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Phase transition in an O(N) gauge model in two dimensions*

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We study the phase transition properties of the nonlinear O(N) σ model in two dimensions when O(N) gauge interactions are included. With nonzero gauge coupling, this theory exhibits a first-order phase transition in the large-N limit. The broken-symmetry phase is stabilized by the Higgs mechanism and Goldstone bosons do not appear.

Recently, Bardeen and Pearson[1] have proposed a transverse-lattice formulation of quantum chromodynamics.[2] In this theory, color confinement is obtained when the vacuum is invariant under the transverse gauge symmetry. A quark-gluon phase, where color is not confined, can result if this symmetry is spontaneously broken. In this paper we will study some aspects of the mechanisms which are responsible for generating such a phase transition.

The independent gauge degrees of freedom or the transverse-lattice theory are associated with links on the transverse lattice. The longitudinal dynamics of a single transverse link consists of a nonlinear SU(3)×SU(3) σ model with SU(3)×SU(3) gauge interactions. For a single transverse link the longitudinal dynamics is two-dimensional. As the transverse gauge symmetry is a local symmetry on the lattice, the phase-transition properties of a single transverse link are relevant to the phase-transition properties of the full gauge theory.

We shall study a somewhat simpler though analogous version of the single-link problem. The model consists of a two-dimensional nonlinear O(N) σ model with O(N) gauge interactions. This theory has the advantage that the model may be systematically studied in the large-N limit. The nonlinear σ model in 2 and $2+\epsilon$ dimensions has recently been extensively studied by Brezin and Zinn-Justin[3] and by Bardeen, Lee, and Shrock.[4] In this theory, spontaneous symmetry breaking can occur in $2+\epsilon$ dimensions but only the symmetric phase can exist in 2 dimensions. The basic result of our paper is that the broken-symmetry phase can be stabilized when gauge interactions are introduced. The existence of nontrivial phase-transition properties in two dimensions makes this theory interesting in its own right.

The nonlinear O(N) σ model with gauge interactions is described by the Lagrangian

$$\mathcal{L}=\tfrac{1}{2}(D_\mu\vec{\phi})^2-\tfrac{1}{4}(G_{\mu\nu ij})^2, \tag{1}$$

where $\vec{\phi}$ is an N-component scalar field with the constraint $\vec{\phi}^2=f_0{}^2$. The gauge fields may be written as an antisymmetric tensor ($A_{\mu ij}$, $i,j=1,\ldots,N$). The covariant derivative is defined by

$$(D_\mu\phi)_i=\partial_\mu\phi_i+gA_{\mu ij}\phi_j \tag{2}$$

and the Yang-Mills field strength is given by

$$G_{\mu\nu ij}=\partial_\mu A_{\nu ij}-\partial_\nu A_{\mu ij}+gA_{\mu ik}A_{\nu kj}+gA_{\mu jk}A_{\nu ik}. \tag{3}$$

In two dimensions this theory is renormalizable with respect to the dimensionless coupling constant $1/f_0{}^2$, and super-renormalizable with respect to the gauge coupling constant g^2.

This theory may be studied directly using the methods discussed in Ref. 4. Since the nonlinear theory is renormalizable, we must be careful to preserve the symmetry structure of the theory in our calculation. Dimensional regularization is not particularly convenient in this case as we would confront the necessity of including contributions from the transverse gauge fields. Instead, we choose to regularize the theory by considering the linear σ model in precisely two dimensions. The nonlinear theory is recovered as a limit of the linear theory.[5]

The linearized theory is described by the Lagrangian

$$\mathcal{L}=\tfrac{1}{2}(D_\mu\vec{\phi})^2-\tfrac{1}{4}\lambda_0(f_0{}^2-\vec{\phi}^2)^2-\tfrac{1}{4}(G_{\mu\nu ij})^2, \tag{4}$$

where the constraint $\vec{\phi}^2=f_0{}^2$ has been relaxed. The nonlinear model is obtained by taking the limit $\lambda_0\to\infty$ with $f_0{}^2$ and g^2 fixed. Since the nonlinear theory is renormalizable, only a logarithmic dependence on λ_0 can occur and is absorbed by the renormalization of $f_0{}^2$. In the large-N limit no such logarithms appear and the limit may be taken without the renormalization involving λ_0.

The theory is most easily studied in the light-cone gauge, $A_{-ij}=0$, $A_{\pm}=(1/\sqrt{2})(A_0\pm A_1)$. The gauge fields A_{+ij} are dependent and may be eliminated in favor of a "Coulomb" interaction. In this gauge, the Lagrangian of Eq. (4) becomes

$$\mathcal{L}=\tfrac{1}{2}(\partial_\mu\vec{\phi})^2-\tfrac{1}{4}\lambda_0(f_0{}^2-\vec{\phi}^2)^2 -\tfrac{1}{4}g^2(\phi_i\overleftrightarrow{\partial}_-\phi_j)(-\partial_-{}^2)^{-1}(\phi_i\overleftrightarrow{\partial}_-\phi_j), \quad (5)$$

where

$$\partial_\pm=\frac{1}{\sqrt{2}}(\partial_0\pm\partial_1).$$

We wish to consider the possibility of spontaneous symmetry breaking of the O(N) symmetry where the field, $\vec{\phi}$, acquires a nonzero vacuum expectation value. The direction of the symmetry breaking may be rotated to any fixed axis. There remains an O($N-1$) symmetry. We rewrite the field, $\vec{\phi}$, as a field, σ, which can have a vacuum expectation value, and fields $[\pi_i,\ i=1,\ldots,(N-1)]$ which have zero vacuum expectation value. We may probe the broken phase by adding a Lagrange multiplier, J, to force the σ field to have a given expectation value, f. A spontaneously broken symmetry phase exists if $f\neq 0$ for $J=0$ and if this phase has the lowest vacuum energy. The Lagrangian of Eq. (5) may be rewritten as

$$\mathcal{L}=\tfrac{1}{2}(\partial\sigma)^2+\tfrac{1}{2}(\partial\vec{\pi})^2-\tfrac{1}{4}\lambda_0(f_0{}^2-\sigma^2-\vec{\pi}^2)^2+J(\sigma-f) -\tfrac{1}{2}g^2(\sigma\overleftrightarrow{\partial}_-\pi_i)(-\partial_-{}^2)^{-1}(\sigma\overleftrightarrow{\partial}_-\pi_i) -\tfrac{1}{4}g^2(\pi_i\overleftrightarrow{\partial}_-\pi_j)(-\partial_-{}^2)^{-1}(\pi_i\overleftrightarrow{\partial}_-\pi_j). \quad (6)$$

The large-N limit of this theory is obtained by letting $N\to\infty$ while holding $\lambda_0 N$, g^2N, $f_0{}^2/N$, and f^2/N fixed. In this limit, a Hartree calculation of the σ and $\vec{\pi}$ propagators becomes exact, with the π mass being determined self-consistently.

The propagators may be computed using the Lagrangian of Eq. (6):

$$\Gamma_\sigma{}^2(P)=P^2+\lambda_0 f_0{}^2-\lambda_0\langle 3\sigma^2+\vec{\pi}^2\rangle_0 +g^2\sum_i\langle(\overleftrightarrow{\partial}_-\pi_i)(-\partial_-{}^2)^{-1}(\pi_i\overleftrightarrow{\partial}_-)\rangle_0 +O(1/N), \quad (7)$$

$$\Gamma_\pi{}^2(P)=P^2+\lambda_0 f_0{}^2-\lambda_0\langle\sigma^2+\pi^2\rangle_0 +g^2\langle(\overleftrightarrow{\partial}_-\sigma)(-\partial_-{}^2)^{-1}(\sigma\overleftrightarrow{\partial}_-)\rangle_0 +g^2\sum_i\langle(\overleftrightarrow{\partial}_-\pi_i)(-\partial_-{}^2)^{-1}(\pi_i\overleftrightarrow{\partial}_-)\rangle_0 +O(1/N). \quad (8)$$

The vacuum expectation value of the σ-field equation of motion may be used to determine the value of the Lagrange multiplier, J, such that $\langle\sigma\rangle_0=f$. We obtain the expression

$$0=J+\lambda_0 f_0{}^2\langle\sigma\rangle_0-\lambda_0\langle\sigma(\sigma^2+\vec{\pi}^2)\rangle_0 +g^2\sum_i\langle(\overleftrightarrow{\partial}_-\pi_i)(-\partial_-{}^2)^{-1}(\pi_i\overleftrightarrow{\partial}_-\sigma)\rangle_0. \quad (9)$$

The various vacuum expectation values in Eqs. (7), (8), and (9) may be evaluated in leading order N by using the full propagators for the σ and π fields. We obtain the following results:

(a) $\langle\sigma\rangle_0=f$,

(b) $\langle\pi_i\rangle_0=0$,

(c) $\langle\sigma^2\rangle_0=\langle\sigma\rangle^2+\dfrac{i}{(2\pi)^2}\displaystyle\int d^2k(k^2-m_\sigma{}^2)^{-1}=f^2+(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\sigma{}^2}$,

(d) $\langle\vec{\pi}^2\rangle_0=(N-1)\dfrac{i}{(2\pi)^2}\displaystyle\int d^2k(k^2-m_\pi{}^2)^{-1}=(N-1)(4\pi)^{-1}\ln\frac{\Lambda^2}{m_k{}^2}$,

$$\text{(e) } \langle(\overleftrightarrow{\partial}_-\sigma)(-\partial_-{}^2)^{-1}(\sigma\overleftrightarrow{\partial}_-)\rangle_0=-\langle\sigma\rangle^2-\frac{i}{(2\pi)^2}\int dk(k^2-m_\sigma{}^2)^{-1}(k_-+P_-)^2(k_--P_-)^{-2}$$
$$=-f^2-\frac{i}{(2\pi)^2}\int dk(k^2-m_\sigma{}^2)^{-1}-\frac{i}{(2\pi)^2}\int dk(k^2-m_\sigma{}^2)^{-1}\frac{2k_-2P_-}{(k_--P_-)^2} \quad (10)$$
$$=-f^2-(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\sigma{}^2}+\frac{1}{\pi},$$

(f) $\displaystyle\sum_i\langle(\overleftrightarrow{\partial}_-\pi_i)(-\partial_-{}^2)^{-1}(\pi_i\overleftrightarrow{\partial}_-)\rangle_0=-(N-1)\frac{i}{(2\pi)^2}\int dk(k^2-m_\pi{}^2)^{-1}(k_-+P_-)^2(k_--P_-)^{-2}=(N-1)(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi{}^2}+\frac{N-1}{\pi}$,

(g) $\langle\sigma^3\rangle_0=f^3+3f(4\pi)^{-1}\ln\dfrac{\Lambda^2}{m_\sigma{}^2}$,

(h) $\langle\sigma\vec{\pi}^2\rangle_0=f(N-1)(4\pi)^{-1}\ln\dfrac{\Lambda^2}{m_\pi{}^2}$,

(i) $\displaystyle\sum_i\langle(\partial_-\pi_i)(-\partial_-{}^2)^{-1}(\pi_i\partial_-\sigma)\rangle_0=f\left(-(N-1)(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi{}^2}+\frac{(N-1)}{\pi}\right)$.

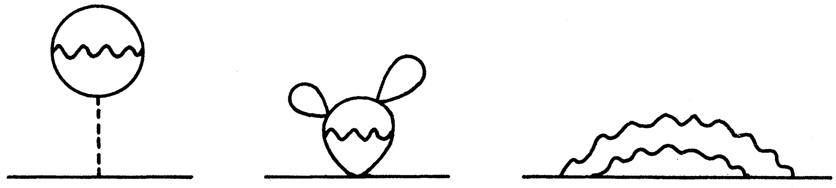

FIG. 1. Diagrams summed in Hartree calculation: (a) tadpole, (b) cactus, and (c) rainbow.

The divergent self-energy integrals have been computed using a Wick rotation and a cutoff, Λ^2.

Using the results of Eq. (10), we may compute all the large-N contributions to Eqs. (7), (8), and (9). The propagators are given by

$$\Gamma_\sigma{}^2(P^2)=P^2-m_\sigma{}^2,$$
$$\Gamma_\pi{}^2(P^2)=P^2-m_\pi{}^2,$$

where

$$\begin{aligned} m_\sigma{}^2&=-\lambda_0 f_0{}^2+3\lambda_0 f^2+\lambda_0 N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi{}^2}+g^2N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi{}^2}-\frac{g^2N}{\pi},\\ m_\pi{}^2&=-\lambda_0 f_0{}^2+\lambda_0 f^2+\lambda_0 N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi{}^2}+g^2f^2+g^2N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi{}^2}-\frac{g^2N}{\pi}. \end{aligned}\tag{11}$$

The Lagrange multiplier is determined by the equation

$$\begin{aligned} J&=-\lambda_0 f_0{}^2 f+\lambda_0 f^3+f\lambda_0 N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi{}^2}+fg^2N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi{}^2}-\frac{fg^2N}{\pi}\\ &=f(m_\pi{}^2-g^2f^2). \end{aligned}\tag{12}$$

The Hartree calculation sums consistently, in leading order N, all tadpole graphs [Fig. 1(a)], all cactus graphs [Fig. 1(b)], and all rainbow graphs [Fig. 1(c)]. All other graphs are nonleading in the large-N limit.

The π mass, $m_\pi{}^2$, must be determined self-consistently from Eq. (11) and Eq. (12). In the case $J=0$, there are two possible solutions corresponding to the symmetric phase, $f=0$, and a spontaneously broken phase, $f\neq 0$. In the symmetric phase we have $f=0$,

$$m_\pi{}^2=-\lambda_0 f_0{}^2+\lambda_0 N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi{}^2}+g^2N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi{}^2}-\frac{g^2N}{\pi}.\tag{13}$$

In the broken-symmetry phase we have

$$\begin{aligned} f^2&=m_\pi{}^2/g^2,\\ -(\lambda_0/g^2)m_\pi{}^2&=-\lambda_0 f_0{}^2+\lambda_0 N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi{}^2}+g^2N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi{}^2}-\frac{g^2N}{\pi}. \end{aligned}\tag{14}$$

Now we wish to compute the dependence of the vacuum energy density, V, on the order parameter, f. The vacuum energy may be computed from

$$\exp\left(-i\int dx\, V\right)=\left\langle \exp\left(i\int dx\,\mathcal{L}\right)\right\rangle_0,\tag{15}$$

where $\mathcal{L}$ is the Lagrangian of Eq. (6). Instead of computing V directly, we shall evaluate $\partial_f V$ using the result of Eq. (10). This ensures a systematic treatment of all divergences. We obtain

$$-\partial_f V=-J+\langle\partial_f J\rangle\langle\sigma-f\rangle_0=-J.\tag{16}$$

This expression may be integrated by considering a similar derivative of the mass equation for $m_\pi{}^2$, Eq. (11). We obtain

$$[1+(\lambda_0+g^2)N(4\pi)^{-1}(m_\pi{}^2)^{-1}]\partial_f m_\pi{}^2=2f(\lambda_0+g^2).\tag{17}$$

When Eqs. (16), (12), and (17) are combined, the derivative of the vacuum energy becomes

$$\begin{aligned} \partial_f V=J&=f(m_\pi{}^2-g^2f^2)\\ &=\tfrac{1}{2}[m_\pi{}^2(\lambda_0+g^2)^{-1}+N(4\pi)^{-1}]\partial_f m_\pi{}^2-g^2f^3. \end{aligned}\tag{18}$$

Equation (18) may be directly integrated to yield the vacuum energy:

$$V=\tfrac{1}{4}m_\pi{}^2(\lambda_0+g^2)^{-1}+\tfrac{1}{2}N(4\pi)^{-1}m_\pi{}^2-\tfrac{1}{4}gf^4+V_0,\tag{19}$$

where V_0 can depend on λ_0, g^2, and $f_0{}^2$ but not on f.

By using our results of Eq. (11), Eq. (12), and Eq. (19) we may study the phase properties of the linear O(N) gauge theory in two dimensions. We summarize the results in Eq. (20):

$$\begin{aligned} m_\sigma^2 &= -\lambda_0 f_0^2 + 3\lambda_0 f^2 + \lambda_0 N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi^2} \\ &\quad + g^2 N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi^2} - \frac{g^2 N}{\pi}, \\ m_\pi^2 &= -\lambda_0 f_0^2 + \lambda_0 f^2 + \lambda_0 N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi^2} \\ &\quad + g^2 f^2 + g^2 N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi^2} - \frac{g^2 N}{\pi}, \\ J &= f(m_\pi^2 - g^2 f^2), \\ V &= \tfrac{1}{4} m_\pi^4(\lambda_0 + g^2)^{-1} + \tfrac{1}{2}N(4\pi)^{-1}m_\pi^2 - \tfrac{1}{4}g^2 f^4. \end{aligned} \tag{20}$$

The two distinct phases occur when $J=0$. The symmetric phase is determined by

$$\begin{aligned} f &= 0, \\ m_\pi^2 &= -\lambda_0 f_0^2 + \lambda_0 N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi^2} \\ &\quad + g^2 N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi^2} - \frac{g^2 N}{\pi}, \\ m_\sigma^2 &= m_\pi^2, \\ V &= \tfrac{1}{4} m_\pi^4(\lambda_0 + g^2)^{-1} + \tfrac{1}{2}N(4\pi)^{-1}m_\pi^2. \end{aligned} \tag{21}$$

The broken phase is determined by

$$\begin{aligned} f^2 &= m_\pi^2/g^2, \\ -(\lambda_0/g^2)m_\pi^2 &= -\lambda_0 f_0^2 + \lambda_0 N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi^2} \\ &\quad + g^2 N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi^2} - \frac{g^2 N}{\pi}, \\ m_\sigma^2 &= m_\pi^2 + (2\lambda_0 - g^2)f^2 \\ &= 2(\lambda_0/g^2)m_\pi^2, \\ V &= \tfrac{1}{4} m_\pi^4(\lambda_0 + g^2)^{-1} + \tfrac{1}{2}N(4\pi)^{-1}m_\pi^2 - \tfrac{1}{4}m_\pi^4/g^2. \end{aligned} \tag{22}$$

Since we are using the linear $O(N)$ model to regularize the nonlinear $O(N)$ model, we will not discuss the linear theory but proceed to a discussion of the nonlinear theory. We note that all expressions we have have used are unrenormalized. The only divergent renormalization necessary in the large-N limit is a logarithmic divergence in f_0^2.

The nonlinear $O(N)$ gauge theory is obtained by taking the limit $\lambda_0 \to \infty$ with all other parameters held fixed. One might worry that such a limit might reorder the large-N expansion. However, we have noted that in two dimensions the nonlinear theory is renormalizable with only logarithmic divergences. Hence, the large-N limit cannot be modified by powers of N and no reordering can occur.

The phase-transition properties of the nonlinear theory may be obtained by taking the nonlinear limit of Eq. (21) and Eq. (22). The symmetric phase is determined by

$$\begin{aligned} f &= 0, \\ f_0^2 &= N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi^2}, \\ m_\sigma^2 &= m_\pi^2, \\ V &= \tfrac{1}{2}N(4\pi)^{-1}m_\pi^2. \end{aligned} \tag{23}$$

This phase is characterized by a full restoration of the $O(N)$ symmetry. As discussed in Ref. 4, the σ particle forms as a bound state of the nonlinear degrees of freedom. Since the full $O(N)$ Coulomb interaction is still operative, only $O(N)$-singlet bound states of σ and $\vec{\pi}$ exist in the physical spectrum.[6] The broken-symmetry phase is determined by

$$\begin{aligned} f^2 &= \frac{m_\pi^2}{g^2}, \\ f_0^2 &= \frac{m_\pi^2}{g^2} + N(4\pi)^{-1}\ln\frac{\Lambda^2}{m_\pi^2}, \\ m_\sigma^2 &= \infty, \\ V &= \tfrac{1}{2}N(4\pi)^{-1}m_\pi^2 - \tfrac{1}{4}\frac{m_\pi^4}{g^2}. \end{aligned} \tag{24}$$

In this phase the σ is not formed as a bound state. Only the $O(N-1)$ Coulomb interactions are operative with the remaining Coulomb interactions being screened. The physical states are $O(N-1)$-singlet bound states of $\vec{\pi}$. The broken-symmetry phase avoids Coleman's theorem[7] as the Higgs mechanism does not permit the existence of Goldstone bosons.

We must now discuss the stability of the two phases to see the range of couplings where each phase can exist and the character of the phase transition. We introduce a renormalized coupling constant, h, through the relation

$$\frac{N(4\pi)^{-1}}{h} = f_0^2 - N(4\pi)^{-1}\ln\frac{\Lambda^2}{M^2},$$

where M^2 is a normalization scale, and we define the fine-structure constant, $\alpha = g^2N/4\pi$. Equations (23) and (34) become the following.

(a) symmetric phase

$$\begin{aligned} \frac{1}{h} &= \ln\frac{M^2}{m_\pi^2} \quad \text{or} \quad m_\pi^2 = M^2 e^{-1/h}, \\ V &= \tfrac{1}{2}N(4\pi)^{-1}m_\pi^2. \end{aligned} \tag{25}$$

(b) broken-symmetry phase

$$\begin{aligned} \frac{1}{h} &= \frac{m_\pi^2}{\alpha} + \ln\frac{M^2}{m_\pi^2}, \\ V &= \tfrac{1}{2}N(4\pi)^{-1}(m_\pi^2 - \tfrac{1}{2}m_\pi^4/\alpha). \end{aligned} \tag{26}$$

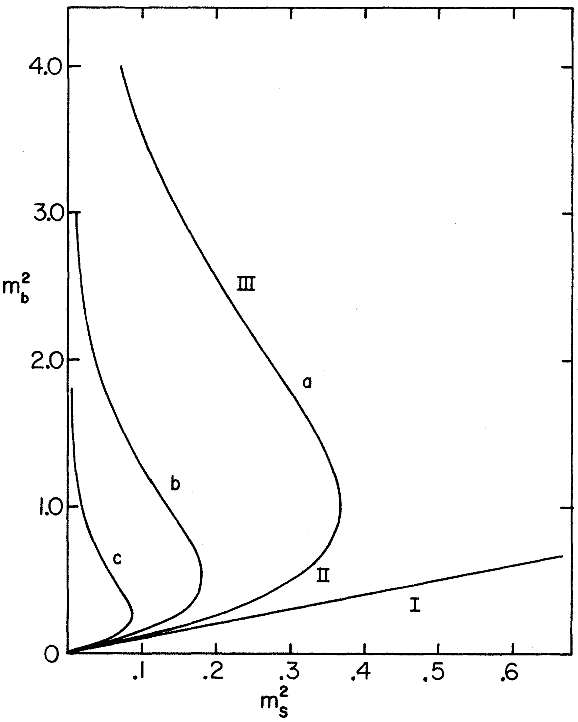

FIG. 2. π masses in symmetric phase (I) and broken-symmetry phases (II, III) as a function of mass in symmetric phase for (a) $\alpha=1$, (b) $\alpha=\frac{1}{2}$, and (c) $\alpha=\frac{1}{4}$.

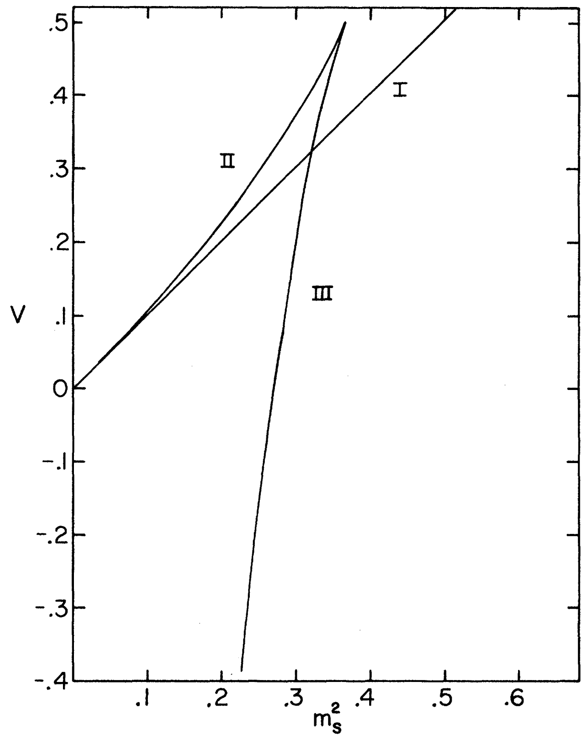

FIG. 3. Vacuum energy as a function of mass in symmetric phase for symmetric phase (I) and broken-symmetry phases (II, III).

The symmetric phase can exist for all values of the coupling constants. The broken-symmetry phase can exist only if the coupling constant, h, is sufficiently small. Since the σ-model coupling constant is directly related to the mass in the symmetric phase, we consider the phase properties of the theory expressed in terms of this mass. We denote the π masses in the symmetric and broken phases by m_s^2 and m_b^2, respectively. As mentioned above we use m_s^2 to parametrize theory in both phases. In Fig. 2 we plot the π masses in each phase for different values of the fine-structure constant. In Fig. 3 we plot the vacuum energy for a fixed value of the fine-structure constant. In Fig. 4, we plot the phase-transition line in coupling-constant space.

By examining Fig. 2, we see that only the symmetric phase (I) can exist for $m_s^2 > \alpha/e$. For $m_s^2 < \alpha/e$, the broken-symmetry phase can also exist with two possible branches (II, III). As we decrease the gauge coupling constant ($a \to b \to c$), the region where the broken phase can exist is restricted to smaller values of m_s^2 or, equivalently, smaller values of the coupling constant h. When the gauge coupling constant vanishes ($\alpha = 0$), only the symmetric phase can exist, in agreement with the discussions of Ref. 3 and Ref. 4.

Although two broken-symmetry phases can exist for $m_s^2 < \alpha/e \cong 0.368\alpha$, the phase transition does not occur until $m_s^2 \lesssim 0.321\alpha$, as seen in Fig. 3. For small m_s^2 ($\lesssim 0.321\alpha$), the stable phase is the larger-mass broken-symmetry phase (III). The vacuum energy is, of course, continuous through the phase transition. However, both m_π^2 and f^2 are discontinuous, which indicates that it is a first-order phase transition. In Fig. 4 we plot the phase-transition line in coupling-constant space where region A is the symmetric phase and region B is the broken-symmetry phase.

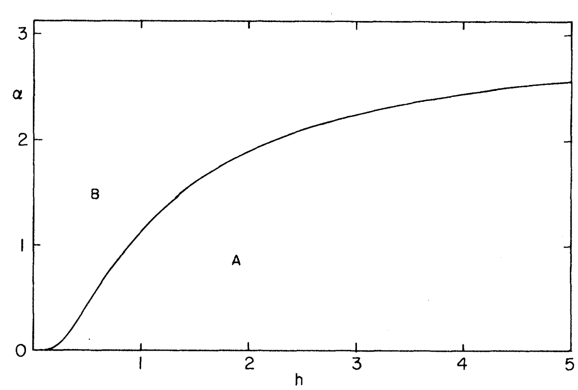

FIG. 4. Phase diagram in terms of coupling constants for symmetric phase (A) and broken-symmetry phase (B).

In this paper we have studied the phase-transition properties of a nonlinear $O(N)$ σ model with gauge interactions. When studied in the large-N limit the theory exhibits a first-order phase transition. The symmetric phase is characterized by the generation of a bound-state σ particle degenerate with the π's and by all physical states being $O(N)$ singlet bound states of σ and π's. The broken-symmetry phase is characterized by a residual $O(N-1)$ symmetry. The σ bound states are not formed and all physical states are $O(N-1)$ singlet bound states. The Higgs mechanism avoids the necessity of Goldstone bosons and stabilizes the broken-symmetry phase in two dimensions.

These results indicate that the longitudinal dynamics of the gluon fields in the transverse-lattice theory of Bardeen and Pearson[1] is rich enough to support a phase transition in the quark-gluon theory, as would be expected in $4+\epsilon$ dimensions or if the number of quarks were to be sufficiently large as to destabilize confinement phase. The phase-transition properties of the full gauge theory are of course much more complex than the simple model studied in this paper. We do think that the results of this paper shed some light on the mechanisms which operate in a gauge theory.

Note added. After this paper was completed, we received a report by J. S. Kang [Phys. Rev. D 14, 1587 (1976)], who studies the linear $O(N)$ gauge model. His calculations include the leading-N contribution from the meson self-interaction but are to first order in the gauge coupling. Our results represent the full leading-N calculation of the properties of the linear and nonlinear $O(N)$ gauge theories in two dimensions.

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[4]W. A. Bardeen, B. W. Lee, and R. E. Shrock, Phys. Rev. D 14, 985 (1976).

[5]Without gauge couplings, the linear model has been studied in the large-N limit by S. Coleman, R. Jackiw, and H. D. Politzer [Phys. Rev. D 10, 2491 (1974)], R. G. Root [*ibid.* 10, 3322 (1974)]; and L. F. Abbott *et al.* [*ibid.* 13, 2212 (1976)].

[6]To establish this intuitive result rigorously, we would have to study a Bethe-Salpeter equation analogous to that proposed by G. 't Hooft [Nucl. Phys. B75, 461 (1964)] and further elaborated upon by C. G. Callan, N. Coote, and D. Gross [Phys. Rev. D 13, 1649 (1976)] and by M. E. Einhorn [Fermilab Report No. Fermilab-Pub-76/22-THY (unpublished)].

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