LBNL-59525
LCC-0147
CBP Tech Note-319

Intrabeam Scattering in the NLC Main Damping Rings

A. Wolski
Lawrence Berkeley National Laboratory, Berkeley, California, 94720.

June 8th, 2004

Abstract

We use Bane's approximation to the Bjorken-Mtingwa theory of intrabeam scattering to calculate the emittance growth as a function of bunch charge in the KEK ATF. We find that our results are consistent with the experimental data. We then calculate the emittance growth in the NLC Main Damping Rings using the same formulae; we allow for some uncertainty in the ATF data by using two different values for the Coulomb log factor in the formulae for the emittance growth rates. We find that despite the IBS emittance growth, it should still be possible to achieve the specified transverse and longitudinal emittances in the NLC Main Damping Rings at the specified bunch charge.

This work was supported by the Director, Office of Science, High Energy Physics, U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

1 Introduction

In electron storage rings, intrabeam scattering (IBS) can lead to an increase in the six dimensional emittance of the bunch as the bunch charge is increased. Most existing storage rings, e.g. for third generation synchrotron light sources, operate in a regime where IBS is a very small effect compared with other effects, such as impedance, tending to increase the emittance. However, the main damping rings (MDRs) of the Next Linear Collider (NLC) need to operate in a regime where the beam energy, bunch charge, and transverse and longitudinal emittances make IBS a relatively large effect. If the emittance of the beam from the storage ring is larger than the design specification, then it could be difficult for the linear collider to achieve its design luminosity. The emittance growth from IBS therefore needs to be carefully quantified.

The theory commonly used to calculate IBS effects is that of Bjorken and Mtingwa [1]. The fact that IBS is a weak effect for most operating storage rings has made it difficult to validate this theory; however, the KEK Accelerator Test Facility (ATF) prototype damping ring has now achieved the vertical emittance where measurements of IBS emittance growth can be made with reasonable accuracy, and have been shown to be consistent with the Bjorken-Mtingwa theory [2].

In Section 2, we present the formulae that we shall use for our calculations. Calculation of the equilibrium beam emittances in a storage ring using a detailed lattice model and the full Bjorken-Mtingwa theory is difficult, because of the complicated integrals that need to be performed. We therefore use an approximation due to Bane [3], which is valid in the parameter regime of interest. In Section 2, we present the results of our calculations of the expected IBS emittance growth in the ATF. Our results are consistent with the experimental data, and in agreement with previous calculations based on the same theory. Agreement with previous calculations is not exact, since different approximations have been used in each case.

Finally, in Section 4, we present the results of our calculations of IBS emittance growth in the MDRs, using the latest lattice, from February 2003 [4]. We find that the emittance growth is tolerable, in that the emittances are within or very close to the specified limits at the specified bunch charge. However, there is little margin to allow for other effects increasing the emittance.

2 Intrabeam Scattering Formulae

Our calculations use the simplified formulae, derived by Bane [3] using certain approximations to the Bjorken-Mtingwa theory. We follow Bane's notation. The approximations needed are all valid in the parameter regimes of the rings we consider here, namely, the ATF and the NLC MDR. In particular, in each case, the beam is at a relativistic energy and the beam is cooler longitudinally than transversely. In the presence of IBS, the equilibrium horizontal emittance and relative energy spread are ε_x and σ_δ respectively, given by:

$$\varepsilon_x = \frac{T_x}{T_x - \tau_x}\varepsilon_{x0} \qquad \sigma_\delta = \frac{T_p}{T_p - \tau_p}\sigma_{\delta 0} \tag{1}$$

where T_x and T_p are the horizontal and longitudinal IBS growth rates, τ_x and τ_p are the horizontal and longitudinal radiation damping times, and ε_{x0} and $\sigma_{\delta 0}$ are the zero-current horizontal emittance and relative energy spread. The longitudinal IBS growth rate is given by:

$$\frac{1}{T_p} \approx \frac{r_0^2 cN(\log)}{16\gamma^3 \varepsilon_x^{3/4} \varepsilon_y^{3/4} \sigma_s \sigma_\delta^3} \left\langle \sigma_H g(a/b) (\beta_x \beta_y)^{-1/4} \right\rangle \tag{2}$$

where r_0 is the classical radius of the electron; c is the speed of light; N is the number of particles in the bunch; γ is the relativistic factor; ε_y is the vertical emittance; σ_s is the rms bunch length; β_x and β_y are the horizontal and vertical lattice beta-functions. The brackets $\langle\ \rangle$ indicate an average over the lattice. Other quantities are defined as follows:

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_\delta^2} + \frac{\mathrm{H}_x}{\varepsilon_x} + \frac{\mathrm{H}_y}{\varepsilon_y}$$

$$g(\alpha) \approx \alpha^{(0.021 - 0.044 \ln \alpha)} \qquad 0.01 < \alpha < 1$$

$$g(1/\alpha) = g(\alpha)$$

$$a/b = \sqrt{\frac{\beta_x \varepsilon_y}{\beta_y \varepsilon_x}}$$

The horizontal dispersion invariant is defined:

$$\mathrm{H}_x = \gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x' + \beta_x \eta_x'^2$$

where η_x is the horizontal dispersion; the prime denotes the gradient of the dispersion with respect to distance along the orbit; α_x, β_x and γ_x are the horizontal Twiss parameters. A similar definition holds for the vertical dispersion invariant. Note that in the lattices we are considering, the vertical dispersion in the design lattice is zero, but some vertical dispersion will arises in the real lattices, because of magnet misalignments. The quantity (log) is the "Coulomb log", the logarithm of the ratio of the maximum to the minimum impact parameter in the collision of two electrons in the bunch. This may be estimated as:

$$(\log) \approx f_{\mathrm{CL}} \ln\left(\frac{\sqrt{\beta_y \varepsilon_y}\, \gamma^2 \varepsilon_x}{r_0 \beta_x} \right) \tag{3}$$

For gaussian bunches, the factor $f_{\mathrm{CL}} = 1$. However, IBS populates the tails of the bunch distribution, and this leads to a reduction in the growth rates of the core emittances; this may be represented by a reduction in the factor f_{CL} to a value as low as 0.5 [5].

The horizontal IBS growth rate is given in terms of the longitudinal growth rate:

$$\frac{1}{T_x} \approx \frac{\sigma_\delta^2 \langle \mathrm{H}_x \rangle}{\varepsilon_x} \frac{1}{T_p} \tag{4}$$

In the vertical plane, the emittance growth depends on whether the vertical emittance is generated principally by vertical dispersion, or by vertical coupling. In the case that most of the vertical emittance comes from vertical dispersion, we can write equations analogous to equations (1) and (4). In the case that the vertical emittance comes from betatron coupling, then we write simply:

$$\varepsilon_y = \kappa \varepsilon_x \tag{5}$$

for a fixed value of the emittance ratio κ.

Since the growth rates depend on the emittances, it is necessary to solve the above equations iteratively, to find the equilibrium emittances for a given bunch charge.

Of the other effects tending to increase the emittances with bunch charge, it is important to include potential well distortion (PWD). In the ATF, this leads to a significant bunch lengthening with bunch charge, and is observed even when the vertical emittance is large and IBS emittance growth is negligible. Calculating the distortion in the longitudinal profile from PWD requires detailed knowledge of the broadband impedance of the ring. However, with the assumption that the impedance is dominated by an inductive component, the bunch lengthening from PWD takes the form [6]:

$$\left(\frac{\sigma_s}{\sigma_{s0}}\right)^3 - \frac{\sigma_s}{\sigma_{s0}} - f_{\mathrm{pw}} N = 0 \tag{6}$$

where the numerical factor f_{pw} is a function of the momentum compaction factor, the natural bunch length, the beam energy, the synchrotron tune and the impedance. The solution to equation (6) for the bunch length in terms of the bunch charge is:

$$\frac{\sigma_s}{\sigma_{s0}} = \frac{1}{3}\sqrt[3]{\frac{\xi}{2}} + \sqrt[3]{\frac{2}{\xi}}$$

where:

$$\xi = 27 f_{\mathrm{pw}} N + \sqrt{729\left(f_{\mathrm{pw}} N\right)^2 - 108}$$

For calculating IBS emittance growth in the ATF, we choose the value of f_{pw} to fit the data. Our results are not sensitive to the exact form of the bunch lengthening or to the value of f_{pw} (note that the bunch length depends on the cube root of this parameter), though it is important to get the overall growth correct.

3 Intrabeam Scattering in the KEK-ATF

A comparison between emittance growth predicted from the Bjorken-Mtingwa IBS theory and observed emittance growth with bunch charge in the ATF is reported in reference [2]. For comparison, we show the results of our own calculations, using the formulae given in Section 2. We attempted to reproduce the conditions of the lattice during the experiments; there is a slight discrepancy in the horizontal emittance, which is 1.2 nm at zero current in the lattice file we have available, and is 1.1 nm in reference [2]. The natural energy spread in our version of the lattice is in good agreement with reference [2], and we adjusted the RF voltage to give the correct natural bunch length. For our calculations, we used a Coulomb log given by (3), with the factor $f_{\mathrm{CL}} = 0.5$ to allow for the effect of the tails in the distribution. We used a potential well factor f_{pw} of 7×10^{-11}. We assume that the vertical emittance is dominated by betatron coupling, and the growth in vertical emittance is therefore simply given by (5).

The variation with bunch charge of the transverse emittances, energy spread and bunch length, as calculated using the formulae in Section 2, are shown in Figure 1. There is good agreement between the results of our calculations using Bane's approximation to the Bjorken-Mtingwa theory, and the experimental data and calculations reported in reference [2]. The longitudinal emittance growth is somewhat larger in our calculations, and the vertical emittance growth a little smaller.

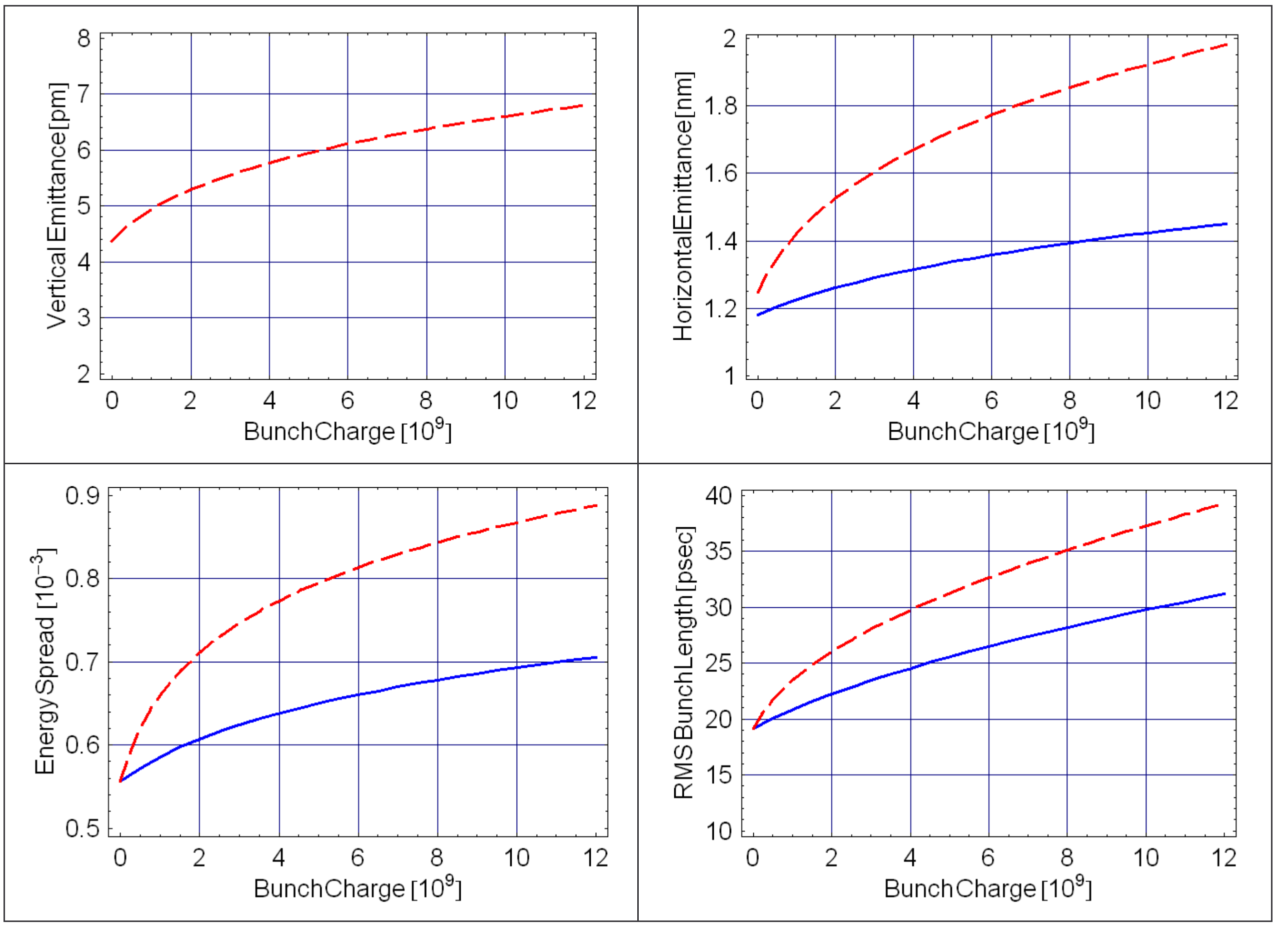

Figure 1

Emittance growth with bunch charge in the ATF calculated using Bane's approximation for the Bjorken-Mtingwa theory. The solid line shows the emittance growth at 6% transverse emittance ratio; the broken line shows the emittance growth at 0.4% emittance ratio.

4 Intrabeam Scattering in the NLC Main Damping Rings

The MDR lattice we have used for our IBS calculations is described in reference [4]. Table 1 shows a comparison of some of the relevant parameters for IBS emittance growth in the ATF and the MDR. The higher energy of the MDR leads to a significant reduction in the IBS emittance growth – note that in equation (2), the growth rate has a (direct) $1/E^3$ dependence on the energy. Similarly, the shorter radiation damping times will reduce the IBS emittance growth. Only the natural emittance, which is a factor of two smaller in the MDR, tends to increase the IBS emittance growth in the NLC MDR compared to the ATF.

Table 1

Parameters affecting IBS emittance growth in the ATF and the MDR.

	KEK ATF	NLC MDR
Beam energy, E	1.28 GeV	1.98 GeV
Bunch charge, N	$< 1.2\times10^{10}$	0.75×10^{10}
Natural emittance, ε_0	1.2 nm	0.61 nm
Vertical emittance, ε_y	4.4 pm	5.0 pm
Natural rms bunch length, σ_{s0}	19 ps	16 ps
Natural rms energy spread, $\sigma_{\delta 0}$	0.56×10^{-3}	1.0×10^{-3}
Mean dispersion invariant, $\langle \mathcal{H}_x \rangle$	3.2 mm	3.2 mm
Horizontal damping time, τ_x	17 ms	3.6 ms
Vertical damping time, τ_y	27 ms	4.1 ms
Longitudinal damping time, τ_p	19 ms	2.2 ms

An impedance model for the vacuum chamber has been constructed, so the bunch lengthening resulting from potential well distortion can in principle be calculated. However, recent studies [7] suggest that PWD bunch lengthening will be very small at the specified bunch charge, so we have neglected this effect here.

The specified rms bunch length in the MDR is 16 ps, or 5 mm. This can be achieved with an RF voltage of 2.5 MV, which can be provided by the 5 RF cavities in the present design. Depending on collective effects in the damping rings, it may be desirable to reduce the RF voltage to lengthen the bunch and reduce the peak current. An upper limit on the bunch length is likely to be set by the bunch compressors, at around 18 ps, or 5.5 mm, corresponding to an RF voltage of 2.0 MV. We have calculated the IBS emittance growth for three different RF voltages: 2.0 MV, 2.25 MV and 2.5 MV.

There is some uncertainty as to the appropriate value to use for the Coulomb log in equation (2). For the ATF, it was found that a good fit to the data could be obtained by using $f_{\mathrm{CL}} = 0.5$ in equation (3). This was justified by considering the different contribution to the core emittance growth from the particles in the core and the tails of the distribution. It is not immediately clear that a similar factor will be appropriate for the MDR. We have therefore calculated the IBS emittance growth for $f_{\mathrm{CL}} = 1$ and $f_{\mathrm{CL}} = 0.5$. We expect that the real emittance growth in the MDR will lie somewhere between the two cases.

We again assume that the vertical emittance is dominated by betatron coupling, so that the vertical emittance is given by (5). This will be the case if the rms vertical dispersion is corrected to around 1.5 mm or better.

The emittance growth with bunch charge for the MDR with $f_{\mathrm{CL}} = 0.5$ is shown in Figure 2, and with $f_{\mathrm{CL}} = 1$ in Figure 3. The relative emittance growth is much less than in the ATF, as expected from the higher energy and much shorter damping times. Even in the most pessimistic case, the horizontal emittance stays below the specified upper limit. With the nominal coupling used in the calculations, the vertical emittance may be a little above the specified equilibrium value with the fastest IBS growth rates. This is unlikely to impact machine performance, since the increase is small compared to emittance dilutions that occur between damping ring extraction and the interaction point. IBS may increase the longitudinal emittance above the specified values, but again the effect is small and will likely be accommodated by the bunch compressors.

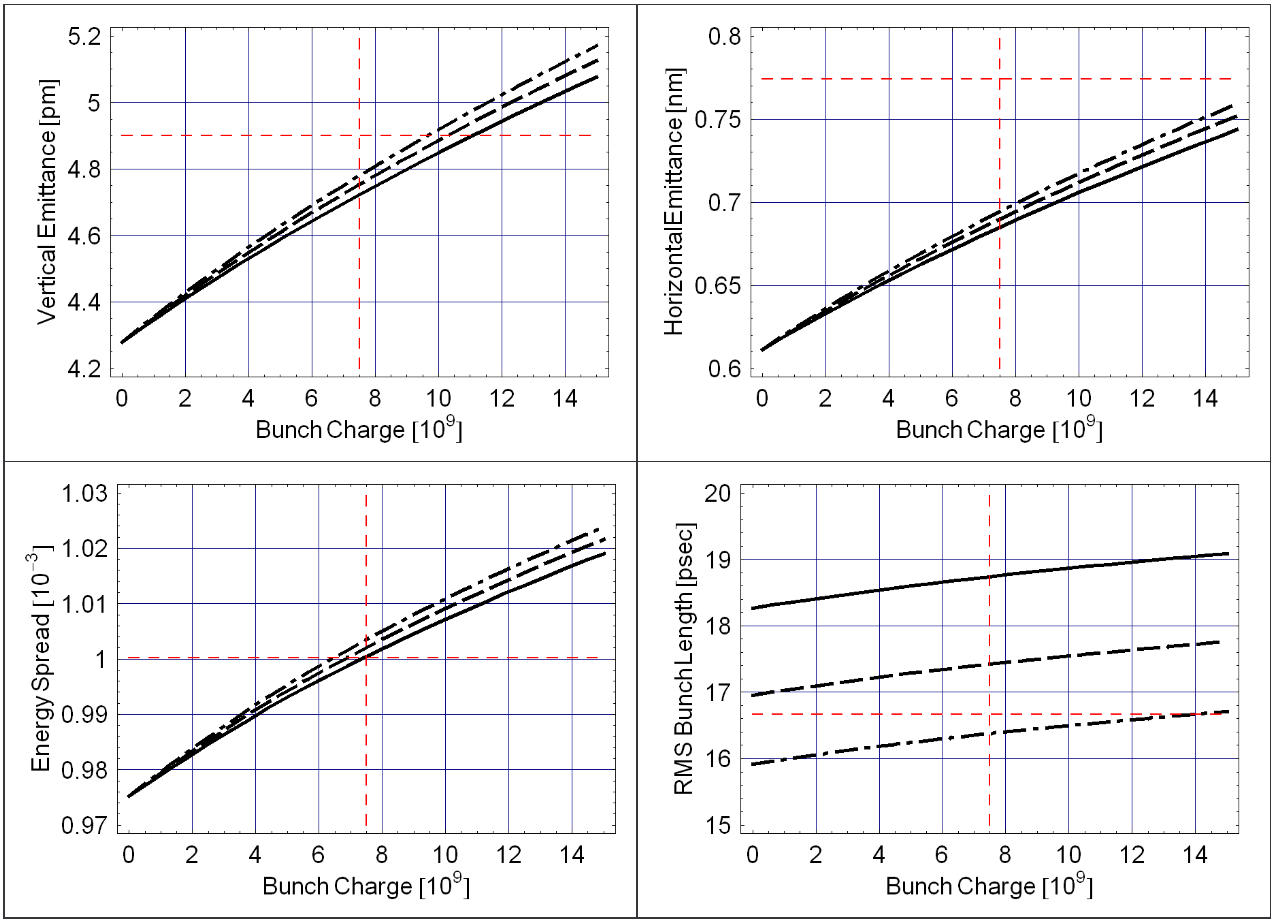

Figure 2

IBS emittance growth in the NLC Main Damping Rings, with growth rate factor $f_{CL} = 0.5$. The different curves are for different RF voltages: solid line, 2.00 MV; dashed line, 2.25 MV; dot-dashed line, 2.50 MV. The broken vertical line shows the bunch charge specified in the machine design, and the broken horizontal lines the specified upper limits on the various parameters.

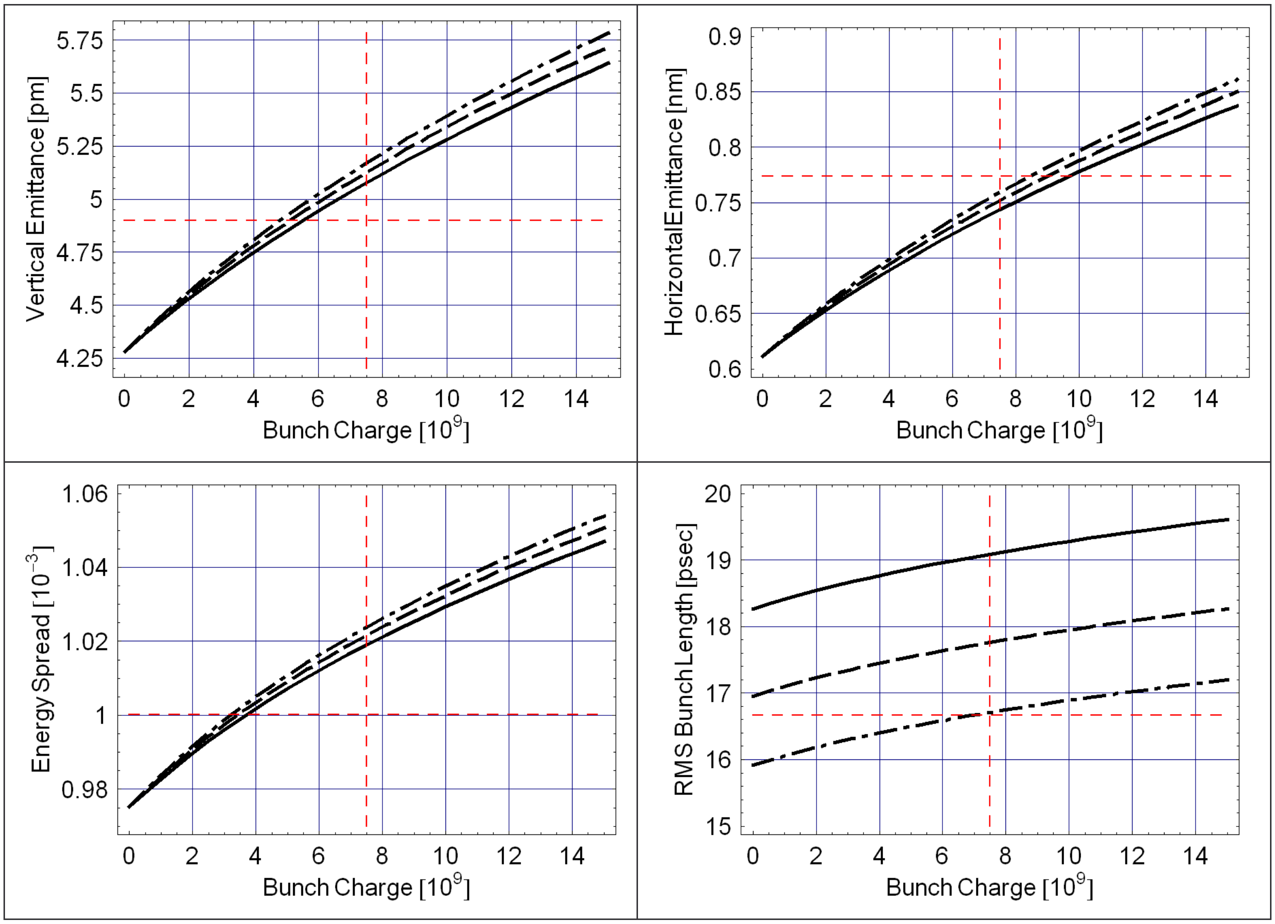

Figure 3

IBS emittance growth in the NLC Main Damping Rings, with growth rate factor $f_{CL} = 1$. The different curves are for different RF voltages: solid line, 2.00 MV; dashed line, 2.25 MV; dot-dashed line, 2.50 MV. The broken vertical line shows the bunch charge specified in the machine design, and the broken horizontal lines the specified upper limits on the various parameters.

5 Conclusions

We have calculated IBS emittance growth in the ATF using Bane's approximation to the Bjorken-Mtingwa theory. Using a reduced value for the Coulomb log, we find that the results of the calculations are consistent with the experimental data.

The IBS emittance growth in the NLC Main Damping Rings will be smaller than in the ATF, because of the higher energy and shorter radiation damping times. Applying the same calculations to the NLC Main Damping Rings that we used for the ATF (with two different values used for the Coulomb log), we find that the horizontal emittance stays below the specified upper limit even with a pessimistic assumption for the IBS growth rates. The specified vertical emittance can also be achieved in the presence of IBS. The bunch length and energy spread may be slightly above the values given by the present machine specification; however, the increases can likely be accommodated by the bunch compressors, and there does not appear to be a strong case for addressing this through the damping ring design at the present time.

References

[1] J.D. Bjorken and S.K. Mtingwa, “Intrabeam Scattering”, Particle Accelerators **13** p.115, 1983.

[2] K. Kubo et al, “Achievement of Ultra-Low Emittance Beam in the KEK ATF Damping Ring”, Phys. Rev. Lett. **92**, 054802 (2004).

[3] K.L.F. Bane, “A Simplified Model of Intrabeam Scattering”, Proceedings of EPAC Paris, 2002.

[4] M.D. Woodley and A. Wolski, “The NLC Main Damping Ring Lattice, February 2003”, LCC-0113, February 2003.

[5] K.L.F. Bane, “Intrabeam Scattering Analysis of Measurements at KEK’s ATF Damping Ring”, SLAC-PUB-9227, June 2002.

[6] H. Wiedemann, “Particle Accelerator Physics”, Volume II, page 207. Springer, 1995.

[7] M. Venturini, “Longitudinal Single-Bunch Instabilities in the NLC Main Damping Rings”, LBNL-55103, 2004.