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Identifying Better Effective Higgsless Theories via W_L W_L Scattering

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The three site Higgsless model has been offered as a benchmark for studying the collider phenomenology of Higgsless models. In this talk, we present how well the three site Higgsless model performs as a general representative of Higgsless models, in describing $W_L W_L$ scattering, and which modifications can make it more representative. We employ general sum rules relating the masses and couplings of the Kaluza-Klein (KK) modes of the gauge fields in continuum and deconstructed Higgsless models as a way to compare the different theories. After comparing the three site Higgsless model to flat and warped continuum Higgsless models, we analyze an extensions of the three site Higgsless model, namely, the Hidden Local Symmetry (HLS) Higgsless model. We demonstrate that $W_L W_L$ scattering in the HLS Higgsless model can very closely approximate scattering in the continuum models, provided that the parameter 'a' is chosen to mimic ρ-meson dominance of $\pi\pi$ scattering in QCD.

Keywords: Higgsless model; $W_L W_L$ scattering; Sum rule; Hidden Local Symmetry.

1. Introduction

Higgsless model[1] is an attractive alternative to the Standard Model (SM) for describing the Electroweak symmetry breaking, in which the symmetry is broken by the boundary conditions of the five dimensional gauge theory. It turned out that dimensional deconstruction[2] is quite useful to understand the important nature of the model, such as the delay of perturbative unitarity violation, boundary conditions, etc. (See Refs.[3,4]) It was also extensively used to study the constraints from the electroweak precision measurements.[5]

The three site Higgsless model[6] was proposed as an extremely deconstructed version of five dimensional Higgsless models, in which only one copy of weak gauge boson (W', Z') is introduced as a new resonance. Such a model contains sufficient

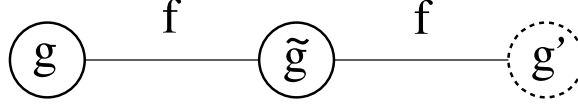*[*]Speaker, Email: kurachi@tuhep.phys.tohoku.ac.jp

Fig. 1.   Gauge sector of the three site Higgsless model. The solid circles represent $SU(2)$ gauge groups, with coupling strengths g_0 and g_1, and the dashed circle is a $U(1)$ gauge group with coupling g_2.

complexity to incorporate interesting physics issues related to fermion masses and electroweak observables, yet remains simple enough that it could be encoded in a Matrix Element Generator program for use with Monte Carlo simulations. Such program was already done by several groups.[7] In this talk, we present how well the three site Higgsless model performs as a general representative of Higgsless models, in describing $W_L W_L$ scattering, and which modifications can make it more representative. After briefly reviewing the three site Higgsless model, we compare the three site Higgsless model to flat and warped continuum Higgsless models. Then, we analyze a Hidden Local Symmetry (HLS) generalization of the three site Higgsless model.[a]

2. The three site Higgsless model

In this section, we briefly review the three site Higgsless model.[6] The gauge sector of the three site Higgsless model is illustrated in Fig. 1 using "Moose notation".[9] The model incorporates an $SU(2) \times SU(2) \times U(1)$ gauge group (with couplings g_0, g_1 and g_2, respectively), and two nonlinear $(SU(2) \times SU(2))/SU(2)$ sigma models in which the global symmetry groups in adjacent sigma models are identified with the corresponding factors of the gauge group. The symmetry breaking between the middle $SU(2)$ and the $U(1)$ follows an $SU(2)_L \times SU(2)_R/SU(2)_V$ symmetry breaking pattern with the $U(1)$ embedded as the T_3-generator of $SU(2)_R$. This extended electroweak gauge sector is in the same class as models of extended electroweak gauge symmetries,[10] which are considered as an application of the hidden local symmetry[11] to the electroweak sector.[b] The decay constants, f_1 and f_2, of two nonlinear sigma models can be different in general, however, we take $f_1 = f_2 = f (= \sqrt{2}v)$ for simplicity. Also, we work in the limit $x \equiv g_0/g_1 \ll 1$, $y \equiv g_2/g_1 \ll 1$, in which case we expect a massless photon, light W and Z bosons, and a heavy set of bosons W' and Z'. Numerically, then, $g_{0,2}$ are approximately equal to the standard model $SU(2)_W$ and $U(1)_Y$ couplings, and we therefore denote $g_0 \equiv g$ and $g_2 \equiv g'$, and define an angle θ such that $\frac{g'}{g} = \frac{\sin\theta}{\cos\theta} \equiv \frac{s}{c} (\equiv t)$. In addition, we denote $g_1 \equiv \tilde{g}$.

[a] This talk is based on the work done in Ref.[8]
[b] The new physics discussed in Ref.[6] is related to the fermion sector.

Table 1.   Leading expressions of $Z'WW$ coupling and $ZW'W$ coupling in each models.

	Three Site	5D Flat	5D Warped
$g_{Z'WW}$	$-\frac{1}{2}\frac{e}{s}\left(\frac{M_W}{M_{W'}}\right)$	$-\frac{4\sqrt{2}}{\pi^2}\frac{e}{s}\left(\frac{M_W}{M_{W'}}\right)$	$-0.36\left(\frac{M_W}{M_{W'}}\right)$
$g_{ZW'W}$	$-\frac{1}{2}\frac{e}{sc}\left(\frac{M_W}{M_{W'}}\right)$	$-\frac{4\sqrt{2}}{\pi^2}\frac{e}{sc}\left(\frac{M_W}{M_{W'}}\right)$	$-0.36\frac{1}{c}\left(\frac{M_W}{M_{W'}}\right)$

3. Comparison between continuum and the three site Higgsless models

The three site Higgsless model can be viewed as an extremely deconstructed version of 5-dimensional $SU(2)\times SU(2)$ Higgsless model in which $SU(2)\times SU(2)$ symmetry is broken down to its diagonal $SU(2)$ by the boundary condition (BC) at one end of the extra dimension, while one of $SU(2)$ is broken down to its $U(1)$ subgroup by the BC at the other end. Thus, it is tempting to investigate how well the three site Higgsless model performs as a low energy effective theory of continuum Higgsless models. For this purpose, we consider $SU(2)\times SU(2)$ Higgsless model in the flat and the warped extra dimension as example models to compare.

The electroweak gauge sector of $SU(2)\times SU(2)$ Higgsless models in both the flat and the warped extra dimension can be characterized (with certain simplifications) by four free parameters. In flat case, those are R (the size of the extra dimension), g_5 (the bulk gauge coupling), g_0 and g_Y (couplings for brane localized kinetic terms of $SU(2)\times U(1)$ gauge bosons). In warped case, those are R' (the seize of extra dimension), b (warp factor), g_5 and g_Y. (See Ref.[12] for detailed descriptions of the models.) Since the gauge sector of the three site Higgsless model is also characterized by four free parameter (see, Fig. 1), these three models can be compared by choosing four parameters so that they reproduce same values of four physical quantities. Three of four parameters should be chosen in a way that electroweak observables (e, M_Z and M_W, for example) are correctly reproduced. Then, there is still one free parameter to fix. Here, we use that parameter to fix the scale of KK mode (the mass of W' boson, for example). Now, there are no free parameter left in each model, and any physical quantities other than those four quantities can be calculated as predictions in each models, which should be compared among three models to see how much the three site Higgsless model approximates continuum Higgsless models.

In the present analysis, we focus on the values of triple gauge boson couplings each model predicts. As for triple gauge boson couplings which involve only SM gauge bosons, it can be shown that those are of the same form as in the SM at the leading order in the expansion of $M_W^2/M_{W'}^2$, and we require that the deviation from the SM value, which appears as next to leading order, should be within experimental bound (which put a lower bound on $M_{W'}$).

In Table 1, we listed the leading expressions of $Z'WW$ coupling and $ZW'W$ coupling in each models. Making the numerical approximations $\frac{4\sqrt{2}}{\pi^2}\frac{e}{s}\simeq 0.36$ and $\frac{4\sqrt{2}}{\pi^2}\frac{e}{sc}\simeq 0.41$, we find, $\frac{g_{Z'WW}|_{\text{warped-5d}}}{g_{Z'WW}|_{\text{flat-5d}}}\simeq\frac{g_{ZW'W}|_{\text{warped-5d}}}{g_{ZW'W}|_{\text{flat-5d}}}\simeq 1$. In other words, the values of $g_{Z'WW}$ and $g_{ZW'W}$ in these continuum models are essentially independent

4

of the geometry of the extra dimension to leading order. Then we can compare the couplings in continuum models to those in the three site Higgsless model, assuming a common value for $M_W/M_{W'}$: $\frac{g_{Z'WW}|_{\text{three-site}}}{g_{Z'WW}|_{\text{flat-5d}}} \simeq \frac{g_{ZW'W}|_{\text{three-site}}}{g_{ZW'W}|_{\text{flat-5d}}} \simeq \frac{\pi^2}{8\sqrt{2}} \simeq 0.87$. The values of $g_{Z'WW}$ and $g_{ZW'W}$ in the three site Higgsless model are about 13% smaller than those values in 5-dimensional $SU(2) \times SU(2)$ Higgsless models. Then, the question which naturally arises is — "Why do $g_{Z'WW}$ and $g_{ZW'W}$ take similar values in different models?" There are two keywords to be addressed to answer to this question: sum rules and the lowest KK mode dominance.

In any continuum five-dimensional gauge theory, the sum rules that guarantee the absence, respectively, of $\mathcal{O}(E^4)$ and $\mathcal{O}(E^2)$ growth in the amplitude for $W_L^+ W_L^- \to W_L^+ W_L^-$ elastic scattering have the following form,[1,13]

$$\sum_{i=1}^{\infty} g_{Z_i WW}^2 = g_{WWWW} - g_{ZWW}^2 - g_{\gamma WW}^2 , \tag{1}$$

$$3\sum_{i=1}^{\infty} g_{Z_i WW}^2 M_{Z_i}^2 = 4g_{WWWW} M_W^2 - 3g_{ZWW}^2 M_Z^2 , \tag{2}$$

where Z_i represents the i-th KK mode of the neutral gauge boson. (Z_1 is identified as Z'.) We focuses on the degree to which the first KK mode saturates the sum on the LHS of the identities (1) and (2). Suppose that we form the ratio of the $n = 1$ term in the sum on the LHS to the full combination of terms on the RHS, evaluated to leading order in $(M_W/M_{W'})^2$. The ratios derived from Eqs. (1) and (2) are:

$$\frac{g_{Z'WW}^2}{g_{WWWW} - g_{ZWW}^2 - g_{\gamma WW}^2} , \tag{3}$$

$$\frac{3g_{Z'WW}^2 M_{Z'}^2}{4g_{WWWW} M_W^2 - 3g_{ZWW}^2 M_Z^2} . \tag{4}$$

If the $n = 1$ KK mode saturates the identity, then the related ratio will be 1.0; ratio values less than 1.0 reflect contributions from higher KK modes. We see from Table 2 that each of these ratios is nearly 1.0 in both the $SU(2) \times SU(2)$ flat and warped Higgsless models, confirming that the first KK mode nearly saturates the sum rules in these continuum models. The similar behavior of the two extra dimensional models is consistent with our finding that the $g_{Z'WW}$ coupling is relatively independent of geometry.

Because the first KK mode nearly saturates the identities (1) and (2) in these continuum models, the ratios (3) and (4) should be useful for drawing comparisons with the three site Higgsless model, which only possesses a single KK gauge mode. As shown in the 3rd column of Table 2, the first ratio has the value one in the three site Higgsless model, meaning that the identity (1) is still satisfied. The ratio related to identity (2), however, has the value 3/4 for the three site Higgsless model, meaning that the second identity is not satisfied; the longitudinal gauge boson scattering amplitude continues to grow as E^2 due to the underlying non-renormalizable

Table 2.   Ratios relevant to evaluating the degree of cancellation of growth in the $W_L W_L$ scattering amplitude from the lowest lying KK resonance at order E^4 (top row, from Eq. (3)), and at order E^2 (second row, from Eq. (4)). A value close to one indicates a high degree of cancellation from the lowest lying resonance. Shown in successive columns for the $SU(2) \times SU(2)$ flat and warped continuum models discussed in the text, and the three site Higgsless model.

	5d 2×2 Flat	5d 2×2 Warped	Three-site
$\dfrac{g_{Z'WW}^2}{g_{WWWW} - g_{ZWW}^2 - g_{\gamma WW}^2}$	$\frac{960}{\pi^6} \simeq 0.999$	0.992	1
$\dfrac{3g_{Z'WW}^2 M_{Z'}^2}{4g_{WWWW} M_W^2 - 3g_{ZWW}^2 M_Z^2}$	$\frac{96}{\pi^4} \simeq 0.986$	0.986	3/4

interactions in the three site Higgsless model. Since the value of the denominator in Eq. (4) has not changed appreciably, this indicates a difference between the values of the $g_{Z'WW}$ couplings in the continuum and three site Higgsless models. This is the reason why $g_{Z'WW}$ in the three site Higgsless model is 13% (or 25% in $g_{Z'WW}^2$) smaller than the value of $g_{Z'WW}$ in continuum models as we have shown in the previous section.

4. Hidden Local Symmetry generalization of the three site Higgsless models

In this section, we consider an Hidden Local Symmetry (HLS) generalization of the three site Higgsless model, in which an extra parameter a is introduced and the Lagrangian takes the following form:[11]

$$
\begin{aligned}
\mathcal{L}^{\text{HLS}} &= -\frac{v^2}{4}\text{Tr}\left[(D^\mu \Sigma_1^\dagger)\Sigma_1 - (D^\mu \Sigma_2)\Sigma_2^\dagger\right]^2 - a\frac{v^2}{4}\text{Tr}\left[(D^\mu \Sigma_1^\dagger)\Sigma_1 + (D^\mu \Sigma_2)\Sigma_2^\dagger\right]^2, \\
&= \frac{v^2}{4}(1+a)\text{Tr}\left[(D_\mu \Sigma_1)^\dagger (D^\mu \Sigma_1) + (D_\mu \Sigma_2)^\dagger (D^\mu \Sigma_2)\right] \\
&\quad + \frac{v^2}{2}(1-a)\text{Tr}\left[(D_\mu \Sigma_1)^\dagger \Sigma_1 (D_\mu \Sigma_2)\Sigma_2^\dagger\right], \\
&= \frac{av^2}{2}\text{Tr}\left[(D_\mu \Sigma_1)^\dagger (D^\mu \Sigma_1) + (D_\mu \Sigma_2)^\dagger (D^\mu \Sigma_2)\right] \\
&\quad + \frac{v^2}{4}(1-a)\,\text{Tr}\left[(D_\mu(\Sigma_1 \Sigma_2))^\dagger D^\mu(\Sigma_1 \Sigma_2)\right].
\end{aligned}
\tag{5}
$$

Note that taking $a = 1$ reproduce the Lagrangian of the three site Higgsless model. Fermion sector of the model is also discussed in detail in Ref.,[8] in which we showed that the model can accommodate the ideal fermion delocalization which is needed for the consistency with the precision EW experiments.

It is straightforward to calculate triple gauge boson couplings of this model, from which we can evaluate the values in Eqs. (3) and (4):

$$
\frac{g_{Z'WW}^2}{g_{WWWW} - g_{ZWW}^2 - g_{\gamma WW}^2} = 1, \qquad \frac{3g_{Z'WW}^2 M_{Z'}^2}{4g_{WWWW} M_W^2 - 3g_{ZWW}^2 M_Z^2} = \frac{3}{4}\,a\,.
\tag{6}
$$

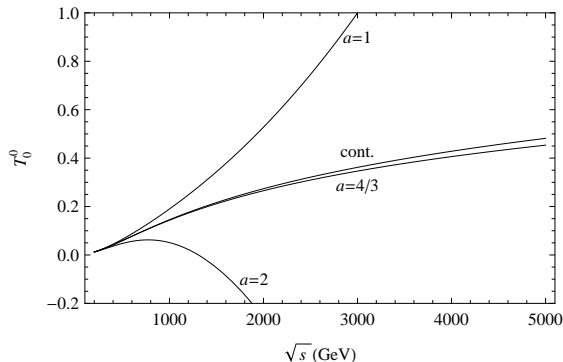By comparing this result with the ones shown in Table 2, we see that the HLS

6

Fig. 2.   Behavior of the partial wave amplitude T_0^0 for NG boson scattering in the triangular moose model with various a. The values $v = 250$ GeV, $M_1 = 500$ GeV are assumed. The curve labeled "cont." shows T_0^0 in the continuum flat $SU(2) \times SU(2)$ model for $M_1 = 500$ GeV.

Higgsless model can very closely approximate scattering in the continuum models if we take $a = \frac{4}{3}$.

It is interesting to note that $a = \frac{4}{3}$ is the choice in which the four-point Nambu-Goldstone (NG) boson coupling, $g_{\pi\pi\pi\pi} = 1 - \frac{3}{4}a$, vanishes in this model.[14] Considering that, in the language of dimensional deconstruction,[2] NG bosons are identified as a fifth-component of the gauge boson field (A_5) in a discretized five-dimensional gauge theory, and also considering the fact that there is no four-point A_5 coupling in five dimensional gauge theories, it is natural that the parameter choice in which the four-point NG boson coupling vanishes well approximates the continuum models.

Fig. 2 shows the partial wave amplitude T_0^0 for NG boson scattering in the global (which means we set $g = g' = 0$ for simplicity) continuum flat $SU(2) \times SU(2)$ model (with the mass of the first KK mode, M_1, taken to be $M_1 = 500$ GeV) compared with T_0^0 in the HLS Higgsless model for several values of the parameter a. The result in the globalthree site Higgsless model is shown by the curve labeled $a = 1$; the value $a = 2$ is motivated by the phenomenological KSRF relation.[15] This plot ties our results together quite neatly: while the curves with three different values of a all give a reasonable description of T_0^0 at very low energies, the best approximation to the continuum behavior of T_0^0 over a wide range of energies is given by the HLS Higgsless model curve with $a = 4/3$. At low energies, the fact that the three-site and the HLS Higgsless models both prevent E^4 growth of the amplitude suffices; but at higher energies, the fact that the HLS Higgsless model with $a = 4/3$ has $g_{\pi\pi\pi\pi} = 0$ and enables it to cut off the E^2 growth of the amplitude as well, as consistent with the behavior in the continuum model.

5. Conclusions

We have considered how well the three site Higgsless model performs as a general representative of Higgsless models, and have studied HLS generalization have the

potential to improve upon its performance. Our comparisons have employed sum rules relating the masses and couplings of the gauge field KK modes. We find that the tendency of the sum rules to be saturated by contributions from the lowest-lying KK resonances suggests a way to quantify the extent to which a highly-deconstructed theory like the three site Higgsless model can accurately describe the low-energy physics. We demonstrated that $W_L W_L$ scattering in the HLS Higgsless model can very closely approximate scattering in the continuum models, provided that the HLS parameter a is chosen appropriately. This observation confirms that the collider phenomenology studied, such as in Ref.[7] , are applicable not only to the three site Higgsless model, but also to extra-dimensional Higgsless models.

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