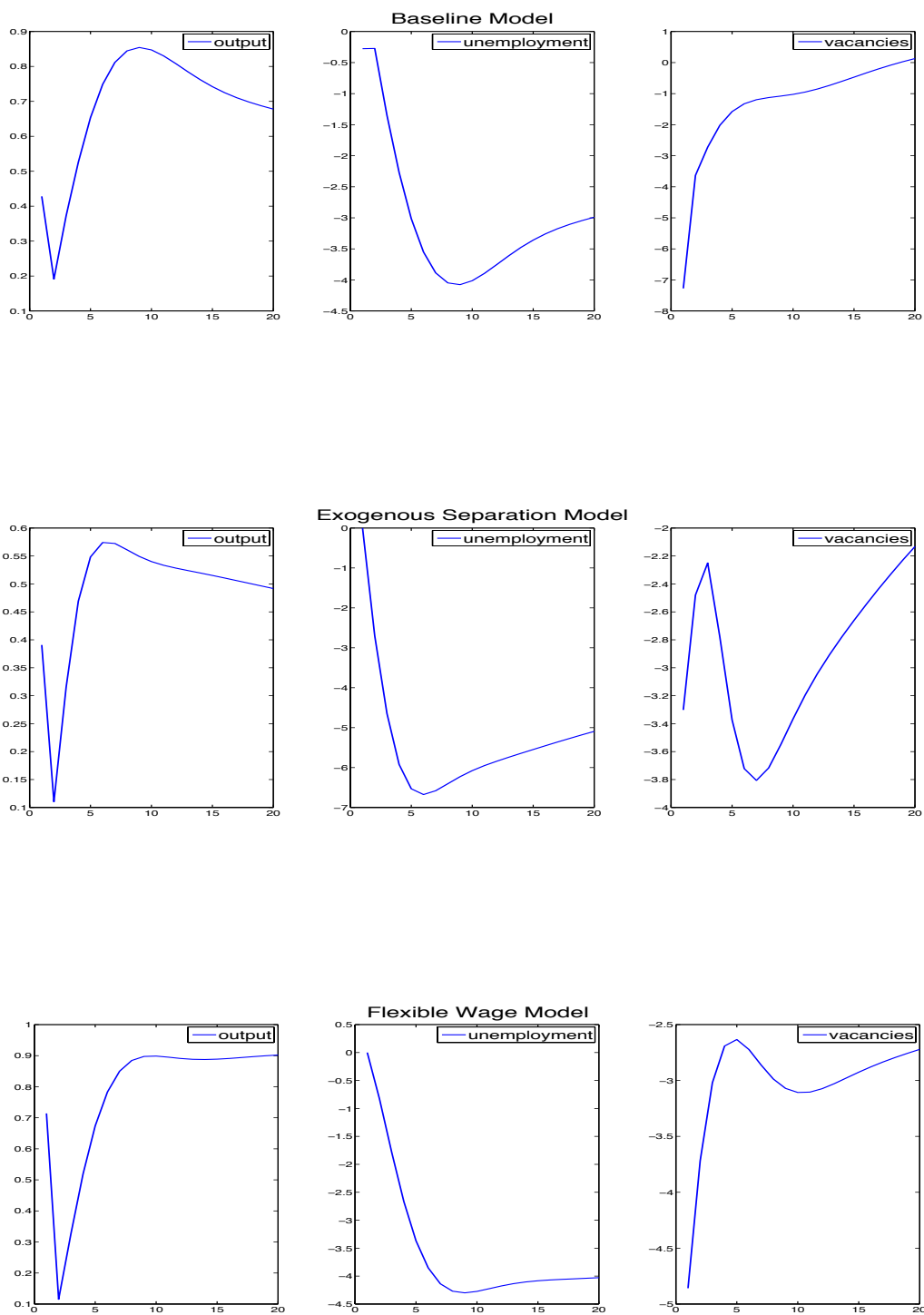
Figure 1.9: Comparison for Impulse Responses to a Matching Efficiency Shock
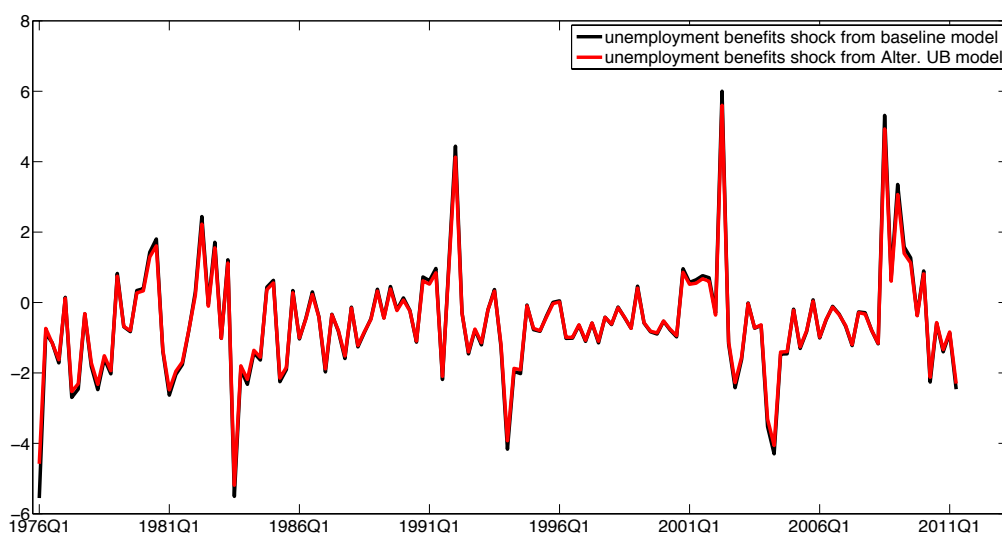
Figure 1.10: Unemployment Benefits Shocks Implied by the Baseline Model and the Model with the Alternative Unemployment Benefits Policy
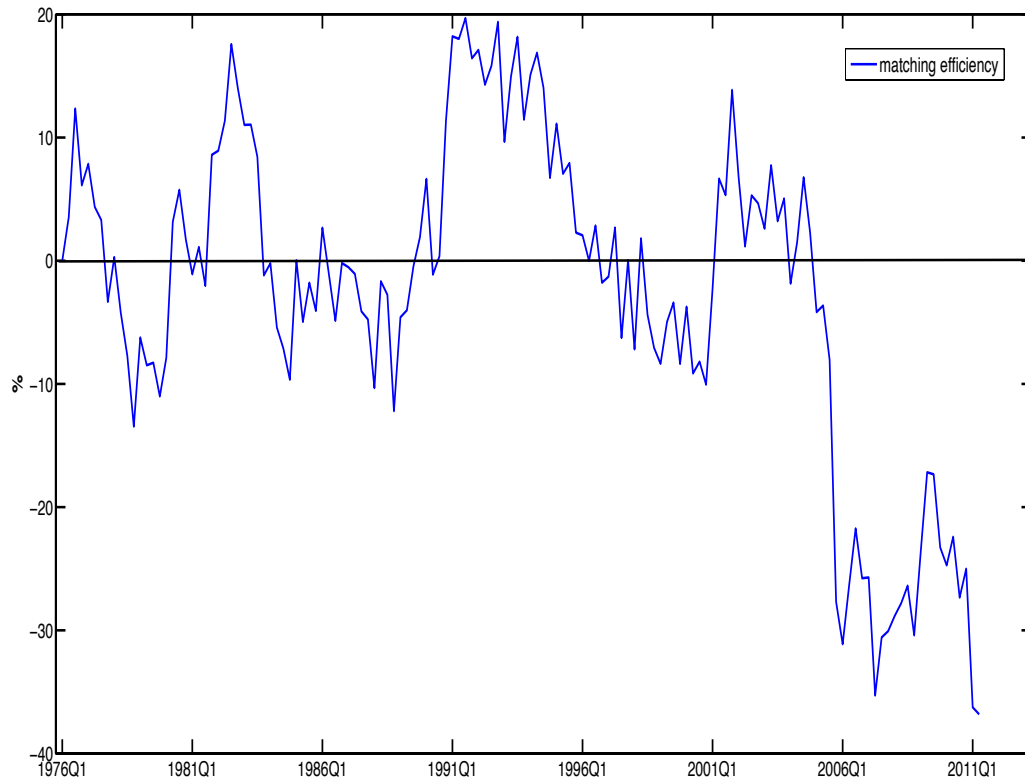
Figure 1.11: Matching Efficiency Implied by the Estimated Model When the
Job-finding Rate and Labor Market Tightness are Used as Observables

Acknowledgement

Chapter 2

Liquidity Shocks and Macroeconomic Policies in a Model with Labor Market Search Frictions

Abstract. By introducing a labor market with search frictions into a Kiyotaki-Moore model, I study the effect of liquidity shocks and several policies. I find that in the model with endogenous separation and real wage rigidity, extended unemployment benefits could slightly alleviate the big decline in output caused by a liquidity shock through mitigating current consumption decline, but raise unemployment and slow the recovery of the labor market. Unconventional credit policy is very effective in stabilizing output. Fiscal expansion has positive on impact effect but negative cumulative effect. The presence of the zero lower bound on the nominal interest rate is needed to get the above results. The importance and the length of staying at the zero lower bound depend on the type of labor market rigidities.

2.1   Introduction

Most people believe that the recent recession was aggravated by the crash in financial markets and a liquidity shock resulting from the bankruptcy of several financial intermediaries such as Lehman Brothers. Output decreased significantly, unemployment climbed to a surprisingly high level and is experiencing a very slow recovery, and the stock market declined dramatically. The federal funds rate collapsed to zero at the end of 2008, which made the traditional tools of monetary policy ineffective. In order to stimulate the economy and decrease the unemployment rate, various alternative policies have been used. The Federal Reserve injected liquidity into the economy by large scale purchases of agency debt, mortgage-backed securities and Treasuries, and its balance sheet has been expanded from $800 billion to over $2 trillion by January 2009. Fiscal expansion is also used to help the economy go out of the recession. President Obama's American Recovery and Reinvestment Act of February 2009 appropriated $787 billion to stimulate the economy. A third policy tool has involved unemployment benefits, which were extended to 99 weeks from 26 weeks. Net reserves of the State Unemployment Insurance program trust fund balance decreased to $-\$25$ billion at the end of June 2011 from $39.7 billion at the end of June 2008, and total unemployment benefits paid by the government increased from $13.6 billion in 2008Q3 to $40.4 billion in 2010Q2.

The impact of a liquidity shock as well as the effectiveness of the above policies are widely debated. Was the Great Recession caused by a negative liquidity shock alone? Is the presence of the zero lower bound (ZLB) on the nominal interest rate important? Are the policies implemented effective in rescuing our economy from the crisis? Does providing more benefits to the unemployed workers contribute to the persistently high unemployment rate? Or does a decrease in matching efficiency contribute more to the slow recovery?

Del Negro et al. (2011) developed a useful New Keynesian model for addressing some of these questions, extending the ideas in Kiyotaki and Moore (2008) to develop an empirically usable dynamic stochastic general equilibrium model. However, there is no unemployment in that model and no basis for discussing policy tools such as extended unemployment benefits. In this paper, I add a labor market with matching

frictions to their model, using two different approaches. The first approach follows Zhang (2012) allowing an endogenous job separation rate and real wage rigidity. The second approach follows Gertler, Sala and Trigari (2008) using exogenous separation and nominal wage rigidity instead.

Financial market frictions in this paper take the same form as in Del Negro et al. (2011). Entrepreneurs face two constraints on the financing of new investment projects: a borrowing constraint on issuing new equity and a resaleability constraint on selling existing equity holdings. A shock to the resaleability constraint is referred as a liquidity shock. Baseline policies are implemented via a simple interest rate rule constrained by the zero bound, constant government spending and constant unemployment benefits. In response to the liquidity shock, government could supplement the baseline policies with one of several alternative policies, including unconventional credit policy (government purchases of private assets), fiscal expansion and extended unemployment benefits, to stimulate the economy.

Besides Del Negro et al. (2011), there have been many influential studies government policies implemented during the Great Recession. For example, Gertler and Kiyotaki (2009), Gertler and Karadi (2011), Chen, Curdia, and Ferrero (2011), Curdia and Woodford (2009a,b), and Eggertsson and Woodford (2003) study unconventional monetary policies, while Christiano and Rebelo (2009), Eggertsson (2009), Cogan, Cwik, Taylor and Wieland (2010), and Woodford(2010) focus on the fiscal expansion. Papers like Moyen and Stahler (2012), Nakajima (2012), and Valletta and Kuang (2010) examine the effect of unemployment benefits on unemployment dynamics. In contrast to my paper, none of the studies on unconventional monetary policy and fiscal policy takes labor market with search and matching frictions into consideration, while papers studies unemployment benefits are either purely empirical or in an environment without the presence of the zero lower bound on the nominal interest rate, which is essential for the effect of policies both in the real economy and in the models.

The main contributions of this paper are summarized as follows. I introduce labor market search frictions into a New Keynesian model with financial frictions in two different approaches, so that the impacts of the liquidity shocks on the labor market, and the effectiveness of the unemployment policy could be studied. Com-

paring the results derived from models with different labor market setups helps us understand the importance of different features of the labor market.

The main results from the simulation with endogenous separation and real wage rigidity are as follows. First, I confirm that as in Del Negro et al. (2011), a liquidity shock can have a big impact on the economy, and depress equity prices at the zero lower bound. The latter is an important feature, since in Shi's (2011) adaption of Kiyotaki and Moore (2008), a liquidity shock leads to an increase in equity prices, contrary to what has always been observed historically. Second, I find as Del Negro et al. (2011) that when the zero lower bound is binding, unconventional credit policy can be particularly effective in mitigating the magnitude of the economic downturn. Third, I find that fiscal expansion can prevent the decline of output effectively at the zero lower bound; however, its effect on investment is small, and its cumulative effect on consumption is negative. Fourth, I find that at the zero lower bound, extended unemployment benefits could also be a useful tool for mitigating the decline in consumption and benefiting output slightly, though at the cost of raising the unemployment rate and slowing the recovery in the labor market, goods market and financial market. The longer the extended unemployment benefits program lasts, the greater the cost is. Meanwhile, a lower matching efficiency contributes less to the slow recovery. Fifth, away from the zero lower bound, the impact of a liquidity shock is smaller, and government policies are also much less effective.

The model with exogenous separation and nominal wage rigidity has similar results at the zero lower bound. Without the presence of the zero lower bound, results derived from the two models are different. The importance and the length of staying at the zero lower bound depend on the type of labor market frictions and rigidities. Different labor market setups cause the differences in the predicted responses of wages and productivity following a liquidity shock, and in turn cause the differences in implied inflation dynamics and the severity of the zero lower bound problem.

The rest of the paper is organized as follows: Section 2 is the model setup, Section 3 is the calibration, Section 4 is the analysis and results, and Section 5 is the conclusion.

2.2 Model

The main framework of the model comes from Kiyotaki and Moore (2008) and Del Negro et al. (2011). There are five types of agents: entrepreneurs in household, workers in household, capital producers, intermediate good firms, and final good firms.

2.2.1 Households

There is a representative household in the economy, and there are a continuum of members, indexed by i, measured on $[0, 1]$, in the household. At the beginning of each period, χ fraction of the household members are selected to be entrepreneurs through an i.i.d. random draw, and the rest $1 - \chi$ fraction of members are workers. That is, the probability of becoming an entrepreneur for a household member in a particular period is χ. In each period, the household members are re-numbered, so that a member $i \in [0, 1-\chi]$ is a worker and a member $i \in (1-\chi, 1]$ is an entrepreneur. Entrepreneurs have the opportunity to invest on capital, but they cannot supply labor. Workers supply labor but have no chance to invest on capital. Since not all workers are hired in each period, the workers are also re-numbered, so that a worker $i \in [0, U_t)$ is unemployed, and a worker $i \in [U_t, 1 - \chi]$ is employed.

People in a household bring back their purchases on consumption goods $C_t(i)$, and these goods are equally distributed among all members. Utility thus depends on the sum of all the consumption goods in the household:

$$C_t = \int_0^1 C_t(i)di. \tag{2.1}$$

Consumption C_t is also a CES function over a continuum of goods with elasticity of substitution ϵ^p,

$$C_t = [\int_0^1 (C_{\widetilde{j}t})^{\frac{\epsilon^p-1}{\epsilon^p}} d\widetilde{j}]^{\frac{\epsilon^p}{\epsilon^p-1}}, \epsilon^p > 1,$$

where $\widetilde{j}$ is the index of the differentiated final consumption goods.

Each member holds an equal share of the household's assets (bonds and equities). The household does not make the labor supply decision. All unemployed

members search on the job market and the frictional search and matching process determines who is employed. The representative household maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}}{1-\sigma} \tag{2.2}$$

s.t.

$$
\begin{aligned}
&C_t + p_t^I I_t + q_t(S_t - I_t - (1-\delta)S_{t-1}) + \frac{B_t}{p_t} = \\
&r_t^K S_{t-1} + \int_{U_t}^{1-\chi} Y_{it}^L di + \int_0^{U_t} (A + G_t^u) di + \frac{r_{t-1}B_{t-1}}{p_t} + D_t + D_t^I - T_t
\end{aligned}
\tag{2.3}
$$

The inter-temporal discount factor is β, and the relative risk aversion is σ.

Unlike Smets and Wouters (2007), I don't include the intensive margin of employment because Gertler, Sala and Trigari (2008) find that most of the cyclical variation in employment in the United States is on the extensive margin and including the intensive margin does not affect the model very much. Leisure and home production of the unemployed are not considered in the utility function. Following the convention in the literature (such as de Hann et al. (2000)), they are considered as a part of the total unemployment compensation which appears in the household's budget constraint as part of the household's income. The total unemployment compensation includes G_t^u, unemployment benefits paid by the government, as well as A, which represents other factors that can be measured in units of consumption goods, such as leisure and home production of the unemployed.

The price for the consumption good is p_t, and the gross nominal interest rate controlled by the Federal Reserve is r_t. The investment on capital is represented by I_t, and the cost of a unit of new investment in terms of the consumption goods is p_t^I. The dividends from the final good sector and the capital producers are D_t and D_t^I respectively, and the lump-sum tax is T_t. Household's disposable real labor income earned by member i is represented by Y_{it}^L.

In period t, the household's assets include government bonds B_{t-1}, capital K_{t-1}^H, and claims on other households' capital S_{t-1}^O. Households' liabilities include claims on own capital sold to other households S_{t-1}^I. The net equity held by the

household S_{t-1} is defined as

$$S_{t-1} = S^O_{t-1} + K^H_{t-1} - S^I_{t-1} \tag{2.4}$$

The rental return on capital is r^K_t, and the price of capital and equities in terms of a consumption good is q_t. The depreciation rate of capital is δ, which means both the claims on own capital and other households' capital depreciate at rate δ in each period.

There are two financial frictions, which are the same as those proposed by Del Negro et al. (2011). One is the borrowing constraint, which means each entrepreneur can only issue new equities up to a fraction θ of his investment $I_t(i)$. The other is the resaleability constraint, which implies that a household member can only sell a fraction ϕ_t of his equity holdings. The smaller θ and ϕ_t are, the more frictional the financial market is. The liquidity shock mentioned in this paper is a shock to ϕ_t.

The equity on own capital held by member $i \in (1 - \chi, 1]$ evolves according to

$$S^I_t(i) \le (1 - \delta)S^I_{t-1} + \theta I_t(i) + (1 - \delta)\phi_t(K^H_{t-1} - S^I_{t-1}), \tag{2.5}$$

where $\theta I_t(i) + (1-\delta)\phi_t(K^H_{t-1} - S^I_{t-1})$ is the maximum amount of the new issued equity, and the new issued equity could be separated into two parts: the claims on new investment, which gives up to $\theta I_t(i)$, and mortgaging capital that is not mortgaged before, which gives up to $(1-\delta)\phi_t(K^H_{t-1} - S^I_{t-1})$. And the equity on other households' capital held by member i evolves according to

$$S^O_t(i) \ge (1 - \delta)S^O_{t-1} - (1 - \delta)\phi_t S^O_{t-1}, \tag{2.6}$$

since the entrepreneur cannot sell more than a fraction ϕ_t of holdings of others' equity. The above two inequalities on equity together with the definition of the net equity give us the evolution of the net equity

$$S_t(i) \ge (1 - \theta)I_t(i) + (1 - \phi_t)(1 - \delta)S_{t-1} \tag{2.7}$$

The government bond is "liquid" and not constrained by the resaleability

constraint. Only the government can issue the liquid asset and households can only take a long position in it:

$$B_t(i) \geq 0. \tag{2.8}$$

The equity holdings, bond holdings, and capital stock of the household depend on each member's decision:

$$S_t = \int_0^1 S_t(i)di = (1-\theta)I_t + (1-\phi_t)(1-\delta)S_{t-1} \tag{2.9}$$

$$B_t = \int_0^1 B_t(i)di \tag{2.10}$$

$$K_t^H = (1-\delta)K_{t-1}^H + \int_0^1 I_t(i)di \tag{2.11}$$

where $I(i) = 0$ for $i \in [0, 1-\chi]$.

The amount of investment can be derived from the entrepreneurs decision. The budget constraint for the entrepreneurs is:

$$
\begin{aligned}
C_t(i) &+ p_t^I I_t(i) + q_t(S_t(i) - I_t(i) - (1-\delta)S_{t-1}(i)) + \frac{B_t(i)}{p_t} \\
&= r_t^K S_{t-1}(i) + \frac{r_{t-1}B_{t-1}(i)}{p_t} + D_t + D_t^I - T_t
\end{aligned} \tag{2.12}
$$

Since we have the assumption that the equity price q_t is greater than the cost of newly produced capital p_t^I, in order to maximize the household's utility, the entrepreneurs try their best to invest on new capitals. That is, they sell all bond holdings, borrow until the borrowing constraint binds, mortgage equity holdings to the upper bound, and buy no consumption good:

$$S_t(i) = (1-\theta)I_t(i) + (1-\phi_t)(1-\delta)S_{t-1} \tag{2.13}$$

$$B_t(i) = 0 \tag{2.14}$$

$$C_t(i) = 0 \tag{2.15}$$

where $i \in (1-\chi, 1]$.

Substituting these into the budget constraint for the entrepreneurs gives us

the investment of each entrepreneur:

$$I_t(i) = \frac{[r_t^K + (1 - \delta)q_t\phi_t]S_{t-1} + \frac{r_{t-1}B_{t-1}}{p_t} + D_t + D_t^I - T_t}{p_t^I - \theta q_t}. \tag{2.16}$$

Since only the entrepreneurs can invest, the aggregate investment is

$$I_t = \int_{1-\chi}^1 I_t(i)di = \chi\frac{[r_t^K + (1 - \delta)q_t\phi_t]S_{t-1} + \frac{r_{t-1}B_{t-1}}{p_t} + D_t + D_t^I - T_t}{p_t^I - \theta q_t}. \tag{2.17}$$

Households choose C_t, I_t, S_t and B_t to maximize the utility. The first order conditions are:

$$C_t : \ C_t^{-\sigma} = \lambda_{1t} \tag{2.18}$$

$$I_t : \ \lambda_{1t}(q_t - p_t^I) = \lambda_{2t} \tag{2.19}$$

$$S_t : \ q_t\lambda_{1t} = \beta\mathbb{E}_t\{\lambda_{1t+1}[r_{t+1}^K + (1 - \delta)q_{t+1}] + \lambda_{2t+1}\frac{\chi[r_{t+1}^K + (1 - \delta)\phi_{t+1}q_{t+1}]}{p_{t+1}^I - \theta q_{t+1}}\} \tag{2.20}$$

$$B_t : \ \lambda_{1t} = \beta\mathbb{E}_t[\frac{r_t}{\pi_{t+1}}(\lambda_{1t+1} + \lambda_{2t+1}\frac{\chi}{p_{t+1}^I - \theta q_{t+1}})] \tag{2.21}$$

where λ_{1t} and λ_{2t} are the Lagrangian multipliers for the budget constraint (3) and the equity evolution (9) respectively. Our previous assumption $q_t > p_t^I$ ensures that λ_{2t} is positive, I need to assume $q_t > p_t^I$. This means the price of equity is bigger than the newly installed capital.

Since we know the household decision on S_t, B_t, and C_t from the first order conditions, as well as the solution for entrepreneurs from (13) to (15), constraints (1), (9), and (10) determine workers' choices on consumption, equity holding, and bond holding. We can check that these choices satisfy the constraint (7) and (8) for workers.

2.2.2 Capital Producer

Capital producers can convert consumption goods into investment goods. Producing capital is costly, and the adjustment cost $\psi(\cdot)$ depends on the deviations of actual investment from its steady-state value I. The adjustment cost function also satisfies $\psi(1) = 0$, $\psi'(1) = 0$, and $\psi''(1) > 0$. The capital producers choose the

amount of investment goods produced I_t to maximize their profits

$$D_t^I = \{p_t^I - [1 + \psi(\frac{I_t}{I})]\}I_t. \tag{2.22}$$

Capital producers are perfectly competitive, and sell the investment goods to entrepreneurs at given price p_t^I.

The first order condition for capital producers maximization problem is:

$$p_t^I = 1 + \psi(\frac{I_t}{I}) + \psi'(\frac{I_t}{I})\frac{I_t}{I} \tag{2.23}$$

2.2.3   Intermediate Good Sector

2.2.3.1   First Specification: Labor Market with Endogenous Separation and Real Wage Rigidity

The intermediate good sector is perfectly competitive, and each firm hires one worker and rents capital to produce identical intermediate goods. The production function of the matched firms follows

$$Y(a_{jt}) = z a_{jt} K_{jt}^\alpha. \tag{2.24}$$

The common technology z is normalized to be 1. Match-specific productivity a_{jt} is a random variable, which follows a Lognormal distribution with mean 0 and standard deviation 0.15 (den Hann et al., 2000). Intermediate goods are sold in a competitive market at given price p_t'.

At the beginning of period t, there are N_t matched workers and firms retaining from last period; $U_t = 1 - \chi - N_t$ workers are unmatched. First, new entrepreneurs are randomly selected from all household members. χ fraction of old entrepreneurs are still entrepreneurs, and the rest $(1 - \chi)\chi$ old entrepreneurs become unemployed workers. χ fraction of matched workers become entrepreneurs, and the number of remaining matches becomes $(1 - \chi)N_t$. The number of new entrepreneurs from originally unemployed workers is χU_t. Now the number of new entrepreneur is still χ, and the new unemployment is $(1 - \chi)\chi + U_t - \chi U_t = U_t + \chi N_t$.

The remaining $(1 - \chi)N_t$ matched workers at the start of period t travel to

their places of employment. At that point, with an exogenous probability $0 \leq \rho^x < 1$ the match is terminated. The remaining $(1 - \rho^x)(1 - \chi)N_t$ pairs of matched workers and firms, indexed by j, jointly observe the match-specific productivity a_{jt}, and then decide whether to continue the match. If a_{jt} is larger than some threshold $\tilde{a}_{jt}$, the match continues and production occurs. Since all the intermediate good firms are identical ex ante, we can eliminate the subscript j. All the matches with match-specific productivity lower than $\tilde{a}_t$ are endogenously terminated. So the endogenous separation rate is given by

$$\rho_t^n = F(\tilde{a}_t) = \int_{-\infty}^{\tilde{a}_t} f(a_t)da_t \tag{2.25}$$

Finally, the number of remaining matches is $(1 - \chi)(1 - \rho^x)(1 - \rho_t^n)N_t$. The total separation rate is $\rho_t = 1 - (1 - \chi)(1 - \rho^x)(1 - \rho_t^n)$.

The number of new matches in period t is M_t. These new matches don't produce any good in the current period, but could only enter production in the next period after surviving from both exogenous and endogenous separations. The total number of matches evolves according to:

$$N_{t+1} = (1 - \rho_{t+1})(N_t + M_t). \tag{2.26}$$

The number of new matches in period t depends on the amount of vacancies posted by the firms, V_t, and the number of unemployed workers, U_t. The matching function $M_t(U_t, V_t)$ takes the form $EU_t^\zeta V_t^{1-\zeta}$, where E is the scale parameter standing for the aggregate matching efficiency.

The probability of a worker finding a job (the job-finding rate) is given by

$$\rho_t^w = \frac{M_t(U_t, V_t)}{U_t} = E\tau_t^{1-\zeta}, \tag{2.27}$$

and the probability of a vacancy being filled (the vacancy-filling rate) is

$$\rho_t^f = \frac{M_t(U_t, V_t)}{V_t} = E\tau_t^{-\zeta}, \tag{2.28}$$

where $\tau_t = V_t/U_t$ represents the labor market tightness.

Firms survived from separations choose capital optimally by maximizing

$$\frac{z_t a_{jt} K_{jt}^{\alpha}}{\mu_t} - r_t^k K_{jt}.$$

where $\mu_t = p_t/p_t'$ is the markup. The optimal capital is

$$K^*(a_{jt}) = (\frac{\alpha z a_{jt}}{\mu_t r_t^k})^{\frac{1}{1-\alpha}}. \tag{2.29}$$

Unmatched firms seeking workers have to pay a cost, γ, to post a vacancy. The vacancy could be filled with probability ρ_t^f, and the filled vacancy could be separated with probability $1 - \rho_{t+1}$ before entering production. The unmatched firm will post a vacancy only when the discounted expected future value of doing so is bigger than or equal the cost. Free entry ensures that unmatched firms post vacancies until

$$\gamma = \beta \rho_t^f \mathbb{E}_t[\frac{\widetilde{\lambda}_{1t+1}}{\widetilde{\lambda}_{1t}}(1 - \rho_{t+1})J_{t+1}], \tag{2.30}$$

where J_{t+1} is the expected future value of a matched firm, which is identical for all firms.

The value of a matched firm with match-specific productivity a_{jt} could be expressed as the net profit obtained from this period's production plus the discounted expected future value of the firm:

$$J_t(a_{jt}) = \frac{Y(a_{jt})}{\mu_t} - Y^L(a_{jt}) - r_t^k K^*(a_{jt}) + \beta \mathbb{E}_t[\frac{\widetilde{\lambda}_{1t+1}}{\widetilde{\lambda}_{1t}}(1 - \rho_{t+1})J_{t+1}], \tag{2.31}$$

where $Y(a_{jt})/\mu_t$ is the firm's revenue from selling the intermediate goods evaluated in terms of final goods, and $Y^L(a_{jt})$ is the real wage of the worker in terms of final goods.

A matched worker's value, $H_t(a_{jt})$, is equal to the real wage he can get from the work this period, and plus the discounted future value of the work:

$$H_t(a_{jt}) = Y^L(a_{jt}) + \beta \mathbb{E}_t\{\frac{\widetilde{\lambda}_{1t+1}}{\widetilde{\lambda}_{1t}}[(1 - \rho_{t+1})H_{t+1} + \rho_{t+1}W_{t+1}]\}, \tag{2.32}$$

where W_t is the value of an unemployed worker:

$$W_t = G_t^u + A + \beta \mathbb{E}_t \{ \frac{\widetilde{\lambda}_{1t+1}}{\widetilde{\lambda}_{1t}} [(1 - \rho_{t+1}) \rho_t^w H_{t+1} + (1 - (1 - \rho_{t+1}) \rho_t^w) W_{t+1}] \}. \quad (2.33)$$

The value of the unemployed worker includes the total unemployment compensation this period and expected income either being employed or not in the future.

The economic surplus of a match is $J_t(a_{jt}) + H_t(a_{jt}) - W_t$. When there is no real wage rigidity, the surplus is divided between the firm and worker through Nash bargaining, and the bargaining power of the worker is Θ. The notional real wage resulting from the Nash bargaining is:

$$Y^{L^*}(a_{jt}) = \Theta [\frac{Y(a_{jt})}{\mu_t} - r_t^k K^*(a_{jt}) + \gamma \tau_t] + (1 - \Theta)(G_t^u + A).$$

However, when there exists a wage norm, and the real wage is rigid in the sense that it depends on the wage norm, the real wage could be expressed as weighted average of the notional wage and the steady state value of the real wage:

$$Y^L(a_{jt}) = \eta [\Theta (\frac{Y(a_{jt})}{\mu_t} - r_t^k K^*(a_{jt}) + \gamma \tau_t) + (1 - \Theta)A] + (1 - \eta)Y^L.$$

The real wage rigidity index is η. If $\eta = 0$, the real wage is solely determined by the steady state surplus, and if $\eta = 1$, the real wage is perfectly flexible.

How is the endogenous separation decision made? That is, how is the threshold of match-specific productivity, $\widetilde{a}_t$, determined? The critical value of a_t below which separation takes place is given by $J_t(\widetilde{a}_t) = 0$. Substituting the real wage and capital used at $\widetilde{a}_t$ into firm's value, the separation threshold is determined by the following equation:

$$\frac{Y(\widetilde{a}_t)}{\mu_t} - Y^L(\widetilde{a}_t) - r_t^k K^*(\widetilde{a}_t) + \frac{\gamma}{\rho_t^f} = 0. \quad (2.34)$$

Define the average capital used in production as follows:

$$K_t^* = \int_{\widetilde{a}_t}^{a_{max}} K^*(a_{jt}) \frac{f(a_t)}{1 - F(\widetilde{a}_t)} da_t. \quad (2.35)$$

The aggregate output of the intermediate good sector is:

$$Y_t = N_t \frac{\mu_t r_t^k}{\alpha} K_t^*. \tag{2.36}$$

The average real wage is defined as:

$$
\begin{aligned}
Y_t^L &= \int_{\tilde{a}_t}^{a_{max}} Y^L(a_{jt}) \frac{f(a_t)}{1 - F(\tilde{a}_t)} da_t \\
&= \eta[\Theta(\frac{1-\alpha}{\alpha} r_t^k K_t^* + \gamma \tau_t) + (1 - \Theta)(G_t^u + A)] + (1 - \eta) Y^L.
\end{aligned}
\tag{2.37}
$$

2.2.3.2 Alternative Specification: Labor Market with Exogenous Separation and Nominal Wage Rigidity

In this subsection, I discuss an alternative description of the labor market following Gertler et al. (2008) with staggered nominal wage contracting and exogenous separation.

The intermediate good firm $j \in [0,1]$ produce output Y_{jt} using capital K_{jt} and labor N_{jt} according to the following production function:

$$Y_{jt} = K_{jt}^\alpha (z N_{jt})^{1-\alpha}.$$

Matches are exogenously separated with probability ρ in each period. Matching function is the same as before. Define the hiring rate X_{jt} as the ratio of new hire to the existing workforce:

$$X_{jt} = \frac{\rho_t^f V_{jt}}{N_{jt-1}}. \tag{2.38}$$

The workforce evolves following:

$$N_{jt} = (1 - \rho + X_{jt}) N_{jt-1}. \tag{2.39}$$

Unlike the fixed vacancy posting cost I used before, the labor adjustment cost is quadratic and depends on the hiring rate X_{jt}.

A firm chooses capital and hiring rate to maximize its value:

$$J_t(Y_t^{NL}, N_{jt-1}) = \frac{Y_{jt}}{\mu_t} - \frac{Y_{jt}^{NL}}{p_t}N_{jt} - \frac{\kappa}{2}X_{jt}^2 N_{jt-1} - r_t^k K_{jt} + \beta\mathbb{E}_t\frac{\lambda_{1t+1}}{\lambda_{1t}}J_{t+1}(Y_{jt+1}^{NL}, N_{jt})$$

where Y_{jt}^{NL} is the nominal wage decided by the staggered Nash bargaining. The firm maximizes its value by choosing optimal capital K_{jt}^* and number of workers N_{jt}.

The value of a matched worker at firm j, $H_t(Y_{jt}^{NL})$, and the value of an unemployed worker W_t are defined as below:

$$H_t(Y_{jt}^{NL}) = \frac{Y_{jt}^{NL}}{p_t} + \beta\mathbb{E}_t\frac{\lambda_{1t+1}}{\lambda_{1t}}[\rho H_{t+1}(Y_{jt+1}^{NL}) + (1-\rho)W_{t+1}],$$

$$W_t = G_t^u + A + \beta\mathbb{E}_t\frac{\lambda_{1t+1}}{\lambda_{1t}}[\rho_{t+1}^w H_{t+1} + (1-\rho_{t+1}^w U_{t+1})].$$

Nominal wages are determined by staggered Nash bargaining. Each period a firm may renegotiate the wage with a fixed probability η. Let Y_t^{NL*} denote the nominal wage of a firm that re-negotiate at t. For those who cannot renegotiate, the wage keeps the same as last period. The re-negotiated nominal wage Y_t^{NL*} is chosen to solve:

$$\max\left(H_t(Y_{jt}^{NL}) - W_t\right)^\Theta J_t(Y_t^{NL})^{1-\Theta}$$

s.t.

$$Y_{jt+s}^{NL} = \begin{cases} Y_{jt+s-1}^{NL} & \text{with probability 1-}\eta \\ Y_{jt+s}^{NL*} & \text{with probability } \eta \end{cases}.$$

The average nominal wage across workers is given by

$$Y_t^{NL} = \int_0^1 Y_{jt}^{NL}\frac{N_{jt}}{N_t}dj. \tag{2.40}$$

The evolution of the average nominal wage is a linear combination of the renegotiated nominal wage and last period's nominal wage:

$$Y_{t+1}^{NL} = \eta Y_{t+1}^{NL*} + (1-\eta)Y_t^{NL}. \tag{2.41}$$

2.2.4 Final Good Sector

The final good sector is monopolistically competitive. Each final good firm, indexed by $\widetilde{j}$, buys output of the intermediate good firms at the price p'_t, converts this output into a differentiated final good, $Y_{\widetilde{j}t}$, with no cost and sells the final good in the market at the price $p_{\widetilde{j}t}$. The demand for each variety is

$$Y_{\widetilde{j}t} = (\frac{p_{\widetilde{j}t}}{p_t})^{-\epsilon_t^p} Y_t, \tag{2.42}$$

and the aggregate price is

$$p_t = [\int_0^1 (p_{\widetilde{j}t})^{1-\epsilon^p} d\widetilde{j}]^{\frac{1}{1-\epsilon^p}}, \tag{2.43}$$

where ϵ_t^p is the elasticity of demand.

Prices are sticky in the final good sector. In the following analysis, the index $\widetilde{j}$ is eliminated because every firm faces an identical problem. Following Calvo(1983), in each period only a fraction of $(1-\omega)$ firms can choose their prices optimally. Let p_t^* be the optimal price set by firms that can reoptimize prices in period t, and the optimization problem for the final good firm is:

$$\max_{p_t^*} \sum_{s=0}^{\infty} \omega^s \mathbb{E}_t \{\Lambda_{t,t+s}[p_t^* Y_{t,t+s} - p'_{t+s} Y_{t,t+s}]\}$$

where

$$Y_{t,t+s} = (\frac{p_t^*}{p_{t+s}})^{-\epsilon_{t+s}^p} C_{t+s}.$$

The result of the optimization problem is:

$$p_t^* = \frac{\mathbb{E}_t \sum_{s=0}^{\infty} \omega^s \Lambda_{t,t+s} C_{t,t+s} \epsilon_{t+s}^p \mu_{t+s}^{-1} p_{t+s}^{1+\epsilon_{t+s}^p}}{\mathbb{E}_t \sum_{s=0}^{\infty} \omega^s \Lambda_{t,t+s} C_{t,t+s} (\epsilon_{t+s}^p - 1) p_{t+s}^{\epsilon_{t+s}^p}}, \tag{2.44}$$

where $\mathbb{E}_t \Lambda_{t,t+s} \equiv \beta^s \mathbb{E}_t[(\widetilde{\lambda}_{1t+s}/\widetilde{\lambda}_{1t})(p_t/p_{t+s})]$ is the stochastic discount factor for nominal payoffs. So the aggregate price is given by

$$p_t = [\omega(p_{t-1})^{1-\epsilon_t^p} + (1-\omega)(p_t^*)^{1-\epsilon_t^p}]^{\frac{1}{1-\epsilon_t^p}}. \tag{2.45}$$

2.2.5   Government

In order to close the model, we need to specify government policies.

Baseline policies include conventional monetary policy, constant government spending and constant unemployment benefits.

Conventional monetary policy follows a standard feedback rule, and the nominal interest rate cannot be lower than 0:

$$\widehat{r}_t = \max\{\phi_\pi \widehat{\pi}_t, -\overline{r} + 1\}, \tag{2.46}$$

where $\widehat{x}_t$ is the log-deviation from steady state value. Government spending is assumed to be proportional to output, $G = g_y Y$. Regular unemployment benefits paid by the government is also constant and proportional to the steady state average real wage, $G^u = g^u_{y^L} Y^L$.

Besides the baseline policies, the government could also use three separate alternative policies to give the economy extra stimulus.

Unconventional credit policy corresponds to government purchases of private equities as a function of its liquidity:

$$\frac{S^g_t}{K^H} = \psi_K (\frac{\phi_t}{\phi} - 1), \tag{2.47}$$

where ψ_K equals 0 when there is no unconventional credit intervention. When the monetary authority implements unconventional credit intervention in response to the liquidity shock, ψ_K is smaller than 0. At steady state, the Federal Reserve does not buy any private equities, that is, $S^g = 0$.

Fiscal expansion follows an AR(1) process:

$$\widehat{g}_t = \rho^G \widehat{g}_{t-1} + \epsilon^G_t. \tag{2.48}$$

where ϵ^G_t is the initial response of government spending to the liquidity shock,

$$\epsilon^G_t = \begin{cases} \epsilon^G_1 (> 0) & t = 1 \\ 0 & \text{otherwise} \end{cases}.$$

Extended unemployment benefits also follows an AR(1) process,

$$\widehat{g}_t^u = \rho^{G^u} \widehat{g}_{t-1}^u + \epsilon_t^{G^u}, \tag{2.49}$$

where $\epsilon_t^{G^u}$ is the initial response of unemployment benefits to the liquidity shock,

$$\epsilon_t^{G^u} = \begin{cases} \epsilon_1^{G^u}(>0) & t = 1 \\ 0 & \text{otherwise} \end{cases}.$$

The government could either use baseline policies only, or use the combinations of baseline policies and one of the alternative policies.

Government budget constraint is of the form:

$$G_t + G_t^u + q_t S_t^g + \frac{r_{t-1} B_{t-1}}{p_t} = T_t + [r_t^K + (1-\delta)q_t] S_{t-1}^g + \frac{B_t}{p_t}. \tag{2.50}$$

Taxes adjust to the government net debt position:

$$T_t - T = \psi^T [(\frac{r_{t-1} B_{t-1}}{p_t} - \frac{rB}{p}) - q_t S_{t-1}^g]. \tag{2.51}$$

Since ψ^T is small, the adjustment of taxes is gradual, government needs to finance its purchases of private equity, fiscal expansion and extended unemployment benefits by issuing government bonds.

2.2.6 Market Equilibrium

In equilibrium, capital stock is owned by households and government:

$$K_t^H = S_t + S_t^g, \tag{2.52}$$

and capital stock equals the capital used by the intermediate good firms:

$$K_{t-1}^H = N_t K_t^*. \tag{2.53}$$

Output equals households' demand for consumption, investment and government consumption and the cost of posting vacancies:

$$Y_t = C_t + [1 + \Psi(\frac{I_t}{I})]I_t + G_t + \gamma V_t. \tag{2.54}$$

2.3   Calibration

Table 2.1 gives the calibrated values of parameters that are standard in other New Keynesian models with financial frictions. All these parameters are chosen following Del Negro et al. (2011). The discount factor β is 0.99, capital share α in the production function is 0.4, and the relative risk aversion parameter in the utility function is set to be 1. The quarterly depreciation rate of capital is 0.025, and investment adjustment cost parameter $\psi''(1)$ is 1. The coefficient on inflation in the interest rate rule is 1.5. The average duration of price and wage stickiness is four quarters, so the Calvo parameter for price setting ω is 0.75 and wage rigidity parameter η is 0.25. The elasticity of substitution among differentiated final goods ϵ^p is 11, which implies the steady state markup μ is 1.1. 5% of the population are randomly chosen to be entrepreneurs in each period. Let $L_t \equiv \frac{B_t}{p_t}$ be the real value of the government bonds. The ratio between real value of the government bonds and annual GDP $\frac{L}{4Y}$ is 0.4 at steady state. The coefficient in the tax rule is 0.1. Financial friction parameters ϕ and θ are set to 0.207, so that the annual steady-state interest rate is 2.2%.

Table 2.2 includes parameters characterizing labor market frictions and alternative policy interventions, most of which are set according to Zhang (2012). In that paper, I used Bayesian methods to estimate a monetary DSGE model with endogenous separation, frictional labor market, and several shocks. I used 9 key macroeconomic quarterly US time series from 1976Q1 to 2011Q2 as observable variables to estimate the standard parameters in the New Keynesian models, parameters characterizing labor market frictions, as well as the parameters for the shock processes. The data I used includes: log difference of real GDP, log difference of real consumption, log difference of real investment, log difference of the real wage, log difference of the GDP deflator, the federal funds rate, log deviation of the unemployment rate from its mean, log deviation of the vacancies from its mean, and log difference of the total

government unemployment benefits[1]. The values of most parameters characterizing the labor market in this paper are taken from the estimation results in Zhang (2012): the total job separation rate is 10.5%, the threshold of match-specific productivity for the endogenous separation is 0.74, workers' bargaining power on wages is 0.36, quarterly job-finding rate is 0.7, steady state labor market tightness is 0.74, and the replacement rate of the unemployment benefits is 0.2. The total unemployment compensation implied by the estimation is 0.72, and this value is used here in the model with exogenous separation and nominal wage rigidity. The elasticity of the matching function is set to 0.5, and matching specific productivity is assumed to be log normally distributed with zero mean and 0.15 standard deviation.

The only parameter related to the labor market that is different from the estimation result in Zhang (2012) is the real wage rigidity parameter η. The estimation result for η is 0.38, but in this paper I use 0.25 to match the wage Calvo parameter in Del Negro et al. (2011) and Gertler et al. (2008). Different values for η don't change the results much.

Parameters for alternative policies will be described in detail later.

2.4 Results

In this section, I simulate models with different features and policies, and report the dynamics of the main variables in response to a negative liquidity shock in separate figures. I also report the on impact responses from different models in one table (Table 4) for the sake of easy comparison.

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[1]In order to ensure the robustness of the estimation results, I also substituted the total government unemployment benefits data with the replacement rate data, and re-estimated the model. Similar results are obtained through the robustness check.

2.4.1 Liquidity Shocks and Policies in the Model with Endogenous Separation and Real Wage Rigidity at the Zero Lower Bound

2.4.1.1 The Impact of a Liquidity Shock

Here I assume a deterministic economy. At $t = 1$, a one-time unexpected negative liquidity shock hits the economy, and after that there is no other shock. And the liquidity evolves following a known AR(1) process: $\widehat{\phi}_t = \rho^\phi \widehat{\phi}_{t-1} + \epsilon_t^\phi$. Participants in the economy perfectly know what will happen in the future. In this case the nonlinearity caused by the zero lower bound on the nominal interest rate won't be a problem in solving the model. The shock tightens the entrepreneurs' resaleability constraint, and the fraction of existing equity holdings that can be sold, ϕ_t, drops by 60%. The autocorrelation parameter of the exogenous shock is 0.833, which means the expected duration of the shock is 6 quarters.

The solid lines in Figure 2.1 are the responses of the main macroeconomic variables to this negative liquidity shock when the government implements baseline policies. The fall in liquidity limits the entrepreneurs' ability to buy investment capital, so investment drops by 17%. Less liquidity also decreases the value of equities held by the households, q_t, which is consistent with what we observe during the Great Recession. The zero lower bound and sticky prices together will lead to expectations of deflation, and inflation falls by almost 4%. Since the nominal interest rate is bounded at zero, the real interest rate, $\widehat{rr}_t = \widehat{r}_t - \mathbb{E}_t \widehat{\pi}_{t+1}$, increases, which depresses consumption. The decrease in investment and consumption together causes the huge drop in output (more than 8%). This decline in output is accompanied by a 10-percentage-point rise in the unemployment rate.

2.4.1.2 The Effectiveness of the Credit Policy

Suppose that in response to the liquidity shock and the zero lower bound on the nominal interest rate, the Federal Reserve immediately implements unconventional credit policy by purchasing private paper. The value of the parameter ψ_K determines the size of this unconventional credit intervention at time $t = 1$. We set

$\psi_K = -0.0672$, and this means the size of the intervention is 10% of annual GDP, which is consistent with the $1.4 trillion increase in the Fed's balance sheet in 2008Q4. The dashed lines in Figure 2.1 are the impulse responses of the macro variables when unconventional credit policy is implemented in response to the liquidity shock. Comparing with the case without any policy intervention, output is more than 2% higher, consumption is 3% higher, and the unemployment rate is 4% lower. Since the central bank exchanges government liquidity for illiquid private equities, entrepreneurs have more liquid assets that are not subject to the resaleability constraint, and could invest more. Under the credit policy, entrepreneurs' liquidity is more than 10% higher. This explains the big increase in investment in period $t = 2$ after the implementation of unconventional credit policy, and this increase in investment boosts the aggregate demand effectively. As a result, both the decline of output and the rise of unemployment are much smaller than the case with no government intervention.

2.4.1.3 The Effectiveness of Fiscal Expansion

Instead of using unconventional credit policy, the government could also expand government spending to stimulate the economy. In this subsection, in response to the liquidity shock the government expands government spending by 4% at $t = 1$, and the size of expansion decreases as the effect of the liquidity shock diminishes. The cumulative increase in government spending in response to the liquidity shock over the time amounts to 6% of annual GDP, consistent with the $787 billion stimulus package. The size of initial fiscal expansion $\epsilon_1^G = 0.075$ and the persistence parameter of fiscal expansion $\rho^G = 0.935$ together give us the fiscal expansion with exactly the same size as that in data. Intuitively, increasing government spending is the most direct way to increase the aggregate demand of the economy. From Figure 2.2, we notice that in the model with endogenous separation and real wage rigidity fiscal expansion is indeed very effective in preventing the decrease in output. Expanding government spending also helps the economy get out of the liquidity trap much more quickly than implementing unconventional credit policy. The unemployment rate, which increased 10% in the absence of policy actions, only rises 6% with the fiscal expansion. Fiscal expansion is more effective than unconventional credit policy in these aspects.

However, its effect on investment and consumption is not as striking. Most of the improvement of output comes from the increase in government spending but not the increase in demand of the private sector. Unlike the credit policy, fiscal expansion basically has no effect on investment, because the entrepreneurs don't get extra liquidity from the policy, and their assets are still subject to the resaleability constraint which is tightened due to the negative liquidity shock. It is worth noting that the on-impact and cumulative effects of fiscal expansion are different. The effects are summarized in Table 2.3. The fall in consumption on impact is less than would be the case without any policy. However, consumption recovers much more slowly because government spending crowds out private spending. When calculating the cumulative effect of fiscal expansion on consumption and output, I find that it actually is negative. The cumulative decrease in consumption is 5% more if fiscal expansion is implemented, and the cumulative decrease in annual output is 5.4% less. But remember the government purchases increase by 6% annual GDP, so excluding that, total private spending decreases because of the fiscal expansion. So at the zero lower bound, the government spending multiplier is larger than 1 on impact, but slightly less than 1 cumulatively.

2.4.1.4 The Effectiveness of Extended Unemployment Benefits Program

Most previous studies have concluded that extended unemployment benefits would be harmful in the sense that higher government compensation for unemployment may reduce the effort the unemployed put into finding a new job, increase workers' required wage, and discourage firms from hiring more workers. Surprisingly, our model implies the opposite result when the zero lower bound is binding. In response to a negative liquidity shock, an increase in unemployment benefits has a small positive effect on output. Although this seems to contradict the previous studies and our intuition, it can still be well explained within our model.

The positive effect of extended unemployment benefits comes from its ability to pull the nominal interest rate away from the zero lower bound more quickly. Extended unemployment benefits can affect the nominal interest rate in a positive way, and this is not a special characteristic of my model, but very common and straightforward in any standard New Keynesian framework. When there is an increase in unemployment

benefits, the required wage will increase correspondingly, which causes a rise in real marginal cost. The increase in the real wage makes labor relatively more expensive than capital, hence firms prefer to use more capital to substitute for labor. As a result, the rental return for capital increases and raises the real marginal cost further. From the New Keynesian Phillips Curve that represents inflation as an increasing function of real marginal cost, we can expect an increase in inflation, and this increase in inflation transfers to an increase in the nominal interest rate through the Taylor' rule. So when an extended unemployment benefits program is implemented in response to a negative liquidity shock, there will be less deflation and hence less downward pressure on the nominal interest rate. This shortens the time the economy spends at the zero lower bound and helps the economy get out of the liquidity trap more quickly. The dashed lines in Figure 2.3 are the responses of the model when the extended unemployment benefits program is implemented. The size of the program, determined by the parameter $\epsilon_1^{G^u}$, is the same as that in the real world. That is, the total amount of the unemployment benefits paid by the government increases by 110%. Output decreases less and consumption is 2% higher immediately after the liquidity shock if unemployment benefits are extended. The time the economy spends at the zero lower bound shortens from 8 quarters to 2 quarters.

Although extended unemployment benefits have a positive effect on the level of output and consumption immediately after the negative liquidity shock, its effect on the labor market is adverse. Extended unemployment benefits program with calibrated size and persistence could increase the unemployment rate instead of decreasing it, and slow down the recovery on the labor market. This is firstly because extended unemployment benefits rise the cost of using labor. This is because the labor market frictions give rise to long-run employment relationships, and it is costly to maintain this profitable long-run attachment between workers and firms. The cost depends on the workers' value which is negatively related to the unemployment benefit. The longer the extended unemployment benefits programs lasts, the more costly to keep this relationship and the bigger the negative effect is. The second source of slow recovery on the labor market is the slow recovery in consumption, that is inadequate aggregate demand. After escaping from the zero lower bound, the effect of extended unemployment benefits keeps working and even drives the nominal interest rate to be

higher than the level before the crisis. This induces higher real interest rate, increases the demand for government bond, and prevents the increase in consumption.

Besides the effect on labor market and goods market, extended unemployment benefits also affect the financial market. Higher demand for government bond induced by the higher real interest rate will crowd out people's demand for private equity. So equity price recovers very slowly. Investment could also be affected. Considering the high real wage caused by extended unemployment benefits, firms prefer to use more capital to substitute workers. This increases capital demand and lead to more investment as the economy is recovering. This explains why investment and go above the steady state years after the liquidity shock. Although equity price keeps low, entrepreneurs' liquidity even increases above steady state because of the increase in government bond holdings and investment. These don't happen when credit policy and fiscal expansion are implemented, because these two policies won't change the relative price of labor and capital.

2.4.1.5 The Role of Matching Efficiency

I also consider the case with a temporary but big and persistent decrease in matching efficiency when the liquidity shock hits the economy, and compare labor market responses with those under extended unemployment benefits.

The size of the initial unexpected decrease in matching efficiency is -20%, which is consistent to Barnichon and Figura (2011), and the persistence of the sudden decrease is 0.93, which is the same as the estimation result in Furlanetto and Groshenny (2012) and Zhang (2012). Figure 2.4 compares the responses of unemployment in the 3 cases, which include the case with a liquidity shock but no policy intervention, the case with a liquidity shock and extended unemployment benefits, and the case with a liquidity shock and a decrease in matching efficiency. I find that extended unemployment benefits significantly slow down the labor market recovery, while a negative matching efficiency shock reduces the big rise in unemployment, and also slows down the recovery, however, only slightly.

Let's first examine why the big decrease in matching efficiency will mitigate the big rise in unemployment. A decrease in matching efficiency has two opposite effects on unemployment. The first one is unemployment increases because it takes

a longer time for the unemployed to find new jobs. The second one is the decline in vacancy filling rate caused by the decrease in matching efficiency will increase the value of existing matches, decrease the threshold for endogenous separation and in turn decrease unemployment. The second effect is very small when the endogenous separation is close to its steady state level, but becomes bigger and bigger as the endogenous separation rate increases. We assume match-specific productivity follows a lognormal distribution, and when the endogenous separation is close to its steady state value, the threshold of matching-specific productivity for endogenous separation is still at the very left tail of the lognormal distribution, where the density is fairly low. That means a decrease in the threshold of endogenous separation caused by the decrease in matching efficiency will not reduce the endogenous separation rate much, and in turn will not cause a big decrease in unemployment. In this case, the first effect of a decrease in matching efficiency dominates, and unemployment increases. However, when the economy is facing a big negative shock like the liquidity shock in our model, the endogenous separation rate rises a lot and is far above its steady state value, which means the threshold for separation is much closer to the middle of the lognormal distribution, where the density is also higher. In this case, a very small decrease in the threshold caused by the decrease in matching efficiency will cause a relatively bigger decrease in separation rate and unemployment, and the second effect of a decrease in matching efficiency will dominate the first one. Our model with the liquidity shock belongs to the second case, so unemployment is lower when matching efficiency decreases.

The reason a big decrease in matching efficiency could not significantly slows down labor market recovery is when they need more workers due to the increase in aggregate demand, the firms could adjust their separation and vacancy posting decisions to partly offset the negative effect of the decrease in matching efficiency.

2.4.2 The Role of the Zero Lower Bound in the Model with Endogenous Separation and Real Wage Rigidity

The zero lower bound plays a very important role both theoretically and practically. Previous studies (such as Del Negro et al. (2011)) have found that away

from the zero lower bound, a negative liquidity shock will not cause a large decrease in output, and unconventional credit policy will not be effective in stimulating the economy. In this sense, the zero lower bound works as an amplification mechanism for the liquidity shock. Intuitively, without the zero lower bound, the conventional monetary policy could achieve the goal of boosting demand and stabilizing the economy by lowering the nominal interest rate. Hence, little room is left for other policies to play a role.

Does this also happen in the models with frictional labor market? Do the results obtained in the previous subsections depend on the presence of the zero lower bound? The answer is yes in the model with endogenous separation and real wage rigidity.

2.4.2.1 The Impact of Liquidity Shocks

I find the zero lower bound still works as an amplification mechanism for the liquidity shock by comparing Panel A and B of Table 2.4 and the solid and dashed lines in Figure 2.5. Without the zero lower bound, the negative liquidity shock has much less impact on the economy. The solid lines are the responses to the liquidity shock without any alternative policy actions when the zero lower bound is binding. Baseline monetary policy loses its power in stimulating the economy because of the zero bound on the nominal interest rate. The dashed lines are the responses to the liquidity shock without any alternative policy actions when the zero lower bound is not binding. In this case, conventional monetary policy becomes useful in stimulating the economy, because the monetary authority can reduce the nominal interest rate as low as needed. Output is 5% higher than the case at the zero lower bound (compare the), consumption slightly decreases, and the increase in the unemployment rate is only 4% (versus 10% at the zero lower bound).

Besides these quantitative differences, the effect on one variable even changes its sign. Comparing the numbers in orange and italic style in Panel A and B of Table 4, we can find that when the zero lower bound is binding, the equity price decreases largely in response to a negative liquidity shock. But when there is no lower bound for the nominal interest rate, the equity price increases after the shock. Why does this happen?

From the first order conditions of the household's problem, we can get the following equation:

$$q_t - p_t^I = (p_t^I - \theta q_t)\lambda_t^c, \qquad (2.55)$$

where

$$\lambda_t^c = \frac{\frac{r_t^K - (1-\delta)q_t}{q_{t-1}} - \frac{r_{t-1}}{\pi_t}}{\chi\left(\frac{r_{t-1}}{\pi_t} - \frac{r_t^K + (1-\delta)\phi_t q_t}{q_{t-1}}\right)}.$$

So the left hand side of the equation is the benefit from issuing equity to finance a unit of investment. Since only θq_t fraction of investment could be financed by issuing new equities, the entrepreneurs have to finance the rest of investment, $p_t^I - \theta q_t$, by liquid assets, which is called the downpayment on a unit of investment in Shi (2011). The cost of the downpayment is measured by the factor λ_t^c. So the right hand side of the equation is the marginal cost of a unit of investment. The equation requires the net marginal benefit of investment to be zero.

The marginal benefit is strictly increasing in q_t and deceasing in p_t^I, and the downpayment is strictly decreasing function in q_t and increasing in p_t^I. That is, for a given λ_t^c, the net marginal benefit of investment is strictly increasing in q_t and decreasing in p_t^I. On one hand, when there is a negative liquidity shock, price of investment drops largely, which leads to a large rise in the net marginal benefit. Without considering the change in λ_t^c, in order to restore the balance between the marginal benefit and marginal cost, q_t has to decrease. On the other hand, the negative liquidity shock tightens the liquidity constraint, and increases the cost of the downpayment λ_t^c. The higher λ_t^c reduces the net marginal benefit of investment given the equity price and investment price, and this requires q_t to increase to maintain the balance between the marginal benefit and marginal cost. Whether q_t increases or decreases depends on the tradeoff between the above two opposite effects on it. Since λ_t^c is strictly decreasing in the real interest rate, the increased real interest rate at the zero lower bound dampens the increase λ_t^c. A smaller increase in λ_t^c leads to a smaller increase in q_t, which cannot fully offset the decrease caused by the fall in p_t^I. As a result, q_t decreases in response to a negative liquidity shock at the zero lower bound. This is consistent with what we observed in the Great Recession. But when there is no lower limit for the nominal interest rate, the real interest rate declines after a

negative liquidity shock, which leads to a larger increase in λ_t^c, and hence a larger positive effect on the equity price. This positive effect is so large that it dominates the negative effect of the fall in p_t^I. So when the zero lower bound is not binding, q_t increases even there is a negative shock on liquidity. This is consistent with the finding in Shi (2011). So without the presence of the zero lower bound, the liquidity shock couldn't be the only shock that induced the Great Recession.

More intuitively, we can also simplify the problem and explain it from the aspect of the balance of equity supply and demand. When there is a negative liquidity shock, the equity supply is directly affected due to the tightened resaleability constraint. That is, entrepreneurs are not allowed to sell as many equities as before. Then how about the equity demand? If the equity demand is also largely decreasing, the equity price cannot increase. And if the equity demand is not affected much, there will be a big gap between equity supply and equity demand, and equity price will be pushed up. The liquidity shock does not affect equity demand directly, but only indirectly from the decline of the fundamental economic activities. When the zero lower bound is binding, the increase in the real interest rate makes the government bonds much more valuable, hence largely increases the demand for the government bonds. Given the household wealth level, this crowds out the demand for equities and causes large decreases in equity demand and equity price. By contrast, when there is no zero lower bound on nominal interest rate, the real interest rate declines after the shock, which causes a fall in the bond demand. In this case, equity demand is not affected much, and equity price increases.

2.4.2.2   The Effectiveness of Government Policies

Because conventional monetary policy can effectively stimulate the economy when the zero lower bound is not binding, other policies could contribute little. The on impact responses under different policies are listed in Panel B of Table 2.4.

When the zero lower bound is not binding, the effect of unconventional credit policy becomes much smaller than before. Output is increased by 0.4% only, much less than the 2% increase when the zero lower bound is binding.

Fiscal policy is still effective in stimulating output. However, although it is still good for output, it has a really bad effect on private consumption and investment.

With fiscal expansion, consumption is 6% lower and investment is 3% lower compared with the case without any policy. The crowding-out effect becomes much more severe when the zero lower bound is not binding. This is consistent with the result in Christiano et al. (2012) that fiscal policy is more effective when the zero lower bound is binding.

The presence of the zero lower bound is very important for the effect of extended unemployment benefits in the model with endogenous separation and real wage rigidity. As seen above, in the presence of the zero lower bound, extended unemployment benefits can be helpful for preventing output and consumption from decreasing further because it can help the economy get out of the liquidity trap more quickly. But when the zero lower bound is not binding, this advantage disappears. So, instead of benefiting output, it even amplifies the shock and causes output to decrease more. However, when the zero lower bound is not binding, extended unemployment benefits seem to speed up the recovery of the economy. This could be explained by the positive relationship between consumption growth and the real interest rate derived from the Euler equation as well. Without the zero lower bound, the real interest rate deceases and has a negative effect on consumption growth after the liquidity shock no matter whether there are extended unemployment benefits or not. Meanwhile, a low equity price has a negative effect on consumption growth, and the low price on investment goods has a positive effect on it. When there is no extended unemployment benefits, the negative effect dominates, so consumption continues decreasing from $t = 2$. However, due to its positive effect on the real wage, implementing extended unemployment benefits results in a smaller magnitude of deflation. Less deflation induces less decrease in the real interest rate and a smaller negative effect on consumption growth. In this case, the positive effect of the low investment price dominates. This is why consumption grows, and output and unemployment recover faster when unemployment benefits are extended after the liquidity shock.

2.4.3 Required Size of Alternative Policies to Mimic the Case without the ZLB

Since under the liquidity shock output has a bigger decline when the zero lower bound is binding comparing with the case without the zero lower bound, then how large should the alternative policies be in order to offset the extra decrease in output caused by the zero lower bound?

Figure 2.6 shows, at the zero lower bound, credit policy with a size of 20% of annual GDP could prevent the big decline in output and keep the output level the same as the case without the zero lower bound. The size of credit policy used in previous analysis, which is calibrated according to the emergency lending facilities implemented during the Great Recession, is only 10% of annual GDP.

Figure 2.7 shows, in order to prevent the extra decrease in output caused by the zero lower bound, fiscal expansion has to be 4 times as large as President Obama's stimulus package, that is, increasing government spending by amount of 25% of annual GDP. However, such a large fiscal expansion will prevent the nominal interest rate to decline to zero, increase the real interest rate and in turn crowd out people's private consumption. Although output is largely increased, consumption or people's lifetime utility decreases a lot.

So under the liquidity shock, the current government intervention is far from enough to eliminate the negative effect on initial output decline caused by the zero lower bound.

2.4.4 Results in the Model with Exogenous Separation and Nominal Wage Rigidity

The same analysis and policy experiments are repeated in the model with exogenous separation and nominal wage rigidity. And the results derived from models with and without the zero lower bound are listed in Panel C and D of Table 2.4 respectively.

A negative liquidity shock affects the real economy significantly in this case as well, and both unconventional credit policy and fiscal expansion are still very effective

in stabilizing output when the zero lower bound is binding. These results are basically similar to what I get in the model with endogenous separation and real wage rigidity.

Comparing the numbers with waved underline in Panel A and C of Table 2.4, we can find that the main difference is the inflation dynamics in response to the liquidity shock. In the model with endogenous separation and real wage rigidity, inflation decreases by 4%; however, in the model with exogenous separation and nominal wage rigidity, it decreases by only less than 1%. Why do we get such a large gap between inflation changes in the two models? Comparing with the model with exogenous separation and nominal wage rigidity, real marginal cost in the model with endogenous separation and real wage rigidity has bigger fluctuations, which causes the inflation rate to have a much bigger decrease under a negative liquidity shock.

Why are there differences in inflation dynamics or real marginal cost fluctuations in the two models? The difference in modeling wage rigidities is the main reason. In the model with endogenous separation and real wage rigidity, real wages are sticky, so when there is a negative shock that reduces the economy surplus of production, the real wage will decrease definitely (although the degree of real rigidity may affect how much it falls). This fall in the real wage will lead to a decrease in real marginal cost as well as the inflation rate. In the model with exogenous separation and nominal wage rigidity, the nominal wage is sticky, and firms and workers bargain on their nominal wages. So when there is a negative liquidity shock, the newly re-optimized nominal wage decreases, however, the deflation makes the average real wage increase slightly. The increase in the real wage has a positive effect directly on real marginal cost and inflation. It also prevents large deflation indirectly by affecting rental return of capital. Since the cost of using labor becomes higher due to the rise of the real wage, firms prefer to use more capital to substitute labor. As a result, unemployment will increase more and capital demand will decrease less in the model with exogenous separation and nominal wage rigidity. A smaller decrease in capital demand means a smaller decrease in the rental return of capital (because the capital stock is already determined in the previous period), which also causes a smaller decrease in real marginal cost and inflation. Besides the wage setup, endogenous separation also contributes to the more volatile inflation dynamics. This is because after a negative shock, the firms' surplus decreases and then firms will raise the threshold

for endogenous separation, which leads to an increase in the average match-specific productivity. This will cause a further decrease in real marginal cost, and in turn, a further decrease in inflation.

Other results are completely opposite to those obtained in the model with endogenous separation and real wage rigidity. Surprisingly, in the model with exogenous separation and nominal wage rigidity extended unemployment benefits are harmful for output even with the presence of the zero lower bound. More generous unemployment benefits lead to a larger decrease in output and larger increase in unemployment. Why does this happen? The effect of extended unemployment benefits in moderating deflation could benefit the economy little in this model, because the deflation problem is not as severe as in the model with endogenous separation and real wage rigidity. In this case, the disadvantages of extended unemployment benefits dominate and result in a decrease in output than in the absence of the policies.

Moreover, the zero lower bound has tiny effects in the model with exogenous separation and nominal wage rigidity. The dynamic differences of main variables between the two cases are negligible. The difference between decreases in output with double underlines in Panel C and D of Table 2.4 is small, and this is true for other main variables as well. Since the nominal interest rate follows $\widehat{r}_t = \phi_\pi \widehat{\pi}_t (\phi_\pi > 1)$, larger deflation means a larger decrease in the nominal interest rate when we are away from the zero lower bound. This leads to a big fall in the real interest rate, which can largely stimulate demand. In the model with endogenous separation and real wage rigidity, demand is boosted sufficiently through the decrease in the real interest rate, so output does not respond much to the negative liquidity shock. However, in the model with exogenous separation and nominal wage rigidity, the drop in inflation is much smaller, so the decrease in the real interest rate is not big enough to boost demand sufficiently. As a result, even without the zero lower bound, output still has a big fall in response to a negative liquidity shock.

The equity price q_t still decreases in response to the liquidity shock without the presence of the zero lower bound. The small decline in real interest rate could not cause a big enough increase in λ_t^c neither, hence, q_t cannot increase even without the zero lower bound.

Since the zero lower bound plays a relatively unimportant role in the model,

the effects of the policies are not affected much when the economy is away from the zero lower bound.

In a word, nominal wage rigidity and exogenous separation smooth the reaction of inflation to a liquidity shock and cause the differences in results derived from the two models. We can say that the search frictions on the labor markets amplify the liquidity shock while the endogenous separation and real wage rigidity dampen it through lowering inflation and real interest rate. However, this dampen effect is valid only when the zero lower bound is not binding, because Taylor rule is a crucial channel for the mechanism to work.

2.5   Conclusion

From studying two models with liquidity friction and labor market frictions, the results I can get are in three folds. First, in the model with endogenous separation, real wage rigidity and zero lower bound, a liquidity shock can lead to a large decline of the whole economy, unconventional credit policy is effective in preventing large drops in output through providing more liquidity to the entrepreneurs, and fiscal expansion increases output on impact, however, has negative effects on the economy cumulatively. Extended unemployment benefits slightly mitigating the decline in output and consumption at the cost of raising the unemployment rate and slowing down the recovery of the labor market. Moreover, the longer the extended unemployment benefits program lasts, the greater the cost. Without the zero lower bound on the nominal interest rate, the above results don't hold any more. Second, in the model with exogenous separation and nominal wage rigidity, besides the results similar to what I get from the other model, such as the large impact of a liquidity shock, and the effectiveness of unconventional credit policy and fiscal expansion, I also get some opposite results. The zero lower bound doesn't play an important role any longer, and extended unemployment benefits could not benefit the economy anymore even at the zero lower bound. Third, different responses of inflation resulting from different setups in wage rigidity and job separation lead to these different results in the two models.

2.6 Appendix

2.6.1 Equation System for the Model with Endogenous Separation and Real Wage Rigidity

2.6.1.1 Nonlinear Equations

$$N_t = 1 - \chi - U_t \tag{2.56}$$

$$N_t = (1 - \rho_t)[N_{t-1} + (1 - \chi)EU_{t-1}^\zeta V_{t-1}^{1-\zeta}] \tag{2.57}$$

$$\rho_t^w = m(U_t, V_t)/U_t = EU_t^\zeta V_t^{1-\zeta}/U_t = E\tau_t^{1-\zeta} \tag{2.58}$$

$$\rho_t^f = m(U_t, V_t)/V_t = EU_t^\zeta V_t^{1-\zeta}/V_t = E\tau_t^{-\zeta} \tag{2.59}$$

$$
\begin{aligned}
\rho_t &= 1 - (1 - \chi)(1 - \rho^x)(1 - \rho_t^n) \\
&= 1 - (1 - \chi)(1 - \rho^x)(1 - \int_{-\infty}^{\tilde{a}_t} f(a_t)da_t) \\
&= 1 - (1 - \chi)(1 - \rho^x)(1 - F(\tilde{a}_t))
\end{aligned}
\tag{2.60}
$$

$$\frac{Y(\tilde{a}_t)}{\mu_t} - Y^L(\tilde{a}_t) - r_t^K K^*(\tilde{a}_t) + \frac{\gamma}{\rho_t^f} = 0 \tag{2.61}$$

$$\beta\mathbb{E}_t\{\frac{\lambda_{1t+1}}{\lambda_{1t}}(1 - \rho_{t+1})\rho_t^f[\frac{Y_{t+1}}{\mu_{t+1}} - r_{t+1}^K K_{t+1}^* - Y_{t+1}^L + \frac{\gamma}{\rho_{t+1}^f}]\} = \gamma \tag{2.62}$$

$$Y_t^L = \eta[\Theta(\frac{1-\alpha}{\alpha}r_t^K K_t^* + \gamma\tau_t) + (1 - \Theta)A] + (1 - \eta)Y^L \tag{2.63}$$

$$\pi_t = \frac{p_t}{p_{t-1}} \tag{2.64}$$

$$p_t^{1-\epsilon_t^p} = \omega(p_{t-1}\pi_{t-1}^\xi)^{1-\epsilon_t^p} + (1 - \omega)(p_t^*)^{1-\epsilon_t^p} \tag{2.65}$$

$$p_t^* = \frac{\mathbb{E}_t \sum_{s=0}^\infty \omega^s \Lambda_{t,t+s} c_{t,t+s} \epsilon_{t+s}^p \mu_{t+s}^{-1} p_{t+s}^{1+\epsilon_{t+s}^p} \pi_{t+s-1,t-1}^{-\xi\epsilon_{t+s}^p}}{\mathbb{E}_t \sum_{s=0}^\infty \omega^s \Lambda_{t,t+s} c_{t,t+s} (\epsilon_{t+s}^p - 1) p_{t+s}^{\epsilon_{t+s}^p} \pi_{t+s-1,t-1}^{\xi(1-\epsilon_{t+s}^p)}} \tag{2.66}$$

$$\mathbb{E}_t \Lambda_{t,t+s} = \beta^s \mathbb{E}_t \frac{\lambda_{1t+s}}{\lambda_{1t}} \frac{p_t}{p_{t+s}} \tag{2.67}$$

$$\widetilde{K}_t^* = (\frac{\alpha z_t \widetilde{a}_t}{\mu_t r_t^K})^{\frac{1}{1-\alpha}} \tag{2.68}$$

$$\begin{aligned} K_t^* &= \int_{\widetilde{a}_t}^{a_{max}} K_{jt}^* \frac{f(a)}{1 - F(\widetilde{a}_t)} da \\ &= (\frac{\alpha z_t}{\mu_t r_t^K})^{\frac{1}{1-\alpha}} \int_{\widetilde{a}_t}^{a_{max}} a_t^{\frac{1}{1-\alpha}} \frac{f(a)}{1 - F(\widetilde{a}_t)} da \\ &= (\frac{\alpha z_t}{\mu_t r_t^K})^{\frac{1}{1-\alpha}} X(\widetilde{a}_t) \end{aligned} \tag{2.69}$$

where

$$\begin{aligned} X(\widetilde{a}_t) &= \int_{\widetilde{a}_t}^{a_{max}} a^{\frac{1}{1-\alpha}} f(a) da \\ &= e^{\frac{\mu_a}{1-\alpha} + \frac{\sigma_a^2}{2(1-\alpha)^2}} \phi(\frac{\mu_a + \sigma_a^2/(1-\alpha) - \log \widetilde{a}}{\sigma_a}) \end{aligned}$$

$$Y_t = N_t \frac{\mu_t r_t^K}{\alpha} K_t^* \tag{2.70}$$

$$p_t^I = 1 + \Psi(\frac{I_t}{I}) + \Psi'(\frac{I_t}{I})\frac{I_t}{I} \tag{2.71}$$

$$\lambda_{1t} = C_t^{-\sigma} \tag{2.72}$$

$$\lambda_{1t} = \beta \mathbb{E}_t \{\lambda_{1t+1}[\frac{r_t}{\pi_{t+1}} + \frac{\chi(q_{t+1} - p_{t+1}^I)}{p_{t+1}^I - \theta q_{t+1}} \frac{r_t}{\pi_{t+1}}]\} \tag{2.73}$$

$$\lambda_{1t} = \beta \mathbb{E}_t \{\lambda_{1t+1}[\frac{r_{t+1}^K + (1-\delta)q_{t+1}}{q_t} + \frac{\chi(q_{t+1} - p_{t+1}^I)}{p_{t+1}^I - \theta q_{t+1}} \frac{r_{t+1}^K + (1-\delta)\phi_{t+1}q_{t+1}}{q_t}]\} \tag{2.74}$$

$$\lambda_{1t}(q_t - p_t^I) = \lambda_{2t} \tag{2.75}$$

$$I_t = \chi \frac{[r_t^K + (1-\delta)q_t \phi_t]S_{t-1} + \frac{r_{t-1}L_{t-1}}{\pi_t} + Y_t(1 - \frac{1}{\mu_t}) + p_t^I I_t - I_t[1 + \Psi(\frac{I_t}{I})] - T_t}{p_t^I - \theta q_t} \tag{2.76}$$

$$L_t = \frac{B_t}{p_t} \tag{2.77}$$

$$K_t^H = (1-\delta)K_{t-1}^H + I_t \tag{2.78}$$

$$\widehat{r}_t = \phi_\pi \widehat{\pi}_t \tag{2.79}$$

$$\frac{S_t^g}{K} = \psi_K(\frac{\phi_t}{\phi} - 1) \tag{2.80}$$

$$\widehat{g}_t = \rho^g \widehat{g}_{t-1} + \epsilon_t^g \tag{2.81}$$

$$\widehat{g}_t^u = \rho^{g^u} \widehat{g}_{t-1}^u + \epsilon_t^{g^u} \tag{2.82}$$

$$q_t S_t^g + \frac{r_{t-1}B_{t-1}}{p_t} + G_t + G_t^u = T_t + [r_t^K + (1-\delta)q_t]S_{t-1}^g + \frac{B_t}{p_t} \tag{2.83}$$

$$T_t - T = \psi^T[(\frac{r_{t-1}B_{t-1}}{p_t} - \frac{rB}{p}) - q_t S_{t-1}^g] \tag{2.84}$$

$$K_t^H = S_t + S_t^g \tag{2.85}$$

$$K_{t-1}^H = N_t K_t^* \tag{2.86}$$

$$Y_t = C_t + [1 + \Psi(\frac{I_t}{I})]I_t + G_t + \gamma V_t \tag{2.87}$$

$$LS_t = \frac{B_t}{B_t + p_t q_t K_t^H} = \frac{L_t}{L_t + q_t K_t^H} \tag{2.88}$$

There are 33 equations and 33 endogenous variables (Y_t, C_t, I_t, G_t, G_t^u, K_t^H, S_t, S_t^g, B_t, L_t, T_t, $\widetilde{K}_t^*$, K_t^*, q_t, p_t^I, p_t, p_t^*, π_t, μ_t, r_t, r_t^K, Y_t^L, u_t, n_t, v_t, ρ_t, $\widetilde{a}_t$, ρ_t^f, ρ_t^w, λ_{1t}, λ_{2t}, $\Lambda_{t,t+s}$, LS_t).

2.6.1.2 Steady State

$$N = 1 - \chi - U \tag{2.89}$$

$$\rho N = m(U, V) = (1-\rho)EU^\zeta V^{1-\zeta} \tag{2.90}$$

$$\rho^w = \frac{m(U,V)}{U} = E\tau^{1-\zeta} \tag{2.91}$$

$$\rho^f = \frac{m(U,V)}{V} = E\tau^{-\zeta} \tag{2.92}$$

$$\rho = 1 - (1-\chi)(1-\rho^x)(1-F(\widetilde{a})) \tag{2.93}$$

$$(1-\eta W)\frac{1-\alpha}{\alpha}r^K\widetilde{K}^* - W\gamma\tau - (1-W)A - (1-\eta)W\frac{1-\alpha}{\alpha}r^K K^* + \frac{\gamma}{\rho^f} = 0 \tag{2.94}$$

$$\beta\rho^f(1-\rho)(\frac{1-\alpha}{\alpha}r^K K^* - Y^L + \frac{\gamma}{\rho^f}) = \gamma \tag{2.95}$$

$$Y^L = W(\frac{1-\alpha}{\alpha}r^K K^* + \gamma\tau) + (1-W)A \tag{2.96}$$

$$\pi = 1 \tag{2.97}$$

$$\widetilde{K}^* = (\frac{\alpha \widetilde{a}}{\mu r^K})^{\frac{1}{1-\alpha}} \tag{2.98}$$

$$K^* = \frac{1}{1 - F(\widetilde{a})} (\frac{\alpha}{\mu r^K})^{\frac{1}{1-\alpha}} \int_{\widetilde{a}}^{a_{max}} a^{\frac{1}{1-\alpha}} f(a) da \tag{2.99}$$

$$Y = \frac{N \mu r^K K^*}{\alpha} \tag{2.100}$$

$$p_I = 1 \tag{2.101}$$

$$\lambda_1 = C^{-\sigma} \tag{2.102}$$

$$\beta^{-1} = r(1 + \chi \frac{q-1}{1-\theta q}) \tag{2.103}$$

$$\beta^{-1} = \frac{r^K + (1-\delta)q}{q}(1 + \frac{\chi(q-1)}{1-\theta q}) - \frac{\chi(1-\delta)(1-\phi)(q-1)}{1-\theta q} \tag{2.104}$$

$$I = \chi \frac{[r_t^K + (1-\delta)q\phi]S + (1 - \frac{1}{\mu})Y + L}{1 - \theta q} \tag{2.105}$$

$$L = \frac{B}{p} \tag{2.106}$$

$$\delta K^H = I \tag{2.107}$$

$$S^g = 0 \tag{2.108}$$

$$G = g_y Y \tag{2.109}$$

$$G^u = g_y^u Y \tag{2.110}$$

$$T = (r-1)L \tag{2.111}$$

$$K^H = S \tag{2.112}$$

$$K^H = N K^* \tag{2.113}$$

$$Y = C + I + G + \gamma V \tag{2.114}$$

$$LS = \frac{L}{L + q K^H} \tag{2.115}$$

2.6.1.3 Log-Linearized Equations

$$\widehat{n}_t = -\frac{U}{N}\widehat{u}_t \tag{2.116}$$

$$\widehat{n}_t = (1-\rho)\widehat{n}_{t-1} - \frac{\rho}{1-\rho}\widehat{\rho}_t + \rho[\zeta\widehat{u}_{t-1} + (1-\zeta)\widehat{v}_{t-1}] \tag{2.117}$$

$$\widehat{\rho}_t^w = (\zeta - 1)\widehat{u}_t + (1-\zeta)\widehat{v}_t \tag{2.118}$$

$$\widehat{\rho}_t^f = \zeta\widehat{u}_t - \zeta\widehat{v}_t \tag{2.119}$$

$$\widehat{\rho}_t = [\frac{(1-\chi)(1-\rho^x)\rho^n}{\rho}]\widehat{\rho}_t^n = [\frac{(1-\chi)(1-\rho^x)\rho^n}{\rho}]\frac{f(\widetilde{a})\widetilde{a}}{F(\widetilde{a})}\widehat{\widetilde{a}}_t \tag{2.120}$$

$$(1-\eta W)\frac{1-\alpha}{\alpha}r^K\widetilde{K}^*(\widehat{r}_t^K + \widehat{\widetilde{k}}_t^*) = \eta W\gamma\tau(\widehat{v}_t - \widehat{u}_t) + \frac{\gamma}{\rho^f}\widehat{\rho}_t^f \tag{2.121}$$

$$-\widehat{\rho}_t^f = \mathbb{E}_t[\widehat{\lambda}_{1t+1} - \widehat{\lambda}_{1t} - \frac{\rho}{1-\rho}\widehat{\rho}_{t+1} + \frac{\frac{1-\alpha}{\alpha}r^K K^*(\widehat{r}_{t+1}^k + \widehat{k}_{t+1}^*) - Y^L\widehat{y}_{t+1}^L - \frac{\gamma}{\rho^f}\widehat{\rho}_{t+1}^f}{\frac{1-\alpha}{\alpha}r^K K^* - Y^L + \frac{\gamma}{\rho^f}}] \tag{2.122}$$

$$Y^L\widehat{y}_t^l = \eta\Theta[\frac{1-\alpha}{\alpha}r^K K^*(\widehat{r}_t^k + \widehat{k}_t^*) + \gamma\tau(\widehat{v}_t - \widehat{u}_t)] \tag{2.123}$$

$$\widehat{\pi}_t = \frac{\beta}{1+\beta\xi}\mathbb{E}_t[\widehat{\pi}_{t+1}] + \frac{\xi}{1+\beta\xi}\widehat{\pi}_{t-1} - \frac{(1-\beta\omega)(1-\omega)}{\omega(1+\beta\xi)}\widehat{\mu}_t \tag{2.124}$$

$$\widehat{\widetilde{k}}_t^* = \frac{1}{1-\alpha}(\widehat{z}_t + \widehat{\widetilde{a}}_t - \widehat{\mu}_t - \widehat{r}_t^K) \tag{2.125}$$

$$\widehat{k}_t^* = \frac{\rho^n}{1-\rho^n}\widehat{\rho}_t^n + \frac{1}{1-\alpha}(\widehat{z}_t - \widehat{\mu}_t - \widehat{r}_t^K) + \frac{X'(\widetilde{a})}{X(\widetilde{a})}\widetilde{a}\widehat{\widetilde{a}}_t \tag{2.126}$$

$$\widehat{y}_t = \widehat{n}_t + \widehat{\mu}_t + \widehat{r}_t^K + \widehat{k}_t^* \tag{2.127}$$

$$\widehat{p}_t^I = \Psi''(1)\widehat{i}_t \tag{2.128}$$

$$\widehat{\lambda}_{1t} = -\sigma\widehat{c}_t \tag{2.129}$$

$$\widehat{\lambda}_{1t} - \mathbb{E}_t[\widehat{\lambda}_{1t+1}] = \widehat{r}_t - \mathbb{E}_t[\widehat{\pi}_{t+1}] + \beta\chi rq\frac{1-\theta}{(1-\theta q)^2}\mathbb{E}_t[\widehat{q}_{t+1}] - \beta\chi rq\frac{1-\theta}{(1-\theta q)^2}\mathbb{E}_t[\widehat{p}_{t+1}^I] \tag{2.130}$$

$$\widehat{q}_t = \widehat{\lambda}_{1t} - \mathbb{E}_t[\widehat{\lambda}_{1t+1}] + \beta\frac{r^K}{q}(1 + \chi\frac{q-1}{1-\theta q})\mathbb{E}_t[\widehat{r}^K_{t+1}] + \beta(1-\delta)\chi\frac{q-1}{1-\theta q}\phi\mathbb{E}_t[\widehat{\phi}_{t+1}]$$

$$- \beta\chi[\frac{r^K}{q} + (1-\delta)\phi]\frac{q(1-\theta)}{(1-\theta q)^2}\mathbb{E}_t[\widehat{p}^I_{t+1}]$$

$$+ \beta[(1-\delta) + \chi(1-\delta)\frac{q-1}{1-\theta q}\phi + \chi(r^K + (1-\delta)\phi q)\frac{1-\theta}{(1-\theta q)^2}]\mathbb{E}_t[\widehat{q}_{t+1}]$$

$$(2.131)$$

$$(1-\theta q)\delta\widehat{i}_t + \delta(1-\chi)\widehat{p}^I_t - (\theta\delta + \chi(1-\delta)\phi)q\widehat{q}_t - \chi\frac{rL}{K^H}(\widehat{r}_{t-1} + \widehat{l}_{t-1} - \widehat{\pi}_t)$$

$$- \chi(1-\delta)q\phi\widehat{\phi}_t - \chi[r^K + (1-\delta)q\phi]\widehat{s}_{t-1} - \chi r^K\widehat{r}^K_t - \chi\frac{Y}{K^H}(1 - \frac{1}{\mu})\widehat{y}_t \qquad (2.132)$$

$$- \chi\frac{Y}{K^H\mu}\widehat{\mu}_t + \chi\frac{T}{K^H}\widehat{t}_t = 0$$

$$\widehat{k}^H_t = (1-\delta)\widehat{k}^H_{t-1} + \delta\widehat{i}_t \qquad (2.133)$$

$$\widehat{r}_t = \max\left(-\log r, \phi_\pi\widehat{\pi}_t\right) \qquad (2.134)$$

$$\widehat{s}^g_t = \psi_K\widehat{\phi}_t \qquad (2.135)$$

$$\widehat{g}_t = \rho^G\widehat{g}_{t-1} + \epsilon^G_t \qquad (2.136)$$

$$\widehat{g}^u_t = \rho^{G^u}\widehat{g}^u_{t-1} + \epsilon^{G^u}_t \qquad (2.137)$$

$$\frac{T}{K^H}\widehat{t}_t = q\widehat{s}^g_t + \frac{rL}{K^H}(\widehat{r}_{t-1} - \widehat{\pi}_t + \widehat{l}_t) - [r^K + (1-\delta)q]\widehat{s}^g_{t-1} - \frac{L}{K^H}\widehat{l}_t + \frac{G}{K^H}\widehat{g}_t + \frac{G^u}{K^H}\widehat{g}^u_t \quad (2.138)$$

$$\frac{T}{K^H}\widehat{t}_t = \psi^T[\frac{rL}{K^H}(\widehat{r}_{t-1} - \widehat{\pi}_t + \widehat{l}_t) - q\widehat{s}^g_t] \qquad (2.139)$$

$$\widehat{k}^H_t = \widehat{s}_t + \widehat{s}^g_t \text{ where } \widehat{s}^g_t = S^g_t/K^H \qquad (2.140)$$

$$\widehat{k}^H_{t-1} = \widehat{n}_t + \widehat{k}^*_t \qquad (2.141)$$

$$\widehat{y}_t = (1 - \frac{I}{Y} - g^y - \frac{\gamma V}{Y})\widehat{c}_t + \frac{I}{Y}\widehat{i}_t + g^y\widehat{g}_t + \frac{\gamma V}{Y}\widehat{v}_t \qquad (2.142)$$

$$\widehat{ls}_t = \frac{1}{1 + \frac{L}{qK^H}}[\widehat{l}_t - (\widehat{q}_t + \widehat{k}^H_t)] \qquad (2.143)$$

2.6.2 Equations for the Labor Market with Exogenous Separation and Nominal Wage Rigidity

2.6.2.1 Staggered Wage Bargaining

The first order condition with respect to for Nash Bargaining:

$$\Upsilon_t(Y_t^{NL^*})J_t(Y_t^{NL^*}) = [1 - \Upsilon_t(Y_t^{NL^*})][H_t(Y_t^{NL^*}) - W_t], \tag{2.144}$$

where

$$\Upsilon_t(Y_{jt}^{NL^*}) = \frac{\Theta}{\Theta + (1 - \Theta)\xi_t(Y_{jt}^{NL^*})/\iota_t}, \tag{2.145}$$

$$\iota_t = 1 + \mathbb{E}_t \frac{\lambda_{1t+1}}{\lambda_{1t}}(1 - \rho)(1 - \eta)\beta/\pi_{t+1}\iota_{t+1}, \tag{2.146}$$

and

$$\xi_t(Y_{jt}^{NL^*}) = 1 + \mathbb{E}_t \frac{\lambda_{1t+1}}{\lambda_{1t}}[(1 - \rho) + X_{t+1}Y_{jt}^{NL^*}](1 - \eta)\beta/\pi_{t+1}\xi_{t+1}Y_{jt}^{NL^*}. \tag{2.147}$$

The log-linearized first order condition is

$$\widehat{J}_t(Y_t^{NL^*}) + (1 - \Upsilon)^{-1}\widehat{\Upsilon}_t(Y_t^{NL^*}) = \widehat{\Delta}_t(Y_t^{NL^*}), \tag{2.148}$$

where

$$\Delta_t(Y_t^{NL^*}) = H_t(Y_t^{NL^*}) - W_t. \tag{2.149}$$

Substituting the log-linearized expressions for $\widehat{J}_t(Y_t^{NL^*})$ and $\widehat{\Delta}_t(Y_t^{NL^*})$ and rearranging yields

$$\widehat{y}_t^L + [\Upsilon\beta(1 - \eta)\xi + (1 - \Upsilon)(1 - \rho)\beta(1 - \eta)\iota]\mathbb{E}_t(\widehat{y}_t^L - \widehat{\pi}_{t+1} - \widehat{y}_{t+1}^L)$$
$$= \Upsilon(1 - \alpha)(Y/N)(1/\mu)(1/Y^L)(-\widehat{\mu}_t + \widehat{y}_t - \widehat{n}_t) +$$
$$\Upsilon X\beta(J/Y^L)\mathbb{E}_t[\widehat{x}_{t+1} + 1/2(\widehat{\lambda}_{1t+1} - \widehat{\lambda}_{1t})] +$$
$$(1 - \Upsilon)(G^u + A)/Y^L\widehat{g}_t^u + (1 - \Upsilon)\Delta/Y^L\beta\rho^w\mathbb{E}_t[\widehat{\rho}_{t+1}^w + \widehat{\Delta}_{x,t+1} + \widehat{\lambda}_{1t+1} - \widehat{\lambda}_{1t}] +$$
$$\Upsilon J/Y^L(1 - \Upsilon)^{-1}[\widehat{\Upsilon}_t(Y_t^{NL^*}) - (1 - \rho)\beta\mathbb{E}_t\widehat{\Upsilon}_{t+1}(Y_{t+1}^{NL^*})], \tag{2.150}$$

where

$$\Delta_{x,t} = H_{x,t} - W_t = \int_0^1 H_t(Y_{jt}^{NL}) \frac{X_{jt} N_{jt-1}}{X_t N_{t-1}} dj \tag{2.151}$$

is the average surplus of employment conditional on being a new worker at t.

2.6.2.2 Log-linearized Equations

Aggregate hiring rate

$$\widehat{x}_t = \frac{(1-\alpha)Y}{N\mu\kappa X}(-\widehat{\mu}_t + \widehat{y}_t - \widehat{n}_t) - \frac{Y^L}{\kappa X}\widehat{y}_t^L + \beta(1-\rho)/2\mathbb{E}_t(\widehat{\lambda}_{1t+1} - \widehat{\lambda}_{1t}) + \beta\mathbb{E}_t\widehat{x}_{t+1} \tag{2.152}$$

Weight in Nash bargaining

$$\widehat{\Upsilon}_t = -(1-\Upsilon)(\widehat{\xi}_t - \widehat{\iota}_t) \tag{2.153}$$

with

$$\widehat{\iota}_t = (1-\rho)(1-\eta)\beta\mathbb{E}_t(\widehat{\lambda}_{1t+1} - \widehat{\lambda}_{1t} - \widehat{\pi}_{t+1} + \widehat{\iota}_{t+1}) \tag{2.154}$$

$$\begin{aligned}
\widehat{\xi}_t =& X(1-\eta)\beta\mathbb{E}_t\widehat{x}_{t+1} - X(1-\eta)\beta\frac{Y^L\xi^2}{\kappa X}\mathbb{E}_t(\widehat{y}_t^L - \widehat{\pi}_{t+1} - \widehat{y}_t^L) \\
&+ (1-\eta)\beta\mathbb{E}_t(\widehat{\xi}_{t+1} + \widehat{\lambda}_{1t+1} - \widehat{\lambda}_{1t} - \widehat{\pi}_{t+1})
\end{aligned} \tag{2.155}$$

Target wage

$$\begin{aligned}
\widehat{y}_t^{L*} =& \frac{\Upsilon(1-\alpha)Y}{N\mu Y^L}(-\widehat{\mu}_t + \widehat{y}_t - \widehat{n}_t) + (\Upsilon\beta\kappa x^2/Y^L + (1-\Upsilon)\rho^w\beta H/Y^L)\mathbb{E}_t\widehat{x}_{t+1} \\
&+ (1-\Upsilon)\rho^w\beta H/Y^L\mathbb{E}_t\widehat{\rho}_{t+1}^w + (1-\Upsilon)(G^u + A)/Y^L\widehat{g}_t^u \\
&+ (\Upsilon\beta\kappa x^2/Y^L/2 + (1-\Upsilon)\rho^w\beta H/Y^L)\mathbb{E}_t(\widehat{\lambda}_{1t+1} - \widehat{\lambda}_{1t}) \\
&+ \Upsilon(1-\Upsilon)^{-1}\kappa X/Y^L(\widehat{\Upsilon}_t - (1-\rho-\rho^w)\beta\widehat{\Upsilon}_{t+1})
\end{aligned} \tag{2.156}$$

Aggregate wage

$$\widehat{y}_t^L = \gamma_b(\widehat{y}_{t-1}^L - \widehat{\pi}_t) + \gamma_o\widehat{y}_t^{L*} + \gamma_f\mathbb{E}_t(\widehat{y}_{t+1}^L - \widehat{\pi}_{t+1}) \tag{2.157}$$

where

$$\gamma_b = (1+\tau_2)\tau_3^{-1}$$

$$\gamma_o = \varsigma\tau_3^{-1}$$

$$\gamma_f = (\tau_4/(1-\eta) - \tau_1)\tau_3^{-1}$$

$$\varsigma = \eta(1-\tau_4)/(1-\eta)$$

$$\tau_1 = [\xi\Upsilon\beta X + \Upsilon X\beta^2(1-\eta)\xi^2(1-\rho) + (1-\Upsilon)\rho^w\beta H/Y^L\Gamma](1-\tau_4)$$

$$\tau_2 = -\xi^2\Upsilon X\beta(1-\eta)(1-\tau_4)$$

$$\tau_3 = (1+\tau_2) + \varsigma + (\tau_4/(1-\eta) - \tau_1)$$

$$\tau_4 = \frac{\Upsilon\beta(1-\eta)\xi + (1-\Upsilon)(1-\rho)\beta(1-\eta)\iota}{1 + \Upsilon\beta(1-\eta)\xi + (1-\Upsilon)(1-\rho)\beta(1-\eta)\iota}$$

$$\Gamma = (1 - \Theta X\beta(1-\eta)\xi)\Theta^{-1}\xi Y^L/(\kappa X)$$

2.6.3 Tables

Table 2.1: Calibrated Values for Parameters Agreed with Del Negro et al. (2011)

Standard parameters		
β	=0.99	Discount factor
α	=0.4	Capital share
σ	=1	Relative risk aversion
δ	=0.025	Depreciation rate
ϵ^p	=11	Elasticity of substitution
ω	=0.75	Price Calvo probability
η	=0.25	Real/Nominal wage rigidity
Financial friction parameters		
$\phi(=\theta)$	=0.207	Resaleability/Borrowing constraint parameter for entrepreneurs
$\psi''(1)$	=1	Investment adjustment cost parameter
χ	=0.05	Probability of investment opportunity
$\frac{L}{4Y}$	=0.4	Steady-state liquidity-GDP ratio
ϵ_1^ϕ	=0.6	Size of the liquidity shock
ρ^ϕ	=0.833	Persistence of the liquidity shock
Baseline policy parameters		
ϕ_π	=1.5	Taylor rule coefficient
ψ^T	=0.1	Transfer rule coefficient

NOTE: All the above parameters are calibrated following Del Negro et al. (2011).

Table 2.2: Calibrated Values of Non-Standard Parameters

Labor market parameters (for both models)		
ζ	=0.5	Elasticity of matching function
W	=0.36	Workers' Bargaining power
ρ	=0.105	Total separation rate
ρ^w	=0.7	Steady-state job-finding rate
Labor market parameters		
(for model with endogenous separation and real wage rigidity only)		
σ_a	=0.15	Standard deviation of match-specific productivity
$\widetilde{a}$	=0.74	Steady-state threshold of productivity
τ	=0.74	Steady-state labor market tightness
Labor market parameters		
(for model with exogenous separation and nominal wage rigidity only)		
$\widetilde{b}$	=0.72	Total unemployment compensation
Steady state policy parameters		
g_y	=0.2	Steady state government spending - output ratio
$g_{y^L}^u$	=0.2	Replacement rate
ρ^G	=0.9375	Persistence of the fiscal expansion
ρ^{G^u}	=0.97	Persistence of the unemployment benefits change
Policy parameters when there is no alternative intervention		
ψ_K	=0	Unconventional credit intervention parameter
ϵ_1^G	=0	Initial fiscal expansion
$\epsilon_1^{G^u}$	=0	Initial change in the unemployment benefits
Policy parameters when alternative policies are implemented separately		
ψ_K	=-0.0672	Unconventional credit intervention parameter
ϵ_1^G	=0.075	Initial fiscal expansion
$\epsilon_1^{G^u}$	=0.5	Initial change in the unemployment benefits

NOTE: The size of initial unemployment benefits change, unconventional credit intervention and fiscal intervention parameters (which didn't appear in Zhang (2012)) are calibrated to match the data. All other parameters are calibrated following Zhang (2012).

Table 2.3: Effect of Fiscal Expansion

Effect	$Y^{FP} - Y^{NP}$	$C^{FP} - C^{NP}$	$\widetilde{Y}^{FP} - Y^{NP}$	multiplier
On impact	2.28%	1.17%	0.78%	> 1
Cumulative	5.4%	-5%	-0.6%	< 1

NOTE: X^{FP} represents the variable when fiscal expansion is implemented, X^{NP} represents the variable without fiscal expansion, and $\widetilde{Y}^{FP}$ represents the output net of increased government spending when fiscal expansion is implemented.

Table 2.4: Results from Different Models

	y	c	i	r¹	π	q	liq.	u¹	w
A. Endo. Separation & Real Wage Rigidity + ZLB									
baseline policies	-8.15	-7.24	-16.84	-0.55	-3.63	-5.95	-36.97	9.57	-3.29
credit policy	-5.87	-4.02	-15.39	-0.55	-2.53	-3.04	-23.30	6.00	-2.28
fiscal expansion	-5.87	-6.07	-16.26	-0.55	-2.52	-4.22	-35.89	6.19	-3.56
ext. unemp. benefits	-7.72	-5.98	-17.48	-0.55	-2.16	-6.68	-36.39	10.37	-1.48
B. Endo. Separation & Real Wage Rigidity + NZLB									
baseline policies	-3.67	-1.89	-14.01	-3.49	-2.33	1.76	-32.76	4.71	-1.57
credit policy	-3.24	-1.38	-13.69	-2.73	-1.82	1.54	-20.92	3.15	-1.29
fiscal expansion	-3.45	-3.19	-14.70	-2.88	-1.92	0.03	-33.73	4.23	-2.21
ext. unemp. benefits	-6.42	-5.65	-16.55	-2.86	-1.91	-5.95	-34.92	10.32	-1.05
C. Exo. Separation & Nominal Wage Rigidity + ZLB									
baseline policies	-8.24	-5.97	-16.66	-0.55	-1.54	-3.80	-36.44	9.86	0.77
credit policy	-6.80	-4.10	-15.66	-0.55	-1.20	-2.52	-23.47	7.39	0.56
fiscal expansion	-6.10	-5.33	-16.38	-0.55	-1.05	-3.13	-35.88	6.64	0.42
ext. unemp. benefits	-11.11	-10.00	-18.05	-0.55	-1.56	-7.04	-37.92	16.68	1.71
D. Exo. Separation & Nominal Wage Rigidity + NZLB									
baseline policies	-7.13	-5.25	-15.98	-2.16	-1.44	-2.79	-35.52	9.72	0.78
credit policy	-6.05	-3.94	-15.21	-1.70	-1.13	-2.12	-22.85	6.56	0.57
fiscal expansion	-5.49	-4.91	-16.00	-1.49	-0.99	-2.56	-35.38	6.51	0.42
ext. unemp. benefits	-10.03	-9.82	-17.40	-2.21	-1.47	-6.48	-37.04	16.63	1.75

NOTE: The table reports the on impact responses of main variables after a negative liquidity shock in different models.

¹ The numbers for the nominal rate and unemployment rate represent level deviations, and for other variables represent log-deviations.

2.6.4   Figures
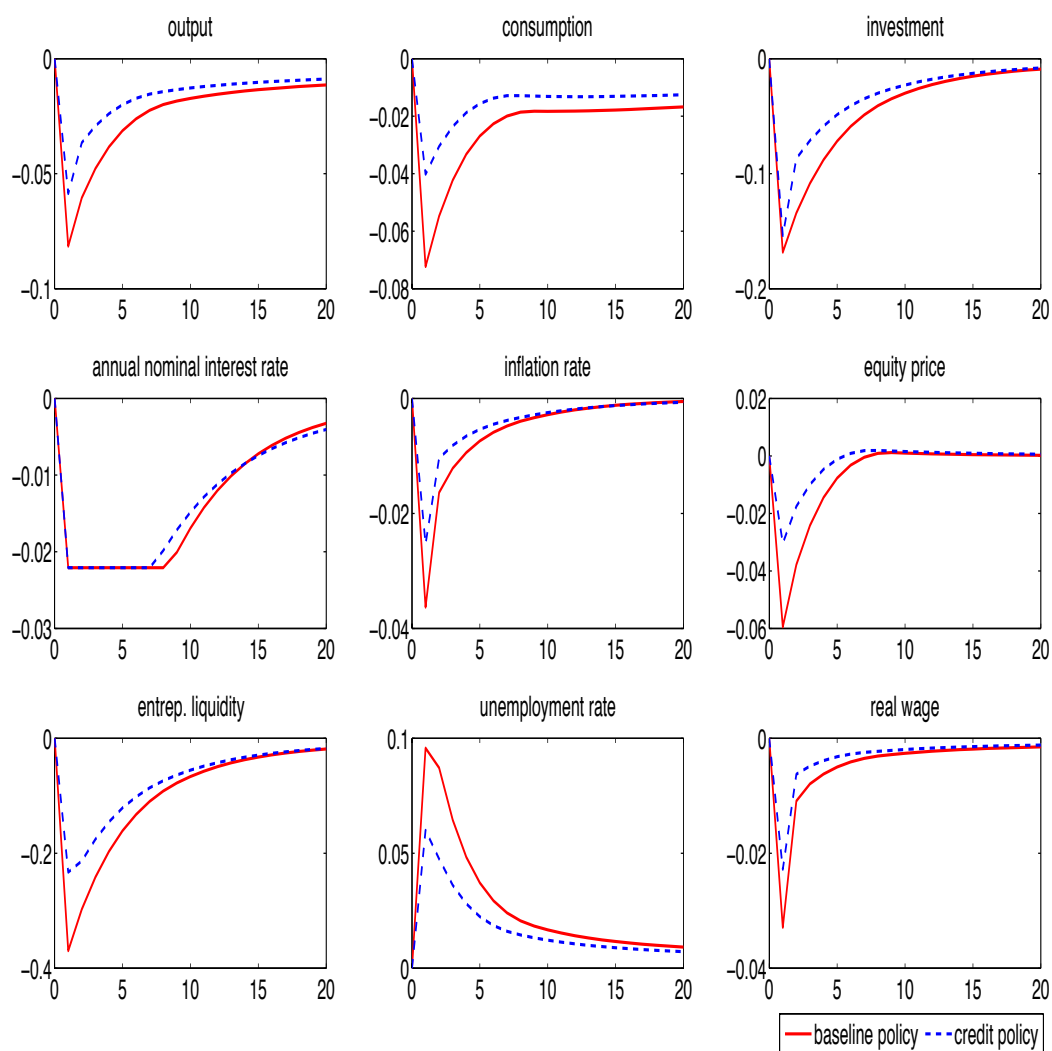
Figure 2.1: The Role of Credit Policy in the Response to a Liquidity Shock at the ZLB in a Model with Endogenous Separation and Real Wage Rigidity.

NOTE: The solid lines show the impulse responses after a liquidity shock in the model with endogenous separation, real wage rigidity and zero lower bound for the nominal interest rate. The dashed lines are the corresponding responses with unconventional credit intervention after the liquidity shock. The X-axis gives time horizon in quarters. In graphs for the annual nominal interest rate and unemployment rate, the Y-axis represents level deviation, and in graphs for other variables, it represents log-deviation.
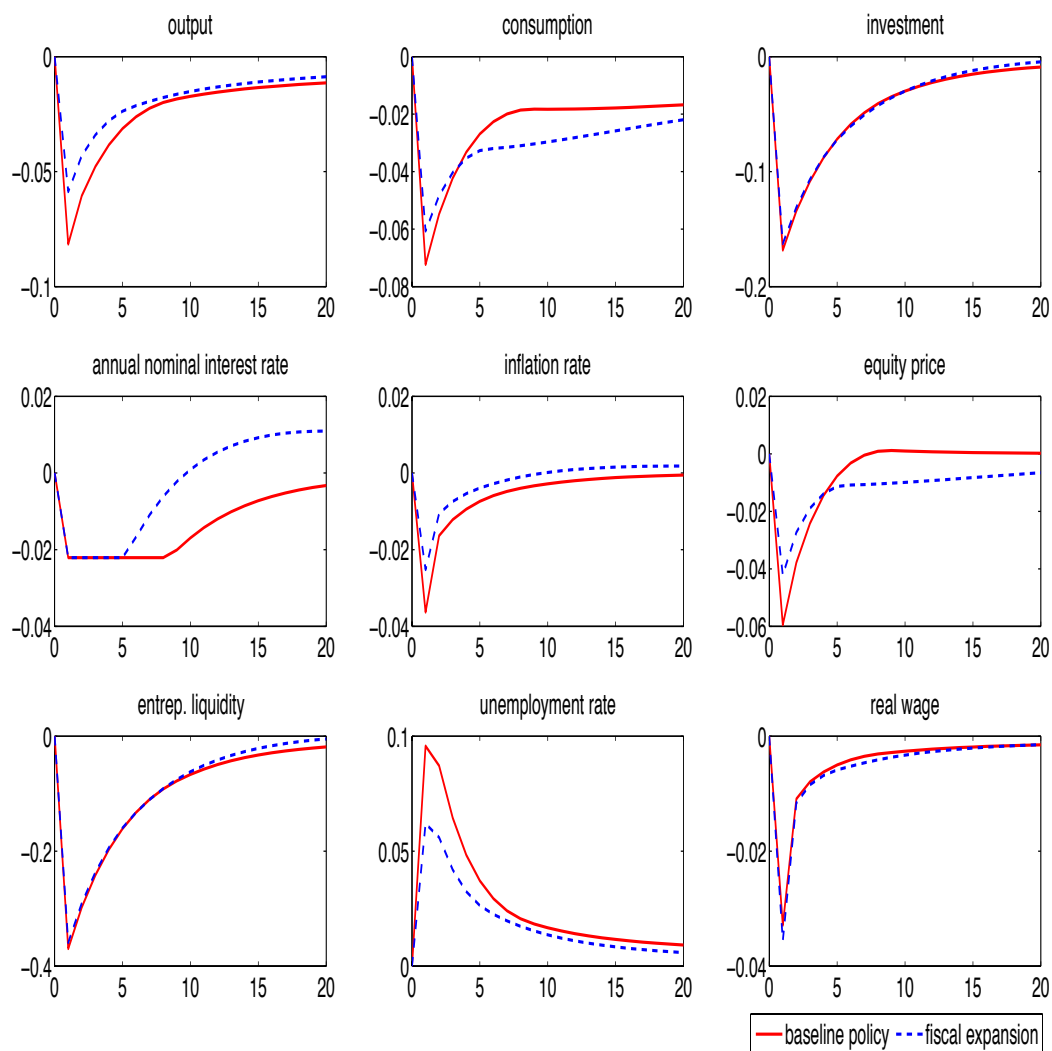
Figure 2.2: The Role of Fiscal Expansion in the Response to a Liquidity Shock at the ZLB in a Model with Endogenous Separation and Real Wage Rigidity.

NOTE: The solid lines show the impulse responses after a liquidity shock in the model with endogenous separation, real wage rigidity and zero lower bound for the nominal interest rate. The dashed lines are the corresponding responses with fiscal expansion after the liquidity shock. The X-axis gives time horizon in quarters. In graphs for the annual nominal interest rate and unemployment rate, the Y-axis represents level deviation, and in graphs for other variables, it represents log-deviation.

Figure 2.3: The Role of Extended Unemployment Benefits in the Response to a Liquidity Shock at the ZLB in a Model with Endogenous Separation and Real Wage Rigidity.

NOTE: The solid lines show the impulse responses after a liquidity shock in the model with endogenous separation, real wage rigidity and zero lower bound for the nominal interest rate. The dashed lines are the corresponding responses with extended unemployment benefits after the liquidity shock. The X-axis gives time horizon in quarters. In graphs for the annual nominal interest rate and unemployment rate, the Y-axis represents level deviation, and in graphs for other variables, it represents log-deviation.

Figure 2.4: The Role of Extended Unemployment Benefits and Matching Efficiency in Response to Liquidity Shocks at the ZLB in a Model with Endogenous Separation and Real Wage Rigidity.

NOTE: The solid line shows the impulse response of the unemployment rate after a liquidity shock in the model with endogenous separation and real wage rigidity when the zero lower bound for the nominal interest rate is binding. The dashed line is the corresponding response with extended unemployment benefits after the liquidity shock. The dash-dot line is the corresponding response with a decline in matching efficiency. The X-axis gives time horizon in quarters. and the Y-axis represents level deviation.
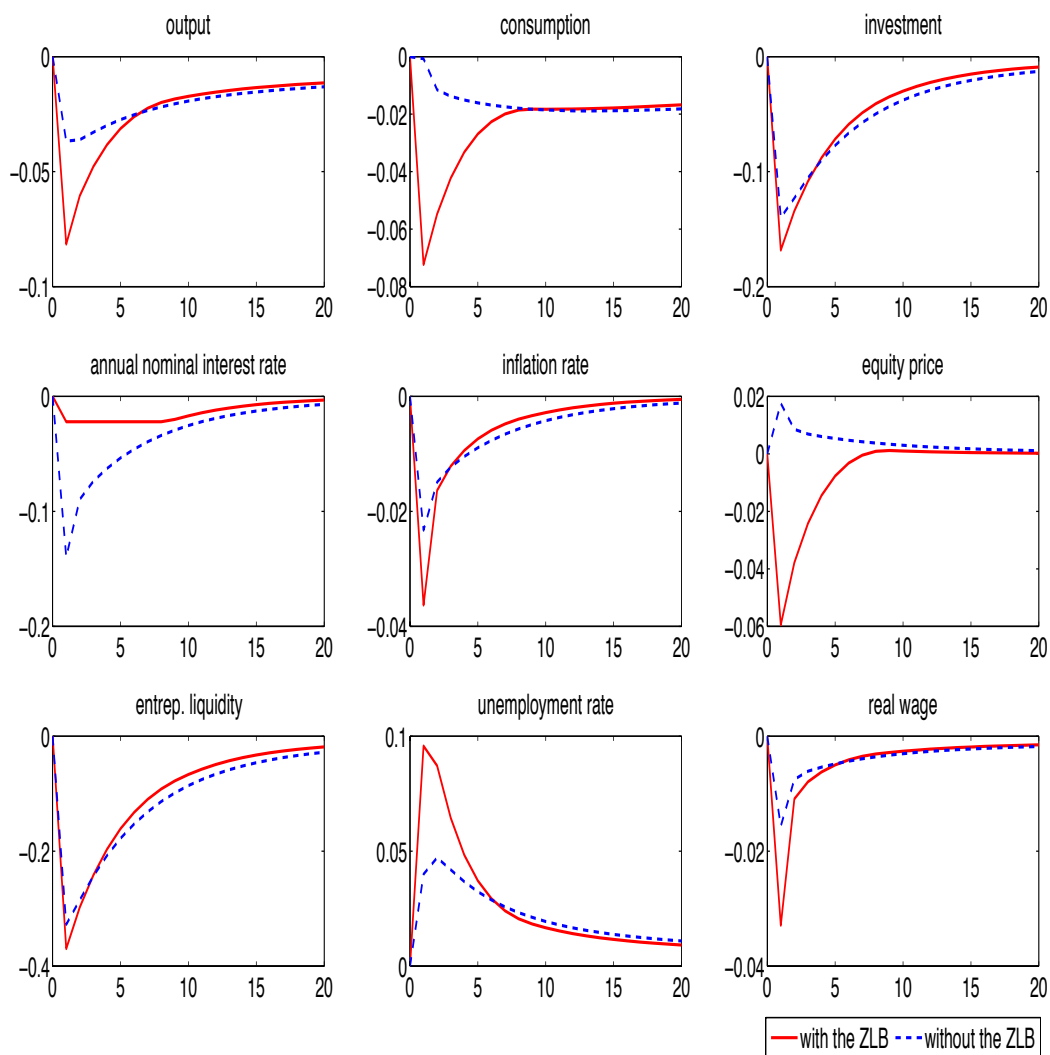
Figure 2.5: The Role of the ZLB in the Response to a Liquidity Shock in a Model with Endogenous Separation and Real Wage Rigidity.
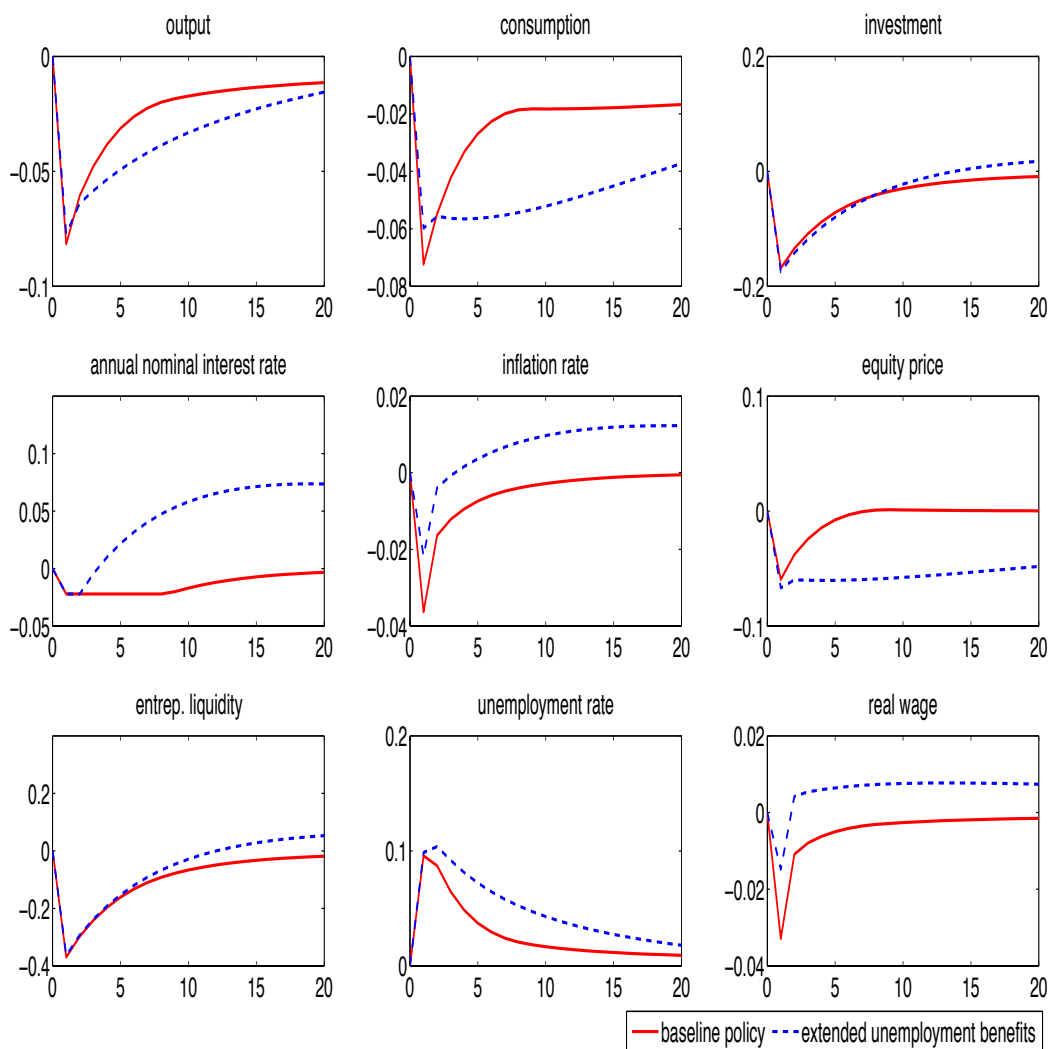
NOTE: The solid lines show the impulse responses after a liquidity shock in the model with endogenous separation, real wage rigidity and zero lower bound for the nominal interest rate. The dashed lines are the corresponding responses after the liquidity shock away from the zero lower bound. The X-axis gives time horizon in quarters. In graphs for the annual nominal interest rate and unemployment rate, the Y-axis represents level deviation, and in graphs for other variables, it represents log-deviation.

Figure 2.6: Using Credit Policy to Mimic the Response to Liquidity Shocks without the ZLB in a Model with Endogenous Separation, Real Wage Rigidity and ZLB.
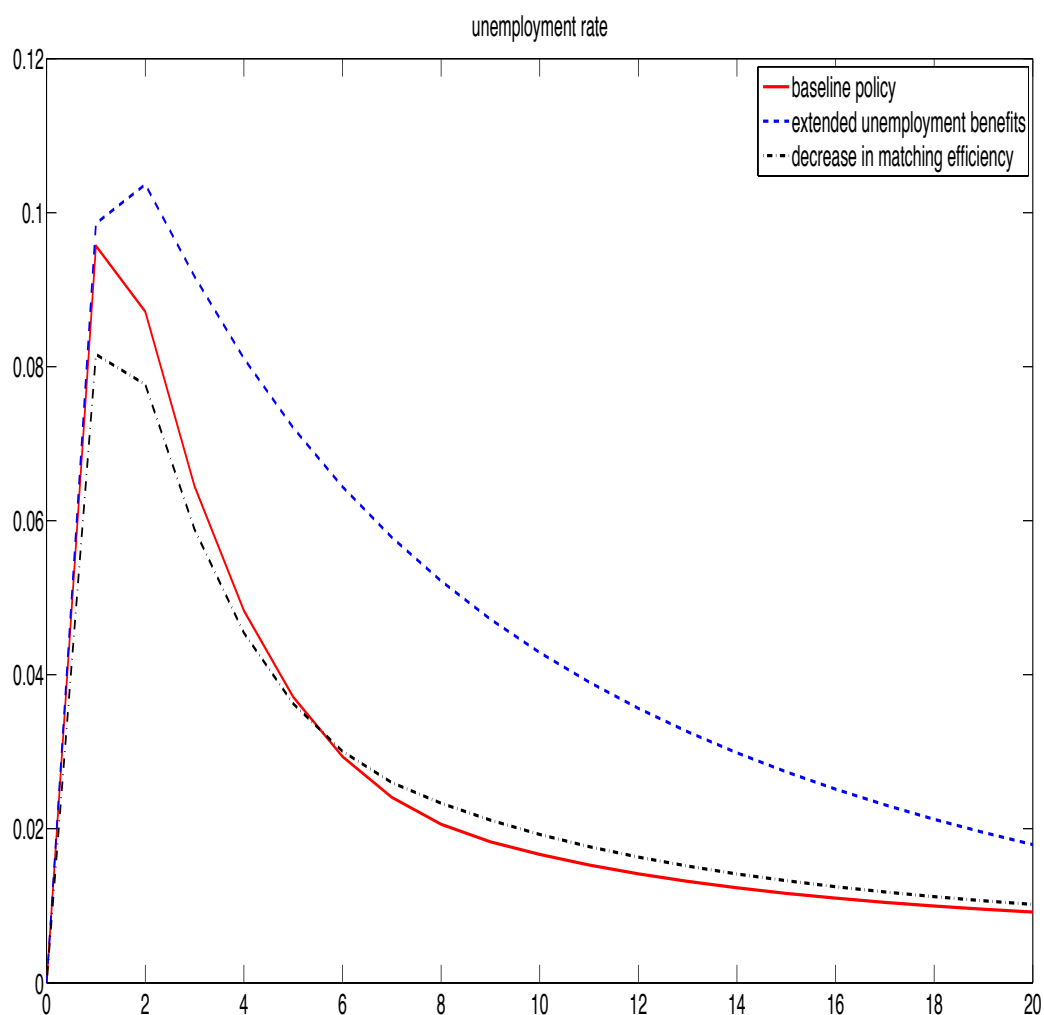
NOTE: The solid line shows the impulse response of output after a liquidity shock in the model with endogenous separation and real wage rigidity when the zero lower bound for the nominal interest rate is binding. The dashed line is the corresponding response without the ZLB. The dash-dot line is the corresponding response with a larger credit policy that helps the output mimic the initial decline without the ZLB when the ZLB is binding. The X-axis gives time horizon in quarters. and the Y-axis represents log deviation.
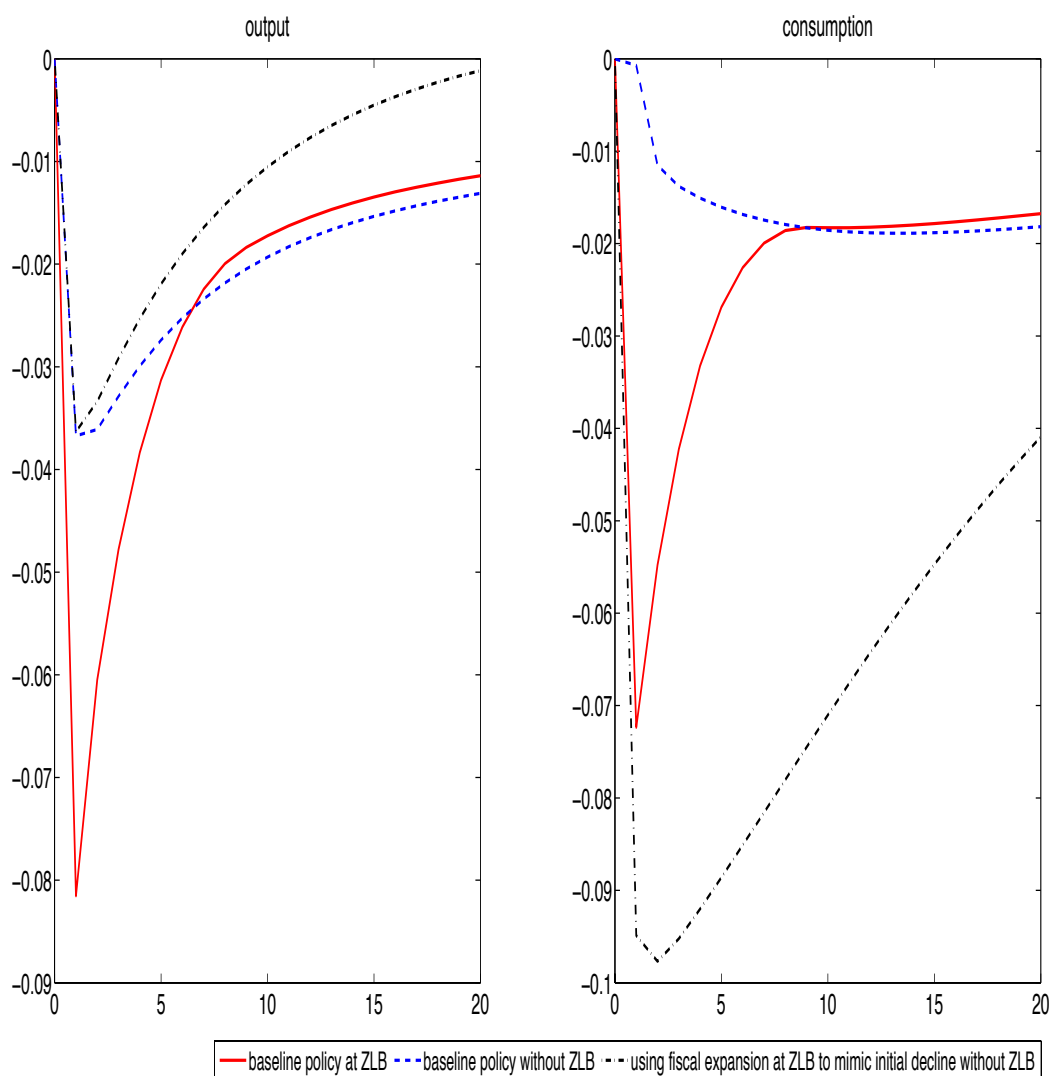
Figure 2.7: Using Fiscal Expansion to Mimic the Response to Liquidity Shocks without the ZLB in a Model with Endogenous Separation, Real Wage Rigidity and ZLB.

NOTE: The solid line shows the impulse response of output after a liquidity shock in the model with endogenous separation and real wage rigidity when the zero lower bound for the nominal interest rate is binding. The dashed line is the corresponding response without the ZLB. The dash-dot line is the corresponding response with a larger fiscal expansion that helps the output mimic the initial decline without the ZLB when the ZLB is binding. The X-axis gives time horizon in quarters. and the Y-axis represents log deviation.
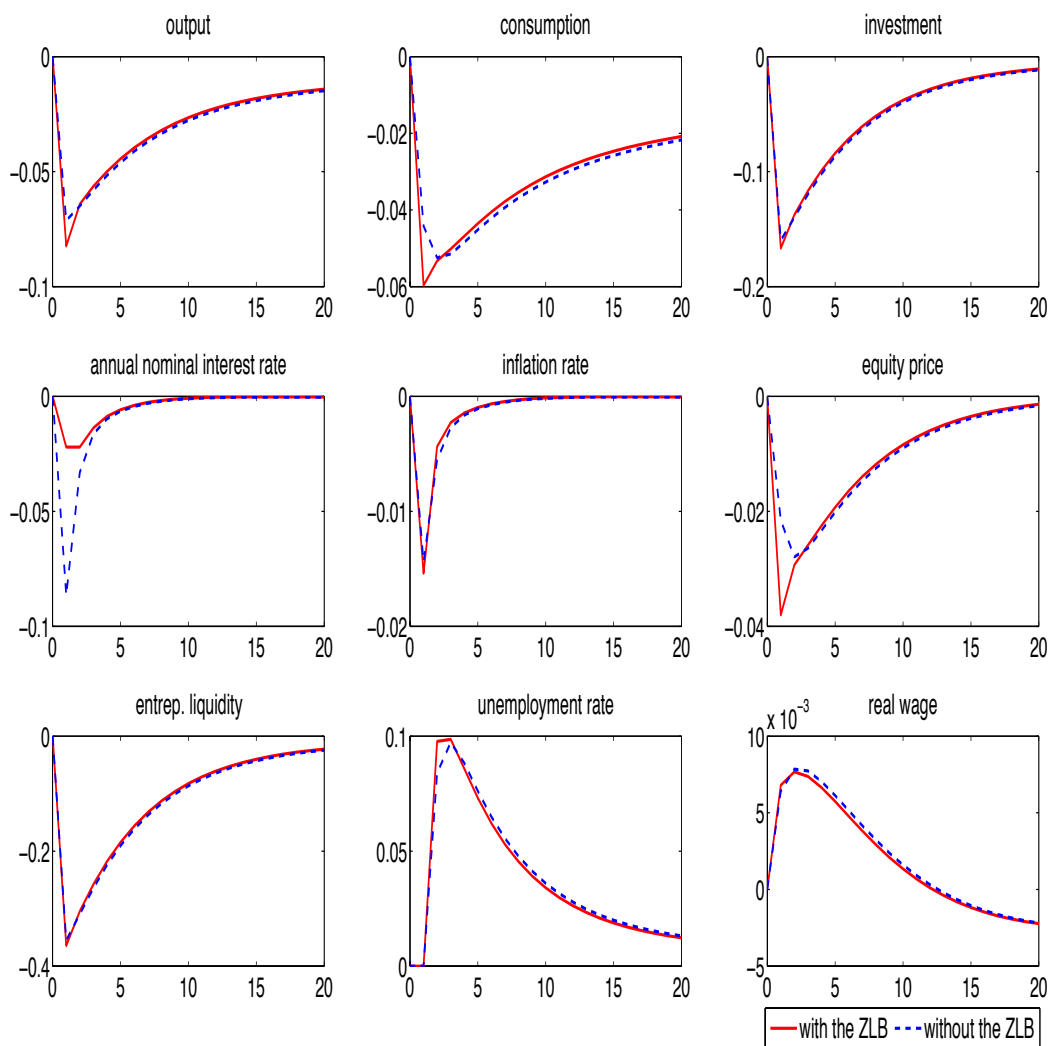
Figure 2.8: The Role of the ZLB in Response to a Liquidity Shock in a Model with Exogenous Separation and Nominal Wage Rigidity.

NOTE: The solid lines show the impulse responses after a liquidity shock in the model with exogenous separation, nominal wage rigidity and zero lower bound for the nominal interest rate. The dashed lines are the corresponding responses after the liquidity shock away from the zero lower bound. The X-axis gives time horizon in quarters. In graphs for the annual nominal interest rate and unemployment rate, the Y-axis represents level deviation, and in graphs for other variables, it represents log-deviation.
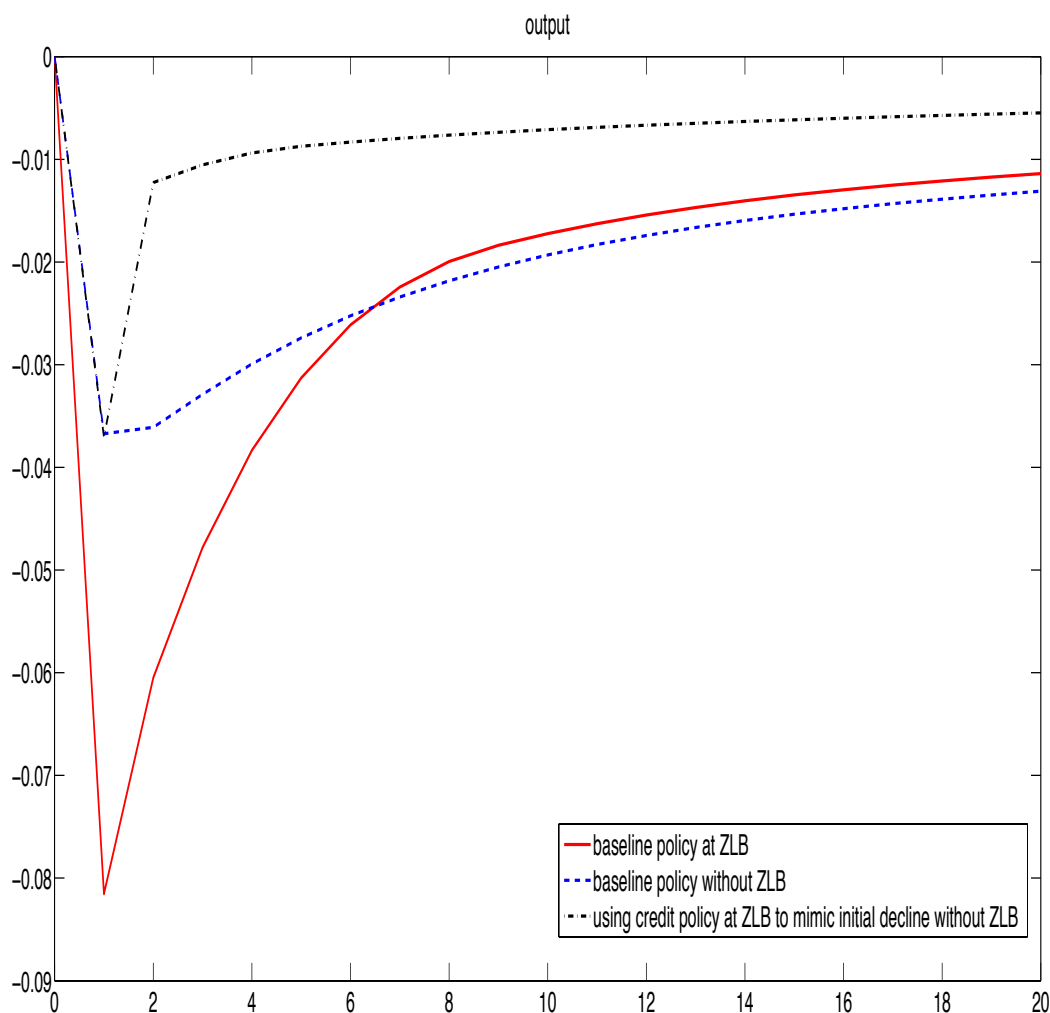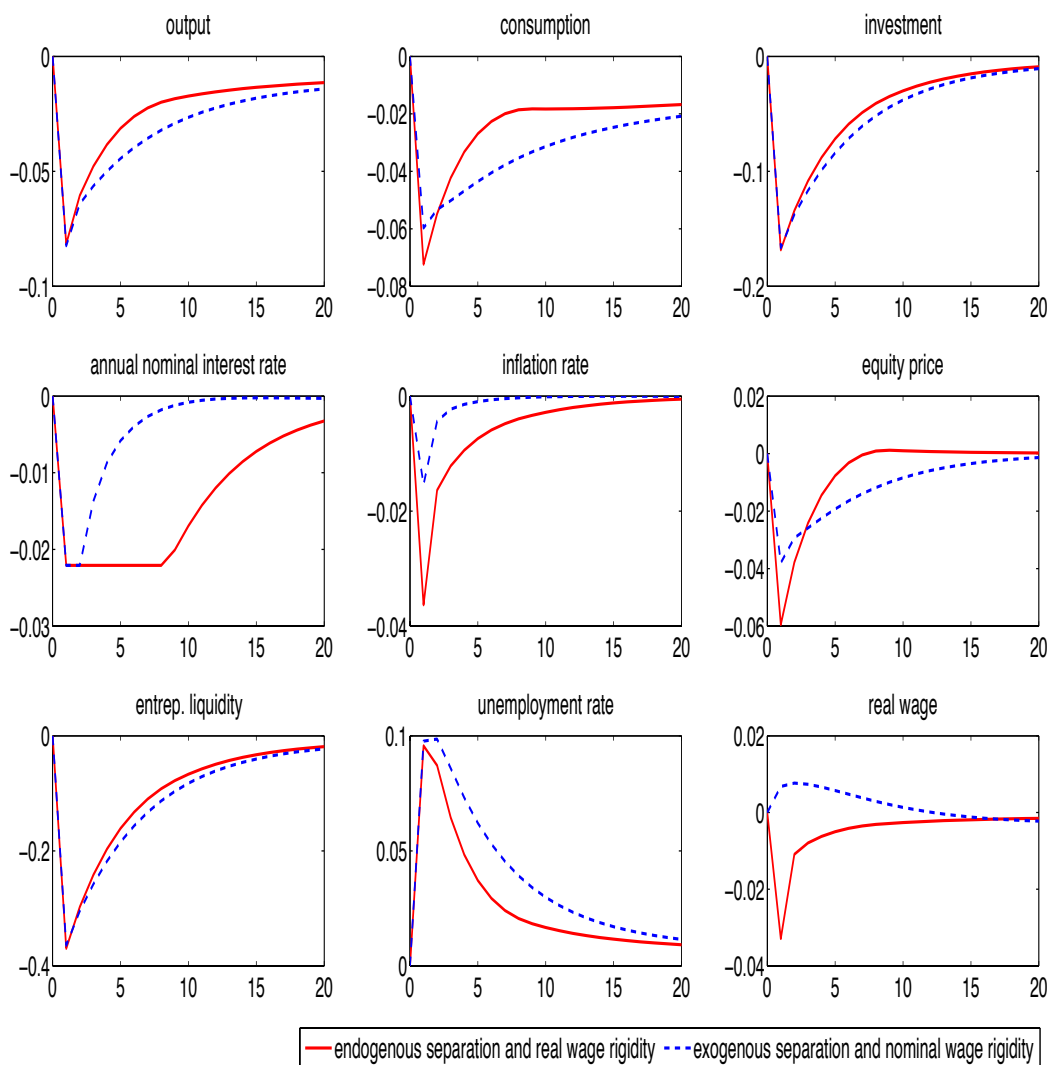
Figure 2.9: The Impact of a Liquidity Shock: Model with Endogenous Separation and Real Wage Rigidity Vs Model with Exogenous Separation and Nominal Wage Rigidity.

NOTE: The solid lines show the impulse responses after a liquidity shock in the model with endogenous separation, real wage rigidity and zero lower bound for the nominal interest rate. The dashed lines are the corresponding responses after the liquidity shock in the model with exogenous separation and nominal wage rigidity. The X-axis gives time horizon in quarters. In graphs for the annual nominal interest rate and unemployment rate, the Y-axis represents level deviation, and in graphs for other variables, it represents log-deviation.

Acknowledgement

Chapter 2, in full, has been submitted for publication of the material. The dissertation author was the primary author of this material.

Chapter 3

Macroeconomic News, Monetary Policy and the Real Interest Rate at the Zero Lower Bound

Abstract. Policy implications of New Keynesian models at the zero lower bound depend on how interest rates respond to various shocks. Through analyzing the responses of various yields to macroeconomic announcements, I find that the predictions of New Keynesian models for the behavior of interest rates when the zero lower bound is binding are reliable: nominal rates are less sensitive to news, and real rates respond to shocks in opposite directions from their behavior away from the zero lower bound. This suggests that at least in the short run, policies that push up inflation are favorable to the economy at the zero lower bound. By this mechanism, policies such as fiscal expansion can be more beneficial for an economy at the zero lower bound compared to their effects at normal times. I also find using an identification strategy based on heterogeneity that at the zero lower bound, monetary policy shocks account for less variation of both nominal and real rates, monetary policy is less effective in affecting short- and medium-term real rates, and the effect dies off faster.

3.1 Introduction

Since the federal funds rate hit the zero lower bound at the end of 2008, there have been a lot of studies on how the economy reacts to government policies at the zero lower bound, especially how effective fiscal expansion and nontraditional monetary policy are in stimulating the economy.

Numerous papers have studied the fiscal multiplier at the zero lower bound, for example Christiano, Eichenbaum, and Rebelo (2011), Woodford (2011), and Eggertsson (2009). Most of these papers employ New Keynesian models, and find that fiscal expansion is particularly effective at the zero lower bound since it decreases the overnight real interest rate through raising expected inflation. However, in normal times fiscal expansion which increases people's inflation expectation will cause an even larger increase in the nominal rate, and in turn cause an increase in the overnight real interest rate. For this reason, the overnight real interest rate reacts to fiscal policy shocks in the opposite direction from normal times when the economy is at the zero lower bound. In New Keynesian models, this is true not only for fiscal policy shocks, but also for any shocks that affect inflation expectation. Is this behavior of short-term real interest rates supported by data or does it only exist in theorecital models? Since rational agents in the economy make their consumption and investment decisions according to the entire future path of the interest rates instead of only considering the overnight real rate, in order for fiscal policy to be more effective at the zero lower bound, the medium- and long-term real interest rates should also react less to shocks. Is this true in the data?

Swanson and Williams (2013) provided insight into these questions by estimating the time-varying sensitivity of interest rates to macroeconomic announcements relative to a benchmark period in which the zero bound was not a concern. They find that yields on Treasury securities with a year or more to maturity were surprisingly responsive to news through 2008-2010, suggesting that fiscal policy was likely to have been about as effective as usual during this period. Only Treasury yields are considered in their paper. However, the real interest rate is the rate that matters for agents' decisions. In my paper, I consider both Treasuries and TIPS, and study whether at the zero lower bound, the short-, medium- and long-term real interest

rates react differently from the normal time to macroeconomic announcements.

Besides fiscal policy, unconventional monetary policies are also widely studied. The effectiveness of Fed's policy in stimulating the economy depends on its effect on interest rates. Could the Fed policy affect interest rates significantly at the zero lower bound?

This question has been studied both theoretically and empirically. With a theoretical model, we can directly study the effect of unconventional monetary policy on output. Important contributions in this literature include Del Negro et al (2011), Chen, Curdia, and Ferrero (2009), Curdia and Woodford (2009a,b), Gertler and Karadi (2011), and Eggertsson and Woodford (2009). Empirical research has studied the effect of monetary policy on medium- and long-term yields. Wright (2012) uses a structural VAR with daily data to identify the effects of monetary policy shocks on longer-term interest rates after the federal funds rate hit the zero bound. Identification through heteroskedasticity is employed to identify the VAR. He finds that stimulative monetary policy shocks lower Treasury and corporate bond yields, but the effects die off fairly fast. Hamilton and Wu (2012) estimate an affine term structure model, and find that if in December of 2006 the Fed were to have sold off all its Treasury holdings of less than one-year maturity and use the proceeds to retire Treasury debt from the long end, this might have resulted in a 14-basis-pint drop in the 10-year rate and an 11-basis-point increase in the 6-month rate. D'Amico, and King (2010), Doh (2010), Gagnon et. al (2010), Hancock and Passmore (2011), Neely (2010), Krishnamurthy and Vissing-Jorgenson (2011) also have very important contributions in this field.

However, without comparing the effects of monetary policies both in normal times and at the zero lower bound, it is hard to evaluate the effectiveness of unconventional monetary policy objectively. In addition, by comparing the effects in both periods, we can clearly know how each yield changes its responses to monetary policy at the zero lower bound, which gives us a clue to what's going on under the change in the effectiveness of monetary policy and may suggest a good way to react to the change. In this paper I use the same identification scheme as Wright (2012) to study the effectiveness of monetary policies on both Treasuries and TIPS in various maturities both before and after the federal funds rate hit the zero lower bound, and

compare the differences between this two periods.

Through estimating the reactions of various interest rates to macroeconomic news announcements both before and after the nominal interest rate hit the zero lower bound , I find that the data support the implications of New Keynesian models on interest rates at the zero lower bound. More specifically, we can summarize the results as follows. First, both nominal rates and real rates behave differently at the zero lower bound. Second, nominal interest rate will be less sensitive to shocks at the zero lower bound, although this change may be not significant for longer-term rates since the possibility of staying at the zero lower bound for 10 years is low. Third, real interest rates will change in the opposite direction in response to shocks at the zero lower bound. This is true at least for short-term real rate. These results suggest that government policies that raise inflation, such as fiscal expansion and extended unemployment benefits, are more effective at the zero lower bound.

By identifying the monetary policy shocks, I find that at the zero lower bound, short-term and medium-term Treasury yields are less sensitive to monetary policy shocks, while TIPS rates become more sensitive. The effect of monetary policy dies off faster, and monetary policy shock accounts for less of the variation in interest rates at the zero lower bound. The central bank has to take more aggressive actions to achieve the goal of stimulating the economy.

The remainder of the paper is structured as follows: Section 2 estimates the responsiveness of interest rates on macroeconomic news announcements. Section 3 studies the effect of monetary policies on interest rates. Section 4 is the conclusion.

3.2   Predictions of a New Keynesian Model

In a standard New Keynesian DSGE model, monetary policy follows a Taylor rule:

$$i_t = \max\left\{\rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t, 0\right\} \tag{3.1}$$

where i_t is the nominal interest rate, ρ is the equilibrium real interest rate, π_t is the deviation from long-run target, and $\widetilde{y}_t$ is the output gap. By specifying $\phi_\pi > 1$, the Taylor rule says that an increase in inflation by one percentage point will prompt the

central bank to raise the nominal interest rate by more than one percentage point. New Keynesian Phillips Curve relates inflation π_t to inflation expectation $\mathbb{E}_t \pi_{t+1}$ and output gap:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \widetilde{y}_t \tag{3.2}$$

Combining the Taylor rule and New Keynesian Phillips Curve, we can get:

$$i_t = \max \left\{ \rho + \beta \phi_\pi \mathbb{E}_t \pi_{t+1} + (\kappa \phi_\pi + \phi_y) \widetilde{y}_t, 0 \right\} \tag{3.3}$$

Typical calibration use $\beta = 0.99$ and $\phi_\pi = 1.5$, implying $\beta \phi_\pi > 1$, which means, an one percentage point increase in inflation expectation will cause a more than one percentage point increase in the nominal rate. The real interest rate is related to the nominal rate and inflation expectation by Fisher's equation:

$$r_t = i_t - \mathbb{E}_t \pi_{t+1} \tag{3.4}$$

Combining Equation (3) and (4), we can get

$$r_t = \begin{cases} \rho + (\beta \phi_\pi - 1)\mathbb{E}_t \pi_{t+1} + (\kappa \phi_\pi + \phi_y)\widetilde{y}_t \\ \qquad \text{or} & \text{at normal times} \\ (1 - \frac{1}{\beta \phi_\pi})i_t + \frac{\rho}{\beta \phi_\pi} + \frac{\kappa \phi_\pi + \phi_y}{\beta \phi_\pi} \widetilde{y}_t \\ -\mathbb{E}_t \pi_{t+1} & \text{at the ZLB} \end{cases} \tag{3.5}$$

That means the real rate will also change in the same direction as the nominal rate and inflation expectation, but by a smaller amount than the nominal rate. However, at the zero lower bound, the overnight nominal interest rate stays constant, which changes the pattern of comovement among the nominal rate, inflation, and the real rate. We can summarize the implications of New Keynesian models at the zero lower bound into two claims:

Claim 1: Nominal interest rates will be less sensitive to shocks at the zero lower bound.

Claim 2: Real interest rates will respond in the opposite direction from inflation expectation in response to shocks at the zero lower bound.

Since the satisfaction of *Claim 2* requires nominal rates do not respond to shocks, a weak version of *Claim 2* when nominal rates do respond to shocks but less sensitively is: real interest rates will be less sensitive to shocks at the zero lower bound.

Since the predictions of the New Keynesian models are on overnight nominal and real rates, it is not clear what will happen to longer-term yields. We cannot deny that it is reasonable to expect that long-term yields may not be very different at the zero lower bound, because the possibility of staying at the zero lower bound for 10 years is very small. However, all claims should be true at least for short-term interest rates.

With these predictions for interest rates, what we can infer about the effectiveness of fiscal and monetary policy? From the pattern of interest rates' reactions to shocks at the zero lower bound, we can get meaningful policy implications. Policies that lead to higher inflation expectation can be more effective at the zero lower bound. One example of this kind of policies is fiscal expansion. In the normal time, fiscal expansion will increase inflation expectation, and in turn increase the nominal and real interest rate. So private demand will be crowded out by the increase in public spending. This weakens the effect of fiscal expansion on stimulating the economy. However, at the zero lower bound, an increase in inflation expectation resulting from fiscal expansion will not lead to a corresponding increase in the nominal rate, hence, the real rate decreases instead of increasing. Without the crowding-out effect, fiscal expansion is more effective at the zero lower bound. While fiscal policy becomes more effective, traditional monetary policy becomes useless at the zero lower bound since there is no room for lowering federal funds rate further. "Nontraditional" monetary policies, such as balance sheet tools and communication tools, have to be implemented. Standard New Keynesian models have nothing to say about the effectiveness of these nontraditional tools, and different variations of the standard model may have different conclusions. However, we can at least know that with short-term nominal interest rates stuck at the zero bound, longer-term rates are also less flexible, so it is harder for monetary policies to work effectively.

In the next two sections, I will use data to verify the predictions of New Keynesian models.

3.3 Effects of Macro News on Interest Rates

I examine the effectiveness of fiscal policy through studying the responses of 9 different interest rates to macroeconomic news announcements, which include both "real-side" news and "price" news, from June 2004 to September 2012. The 9 interest rates are 2-year, 5-year, and 10-year Treasury yields, TIPS yields and TIPS breakeven from Gürkaynak, Sack and Wright (2007, 2010).

The news or the surprise component of an announcement is defined to be the difference between the released value of a macroeconomic variable and expected value of the announcement. 13 macroeconomic announcements are included: capacity utilization, consumer confidence, core CPI, durable orders, ECI, GDP advance report, initial claims, ISM, new home sales, nonfarm payrolls, core PPI, retail sales, and unemployment rate. Table 3.1 reports the basic information of these announcements.

A_{it} is the released value of an announcement of type i at time t, and E_{it} is the expected value of the release at time t. The expected values come from Money Market Services data. The news, s_{it}, is defined as the difference between the release value and expected value normalized by its standard deviation:

$$s_{it} = \frac{A_{it} - E_{it}}{\sigma(A_{it} - E_{it})}.$$
(3.6)

The sign of surprises are normalized such that positive surprises increase short-term Treasury yields. That is, the signs of surprises of countercyclical variables, such as the unemployment rate and initial claims, are flipped. Define the time indicator as

$$\delta_t = \begin{cases} 0 & \text{if } t \in [May\ 2004, Nov.\ 2008] \\ 1 & \text{if } t \in [Dec.\ 2004, Sept.\ 2012], \end{cases}$$

which separates the full sample into two subsamples: one is in normal times and the other is at the zero lower bound.

For yields curve j, we do the following regression:

$$\Delta y_t^j = \alpha_1^j + \Sigma_{i=1}^{13} \beta_{i1}^j s_{it} + \alpha_2^j \delta_t + \Sigma_{i=1}^{13} \Delta \beta_i^j s_{it} \delta_t + \epsilon_t^j$$
(3.7)

where Δy_t^j is the change in yield j in a new announcement day t. The coefficients in normal times and at the zero lower bound are represented by β_{i1}^j and β_{i2}^j respectively, and $\Delta \beta_i^j$ represent the difference between coefficients in these two separate periods, that is,

$$\Delta \beta_i^j = \beta_{i2}^j - \beta_{i1}^j. \tag{3.8}$$

The sample size of each yield is $T = 1064$, and the number of regressors including the constant terms is $K = 28$.

Tables 3.2 and 3.3 report the estimated coefficients β_{i1}^j and $\Delta \beta_i^j$ separately. Numbers in the parentheses are the t-statistics, and coefficients that are significant at the 5% significance level are represented in bold font. Since $\Delta \beta_i^j = \beta_{i2}^j - \beta_{i1}^j$, $\Delta \beta_i^j < 0$ indicates $\beta_{i2}^j < \beta_{i1}^j$, that is, interest rates are less sensitive to news at the zero lower bound. For 2-year Treasury bond, this is true except for news on Core PPI. The short-term nominal interest rate seems to be more sensitive to Core PPI at the zero lower bound. One possibility of this is since the Great recession, production has been far below the capacity, which means there is bigger room for producers to increase their output when facing higher selling prices compared to normal times. All other coefficients support *Claim 1* that short term nominal interest rate is less sensitive to news at the zero lower bound.

2-year TIPS are also less sensitive to most of the news, except for capacity utilization and nonfarm payrolls. All other coefficients support the weak version of *Claim 2* that short term real interest rate is less sensitive to news at the zero lower bound.

In order to verify the two claims further, we need to do two hypothesis tests.

Test I: $H_0 : \beta_{i1}^j = \beta_{i2}^j$ ($\Delta \beta_i^j = 0$) for all $i = 1, ..., 13$, given j.

This test studies the overall behavior of each yield in response to the 13 news items. In each regression, 13 slope coefficients in each of the two periods are jointly tested for a structural break. Intercepts are excluded in the test, so in each test there are 13 restrictions, and the statistic follows an F-distribution with degrees of freedom (13, T-K).

From the top panel of Table 3.4, we see that the null hypotheses for 2-year Treasury, 2, 5-, and 10-year TIPS, and 10-year TIPS breakeven are rejected at the

5% significance level. This means the real interest rate in all three maturities behaves differently at the zero lower bound, and only the short-term nominal rate is different from before. This tells us that interest rates do behave differently at the zero lower bound compared to normal times, which is the prerequisite of the two Claims. Why doesn't the response of 5- and 10-year Treasuries change at the zero lower bound? This is because people see a significant possibility of escaping from the zero lower bound over most of the term of the securities. As the maturity increases, the responses of the yields will be closer to the responses in normal times, because the possibility of getting away from the zero lower bound increases. This is very similar to the result in Hamilton and Wu (2012), in which they find that at the short end of the yield are essentially unresponsive to any macroeconomic development at the zero lower bound, but as the maturity increases this effect of the zero lower bound dies out and almost disappears when the maturity approaches 4 year.

Test II: $H_0 : \beta_{i1}^j = \beta_{i2}^j$ ($\Delta\beta_i^j = 0$) for given i, j.

This type of test studies whether the response of yield j to each announcement is different at the zero lower bound. So for each yield, there are 13 Type II tests, each test has only one restriction, and the statistic follows an F-distribution with degrees of freedom (1, T-K). The F-statistics are equivalent to the t-statistics reported in Table 3.3.

From the bottom panel of Table 3.4, the coefficients for GDP and New Home Sales are significantly different at the zero lower bound for TIPS in all three maturities, and coefficients for ISM are different at the zero lower bound for 2- and 5-year TIPS at the 5% significance level. Moreover, coefficients for these three variables at the zero lower bound, β_{i2}^j, can be obtained using Equation (3.8) and results in Table 3.2 and 3.3, and as shown in Table 3.5 we can find that most of the coefficients on these three variables flip signs. In the normal time, higher-than-expected GDP, ISM and New Home Sales indicate the economy is booming, which will cause a rise in inflation expectation and the real interest rate. However, at the zero lower bound, although changes on inflation expectation under higher-than-expected GDP, ISM, and New Home Sales are not different from the normal time, the nominal interest rate cannot freely increase. This will lead the real rate to decrease, which is opposite to the response in normal times. This result is consistent with *Claim 2*. The only

exception is the ISM coefficient for 5-year TIPS. However, it is hard to treat it as a violation of *Claim 2*. We can examine the significance of these β_{i2}^{j}'s through testing the restriction $\beta_{i1}^{j} + \Delta\beta_{i}^{j} = 0$. We can find that the ISM coefficient for 5-year TIPS itself is not statistically different from zero at the zero lower bound, which makes it ambiguous whether it is positive or negative. Apart from the coefficients for the above three news, other coefficients cannot be treated different from before according to the test. The first reason that so many coefficients do not differ from before is the available sample size is not large enough, so many of the coefficients themselves are not significant, let alone the comparison between different periods. The second reason is the time break may be more apparent for shorter-term TIPS rate since the model's prediction is on overnight rate, however high-frequency data on TIPS yields with less than 2 year maturity are not available.

The above analysis tells us the predictions of New Keynesian models on interest rates at the zero lower bound summarized as the three claims are supported by the data, and fiscal policy can be more effective at the zero lower bound.

In addition to the effects of the real side news and price news on the yields, the effect of monetary policy news also draws close attentions from both economists and policy makers. This will be analyzed in the next section.

3.4 Effects of Monetary Policy Affects on Interest Rates

Besides the macroeconomic announcements, another source of interest rates fluctuations is monetary policy shocks. The surprise element of FOMC decisions about the target federal funds rate is not considered in the above analysis because firstly, it is controversial to use it as a monetary policy shock, and secondly, the target rate and the expectation are consistent all the time after we entered the zero lower bound, which could provide no information on how the yields respond to "news" on monetary policies. So the effect of a monetary policy shock will be studied separately in this section. Instead, the effect of monetary policy is studied using identification through heteroskedasticity. This idea was first proposed by Rigobon (2003), and then

applied to identify structural VARs. Wright (2012) makes a great advance in using this methodology to study the effect of monetary policy on long-term interest rates at the zero lower bound. This identification method helps us overcome several challenges in measuring the effects of monetary policy shocks at the zero lower bound. Here, I follows Wright's procedure to identify monetary policy shocks both before and after the federal funds rate hit the zero lower bound. I assume that on FOMC days or days with important Fed announcements, the variance of changes in interest rates is larger, and that monetary policy announcements make a bigger contribution to the change on those days, compared to others.

Six yields, 2-, 5,-, and 10-years for Treasury and TIPS are used here. Two separate regressions are conducted using data from two subsamples. One subsample is from Jan. 2004 to June 2008, and the other is from Dec. 2008 to Jan. 2013.

Suppose the 6 yields have the following reduced form VAR representation:

$$y_t = \Pi' x_t + \epsilon_t \tag{3.9}$$

where y_t denotes an (6×1) vector containing the values that 6 variables at time t, $\Pi' = [c \ \Phi_1 \ ... \ \Phi_p]'$, and $x_t = [1 \ y'_{t-1} \ ... \ y'_{t-p}]'$.

The residual ϵ_t is linear combination of 6 structural shocks: $\epsilon_t = R[\eta_t^1 \ ... \ \eta_t^6]'$. We assume η^1 represent monetary policy shocks, and R_1 is the first column of R, the elements of which represent the effects of monetary policy shocks on each yields. R is unidentified without extra restrictions. Since we are interested in the effect of monetary policy shocks, we will try to identify R_1 only.

The first step is estimating the regular VAR, to get the estimated coefficient matrix $\widehat{\Pi}'$ and residual $\widehat{\epsilon}_t$.

The next step is using identification through heteroskedasticity to get $\widehat{R}_1$. We assume that monetary policy shocks have mean zero, and variance σ_H^2 on monetary policy announcement days, and variance σ_L^2 on regular days, and $\sigma_H^2 > \sigma_L^2$. The covariance matrices of residuals on announcement days and regular days are Σ_H and Σ_L respectively. The difference between theses two matrices satisfies

$$\Sigma_H - \Sigma_L = R_1 R_1' \sigma_H^2 - R_1 R_1' \sigma_L^2 \tag{3.10}$$

Here for purpose of estimation, we normalize $\sigma_H^2 - \sigma_L^2 = 1$ and other structural shocks follow the same distribution with mean 0 and variance 1 on all days.

Minimum distance estimation is used to estimate R_1, and the weighting matrix is constructed from the inverse of the sum of estimated covariance matrices of $vech(\widehat{\Sigma}_H)$ and $vech(\widehat{\Sigma}_L)$, denoted $\widehat{V}_H$ and $\widehat{V}_L$. The minimum distance estimator of R_1 can be expressed as

$$\widehat{R}_1 = \arg\min_{R_1}[vech(\widehat{\Sigma}_H - \widehat{\Sigma}_L - R_1 R_1')]'[\widehat{V}_H + \widehat{V}_L]^{-1}[vech(\widehat{\Sigma}_H - \widehat{\Sigma}_L - R_1 R_1')] \quad (3.11)$$

Two tests are used to verify the identification.

Test III: $H_0 : \Sigma_H = \Sigma_L$.

This test tells us whether the condition of using identification through heteroskedasticity is satisfied. Rejection of the null hypothesis means the variances of monetary policy shocks are different on announcement days and non-announcement days, and in turn identification through heteroskedasticity can be used to identify monetary policy shocks. Otherwise, identification through heteroskedasticity is not usable.

A Likelihood ratio test is used for the test,

$$LR = 2(\mathbb{L}_1^* - \mathbb{L}_0^*) = T\log|\widehat{\Sigma}| - (T_L \log|\widehat{\Sigma}_L^{GLS}| + T_H \log|\widehat{\Sigma}_H^{GLS}|) \quad (3.12)$$

where $\mathbb{L}_1^*$ is the likelihood under H_1, $\mathbb{L}_0^*$ is the likelihood under H_0, $\widehat{\Sigma}$ is the covariance matrix under the null hypothesis without differing the announcement and non-announcement days, and $\widehat{\Sigma}_L^{GLS}$ and $\widehat{\Sigma}_H^{GLS}$ are iterated GLS estimators of the covariance matrices of non-announcement and announcement days. A large LR means the likelihood under H_1 is significantly larger than that under H_0, that is, H_0 should be rejected. Comparing the distribution of bootstrap under the null hypothesis, we can get the bootstrap p-value.

Test IV: $H_0 : R_1 R_1' = \Sigma_H - \Sigma_L$.

This test tells us whether there is a single monetary policy shock.

Comparing the statistic

$$\widehat{M} = [vech(\widehat{\Sigma}_H^* - \widehat{\Sigma}_L^* - \widehat{R}_1^*\widehat{R}_1^{*'})]'[\widehat{V}_H^* + \widehat{V}_L^*]^{-1}[vech(\widehat{\Sigma}_H^* - \widehat{\Sigma}_L^* - \widehat{R}_1^*\widehat{R}_1^{*'})]$$

and the bootstrap distribution of

$$\widehat{M}^* = [vech(\widehat{\Sigma}_H - \widehat{\Sigma}_L - \widehat{R}_1\widehat{R}_1')]'[\widehat{V}_H + \widehat{V}_L]^{-1}[vech(\widehat{\Sigma}_H - \widehat{\Sigma}_L - \widehat{R}_1\widehat{R}_1')].$$

where variables with $*$ are bootstrap analogs of corresponding estimated variables,gives us the bootstrap p-value. If we cannot reject the null hypothesis, the difference between the two covariance matrices can be factored in the form R_1R_1'.

Table 3.6 reports the results of the two hypothesis testings. No matter in normal times or at the zero lower bound, the null hypothesis of *Test III* is rejected, and the null hypothesis of *Test IV* cannot be rejected. The results justify our usage of identification through heteroskedasticity.

Using the estimated VAR and $\widehat{R}_1$, we can derive the impulse responses of the yields to a monetary policy shock. Figure 3.1 shows the impulse responses before the fed funds target rate hit the zero lower bound under a monetary policy shock whose size is normalized to increase 10-year treasury yields by 0.25%. The 68% confidence intervals of the impulse responses are constructed through the stationary bootstrap (Politis and Romano (1994)). The stationary bootstrap is a resampling methods similar to the block resampling techniques, and this resampling procedure is repeated to build up an approximation to the sampling distribution of the statistics. In contrast to the block resampling methods, the pseudo-time series generated by the stationary bootstrap method is actually a stationary time series. Figure 3.2 shows the impulse responses at the zero lower bound, where the size of the monetary policy shock is also normalized to increase 10-year Treasury yields by 25 basis points.

Comparing these two figures, we can find that there are several differences between the two periods. First, shorter-term Treasuries are less sensitive to monetary policy shocks at the zero lower bound. This is easy to understand, because federal funds rate is zero over most of the term of shorter-term Treasuries.

Second, TIPS rates, especially shorter-term TIPS rates, are more sensitive than Treasury rates with the same maturity at the zero lower bound. This is a feature of a "price puzzle" effect that this identification strategy produces when applied to the normal periods: inflation expectation increases with a contractionary monetary policy. This is also a common finding in the literature. One proposed solution (Sims

(1992), Christiano, Eichenbaum, and Evens (1999) is improving the information set of the VAR by including commodity prices. Barth and Ramey (2001) argue that this is not a puzzle, but just because monetary policy transmit its effects on real variables trough a cost channel. Since we see "price puzzle" in normal times, but we don't see a statistically significant initial increase in inflation expectation at the zero lower bound in response to contrationary monetary policy, one possibility is that this results from asymmetric information. As mentioned in Tas (2011), people believe the central bank is more informative in forecasting future inflation, and tend to perceive the contractionary monetary policy as a signal of a high probability of higher future inflation according to the central bank's information set. This will lead people to form higher inflation expectations. However, after the conventional monetary policy became useless, communication tools have been used by Fed much more than before, and this is helpful in eliminating the asymmetric information problem.

Third, 2- and 5-year TIPS rates have smaller responses at the zero lower bound than in normal times, even though in both periods the monetary policy shocks have the same effect on 10-year Treasury yields. If we assume the zero lower bound puts little constraint on the changes of the TIPS rates and 10-year Treasury yields (which is a reasonable assumption), then monetary policy shocks of the same size should cause TIPS rates to change in the same amount both at the zero lower bound and in normal times. Since now, TIPS rates have smaller changes at the zero lower bound, under the above assumption, we can say that monetary policy can do less in affecting short- and medium- term real interest rates at the zero lower bound.

Fourth, the effect of monetary policy shocks, especially for shorter-term TIPS, dies off faster at the zero lower bound. In the normal time, the estimated half-life of the monetary policy effect for 2- and 5-year TIPS is round 300 days, while this number reduces to only 100 day at the zero lower bound.

Table 3.7 and 3.8 report the results of forecast error variance decompositions in normal times and at the zero lower bound. In both cases, the size of the monetary policy shock is normalized to account for 10% of the variance of the one-day ahead forecast of 10-year Treasury. At the zero lower bound, monetary policy shocks become much less important for the one-day ahead forecast variance of Treasuries and 2-year TIPS. Although the one-day ahead forecast variance of 5- and 10-year TIPS, and

10-year Treasury are very similar in the two periods, the differences become bigger at longer horizons. At the zero lower bound, monetary policy shocks are less important for the variation in both nominal and real rates, and it requires larger efforts of the central bank to simulate the economy through affecting the interest rates. This is consistent with the result from analyzing the impulse responses.

3.5   Conclusion

Since the changes in the interest rates in New Keynesian models with the zero lower bound have very rich policy implications, and many evaluations on the effectiveness of government policies rely on these models, it is necessary to verify these implications using data.Empirical studies in this paper find that at the zero lower bound, fiscal policy is more effective and monetary policy is less effective.

Through analyzing the responses of various yields to real-side news, price news and monetary policy shocks, I find that Treasuries are less sensitive to news and the behavior of 2-year Treasury is significantly different at the zero lower bound. More importantly, TIPS yields in all maturities are different from before, and respond to the news whose effect son TIPS are significantly different in the opposite direction. This means TIPS rates respond to these news in the opposite direction to inflation expectations at the zero lower bound. Less sensitive nominal rates together with real rates changing in the opposite direction at the zero lower bound ensure the effectiveness of fiscal policy. However, monetary policy at the zero lower bound is not as effective in affecting shorter-term Treasury yields as in normal times: the TIPS rates are less sensitive to monetary policy, and the effect of monetary policy dies off faster, and less variations in the yields can be explained by monetary policy shocks.

3.6   Appendix

3.6.1   Table

<div align="center">

Table 3.1: Macroeconomic Announcements[1]

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Name	Notation	Frequency	Std	Units
Capacity	Cap	Monthly	0.39	Percent
Consumer Confidence	Conf	Monthly	5.29	Index
Core CPI	CPI	Monthly	0.09	% change mom[2]
Durable Orders	Dura	Monthly	2.44	% change mom
Employment Cost Index	ECI	Quarterly	0.13	% change mom
GDP Advance Report	GDP	Quarterly	0.63	% change mom
Initial Claims	IC	Weekly	19.10	Thousands
ISM Manufacturing	ISM	Monthly	2.05	Index
New Home Sales	NHS	Monthly	67.10	Thousands
Nonfarm Payrolls	NFP	Monthly	70.40	Thousands
Core PPI	PPI	Monthly	0.26	% change mom
Retail Sales	Retail	Monthly	0.55	% change mom
Unemployment Rate	Un	Monthly	0.16	Percent

[1] Sources of these announcements include Bureau of Economic Analysis, Bureau of Labor Statistics, Bureau of Census, Employment and Training Administration, University of Michigan, and Institute for Supply Management.

[2] Percentage change month-over-month.

Table 3.2: Regression of Daily Yield Changes on Macro Surprises I
– Results In Normal Times ($\delta_t = 1$): β_{i1}^{j}

	Nominal			TIPS			Breakeven Inflation		
	2yr	5yr	10yr	2yr	5yr	10yr	2yr	5yr	10yr
Cap	2.89	**2.84**	1.67	-6.10	-0.50	0.27	**8.99**	**3.34**	1.40
	(1.75)	(2.11)	(1.67)	(-1.49)	(-0.48)	(0.53)	(2.35)	(2.73)	(1.76)
Conf	0.57	-0.14	-0.78	1.10	-0.37	-0.59	-0.53	0.23	-0.19
	(0.62)	(-0.12)	(-0.67)	(1.13)	(-0.32)	(-0.59)	(-0.91)	(0.53)	(-0.35)
CPI	**1.82**	**2.11**	**2.15**	**-1.88**	-0.65	-0.33	**3.70**	**2.77**	**2.48**
	(2.05)	(2.26)	(2.56)	(-2.29)	(-1.20)	(-0.68)	(3.32)	(3.50)	(3.17)
Dura	1.11	0.88	0.84	1.02	0.87	0.90	0.09	0.00	-0.06
	(1.31)	(1.10)	(1.26)	(1.13)	(1.47)	(1.45)	(0.13)	(0.01)	(-0.18)
ECI	1.10	0.95	0.60	0.25	-0.49	-0.55	0.85	**1.44**	**1.16**
	(0.92)	(0.85)	(0.63)	(0.16)	(-0.41)	(-0.59)	(0.79)	(2.43)	(2.05)
GDP	1.81	2.10	2.20	**2.17**	**2.36**	**2.25**	-0.36	-0.26	-0.04
	(1.35)	(1.50)	(1.75)	(2.16)	(2.82)	(2.87)	(-0.21)	(-0.24)	(-0.06)
IC	**1.84**	**1.98**	**1.73**	1.52	**1.19**	**0.94**	0.32	0.79	0.79
	(3.72)	(3.92)	(3.70)	(1.75)	(2.38)	(2.19)	(0.40)	(0.14)	(0.10)
ISM	**3.33**	**2.40**	**1.86**	**3.63**	**2.49**	**1.81**	-0.29	-0.08	0.05
	(4.20)	(3.29)	(3.32)	(3.57)	(3.68)	(3.36)	(-0.39)	(-0.14)	(0.10)
NHS	0.89	0.87	0.97	0.84	0.81	0.66	0.05	0.07	0.30
	(1.74)	(1.58)	(1.90)	(1.56)	(1.74)	(1.72)	(0.13)	(0.24)	(1.31)
NFP	**3.73**	3.32	2.76	2.16	2.05	1.96	**1.57**	**1.27**	**0.80**
	(2.06)	(1.58)	(1.67)	(1.03)	(1.01)	(1.36)	(2.14)	(3.57)	(2.33)
PPI	0.98	**1.29**	**1.28**	-0.22	0.54	**0.59**	1.20	0.75	0.69
	(1.54)	(2.06)	(2.35)	(-0.39)	(1.43)	(2.22)	(1.71)	(1.38)	(1.48)
Retail	**2.44**	**2.21**	1.55	1.54	**1.80**	**1.52**	0.89	0.41	0.02
	(2.79)	(2.51)	(1.72)	(1.76)	(2.41)	(2.11)	(0.81)	(0.82)	(0.06)
Un	**3.02**	2.71	**2.27**	3.35	2.23	1.77	-0.32	0.48	0.50
	(2.31)	(1.94)	(1.97)	(1.74)	(1.45)	(1.62)	(-0.23)	(0.75)	(0.99)

[1] Coefficients before the federal funds target rate hit the zero lower bound (β_{i1}^{j}) are shown in the table. Constant terms are not shown.

[2] Heteroskedasticity-consistent t-statistics are shown in parentheses.

[3] Numbers in bold font indicate statistical significance at the 5% significance level.

Table 3.3: Regression of Daily Yield Changes on Macro Surprises II
– Difference between Coefficients at the ZLB and in Normal Times: $\Delta \beta_i^j$

	Nominal			TIPS			Breakeven Inflation		
	2yr	5yr	10yr	2yr	5yr	10yr	2yr	5yr	10yr
Cap	-2.91	**-3.55**	**-2.92**	8.48	1.19	-0.34	**-11.39**	**-4.74**	**-2.57**
	(-1.67)	(-2.15)	(-2.04)	(1.88)	(0.93)	(-0.45)	(-2.62)	(-2.78)	(-2.05)
Conf	-0.45	0.65	1.90	-0.76	0.43	0.94	0.30	0.22	0.96
	(-0.43)	(0.45)	(1.29)	(-0.64)	(0.30)	(0.75)	(0.34)	(0.30)	(1.23)
CPI	-2.47	**-3.52**	-3.60	-0.92	-2.17	-1.90	-1.55	-1.35	-1.70
	(-1.93)	(-1.98)	(-1.91)	(-0.54)	(-1.28)	(-1.22)	(-0.98)	(-1.21)	(-1.59)
Dura	-1.09	-0.14	0.19	-0.24	0.29	0.56	-0.84	09.32	-0.37
	(-0.90)	(-0.09)	(0.12)	(-0.18)	(0.26)	(0.50)	(-0.67)	(-0.42)	(-0.45)
ECI	-0.45	0.27	0.88	-0.50	1.15	2.42	0.04	-0.87	-1.54
	(-0.27)	(0.11)	(0.33)	(-0.23)	(0.58)	(1.12)	(0.02)	(-0.73)	(-1.51)
GDP	-2.00	-2.77	-2.60	**-5.61**	**-4.36**	**-4.43**	3.61	1.59	1.84
	(-1.28)	(-1.42)	(-1.27)	(-3.05)	(-3.01)	(-3.31)	(1.59)	(1.03)	(1.64)
IC	-0.92	-0.57	-0.22	-0.87	-0.20	-0.20	-0.05	-0.37	-0.01
	(-1.69)	(-0.87)	(-0.31)	(-0.85)	(-0.28)	(-0.27)	(-0.05)	(-0.69)	(-0.02)
ISM	**-3.26**	-1.56	-0.21	**-4.43**	**-2.35**	-1.01	1.17	0.79	0.81
	(-3.14)	(-1.19)	(-0.15)	(-3.73)	(-2.17)	(-0.95)	(1.07)	(1.01)	(0.99)
NHS	-0.76	-1.04	-1.32	**-2.96**	**-2.86**	**-2.79**	2.20	1.82	1.47
	(-0.59)	(-0.57)	(-0.71)	(-2.44)	(-2.68)	(-2.90)	(1.50)	(1.26)	(1.09)
NFP	-0.01	1.24	1.28	0.05	0.66	0.68	-0.06	0.58	0.60
	(-0.01)	(0.51)	(0.66)	(0.02)	(0.30)	(0.42)	(-0.04)	(0.58)	(0.72)
PPI	0.49	1.30	2.28	-1.08	0.09	0.77	1.56	1.21	1.50
	(0.61)	(1.13)	(1.77)	(-1.21)	(0.10)	(1.04)	(1.34)	(1.09)	(1.47)
Retail	-1.26	-0.04	1.41	-1.83	-0.60	-0.81	0.58	0.56	**2.22**
	(-1.27)	(-0.04)	(1.08)	(-1.61)	(-0.61)	(-0.87)	(0.39)	(0.46)	(2.72)
Un	**-3.50**	-3.07	-2.48	-2.86	-1.72	-1.42	-0.64	-1.35	-1.06
	(-2.22)	(-1.70)	(-1.59)	(-1.32)	(-0.97)	(-1.05)	(-0.38)	(-1.37)	(-1.34)

[1] Difference between coefficients at the zero lower bound and in normal times (β_{i2}^j) are shown in the table. Constant terms are not shown.

[2] Heteroskedasticity-consistent t-statistics are shown in parentheses.

[3] Numbers in bold font indicate statistical significance at the 5% significance level.

Table 3.4: Testing the Structural Change at the ZLB

Test I	$H_0 : \beta_{i1}^j = \beta_{i2}^j \ (\Delta\beta_i^j = 0)$ for all i, given j (testing all pairs of β_i^j's together, $F(13, T-K)$))								
	Nominal			TIPS			Breakeven Inflation		
	2yr	5yr	10yr	2yr	5yr	10yr	2yr	5yr	10yr
	2.48	1.61	1.66	**3.31**	**1.97**	**2.07**	1.33	1.46	**2.25**
Test II	$H_0 : \beta_{i1}^j = \beta_{i2}^j \ (\Delta\beta_i^j = 0)$ for given i, j (testing each pair of β_i^j, $F(1, T-K)$)								
	Nominal			TIPS			Breakeven Inflation		
	2yr	5yr	10yr	2yr	5yr	10yr	2yr	5yr	10yr
Cap	2.79	4.60	**4.17**	3.53	0.87	0.21	**6.89**	**7.73**	**4.20**
Conf	0.18	0.20	1.67	0.43	0.09	0.56	0.12	0.09	1.52
CPI	3.74	**3.93**	3.65	0.29	1.64	1.49	0.96	1.47	2.52
Dura	0.81	0.01	0.02	0.03	0.07	0.25	0.45	0.17	0.20
ECI	0.07	0.01	0.11	0.05	0.33	1.25	0.00	0.54	2.27
GDP	1.65	2.01	1.62	**9.30**	**9.07**	**10.94**	2.52	1.07	2.69
IC	2.86	0.75	0.10	0.72	0.08	0.07	0.00	0.47	0.00
ISM	**9.86**	1.42	0.02	**13.93**	**4.70**	0.90	1.14	1.02	0.97
NHS	0.35	0.33	0.51	**5.94**	**7.19**	**8.41**	2.26	1.60	1.20
NFP	0.00	0.26	0.43	0.00	0.09	0.17	0.00	0.34	0.52
PPI	0.38	1.28	3.14	1.47	0.01	1.07	1.79	1.19	2.15
Retail	1.61	0.00	1.17	2.60	0.38	0.76	0.15	0.21	**7.41**
Un	**4.94**	2.88	2.52	1.75	0.94	1.09	0.14	1.89	1.79

[1] A number in bond font means the null hypothesis is rejected at the 5% significance level.

Table 3.5: Coefficients for TIPS with Structural Change at the ZLB – β_{i2}^j [1]

	2-year TIPS	5-year TIPS	10-year TIPS
GDP	-3.44	-2.00	-2.19
ISM	-0.81	**0.14**[2]	–
NHS	-2.11	-2.05	-2.13

[1] $\beta_{i2}^j = \beta_{i1}^j + \Delta\beta_i^j$.
[2] ISM coefficient for 5-year TIPS does not flip sign at the zero lower bound.

Table 3.6: Testing the Identification through
Heteroskedasticity – *Test III and Test IV*

	Null Hypothesis	Statistic	Bootstrap p-value
Normal time	$\Sigma_H = \Sigma_L$[1] (LR test)	110	**0.01**
	$R_1 R_1' = \Sigma_H - \Sigma_L$[2] (Wald test)	78	0.39
at the ZLB	$\Sigma_H = \Sigma_L$ (LR test)	120	**0.04**
	$R_1 R_1' = \Sigma_H - \Sigma_L$ (Wald test)	30	0.39

[1] This tests the hypothesis that the covariance matrices of reduced form errors are the same on announcement and non-announcement days.

[2] This tests the hypothesis that monetary policy shocks can account for the difference between the covariance matrices of reduced form errors are the same on announcement and non-announcement days.

[3] A bold number means the null hypothesis is rejected.

Table 3.7: Forecast Error Variance Decomposition –
Normal Times (Jan. 2004 - Jun. 2008)

	1	50	100	200	500
2 year Treasury	46.3 (37.2,69.0)	64.3 (45.5,94.0)	66.5 (44.2,95.8)	57.5 (38.0,89.5)	47.6 (32.8,84.3)
5 year Treasury	26.7 (23.1,34.7)	40.8 (28.7,55.2)	46.2 (29.9,62.7)	44.7 (28.6,64.2)	40.1 (26.9,64.0)
10 year Treasury	10.0	16.5 (9.6,21.2)	20.4 (10.2,26.9)	22.7 (10.9,30.2)	22.3 (11.4,31.9)
2 year TIPS	14.6 (9.3,25.2)	26.9 (14.4,47.5)	41.0 (19.9,64.4)	51.5 (25.7,74.2)	44.6 (25.5,73.3)
5 year TIPS	18.2 (9.9,25.0)	26.8 (15.3,40.5)	36.2 (19.2,53.0)	43.3 (22.5,61.9)	40.0 (22.8,62.8)
10 year TIPS	14.7 (2.8,21.2)	18.1 (8.2,25.8)	23.0 (10.1,32.8)	28.6 (12.5,40.0)	28.9 (13.8,42.8)

[1] This table shows the estimates of the share of the forecast error variance of the 6 variables in the system at the selected horizons that is due to the monetary policy shock. The size of the monetary policy shock is normalized to account for 10 percent of the variance of the one-day ahead forecast of 10-year Treasury yields.
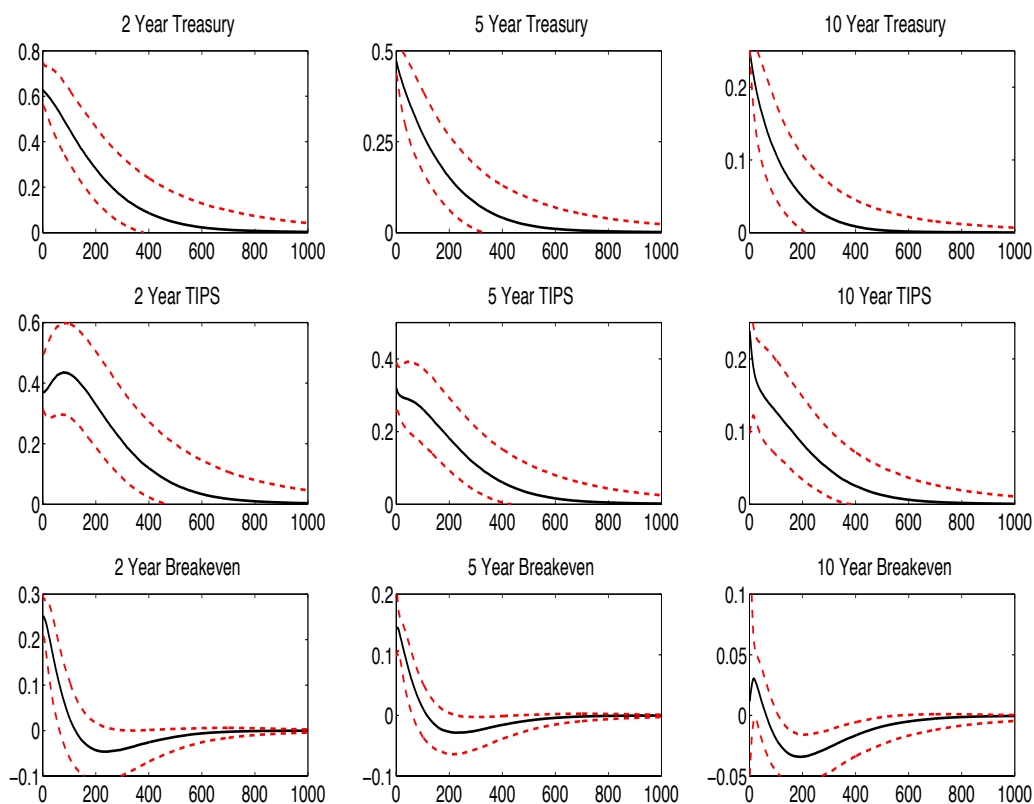[2] 68 percent bootstrap confidence intervals are shown in parentheses.

Table 3.8: Forecast Error Variance Decomposition –
at the ZLB (Dec. 2008 - Jan. 2013)

	1	50	100	200	500
2 year Treasury	7.4 (5.1,11.7)	10.1 (6.6,14.3)	11.2 (6.7,15.5)	12.0 (6.4,16.4)	12.6 (6.3,17.0)
5 year Treasury	11.3 (9.2,19.2)	12.9 (9.9,21.1)	13.6 (9.3,21.4)	14.0 (3.9,22.5)	14.2 (8.1,22.7)
10 year Treasury	10.0	12.2 (10.4,16.7)	13.1 (9.5,18.7)	13.7 (5.0,19.4)	14.0 (8.2,19.9)
2 year TIPS	9.0 (5.3,24.1)	9.5 (3.8,26.9)	9.1 (3.1,28.4)	9.0 (1.4,28.4)	9.7 (3.0,28.4)
5 year TIPS	19.2 (14.0,36.3)	16.9 (10.0,29.1)	15.8 (7.9,26.2)	15.1 (2.4,25.2)	14.8 (6.4,25.4)
10 year TIPS	14.6 (10.4,19.3)	15.4 (10.3,24.9)	15.3 (8.8,24.1)	15.0 (3.1,23.2)	14.8 (7.2,27.6)

[1] This table shows the estimates of the share of the forecast error variance of the 6 variables in the system at the selected horizons that is due to the monetary policy shock. The size of the monetary policy shock is normalized to account for 10 percent of the variance of the one-day ahead forecast of 10-year Treasury yields.
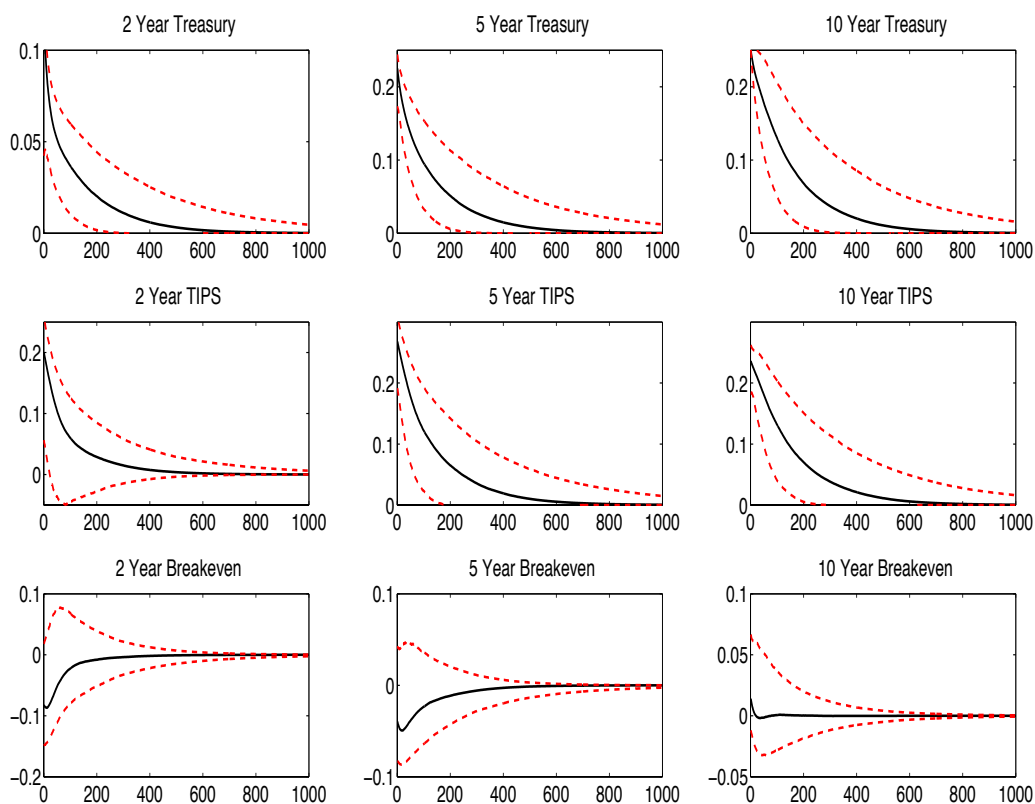[2] 68 percent bootstrap confidence intervals are shown in parentheses.

3.6.2   Figures

Figure 3.1: Impulse Responses in Normal Times.

NOTE: Impulse responses to a monetary policy shock from 0 to 500 days, estimated from data in normal times. The solid lines are bootstrap median, and the dashed lines are 68 percent bootstrap confidence intervals. The monetary policy shock is normalized to raise the 10-year Treasury yields by 25 basis points.

Figure 3.2: Impulse Responses at the ZLB.

NOTE: Impulse responses to a monetary policy shock from 0 to 500 days, estimated from data at the zero lower bound. The solid lines are bootstrap median, and the dashed lines are 68 percent bootstrap confidence intervals. The monetary policy shock is normalized to raise the 10-year Treasury yields by 25 basis points.

References

[1] Andolfatto, D.. (1996). "Business Cycles and Labor Market Search." *American Economic Review*, 86, 112-132.

[2] Andrea Ajello. "Financial Intermediation, Investment Dynamics and Business Cycle Fluctuations." *Working Paper Series, Northwestern University.*

[3] Andres, J., Lopes-Salido,D., and Nelson, E.. (2005). "Sticky-price Models and the Natural Rate Hypothesis." *Journal of Monetary Economics*, 52,1025-1053.

[4] Barlevy, Gadi. (2011). "Evaluating the Role of Labor Market Mismatch in Rising Unemployment." *Economic Perspectives*, 35(3), 82-96.

[5] Barnichon, Regis. (2010). "Building a Composie Help-Wanted Index." *Economics Letters*, 109(2010), 175-178.

[6] Barnichon, Regis, and Andrew Figura. (2011). "What Drives Matching Efficiency? A Tale of Composition and Dispersion." *Finance and Economics Discussion 2011-10*, Board of Governors of the Federal Reserve System.

[7] Barth, Marvin J, and Valerie A. Ramey. (2001). "The Cost Channel of Monetary Transmission." *NBER Macroeconomics Annual*, 16, 199-240.

[8] Beechey, Meredith J., and Jonathan H. Wright. (2009). "The High-Frequency Impact of News on Long-Term Yields and Forward Rates: Is It Real?" *Journal of Monetary Economics*, 56, 535-544.

[9] Blanchard, Olivier, and Jordi Gali. (2010). "Labor Markets and Monetary Policy: A New-Keynesian Model with Unemployment." *American Economic Journal: Macroeconomics*, 2(2), 1-30.

[10] Chen, Han, Vasco Curdia, and Andrea Ferrero. (2011). "The Macroeconomic Effects of Large Scale Assets Purchase Programs." *Working Paper Series, Federal Reserve Bank of New York.*

[11] Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. (1999). " Monetary Policy Shocks: What Have We Learned and to What End?" *Handbook of Macroeconomics*, 1A, 65-148.

[12] Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. (2005). "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*, 113(1), 1-45.

[13] Christiano, Lawrence J., Martin Eichenbaum, and Sergio Rebelo. 2012. "When is the Government Spending Multiplier Larger?" *working paper*.

[14] Christiano, Lawrence J., Mathias Trabandt, and Karl Walentin. (2011). "Introducing Financial Frictions and Unemployment into a Small Open Economy Model." *Journal of Economic, Dynamics and Control*, forthcoming.

[15] Cogan, John F., Tobias Cwik, John B. Taylor, and Volker Wieland. (2010). "New Keynesian versus Old Keynesian Government Spending Multipliers." *Journal of Economic Dynamics and Control*, 34(3), 281-295.

[16] Costain, James S., and Michael Reiter. (2008). "Business Cycles, Unemployment Insurance, and the Calibration of Matching Models." *Journal of Economic Dynamics and Control*, 32(4), 1120-1155.

[17] Curdia, Vasco, and Michael Woodford. (2009a). "Credit Frictions and Optimal Monetary Policy." *Mimeo, Federal Reserve Bank of New York and Columbia University.*

[18] Curdia, Vasco, and Michael Woodford. (2009b). "Conventional and Unconventional Monetary Policy." *Federal Reserve Bank of New York Staff Reports 404.*

[19] Cwik, Tobias, and Volker Wieland. (2011). "Keynesian Government Spending Multipliers and Spillovers in the Euro Area". *Economic Policy*, 26(67), 493-549.

[20] Daly, Mary C., Bart Hobijn, and Robert G. Valletta. (2012). "A Search and Matching Approach to Labor Markets: Did the Natural Rate of Unemployment Rise?" *Economic Perspectives*, 26(3), 3-26.

[21] D'Amico, Stefania, and Thomas B. King. (2010). "Flow and Stock Effects of Large-Scale Treasury Purchases." *Finance and Economics Discussion Series.*

[22] Del Negro, Marco, Gauti Eggertsson, Andrea Ferrero, and Nobuhiro Kiyotaki. (2011). "The Great Escape? A Quantitative Evaluation of the Fed's Liquidity Facilities." *Federal Reserve Bank of New York, Staff Reports*, No. 520.

[23] Den Hann, Wouter, Garey Ramey, and Joe Watson. (2000). "Job Destruction and Propagation of Shocks." *American Economic Review*, 90(3), 482-498.

[24] Diamond, Peter. (2011). "Unemployment, Vacancies, Wages." *American Economic Review*, 101(4), 1045-1072.

[25] Dickens, William T.. (2010). "Has the Recession Increased the NAIRU?" *working paper.*

[26] Doh, Taeyoung/ (2010). "The Efficacy of Large-Scale Asset Purchases at the Zero Lower Bound." *Federal Reserve Bank of Kansas City Economic Review.*

[27] Eggertsson, Gauti. (2008). "Great Expectations and the End of the Depression." *American Economic Review*, 98(4), 1476-1516.

[28] Eggertsson, Gauti. (2009). "What Fiscal Policy is Effective at Zero Interest Rate?" *New York Fed Staff Report.*

[29] Eggertsson, Gauti, and Michael Woodford. (2003). "The Zero Bound on Interest Rates and Optimal Monetary Policy." *Brookings Papers on Economic Activity*, 34(1), 139-235.

[30] Elsby, Michael, Bart Hobijn, Aysegul Sahin, and Robert G. Valletta. (2011). "The Labor Market in the Great Recession: an Update." *Brookings Papers on Economic Activity*, 103, 353-371.

[31] Faust, Jon, Eric T. Swanson, and Jonathan H. Wright. (2011). "Identifying VARs Based on High Frequency Futures Data." *Journal of Monetary Economics*, 51, 1107-1131.

[32] Fujita, Shigeru. (2011). "Effects of Extended Unemployment Insurance Benefits: Evidence from the Monthly CPS." *Federal Reserve Bank of Philadelphia Working Paper*, 10-35 R.

[33] Fujita, Shigeru, and Garey Ramey. (2009). "The Cyclicality of Separation and Job Finding Rates." *International Economic Review*, May 2009, 415-430.

[34] Fujita, Shigeru, and Garey Ramey. (2011). "Exogenous vs. Endogenous Separation." *American Economic Journal - Macroeconomics*, forthcoming.

[35] Furlanetto, Francesco, and Nicolas Groshenny. (2012). "Mismatch Shocks and Unemployment During the Great Recession." *CAMA working paper series*, 34/2012.

[36] Gagnon, Joseph, Matthew Raskin, Julie Remache, and Brian Sack. (2010). "Large-Scale Asset Purchases by the Federal Reserve: Did They Work?" *Federal Reserve Bank of New York Staff Report*, 441.

[37] Gali, Jordi, Frank Smets, and Rafael Wouters. (2010). "Unemployment in an Estimated New Keynesian Model." prepared for the *NBER Macroeconomics Annual 2011.*

[38] Gertler, Mark, and Nobuhiro Kiyotaki. (2010). "Financial Intermediation and Credit Policy in Business Cycle Analysis." *Handbook of Monetary Economics*, Vol. 3, Elsevier, Chapter 11, 547-599.

[39] Gertler, Mark, and Peter Karadi. "A Model of Unconventional Monetary Policy." (2011). *Journal of Monetary Economics*, 58(1), 17-34.

[40] Gertler, Mark, Luca Sala, Antonella Trigari. (2008). "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining." *Journal of Money, Credit and Banking*, 40(8), 1713-1764.

[41] Gertler, Mark, and Antonella Trigari. (2009). "Unemployment Fluctuations with Staggered Nash Wage Bargaining." *Journal of Political Economy*, 117(1), 38-86.

[42] Groshenny, Nicolas. (2012). "Monetary Policy, Inflation and Unemployment: In Defense of the Federal Reserves." *Macroeconomic Dynamics*, forthcoming.

[43] Gürkaynak, Refet, Brian Sack, and Eric Swanson. (2005a). "The Sensitivity of Long-Term Interest Rates to Economic News: Evidence and Implication for Macroeconomic Models." *American Economic Review*, 95, 426-436.

[44] Gürkaynak, Refet, Brian Sack, and Eric Swanson. (2005b). "Do Actions Speak Louder than Words? The Response of Asset Prices to Monetary Policy Actions and Statements." *International Journal of Central Banking*, 1, 55-93.

[45] Gürkaynak, Refet, Brian Sack, and Jonathan H. Wright. (2007). "The U.S. Treasury Yield Curve: 1961 to the Present." *Journal of Monetary Economics*, 54, 2291-2304.

[46] Gürkaynak, Refet, Brian Sack, and Jonathan H. Wright. (2010). "The TIPS Yield Curve and Inflation Compensation". *American Economic Journal*, 2, 70-92.

[47] Hall, Robert. (2005). "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review*, Vol. 95, No. 1, 50-65.

[48] Hall, Robert. (2005b). "Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market." *Review of Economics and Statistics*, 87(3), 397-407.

[49] Hamilton, James D.. (1994). "Time Series". *Princeton University Press.*

[50] Hamilton, James D., and Jing Cynthia Wu. (2012). "The Effectiveness of Alternative Monetary Policy Tools in a Zero Lower Bound Environment." *Journal of Money, Credit and Banking*, 44, 3-46.

[51] Kiyotaki, Nobuhiro, and Jonh Moore. (1998). "Credit Cycles." *Journal of Political Economy*, 1997, 105(2), 211-248.

[52] Kiyotaki, Nobuhiro, and Jonh Moore. (2008). "Liquidity, Business Cycles, and Monetary Policy." *Mimeo, Princeton University.*

[53] Krause, Michael, and Thomas Lubik. (2007). "The (Ir)relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions." *Journal of Monetary Economics*, 54, 706-727.

[54] Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. (2011). "The Effects of Quantitative Easing on Interest Rates: Channels and Implications for Policy." *Brookings Papers on Economic Activity*, 2, 215-265.

[55] Kohn, Don. (2009). "Monetary Policy in the Financial Crisis." *speech*.

[56] Kuester, Keith. (2010). "Real Price and Wage Rigidities with Matching Frictions." *Journal of Monetary Economics*, 57, 466-477.

[57] Kuttner, Kenneth N. (2001). "Monetary Policy Surprises and Interest Rates: Evidence from the Fed Funds Futures Market." *Journal of Monetary Economics*, 47 523-544.

[58] Liu, Zheng, Pengfei Wang, and Tao Zha. (2010). "Do Credit Constraints Amplify Macroeconomic Fluctuations?" *working paper*, Federal Reserve Bank of Atlanta.

[59] Mian, Atif, and Amir Sufi. (2012). "What Explains High Unemployment? The Aggregate Demand Channel." *NBER working paper*.

[60] Michaillat, Pascal. (2011). "Do Matching Frictions Explain Unemployment? Not in Bad Times." *American Economic Review*, forthcoming.

[61] Mortensen, Dale, and Christopher Pissarides. (1994). "Job Creation and Job Destruction in the Theory of Unemployment." *Review of Economic Studies*, 61, 397-415.

[62] Moyen, Stephane, and Nikolai Stahler. (2012). "Unemployment Insurance and the Business Cycle: Should Benefit Entitlement Duration React to the Cycle?" *Macroeconomic Dynamics*, forthcoming.

[63] Nakajima, Makoto. "A Quantitative Analysis of Unemployment Benefits Extensions." *Journal of Monetary Economics*, 59(7), 686-702.

[64] Neely, Christopher. (2010). "The Large-Scale Asset Purchases Had Large International Effects." *working paper*, Federal Reserve Bank of St. Louise.

[65] Pissarides, Christopher A. (2000). "Equilibrium Unemployment Theory." Second edition. *The MIT Press*.

[66] Politis, Dimitris N., and Joseph P. Romano. (1994). "The Stationary Bootstrap". *Journal of the American Statistical Association*, 89, 1303-1313.

[67] Ravenna, Federico, and Carl Walsh. (2008). "Vacancies, Unemployment and the Phillips Curve." *European Economic Review*, 52, 1494-1521.

[68] Ravenna, Federico, and Carl Walsh. (2011). "Welfare-based Optimal Monetary Policy with Unemployment and Sticky Prices: a Linear-quadratic Framework." *American Economic Journal: Macroeconomics*, 3, 130-162.

[69] Reifschneider, David, and John C. Williams. (2000). "Three Lessons for Monetary Policy in a Low Inflation Era." *Journal of Money, Credit and Banking*, 32. 936-996.

[70] Rigobon, Roberto. (2003). "Identification Through Heteroskedasticity." *Review of Economics and Statistics*, 85, 777-792.

[71] Rigobon, Roberto, and Brian Sack. (2003). "Measuring the Response of Monetary Policy to the Stock Market." *Quarterly Journal of Economics*, 118, 639-669.

[72] Rigobon, Roberto, and Brian Sack. (2004). "The Impact of Monetary Policy on Aset Prices." *Journal of Monetary Economics*, 51, 1553-1575.

[73] Rigobon, Roberto, and Brian Sack. (2005). "The Effects of War Risk on US Financial Market." *Journal of Banking and Finance*, 29, 1769-1789.

[74] Sahin, Aysegul, Joseph Song, Giorgio Topa, and Giovanni L. Violante. (2012). "Mismatch Unemployment." *working paper*, Federal Reserve Bank of New York.

[75] Rudebusch, Glenn D. (2009). "The Fed's Monetary Policy Response to the Current Crisis." *Federal Reserve Bank of San Francisco Economic Letter* .

[76] Shi, Shouyong. (2011). "Liquidity, Assets, and Business Cycles." *Mimeo*, University of Toronto.

[77] Shimer, Rober. (2005). "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review*, 95(1), 25-49.

[78] Smets, Frank, and Raf Wouters. (2003). "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area." *Journal of the European Economic Association*, 2003 1(5), 1123-1175.

[79] Smets, Frank, and Rafael Wouters. (2007). "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, Vol 97 No. 3, 586-606.

[80] Sveen, Tommy, and Lutz Weinke. (2007). "New Keynesian Perspectives on Labor Market Dynamics." *Journal of Monetary Economics*, 55, 921-930.

[81] Sveen, Tommy, and Lutz Weinke. (2009). "Inflation and Labor Market Dynamics Revisited." *Journal of Monetary Economics*, 56,1096-1100.

[82] Swanson, Eric T.. (2011). "Let's Twist Again: A High-Frequence Even-Study Analysis of Operation Twist and its Implications for QE2." *Brookings Papers on Economic Activity*, 1, 151-188.

[83] Swanson, Eric T., and John C. Williams. (2013). "Measuring the Effect of the Zero Lower Bound on Medium- and Longer-Term Interest Rates." *working paper, Federal Reserve Bank of San Francisco*.

[84] Tasci, Murat, and John Lindner. (2010). "Has the Beveridge Curve Shifted?" *Economic Trends, Federal Reserve Bank of Cleveland.*

[85] Taylor, John B., and John C. Williams. (2009). "A Black Swan in the Money Market." *American Economic Journal: Macroeconimcs*, 1(1), 58-83.

[86] Trigari, Antonella. (2006). "The Role of Search Frictions and Bargaining for Inflation Dynamics." *IGIER working paper*, No.304.

[87] Trigari, Antonella. (2009). "Equilibrium Unemployment, Job Flows, and Inflation Dynamics." *Journal of Money, Credit and Banking*, Vol. 41, Issue 1, 1-33.

[88] Valletta, Robert G., and Katherine Kuang. (2010). "Is Structural Unemployment on the Rise?" *FRBSF Economic Letter*, 2010-34.

[89] Veracierto, Marcelo. (2011). "Worker Flows and Matching Efficiency." *Economic Perspectives*, 35(4), 147-169.

[90] Walsh, Carl E.. (2003). "Labor Market Search and Monetary Policy." *Elements of Dynamic Macroeconomic Analysis*, 451-486.

[91] Walsh, Carl E.. (2005). "Labor market Search, Sticky Prices, and Interest Rate Policies." *Review of Economic Dynamics*, 8 (2005), 829-849.

[92] Whittaker, Julie. (2008). "Extending Unemployment Compensation Benefits During Recessions." *Congressional Research Service Report*, RL34340.

[93] Weidner, Justin, and John C. Williams. (2011). "What is the New Normal Unemployment Rate?" *FRBSF Economic Letter*. 2011-05.

[94] Woodford, Michael. (2011). "Simple Analytics of the Government Expenditure Multiplier". *American Economic Journal: Macroeconomics*, 3, 1-35.

[95] Wright, Jonathan H.. (2012). "What does Monetary Policy do to Long-Term Interest Rates at the Zero Lower Bound?" *working paper, Johns Hopkins University.*

[96] Yashiv, Eran. (2007). "Labor Search and Matching in Macroeconomics." *European Economic Review*, 51, 1859-1895.