

# UC Berkeley

## Building Efficiency and Sustainability in the Tropics (SinBerBEST)

### Title

Comparative Study of Correlated Shadowing Loss Model for Wireless Sensor Networks

### Permalink

<https://escholarship.org/uc/item/1tf7559b>

### Authors

Chan, Edwin  
Boon Hee, Soong  
Chung La, Cam

### Publication Date

2013-12-01

Peer reviewed

# Comparative Study of Correlated Shadowing Loss Model for Wireless Sensor Networks

Yiu Wing Edwin Chan<sup>\*†</sup>, Cam Chung La<sup>\*</sup> and Boon-Hee Soong<sup>\*†</sup>

<sup>\*</sup>School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798

<sup>†</sup>BEARS, CREATE Tower, Singapore 138602

Email: {edwinchan, LACA0001, ebhsoong}@ntu.edu.sg

**Abstract**—Accurate prediction of received signal strength is pivotal to reliable wireless communications. Unfortunately, wireless signals are often subject to random attenuation due to the environment around the receivers. One of the degradation factors that contributes to such loss is shadowing loss. It is well known that the shadowing losses of two nearby radio links are correlated. In this paper, we study the features of three existing models for correlated shadowing loss, namely autocorrelation model, cross-correlation model and joint path loss model, and establish the relationship between them.

## I. INTRODUCTION

Shadowing loss is the attenuation of signal strength due to the environment. It results from some phenomena: reflection, refraction and diffraction. Depending on territories and geographical characteristics, it can vary widely from one area to another. Furthermore, shadow fading in one place can change a lot year by year since new houses may be built, or old factories may be demolished. Hence, it is almost impossible to derive a specific value of shadowing loss for every kind of environment.

Instead, researchers attempt to investigate the correlation of shadowing loss. The underlying assumption is that when the structure of environment changes, all signals transmitting across it might be affected in a similar fashion. For instance, if the shadowing loss of a signal received at base station  $A$  is known, given the correlation among all transmitting paths in the wireless network, that of the signal received at base station  $B$  can be predicted as well. A number of works on correlated shadowing loss have been published, however, there is no one well-agreed model for correlation prediction [1].

In this paper, we choose to study three well-known correlation models:

- Autocorrelation model (ACM) [2]
- Cross-correlation model (CCM) [1]
- Joint Path Loss model (JPL) [3]

Each of them was proposed at different times in order to deal with different scenarios that range from simple to complicated. The detail of each model as well as their connections will be studied and presented in the following sections of this paper.

The organization of this paper is as follows. In Section II, a brief overview of related works is presented. Section III describes the models in detail and highlights their behavior under different conditions. Comparison of the models and

the respective results are presented in Section IV. Finally, Section V concludes the paper.

## II. LITERATURE REVIEW

Research in shadowing loss in wireless communications has been studied for decades. For indoor communications, Hashemi et al. [4] take many measurements and find that path loss in indoor scenario cannot be represented by a simple equation (e.g. [2], [5]) since it is fairly sensitive to size, shape and movement of objects in the building. As it is too complicated to study the loss in an arbitrary environment, researchers so far try to focus on some typical conditions from which they hope to learn the basic and common properties of such loss, and generalize them for other types of environments or multi-hop networks.

For mobile radio system, Gudmundson [2] suggests a correlation model to characterize the shadow fading of signals received by a mobile receiver from a base station. Wang et al. [5] later extend this model to scenarios where both ends of the link are mobile, and show that the correlation coefficient is simply a product of two correlation values obtained from single end mobility case. Saunders and Aragón-Zavala [1] also propose a cross-correlation model for cases where two links have one common end. Agrawal and Patwari [3] later propose a joint path loss model for arbitrary pair of links which do not share a common end point. Kasiri et al. adopt the same approach in [6].

At first glance, the three models [1]–[3] seem to be unrelated. However, as they were proposed to evaluate the same quantity (i.e. shadowing loss correlation), they should produce similar results under the same scenario. This motivates us to investigate further. If we are able to establish the relationships, we can then conclude many properties of shadowing loss correlation, and are one step closer to formulate a more uniform model to describe the shadowing loss.

## III. MATHEMATICAL MODELS

We begin by presenting a general model for signal strength in wireless communications. Suppose there is a wireless link  $(i, j)$  between transmitter  $i$  and receiver  $j$ , the received signal power  $P_{ij}$  can be described by:

$$P_{ij} = P_T - P_L - Z_{ij} \quad (1)$$

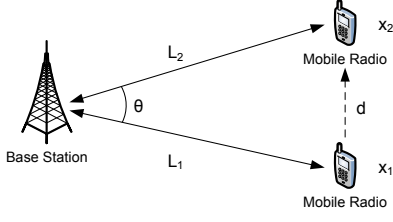


Fig. 1. Two links scenario studied for ACM.

where  $P_T$  is the transmitted power in dBm,  $P_L$  is the distance-dependent path loss, and  $Z_{ij}$  is the total fading loss along the link  $(i, j)$  in dB.  $Z_{ij}$  can be further broken down into two components:

$$Z_{ij} = X_{ij} + Y_{ij} \quad (2)$$

where  $X_{ij}$  is the shadowing loss along link  $(i, j)$  in dB and  $Y_{ij}$  is the non-shadowing loss along link  $(i, j)$  in dB.

$X_{ij}$  is usually considered as the sum of losses due to obstacles (in dB) along the propagation path:

$$X_{ij} = \sum (\text{loss due to obstacle}) \quad (3)$$

Hence  $X_{ij}$  can be modeled as a Gaussian random variable [1]. Since  $Y_{ij}$  can be considered independent across link pairs in typical wireless systems [3], we only concentrate on  $X_{ij}$  in this paper.

#### A. Autocorrelation Model

When a mobile radio moves from  $x_1$  to  $x_2$ , its link to a base station can be considered as two separate links, as shown in Fig. 1. Autocorrelation model [2] is proposed for such a scenario to model the correlation of the shadowing losses associated with these two links, and is given below:

$$\rho_a(d) = e^{-d/r_c} \quad (4)$$

where

- $d$ : distance between the two positions
- $r_c$ : decorrelation distance; the distance where correlation drops to  $e^{-1}$ .

From (4), it is clear that the correlation depends on  $d$  and is independent of the distance between the mobile and static nodes. In addition, it is usually assumed that the distance the mobile node moved is small compared to the length of either link, i.e.  $\min\{L_1, L_2\} \gg d$ , where  $L_1$  and  $L_2$  are the lengths of the two links. In other words, the two links are subject to similar terrain condition. An example for ACM is plotted in Fig. 2.

#### B. Cross-Correlation Model

In the case where two disparate links share one common end (node 3 in Fig. 3), the shadowing loss correlation can be predicted by [1]:

$$\rho_c = \begin{cases} \sqrt{\frac{D_1}{D_2}} & \text{for } 0 \leq \phi < \phi_T \\ \left(\frac{\phi_T}{\phi}\right)^\gamma \sqrt{\frac{D_1}{D_2}} & \text{for } \phi_T \leq \phi \leq \pi \end{cases} \quad (5)$$

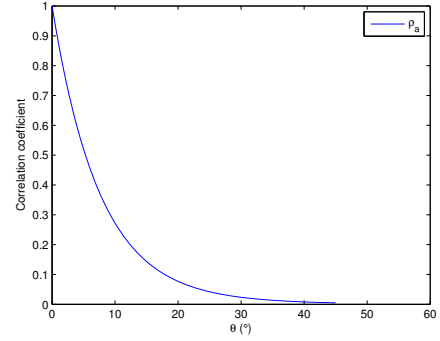


Fig. 2. Autocorrelation coefficient when  $L_1 = L_2 = 3000\text{m}$  and  $r_c = 800\text{m}$ .

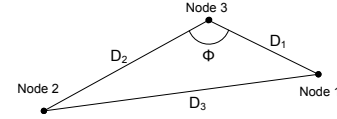


Fig. 3. Two links scenario studied for CCM.

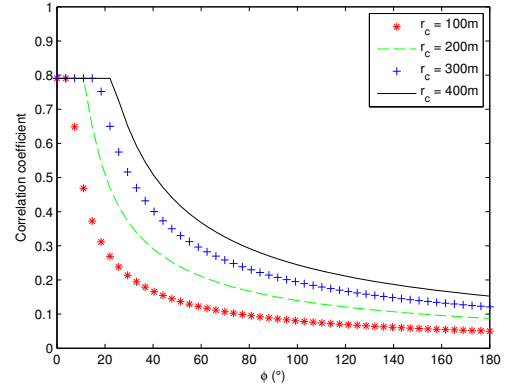


Fig. 4. Effect of  $r_c$  on CCM when  $D_1 = 1000\text{m}$ ,  $D_2 = 1600\text{m}$  and  $\gamma = 0.8$ .

where

- $D_1$ : the smaller of the two path lengths
- $D_2$ : the larger of the two path lengths
- $\gamma$ : exponential constant
- $\phi$ : angle between the two links
- $\phi_T$ : threshold angle which is defined as:

$$\phi_T = 2 \sin^{-1} \frac{r_c}{2D_1} \quad (6)$$

The distance constant  $r_c$  in CCM is the same as the one in ACM.  $\gamma$  will depend on the terrain, obstacle arrangements, height of antenna with respect to surroundings.

Changing  $r_c$  only affects  $\phi_T$  as shown in (6). When  $r_c$  increases (decreases),  $\phi_T$  increases (decreases), and so does the correlation coefficient  $\rho_c$ . The effect is shown in Fig. 4.

From (5), we note that for a particular node configuration (i.e.  $D_1, D_2$  are specified) and when  $\phi_T \leq \phi < \pi$ , as  $\gamma$  increases, the correlation  $\rho_c$  will drop faster and vice versa (shown in Fig. 5). Note that  $\phi_T/\phi$  is typically smaller than

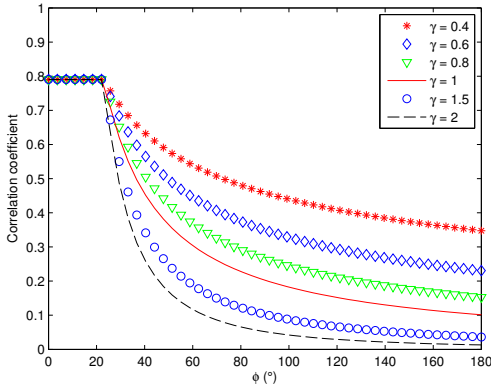


Fig. 5. Effect of  $\gamma$  on CCM when  $D_1 = 1000\text{m}$ ,  $D_2 = 1600\text{m}$  and  $r_c = 400\text{m}$ .

1. Moreover, when  $\gamma > 1$ , the value of  $\rho_c$  will decline very fast. Hence,  $\gamma$  should be practically in the range of  $(0,1)$ .

### C. Joint Path Loss Model

1) *Single Link*: The spatial loss field  $p(x)$  [3] is used to quantify the shadowing loss on a link. The loss is higher if the link passes through area of large  $p(x)$ . Furthermore,  $p(x)$  is assumed to be an isotropic wide-sense stationary Gaussian random field, while its spatial correlation is zero mean and exponentially-decaying.

The shadowing loss on a single link between node pair  $(j, k)$  is a weighted integral of spatial loss field:

$$L_{j,k} = \frac{1}{\|x_k - x_j\|^{1/2}} \int_{x_j}^{x_k} p(x) dx \quad (7)$$

The variance of shadowing for link  $a = (j, k)$  is:

$$\text{Var} [L_{j,k}] = \sigma_x^2 \left[ 1 + \frac{\delta}{\|x_k - x_j\|} e^{-\|x_k - x_j\|/\delta} - \frac{\delta}{\|x_k - x_j\|} \right] \quad (8)$$

When  $\|x_k - x_j\| \gg \delta$ ,

$$\text{Var} [L_{j,k}] \approx \sigma_x^2 \quad (9)$$

where  $\delta$  is the space constant and  $\sigma_x$  is the standard deviation of shadow fading.

2) *Shadowing Correlation*: Consider two links  $a = (i, j)$  and  $b = (k, l)$  in the field  $p(x)$  with shadowing  $L_a$  and  $L_b$  respectively. The covariance of  $L_a$  and  $L_b$  is:

$$\text{Cov} (L_a, L_b) = \frac{\sigma_x^2}{\delta d_{i,j}^{1/2} d_{k,l}^{1/2}} \int_{C_{i,j}} \int_{C_{k,l}} e^{-\frac{\|\beta - \alpha\|}{\delta}} d\alpha^T d\beta \quad (10)$$

where  $d_{i,j} = \|x_i - x_j\|$  and  $C_{m,n}$  is the line between points  $x_m$  and  $x_n$ .

The shadowing correlation coefficient between the two links is therefore:

$$\rho = \frac{\text{Cov} (L_a, L_b)}{\sqrt{\text{Var} [L_a] \text{Var} [L_b]}} \quad (11)$$

Using a network configuration similar to Fig. 3, the effect of each parameter is investigated. As shown in Fig. 6,  $\rho$

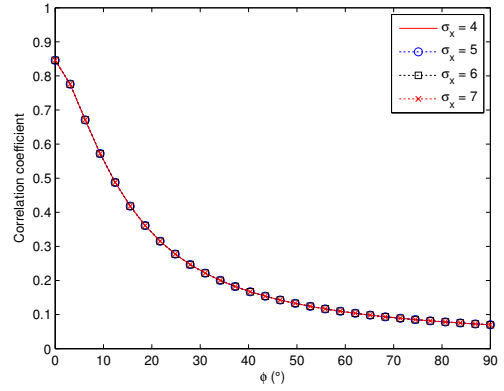


Fig. 6. Effect of  $\sigma_x$  on JPL when  $D_1 = 2000\text{m}$ ,  $D_2 = 3000\text{m}$  and  $\delta = 200$ .

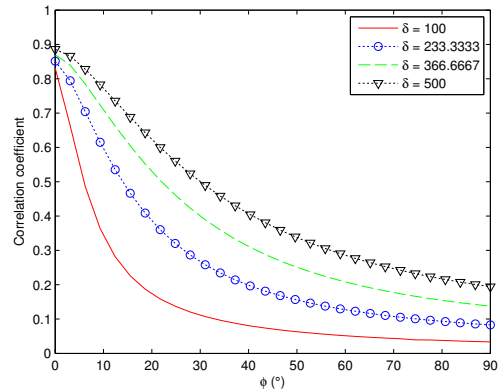


Fig. 7. Effect of  $\delta$  on JPL when  $D_1 = 2000\text{m}$ ,  $D_2 = 3000\text{m}$  and  $\sigma_x = 0.3$ .

remains the same when  $\sigma_x$  changes. It confirms that the standard deviation  $\sigma_x$  does not affect the correlation coefficient at all. As observed in Fig. 7, when  $\delta$  increases, the correlation coefficient  $\rho$  increases (when all other parameters are kept constant).

## IV. COMPARISON RESULTS

### A. Strategy

We again consider the two links configuration illustrated in Fig. 3.  $D_1$  and  $D_2$  are kept constant while the angle  $\phi$  varies from  $0^\circ$  to  $180^\circ$ . The shadowing correlation will be estimated according to each studied model, and the set of parameters  $(r_c, \gamma, \delta)$  will be adjusted until they produce approximately the same results. The parameter adjustment bases on how each parameter influences the correlation values as discussed in the previous section. Note that node 2 and node 3 in Fig. 3 are equivalent to  $x_1$  and  $x_2$  in Fig. 1.

### B. Results and Discussions

1) *CCM vs JPL*: Using a specific pair of  $D_1$  and  $D_2$ , we follow the strategy described in Section IV-A and obtain some suitable parameter set  $(r_c, \gamma, \delta)$ . One example is shown in Fig. 8. The procedure is then continued for other pairs of  $D_1$  and  $D_2$ , and the respective results are listed in Table I.

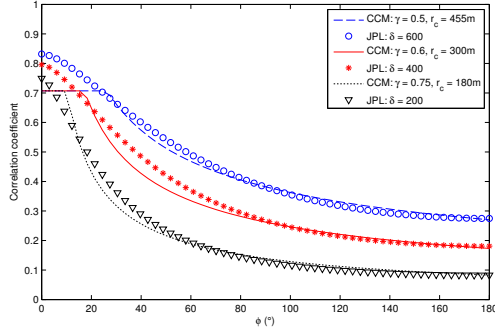


Fig. 8. CCM vs JPL when  $D_1 = 1000\text{m}$  and  $D_2 = 2000\text{m}$ .

TABLE I  
SIMULATION RESULTS FOR CCM VS JPL

$D_1$ (m)	$D_2$ (m)	$r_c$ (m)	$\gamma$	$\delta$ (m)
2000	2000	70	0.65	150
		300	0.7	350
		600	0.7	650
1000	1500	100	0.7	150
		300	0.7	320
		600	0.6	620
1000	2000	455	0.5	600
		300	0.6	400
		200	0.65	275

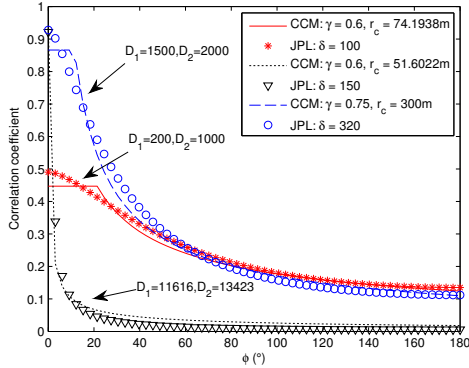


Fig. 9. CCM vs JPL with different sets of values for  $D_1$  and  $D_2$ .

With the aids of mathematical regression and wide range of parameter values, the relationship between parameters is established in the following form:

$$\frac{r_c}{\gamma} = a\delta^2 + b\delta + c \quad (12)$$

where  $a$ ,  $b$  and  $c$  are coefficients related to the ratio  $n = D_1/D_2$  as follows:

$$\begin{aligned} a &= -0.3141n^4 + 0.7501n^3 - 0.5913n^2 + 0.16n - 0.005 \\ b &= -32.372n^3 + 52.756n^2 - 20.266n + 1.4744 \\ c &= 7262.8n^4 - 11700n^3 + 4521.6n^2 - 2778n + 61.06 \end{aligned} \quad (13)$$

Further verification with new set of values of  $D_1$  and  $D_2$

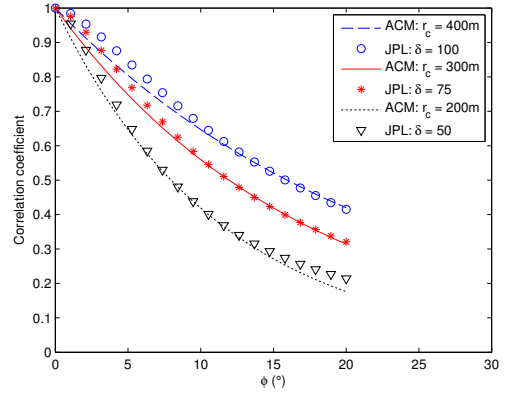


Fig. 10. ACM vs JPL when  $D_1 = D_2 = 1000\text{m}$ .

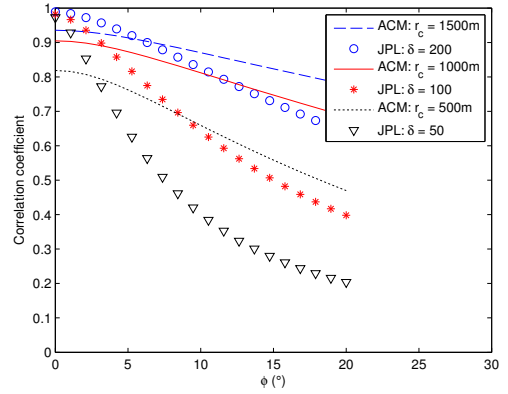


Fig. 11. ACM vs JPL when  $D_1 = 1000\text{m}$  and  $D_2 = 1100\text{m}$ .

are shown in Fig. 9. The values for the parameter sets are selected according to (12) and (13).

2) *ACM vs JPL*: As explained before, when ACM is applied on a configuration such as Fig. 3,  $D_3$  should be small compared to  $D_1$  and  $D_2$ . This means that the angle  $\phi$  should also be small. Therefore we are only interested in the range of  $0^\circ < \phi < 20^\circ$  in the following studies.

- For the case  $D_1 = D_2$ :

Since  $D_1 = D_2$ ,  $n = D_1/D_2 = 1$ . In this case the two models will match when

$$r_c = 4\delta \quad (14)$$

Results are plotted in Fig. 10. As we can see, the two models match very well.

- For the case  $D_1 \neq D_2$ :

When  $D_1$  and  $D_2$  differ by little, there is a large discrepancy between the two models, especially when the angle is very small. In the extreme case where  $\phi = 0$ , in order to match the two models, ACM requires a very large  $r_c$  ( $> 1500\text{m}$ ) while JPL needs  $\delta = 200\text{m}$  as shown in Fig. 11. Further study is required to understand this scenario.

One more thing to take note is that ACM does not take into account the moving direction of mobile node. For instance, when the two links in ACM are collinear (see Fig. 12),

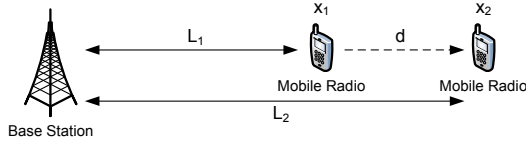


Fig. 12. Two collinear links studied in ACM.

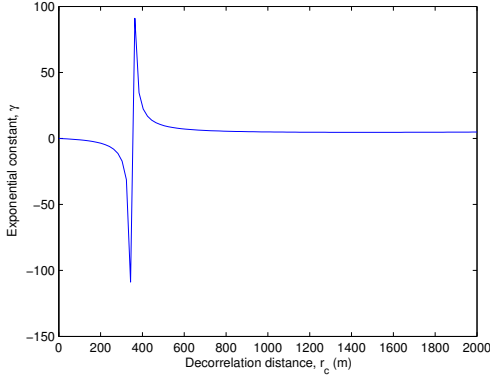


Fig. 13. Relationship of  $\gamma$  and  $r_c$  according to (15).

intuitively, the correlation is high because they experience similar environmental obstruction on the overlapping part. On the other hand, when the overlapping is minimal (cf. Fig. 1), the correlation should be low. However, ACM treats these situations equally and this limits its application in general.

3) *CCM vs ACM*: As both CCM and ACM use the same parameter  $r_c$ , (12), (13) and (14) provide a pathway to link up the two models.

- For the case  $D_1 = D_2$ :

Substituting (14) into (12), we can obtain:

$$\frac{r_c}{\gamma} = a \frac{r_c^2}{16} + b \frac{r_c}{4} + c \quad (15)$$

Since  $D_1 = D_2$  and from (13), we obtain  $a = -10^{-4}$ ,  $b = 1.5924$  and  $c = -132.505$ . (15) is plotted in Fig. 13. Contrary to our expectation,  $\gamma$  hardly lies in the range of  $(0, 1)$ . As a result, (15) cannot provide valid values of  $r_c$  and  $\gamma$ . Therefore, there is no suitable set of parameters  $(r_c, \gamma, \delta)$  that can combine the three models under the same network configuration. In other words, the condition in which JPL, CCM and ACM produce the same correlation values does not exist.

- For the case  $D_1 \neq D_2$ :

This scenario is even more complicated. ACM and CCM cannot generate agreeable results due to conflicting expected conditions. Mathematically, the constant part of CCM (where  $\phi < \phi_T$ ) cannot be equated to any part of ACM easily, as demonstrated in Fig. 14.

## V. CONCLUDING REMARKS

In this paper, we examine thoroughly three shadowing loss correlation models, namely autocorrelation, cross-correlation,

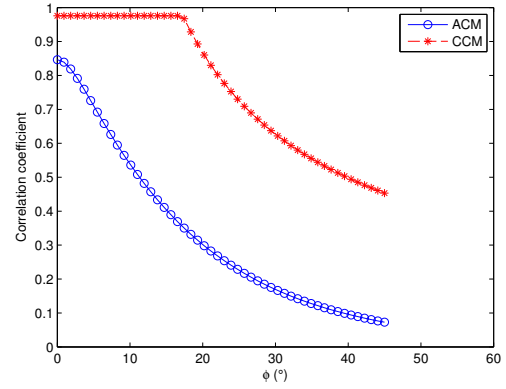


Fig. 14. Sample result of CCM vs ACM when  $D_1 = 2000\text{m}$ ,  $D_2 = 2100\text{m}$ ,  $\gamma = 0.8$  and  $r_c = 600\text{m}$ .

and joint path loss models. Their features, as well as their application areas, are discussed. Moreover, we deduce an equation that relates JPL to CCM, and also present a number of suggestions for further investigations. We are currently developing an algorithm to automatically evaluate the parameters to equate CCM with JPL, in order to facilitate further research. For future work, we will continue our effort in establishing the relationship between ACM and the other two models for different scenarios, and apply the findings to localization for wireless sensor networks (e.g. [7]).

## ACKNOWLEDGMENTS

This research is funded by the Republic of Singapore's National Research Foundation through a grant to the Berkeley Education Alliance for Research in Singapore (BEARS) for the Singapore-Berkeley Building Efficiency and Sustainability in the Tropics (SinBerBEST) Program. BEARS has been established by the University of California, Berkeley as a center for intellectual excellence in research and education in Singapore.

The second author would also like to thank Nanyang Technological University for the URECA project programme.

## REFERENCES

- [1] S. R. Saunders and A. Aragón-Zavala, *Antennas and propagation for wireless communication systems*, 2nd ed. Chichester, England: J. Wiley & Sons, 2007.
- [2] M. Gudmundson, "Correlation model for shadow fading in mobile radio systems," *Electronics Letters*, vol. 27, no. 23, pp. 2145–2146, 1991.
- [3] P. Agrawal and N. Patwari, "Correlated link shadow fading in multi-hop wireless networks," *IEEE Transactions on Wireless Communications*, vol. 8, no. 8, pp. 4024–4036, 2009.
- [4] H. Hashemi, D. Tholl, and G. Morrison, "Statistical modeling of the indoor radio propagation channel. I," in *Proc. IEEE Vehicular Technology Conference*, 1992, pp. 338–342 vol.1.
- [5] Z. Wang, E. K. Tameh, and A. R. Nix, "Joint shadowing process in urban peer-to-peer radio channels," *IEEE Transactions on Vehicular Technology*, vol. 57, no. 1, pp. 52–64, 2008.
- [6] B. Kasiri, M. Naderi, and B. Abolhassani, "A new realistic relay selection method based on correlated shadowing for multihop cellular networks," in *Proc. International Conference on Computer and Electrical Engineering (ICCEE)*, 2008, pp. 669–673.
- [7] Y. W. E. Chan and B. H. Soong, "A new lower bound on range-free localization algorithms in wireless sensor networks," *IEEE Communications Letters*, vol. 15, no. 1, pp. 16–18, 2011.